A Method for Detecting Structural Breaks and an Application to the Turkish Stock Market

Sidika Basci

Estim Forecasting Center, Sairler sok. 32/C, 06700 GOP, Ankara, Turkey

Erdem Basci

Department of Economics, Bilkent University, Bilkent, Ankara 06533, Turkey

Asad Zaman

Lahore University of Management Sciences, Opp. Sector U, LCCHS, Lahore Cantt
Lahore 54792, Pakistan.

Abstract

We suggest a procedure for model update, based on detection of structural breaks at unknown change-points. The procedure makes use of the SupF test introduced by Andrews (1993). We apply this procedure for modelling the common stock index returns in the Ýstanbul Stock Exchange for the 11 year period of 1989 - 1999. The underlying model consists simply of a mean plus noise, with occasional jumps in the level of mean at unknown time instances. The problem is the detection of this jump and the corresponding model update. We find critical values for the SupF test statistic by using the Bootstrap method. A trading rule that uses the forecasts from the suggested procedure is observed to outperform the buy-and-hold strategy.

1. Introduction

Testing for the presence of a structural change in linear econometric models when the change point is unknown, is still an active research area. In this paper we suggest a dynamic real time procedure for the detection of structural breaks and the corresponding update of the parameters of the model at hand. That way, we aim at having an up-to-date operating model to be used for prediction or forecasting purposes.

If in a linear regression model the possible change point is known, the Chow (1960) test can be applied. The Chow test statistic has an F distribution under the null hypothesis of no change, hence the tabulated critical values can be used. However if the change point is not known, one possibility is to calculate the F statistics for each potential change point and find their maximum. The test statistic obtained this way is called SupF. The asymptotic critical values of the SupF test are reported by Andrews (1993).

The detection method we propose here is based on the availability of a test statistic for an unknown change point. The method involves the addition of new observations to the sample, in case the null of no change is not rejected. If there is a rejection, then the change point is estimated and the new starting point for the sample is set as the estimated change point. Then the regression model parameters to be used are estimated on this new and smaller sample.[1]

A simple special case of the regression model consists of just the constant and the noise terms. This special case has been studied by Hawkins (1977), Worsley (1979), James, James and Seigmund (1987) and Chu (1990). Mainly they have obtained asymptotic distributions of Likelihood Ratio (LR) test statistic. The SupF test that we will use here is equivalent to LR test, as also noted in Andrews (1993). Chernoff and Zacks (1964) derive a Bayesian test in this setting by imposing a normal prior on the mean. Hinkley (1970) studies the issue of inference about the location of change point.

We model the weekly common stock index returns as a mean plus noise with occasional changes in the mean and possibly the variance. With this model in mind, we use the proposed method on the Ýstanbul Stock Exchange data by using the SupF test. We also report the profits from a trading strategy based on the method.

The possibility that means of stock returns are changing over time has been explored in the literature. For example Conrad and Kaul (1988, 1989) and Poterba and Summers (1988) model *expected* returns as changing every period according to an autoregressive process. Similarly, the ARCH and GARCH models suggest gradually but continuously changing variances. In contrast, our procedure is that of continually testing for the presence of occasional jumps in the mean and possibly the variance.

---

1 A related procedure for predicting regime changes in macroeconomic data has been introduced and applied by Neftçi (1982). In his case however, there are two regimes and the parameters determining the probability distribution functions under the two regimes are assumed to be known, so that there is no problem of parameter estimation.

The paper is organized as follows. After the introduction, Section 2 describes the data. Section 3 presents the method and the test statistics employed. The final section is devoted to a summary and discussion of the results.

2. The Data

The data we use is the Friday closing values of the ISE composite index expressed in US$ as calculated by the IFC. We loaded it from the DATASTREAM database. The data spans the January 5, 1989 to October 29, 1998 period, which contains 513 observations of the index. The natural logarithm of the level can be seen in Figure 1. There seems to be several time points at which the mean rate of increase has changed. However this may be an illusion and hence a formal statistical testing procedure will be applied.

The first difference of the natural logarithm of index gives us 512 observations on weekly continuously compounded rates of return. This data is plotted in Figure 2. From this data, the eye can not easily detect mean changes, while there seems to be changes in the volatility over time.

[Insert Figures 1 and 2 here.]

3. The Method and Test Statistics

We model the weekly returns from ISE index by an independently and normally distributed error term with occasional changes in mean and possibly variance at the same time instants. Formally we can write

$$R_t = b_{0t} + e_t \qquad (1)$$

where R_t is the return at time t, b_{0t} is the mean valid at time t and $e_t \sim iidN(0, s^2)$.

The method we apply is based on testing for the validity of the most recent parameter estimate after obtaining a new observation.[2] To this end unknown change point tests will be employed. The null hypothesis is no change in the mean and the variance. If the null is rejected, then an estimate of the change point is used as the beginning index of our new sample. The new mean is estimated as the average of returns on the new sample. The algorithm can be described as follows:[3]

---

[2] We chose a window length of 52. The test is conducted if and only if there are at least 52 sample observations from the most recent estimated change point to the current week. For a discussion of the rationale behind fixing such a data window and on the choice of its length, see Section 4.

[3] This algorithm is implemented in GAUSS, and the code is available upon request.

```
10      Let START=1;
20      Let T=1;
30              If T-START>=51, test for the null of no structural break on the most
                                                             recent 52 return data;
40              If rejected set START=Estimated change-point;
50              MEAN=Average of returns from START to T;
60              Obtain the following week's return;
70              Let T=T+1;
80      Go to 30;
```

In implementing our algorithm, we use the SupF test statistic, large sample properties of which are studied by Andrews (1993). This test statistic assumes a regression model of the form

$$Y_t = b_t X_t + e_t \tag{2}$$

Under this setting the null and alternative hypotheses are,

H_0 : no change in the regression model parameters over the observed sample period
H_1 : one change in the regression model parameters

The statistic for the SupF test is calculated as,

$$SupF = \max_{1 \le t \le T-1} F_t \tag{3}$$

where F_t is the usual F statistic calculated at the change point t. If this statistic is greater than some critical value we reject the null hypothesis of no change. The asymptotic critical values are reported in Andrews (1993). The estimated *change point* then is the time index that maximizes the F statistic.

In our case, where the model is that of a mean plus a noise term, we only have a constant term in the regression equation. For this reason, we set the independent variable, X_t equal to one for all t.

In order to make the algorithm described above operational, we need the critical values for the sample size of 52. We calculated the 5% and 10% critical values of the SupF test statistic by using a numerical simulation method called Bootstrap. Diebold and Chen (1995) show that bootstrap critical values for the SupF test are more accurate, in finite samples, than their asymptotic counterparts. This method is a numerical procedure for finding finite sample critical values, where rather than using a pseudo normal random variable generator, the estimated error terms themselves are resampled and used in Monte-Carlo simulations. This method has the advantage of being immune to deviations from the normality assumption on residuals. Details and the justification for the use of Bootstrap can be found in Efron (1982).

The 5% and 10% critical values found for the ISE $ return data are found as 11.13 and 8.82 from simulations with a Monte-Carlo sample size of 10,000. In contrast, for example, the 5% critical value for a usual Chow test would be 4.04. As it should be, the SupF test is observed to be more conservative than the Chow test. That is, it rejects the null less frequently.

4. Results and concluding remarks

The algorithm is applied to the stock index return data by using the two critical values reported in Section 3. As a result, the algorithm produced a list of signalled change points and the time that they are signalled. These times are reported in Table 1. For example we observe from the left panel of Table 1, that the first time the test has rejected the null is in week 99 for observations between 48 to 99, reporting an estimated change point in week 58. In such a case, for forecasting the period 100 return, a new mean would be estimated by using data from period 59 to period 99.

Although there is a large overlap in the signalled and estimated change points by the two significance levels, the use of 10% level increased the number of signalled breaks. This is due to the increase in the power of the test obtained at the cost of increase in the probability of Type 1 error. The signalled change points for the 10% level are plotted in Figure 3.

Table 1
The estimated change points and their signalling times for the period
January 5, 1989- October 29, 1998

At 5% Significance Level		At 10% Significance Level	
Estimated Change Point	Signal Time	Estimated Change Point	Signal Time
58	99	58	98
149	151	149	151
153	201	153	201
-	-	165	213
212	216	212	217
264	265	264	265
265	316	265	316
268	319	268	319
278	320	278	320
329	330	329	330
420	421	420	421
421	472	421	472
-	-	500	504

[Insert Figure 3 here.]

In order to observe the trading profits by using the forecasts generated by our algorithm, we employ the following trading strategy on our sample. If the expected weekly return is positive, we are in the market and obtain the following week's index return. Otherwise, we are out of the market and stay in US$ during the following week to get a $ return of 0%. The resulting buy and sell periods for the procedure using the test at 10% significance level are indicated in Figure 4. The most striking aspect of the Figure is the rather infrequent trading advice provided by the method. Moreover, in most cases the forecasts on the direction of the market seem to be justified.

[Insert Figure 4 here.]

The resulting weekly average returns corresponding to the 5% and 10% crtitical values are reported in Table 2. For the overall sample period, the trading rule yielded weekly $ returns of 0.442% and 0.517% for the 5% and 10% critical values respectively. These values correspond to continuously compounded annual returns of 23.0% and 26.9% respectively. In contrast, the $ returns from a buy and hold strategy for the same period would be 0.178% weekly or 9.2% annual, continuously compounded. Transaction costs are ignored in these calculations. However, as seen from Table 2, these costs are indeed negligible. The calculations were made assuming a transaction cost of 2% per each transaction. Since the total number of transactions in the overall 10 year period does not exceed 7 in either case, we get a negligible effect on average.

Table 2

The US$ returns from various trading rules for the period
January 5, 1989- October 29, 1998

Trading rule	*Weekly return*	*Annual return*
Buy and hold	0.178 %	9.2 %
Always out	0 .000 %	0.0 %
Buy if E(R)>0 (No test)	0.178 %	9.2 %
Buy if E(R)>0 (5% test)	0.443 %	23.0 %
Buy if E(R)>0 (10% test)	0.517 %	26.9 %
Buy if E(R)>0 (5% test, net of trans.costs)	0.440 %	22.9 %
Buy if E(R)>0 (10% test, net of trans.costs)	0.514 %	26.7 %

A weakness of this procedure is that it assumes homoskedastic noise terms under the null. Figure 2, however, suggests that our sample has heteroskedasticity. This weakness could be remedied as soon as a test for stuctural test in GARCH models is made available. Nevertheless, the overall performance of the procedure seems quite satisfactory. This can be attributed to our fixing of a test window of 52 weeks.[4] In a preliminary version of this paper we did not fix a test window and tested for the presence of structural breaks each and every week, as new data comes. This resulted in a too frequent rejection of the null, possibly due to changes in the variance of the data rather than changes in the mean. A large window length seems to have helped in reducing such undesirable rejections due to variance changes. But at the same time, of course, larger window lengths have the disadvantage of delaying the detection time of a possible change in mean that can actually take place before the window duration is reached.

These results indicate the presence of occasional changes in the mean of the underlying return generating process in the Turkish stock market. The question of whether these changes in mean can be attributed to changes in equilibrium expected returns and hence are consistent with the efficient markets hypothesis or whether they are due to the presence of rational of irrational bubbles in stock prices is a debatable issue and needs further study.

References

ANDREWS, D. W. (1993), "Tests for Parameter Instability and Structural Change with Unknown Change Point," *Econometrica*, 61, 4, 821-856.

CHERNOFF, H and S. ZACKS (1964), "Estimating the Current Mean of a Normal Distribution which is Subjected to Changes in Time," *Annals of Mathematical Statistics,* 35, 999-1018.

CHOW, G. C. (1960), "Tests of Equality Between Subsets of Coefficients in Two Linear Regression Models," *Econometrica*, 591-605.

CHU, C., J. (1990), " The Econometrics of Structural Change," Dissertation, University of California, San Diego.

CONRAD, J. and G. KAUL (1988), "Time-Variation in Expected Returns," *Journal of Business*, 61, 409-425.

CONRAD, J. and G. KAUL (1989), "Mean Reversion in Short-Horizon Expected Returns," *The Review of Financial Studies*, 2, 2, 225-240.

[4] We thank an anonymous referee of this journal for suggesting the use of a fixed window length in our testing procedure.

DIEBOLD, F.X. and C. CHEN (1996), "Testing Structural Stability with Endogenous Break Point: A Size Comparison of Analytic and Bootstrap Procedures," *Journal of Econometrics*, 70, 221-241.

EFRON, B. (1982), *The Jacknife, the Bootstrap and Other Resampling Plans,* SIAM, Philadelphia, Pennsylvania.

HAWKINS, D. M. (1977), "Testing a Sequence of Observations for a Shift in Location," *Journal of the American Statistical Association*, 72, 357, 180-186.

HINKLEY, D. V. (1970), "Inference about the Change Point in a Squence of Random Variables," *Biometrika*, 57, 1, 1-17.

JAMES B., K. L. JAMES and D. SIEGMUND (1987), "Test for a Change Point," *Biometrika*, 74, 1, 71-83.

NEFTÇÝ, S. N. (1982), "Optimal Prediction of Cyclical Downturns," *Journal of Economic Dynamics and Control*, 4, 225-241.

POTERBA, J. A. and L. M. SUMMERS (1988), "Mean Reversion in Stock Prices: Evidence and Implications," *Journal of Financial Economics*, 22, 27-59.

WORSLEY, K. J., (1979), "On the Likelihood Ratio Test for a Shift In Location of Normal Populations," *Journal of the American Statistical Association*, 74, 366, 365-367.

Özet

Yapýsal kýrýlmalarýn farkedilmesi için bir yöntem
ve Türk hisse senedi piyasasýna uygulanmasý

Bu makalede bilinmeyen deðiþme noktalarýnda gerçekleºen yapýsal kýrýlmalarýn farkedilmesine dayanan bir model yenileme yöntemi önerilmektedir. Yöntem Andrews (1993) tarafýndan literatüre sunulan SupF testini kullanmaktadýr. Bu yöntem 1989-1998 arasýndaki 10 yýllýk dönemde Ýstanbul Menkul kýymetler borsasýnýn endeks getirilerini modellemede kullanýlmýþtýr. Temel model bir ortalama getiri ve ona eklenen rassal gürültüden ibarettir. Ancak ortalama getirinin arada sýrada bilinmeyen zamanlarda sýçramalar yapma olasýlýðýný dikkate alýnarak, problem, bu tür sýçramalarýn farkýna varýlmasý ve modelin buna göre yeniden tahmini olarak tanýmlanmýþtýr. Kullanýlan SupF test istatistiðinin kritik deðerleri bir nümerik benzetme algoritmasý ile bulunmuºtur. Yöntemden elde edilen kestirimlerin kullanýlmasýna dayanan bir alýþ-veriþ kuralýnýn 'al ve tut' kuralýndan daha iyi getiri saðladýðýný gözlemlenmiºtir.

Figure 1: ISE index (ln of $ value)

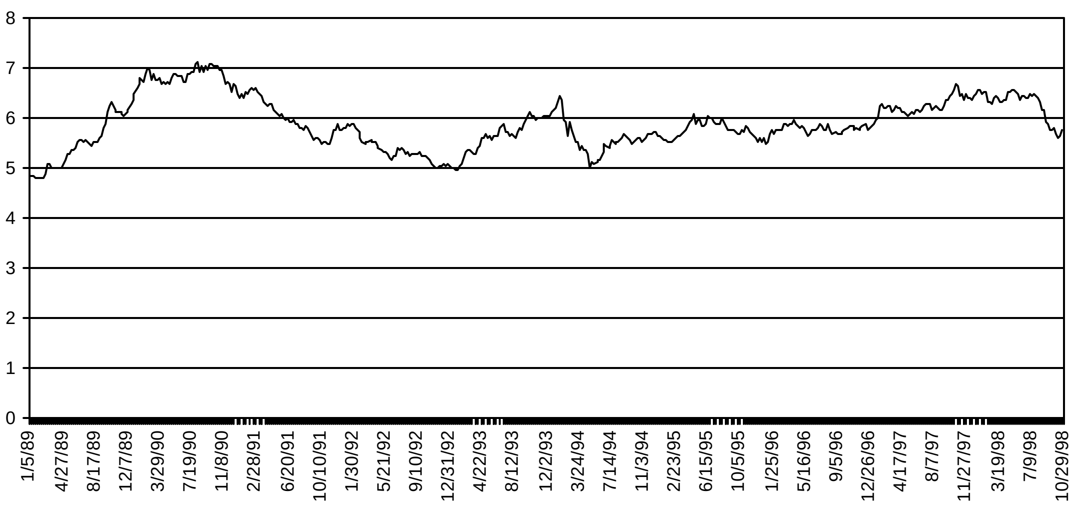

Figure 2: Weekly rate of return series of ISE index in US$ (Jan 1989-Oct 1998)

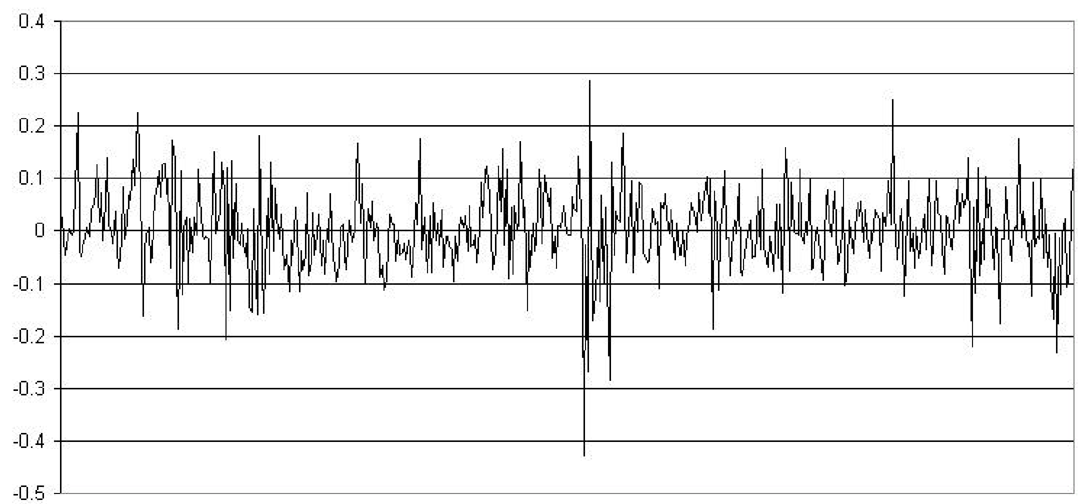

Figure 3: ISE index (ln of $ value) and the signalled break times (10% level)

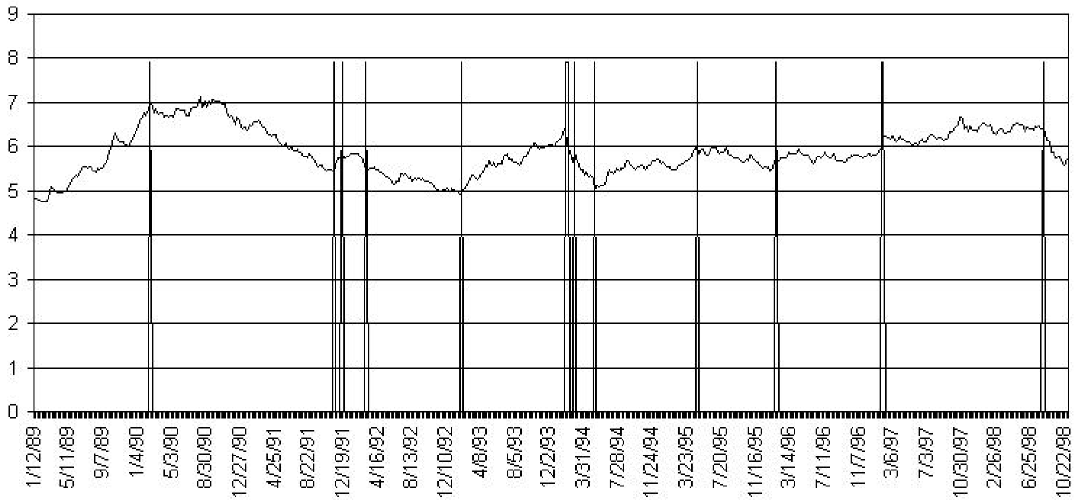

Figure 4: Buy and sell periods suggested by the trading rule (Buy: 8.5, Sell: 0)