Models and methods for supporting lot sizing decisions in product recovery systems

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Tobias Schulz

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1. Introduction

In recent years, the efficient management of closed-loop supply chains has been given increasing attention in theory and industry. By closing the loop, many manufacturers extend their originally designed, forward-oriented logistic activities to integrate the backward flow of products from their customers. This creates new opportunities to generate value by recovering products, components, or materials. When the returned product is properly functioning, it can be almost directly resold to the customers (in some cases, a repackaging is required). Yet, firms can also create value from the recovery of broken products.

Thierry et al. (1995) categorize five different options on how product recovery for returned broken products can be organized. The least complex option to restore the product's functionality is to repair it. There, broken parts are simply replaced or fixed which results in a lower product quality compared to a new product. The second option named refurbishing brings products to a predefined quality standard. Instead of replacing or fixing broken parts, technologically superior parts can be used to achieve a prespecified target quality level. However, the quality of a refurbished product does not have to concur with the quality of a new product. Bringing a returned product to a quality standard comparable to a new product is the objective of the third option of product recovery, remanufacturing. The process of remanufacturing pursues this objective by disassembling the returned product, thoroughly inspecting all components obtained and replacing/mechanically remanufacturing broken components. These components (remanufactured and new) are assembled into the remanufactured product. While this is the most sophisticated form of product recovery which aims to save a large part of the product's value, the final two options mentioned by Thierry et al. (1995) cannibalization and recycling focus more on harvesting the components and materials, respectively. Among these options, remanufacturing has become an interesting option not only for original equipment manufacturers (OEMs) due to the potential benefits it can create. Until today, remanufacturing operations have been established in a large variety of industrial applications and account for total sales of more than $50 billion a year (see, e.g., Guide, 2000).

Akçalı and Çetinkaya (2011) identify in their literature review four basic remanufacturable product categories that can be found in industry: Refillable containers, durable products, technology products, and recoverable materials. The remanufacturing process for refillable containers (for liquid gases and beverages) is described, for instance, by Kelle and Silver (1989). Moreover, toner cartridges and single-use cameras can be interpreted as refillable containers as well (see, for instance, in Majumder and Groenevelt, 2001). Next to the comparably simple remanufacturing of refillable containers, there is a multitude of durable products which are remanufactured. Seitz and Wells (2006) present, for instance, the case of automotive engine remanufacturing. The importance of remanufacturing for automotive manufacturers can be highlighted by the following figures. In 2008, Volkswagen remanufactured 3.83 million components (mostly engines and transmissions) and generated a revenue of around 600 million € with their remanufacturing activities (see Volkswagen, 2010). Two additional examples for the remanufacturing of complex durable products are photocopiers (as presented in Thierry et al., 1995) and various medical equipment (see Ferrer and Ketzenberg, 2004). However, not only durables are remanufactured in practice but also high-end technology products like cellular phones (as presented in Guide and Van Wassenhove, 2001) and PC components (as in Ashayeri et al., 1996). Finally, Akçalı and Çetinkaya (2011) name recoverable materials (like steel and glass) as a remanufacturable product category although it may also be classified as a recycling process since no disassembly operation needs to be performed and solely the relevant materials are recovered. All industrial remanufacturing processes named above have in common that a remanufacturer has to consider a plethora of different tasks during the planning process.

In his seminal work, Guide (2000) describes the complicating characteristics of remanufacturing in industry. An important planning task for a remanufacturer (or an OEM) is to adapt his logistics network to handle the return flow of products from his customers to his remanufacturing facilities efficiently. Yet, even the best organized system cannot obviate all uncertainties regarding the timing and quantity of returns. Furthermore, the quality of returns is uncertain as well, i.e. the remanufacturer does often not know before disassembling his returns which components can be properly recovered. Altogether, these characteristics imply an uncertain supply of recoverable components. This uncertain supply aggravates the planning process substantially as the remanufacturer intends to satisfy customer demand at least partly by remanufacturing returned products. For all planning tasks, Guide elaborates a number of possible research questions that require further attention. Due to the complexity of an industrial remanufacturing system, all research questions can only be formulated to focus on a small part of the entire system. One of the most important questions to ask in this context contains the timing and sizing of remanufacturing and manufacturing decisions when substantial setup costs prevail for each process. To present different solution approaches to answer this question is the main focus of the following work.

In order to do this, a simplified model of a remanufacturing system needs to be formulated that includes all relevant decisions but can be adapted easily to different scenarios. Next to the decisions on when and how much to (re)manufacture, the corresponding inventory levels play an important role in this context. Due to their importance, there are many possibilities to simplify the existing interdependencies of all relevant inventory levels in a remanufacturing system. Akçalı and Çetinkaya (2011) classify 14 different modelling approaches that can be found in literature. Although some of these approaches are rarely used, one approach (named 2SP-c in their work) has been applied in 41 scientific contributions. Throughout this work, we will use this approach as well. Figure 1.1 depicts the interdependencies of all relevant stocking points and processes.

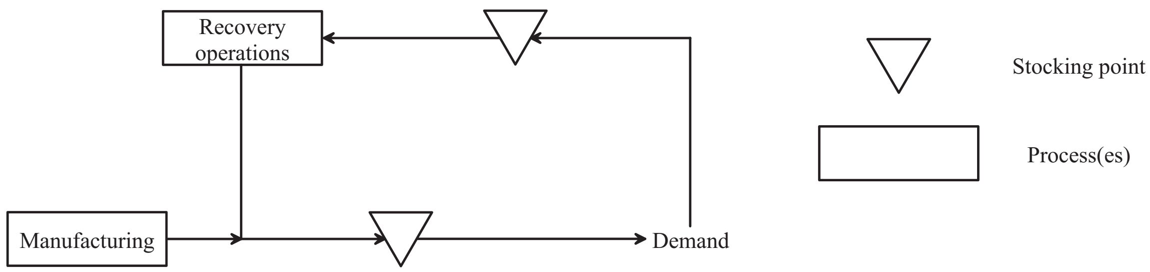

Fig. 1.1: Simplified inventory model for a remanufacturing system

In this simplified model, a single final product (containing a single, vital part) is demanded by the remanufacturer's customers. Regarding demand, we assume that the remanufacturer always knows in advance what his customers request, i.e. the demand is deterministic. When the customers have no further use for their product, they have the possibility to return it to the remanufacturer who stores all returns in a corresponding inventory (at a given holding cost). After a collection interval, the remanufacturer recovers a number of returns from this stock which brings them to an as-good-as-new condition. Each remanufacturing run requires a specific setup, for instance to adjust the required tools. This setup incurs a setup cost that needs to be considered in the decision making process. All successfully recovered products are held in a final product inventory (at a given holding cost) from which final products are delivered to the customers. In this model context, we impose the assumption that the entire demand cannot be satisfied by remanufacturing returns alone. Thus, the additional option of manufacturing new products is required. Likewise, each manufacturing setup incurs a corresponding setup cost. Newly fabricated products are also brought to the final product stock. Therefore, the remanufacturer has two options to serve his customers, remanufacturing returns and manufacturing new products. To facilitate the solution finding process, we further impose the assumption that customers do not care whether they procure a remanufactured or a new product.

Respecting this basic modelling approach, we elaborate four variations of it in the subsequent Chapters and analyze their implications. These variations result from the fact that all processes in the basic model can be interpreted differently. The following listing presents all possible process interpretations used henceforth:

1. ***Recovery process***: The recovery process can be explicitly modelled with separate disassembly and remanufacturing activities but can also be integrated in a single recovery operation. A more detailled process is recommended when both processes differ significantly with respect to their corresponding cost.

2. ***Yield from recovery***: The yield from recovery is, in general, not known in advance. In contrast to stochastic yields, the commonly applied simplification of deterministic yields can be presumed as well to facilitate the solution finding.

3. ***Manufacturing new products***: Manufacturing new products can be allowed throughout the entire planning horizon or only at its beginning. In reality, manufacturing new products becomes in some cases prohibitively expensive after high volume series production runs out. Depending on the respective model setting, one of both interpretations will be allowed.

4. ***Demand and return process***: The modelling of the demand and return processes can differ as well. On the one hand, a static and continuous demand and return can be assumed over the entire planning horizon. As this is only a simplification of a real-life environment, the more realistic dynamic demands and returns can be applied as well. Moreover, we differ between exogeneously given demands and returns or whether the remanufacturer is able to endogeneously influence both parameters.

Having two interpretations of four processes leads in general to 16 different model settings. Yet, as not all models are relevant for a real-life environment, we restrict our attention to four settings. Table 1.1 presents the respective process interpretations of the subsequent Chapters 2-5.

Tab. 1.1: Process interpretations of the following Chapters

	Recovery process	***Yield from recovery***	***Manufacturing new products***	***Demand and return***
Ch. 2	remanufacturing	deterministic	always possible	static and exogenous
Ch. 3	disassembly & remanufacturing	stochastic	always possible	static and exogenous
Ch. 4	remanufacturing	deterministic	always possible	dynamic and exogenous
Ch. 5	remanufacturing	deterministic	only at the beginning of the planning horizon	dynamic and endogenous

Chapter 2 considers a problem that is well known in literature and can be referred to as the basic lot sizing model for a remanufacturing system. In it, remanufacturing and manufacturing decisions have to be aligned over an infinite planning horizon in which all parameters are static and exogenously given. We review at the beginning of Chapter 2 all currently published solution approaches for this basic model setting and propose a more effective new solution approach. Thereafter, the first optimization approach (type: mixed-integer non-linear optimization problem) for this model is introduced that allows to compare all solutions independently. A numerical study concludes this Chapter which reveals two interesting results. At first, the newly proposed solution can improve the currently published solutions by more than 9% for some parameter settings. Moreover, the optimization approach is able to improve the solutions slightly further but only in some cases.

In the following Chapter 3, the basic lot sizing model of the preceding Chapter is extended to incorporate a more detailled recovery process by considering disassembly and remanufacturing activities separately. As mentioned above, the quality of returns cannot always be evaluated by the remanufacturer in advance. Thus, disassembling the returns and inspecting the components obtained thereby is an option to evaluate the quality of the product returns. This Chapter formulates the research question what changes to the currently applied methods have to be made when a stochastic yield from disassembly prevails. In it, three policies are formulated that differ in their degree of sophistication. While the least sophisticated policy ignores the stochastic yield entirely and assumes a deterministic equivalent yield, the most sophisticated policy incorporates the entire yield distribution in its solution finding process. By conducting a numerical study, we find that the error of applying the least sophisticated policy instead of the most sophisticated one is seldomly larger than 2%. This can be interpreted as a very interesting result as it implies that neglecting stochastic yields and assuming a deterministic equivalent yield instead does not necessarily result in a large cost difference.

Chapter 4 adjusts the modelling approaches of the preceding Chapters to incorporate dynamic demand and return patterns in a simplified remanufacturing system. By doing this, a more realistic setting can be established since the number of products returning to the remanufacturer varies, in general, over time. Of course, the same can be said about the number of products demanded from the remanufacturer. In their work, Teunter et al. (2006) extend three well-known heuristic approaches from the single-item dynamic lot sizing problem (Silver Meal, Part Period, and Least Unit Cost) by evaluating two options to satisfy demand, remanufacturing returns and manufacturing new products. In a numerical study, the authors identify their adapted Silver Meal approach as the best performing heuristic. However, this heuristic shows an average error of around 8% when compared to the optimal solution obtained by mixed-integer linear programming. Since this average error appears to be unnecessarily high, Chapter 4 introduces a modification to the Silver Meal approach by Teunter et al. (2006) that is able to reduce the average error to below 2.5% when tested in the same experimental design.

Until now, the remanufacturing system is modelled in a way that the remanufacturer faces an exogenously given demand and return. In Chapter 5, this assumption is intensively discussed. There, a number of model settings are established that describe different options for the remanufacturer to directly control the demand for his products. In general, the preceding Chapters assumed that the remanufacturer is always willing to satisfy the demand for remanufactured products. The model settings introduced in Chapter 5, however, allow the remanufacturer to buy back used products to prevent him from fulfilling a request for a remanufactured product. As this proceeding can be reasonable, Chapter 5 elaborates optimal buy-back strategies for different settings regarding information availability and buy-back flexibility. A numerical study concludes the analysis and presents circumstances under which buy-back seems to be especially beneficial for the remanufacturer. In contrast to the preceding Chapters, these modelling approaches assume that the setup cost for (re)manufacturing are neglible.

As mentioned above, all relevant tasks for a real-life remanufacturing system can never be solved simultaneously. This work addresses, hence, only a small number of relevant tasks. In real-life remanufacturing systems, for instance, a large number of products are disassembled that contain a lot of components. When different products contain the same component, the question arises which kind of product to disassemble in order to obtain a specific component. This planning task, known as disassembly planning, has been discussed intensively, e.g., by Langella and Schulz (2006) as well as Schulz (2007). Moreover, uncertainties do not only prevail on the supply side of industrial remanufacturing systems but also on their demand side as a remanufacturer cannot forecast the demand for remanufactured products perfectly. In this case, an efficient inventory management needs to be established for the final product inventory. Next to defining cost-minimizing service levels, the method of drawing operating curves can help to depict the existing trade-off between mean inventory levels and mean stock-outs. For a detailled discussion on operating curves, please refer to Inderfurth and Schulz (2007a,b, 2008, 2009, 2010). Obviously, a large number of additional planning aspects can be included in a comprehensive analysis of a real-life remanufacturing system (again, we would like to refer the reader to Guide, 2000; Atasu et al., 2010). Yet, to focus on the decisions on when and how much to (re)manufacture we restrict ourselves to the simplified model settings presented above. The next Chapter begins the analysis by introducing the basic lot sizing model for recovery systems and elaborates possible improvements to currently established solution methods.

2. Optimal and predefined policies for the static lot sizing problem in a two stage recovery system

2.1 Introduction

The growing environmental concern of their customers combined with an increasing price consciousness poses a challenging task for many manufacturing companies[1]. This development in customer behavior supports the manufacturing companies to consider product recovery as a viable alternative to satisfy customer demand. Depending on the degree of disassembly and material reuse, Thierry et al. (1995) classify five different recovery options. Among these options, remanufacturing returned products seems to be of special interest since it addresses both issues demanded by their customers. On the one hand, remanufacturing a returned product reduces landfill space as it needs not to be disposed of. On the other hand, as a part of the value embedded in the product is saved, the manufacturer is able to offer his customers a significant price discount on the remanufactured product. When accepting this offer, the customer does not face a disadvantage compared to buying a new product since in general the same warranty is issued for both.

In literature, a variety of real-life industrial applications for remanufacturing has been presented ranging from car engines (as in Seitz and Wells, 2006) over photocopiers (as in Thierry et al., 1995) to water pumps for diesel engines (as in Tang and Teunter, 2006).

[1] This Chapter is based on the work titled 'Optimal and predefined policies for the static lot sizing problem in a two stage recovery system' that has been published in the FEMM working paper series (see Schulz, 2011a).

Common to all industrial applications is that remanufacturing a returned product requires a large number of different processing operations. After return, each product is disassembled to obtain its components. All components are inspected whether they can be reused or not. If necessary, mechanical rework processes ensure the required quality standards. Complemented by new components, the remanufactured components are assembled into remanufactured products which can be offered for sale.

For establishing an efficient remanufacturing system, a multitude of planning tasks have to be taken into account. Guide (2000) illustrates in his work the complexity of possible obstacles to overcome during this planning process. One of the most complex issues mentioned in his work is lot sizing for remanufacturing, i.e. the question of when to remanufacture returned products and how many items to include in each remanufacturing batch. As, in general, the entire customer demand cannot be satisfied by remanufacturing, a number of new products need to be manufactured in addition. Incurring a setup cost for initiating a remanufacturing/manufacturing batch and holding cost for storing a returned/final product, a lot sizing problem results that needs to integrate remanufacturing and manufacturing decisions. This objective represents the main focus of this Chapter.

The first attempt to find a solution to this problem has been proposed by Schrady (1967). He abstracts from a possible real-life remanufacturing system by imposing a number of assumptions to facilitate the solution finding. Most importantly, his assumption of a static product demand and return flow of products over an infinite planning horizon results in a multi-level EOQ problem setting (with EOQ being the Economic Order Quantity). In order to find a solution to this problem, Schrady separates the infinite planning horizon into equal cycles. All cycles contain the same sequence of lot sizing decisions and are repeated identically over the entire planning horizon. As commonly applied to EOQ-type lot sizing problems, the cycle needs to be determined that minimizes the total cost per time unit. Schrady recommends a cyclic solution in which R equal remanufacturing lots precede a single manufacturing lot. For this kind of policy he derives closed-form expressions for the (re)manufacturing batch sizes. Further on, Schrady's proposed solution is referred to as the $(R, 1)$ policy indicating that R remanufacturing batches and one manufacturing batch are set up in a cycle.

Nahmias and Rivera (1979) extend Schrady's contribution by incorporating a finite recovery rate while keeping the production rate infinitely large. In their contribution, they adjust the closed-form expressions for both lot sizes to respect their change to the model setting. Another extension to Schrady's basic model has been proposed by Richter (1996a,b). He includes the option to decide whether to dispose of returned products or not. While in the basic model remanufacturing is assumed to be always beneficial, Richter shows that this solution depends on the size of the variable cost of (re)manufacturing. Therefore, a variable disposal rate can influence the solution to this problem setting significantly when remanufacturing might not be beneficial in general. Coming back to Schrady's original problem setting, Teunter (2001) proposes another policy structure that promises better results for some parameter combinations. Teunter derives closed-form expressions for both lot sizes when one remanufacturing batch is succeeded by M equal manufacturing lots. His solution will, thus, be referred to as the $(1, M)$ policy. Later on, Teunter (2004) extends in another contribution the work of Nahmias and Rivera to include finite recovery and production rates into the closed-form expressions for both the $(R, 1)$ and $(1, M)$ policies. All contributions introduced so far obtain closed-form expressions for the (re)manufacturing batch sizes under the assumption of a non-integer value for R and M, respectively. Since R and M have to be integer to ensure feasibility, Minner (2002) proposes a methodology to correctly consider the issue of integrality.

In his first work, Teunter mentions two opportunities to improve the solutions proposed until then. First, he conjectures a more general (R, M) policy (with $R, M > 1$ simultaneously) that can decrease the total cost incurred compared to the $(R, 1)$ and $(1, M)$ policies. This conjecture has been tested by Choi et al. (2007). They introduce a solution procedure that is able to derive the minimum total cost value for a more general (R, M) solution while keeping all (re)manufacturing batches equal. In addition, a numerical experiment has been conducted to evaluate the possible improvements the more general (R, M) policy offers. In their study, the (R, M) policy is able to improve the currently proposed policies in about 0.2% of all tested instances with a maximum deviation of less than 0.5%. These findings have been, among other things, confirmed by Liu et al. (2009). Moreover, Konstantaras and Skouri (2010) extend the (R, M)

policy to include possible shortages. In order to do that, they adapt and facilitate the solution procedure introduced by Choi et al. As a result, their solution approach is valid for both the non-shortage and the shortage case.

Next to creating a more general (R, M) policy structure, Teunter (2001) conjectures to allow for differently sized remanufacturing batches within a cycle to improve the solution even further. By using a Lagrange-multiplier approach, Minner and Lindner (2004) proved Teunter's conjecture to be true, i.e. policies containing differently sized remanufacturing batches can outperform policies with equal ones. Yet, they have not evaluated the potential gain differently sized remanufacturing batches can have. Feng and Viswanathan (2011) extend in their contribution the general (R, M) policy by Choi et al. to include differently sized remanufacturing batches. Their approach proposes to split the entire (R, M) cycle into two subcycles. Thereafter, an enumerative procedure tests whether the solution can be improved when the remanufacturing lot sizes are altered in both subcycles. Yet, within a subcycle all remanufacturing batch sizes remain equal. The main contribution of this Chapter is to show that scheduling non-equal remanufacturing batches in a cycle proposes a significant cost reduction for some parameter classes. Furthermore, a more general optimization approach is introduced that allows to evaluate the solution quality of the preset policy structures.

The remainder of this Chapter is organized as follows. After elaborating all assumptions required of the general problem setting in Section 2.2.1, Schrady's $(R, 1)$ policy and Teunter's $(1, M)$ policy are presented as in the original contributions in Sections 2.2.2 and 2.2.3. The only difference to their presentations is that a yield parameter β is included in our contribution to consider the influence of an imperfect remanufacturing process. Afterwards, Section 2.2.4 presents the alternative formulation of the total cost function proposed by Minner (2002) to derive a closed-form expression for the integer number of remanufacturing and manufacturing batches in a cycle. Such a formulation has neither been included in Schrady's nor in Teunter's work. While Section 2.2.5 discusses the results of the preceding subsections in greater detail, Section 2.2.6 introduces a new policy structure, the $(R, 1)^g$ policy. Deviating from the formerly introduced $(R, 1)$ policy, this policy allows for differently sized remanufacturing lots in a cycle. More precisely, the amount to be remanufactured in a batch decreases geometrically throughout the cycle. This characteristic permits to fulfill the zero inventory property in both inventory levels, i.e. each remanufacturing lot remanufactures all returns in stock. Contrary, implementing an $(R, 1)$ policy with equal remanufacturing lots means that not necessarily all returns are remanufactured in a batch and a positive number of items can remain in stock. However, the $(R, 1)^g$ policy structure is a predefined structure like the $(R, 1)$ and $(1, M)$ policies which only allows to compare different policies. As no general optimization approach has yet been formulated in literature to evaluate the predefined policy structures properly, Section 2.3 provides an approach to obtain a benchmark solution by solving the underlying problem without presuming predefined structural characteristics. Thereafter, Section 2.4 conducts a numerical study by presenting a base case from literature and varying its parameters in a sensitivity analysis to assess the influence of each parameter on the solution quality. In this study, the simplified policy structures are compared to the benchmark solution in order to evaluate their performance. Finally, this Chapter is concluded in Section 2.5 with a short summary and an outlook on future research opportunities.

2.2 Predefined policy structures for the two stage remanufacturing system

2.2.1 General model setting

Before analyzing the two stage remanufacturing system intensively, all necessary assumptions have to be stated. In general, the model setting presented subsequently concurs (with one exception) to the model setting introduced by Schrady. In it, an original equipment manufacturer (OEM) engaged in the area of remanufacturing represents the background. Figure 2.1 presents its general structure.

The OEM sells one product A to his customers. Demand for product A is assumed to be constant and depletes the finished goods inventory continuously at a rate of λ units per time unit. A fraction α of used products in the market (denoted by A') returns to the manufacturer when his customers have no further use for it. Therefore, a continuous

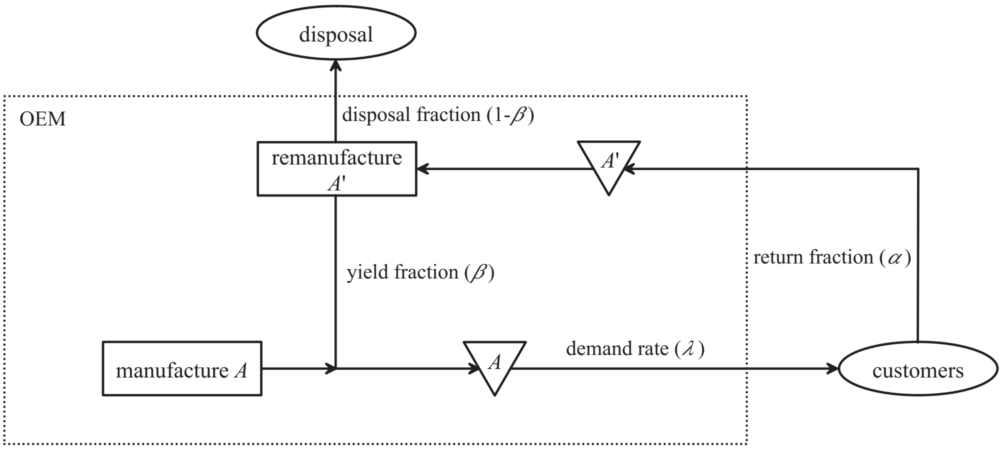

Fig. 2.1: Inventory system in a two stage remanufacturing environment

inflow of $\lambda\alpha$ returned products per time unit is observed for the used product inventory. Storing an unit of A' in this inventory results in a holding cost h_R per time unit. Due to different stages of wear, not all returned products can be brought to an as-good-as-new condition which is a prerequisite to resell the product. Hence, β denotes the deterministic fraction of returned products that can be successfully reworked. Thus, α as well as β must not exceed one while being non-negative. All products that cannot be remanufactured sufficiently are recycled. Recycling a returned product is assumed to be free of charge. This assumption can be imposed when the value of all materials contained in A' is about the same as the value of work required to separate these materials. Setting up the mechanical rework and cleaning tools incurs a setup cost K_R. All successfully remanufactured products are held in a final product inventory at a cost of h_M per unit per time unit. In order to secure that demand for A is always met, some new products have to be manufactured in addition (as α and β are usually smaller than one). The relevant setup cost is denoted by K_M representing the cost for initiating a manufacturing lot for product A. This model includes neither processing nor lead times, i.e. whenever a (re)manufacturing batch is issued it arrives instantly. Newly manufactured products are held in the same serviceables inventory as remanufactured ones. Regarding the cost of storage, both remanufactured and new products are evaluated with the same holding cost parameter h_M. As two levels of inventory are considered (used product and final product) the resulting system is defined as a two stage recovery system.

In general, the holding costs of both inventory levels (when interpreted as opportunity cost of capital) are connected by the following condition. Since an increasing product value indicates more tied-up capital, the holding cost parameter h_M must be larger than h_R as the remanufacturing process provides a significant increase in value. Yet, only the fraction β of all products returned can be sufficiently remanufactured. In other words, at an average $1/\beta$ products have to be remanufactured to obtain one salable product. As it cannot be observed before remanufacturing whether this process is successful, the following condition for both holding cost parameters has to hold to assure validity: $h_R/\beta < h_M$. On the other hand, no condition is imposed for the process related setup costs K_R and K_M. Contrary to these fixed cost parameters, the subsequent model omits the use of variable costs for manufacturing and remanufacturing product A. By assumption, obtaining a unit of A by remanufacturing is always less expensive than manufacturing it. Consequently, the OEM commences the remanufacturing process for all returns (whether it is successful or not) and disposes no return in advance.

Figure 2.2 presents the levels of inventory for the analyzed framework and depicts whether the inflows to and outflows from each level are continuous or discrete. The entire system has a continuous inflow and outflow of goods amounting to $\lambda\alpha$ and λ units per time unit, respectively. All parameters remain constant over an infinite plan-

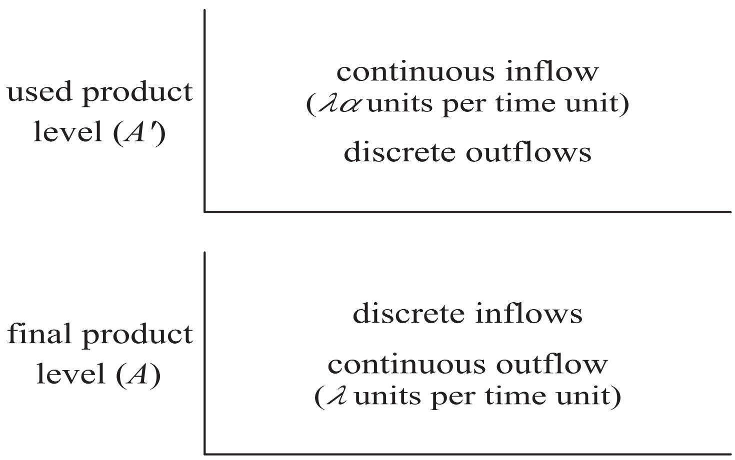

Fig. 2.2: Two stocking points and their inflows and outflows

ning horizon which leads to an EOQ-type model (as setup and holding costs prevail). The standard single level EOQ approach recommends to replenish the inventory with a certain amount (known as the economic order quantity) whenever it is depleted. By following this simple rule over the infinite planning horizon and thereby creating identically repeated cycles, the EOQ approach minimizes the total cost per time unit. This chapter adopts the standard EOQ procedure to the more sophisticated two stage inventory problem presented above. In it, six decisions of interest have to be evaluated: the *length* of a cycle (T) as well as the *number* of lots scheduled therein, i.e. the number of remanufacturing (R) and manufacturing lots (M). Moreover, to define a cycle unambiguously, further information is required on the *sequence* of batch scheduling and on the *quantities* of individual lot sizes (denoted by Q_R for remanufacturing and Q_M for manufacturing lots) that need not be integer. Since all lot sizes within a cycle can be different, a complex policy structure can result. Yet, by imposing restrictions on some of these decisions, simple policy structures can be derived that facilitate finding a solution to this problem setting.

2.2.2 Schrady's $(R, 1)$ policy

The first attempt to define a simple policy structure for this problem has been undertaken by Schrady (1967). In his work, the author elaborates a set of formulae for a cyclic pattern in which one manufacturing lot is succeeded by a number of equally sized remanufacturing lots R. Therefore, this policy is referred to as the $(R, 1)$ policy. The simplifying assumption of having remanufacturing lots of equal size is, among other things, relaxed later on. Before doing this, the $(R, 1)$ policy with equal remanufacturing lots is presented. Figure 2.3 illustrates, for example, a cyclic pattern with one manufacturing and three remanufacturing lots. All lots are arranged in the way that both the used product and the final product inventories are entirely depleted at the beginning of a cycle. Thus, a cycle starts with a remanufacturing batch containing Q_R returned products. Since the fraction β can be brought to an as-good-as-new condition, $Q_R \cdot \beta$ products enter the final product inventory at the beginning of each cycle. After $\frac{Q_R \cdot \beta}{\lambda}$ time units the final product inventory is depleted and the sole manufacturing lot containing Q_M final products is scheduled. Thereafter, the remaining remanufacturing lots are initiated until the end of the cycle is reached and the next, identical cycle commences. Since all remanufacturing lots are presumed to be equal, not all remanufacturable returns available in stock are remanufactured at all times. Hence, the used product level is only depleted at the beginning/end of a cycle.

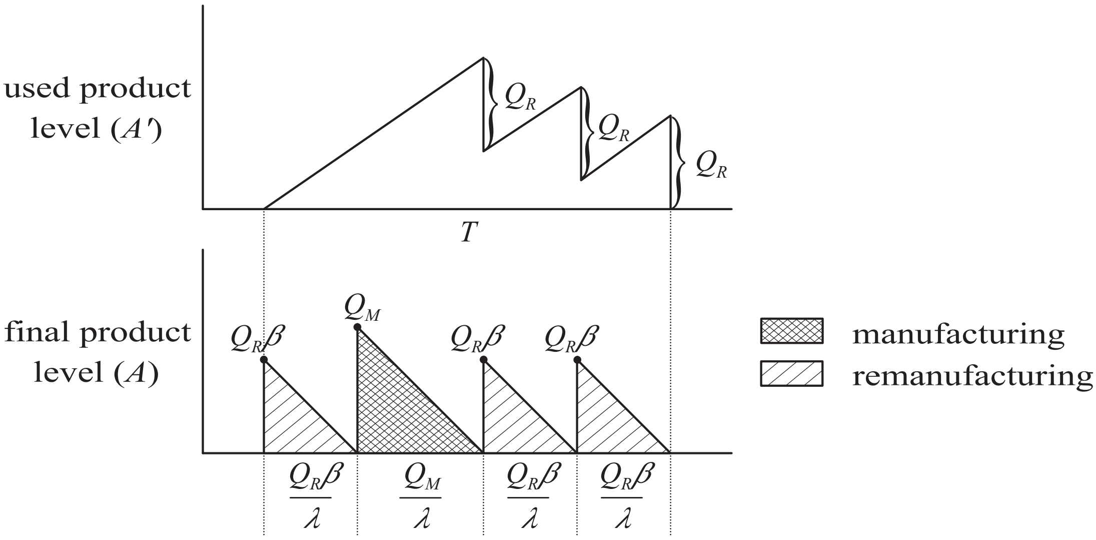

Fig. 2.3: Used product and final product level corresponding to a (3,1) policy

Each $(R, 1)$ policy structure is unambiguously outlined by two decision variables. In his work, Schrady chooses the lot sizes Q_R and Q_M to evaluate the total cost of his policy structure. The remaining relevant decisions (number of remanufacturing lots R and cycle length T) can be deduced from Q_R and Q_M as follows. A cyclic structure results if both inventory levels at the beginning of each cycle are equal to their respective levels at the corresponding cycle's end. To ensure this, the number of returned products collected in a cycle must be equal to the amount of products remanufactured in it. Since the OEM receives $\lambda\alpha$ products per time unit and each cycle has a length of T time units, $\lambda\alpha T$ products are remanufactured in R identical batches of size Q_R, i.e.

$$R \cdot Q_R = \lambda\alpha T. \tag{2.1}$$

As can be derived from Figure 2.3, the length of a cycle T is computed by

$$T(Q_R, Q_M) = \frac{R \cdot Q_R \cdot \beta + Q_M}{\lambda}. \tag{2.2}$$

By combining equations (2.1) and (2.2), analytical expressions can be formulated for both R and T that depend only on the relevant decision variables Q_R and Q_M.

$$R(Q_R, Q_M) = \frac{\alpha \cdot Q_M}{(1-\alpha\beta) \cdot Q_R} \qquad \text{and} \qquad T(Q_M) = \frac{Q_M}{\lambda\,(1-\alpha\beta)}. \tag{2.3}$$

To obtain the smallest total cost of the predetermined $(R, 1)$ policy structure, the sum of a setup and a holding cost term has to be minimized. Starting with the setup cost term, the number of remanufacturing lots R needs to be multiplied by K_R and added to the setup cost for initiating the manufacturing batch K_M. The resulting value needs to be divided by the cycle length T to compute the setup cost per time unit. Using equations (2.2) and (2.3), this results in[2]:

$$\frac{K_m + R \cdot K_R}{T} = \lambda \cdot \left(\frac{(1-\alpha\beta) \cdot K_M}{Q_M} + \frac{\alpha \cdot K_R}{Q_R} \right). \tag{2.4}$$

Regarding the holding cost term, the following analysis considers both inventories separately. The holding cost per time unit for the used product inventory can be determined by evaluating the average inventory during a cycle. In static lot sizing problems, the average inventory can be computed by dividing the maximum inventory level within a cycle y_R^{max} by two. Yet, this can only be done when the inventory level of the corresponding stock is zero at the beginning and at the end of a cycle but never within. Due to the policy prerequisite of having remanufacturing lots of equal size, this is always given for an $(R, 1)$ policy structure in the used product inventory. As depicted in Figure 2.3, the maximum inventory in the used product stock prevails after the products fabricated in the cycle's only manufacturing lot run out. At this point in time, the inventory contains all products returning to the OEM while one remanufacturing and the manufacturing lot have satisfied customer demand. As $\lambda\alpha$ products return per time unit, the holding cost for the used product stock is

$$\frac{1}{2} y_R^{max} \cdot h_R = \frac{1}{2} \cdot \alpha \cdot (Q_R \cdot \beta + Q_M) \cdot h_R. \tag{2.5}$$

The average holding cost in the final product inventory, on the other hand, cannot be determined by dividing the maximum inventory level during a cycle by two since it drops to zero several times in it. Generally speaking, the holding cost in a cycle is determined by multiplying the inventory during this cycle by the corresponding holding

[2] For details, please refer to the Appendix, page 69.

cost. The inventory during a cycle is computed by assessing the region bounded by the inventory level. For instance, to determine the holding cost of the final product level, the area of the observed triangles in Figure 2.3 has to be evaluated. This term has to be multiplied by h_R and divided by T as the holding cost per time unit is required. By using equations (2.2) and (2.3), this gives[3]

$$\left(\frac{1}{2}\cdot\frac{R\cdot(Q_R\cdot\beta)^2}{\lambda}+\frac{1}{2}\cdot\frac{(Q_M)^2}{\lambda}\right)\cdot h_M\cdot\frac{1}{T}=\quad\frac{1}{2}\left(\alpha\beta^2\cdot Q_R+(1-\alpha\beta)\cdot Q_M\right)h_M. \quad (2.6)$$

After establishing the relevant setup and holding cost terms, the total cost function for Schrady's $(R,1)$ policy depending on both lot sizes Q_R and Q_M is formulated by summarizing the cost components in (2.4), (2.5), and (2.6). Henceforth, this total cost function is denoted by TC_{R1}. It is

$$\begin{aligned}TC_{R1}(Q_R,Q_M)=&\lambda\cdot\left(\frac{(1-\alpha\beta)\cdot K_M}{Q_M}+\frac{\alpha\cdot K_R}{Q_R}\right)+\frac{1}{2}\cdot\alpha\cdot(Q_R\cdot\beta+Q_M)\cdot h_R+\\&\frac{1}{2}\cdot\left(\alpha\beta^2\cdot Q_R+(1-\alpha\beta)\cdot Q_M\right)\cdot h_M. \end{aligned}\quad (2.7)$$

This total cost function (2.7) is jointly convex[4] in both decision variables Q_R and Q_M, i.e. the smallest total cost can be determined by exploiting its partial derivatives. For instance, by computing the partial derivative of (2.7) with respect to Q_R, the best remanufacturing lot size Q_R^+ for the $(R,1)$ policy structure is obtained. This gives

$$\begin{aligned}\frac{\partial TC_{R1}}{\partial Q_R}&=-\frac{\lambda\alpha K_R}{(Q_R)^2}+\frac{1}{2}\cdot\alpha\beta\cdot(h_R+\beta\cdot h_M)=0\quad\text{and results in}\\ Q_R^+&=\sqrt{\frac{2\lambda\cdot K_R}{\beta\cdot(h_R+\beta\cdot h_M)}}.\end{aligned}\quad (2.8)$$

Similarly, the best manufacturing lot size Q_M^+ for an $(R,1)$ policy structure is calculated by

$$\begin{aligned}\frac{\partial TC_{R1}}{\partial Q_M}&=-\frac{\lambda\,(1-\alpha\beta)\,K_M}{(Q_M)^2}+\frac{1}{2}\cdot(\alpha\cdot h_R+(1-\alpha\beta)\cdot h_M)=0\quad\text{and results in}\\ Q_M^+&=\sqrt{\frac{2\lambda\cdot(1-\alpha\beta)\cdot K_M}{\alpha\cdot h_R+(1-\alpha\beta)\cdot h_M}}.\end{aligned}\quad (2.9)$$

[3] For details, please refer to the Appendix, page 69.

[4] For the mathematical proof, please refer to the Appendix, page 69.

The information about the best remanufacturing and manufacturing batch sizes can be inserted into the equations (2.3) to obtain the cost minimizing number of remanufacturing lots R^+ and the corresponding cost minimizing cycle length T^+:

$$R^+ = \frac{\alpha}{1-\alpha\beta} \cdot \sqrt{\frac{(1-\alpha\beta) \cdot K_M \cdot \beta \cdot (h_R + \beta \cdot h_M)}{K_R \cdot (\alpha \cdot h_R + (1-\alpha\beta) \cdot h_M)}} \tag{2.10}$$

$$T^+ = \sqrt{\frac{2 \cdot K_M}{\lambda \cdot (1-\alpha\beta) \cdot (\alpha \cdot h_R + (1-\alpha\beta) \cdot h_M)}}. \tag{2.11}$$

When determining the optimal $(R, 1)$ policy, the number of remanufacturing lots needs to be determined as in (2.10). However, the number of remanufacturing lots is not necessarily integer which is a prerequisite for obtaining a feasible solution. In this case, Schrady recommends a simple rounding procedure (without exactly specifying the required rounding operations) to determine the optimal policy. In Section 2.2.4, an exact approach is elaborated to find a solution to this problem.

In his original work, Schrady does not consider an imperfect remanufacturing process as he assumes the yield fraction β to be one. By introducing this fraction in the above analysis, several conclusions can be drawn when comparing a situation with yield loss to a situation without it. All conclusions are supported by analyzing the first derivatives of the respective formulae with respect to β. When β is smaller than one, a shorter cycle is recommended. As the overall number of returns decreases due to a shorter cycle, the number of (equal) remanufacturing lots per cycle decreases as well. Yet, to compensate for the yield loss and to use each remanufacturing setup efficiently, more returns are remanufactured in a setup which decreases the number of remanufacturing lots even further. Regarding the manufacturing lot size Q_M, no general conclusion can be drawn as the sign of the first derivative w.r.t β depends on K_M and both holding cost parameters.

Schrady's idea of creating cycles with one manufacturing lot and at least one remanufacturing lot has been discussed in literature later on. Teunter (2001) extends Schrady's work by proposing that it might be better to deviate from Schrady's $(R, 1)$ policy in some cases. His approach is introduced subsequently.

2.2.3 Teunter's $(1, M)$ policy

Contrary to Schrady's approach, Teunter proposes a preset policy structure which contains one remanufacturing and M (with $M \geq 1$) manufacturing batches. This policy structure is, thus, denoted as the $(1, M)$ policy. To give an example, Figure 2.4 depicts a (1,2) policy. At the beginning of a cycle, the sole remanufacturing lot containing Q_R returned products is initiated. Due to the imperfect remanufacturing process, only the fraction β can be sufficiently remanufactured, i.e. $Q_R \cdot \beta$ products enter the final product stock. Since the OEM's customers request λ products per time unit, this lot lasts for $\frac{Q_R \cdot \beta}{\lambda}$ time units. Thereafter, M manufacturing lots of equal size (each comprehending Q_M final products) are scheduled, each lasting for $\frac{Q_M}{\lambda}$ time units.

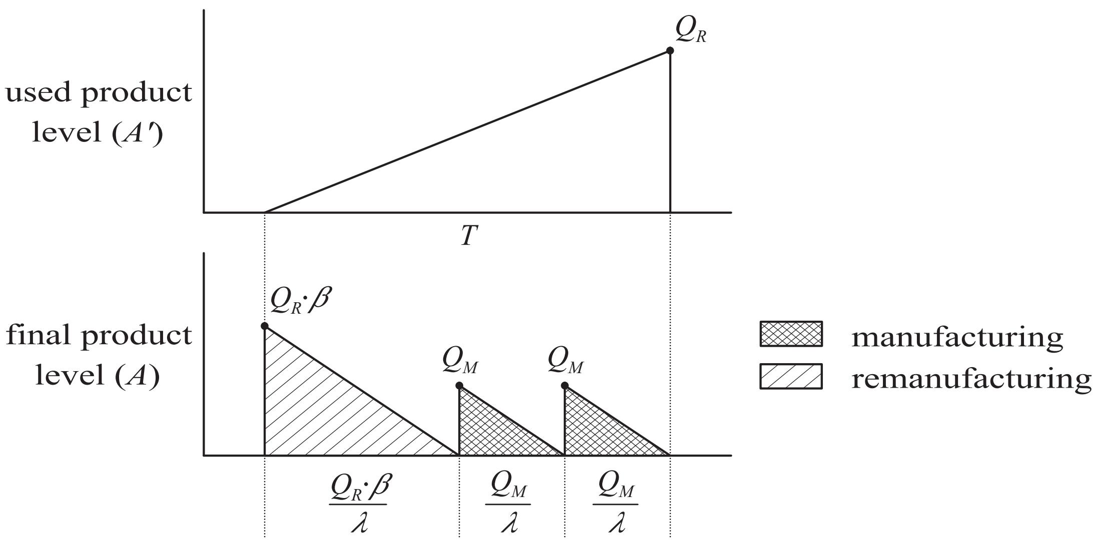

Fig. 2.4: Used product and final product level corresponding to a (1,2) policy

Similar to the $(R, 1)$ policy by Schrady, Teunter uses both lot sizes Q_R and Q_M to formulate the $(1, M)$ policy unambiguously, i.e. the number of manufacturing lots in a cycle (M) and the cycle length (T) can be deduced directly from these lot sizes. To guarantee a perfect cyclic structure, all remanufacturing lots must be of equal size. Therefore, the number of returned products at the end of a cycle is as large as the remanufacturing lot at its beginning. Since $\lambda\alpha$ products return per time unit, the subsequent condition has to hold

$$Q_R = \lambda \alpha T. \tag{2.12}$$

As can be observed in Figure 2.4 the cycle length T is computed by

$$T(Q_R, Q_M) = \frac{Q_R \cdot \beta + M \cdot Q_M}{\lambda}. \tag{2.13}$$

Combining equations (2.12) and (2.13) provides two formulae to describe the number of manufacturing lots M and the cycle length T depending on Q_R and Q_M.

$$M(Q_R, Q_M) = \frac{Q_R \cdot (1 - \alpha\beta)}{\alpha \cdot Q_M} \qquad \text{and} \qquad T(Q_R) = \frac{Q_R}{\lambda \alpha}. \tag{2.14}$$

When comparing conditions (2.3) and (2.14), the number of manufacturing lots M for a $(1, M)$ policy is the inverse of the number of remanufacturing lots R for an $(R, 1)$ policy when both are formulated depending on Q_R and Q_M. In order to pursue the objective of minimizing the total cost per time unit, a setup and a holding cost term have to be assessed again. The former comprises the setup cost of a cycle (M times the setup cost for manufacturing K_M plus once the setup cost for remanufacturing K_R) divided by the cycle length T. By transformation using equations (2.13) and (2.14), the following expression is derived[5]:

$$\frac{M \cdot K_m + K_R}{T} = \lambda \cdot \left(\frac{K_M \cdot (1 - \alpha\beta)}{Q_M} + \frac{K_R \cdot \alpha}{Q_R} \right). \tag{2.15}$$

After formulating the setup cost, the relevant holding cost per time unit is determined. To do this, the formerly applied methodology of calculating the area bounded by both inventories during a cycle has to be used. Hence, by using equations (2.13) and (2.14), the holding cost per time unit for both inventory levels is calculated as[6]:

$$\begin{aligned} &\left[\frac{1}{2} \cdot Q_R \cdot T \cdot h_R + \left(\frac{1}{2} \cdot \frac{(Q_R \cdot \beta)^2}{\lambda} + M \cdot \frac{1}{2} \cdot \frac{(Q_M)^2}{\lambda} \right) \cdot h_M \right] \cdot \frac{1}{T} \\ = \; &\frac{1}{2} \cdot \left(Q_R \cdot h_R + \left(\alpha\beta^2 \cdot Q_R + (1 - \alpha\beta) \cdot Q_M \right) \cdot h_M \right). \end{aligned} \tag{2.16}$$

[5] For details, please refer to the Appendix, page 70.

[6] For details, please refer to the Appendix, page 70.

Next, the total cost function for Teunter's $(1, M)$ policy (indicated by subindex $_{1M}$) is formulated by summarizing the cost components of (2.15) and (2.16). It is

$$\begin{aligned} TC_{1M}(Q_R, Q_M) =& \lambda \cdot \left(\frac{K_M \cdot (1-\alpha\beta)}{Q_M} + \frac{K_R \cdot \alpha}{Q_R} \right) + \\ & \frac{1}{2} \cdot \left(Q_R \cdot h_R + \left(\alpha\beta^2 \cdot Q_R + (1-\alpha\beta) \cdot Q_M \right) \cdot h_M \right). \end{aligned} \tag{2.17}$$

Like the cost function TC_{R1}, the total cost function (2.17) is jointly convex[7] in both Q_R and Q_M. Interestingly, the only difference between both cost functions is the evaluation of the used product's inventory which has no influence on the curvature of the total cost function but on its cost minimizing decision variables. By utilizing calculus, these variables can be computed. For instance, deriving the total cost function (2.17) with respect to Q_R provides the optimal size of the remanufacturing lot Q_R^+ for a $(1, M)$ policy structure:

$$\begin{aligned} \frac{\partial TC_{1M}}{\partial Q_R} &= -\frac{\lambda\alpha K_R}{(Q_R)^2} + \frac{1}{2} \cdot \left(h_R + \alpha\beta^2 \cdot h_M \right) = 0 \quad \text{and, thus,} \\ Q_R^+ &= \sqrt{\frac{2\lambda\alpha \cdot K_R}{h_R + \alpha\beta^2 \cdot h_M}}. \end{aligned} \tag{2.18}$$

Apparently, the same procedure can be applied to determine Q_M as well. Thus, the optimal size of each manufacturing lot Q_M^+ when presuming a $(1, M)$ policy structure is derived from

$$\begin{aligned} \frac{\partial TC_{1M}}{\partial Q_M} &= -\frac{\lambda\,(1-\alpha\beta)\,K_M}{(Q_M)^2} + \frac{1}{2} \cdot (1-\alpha\beta) \cdot h_M = 0 \quad \text{which results in} \\ Q_M^+ &= \sqrt{\frac{2\lambda \cdot K_M}{h_M}}. \end{aligned} \tag{2.19}$$

Inserting the optimal values of Q_R^+ and Q_M^+ into conditions (2.14) gives the cost minimizing number of manufacturing lots per cycle M^+ and cycle length T^+ for a $(1, M)$ policy structure:

$$M^+ = \frac{(1-\alpha\beta)}{\alpha} \cdot \sqrt{\frac{\alpha \cdot K_R \cdot h_M}{K_M \cdot (h_R + \alpha\beta^2 \cdot h_M)}} \tag{2.20}$$

$$T^+ = \sqrt{\frac{2 \cdot K_R}{\lambda\alpha \cdot (h_R + \alpha\beta^2 \cdot h_M)}}. \tag{2.21}$$

[7] For the mathematical proof, please refer to the Appendix, page 71.

Like for the $(R, 1)$ policy structure, the influence of including an imperfect yield β when initiating an $(1, M)$ policy is analyzed. For instance, the cost minimizing manufacturing lot size Q_M^+ is not influenced at all. On the contrary, the remanufacturing lot size Q_R^+ increases to efficiently compensate the yield loss with respect to the setup cost. Hence, the cycle length T^+ increases as more returns need to be collected. A longer cycle means that more new products are required to satisfy demand which results in an increasing number of manufacturing lots per cycle since the manufacturing lot size remains constant. Like in the preceding subsection, these logically drawn conclusions can be derived as well by analyzing the slope of the respective cost minimizing formulae with respect to β.

After establishing the $(1, M)$ and $(R, 1)$ policy structures, it is worth mentioning that both total cost functions yield the same result in a (1,1) scenario. However, using both policies to determine a feasible solution requires both R^+ and M^+ to be integer. Considering the cost minimizing values for R^+ in equation (2.10) and M^+ in equation (2.20) depicts that this is not the case in general. While Teunter omits to discuss this issue in his contribution, Schrady mentions it briefly by proposing a rounding procedure without clearly specifying the exact rounding operation. Minner (2002) continues the discussion and elaborates an interesting result by alternatively formulating the total cost functions of both policy structures. In his contribution, both total cost functions are formulated to depend on only R or M, respectively. By doing this, the obstacle of obtaining non-integer values for R and M is avoided since the total cost function depends on the sole variable that is required to be integer. The next subsection focuses on deriving his findings.

2.2.4 Alternative formulation of the $(R, 1)$ and $(1, M)$ policies

To define the total cost function of their policy structures unambiguously, Schrady and Teunter use both lot sizes Q_R and Q_M as their relevant decision variables. However, by inserting one of the cost minimizing lot sizes Q_R^+ (or alternatively Q_M^+) into the total cost function, the number of relevant decision variables can be reduced by one. Nevertheless, the obstacle of ensuring the number of remanufacturing (or manufacturing) lots to be integer remains to be solved. Therefore, Minner reformulates the total cost functions of both policy structures to depend on either R for an $(R, 1)$ policy or M for a $(1, M)$ policy structure. As two decision variables are required at the beginning, Minner chooses the cycle length T to be the second one.

For the $(R, 1)$ policy, the number of remanufacturing lots per cycle can exceed one while the number of manufacturing lots is exactly equal to one. Since all remanufacturing batches are of equal size, the amount of products returning in a cycle ($\lambda\alpha T$) has to be divided by R to obtain the size of each individual lot. Likewise, the amount to be manufactured in each cycle is given by the demand for the considered product that cannot be met by remanufacturing returned products, i.e. $(1 - \alpha\beta)$ of the entire demand. Therefore, the corresponding lot sizes can be reformulated (depending on R and T) according to formulae (2.1) and (2.3) as

$$Q_R(R, T) = \frac{\lambda\alpha T}{R} \quad \text{and} \quad Q_M(T) = \lambda\,(1 - \alpha\beta)\,T. \tag{2.22}$$

The setup cost per time unit is defined according to formula (2.4) which gives

$$\frac{R \cdot K_R + K_M}{T}. \tag{2.23}$$

Both holding cost elements can be simplified as well. Starting with the holding cost for the used product stock, the maximum inventory level in a cycle has to be evaluated. Corresponding to equation (2.5) this results in

$$\begin{aligned}\frac{1}{2} \cdot \alpha \cdot (Q_R \cdot \beta + Q_M) \cdot h_R =& \frac{1}{2} \cdot \alpha \cdot \left(\frac{\lambda\alpha T}{R} \cdot \beta + \lambda \cdot (1 - \alpha\beta)T\right) \cdot h_R \\ =& \frac{1}{2}\lambda T \left(1 + \alpha\beta\left(\frac{1}{R} - 1\right)\right) \cdot \alpha h_R. \end{aligned} \tag{2.24}$$

In compliance with equation (2.6), the holding cost per time unit for the final product inventory is reformulated as

$$\begin{aligned}\frac{1}{2} \cdot \left(\alpha\beta^2 \cdot Q_R + (1 - \alpha\beta) \cdot Q_M\right) \cdot h_M =& \frac{1}{2} \cdot \left(\alpha\beta^2 \cdot \frac{\lambda\alpha T}{R} + \lambda \cdot (1 - \alpha\beta)^2\, T\right) \cdot h_M \\ =& \frac{1}{2}\lambda T \cdot \left(\frac{\alpha^2\beta^2}{R} + (1 - \alpha\beta)^2\right) \cdot h_M. \end{aligned} \tag{2.25}$$

By adding up the setup and holding cost terms, the total cost function for the $(R,1)$ policy is established such that it depends on both R and T:

$$TC_{R1}(R,T) = \frac{RK_R + K_M}{T} + \frac{1}{2}\lambda T\left(\left(1+\alpha\beta\left(\frac{1}{R}-1\right)\right)\alpha h_R + \left(\frac{\alpha^2\beta^2}{R} + (1-\alpha\beta)^2\right)h_M\right). \tag{2.26}$$

For any given value of R, the optimal cycle length T can be computed by calculus. Thereby, the cycle length needs to be determined for which the partial derivative of the total cost function with respect to T is zero. This gives

$$\frac{\partial TC_{R1}}{\partial T} = -\frac{RK_R + K_M}{T^2} + \frac{1}{2}\lambda\left(\left(1+\alpha\beta\left(\frac{1}{R}-1\right)\right)\alpha h_R + \left(\frac{\alpha^2\beta^2}{R} + (1-\alpha\beta)^2\right)h_M\right) = 0$$

$$\text{and, thus,} \quad T^+_{R1}(R) = \sqrt{\frac{2\,(RK_R + K_M)}{\lambda\left(\left(1+\alpha\beta\left(\frac{1}{R}-1\right)\right)\alpha h_R + \left(\frac{\alpha^2\beta^2}{R} + (1-\alpha\beta)^2\right)h_M\right)}}. \tag{2.27}$$

Inserting T^+_{R1} into the total cost function TC_{R1} yields an expression that only depends on the number of remanufacturing lots R

$$TC^+_{R1}(R) = \sqrt{2\lambda(RK_R+K_M)\left(\left(1+\alpha\beta\left(\frac{1}{R}-1\right)\right)\alpha h_R+\left(\frac{\alpha^2\beta^2}{R}+(1-\alpha\beta)^2\right)h_M\right)}. \tag{2.28}$$

The cost minimizing number of remanufacturing lots R can, thus, be computed by deriving function (2.28) with respect to R. Not surprisingly, this value matches exactly equation (2.10) and is therefore omitted to be presented again. Yet, the reformulation of the total cost function allows to determine the cost minimizing integer value of R. When analyzing function (2.28) in the relevant range ($R > 0$), several characteristics can be derived. First, formula (2.10) proves that there is only a single optimal value for R minimizing the total cost function. Moreover, the total cost function approaches infinity when R moves closer both to zero as well as to $+\infty$[8]. From that it follows that the local minimum determined by (2.10) is a global minimum for the relevant range. Exploiting these characteristics, a general procedure can be applied to determine the cost minimizing integer value R^*. Figure 2.5 depicts the optimal total cost function TC^+_{R1} around its optimal non-integer value R^+. In it, we can observe that R^+ and R^*

[8] For the mathematical proof, please refer to the Appendix, page 71.

are located between $\hat{R}$ and $\hat{R}+1$ which do not have to be integer but have to fulfill the condition $TC^{+}_{R1}(\hat{R}) = TC^{+}_{R1}(\hat{R}+1)$. This means the total cost function yields the same result for both values.

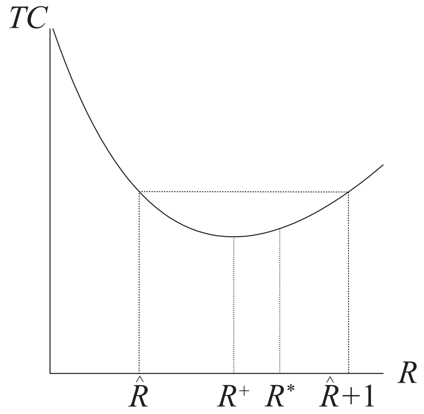

Fig. 2.5: Total cost function TC^{+}_{R1}

There is only one integer value for R between $\hat{R}$ and $\hat{R}+1$. This value must therefore be the cost minimizing integer solution R^*. Consequently, the value of $\hat{R}$ simply needs to be rounded up to compute R^*. In the case that $\hat{R}$ is an integer itself, $\hat{R}$ as well as $\hat{R}+1$ are both cost minimizing. R^* is determined by[9]

$$\begin{aligned} &TC^{+}_{R1}(\hat{R}) = TC^{+}_{R1}(\hat{R}+1) \quad \text{which results in} \\ &R^* = \left\lceil -\frac{1}{2} \pm \sqrt{\frac{1}{4} + \frac{K_M\alpha^2\beta\cdot(h_R + h_M\beta)}{K_R\cdot\left(\alpha\,(1-\alpha\beta)\,h_R + (1-\alpha\beta)^2\,h_M\right)}}\,\right\rceil . \end{aligned} \tag{2.29}$$

Since only a positive number of remanufacturing lots is allowed, an unequivocal value for R^* can be determined. Moreover, the general function $\lceil -0.5 + x \rceil$ describes the same term as if x is rounded to the nearest integer. Thus, the cost minimizing integer number of remanufacturing lots for an $(R,1)$ policy structure is computed by the following value ($\updownarrow$ indicates rounding to the nearest integer)

$$R^* = \sqrt{\frac{1}{4} + \frac{K_M\alpha^2\beta\cdot(h_R + h_M\beta)}{K_R\cdot\left(\alpha\,(1-\alpha\beta)\,h_R + (1-\alpha\beta)^2\,h_M\right)}}\;\updownarrow . \tag{2.30}$$

[9] For details, please refer to the Appendix, page 71.

This value corresponds to the optimal value of R^+ determined by Schrady in equation (2.10) except that a quarter is added to the radicand and the resulting value is rounded to the nearest integer afterwards. The same kind of analysis can be conducted for a $(1, M)$ policy.

For the $(1, M)$ policy structure, the decision variables introduced by Teunter (Q_R and Q_M) are replaced as well by functional expressions depending on the cycle length T and the number of manufacturing lots M. Similar to the adaptations presented above, the (re)manufacturing batch sizes Q_R and Q_M are reformulated according to formulae (2.12) and (2.13) as

$$Q_R(T) = \lambda\alpha T \qquad \text{and} \qquad Q_M(M,T) = \frac{\lambda(1-\alpha\beta)T}{M}. \tag{2.31}$$

By implementing equations (2.31), the reformulation of the setup cost per time unit is facilitated. Analogous to equation (2.15), this results in

$$\frac{K_R + M \cdot K_M}{T}. \tag{2.32}$$

To obtain the holding cost per time unit for a $(1, M)$ policy in the alternative formulation, formulae (2.31) are used to adapt equation (2.16):

$$\begin{aligned} &\left[\frac{1}{2} \cdot Q_R \cdot T \cdot h_R + \left(\frac{1}{2} \cdot \frac{(Q_R \cdot \beta)^2}{\lambda} + M \cdot \frac{1}{2} \cdot \frac{(Q_M)^2}{\lambda}\right) \cdot h_M\right] \cdot \frac{1}{T} \\ = \; &\frac{1}{2}\lambda T \cdot \left(\alpha h_R + \left(\alpha^2\beta^2 + \frac{(1-\alpha\beta)^2}{M}\right) \cdot h_M\right). \end{aligned} \tag{2.33}$$

The total cost per time unit results from the sum of the setup cost (2.32) and holding cost (2.33) per time unit. Hence, we get

$$TC_{1M}(M,T) = \frac{K_R + M \cdot K_M}{T} + \frac{1}{2}\lambda T \cdot \left(\alpha h_R + \left(\alpha^2\beta^2 + \frac{(1-\alpha\beta)^2}{M}\right) \cdot h_M\right). \tag{2.34}$$

In analogy to the procedure for the $(R, 1)$ policy structure, the optimal cycle length T^+_{1M} and the corresponding minimizing total cost function TC^+_{1M} depending only on the number of manufacturing lots M can be determined.

$$\begin{aligned} T^+_{1M}(M) &= \sqrt{\frac{2 \cdot (K_R + M \cdot K_M)}{\lambda \cdot \left(\alpha h_R + \left(\alpha^2\beta^2 + \frac{(1-\alpha\beta)^2}{M}\right) \cdot h_M\right)}} \\ TC^+_{1M}(M) &= \sqrt{2\lambda \cdot (K_R + M \cdot K_M) \cdot \left(\alpha h_R + \left(\alpha^2\beta^2 + \frac{(1-\alpha\beta)^2}{M}\right) \cdot h_M\right)}. \end{aligned} \tag{2.35}$$

The total cost function (2.35) reveals the same characteristics as the total cost function for an $(R,1)$ policy structure, i.e. it has a single minimum and approaches infinity for $M \to 0$ and $M \to \infty$[10]. Therefore, the same methodology can be applied as for the $(R,1)$ policy. Let $\hat{M}$ denote the value of M that needs to be rounded up to obtain the cost minimizing integer number of manufacturing batches in a cycle. We find[11]

$$TC^{+}_{1M}(\hat{M}) = TC^{+}_{1M}(\hat{M}+1) \quad \text{and, thus,}$$
$$M^* = \left\lceil -\frac{1}{2} \pm \sqrt{\frac{1}{4} + \frac{K_R \cdot (1-\alpha\beta)^2 \cdot h_M}{K_M \cdot (\alpha h_R + \alpha^2\beta^2 h_M)}} \right\rceil . \tag{2.36}$$

When comparing the results of the $(R,1)$ policy with the results of the $(1,M)$ policy in equation (2.36), the outcome is quite similar. Thus, the cost minimizing integer number of manufacturing lots in a cycle is computed by adding a quarter to the radicand of Teunter's solution in equation (2.20) and rounding the resulting value to the nearest integer. This means

$$M^* = \sqrt{\frac{1}{4} + \frac{K_R \cdot (1-\alpha\beta)^2 \cdot h_M}{K_M \cdot (\alpha h_R + \alpha^2\beta^2 h_M)}} \updownarrow . \tag{2.37}$$

Deriving closed-form expressions for R^* and M^* has been one of the main results of Minner's contribution. However, both values can never be smaller than 1 (as he presumed) since the radicand is at least 0.25, i.e. its square root is at least 0.5. As this value has to be rounded to the nearest integer afterwards, the optimal values for R^* and M^* are always at least equal to 1.

Concluding, the optimal parameter R^* for an $(R,1)$ policy can be determined by equation (2.30). Likewise, M^* can be computed using (2.37) to get the optimal $(1,M)$ policy. For a given set of parameters, the resulting optimal total cost functions $TC^{+}_{R1}(R^*)$ and $TC^{+}_{1M}(M^*)$ would have to be compared to find the better solution. The next subsection proves that this is not necessary as R^* and M^* cannot exceed a value of one simultaneously when restricting oneself to the $(R,1)$ and $(1,M)$ policies.

[10] We omit to present the mathematical proof as it is similar to the proof for the $(R,1)$ policy.
[11] For details, please refer to the Appendix, page 72.

2.2.5 Comparison of the optimal values for R^* and M^*

At the beginning of this subsection, a small example illustrates the implications when R^* and M^* would not be larger than one at the same time. Assume that by applying formula (2.30) to an exemplary set of parameters, two remanufacturing lots should be initiated when considering an $(R, 1)$ policy structure, i.e. $R^* = 2$. Consequently, M^* would have to be one as the conjecture to be proven states that both R^* and M^* cannot be larger than one simultaneously. Therefore, the best $(1, M)$ policy structure would be a (1,1) policy. This policy is, however, outperformed by the (2,1) policy structure since a (1,1) policy is a possible $(R, 1)$ policy structure as well. Thus, the decision maker would simply need to calculate the optimal values for R^* and M^* using formulae (2.30) and (2.37) to obtain the best policy parameters for the predetermined policy structures. Hence, a comparison of both minimal cost values of the $(R, 1)$ and $(1, M)$ policies could be omitted.

In order to prove the above stated conjecture, two inequalities would have to hold simultaneously. At first, R^* determined by formula (2.30) has to be larger than 1.5 since its value rounded to the nearest integer is consequently greater or equal to two. This gives

$$\sqrt{\frac{1}{4} + \frac{K_M \alpha^2 \beta \cdot (h_R + h_M \beta)}{K_R \cdot \left(\alpha (1 - \alpha\beta) h_R + (1 - \alpha\beta)^2 h_M\right)}} \geq 1.5 \quad \text{and, thus,}$$

$$\frac{1}{2} K_M \alpha^2 \beta \cdot (h_R + h_M \beta) - K_R \cdot \alpha (1 - \alpha\beta) h_R \geq K_R \cdot (1 - \alpha\beta)^2 h_M. \tag{2.38}$$

If condition (2.38) is fulfilled, more than one remanufacturing lot should be initiated in a cycle ($R^* \geq 2$) when applying the $(R, 1)$ policy. In this case, the number of manufacturing lots is set to one due to its predefined policy structure.

Next, the same analysis is put forth for the $(1, M)$ policy by accordingly evaluating condition (2.37). We find

$$\sqrt{\frac{1}{4} + \frac{K_R \cdot (1 - \alpha\beta)^2 \cdot h_M}{K_M \cdot (\alpha h_R + \alpha^2 \beta^2 h_M)}} \geq 1.5 \quad \text{which results in}$$

$$2K_M \cdot \left(\alpha h_R + \alpha^2 \beta^2 h_M\right) \leq K_R \cdot (1 - \alpha\beta)^2 \cdot h_M. \tag{2.39}$$

Condition (2.39) needs to hold if more than one manufacturing lot should be scheduled in a cycle ($M^* \geq 2$) when applying the $(1, M)$ policy. Without loss of generality, the non-strict inequalities are replaced by strict inequalities. If two strict inequalities have to hold at the same time, it is possible to subtract them and analyze the validity of the resulting inequality. This gives

$$\frac{1}{2}K_M\alpha^2\beta\cdot(h_R+h_M\beta)-K_R\cdot\alpha(1-\alpha\beta)h_R-2K_M\cdot\left(\alpha h_R+\alpha^2\beta^2h_M\right)>0$$
$$K_M\alpha\cdot\left(\left(\frac{1}{2}\alpha\beta-2\right)h_R-\frac{3}{2}\alpha\beta^2h_M\right)-K_R\cdot\alpha(1-\alpha\beta)h_R>0. \quad (2.40)$$

As all parameters are positive and $\alpha\beta$ cannot exceed one, the term on the left hand side of inequality (2.40) is always negative. Hence, this inequality is never satisfied, i.e. conditions (2.38) and (2.39) never hold simultaneously. This means, R^* and M^* can never be larger than one at the same time when restricting oneself to the preset $(R,1)$ and $(1,M)$ policies. However, this result is only valid for these two policy structures. Choi et al. (2007) have shown in their work, for instance, that a more general (R,M) policy with both R and M larger than one can reduce the resulting total cost.

After introducing the $(R,1)$ and $(1,M)$ policy structures it has to be mentioned that their solution quality is hardly discussed in literature. Minner and Lindner (2004), for instance, elaborate in their contribution that it might not be optimal to choose remanufacturing lots of equal size in a cycle. This topic is discussed more intensively in the next subsection. There, a third preset policy structure is introduced which allows for different remanufacturing batches in a cycle.

2.2.6 The $(R,1)^g$ policy

When non-equal remanufacturing lots are allowed in a cycle, a multitude of alternative policy structures can be formulated. In their article, Minner and Lindner apply a Lagrange-multiplier approach to investigate the optimality of having remanufacturing lots of equal size when using an $(R,1)$ policy. As a result, they identify three cases which have in common that differently sized remanufacturing batches are initiated within each cycle. The first case is to have $R-1$ remanufacturing lots of equal size which are succeeded by a smaller last one. The second case comprises that all remanufacturing lots in a cycle decrease geometrically. Finally, the third case incorporates a mix of the first two, i.e. a number of equally sized remanufacturing batches are followed by a number of geometrically decreasing ones.

Minner and Lindner restrict their analysis to identifying these three cases. The subsequent analysis focuses, however, only on the second case as this case has a special characteristic. When scheduling geometrically decreasing remanufacturing lots in a cycle, each lot remanufactures all currently available returns. Such a schedule (that fulfills the zero inventory property) is easy to apply and can neither be implemented for a regular $(R, 1)$ policy with equal remanufacturing lots (see, for instance, Figure 2.3) nor for the remaining two cases identified by Minner and Lindner. Figure 2.6 presents the used product and final product inventories in a cycle when two geometrically decreasing remanufacturing lots are initiated. To differ this policy from a regular $(2, 1)$ policy, it is denoted by $(2, 1)^g$ to indicate the geometrically decreasing remanufacturing batches.

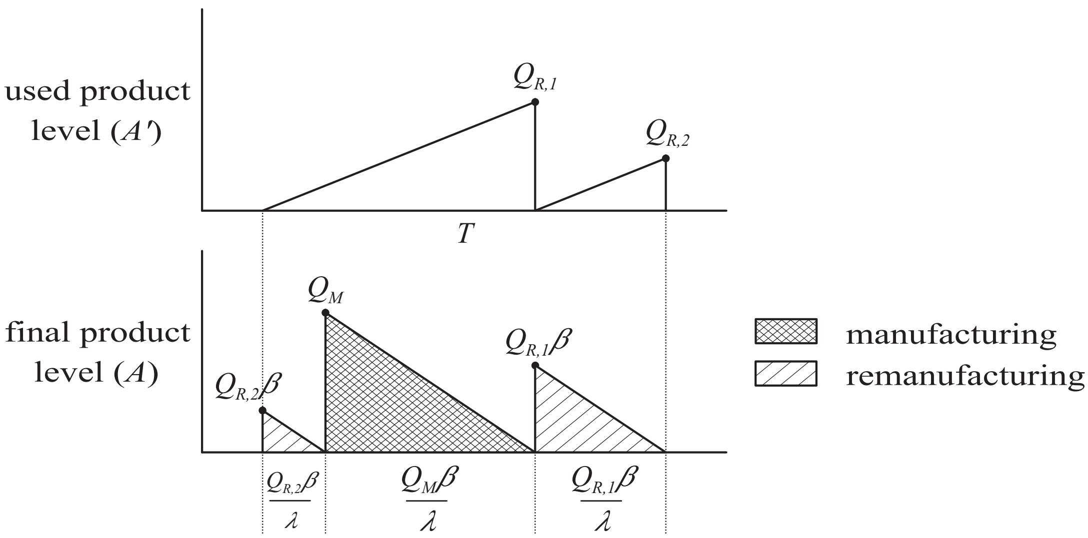

Fig. 2.6: Used product and final product level corresponding to a $(2,1)^g$ policy

By definition, the largest remanufacturing batch in a cycle is denoted by $Q_{R,1}$. It comprehends all products collected by the OEM while the smallest remanufacturing lot (denoted by $Q_{R,R}$) and the manufacturing lot satisfy customer demand. Beginning with the largest remanufacturing batch, each subsequently scheduled lot remanufactures only $\alpha\beta$ of its predecessor's lot size. This fact is illustrated using $Q_{R,1}$ and $Q_{R,2}$ from Figure 2.6. $Q_{R,1}$ satisfies demand for exactly $Q_{R,1} \cdot \beta/\lambda$ time units. During that time interval, the collection of returns for the second remanufacturing lot $Q_{R,2}$ takes place. Over a time span of $Q_{R,1} \cdot \beta/\lambda$ time units $\lambda\alpha$ products are accumulated per time unit. Therefore, as all collected products have to be remanufactured, $Q_{R,2}$ comprises $\lambda\alpha \cdot Q_{R,1} \cdot \beta/\lambda = \alpha\beta \cdot Q_{R,1}$ units. To implement geometrically decreasing lots, two conditions have to be respected. First, as shown previously each remanufacturing lot (except $Q_{R,1}$) remanufactures $\alpha\beta$ of its predecessor's batch size. Second, all returned products must be remanufactured during a cycle, i.e. $\sum_{i=1}^{R} Q_{R,i} = \lambda\alpha T$. Respecting these two conditions, an expression can be derived which describes the size of each remanufacturing lot for the $(R,1)^g$ policy. Hence[12],

$$Q_{R,i} = \frac{\lambda\alpha^i\beta^{i-1}T \cdot (1-\alpha\beta)}{1-\alpha^R\beta^R} \qquad \forall i = 1,..,R. \tag{2.41}$$

After formulating the amount to be remanufactured in each lot, the total cost function is established. The setup cost per time unit can be computed similar to an $(R,1)$ policy structure by the following formula:

$$\frac{R \cdot K_R + K_M}{T}. \tag{2.42}$$

Due to their complexity, the holding cost terms for both inventories are analyzed separately. Starting with the used product inventory and using equations (2.41), the holding cost per time unit for this inventory is formulated as[13]

$$\frac{1}{2}\lambda\alpha T h_R \cdot \left(\frac{1-\alpha\beta}{1+\alpha\beta} \cdot \frac{1+\alpha^R\beta^R}{1-\alpha^R\beta^R}\right). \tag{2.43}$$

Similarly, the holding cost for the final product inventory is computed. The size of the cycle's manufacturing lot corresponds to its size for a regular $(R,1)$ policy, i.e. $Q_M = \lambda(1-\alpha\beta)T$. Therefore, we obtain[14]

$$\frac{1}{2}\lambda T h_M \left(\alpha^2\beta^2 \cdot \frac{1-\alpha\beta}{1+\alpha\beta} \cdot \frac{1+\alpha^R\beta^R}{1-\alpha^R\beta^R} + (1-\alpha\beta)^2\right). \tag{2.44}$$

The total cost per time unit for a $(R,1)^g$ policy is then calculated by summing up the cost components (2.42), (2.43), and (2.44). Until now, the total cost function depends

[12] For details, please refer to the Appendix, page 73.
[13] For details, please refer to the Appendix, page 74.
[14] For details, please refer to the Appendix, page 74.

on both the number of remanufacturing batches R and the cycle length T. It is

$$TC_{R1^g}(R,T) = \frac{RK_R+K_M}{T} + \frac{1}{2}\lambda T\left(\left(\alpha h_R+\alpha^2\beta^2 h_M\right)\cdot\left(\frac{1-\alpha\beta}{1+\alpha\beta}\cdot\frac{1+\alpha^R\beta^R}{1-\alpha^R\beta^R}\right)+h_M\left(1-\alpha\beta\right)^2\right). \tag{2.45}$$

Similar to the approaches presented above, the total cost can be adapted to depend only on the number of remanufacturing lots in a cycle R. For this, the cost minimizing cycle length $T^+_{R1^g}$ needs to be computed and inserted into the total cost function. This gives

$$T^+_{R1^g}(R) = \sqrt{\frac{2\cdot(RK_R+K_M)}{\lambda\left(\left(\alpha h_R+\alpha^2\beta^2 h_M\right)\cdot\left(\frac{1-\alpha\beta}{1+\alpha\beta}\cdot\frac{1+\alpha^R\beta^R}{1-\alpha^R\beta^R}\right)+h_M\left(1-\alpha\beta\right)^2\right)}} \quad \text{and, thus,}$$

$$TC^+_{R1^g}(R) = \sqrt{2\lambda(RK_R+K_M)\left(\left(\alpha h_R+\alpha^2\beta^2 h_M\right)\left(\frac{1-\alpha\beta}{1+\alpha\beta}\cdot\frac{1+\alpha^R\beta^R}{1-\alpha^R\beta^R}\right)+h_M\left(1-\alpha\beta\right)^2\right)}. \tag{2.46}$$

Although depending only on R, no closed-form expression exists to calculate the cost minimizing value of R since it can be found both in the base and exponent of equation (2.46). To obtain a greater insight into the total cost function's behavior, a large set of different problem instances has been created. Without exception, the cost function always had only one cost minimum. Based on this observation, a simple local search method is recommended to determine the optimal value for R.

After formulating the total cost function for an $(R,1)^g$ policy structure, two interesting insights can be derived. At first, the condition required for a $(2,1)^g$ policy to outperform a (1,1) policy is depicted. When setting R equal to one, the total cost function of the $(R,1)^g$ policy matches exactly the total cost function of the $(R,1)$ policy. Therefore, after replacing $\frac{1-\alpha\beta}{1+\alpha\beta}\cdot\frac{1+\alpha^2\beta^2}{1-\alpha^2\beta^2}$ by V, the following condition results[15]

$$\text{from} \quad TC^+_{R1^g}(1) - TC^+_{R1^g}(2) > 0:$$

$$K_R\left(\alpha h_R+\alpha^2\beta^2 h_M\right)(1-2V) - K_R h_M(1-\alpha\beta)^2 + K_M\left(\alpha h_R+\alpha^2\beta^2 h_M\right)(1-V) > 0. \tag{2.47}$$

If condition (2.47) holds, the number of remanufacturing lots scheduled when applying an $(R,1)^g$ policy should be larger than one. The preceding subsection 2.2.5 has proven

[15] For details, please refer to the Appendix, page 75.

that R and M can never be larger than one simultaneously when restricting to the $(R, 1)$ and $(1, M)$ policy structures. An interesting question arises whether the same can be proven for the $(R, 1)^g$ and $(1, M)$ policies. For this to be true, conditions (2.39) and (2.47) must not hold simultaneously. As before, this can be examined by subtracting these inequalities and analyzing the resulting inequality. We find

$$K_R\left(\alpha h_R + \alpha^2\beta^2 h_M\right)(1-2V) - K_M\left(\alpha h_R + \alpha^2\beta^2 h_M\right)(1+V) > 0. \tag{2.48}$$

The resulting inequality (2.48) can never be fulfilled as long as V is always larger than 0.5 since then both terms on the left hand side of (2.48) are strictly negative. The following calculations prove that this is the case[16]:

$$\frac{1-\alpha\beta}{1+\alpha\beta}\cdot\frac{1+\alpha^2\beta^2}{1-\alpha^2\beta^2} > \frac{1}{2} \quad \text{which results in}$$
$$(1-\alpha\beta)^3 > 0. \tag{2.49}$$

Since $\alpha\beta$ always lies between zero and one, $(1-\alpha\beta)^3$ is always positive. Inequality (2.48), thus, is never fulfilled. Therefore, if the local search applied to equation (2.46) computes a cost minimizing value of R larger than one, the best possible $(1, M)$ policy would be a (1,1) structure which is in this case outperformed by the best $(R, 1)^g$ policy.

The $(R, 1)^g$ policy structure cannot only be compared to the $(1, M)$ policy but also to the $(R, 1)$ policy with equal remanufacturing lots. As this cannot be done in general, a condition is derived at which a $(2, 1)^g$ policy outperforms a $(2, 1)$ policy. To do so, the total cost functions (2.28) and (2.46) have to be subtracted for R=2. From[17]

$$TC^+_{R1}(2) - TC^+_{R1^g}(2) > 0 \quad \text{we find}$$
$$\frac{h_M}{h_R} < \frac{3+\alpha\beta}{\beta\left(1-\alpha\beta\right)}. \tag{2.50}$$

Four parameters determine whether a $(2, 1)^g$ policy with geometrically decreasing remanufacturing lots outperforms a (2,1) policy with equal remanufacturing batches: both holding cost parameters and their relation as well as the return and yield fractions α and β. The relation between h_M and h_R influences the result of the analysis substantially. It says that for large values of h_M compared to h_R, equal remanufacturing

[16] For details, please refer to the Appendix, page 75.

[17] For details, please refer to the Appendix, page 75.

batches are preferred. Otherwise, geometrically decreasing remanufacturing batches should be initiated when the ratio between h_M and h_R is comparably small. The exact value is depicted for $R = 2$ in (2.50). Interestingly, the value of the right hand side of (2.50) approaches infinity when α and β move closer to either zero or one. In these settings, geometrically decreasing remanufacturing lots are mostly preferred over lots of equal size. In the following, the right hand side of inequality (2.50) is analyzed in greater detail. By deriving it with respect to α the impact of the return fraction is evaluated. We obtain

$$\frac{\partial \frac{3+\alpha\beta}{\beta(1-\alpha\beta)}}{\partial \alpha} = \frac{4}{(1-\alpha\beta)^2}. \tag{2.51}$$

Since this term is strictly positive, $\frac{3+\alpha\beta}{\beta(1-\alpha\beta)}$ increases if α becomes larger. Hence, a larger return fraction benefits the $(2,1)^g$ policy over the (2,1) structure. The same kind of analysis is also conducted for the yield parameter β

$$\frac{\partial \frac{3+\alpha\beta}{\beta(1-\alpha\beta)}}{\partial \beta} = \frac{-3+6\alpha\beta+\alpha^2\beta^2}{\beta^2\,(1-\alpha\beta)^2}. \tag{2.52}$$

Contrary to the analysis of the return fraction, an ambiguous result is derived for β. While the denominator of (2.52) is positive, the numerator's sign depends on the value of α. If the return fraction α is smaller than $-3+\sqrt{12}$ (around 46.4%), the right hand side of (2.50) decreases continuously as larger β becomes, i.e. a (2,1) policy becomes more attractive as β rises. If, on the other hand, α is larger than $-3+\sqrt{12}$, an increasing β lets the value of $\frac{3+\alpha\beta}{\beta(1-\alpha\beta)}$ fall until it reaches a minimum but begins to rise thereafter until β approaches one.

The results regarding α and β are supported by logical conclusions. Figure 2.7 confronts the result of a (2,1) with a $(2,1)^g$ policy. Both policies represent trade-off solutions when regarding them from an efficiency point of view. Implementing a (2,1) policy, for instance, is the perfect (because least costly) policy with one manufacturing and two remanufacturing batches for the final product level. Yet, the used product level needs to deviate from a good solution by scheduling a remanufacturing lot that does not remanufacture all returned products on hand. Contrary, a $(2,1)^g$ policy accepts a worse solution in the final product stock by allowing to remanufacture in differently sized lots. By doing this, an efficient remanufacturing process from the used product

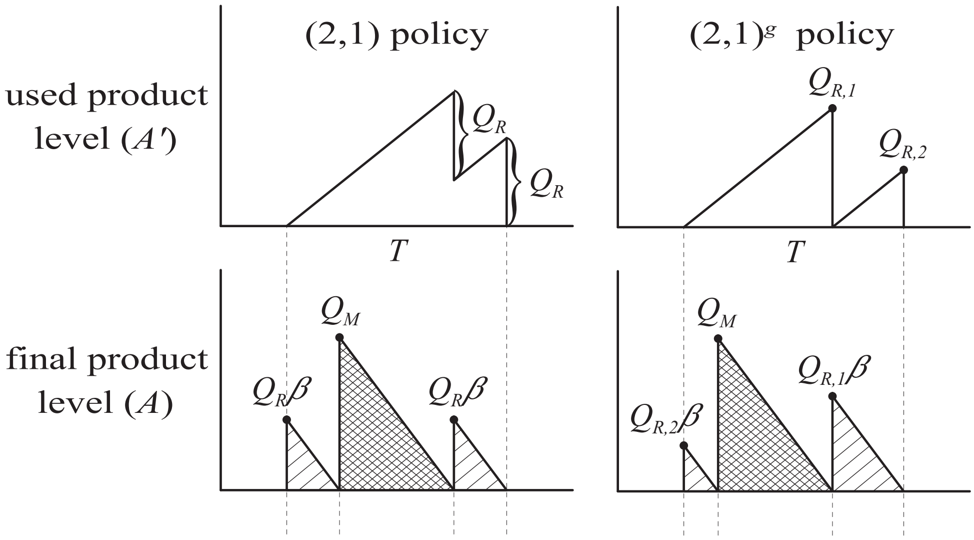

Fig. 2.7: Comparison of a (2,1) policy to a $(2,1)^g$ policy

inventory's point of view is obtained since no return remains in stock after initiating a remanufacturing batch. Hence, if holding returned products in the used product stock is relatively expensive compared to holding finished products in the final product inventory, it becomes more interesting to find an efficient solution for the used product level (geometrically decreasing remanufacturing lots). On the other hand, if holding finished products is very costly compared to holding returned products, the best solution for the final product level is chosen (equal remanufacturing lots).

These conclusions can be used to analyze the results with respect to α and β. When the fraction of returned products per time unit rises, more returns need to be held and, thus, the importance of the used product stock increases. Hence, a $(2,1)^g$ policy becomes more attractive as it focuses on an efficient solution for this inventory level. If the quality parameter β rises, the effects cannot be seen as clearly. An increase in β leads, ceteris paribus, to less manufactured but more remanufactured products to satisfy customer demand. Since this only shifts the batches of the final product level but has no direct influence on the used product level, a logical conclusion cannot be drawn in general.

As mentioned above, comparing a (2,1) to a $(2,1)^g$ policy structure can only give some structural insights since these results cannot be generalized. Yet, similar results as in (2.50) can be derived when comparing other policy structures. In Table 2.1, the conditions for an $(R,1)^g$ policy to dominate an $(R,1)$ policy are presented (for $R \leq 5$).

Tab. 2.1: Comparison of policies with and without remanufacturing lots of equal size

	Condition that needs to hold when $TC^+_{R1}(R) - TC^+_{R1^g}(R) > 0$
For $R = 2$	$\frac{h_M}{h_R} < \frac{3+\alpha\beta}{\beta(1-\alpha\beta)}$
For $R = 3$	$\frac{h_M}{h_R} < \frac{2+\alpha\beta}{\beta(1-\alpha\beta)}$
For $R = 4$	$\frac{h_M}{h_R} < \frac{3\alpha^3\beta^3+9\alpha^2\beta^2+7\alpha\beta+5}{\beta(-3\alpha^3\beta^3-\alpha^2\beta^2+\alpha\beta+3)}$
For $R = 5$	$\frac{h_M}{h_R} < \frac{2\alpha^3\beta^3+4\alpha^2\beta^2+\alpha\beta+3}{\beta(-2\alpha^3\beta^3+\alpha^2\beta^2-\alpha\beta+2)}$

Unfortunately, no bounds can be determined on the maximum error of applying an $(R,1)$ instead of an $(R,1)^g$ policy. To illustrate the complexity of this situation, a small example is presented. For an exemplary parameter set, a policy with one manufacturing and two equal remanufacturing lots is the best option considering all $(R,1)$ and $(1,M)$ policies. For the same parameter set, however, three geometrically decreasing remanufacturing lots are the best solution of all possible $(R,1)^g$ policies, i.e. a $(2,1)$ policy would have to be compared to a $(3,1)^g$ policy for this parameter set. As the interdependence of all policies has to be respected, i.e. general conditions would have to be derived describing which preset policy is the best for each parameter combination, a closed-form expression on the percentage gain cannot be derived. Therefore, a small numerical study is conducted in Section 2.4 to evaluate the $(R,1)^g$ policy with respect to the $(R,1)$ policy structure. In this study, a number of instances revealed a performance gain of more than 5 % when initiating geometrically decreasing instead of equal remanufacturing batches. Yet, no contribution has yet established a methodology to evaluate the performance of the introduced policy structures in general. Hence, the upcoming Section 2.3 establishes a benchmark solution that determines for a given R and M the optimal solution without imposing additional constraints on the lot sizes.

2.3 Establishing a benchmark solution

In order to define a policy structure unambiguously, six decisions need to be determined: the cycle length (T), the number of (re)manufacturing batches (R and M), the sequence of batch scheduling, and the corresponding (re)manufacturing batch sizes (Q_R and Q_M). When establishing a preset policy structure, a number of decisions are fixed in advance. For the $(R,1)^g$ policy, for instance, the number of manufacturing lots per cycle is fixed to one. Furthermore, the batch sequence in a cycle is predefined and the remanufacturing lot sizes are geometrically decreasing. The exact size of the (re)manufacturing batches depends, however, on the not yet known cycle length and the number of remanufacturing lots R. Therefore, by fixing the number of manufacturing lots and assuming a characteristic pattern for all remanufacturing batch sizes, the best $(R,1)^g$ policy is obtained. Likewise, the best $(R,1)$ and $(1,M)$ policy can be determined. The subsequently introduced optimization approach deviates from this procedure as it fixes next to the number (re)manufacturing lots (R and M) also the cycle length. By computing the optimal solutions for a possible set of (R,M) combinations, a benchmark solution can be found.

In general, the total cost of a cycle consists of its total setup cost (SC) that is added to the corresponding total holding cost (HC). As the total cost per time unit (TC) represents the objective of optimization, the sum of both costs has to be divided by the cycle length T:

$$TC = \frac{SC + HC}{T} \tag{2.53}$$

While the setup cost SC depends only on the number of remanufacturing and manufacturing batches in a cycle but not on the cycle length itself, the holding cost HC depends on the cycle length, i.e. $HC = HC(T)$. Obtaining the benchmark solution to this problem exploits the dependency of HC with respect to T. First, by fixing the number of R and M in a cycle, the setup cost is also fixed. Therefore, only the size of the holding cost per cycle needs to be minimized to determine the optimal solution. Interestingly, the holding cost per cycle depends quadratically on the cycle length. This can be explained by the fact that the relative scheduling of remanufacturing and manufacturing batches in a cycle (e.g. remanufacture 20% of all returns after 60% of the cycle has passed) does not depend on its overall length. For instance, if T is doubled all batch sizes are doubled, too. Hence, the time to collect the appropriate returns doubles as well as the time a (re)manufacturing lot is able to satisfy customer demand. Thus, HC is going to be four times its initial value if T is doubled. By defining HC_1 as the holding cost for a cycle length of one time unit, the condition $HC(T) = HC_1 \cdot T^2$ can be established. After inserting this condition into formula (2.53) the optimal cycle length and total cost can be determined by:

$$\begin{aligned} TC &= \frac{SC + HC_1 \cdot T^2}{T} = \frac{SC}{T} + HC_1 \cdot T \\ \Rightarrow T^* &= \sqrt{\frac{SC}{HC_1}} \\ \Rightarrow TC^* &= 2 \cdot \sqrt{SC \cdot HC_1} \end{aligned} \tag{2.54}$$

By fixing the cycle length to one time unit, the optimal batch sequence and the corresponding (re)manufacturing batch sizes can be determined as long as R and M are given. The most interesting aspect of this approach is that no direct relation between the (re)manufacturing batches of a cycle are imposed. In order to calculate the optimal solution for any (R, M) combination, the problem is solved sequentially in two steps. These are:

Step 1: For a given (R, M) combination, minimize HC_1 w.r.t. the lot sequence and (re)manufacturing lot sizes.

Step 2: Compute the optimal total cost and cycle length for HC_1^* using formula (2.54).

To obtain the optimal solution for HC_1, the concept of subcycle-oriented optimization is employed. In this concept, the whole cycle is separated into R subcycles (denoted by s) in which the following presumptions are required to hold. At the beginning of each subcycle the sole remanufacturing lot is initiated. It contains exactly $Q_{R,s}$ items that are remanufactured at once. If the number of remanufactured components is not sufficient to satisfy the subcycle's demand, a number of components (denoted by $\Theta_{M,s}$) have to be manufactured in $\nu_{M,s}$ equal manufacturing lots. All manufacturing lots in a subcycle should be of equal size since deviating from equal manufacturing lots in a subcycle would increase the holding cost incurred. This is shown in the Appendix of Chapter 3, page 107. The individual lot size of a manufacturing lot $Q_{M,s}$ is therefore determined by $\Theta_{M,s}/\nu_{M,s}$. However, it is possible that no new component is fabricated in a subcycle, i.e. $\Theta_{M,s} = 0$. To summarize, each subcycle contains exactly one remanufacturing lot and zero, one, or more manufacturing lots.

To determine the optimal cycle when R and M are given, no further assumptions regarding the (re)manufacturing batches are imposed. This includes the option to have used products left in stock at the end of a subcycle as depicted in Figure 2.3. By including this possibility, not all products available in stock have to be remanufactured at the end of a subcycle. In the following model, V_s denotes the used product inventory level at the end of subcycle s. On the other hand, the final product level has to be depleted at the end of each subcycle. Due to the flexibility in timing and sizing the (re)manufacturing batches, initiating one of these batches before the final product level is empty would increase the holding cost incurred since holding final products is more expensive than holding returns. Figure 2.8 presents a possible solution when the subcycle-oriented optimization approach is applied to a policy structure with three remanufacturing and two manufacturing lots. Without loss of generality, both inventories are set to zero at the beginning/end of a cycle, i.e. $V_R = 0$.

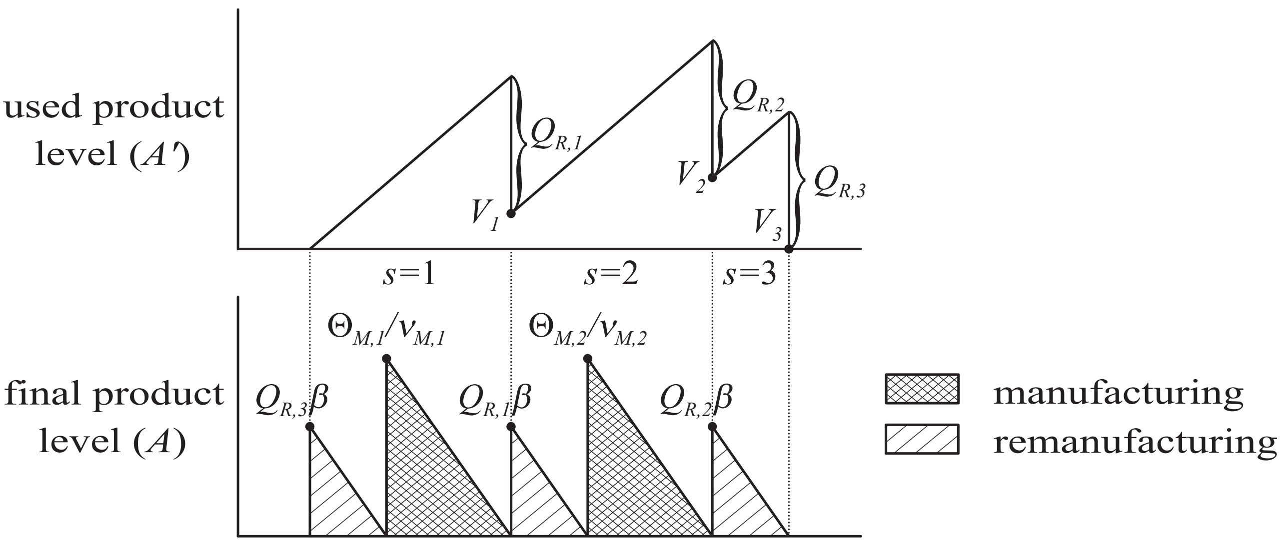

Fig. 2.8: Exemplary cycle with $R = 3$ and $M = 2$

For a given policy structure, the minimal holding cost for a cycle length of one time unit HC_1^* can be determined by the following optimization approach:

$$\min HC_1 = \frac{1}{2\lambda} \cdot \left(\frac{h_R}{\alpha} \cdot \left[(Q_{R,1} + V_1)^2 + \sum_{s=2}^{R} \left[(Q_{R,s} + V_s)^2 - (V_{s-1})^2 \right] \right] + h_M \cdot \sum_{s=1}^{R} \left[(Q_{R,s} \cdot \beta)^2 + \frac{(\Theta_{M,s})^2}{\nu_{M,s}} \right] \right) \tag{2.55}$$

subject to

$$Q_{R,s} = \alpha \left(Q_{R,s-1} \cdot \beta + \Theta_{M,s} \right) - (V_s - V_{s-1}) \qquad \forall s = 2..R \tag{2.56}$$

$$Q_{R,1} = \alpha \left(Q_{R,R} \cdot \beta + \Theta_{M,1} \right) - V_1 \tag{2.57}$$

$$\sum_{s=1}^{R} Q_{R,s} = \lambda\alpha\beta \tag{2.58}$$

$$\sum_{s=1}^{R} \Theta_{M,s} = \lambda (1 - \alpha\beta) \tag{2.59}$$

$$\sum_{s=1}^{R} \nu_{M,s} = \max(R, M) \tag{2.60}$$

$$\sum_{s=1}^{R} \gamma_{M,s} = M \tag{2.61}$$

$$\Theta_{M,s} \leq \lambda\gamma_{M,s} \qquad \forall s = 1..R \tag{2.62}$$

$$\nu_{M,s} \geq 1 \quad \text{and integer} \qquad \forall s = 1..R \tag{2.63}$$

$$\gamma_{M,s} \geq 0 \quad \text{and integer} \qquad \forall s = 1..R \tag{2.64}$$

$$Q_{R,s}, \Theta_{M,s}, V_s \geq 0 \qquad \forall s = 1..R \tag{2.65}$$

The objective function HC_1 (2.55) represents the holding cost of both inventories for a cycle length of one time unit and needs to be minimized. Beginning with the final product level, the relevant area of this inventory has to be calculated which consists of $R + M$ right-angled triangles. Because of the imperfect remanufacturing process, only the fraction β of all remanufactured products meet the requested quality standards to be resold to the customers. Therefore, each subcycle's remanufacturing batch satisfies customer demand for $Q_{R,s} \cdot \beta/\lambda$ time units. On the other hand, the amount to be manufactured in subcycle s is divided into $\nu_{M,s}$ lots of equal size. Each lot contains, thus, $\Theta_{M,s}/\nu_{M,s}$ products to fulfill demand for a period of $\Theta_{M,s}/(\nu_{M,s} \cdot \lambda)$ time units.

Therefore, the area of the final product inventory is computed by

$$\begin{aligned}&\sum_{s=1}^{R}\left[\frac{1}{2}\cdot(Q_{R,s}\cdot\beta)\cdot\frac{(Q_{R,s}\cdot\beta)}{\lambda}+\nu_{M,s}\cdot\frac{1}{2}\cdot\frac{\Theta_{M,s}}{\nu_{M,s}}\cdot\frac{\Theta_{M,s}}{\nu_{M,s}\cdot\lambda}\right]\\&=\frac{1}{2\lambda}\cdot\sum_{s=1}^{R}\left[(Q_{R,s}\cdot\beta)^2+\frac{(\Theta_{M,s})^2}{\nu_{M,s}}\right].\end{aligned}$$

Analyzing the used product level is more complicated due to the possibility of having an initial inventory (V_{s-1}) as well as a final inventory of used products (V_s) in each subcycle s. Therefore, the area to be analyzed can take on a trapezoidal shape when both V_{s-1} and V_s are positive (as for the second subcycle of Figure 2.8). The relevant area is then computed by the general formula $0.5\cdot(a+b)\cdot h$ in which h is the trapezoid's height, and a as well as b represent the lengths of its parallel sides. For subcycle s the parallel sides are V_{s-1} and $Q_{R,s}+V_s$, respectively. The trapezoid's height is equal to the subcycle's duration. As the number of products returning to the OEM in a subcycle is defined by $Q_{R,s}+V_s-V_{s-1}$, a subcycle lasts for $(Q_{R,s}+V_s-V_{s-1})/\lambda\alpha$ time units. The area of the used product inventory can, thus, be computed for each subcycle by:

$$\begin{aligned}&\frac{1}{2}(V_{s-1}+Q_{R,s}+V_s)\cdot\frac{Q_{R,s}+V_s-V_{s-1}}{\lambda\alpha}\\=&\frac{1}{2\lambda\alpha}\left(V_{s-1}Q_{R,s}+V_{s-1}V_s-V_{s-1}^2+Q_{R,s}^2+Q_{R,s}V_s-Q_{R,s}V_{s-1}+V_sQ_{R,s}+V_s^2-V_sV_{s-1}\right)\\=&\frac{1}{2\lambda\alpha}\left((Q_{R,s}+V_s)^2-(V_{s-1})^2\right).\end{aligned}\tag{2.66}$$

Due to the overall cyclic structure, equation (2.66) has to be adapted for the first subcycle. In this case, the predecessor of the first subcycle would be the last subcycle of the preceding cycle. Therefore, equation (2.66) becomes $\frac{1}{2\lambda\alpha}\cdot(Q_{R,1}+V_1)^2$ for $s=1$ since V_R has been fixed to zero. By multiplying each area with the respective holding cost parameter and summing up over all subcycles, the objective function (2.55) is established. It represents for a given policy structure (i.e. R and M are preset) the holding cost HC_1.

In order to guarantee feasibility of the solution, constraints (2.56) to (2.65) have to be met. The restrictions in (2.56) represent the inventory balance constraints of the used product level. They describe the inventory at the end of subcycle s (V_s) as the inventory at its beginning (V_{s-1}) plus its inflows and minus its outflows. The inflows include all used products arriving in this subcycle. As subcycle s has a length of $(Q_{R,s-1} \cdot \beta + \Theta_{M,s})/\lambda$ time units and $\lambda\alpha$ used products arrive per time unit, altogether $\alpha \cdot (Q_{R,s-1} \cdot \beta + \Theta_{M,s})$ used products reach the OEM in subcycle s. The outflows are computed by the remanufacturing lot initiated at the end of subcycle s which comprises $Q_{R,s}$ products to be remanufactured. Using these inventory balance equations, one can derive constraint (2.56) after the following manipulation

$$
\begin{aligned}
V_s &= V_{s-1} + \alpha \cdot (Q_{R,s-1} \cdot \beta + \Theta_{M,s}) - Q_{R,s} \\
Q_{R,s} &= \alpha \left(Q_{R,s-1} \cdot \beta + \Theta_{M,s}\right) - \left(V_s - V_{s-1}\right).
\end{aligned}
$$

Constraint (2.57) has to be incorporated to reflect the cyclic structure of the underlying problem, i.e. a cycle's last subcycle is the predecessor of the successive cycle's first subcycle. Constraint (2.58) guarantees that all products returning during a cycle are remanufactured. Since demand cannot be met solely by remanufacturing, restriction (2.59) assures the missing components to be manufactured.

To apply the subcycle-oriented optimization approach, the number of remanufacturing and manufacturing lots has to be fixed in advance. As R can be smaller than M, not all subcycles have to include a manufacturing lot. If, for instance, no manufacturing lot is scheduled in subcycle s, the value $\nu_{M,s}$ will be zero which would make the objective function infeasible (division by zero). In this case, constraints (2.60) to (2.64) ensure that no new product is fabricated, i.e. $\Theta_{M,s} = 0$. Moreover, $\nu_{M,s}$ is forced to be equal to one to avoid division by zero in the objective function. Forcing $\Theta_{M,s}$ and $\nu_{M,s}$ to zero and one, respectively, can be achieved by introducing another integer decision variable $\gamma_{M,s}$ that decides whether a subcycle contains a manufacturing lot or not. If not, $\gamma_{M,s}$ is zero and constraint (2.62) restricts $\Theta_{M,s}$ to be zero. Otherwise, if a subcycle contains at least one manufacturing lot, $\gamma_{M,s}$ can take on any positive integer value as it does not affect the objective function. Yet, restriction (2.61) ensures the sum of $\gamma_{M,s}$ over all subcycles to equate M. This combined with constraint (2.60) guarantees that at least $R - M$ (if $R > M$) subcycles do not contain a manufacturing batch. Constraints (2.63) to (2.65) restrict all decision variables to non-negative values. While this is sufficient for $Q_{R,s}$,$\Theta_{M,s}$, and V_s, the remaining variables $\nu_{M,s}$ and $\gamma_{M,s}$ have to be integer in addition. Furthermore, $\nu_{M,s}$ must be greater or equal to one to ensure validity.

The subcycle-oriented optimization approach can be applied to a multitude of policy structures to compute a benchmark solution. In order to do that, a number of non-linear optimization problems have to be solved. Although all constraints are linear, the objective function is non-linear because of the $\nu_{M,s}$ decision variables in its denominator. Therefore, a standard linear solver cannot be used to generate the benchmark solution. Instead, the software package GAMS provides a number of solvers that can handle non-linearity quite efficiently. To determine the benchmark solution, the mixed-integer non-linear programming solver SBB has been applied. This solver uses a combination of the Branch&Bound methodology known from linear programming combined with one of the GAMS NLP solvers (for further details on the SBB solver please refer to SBB, 2009). With respect to time and solution quality, the NLP solver CONOPT3 worked best for this problem setting (see Drud, 2009, for additional details). As no NLP solver can guarantee to find the optimal solution to the NLP relaxations (the integrality constraint is relaxed), it cannot be proven that an optimization run provides the true optimal solution to a problem. Nevertheless, it is possible to compare the results of the predefined policy structures $(R,1)$, $(1,M)$, and $(R,1)^g$ with this benchmark solution to get an idea on their performance. To evaluate the potential benefits the benchmark solution is able to offer in comparison to the predefined policy structures, a numerical study is presented in the following Section 2.4.

2.4 Numerical study

Comparing the predefined policy structures with the benchmark solution in a numerical study requires appropriate test instances. Rardin and Uzsoy (2001) describe four different options on how to generate test instances properly. First, they name real world data sets as a viable source of information. Data sets taken from real applications promise the most realistic evaluation of the tested algorithms as all conclusions drawn from the experiments can be almost directly transferred to the real application. However, there are several pitfalls concerning real world data sets. For example, gathering this kind of data can be extraordinarily difficult. In our problem context, estimating exact setup and holding cost parameters is sometimes a challenging task in a real-life environment. Furthermore, as there is only a limited number of real-life problems, the considered algorithms cannot be tested extensively with a large number of different parameter sets. Hence, Rardin and Uzsoy name random variants of real world data sets as a second source of generating problem instances. By maintaining most of the structural properties, the random variation of one or several parameters avoids the pitfall of not having enough real-life parameter sets. If no practical data is available at all, the third option of exploiting published and online libraries becomes interesting. Although being a rich source of different test instances for some problem settings, it may occur that a new algorithm is proposed performing only well on these instances. Thus, it must be ensured that a large number of different instances is tested with a new algorithm. Finally, a random instance generation provides the simplest and fastest way to generate a vast number of test instances. This fourth option becomes interesting when none of the other options (exclusive or combined) is able to establish a comprehensive set of experiments.

The numerical study conducted in this section uses the second methodology, variation of real world data. Yet, regarding this problem setting there are only a few contributions in literature presenting practical data. Tang and Teunter (2006), for instance, analyze the operations of a company that (re)manufactures water pumps for diesel engines. They provide data on five different types of water pumps including setup and holding cost parameters. In another work, Ashayeri et al. (1996) present the case of remanufacturing computers. As well, they illustrate a practical example including setup and holding cost parameters. Their example is taken as a base case scenario in this section. To evaluate the influence of all parameters, the base case scenario is modified in a sensitivity analysis afterwards. Of course, the data published in Tang and Teunter could have been taken as well for a base case scenario. At the end of this section, a short analysis of this data set is presented and compared to the results of the base case. Interestingly, the important parameter constellation describing the ratio of h_M to h_R is the same in both contributions, i.e. holding a final product is twice as expensive as holding a used product in the corresponding inventory. However, both contributions discuss that determining h_R is especially difficult for practical applications.

Base case scenario of Ashayeri et al.

Ashayeri et al. present the following parameters which have been used for the base case scenario. The OEM faces a constant and continuous demand of 100 products per time unit which comprises in this case 3 days. Initiating a remanufacturing batch costs 50 Dutch guilders while setting up a manufacturing lot is a little more expensive with 150 guilders. Holding a used computer for three days costs 1 guilder, while holding a new or remanufactured computer costs 2 guilders. For the sake of simplicity, the currency is omitted in the following analysis. Over the infinite planning horizon, 60 % of the demand per time unit is returned to the computer remanufacturer. As Ashayeri et al. did not include a possible yield loss from remanufacturing, they assumed that all returns can be successfully remanufactured. In order to incorporate an imperfect remanufacturing process, we set β to 80% for the base case scenario. This change to the original Ashayeri et al. scenario can be imposed as this parameter is altered later on to observe its influence on the performance of the preset policy structures compared to the benchmark solution. Table 2.2 summarizes all base case parameters.

Tab. 2.2: Base case parameters

λ	α	β	K_R	K_M	h_R	h_M
100	60 %	80 %	50	150	1	2

Applying equations (2.30) and (2.37) provides the cost minimizing parameters R^* and M^* for the predefined $(R, 1)$ and $(1, M)$ policy structures. We obtain

$$M^* = \sqrt{\frac{1}{4} + \frac{K_R \cdot (1-\alpha\beta)^2 \cdot h_M}{K_M \cdot (\alpha h_R + \alpha^2\beta^2 h_M)}} \updownarrow = 0.648 \updownarrow = 1 \quad \text{and}$$

$$R^* = \sqrt{\frac{1}{4} + \frac{K_M \alpha^2 \beta \cdot (h_R + h_M \beta)}{K_R \cdot \left(\alpha (1-\alpha\beta) h_R + (1-\alpha\beta)^2 h_M\right)}} \updownarrow = 1.698 \updownarrow = 2.$$

While a $(1, 1)$ policy is the cost minimizing of all $(1, M)$ policies for the base case scenario, the best $(R, 1)$ policy structure would be a $(2, 1)$ policy. As elaborated in Section 2.2.5, the $(2, 1)$ policy yields a better solution than the (1,1) policy when

$R^* = 2$ and $M^* = 1$. Thus, by using equations (2.27) and (2.28) the optimal cycle length $T^*_{R1}(2)$ and the optimal total cost $TC^*_{R1}(2)$ are computed

$$T^*_{R1}(2) = \sqrt{\frac{2 \cdot (R \cdot K_R + K_M)}{\lambda \cdot \left(\left(1 + \alpha\beta\left(\frac{1}{R} - 1\right)\right) \cdot \alpha h_R + \left(\frac{\alpha^2\beta^2}{R} + (1-\alpha\beta)^2\right) \cdot h_M\right)}}$$

$T^*_{R1}(2) = 2.0185$ time units ≈ 6 days

$$TC^*_{R1} = \sqrt{2\lambda \cdot (RK_R + K_M) \cdot \left(\left(1 + \alpha\beta\left(\frac{1}{R} - 1\right)\right)\alpha h_R + \left(\frac{\alpha^2\beta^2}{R} + (1-\alpha\beta)^2\right) h_M\right)}$$

$TC^*_{R1}(2) = 247.71$

Regarding the predefined $(R,1)^g$ policy with geometrically decreasing remanufacturing batches, the cost minimizing number of remanufacturing lots R^* is determined by equation (2.46). As no closed-form expression exists to compute R^*, a local search procedure has been proposed to determine this value. This procedure begins to compute the total cost for $R = 1$. Thereafter, the total cost value is computed for $R+1$ until the total cost increases for the first time. When this happens, the local search procedure terminates. The total cost function for an $(R,1)^g$ policy is

$$TC^*_{R1^g} = \sqrt{2\lambda\left(RK_R + K_M\right)\left(\left(\alpha h_R + \alpha^2\beta^2 h_M\right)\left(\frac{1-\alpha\beta}{1+\alpha\beta} \cdot \frac{1+\alpha^R\beta^R}{1-\alpha^R\beta^R}\right) + h_M\left(1-\alpha\beta\right)^2\right)}$$

Table 2.3 presents the results for all R values between one and five. It can be seen that the total cost value for R=2 is the smallest with 238.40 and that it constantly increases for $R \geq 2$. Therefore, the best $(R,1)^g$ policy is a $(2,1)^g$ policy.

Tab. 2.3: Total cost values for the base case scenario for $1 \leq R \leq 5$

R=1	R=2	R=3	R=4	R=5
253.11	238.40	245.71	258.60	273.20

By switching to geometrically decreasing remanufacturing lots, a cost saving of around 3.91% ($\frac{247.71-238.40}{238.40}$-1) is realized. The relevant decision variables are summarized in Table 2.4. The optimal cycle length for the $(2,1)^g$ policy is a little longer than for the

(2,1) policy with remanufacturing lots of equal size. When applying a $(2,1)$ policy all returns in a cycle are remanufactured in two equal lots, i.e. $Q_{R,1} = Q_{R,2} = 0.5 \cdot \lambda\alpha T$. On the other hand, the remanufacturing lot sizes of the $(2,1)^g$ policy are geometrically decreasing and can be computed according to equations (2.41). For both policy structures, the only manufacturing lot in a cycle comprises exactly $\lambda(1-\alpha\beta)T$ newly fabricated products.

Tab. 2.4: Decision variables for the $(2,1)$ and $(2,1)^g$ policy structures

	(2,1) policy	$(2,1)^g$ policy
T	2.0184	2.0973
$Q_{R,1}$	60.552	85.0257
$Q_{R,2}$	60.552	40.8123
$Q_{M,1}$	104.9568	109.061

We omit to present the results of the $(2,1)$ and $(2,1)^g$ policies graphically as they correspond to the inventory developments of Figure 2.7. In order to evaluate whether these policies obtained good solutions, the benchmark solution to the base case has been calculated as well. Altogether, 100 different combinations of R and M have been analyzed in which each parameter could take on integer values between 1 and 10. The result has been that the benchmark solution obtained a solution corresponding to the $(2,1)^g$ policy's solution and is, thus, not able to improve the solution obtained by the predefined policies.

The remainder of this subsection presents the results of a one parameter sensitivity analysis. While keeping six of the seven parameters (please refer again to Table 2.2 for a short overview) constant, the residual parameter is altered in a reasonable range. This sensitivity analysis is conducted for all parameters except λ since this parameter does not influence the solution structure which can be seen in the benchmark solution's objective function (2.55). At first, the influence of the return fraction α is examined.

Return fraction α

The fraction of used products returning to the OEM α can vary theoretically between 0 % and 100%. As the extreme values do not seem to be reasonable since the entire demand would be satisfied by either remanufacturing or manufacturing only, the sensitivity analysis considers all α values between 1 % and 99 % in steps of 0.5%. The three predefined policy structures $(R, 1)$, $(1, M)$, and $(R, 1)^g$ have been tested with this data and the minimum total cost value for each preset policy structure is presented graphically in Figure 2.9. There, the best preset policy structure is indicated below the minimum total cost of all policies.

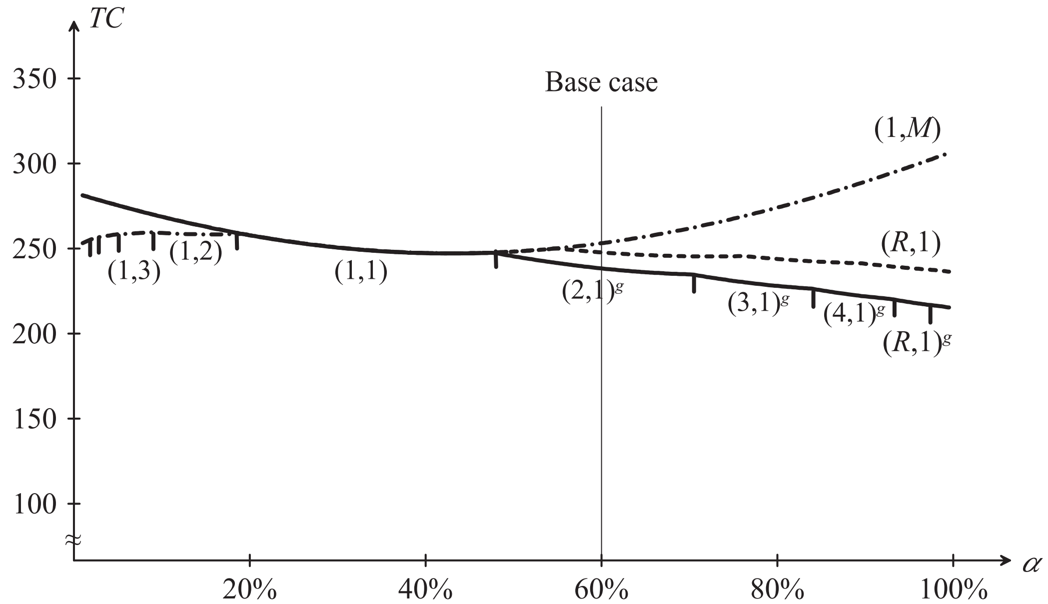

Fig. 2.9: Minimum total cost of the preset policy structures for different α values

For small values of α ($\alpha \leq 19\%$), the $(1, M)$ policy dominates the structures that propose to initiate more than one remanufacturing lot in a cycle. If α is low, there are not many returns to remanufacture. Hence, a large fraction of customer demand needs to be satisfied by manufacturing product A. Since all returned products have to be remanufactured as the option to dispose them of is prohibited, the cycle length is quite long to collect a sufficient number of returns to remanufacture in a single batch. Thus, the total amount to be manufactured increases which lets the number of manufacturing lots become larger in a cycle as well. By exploiting condition (2.39) the exact value of α is calculated at which the optimal number of manufacturing lots switches from two to one. When α is smaller than 19.18% it is better to schedule two manufacturing lots instead of one in a cycle[18].

For α between 19.18 % and around 48 % all preset policies determine the same minimum total cost. In this range, a $(1, 1)$ policy is the best choice for all preset policy structures, i.e. they coincide. For α larger than 48 % the $(R, 1)^g$ policy dominates the other policy structures. Therefore, condition (2.47) provides the exact α value for which an $(R, 1)^g$ policy begins to outperform both the $(1, M)$ and $(R, 1)$ policies. By applying the bisection method to this method an exact α of 47.65 % has been computed.

In order to evaluate whether the total cost values determined by the preset policy structures might be far from optimal, the benchmark solution has been obtained for all problem instances as well. Since the optimization approach requires that R and M are set in advance, the number of examined combinations has to be limited to keep the computational effort controllable. Altogether, the mixed-integer non-linear optimization problem has calculated the benchmark solution for 36 policy structures where R as well as M could take on any integer value between 1 and 6. By restricting the number of combinations and as no NLP solver can guarantee to provide the true optimal solution to a problem, it cannot be guaranteed to find the optimal solution to the entire problem. However, this approach offers an opportunity to evaluate the performance of the preset policies on a general level what is not found in literature up to now.

The benchmark solution is able to improve the preset policies' solutions in some cases but not in general. In order to elaborate the influence of equal remanufacturing lots, the benchmark solution is compared on the one hand to the minimum total cost of the $(R, 1)$ and $(1, M)$ policies. On the other hand, the benchmark solution is confronted with the best result from the $(R, 1)$, the $(1, M)$, and the new $(R, 1)^g$ policy. Figure 2.10 presents these results.

[18] For details on how to determine this value, please refer to the Appendix, page 76.

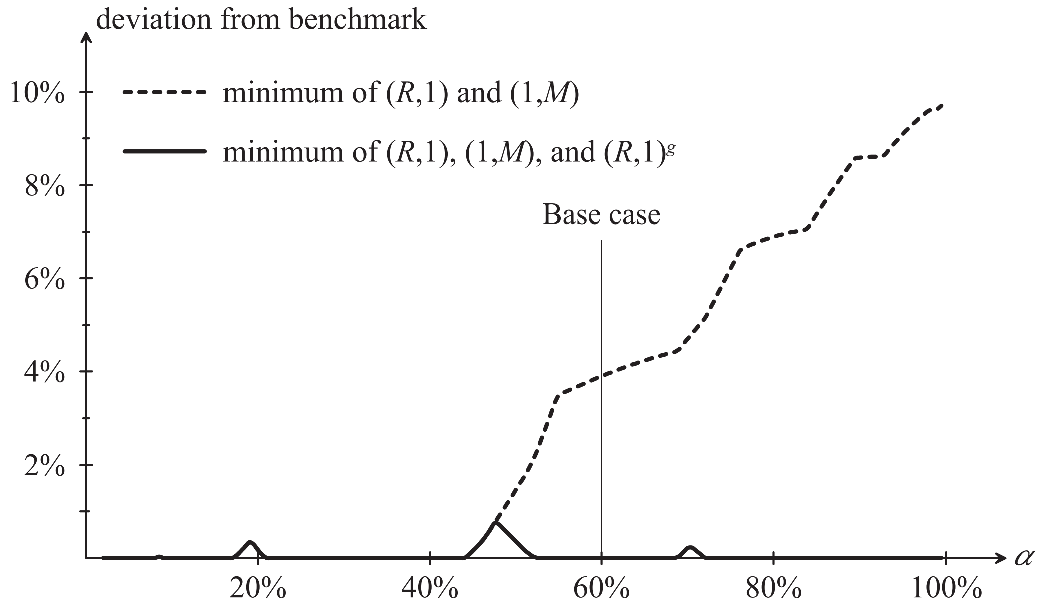

Fig. 2.10: Deviation from benchmark solution for different α values

Applying the $(R, 1)$ and $(1, M)$ policy structures leads to an error of more than 9% for large return fractions when compared to the benchmark. Furthermore, by including the $(R, 1)^g$ policy structure into the decision making process, the deviation from the benchmark solution can be limited to less than 1% over all instances tested when varying α for the base case. For instance, the maximum deviation of the best preset policy structure to the benchmark solution has been around 0.75% for a return fraction of 47.5%. For this return fraction, the benchmark solution proposed a (3,2) policy while a $(1, 1)$ policy has been the best suggestion by the preset policies. The proposed cycles of the $(1, 1)$ and $(3, 2)$ policies are depicted in Figure 2.11 while their relevant decision variables are presented in Table 2.5.

The most striking difference between both solutions is their divergent cycle length. It can be seen that the $(1, 1)$ policy's cycle length is much shorter than the cycle length for the benchmark $(3, 2)$ policy. This is because the benchmark solution needs to divide the much larger setup cost of scheduling and therefore needs a longer cycle to do this efficiently. Comparing the total cost values between both solutions, the $(1, 1)$ policy obtained a total cost of 247.59 per time unit while the $(3, 2)$ structure is able to reduce the total cost to 245.76 per time unit. The relative deviation between both values is, thus, around 0.75%. When analyzing the benchmark in greater detail, the $(3, 2)$

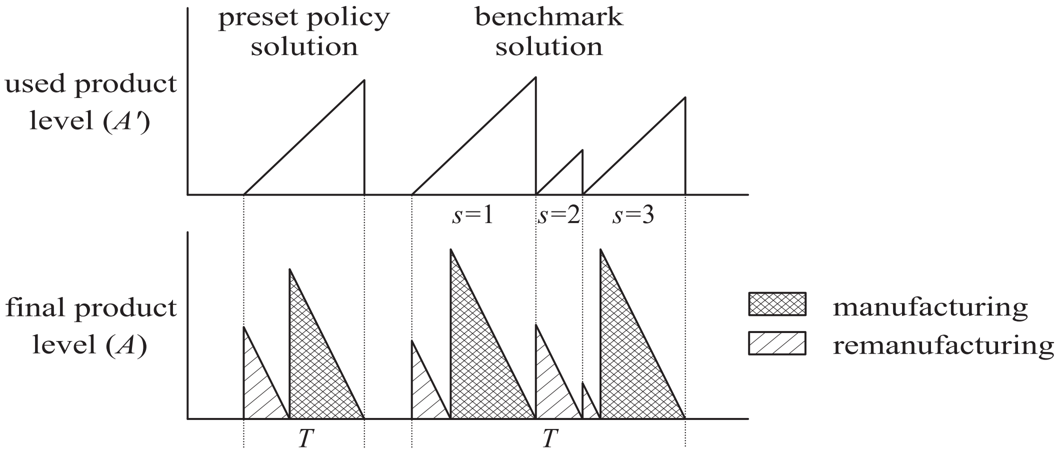

Fig. 2.11: $(1, 1)$ and $(3, 2)$ policies for base case with α=0.475

Tab. 2.5: Decision variables for the $(1, 1)$ and $(3, 2)$ policies

	preset policy structure $(1, 1)$ policy	benchmark solution $(3, 2)$ policy
TC	247.59	245.76
T	1.6155	3.6621
$Q_{R,1}$	76.7363	78.7352
$Q_{R,2}$	/	29.9194
$Q_{R,3}$	/	65.2952
$Q_{M,1}$	100.161	113.5251
$Q_{M,2}$	/	/
$Q_{M,3}$	/	113.5251

solution can be separated into two smaller cycles. The first smaller cycle consists of $Q_{R,3}$, $Q_{R,1}$, and $Q_{M,1}$ which coincide with a $(2, 1)^g$ policy structure, i.e. $Q_{R,2} = \alpha\beta \cdot Q_{R,1}$. Thereafter, $Q_{R,2}$ and $Q_{M,3}$ correspond to a $(1, 1)$ policy. Moreover, the manufacturing lots of both smaller cycles are of equal size.

Interestingly, the deviation of the best preset policy to the benchmark solution follows a characteristic pattern. For a multitude of instances, at least one of the preset structures obtains the benchmark solution. This might not be the case when the overall optimal solution to each instance could be obtained which is not possible due to its computational complexity. However, in some areas the benchmark solution is already better than the best preset policy structures of which four areas can be identified in Figure 2.10. Although hardly recognizable, the first return fraction for which this happens is 8.5%. Moreover, around the return fractions 18%, 45%, and 70% the other deviations can be found.

Without loss of generality, either the ratio of R to M or its inverse is an integer number for all preset policy structures as either R or M is one. Yet, if the benchmark solution deviates from these policy structures, both the ratio of the benchmark's R and M as well as its inverse are not integer. This fact has been depicted in Figure 2.12 which exhibits the benchmark solution's ratio of R to M depending on α.

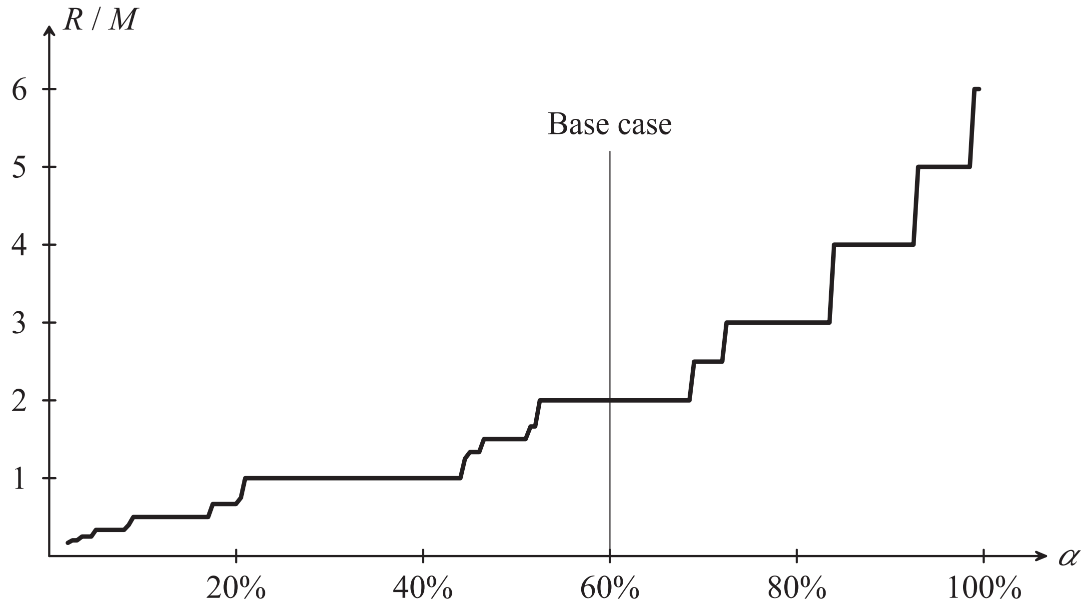

Fig. 2.12: Ratio of R to M for the benchmark solution

It can be seen that the ratio of R to M never decreases when α becomes larger. For instance, around $\alpha = 45\%$ the value of this ratio is between one and two. Due to the experimental design (restricting the maximum value of R and M to six) only five different policy structures can be found that show an R/M ratio between one and two: the (3,2) [and therefore also the (6,4) policy which yields the same result], (4,3), (5,3), (5,4), and (6,5) policies. Except for the last one, all policy structures have been chosen by the benchmark solution for at least one α value. When varying the other parameters in the remaining sensitivity analysis, the same general monotonic behavior can be observed. This leads to the conjecture that the ratio of R to M for the exact solution increases monotonically when α increases. However, this conjecture cannot be proven since it is unlikely to determine the optimal solution, e.g., for a (201, 87) policy with the currently existing optimization software. Next, the influence of the yield parameter β is analyzed.

Yield parameter β

While keeping the remaining base case parameters constant, the fraction of successfully remanufactured products β is altered in the following. This parameter has not been given by Ashayeri et al. and is, thus, of special interest. Like the return fraction, β could be changed between 0 and 100 %. Yet, for the experiments β is alternated between 1 and 100 % in steps of 0.5 %. A β smaller than one does not seem to be reasonable since then almost the entire demand would have to be satisfied by manufacturing new products. Figure 2.13 presents the solutions of the preset policy structures.

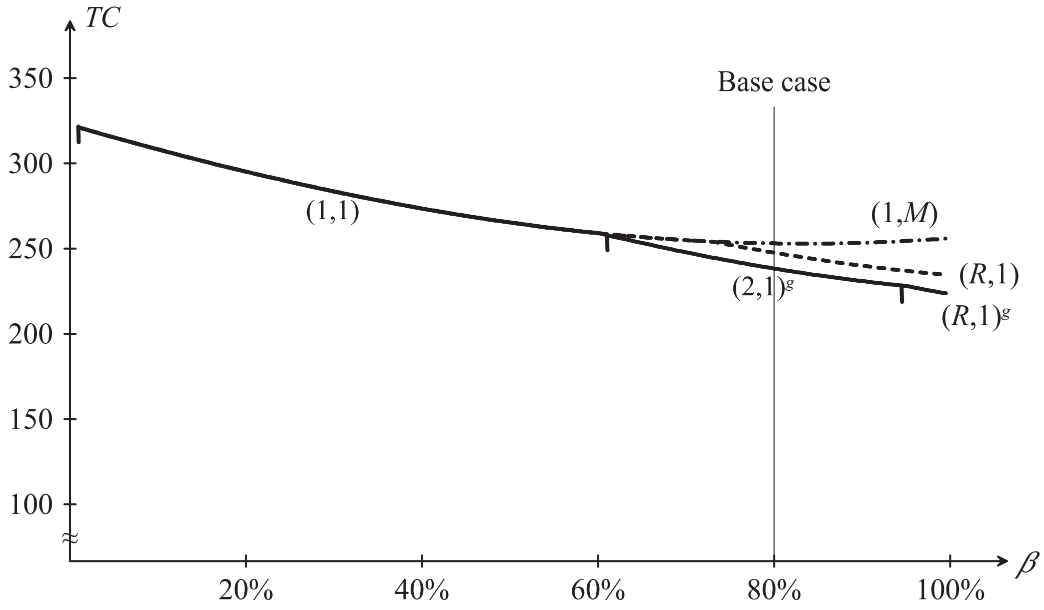

Fig. 2.13: Minimum total cost of the preset policies for different β values

For β smaller than around 60% all preset policies determine the same result, i.e. a (1,1) policy is suggested. As β become larger, the preset policy structures differ in their evaluation. Again, the $(R, 1)^g$ policy outperforms both the $(R, 1)$ and $(1, M)$ policies. By analyzing condition (2.47) the exact value of β is determined at which the $(R, 1)^g$ policy's solution begins to yield a better result than the other two structures. Using the bisection method again derives a β of 60.17%.

To compare the solutions of the preset policy structures to the benchmark solution, Figure 2.14 depicts the percentage error between both methodologies. Interestingly, the performance gain is largest for $\beta = 100\%$ which has been the original assumption of Ashayeri et al. Therefore, the declaration of an imperfect remanufacturing process as base case scenario has been justified. Otherwise, the influence of a large β appears

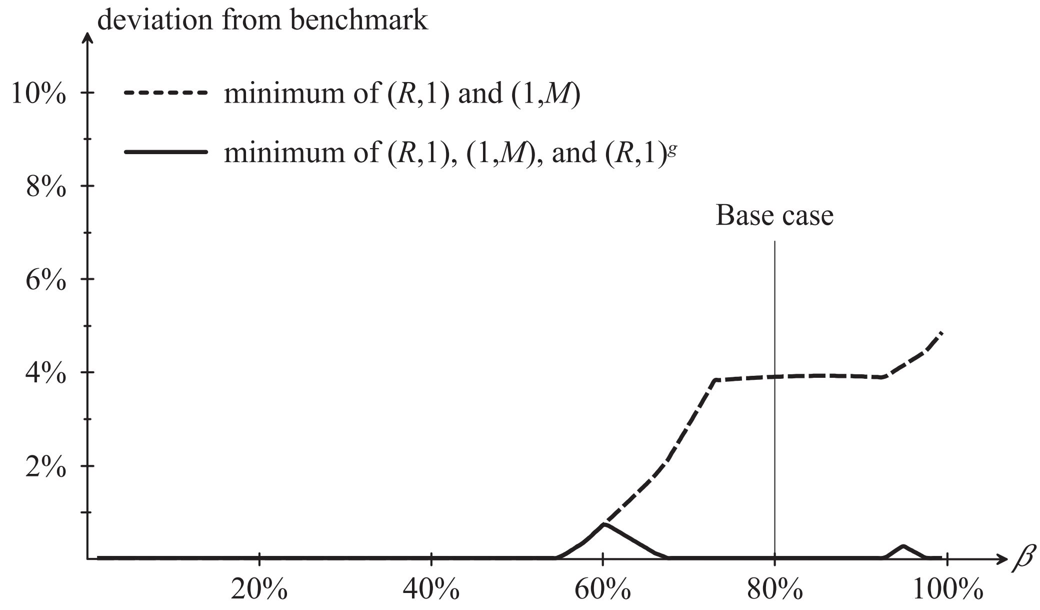

Fig. 2.14: Deviation from benchmark solution for different β values

not to be as strong as for a large return fraction α although a percentage error of more than 5 % compared to the benchmark solution can be observed when the decision maker omits to check the $(R, 1)^g$ policy's solution. However, including the $(R, 1)^g$ policy's solution does not always provide the best solution. In this analysis, there are two regions in which the benchmark solution is better than the preset structures. The first area can be found around β=60% at which the best preset policy structure changes from a (1,1) policy to a $(2, 1)^g$ policy. Here, as well as for the return fraction α, the benchmark obtains a solution for which the ratio of R to M lies between one and two. Consequently, the area around $\beta = 95\%$ shows the transition from a $(2,1)^g$ to a $(3,1)^g$ policy. In the following, the effects of diverse holding cost values are examined.

Holding cost parameters h_R *and* h_M

Regarding the holding cost parameters h_R and h_M, not only the absolute values are of importance but also the ratio of both values. It has been observed in Table 2.1, for instance, that the ratio of the holding cost parameters determines whether an $(R,1)^g$ policy with geometrically decreasing remanufacturing lots finds a better solution than an $(R,1)$ policy with equal remanufacturing lots. At first, the influence of the holding cost for the used product inventory is examined. As h_R must not exceed βh_M, it has been chosen to take on values between 0 and 1.6 in steps of 0.01. Figure 2.15 presents the best solutions obtained by the preset policy structures when h_R is altered. In

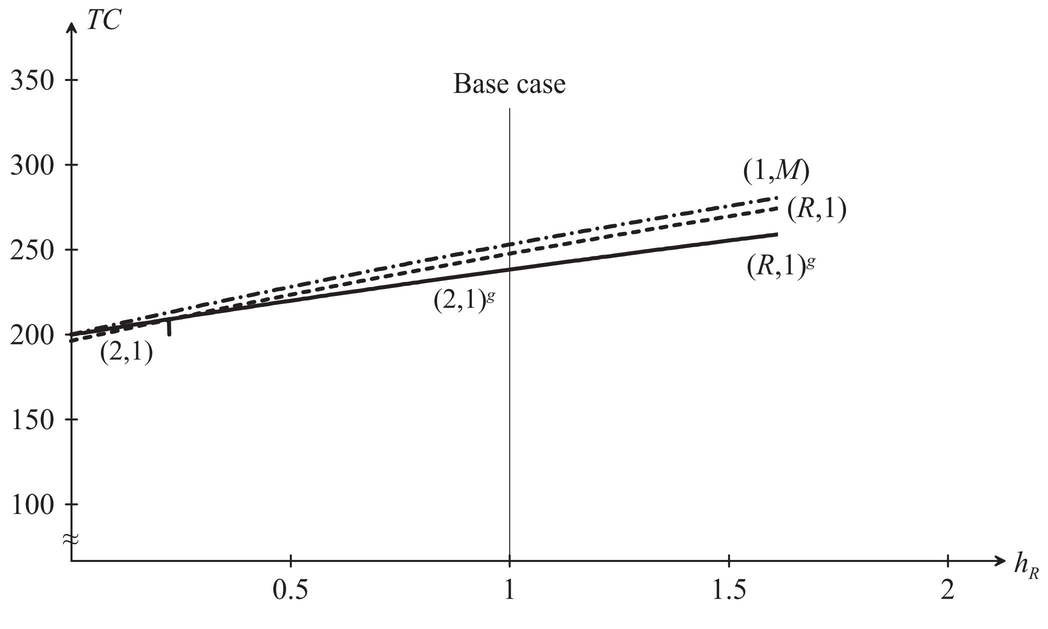

Fig. 2.15: Minimum total cost of the preset policies for different h_R values

contrast to both Figure 2.9 for the return fraction α and Figure 2.13 for the quality parameter β all preset policies determine a different solution, i.e. a (1,1) policy has never been the best proposed solution. Instead, the $(R,1)^g$ policy dominates both competitors for most of the h_R values except for some small ones. There, the $(R,1)$ policy is the best alternative. Considering the exact value of the change, it must be noticed that the switch takes place from a (2,1) to a $(2,1)^g$ policy. Therefore, the value of h_R can be computed by exploiting condition (2.50) which gives an h_R of 0.2391. This means if the holding cost rate for the used product level drops below 0.2391 (which is around 12 % of the final product level's holding cost), a policy structure with equal remanufacturing batches is preferable. As initiating equally sized remanufacturing lots reduces the inefficiency in the final product's stock, it is reasonable to take inefficiencies in the used product level into account when the holding cost h_R is comparatively low.

The percentage error when compared to the benchmark solution is presented in Figure 2.16. For h_R smaller than 0.2391 both curves are identical as including the $(R,1)^g$

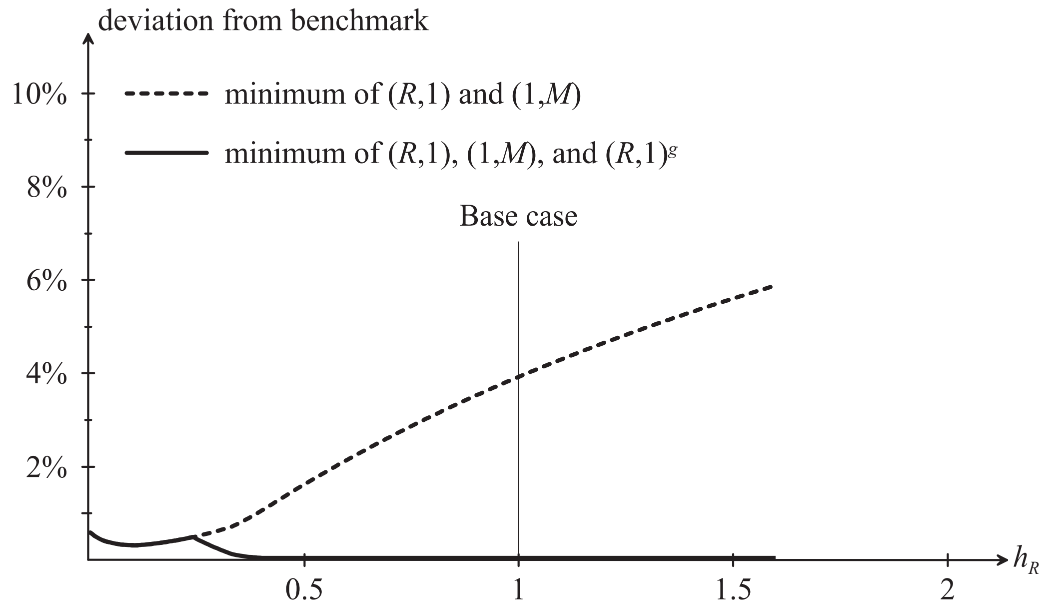

Fig. 2.16: Deviation from benchmark solution for different h_R values

policy into the decision making process does not yield any benefit. However, if h_R approaches βh_M the benefit becomes larger until it reaches around 6 % when they are almost identical. Furthermore, the $(R,1)^g$ policy coincides with the benchmark solution for all h_R values larger than 0.4 as the benchmark always computes a policy structure similar to a $(2,1)^g$ policy.

Regarding the holding cost value for the final product inventory, similar conclusions can be drawn. Since h_M must not be smaller than h_R/β the smallest value for h_M in this sensitivity analysis is 1.2. The maximum value, on the other hand, is set to be three. Within this range all values in steps of 0.01 have been examined. Figure 2.17 presents the results of the three preset policies that reflect the findings of Figure 2.15. Over all instances, the $(R,1)^g$ policy provides the best results of the preset policy

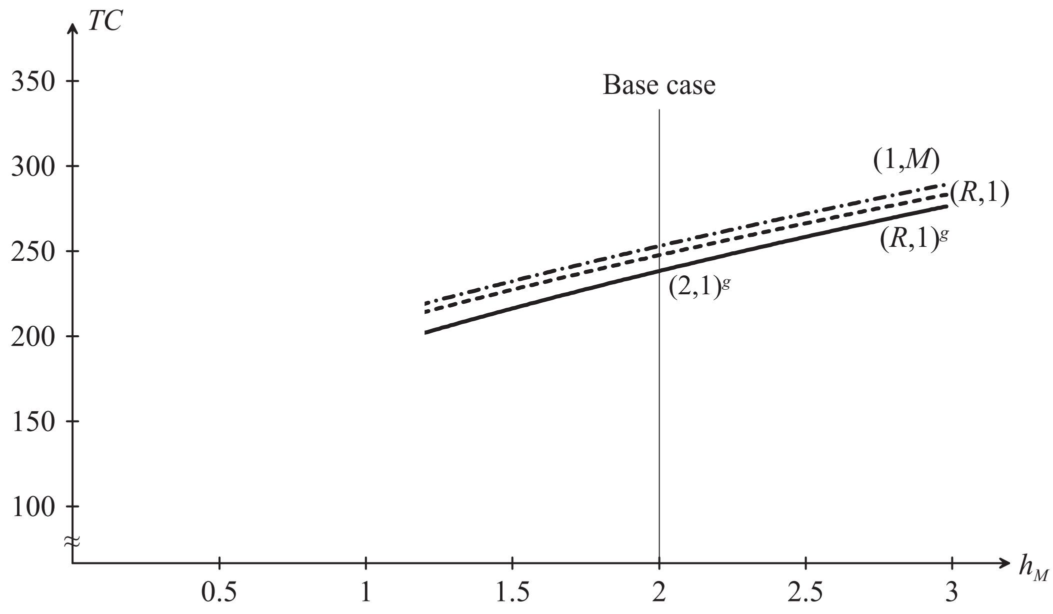

Fig. 2.17: Minimum total cost of the preset policies for different h_M values

structures. To be more precise, the proposed policy has been a $(2,1)^g$ policy for all tested h_M values. Moreover, the absolute deviation between the $(R,1)^g$ and the $(R,1)$ policies is largest for h_M values that lie close to h_R/β. As the total cost increase the larger h_M becomes, the largest relative deviation is observed for small h_M values, too. If the experiments would have been extended to incorporate h_M values larger than 8.3647, the $(R,1)^g$ policy would have been outperformed by the $(R,1)$ policy. This value can be derived from the ratio of h_M to h_R in equation (2.50) that describes for a given value of h_R the value of h_M at which a $(2,1)$ policy is better than a $(2,1)^g$ policy. We omit to present the percentage error with respect to the benchmark for a varying h_M as the results can be derived from Figure 2.16 as well. After analyzing the influence of both holding cost parameters, the influence of the setup cost parameters K_R and K_M is evaluated.

Setup cost parameters K_R ***and*** K_M

Next to the holding cost parameters, the setup cost values have a direct influence on the number of lots in a cycle. In general, when the setup cost falls while keeping all other parameters constant, the number of lots does not decrease. To verify this general thought for the underlying problem, both setup cost parameters have been altered to take on values between 0 and 250 in steps of 1. Starting with the setup cost for remanufacturing K_R, the solutions of the preset policy structures have been depicted in Figure 2.18. For the variation of the base case scenario with respect to the setup

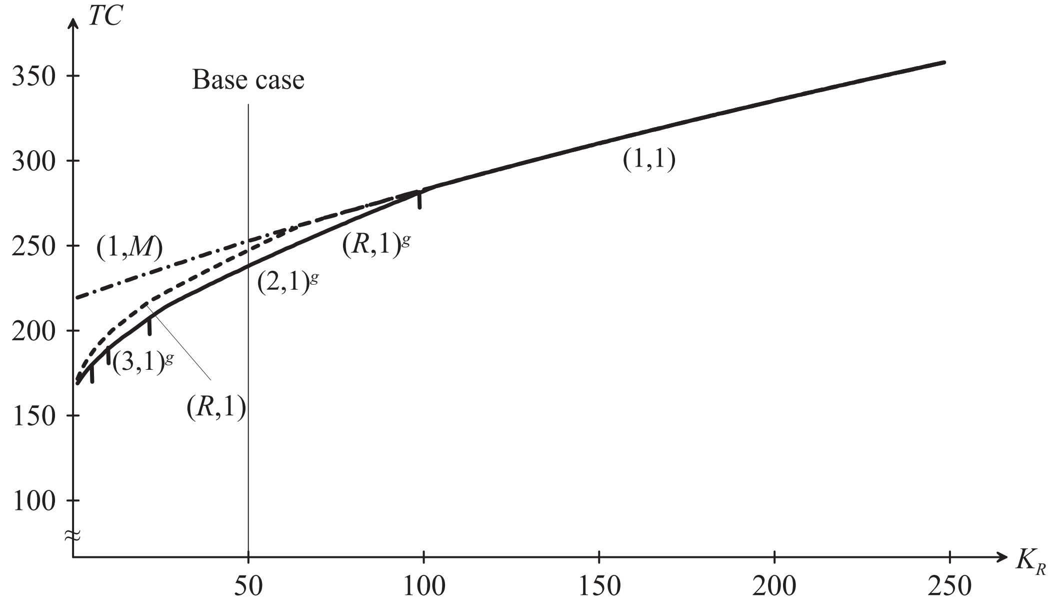

Fig. 2.18: Minimum total cost of the preset policies for different K_R values

cost for remanufacturing, two different phases can be observed. For K_R being larger than 104, all preset policy structures compute the same result, a $(1,1)$ policy. If, on the other hand, K_R is smaller than 104 the $(R,1)^g$ policy yields the minimum total cost of these policies. The exact value can be determined by exploiting condition (2.47) since this condition describes the transition from a (1,1) to a $(2,1)^g$ policy structure. In this case, the exact value for K_R is 103.8156. Otherwise, by manipulating condition (2.39) the exact value of K_R can be determined from which a $(1,2)$ policy dominates the $(1,1)$ policy. The value obtained thereby needs to be larger than 250 as the $(1,M)$ policy does not dominate the other two policy structures in Figure 2.18. The exact value is for the base case scenario a K_R of 588.4615. A (1,1) policy is, therefore, the best preset policy structure for all K_R values between 103.8156 and 588.4615.

In order to evaluate the overall solution quality of the preset policy structures, they are confronted with the benchmark solution as well. Figure 2.19 depicts the results. As

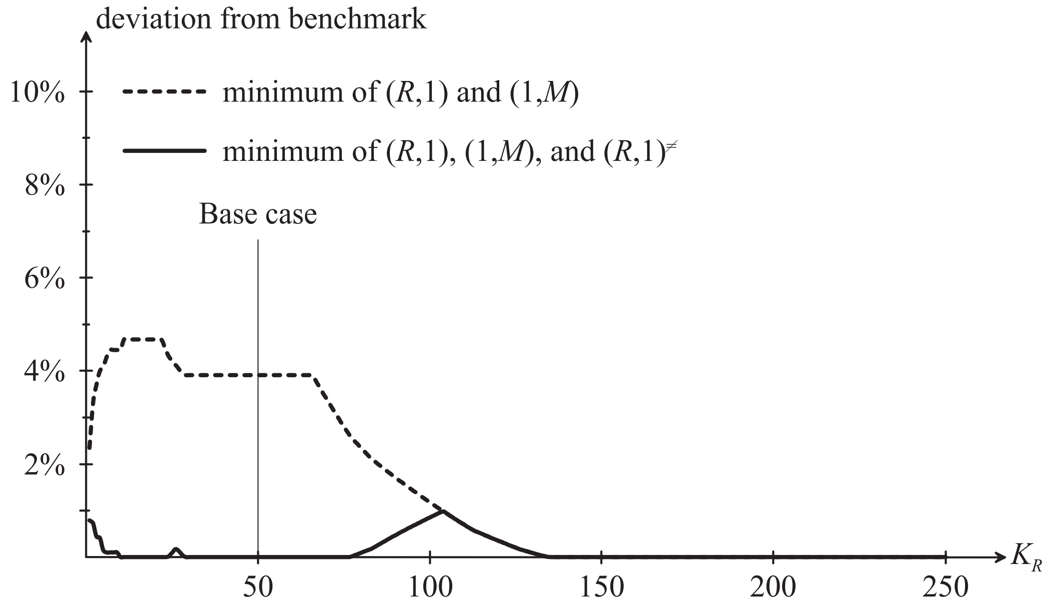

Fig. 2.19: Deviation from benchmark solution for different K_R values

has been observed for all parameters until now, neglecting the opportunity to consider geometrically decreasing remanufacturing lots can lead to a significant error in the problem's solution. When varying K_R, this can be observed for a relatively small setup cost of remanufacturing. Yet, even when incorporating the $(R, 1)^g$ policy, an error of up to 1% prevails when comparing the preset policy structures to the benchmark solution. Especially for K_R between 77 and 134 this error is recognizable.

After elaborating the results for K_R, the analysis is put forth for the setup cost of initiating a manufacturing lot K_M. This parameter is altered as well between 0 and 250 in steps of 1. Figure 2.20 illustrates the results of the experiments regarding the minimum total cost for each preset policy structure. In this Figure, three different sections can be found. When the setup cost for manufacturing is quite small, the $(1, M)$ policy calculates the best results since more than one manufacturing lot in a cycle is beneficial. The second section is represented by a (1,1) policy in which all preset policy

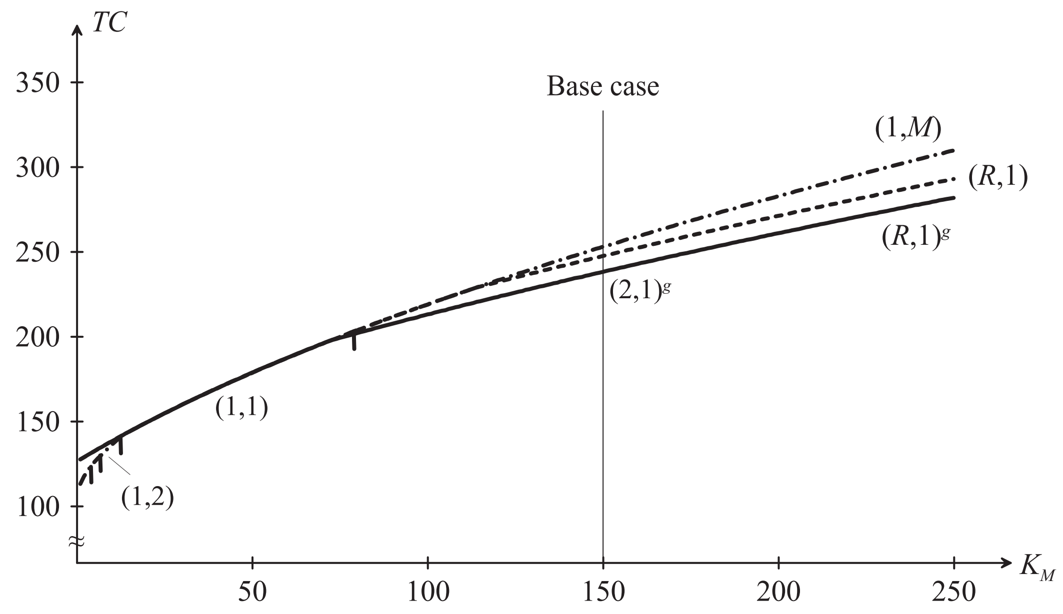

Fig. 2.20: Minimum total cost of the preset policies for different K_M values

structures determine the same result. Finally, the last section is characterized by the $(R,1)^g$ policy dominating both the $(R,1)$ and $(1,M)$ policy structures. The transition values that limit these sections can be determined by exploiting conditions (2.39) and (2.47). Solving these conditions with respect to K_M, condition (2.39) computes a K_M of 12.7451 representing the transition from a $(1,2)$ to a $(1,1)$ policy. A K_M of 72.2341, on the other hand, defines the transition from a $(1,1)$ to a $(2,1)^g$ policy. The second section of Figure 2.20 lies therefore between K_M=12.7451 and K_M=72.2341.

Concluding, the benchmark solution has been obtained as well for all instances regarding a variation of K_M and can now be opposed to the preset policy structures in Figure 2.21. This Figure presents an almost familiar picture. Omitting geometrically decreasing remanufacturing lots results in an error of up to 3.9 %. Interestingly, this percentage deviation has been constant over a multitude of instances from $K_M = 114$ to the upper bound. This is because the best preset policy structures have been the $(2,1)^g$ and the $(2,1)$ policies. Since the holding cost per time unit is not affected by a variation in the setup cost for manufacturing, the total cost value increases proportionally with an increasing K_M value. As a matter of fact, this happens independently from the presumption of considering equal or different remanufacturing lots in a cycle. The same behavior could also be observed in Figure 2.19 for the setup cost for reman-

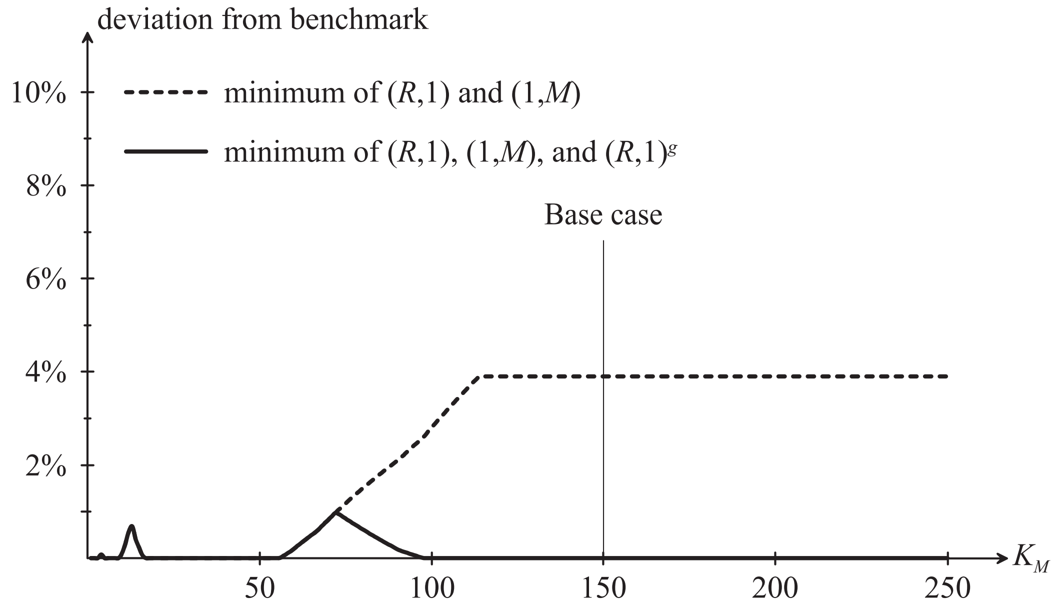

Fig. 2.21: Deviation from benchmark solution for different K_M values

ufacturing. Although not being as prominent as for the setup cost for manufacturing, the same explanation can be used there.

Parameter settings of Tang and Teunter

Before concluding this chapter, the effects of a different base case scenario are analyzed in the following. As mentioned above, Tang and Teunter (2006) present real-life data for the (re)manufacturing process of water pumps for diesel engines. In their contribution, five different types of water pumps (denoted by TT1 to TT5) are considered. Table 2.6 summarizes the relevant setup and holding cost parameters for these products. For all products, the setup cost to initiate a (re)manufacturing batch is 20. Furthermore, holding a final product for one time unit costs twice the amount of holding a returned product for one time unit. The remanufacturer faces only a small return ratio of water pumps amounting to 20% for all analyzed products. Since the yield parameter β has not been included by Tang and Teunter, we fix it to 80% as for the Ashayeri et al. base case scenario. Finally, demand for TT1 to TT5 differs between 3 and 30 units per time unit.

Tab. 2.6: Parameters for TT1 to TT5

Product	λ	α	β	K_R	K_M	h_R	h_M
TT1	9	20 %	80 %	20	20	0.0088	0.0175
TT2	9	20 %	80 %	20	20	0.0132	0.0263
TT3	9	20 %	80 %	20	20	0.0175	0.035
TT4	30	20 %	80 %	20	20	0.0219	0.0438
TT5	3	20 %	80 %	20	20	0.0263	0.0525

In order to determine the best preset policy structure, equations (2.30), (2.37), and (2.46) are evaluated for all products. As a result, a $(1, 2)$ policy outperforms both the $(R, 1)$ and the $(R, 1)^g$ policy structures for all parameter sets examined. Afterwards, by applying the optimization approach presented in Section 2.3 the benchmark solution for all products is obtained. Due to the similar parameter structure, the benchmark solution coincides for all products as well, i.e. a policy with two remanufacturing and five manufacturing lots in a cycle is recommended. Yet, the error of applying a (1,2) policy instead of the benchmark solution is less than 0.01 % for all products. Table 2.7 summarizes the relevant results.

Tab. 2.7: Best preset policy structure and benchmark for TT1 to TT5

	Best preset policy structure			Benchmark solution		
Product	R	M	TC	R	M	TC
TT1	1	2	3.0087	2	5	3.0085
TT2	1	2	3.6877	2	5	3.6873
TT3	1	2	4.2524	2	5	4.2517
TT4	1	2	8.6853	2	5	8.6839
TT5	1	2	3.0075	2	5	3.0071

A sensitivity analysis has been conducted for the base case scenario of Ashayeri et al. to assess the impact of each parameter on the solution structure. This could be done for the parameter sets of TT1 to TT5 as well. As the results do not differ substantially with respect to the Ashayeri et al. base case, only a variation of the return ratio α for product TT1 is presented henceforth. Figure 2.22 compares the minimum total cost of the preset policy structures $(1, M)$, $(R, 1)$, and $(R, 1)^g$ for TT1 when α is altered.

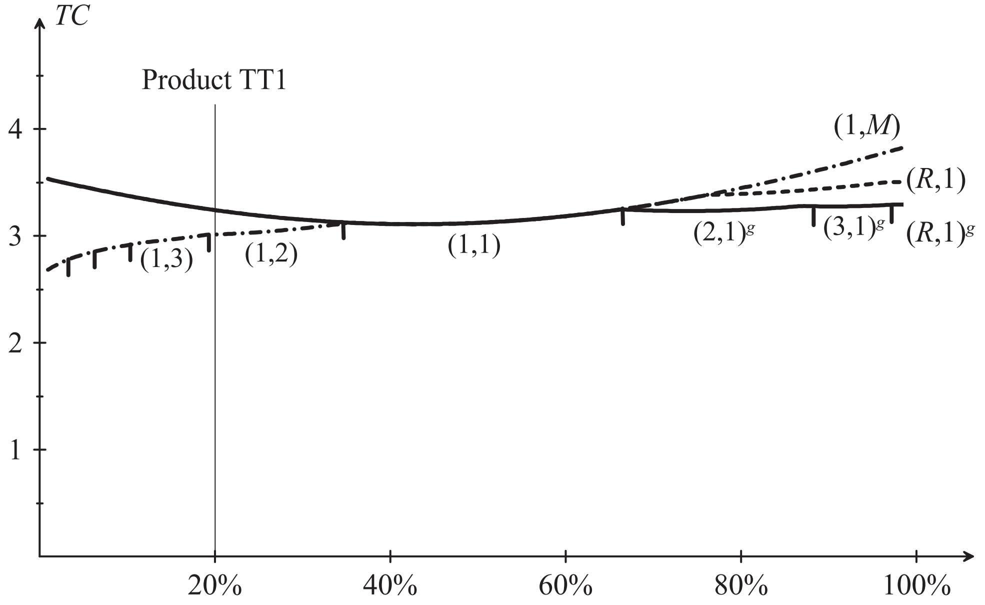

Fig. 2.22: Minimum total cost of the preset policy structures for different α values (TT1)

In correspondence to Figure 2.9, the $(1, M)$ policy dominates the policies that propose to schedule more than one remanufacturing lot for small return rates. When α lies between 35.31% and 66.3%, a $(1, 1)$ policy is suggested by all preset policy structures. The exact values for α are computed, again, by evaluating equations (2.39) and (2.47). When α becomes larger than 66.3%, the $(R, 1)^g$ policy is the best preset policy structure. Thus, Figure 2.23 depicts the deviation of the best preset policy structures from the benchmark solution when including or omitting the $(R, 1)^g$ policy. It can be observed as for the Ashayeri et al. base case that neglecting the $(R, 1)^g$ policy in the decision making process results in a significant error for large return fractions. Moreover, the best preset policy structure does not deviate by more than 1.5% from the benchmark solution. The worst case deviation of 1.02% can be found when α amounts to 66.5%.

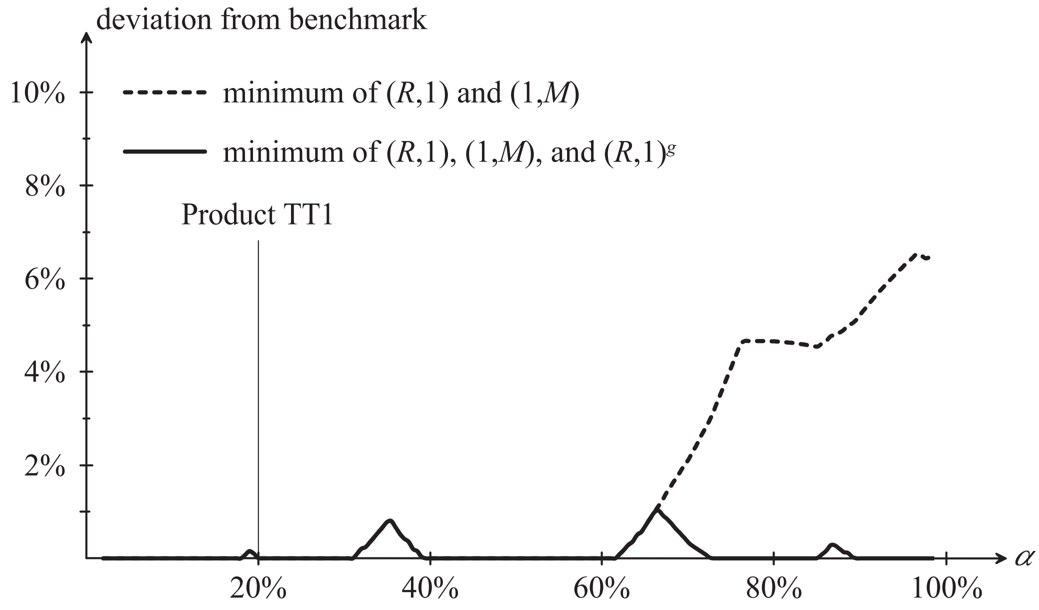

Fig. 2.23: Deviation from benchmark solution for different α values (TT1)

A similar outcome can be observed when the return ratio is altered for products TT2 to TT5. Likewise, the findings of varying the remaining parameters correspond to a large extent to the Ashayeri et al. base case scenario. Therefore, we omit to present the corresponding Figures and conclude this Chapter by a short summary and an outlook on future research options.

2.5 Concluding remarks and outlook

After giving a short introduction to the problem setting and presenting the available literature on this topic in Section 2.1, two policy structures known from literature have been presented in the adjacent Section 2.2, Schrady's $(R, 1)$ policy and Teunter's $(1, M)$ policy. Both policies rely on the assumption of equally sized batches in either the remanufacturing (Schrady) or the manufacturing (Teunter) process and formulate the objective function depending on these lot sizing decisions. By doing this, they neglect that the number of remanufacturing lots R and manufacturing lots M have to be integer. Minner avoids this pitfall by reformulating both policies such that their objective function depends on the cycle length T as well as on R or M, respectively. This reformulation allows to derive closed-form expressions to determine the optimal integer R^* and M^* for both policy structures. Subsequently, it has been proven that it is not possible for R^* and M^* to be larger than one simultaneously when restricting oneself to the $(R, 1)$ and $(1, M)$ policies. This reduces the effort to compute the better of these policies. In contrast to these policy structures, a third approach has been introduced in Section 2.2.6, the $(R, 1)^g$ policy. Instead of initiating remanufacturing lots of equal size as the $(R, 1)$ policy requires, the $(R, 1)^g$ policy schedules geometrically decreasing remanufacturing lots. When doing this, each remanufacturing lot remanufactures all available returns which depletes the used product inventory after each remanufacturing run. Yet, a closed-form expression to generate the optimal integer value for R could not be derived. However, some conditions could be determined at which this preset policy outperforms the other preset policy structures.

So far, the preset policy structures could only be compared to each other. Therefore, the main focus of the subsequent Section 2.3 has been to present an optimization approach in order to compute the optimal cycle. To do this, a mixed-integer non-linear problem is introduced that requires the number of remanufacturing and manufacturing batches in a cycle as input. This model has been solved to generate a benchmark solution that provides an opportunity to evaluate the performance of the three preset policy structures. This evaluation has been the subject of Section 2.4, the numerical study. Starting with the introduction of a base case scenario (taken from Ashayeri et al.), a sensitivity analysis is conducted that modifies each parameter individually. Several interesting aspects have been observed during this study. One of the most important aspects has been that by neglecting the $(R, 1)^g$ policy a significant error of up to 9 % could be made for some parameter combinations (in this study this is the case when the return rate α is large). Furthermore, the benchmark solution has in no instance been worse than the best preset policy structure. This could not be expected beforehand due to the non-linearity of the objective function and the restriction to 36 different parameter combinations of R and M. Finally, the best solutions of the preset policies have never been worse than 1 % compared to the benchmark solution which can be interpreted as a promising result.

Several research questions remain still unanswered and can be addressed in future. At first, the preset policy structures can be extended to include policies having more than one remanufacturing and more than one manufacturing batch in a cycle simultaneously. Choi et al. (2007) have been the first authors to test this assumption. Although they restrict their analysis to general (R, M) policy structures with equal remanufacturing and equal manufacturing batches, they are able to identify problem instances for which the solution can be improved. It would be interesting to evaluate the performance gain for a more general $(R, M)^g$ policy structure in which all remanufacturing batches use all available returns in stock.

Improving the benchmark solution can be another challenging task. In order to do this, the properties of the benchmark solutions have to be analyzed in greater detail to incorporate these findings into an improved optimization approach. Another opportunity would be to drop the integrality constraints of the optimization program and examine the relaxed solution to determine lower bounds for the optimal solution. It would be an interesting insight if the monotonicity in the ratio of R^* and M^* (as depicted in Figure 2.12 for a varying α) can be confirmed when using the relaxed optimization approach instead of the original one.

In addition, several assumptions can be analyzed critically to evaluate their importance on the results presented in this chapter. In our study, remanufacturing has been considered a profitable opportunity for the OEM no matter how long the returns are kept in stock. In reality, disposing of some returns at the beginning of a cycle can be advantageous as they would have to be stored over a long time before remanufacturing. Several contributions have analyzed this setting as well as a setting with a finite production and remanufacturing rate and possible lead times. Future research efforts can examine the effect of introducing differently sized remanufacturing lots for these settings as well. Moreover, the assumption of static demand and return rates can be criticized. Incorporating time variant returns and demand can help to model a more realistic system in this problem context. Chapter 4 is going to address this issue. Concluding, uncertainties can almost never be neglected in real-life systems. Uncertainties prevail for remanufacturing systems regarding their inputs as the OEM does not know how many customers return their product at what time and in which condition. Furthermore, the output is uncertain, too, since the yield of remanufacturing and the customer demand can only be estimated in advance. The next Chapter 2 presents, thus, an adaptation of this Chapter's model to evaluate the error that can be made when stochastic yields prevail.

2.6 Appendix

Derivation of equation (2.4):

$$
\begin{aligned}
\frac{K_m + R\cdot K_R}{T} &= \frac{K_m + R\cdot K_R}{\frac{Q_M + R\cdot Q_R\cdot\beta}{\lambda}} \\
&= \lambda\cdot\frac{K_m + \frac{\alpha\cdot Q_M}{(1-\alpha\beta)\cdot Q_R}\cdot K_R}{Q_M + \frac{\alpha\cdot Q_M}{(1-\alpha\beta)\cdot Q_R}\cdot Q_R\cdot\beta} \\
&= \lambda\cdot\frac{K_m + \frac{\alpha\cdot Q_M}{(1-\alpha\beta)\cdot Q_R}\cdot K_R}{Q_M\cdot\left(1+\frac{\alpha\beta}{1-\alpha\beta}\right)} \\
&= \lambda\cdot\frac{K_m + \frac{\alpha\cdot Q_M}{(1-\alpha\beta)\cdot Q_R}\cdot K_R}{\frac{Q_M}{1-\alpha\beta}} \\
&= \lambda\cdot\left(K_m\cdot\frac{1-\alpha\beta}{Q_M} + \frac{\alpha\cdot Q_M}{(1-\alpha\beta)\cdot Q_R}\cdot K_R\cdot\frac{1-\alpha\beta}{Q_M}\right) \\
&= \lambda\cdot\left(\frac{(1-\alpha\beta)\cdot K_M}{Q_M} + \frac{\alpha\cdot K_R}{Q_R}\right).
\end{aligned}
$$

Derivation of equation (2.6):

$$
\begin{aligned}
&\left(\frac{1}{2}\cdot\frac{R\cdot(Q_R\cdot\beta)^2}{\lambda} + \frac{1}{2}\cdot\frac{(Q_M)^2}{\lambda}\right)\cdot h_M\cdot\frac{1}{T} \\
= \quad &\frac{1}{2\lambda}\cdot\frac{R\cdot(Q_R\cdot\beta)^2 + (Q_M)^2}{\frac{R\cdot Q_R\cdot\beta + Q_M}{\lambda}}\cdot h_M \\
= \quad &\frac{1}{2}\cdot\frac{\frac{\alpha\cdot Q_M}{(1-\alpha\beta)\cdot Q_R}\cdot(Q_R\cdot\beta)^2 + (Q_M)^2}{\frac{\alpha\cdot Q_M}{(1-\alpha\beta)\cdot Q_R}\cdot Q_R\cdot\beta + Q_M}\cdot h_M \qquad \text{manipulation similar to (2.4)} \\
= \quad &\frac{1}{2}\cdot\frac{\frac{\alpha\cdot Q_M}{(1-\alpha\beta)}\cdot Q_R\cdot\beta^2 + (Q_M)^2}{\frac{Q_M}{1-\alpha\beta}}\cdot h_M \\
= \quad &\frac{1}{2}\cdot\left(\alpha\beta^2\cdot Q_R + (1-\alpha\beta)\cdot Q_M\right)\cdot h_M.
\end{aligned}
$$

Proof of convexity of the total cost function in (2.7) In order to prove the convexity of the total cost function TC_{R1}, its Hessian matrix has to be elaborated.

This results in:

$$H(TC_{R1}) = \begin{bmatrix} \frac{\partial^2 TC_{R1}}{\partial (Q_R)^2} & \frac{\partial^2 TC_{R1}}{\partial Q_R \partial Q_M} \\ \frac{\partial^2 TC_{R1}}{\partial Q_M \partial Q_R} & \frac{\partial^2 TC_{R1}}{\partial (Q_M)^2} \end{bmatrix} = \begin{bmatrix} \frac{2\lambda\alpha K_R}{(Q_R)^3} & 0 \\ 0 & \frac{2\lambda(1-\alpha\beta)K_M}{(Q_M)^3} \end{bmatrix}.$$

This Hessian is positive definite as its leading principal minors are strictly positive, i.e. $\frac{2\lambda\alpha K_R}{(Q_R)^3} > 0$ and $\frac{4\lambda^2\alpha K_r K_m(1-\alpha\beta)}{(Q_R)^3\cdot(Q_M)^3} > 0$. Therefore, the total cost function TC_{R1} is jointly convex in both decision variables Q_R and Q_M.

Derivation of equation (2.15):

$$\begin{aligned}
\frac{M\cdot K_m + K_R}{T} &= \frac{M\cdot K_m + K_R}{\frac{M\cdot Q_M + Q_R\cdot\beta}{\lambda}} \\
&= \lambda\cdot\frac{\frac{Q_R\cdot(1-\alpha\beta)}{\alpha\cdot Q_M}\cdot K_m + K_R}{\frac{Q_R\cdot(1-\alpha\beta)}{\alpha\cdot Q_M}\cdot Q_M + Q_R\cdot\beta} \\
&= \lambda\cdot\frac{\frac{Q_R\cdot(1-\alpha\beta)}{\alpha\cdot Q_M}\cdot K_m + K_R}{Q_R\cdot\left(\frac{1-\alpha\beta}{\alpha}+\beta\right)} \\
&= \lambda\cdot\frac{\frac{Q_R\cdot(1-\alpha\beta)}{\alpha\cdot Q_M}\cdot K_m + K_R}{\frac{Q_R}{\alpha}} \\
&= \lambda\cdot\left(\frac{Q_R\cdot(1-\alpha\beta)}{\alpha\cdot Q_M}\cdot K_m\cdot\frac{\alpha}{Q_R} + K_R\cdot\frac{\alpha}{Q_R}\right) \\
&= \lambda\cdot\left(\frac{K_M\cdot(1-\alpha\beta)}{Q_M} + \frac{K_R\cdot\alpha}{Q_R}\right).
\end{aligned}$$

Derivation of equation (2.16):

$$\begin{aligned}
&\left[\frac{1}{2}\cdot Q_R\cdot T\cdot h_R + \left(\frac{1}{2}\cdot\frac{(Q_R\cdot\beta)^2}{\lambda} + M\cdot\frac{1}{2}\cdot\frac{(Q_M)^2}{\lambda}\right)\cdot h_M\right]\cdot\frac{1}{T} \\
= \;& \frac{1}{2}\cdot Q_R\cdot h_R + \frac{\left(\frac{1}{2}\cdot\frac{(Q_R\cdot\beta)^2}{\lambda} + M\cdot\frac{1}{2}\cdot\frac{(Q_M)^2}{\lambda}\right)}{\frac{M\cdot Q_M + Q_R\cdot\beta}{\lambda}}\cdot h_M \\
= \;& \frac{1}{2}\cdot Q_R\cdot h_R + \frac{1}{2}\cdot\frac{(Q_R\cdot\beta)^2 + \frac{Q_R\cdot(1-\alpha\beta)}{\alpha\cdot Q_M}\cdot(Q_M)^2}{\frac{Q_R\cdot(1-\alpha\beta)}{\alpha\cdot Q_M}\cdot Q_M + Q_R\cdot\beta}\cdot h_M \qquad \text{manipulation similar to (2.15)} \\
= \;& \frac{1}{2}\cdot Q_R\cdot h_R + \frac{1}{2}\cdot\frac{(Q_R\cdot\beta)^2 + \frac{Q_R\cdot(1-\alpha\beta)}{\alpha}\cdot Q_M}{\frac{Q_R}{\alpha}}\cdot h_M \\
= \;& \frac{1}{2}\cdot\left(Q_R\cdot h_R + \left(\alpha\beta^2\cdot Q_R + (1-\alpha\beta)\cdot Q_M\right)\cdot h_M\right).
\end{aligned}$$

Proof of convexity of the total cost function in (2.17):

As for the $(R,1)$ policy, the Hessian matrix of the total cost function has to be computed to analyze its properties. The Hessian matrix of TC_{1M} is:

$$H(TC_{1M}) = \begin{bmatrix} \frac{\partial^2 TC_{1M}}{\partial (Q_R)^2} & \frac{\partial^2 TC_{1M}}{\partial Q_R \partial Q_M} \\ \frac{\partial^2 TC_{1M}}{\partial Q_M \partial Q_R} & \frac{\partial^2 TC_{1M}}{\partial (Q_M)^2} \end{bmatrix} = \begin{bmatrix} \frac{2\lambda\alpha K_R}{(Q_R)^3} & 0 \\ 0 & \frac{2\lambda(1-\alpha\beta)K_M}{(Q_M)^3} \end{bmatrix}.$$

The Hessian matrix of TC_{1M} coincides with the matrix for TC_{R1}. Thus, all leading principal minors are strictly positive again and the joint convexity (regarding Q_R and Q_M) of the total cost function TC_{1M} is proven.

Behavior of equation (2.28) near 0 and ∞ :

$$TC^{+}_{R1}(R) = \sqrt{2\lambda(R\cdot K_R + K_M)\left(\left(1+\alpha\beta\left(\frac{1}{R}-1\right)\right)\alpha h_R + \left(\frac{\alpha^2\beta^2}{R} + (1-\alpha\beta)^2\right)h_M\right)}$$

$$TC^{+}_{R1}(R) = \sqrt{2\lambda\left(A\cdot R + B + \frac{C}{R}\right)}$$

$$\text{with} \quad A = K_R\cdot\left(\alpha h_R\,(1-\alpha\beta) + (1-\alpha\beta)^2\right) \geq 0$$

$$B = K_R\alpha^2\beta\,(h_R + \beta h_M) + K_M\left(\alpha h_R\,(1-\alpha\beta) + (1-\alpha\beta)^2\, h_M\right) \geq 0$$

$$C = K_M\alpha^2\beta\,(h_R + \beta h_M) \geq 0$$

$$\lim_{R\to 0} TC^{+}_{R1}(R) = \sqrt{2\lambda\left(A\cdot R + B + \frac{C}{R}\right)} = \infty$$

$$\lim_{R\to \infty} TC^{+}_{R1}(R) = \sqrt{2\lambda\left(A\cdot R + B + \frac{C}{R}\right)} = \infty.$$

Derivation of equation (2.29):

$$TC^{+}_{R1}(\hat{R}) = TC^{+}_{R1}(\hat{R}+1)$$

$$\sqrt{2\lambda\left(\hat{R}K_R + K_M\right)\left(\left(1+\alpha\beta\left(\frac{1}{\hat{R}}-1\right)\right)\alpha h_R + \left(\frac{\alpha^2\beta^2}{\hat{R}} + (1-\alpha\beta)^2\right)h_M\right)}$$

$$= \sqrt{2\lambda\left(\left(\hat{R}+1\right)K_R + K_M\right)\left(\left(1+\alpha\beta\left(\frac{1}{\hat{R}+1}-1\right)\right)\alpha h_R + \left(\frac{\alpha^2\beta^2}{\hat{R}+1} + (1-\alpha\beta)^2\right)h_M\right)}$$

$$\left(\hat{R}K_R + K_M\right)\left(\left(1+\alpha\beta\left(\frac{1}{\hat{R}}-1\right)\right)\alpha h_R + \left(\frac{\alpha^2\beta^2}{\hat{R}} + (1-\alpha\beta)^2\right)h_M\right)$$
$$= \left(\left(\hat{R}+1\right)K_R+K_M\right)\left(\left(1+\alpha\beta\left(\frac{1}{\hat{R}+1}-1\right)\right)\alpha h_R+\left(\frac{\alpha^2\beta^2}{\hat{R}+1}+(1-\alpha\beta)^2\right)h_M\right)$$

$$K_M\cdot\left(\alpha^2\beta\frac{1}{\hat{R}}h_R + \alpha^2\beta^2\frac{1}{\hat{R}}h_M\right) - K_M\cdot\left(\alpha^2\beta\frac{1}{\hat{R}+1}h_R + \alpha^2\beta^2\frac{1}{\hat{R}+1}h_M\right)$$
$$-K_R\cdot\left(\alpha\left(1-\alpha\beta\right)h_R + \left(1-\alpha\beta\right)^2 h_M\right) = 0$$

$$K_M\alpha^2\beta\cdot\left(\frac{h_R}{\hat{R}} - \frac{h_R}{\hat{R}+1} + \frac{h_M\beta}{\hat{R}} - \frac{h_M\beta}{\hat{R}+1}\right) - K_R\cdot\left(\alpha\left(1-\alpha\beta\right)h_R + \left(1-\alpha\beta\right)^2 h_M\right) = 0$$

$$K_M\alpha^2\beta\cdot\left(h_R + h_M\beta\right)\cdot\left(\frac{1}{\hat{R}} - \frac{1}{\hat{R}+1}\right) - K_R\cdot\left(\alpha\left(1-\alpha\beta\right)h_R + \left(1-\alpha\beta\right)^2 h_M\right) = 0$$

$$\frac{\hat{R}+1-\hat{R}}{\hat{R}\cdot\left(\hat{R}+1\right)} - \frac{K_R\cdot\left(\alpha\left(1-\alpha\beta\right)h_R + \left(1-\alpha\beta\right)^2 h_M\right)}{K_M\alpha^2\beta\cdot\left(h_R+h_M\beta\right)} = 0$$

$$\hat{R}^2 + \hat{R} - \frac{K_M\alpha^2\beta\cdot\left(h_R+h_M\beta\right)}{K_R\cdot\left(\alpha\left(1-\alpha\beta\right)h_R + \left(1-\alpha\beta\right)^2 h_M\right)} = 0$$

$$\hat{R} = -\frac{1}{2} \pm \sqrt{\frac{1}{4} + \frac{K_M\alpha^2\beta\cdot\left(h_R+h_M\beta\right)}{K_R\cdot\left(\alpha\left(1-\alpha\beta\right)h_R + \left(1-\alpha\beta\right)^2 h_M\right)}}$$

$$R^* = \left\lceil -\frac{1}{2} \pm \sqrt{\frac{1}{4} + \frac{K_M\alpha^2\beta\cdot\left(h_R+h_M\beta\right)}{K_R\cdot\left(\alpha\left(1-\alpha\beta\right)h_R + \left(1-\alpha\beta\right)^2 h_M\right)}}\ \right\rceil.$$

Derivation of equation (2.36):

$$TC^+_{1M}(\hat{M}) = TC^+_{1M}(\hat{M}+1)$$

$$\sqrt{2\lambda\cdot\left(K_R + \hat{M}\cdot K_M\right)\cdot\left(\alpha h_R + \left(\alpha^2\beta^2 + \frac{\left(1-\alpha\beta\right)^2}{\hat{M}}\right)\cdot h_M\right)}$$
$$= \sqrt{2\lambda\cdot\left(K_R + \left(\hat{M}+1\right)\cdot K_M\right)\cdot\left(\alpha h_R + \left(\alpha^2\beta^2 + \frac{\left(1-\alpha\beta\right)^2}{\hat{M}+1}\right)\cdot h_M\right)}$$

$$\left(K_R + \hat{M} \cdot K_M\right) \cdot \left(\alpha h_R + \left(\alpha^2\beta^2 + \frac{(1-\alpha\beta)^2}{\hat{M}}\right) \cdot h_M\right)$$
$$- \left(K_R + \left(\hat{M}+1\right) \cdot K_M\right) \cdot \left(\alpha h_R + \left(\alpha^2\beta^2 + \frac{(1-\alpha\beta)^2}{\hat{M}+1}\right) \cdot h_M\right) = 0$$

$$K_R \cdot (1-\alpha\beta)^2 \cdot h_M \cdot \left(\frac{1}{\hat{M}} - \frac{1}{\hat{M}+1}\right) - K_M \cdot \left(\alpha h_R + \alpha^2\beta^2 h_M\right) = 0$$

$$\frac{\hat{M}+1-\hat{M}}{\hat{M} \cdot \left(\hat{M}+1\right)} - \frac{K_M \cdot (\alpha h_R + \alpha^2\beta^2 h_M)}{K_R \cdot (1-\alpha\beta)^2 \cdot h_M} = 0$$

$$\hat{M}^2 + \hat{M} - \frac{K_R \cdot (1-\alpha\beta)^2 \cdot h_M}{K_M \cdot (\alpha h_R + \alpha^2\beta^2 h_M)} = 0$$

$$\hat{M} = -\frac{1}{2} \pm \sqrt{\frac{1}{4} + \frac{K_R \cdot (1-\alpha\beta)^2 \cdot h_M}{K_M \cdot (\alpha h_R + \alpha^2\beta^2 h_M)}}$$

$$M^* = \left\lceil -\frac{1}{2} \pm \sqrt{\frac{1}{4} + \frac{K_R \cdot (1-\alpha\beta)^2 \cdot h_M}{K_M \cdot (\alpha h_R + \alpha^2\beta^2 h_M)}} \right\rceil .$$

Derivation of equation (2.41):

$$\begin{aligned}
\lambda\alpha T &= Q_{R,1} + Q_{R,2} + Q_{R,3} + .. + Q_{R,R} \\
\lambda\alpha T &= Q_{R,1} + \alpha\beta Q_{R,1} + \alpha^2\beta^2 Q_{R,1} + .. + \alpha^{R-1}\beta^{R-1} Q_{R,1} \\
\lambda\alpha T &= Q_{R,1} \cdot \left(1 + \alpha\beta + \alpha^2\beta^2 + .. + \alpha^{R-1}\beta^{R-1}\right) \\
\lambda\alpha T &= Q_{R,1} \cdot \sum_{i=0}^{R-1} \alpha^i\beta^i \\
\lambda\alpha T &= Q_{R,1} \cdot \frac{1-\alpha^R\beta^R}{1-\alpha\beta} \\
Q_{R,1} &= \frac{\lambda\alpha T \cdot (1-\alpha\beta)}{1-\alpha^R\beta^R}.
\end{aligned}$$

In the transformations, the convenient formula for a geometric series has been used. In general, this formula states that $s_n = a_0 \sum_{k=0}^{n} q^k = a_0 \frac{1-q^{n+1}}{1-q}$ (for $q \neq 1$), where a_0 denotes the initial value and q the common ratio. Here, the initial value is equal to $Q_{R,1}$ while the common ratio is $\alpha\beta$. The remaining remanufacturing batches can be calculated by using the first condition explained above. Therefore,

$$Q_{R,i} = \frac{\lambda\alpha T \cdot (1-\alpha\beta)}{1-\alpha^R\beta^R} \cdot \alpha^{i-1}\beta^{i-1} = \frac{\lambda\alpha^i\beta^{i-1}T \cdot (1-\alpha\beta)}{1-\alpha^R\beta^R} \qquad \forall i = 1,..,R.$$

Derivation of equation (2.43):

$$\begin{aligned}
&\left[\frac{1}{2}\sum_{i=1}^{R}\left(Q_{R,i}\cdot\frac{Q_{R,i}}{\lambda\alpha}\right)h_R\right]\cdot\frac{1}{T}\\
=&\left[\frac{h_R}{2\lambda\alpha}\sum_{i=1}^{R}\left(\frac{\lambda\alpha^i\beta^{i-1}T\cdot(1-\alpha\beta)}{1-\alpha^R\beta^R}\right)^2\right]\cdot\frac{1}{T}\\
=&\frac{h_RT}{2\lambda\alpha}\frac{\lambda^2\cdot(1-\alpha\beta)^2}{(1-\alpha^R\beta^R)^2}\cdot\sum_{i=1}^{R}\alpha^{2i}\beta^{2\cdot(i-1)}\\
=&\frac{h_RT}{2\alpha}\frac{\lambda\cdot(1-\alpha\beta)^2}{(1-\alpha^R\beta^R)^2}\cdot\frac{1}{\beta^2}\sum_{i=1}^{R}\left(\alpha^2\right)^i\left(\beta^2\right)^i\\
=&\frac{h_RT}{2\alpha}\frac{\lambda\cdot(1-\alpha\beta)^2}{(1-\alpha^R\beta^R)^2}\cdot\frac{\alpha^2\beta^2}{\beta^2}\sum_{i=0}^{R-1}\left(\alpha^2\right)^i\left(\beta^2\right)^i \qquad \text{[formula for geometric series]}\\
=&\frac{h_RT}{2}\frac{\lambda\alpha\cdot(1-\alpha\beta)^2}{(1-\alpha^R\beta^R)^2}\cdot\frac{1-\alpha^{2R}\beta^{2R}}{1-\alpha^2\beta^2}\\
=&\frac{h_RT}{2}\frac{\lambda\alpha\cdot(1-\alpha\beta)^2}{(1-\alpha^R\beta^R)^2}\cdot\frac{\left(1-\alpha^R\beta^R\right)\cdot\left(1+\alpha^R\beta^R\right)}{(1-\alpha\beta)\cdot(1+\alpha\beta)}\\
=&\frac{1}{2}\lambda\alpha Th_R\cdot\left(\frac{1-\alpha\beta}{1+\alpha\beta}\cdot\frac{1+\alpha^R\beta^R}{1-\alpha^R\beta^R}\right).
\end{aligned}$$

Derivation of equation (2.44):

$$\begin{aligned}
&\left[\frac{1}{2}\left(\sum_{i=1}^{R}\left(Q_{R,i}\cdot\beta\cdot\frac{Q_{R,i}\cdot\beta}{\lambda}\right)+Q_M\cdot\frac{Q_M}{\lambda}\right)h_M\right]\frac{1}{T}\\
=&\left[\frac{h_M}{2\lambda}\left(\beta^2\sum_{i=1}^{R}\left(\frac{\lambda\alpha^i\beta^{i-1}T\cdot(1-\alpha\beta)}{1-\alpha^R\beta^R}\right)^2+\lambda^2\cdot(1-\alpha\beta)^2T^2\right)\right]\frac{1}{T}\\
=&\frac{h_MT}{2\lambda}\left(\frac{\lambda^2\cdot(1-\alpha\beta)^2}{(1-\alpha^R\beta^R)^2}\sum_{i=1}^{R}\left(\alpha^2\right)^i\left(\beta^2\right)^i+\lambda^2\cdot(1-\alpha\beta)^2\right)\\
=&\frac{\lambda h_MT}{2}\left(\frac{\alpha^2\beta^2\cdot(1-\alpha\beta)^2}{(1-\alpha^R\beta^R)^2}\sum_{i=0}^{R-1}\left(\alpha^2\right)^i\left(\beta^2\right)^i+(1-\alpha\beta)^2\right)\\
=&\frac{\lambda h_MT}{2}\left(\frac{\alpha^2\beta^2\cdot(1-\alpha\beta)^2}{(1-\alpha^R\beta^R)^2}\cdot\frac{1-\alpha^{2R}\beta^{2R}}{1-\alpha^2\beta^2}+(1-\alpha\beta)^2\right)\\
=&\frac{1}{2}\lambda Th_M\left(\alpha^2\beta^2\cdot\frac{1-\alpha\beta}{1+\alpha\beta}\cdot\frac{1+\alpha^R\beta^R}{1-\alpha^R\beta^R}+(1-\alpha\beta)^2\right).
\end{aligned}$$

Derivation of inequality (2.47):

$$TC^{+}_{R1^g}(1) - TC^{+}_{R1^g}(2) > 0$$

$$\sqrt{2\lambda\cdot(K_R+K_M)\cdot\left(\alpha h_R+\alpha^2\beta^2 h_M+h_M\,(1-\alpha\beta)^2\right)}$$
$$-\sqrt{2\lambda\cdot(2K_R+K_M)\cdot\left((\alpha h_R+\alpha^2\beta^2 h_M)\cdot V+h_M\,(1-\alpha\beta)^2\right)} > 0$$

$$(K_R+K_M)\cdot\left(\alpha h_R+\alpha^2\beta^2 h_M+h_M\,(1-\alpha\beta)^2\right)$$
$$-\,(2K_R+K_M)\cdot\left(\left(\alpha h_R+\alpha^2\beta^2 h_M\right)\cdot V+h_M\,(1-\alpha\beta)^2\right) > 0$$

$$K_R\left(\alpha h_R+\alpha^2\beta^2 h_M\right)(1-2V)-K_R h_M(1-\alpha\beta)^2+K_M\left(\alpha h_R+\alpha^2\beta^2 h_M\right)(1-V) > 0.$$

Derivation of inequality (2.49):

$$\begin{aligned}
\frac{1-\alpha\beta}{1+\alpha\beta}\cdot\frac{1+\alpha^2\beta^2}{1-\alpha^2\beta^2}) &> \frac{1}{2}\\
2\,(1-\alpha\beta)\left(1+\alpha^2\beta^2\right) &> (1+\alpha\beta)\left(1-\alpha^2\beta^2\right)\\
2-2\alpha\beta+2\alpha^2\beta^2-2\alpha^3\beta^3 &> 1+\alpha\beta-\alpha^2\beta^2-\alpha^3\beta^3\\
1-3\alpha\beta-3\alpha^2\beta^2-\alpha^3\beta^3 &> 0\\
(1-\alpha\beta)^3 &> 0.
\end{aligned}$$

Derivation of inequality (2.50):

$$TC^{+}_{R1}(2) - TC^{+}_{R1^g}(2) > 0$$

$$\sqrt{2\lambda\cdot(2K_R+K_M)\cdot\left(\left(1+\alpha\beta\left(\frac{1}{2}-1\right)\right)\cdot\alpha h_R+\left(\frac{\alpha^2\beta^2}{2}+(1-\alpha\beta)^2\right)\cdot h_M\right)}$$
$$-\sqrt{2\lambda\cdot(2K_R+K_M)\cdot\left((\alpha h_R+\alpha^2\beta^2 h_M)\cdot V+h_M\,(1-\alpha\beta)^2\right)} > 0$$

$$\left(1+\alpha\beta\left(\frac{1}{2}-1\right)\right)\cdot\alpha h_R+\frac{\alpha^2\beta^2}{2}\cdot h_M-\left(\alpha h_R+\alpha^2\beta^2 h_M\right)\cdot V > 0$$

$$h_M\alpha^2\beta^2 V-\alpha h_R\left(-1+\frac{1}{2}\alpha\beta+V\right) > 0$$

Since V is larger than 0.5 which has been shown in condition (2.49), the direction of the inequality is reversed. Hence, when replacing V by $\frac{1-\alpha\beta}{1+\alpha\beta} \cdot \frac{1+\alpha^2\beta^2}{1-\alpha^2\beta^2}$

$$\frac{h_M}{h_R} < \frac{\alpha\left(\frac{1-\alpha\beta}{1+\alpha\beta} \cdot \frac{1+\alpha^2\beta^2}{1-\alpha^2\beta^2} - 1 + \frac{1}{2}\alpha\beta\right)}{\alpha^2\beta^2\left(\frac{1}{2} - \frac{1-\alpha\beta}{1+\alpha\beta} \cdot \frac{1+\alpha^2\beta^2}{1-\alpha^2\beta^2}\right)}$$

$$\frac{h_M}{h_R} < \frac{\left(\frac{1-\alpha\beta}{1+\alpha\beta} \cdot \frac{1+\alpha^2\beta^2}{(1-\alpha\beta)\cdot(1+\alpha\beta)} - 1 + \frac{1}{2}\alpha\beta\right)}{\alpha\beta^2\left(\frac{1}{2} - \frac{1-\alpha\beta}{1+\alpha\beta} \cdot \frac{1+\alpha^2\beta^2}{(1-\alpha\beta)\cdot(1+\alpha\beta)}\right)}$$

$$\frac{h_M}{h_R} < \frac{\left(\frac{1+\alpha^2\beta^2}{(1+\alpha\beta)^2} - 1 + \frac{1}{2}\alpha\beta\right)}{\alpha\beta^2\left(\frac{1}{2} - \frac{1+\alpha^2\beta^2}{(1+\alpha\beta)^2}\right)}$$

$$\frac{h_M}{h_R} < \frac{\frac{1+\alpha^2\beta^2-(1+\alpha\beta)^2+\frac{1}{2}\alpha\beta\cdot(1+\alpha\beta)^2}{(1+\alpha\beta)^2}}{\alpha\beta^2\left(\frac{(1+\alpha\beta)^2-2\cdot(1+\alpha^2\beta^2)}{2\cdot(1+\alpha\beta)^2}\right)}$$

$$\frac{h_M}{h_R} < \frac{1+\alpha^2\beta^2-1-2\alpha\beta-\alpha^2\beta^2+\frac{1}{2}\alpha\beta+\alpha^2\beta^2+\frac{1}{2}\alpha^3\beta^3}{\frac{1}{2}\alpha\beta^2\left(1+2\alpha\beta+\alpha^2\beta^2-2-2\alpha^2\beta^2\right)}$$

$$\frac{h_M}{h_R} < \frac{-\frac{3}{2}\alpha\beta+\alpha^2\beta^2+\frac{1}{2}\alpha^3\beta^3}{\frac{1}{2}\alpha\beta^2\left(-1+2\alpha\beta-\alpha^2\beta^2\right)}$$

$$\frac{h_M}{h_R} < \frac{-\frac{1}{2}\alpha\beta\left(3-2\alpha\beta-\alpha^2\beta^2\right)}{-\frac{1}{2}\alpha\beta^2\left(1-\alpha\beta\right)^2}$$

$$\frac{h_M}{h_R} < \frac{(3+\alpha\beta)\cdot(1-\alpha\beta)}{\beta\left(1-\alpha\beta\right)^2}$$

$$\frac{h_M}{h_R} < \frac{3+\alpha\beta}{\beta\left(1-\alpha\beta\right)}.$$

Base case analysis: Determination of α when it is better to initiate two instead of one manufacturing lots in a cycle (page 51)

$$K_R \cdot (1-\alpha\beta)^2 \cdot h_M - 2K_M \cdot \left(\alpha h_R + \alpha^2\beta^2 h_M\right) = 0$$

$$K_R h_M - 2K_R\alpha\beta h_M + K_R\alpha^2\beta^2 h_M - 2K_M\alpha h_R - 2K_M\alpha^2\beta^2 h_M = 0$$

$$\alpha^2\left(K_R h_M\beta^2 - 2K_M h_M\beta^2\right) - \alpha\left(2K_R h_M\beta + 2K_M h_R\right) + K_R h_M = 0$$

$$\alpha_{1,2} = \frac{K_R h_M\beta + K_M h_R}{K_R h_M\beta^2 - 2K_M h_M\beta^2} \pm \sqrt{\left(\frac{K_R h_M\beta + K_M h_R}{K_R h_M\beta^2 - 2K_M h_M\beta^2}\right)^2 - \frac{K_R h_M}{K_R h_M\beta^2 - 2K_M h_M\beta^2}}$$

$$\alpha_{1,2} = -\frac{230}{320} \pm \sqrt{\left(\frac{230}{320}\right)^2 + \frac{100}{320}}$$

$$\alpha_1 = 0.1918 \qquad \alpha_2 = -1.6291.$$

As only α_1 lies within the relevant range between 0 and 1, the value of α at which it is better to have only one manufacturing lot instead of two lies for the base case at 19.18 %.

3. On the alignment of lot sizing decisions in a remanufacturing system in the presence of random yield

3.1 Introduction

The reuse field has grown significantly in the past decades due to its economical benefits and the environmental requirements[1]. Remanufacturing which represents a sophisticated form of reuse focuses on value-added recovery and has been introduced for many different products ranging from car engines (as has been reported in Seitz and Wells, 2006) over photocopiers (as has been reported in Thierry et al., 1995) to water pumps for diesel engines (as has been reported in Tang and Teunter, 2006). Within the process of remanufacturing, products that are returned by the customers to the producer are disassembled to obtain functional components. The obtained components are afterwards cleaned and reworked until an as-good-as-new quality is assured. Having met the required quality standards, these components can be used for the assembly of a remanufactured product that is delivered to the customers with the same warranty as a newly produced one. In addition to the economic profitability, as a part of the embedded economic value can be saved by remanufacturing, there is an increasingly legislative restriction that assigns the producers the responsibility for their used products, for instance the Directive 2002/96/EC related to Waste Electrical and Electronic Equipment and the Directive 2002/525/EC related to End of Life Vehicles. Because of

[1] This Chapter is based on the work titled 'On the alignment of lot sizing decisions in a remanufacturing system in the presence of random yield' that has been published in the FEMM working paper series (see Schulz and Ferretti, 2008).

that, remanufacturing has become an important industry sector to achieve the goal of sustainable development (see, for instance, Webster and Mitra, 2007). Therefore, the management and control of inventory systems that incorporate joint manufacturing and remanufacturing options has received considerable attention in recent literature contributions.

One of the main topics in these contributions is the assessment of joint lot sizing decisions for remanufacturing and manufacturing which has been thoroughly investigated in recent years. One of the first authors who established a basic modeling approach is Schrady (1967) who develops a simple solution procedure for determining the lot sizes of repair and manufacturing lots. He assumes in his work that a constant and continuous demand for a single product has to be satisfied over an infinite planning horizon. Furthermore, a constant return fraction is established that describes the percentage of used products that return to the producer. By using that assumption, a constant and continuous return rate is ensured. Presuming setup costs for remanufacturing and manufacturing as well as different holding costs for repairable and newly manufactured products, a simple EOQ-type formula (with EOQ being the economic order quantity) is proposed that minimizes the sum of setup and holding costs per time unit. As a result, an efficient cyclic pattern is established which is characterized by the fact that within each repair cycle a number of repair lots of equal size succeed exactly one manufacturing lot. By solving the proposed EOQ-formula which can be applied because an infinite production and repair rate is presumed as well, the number of repair lots and the length of a repair cycle can be determined. Teunter (2001) generalizes the results of Schrady in a way that he examines different structures of a repair cycle. He adds to the efficient cycle patterns a cycle in which several manufacturing lots of equal size are followed by exactly one repair lot. The assumption of equal lot sizes is among other aspects critically studied in the contribution of Minner and Lindner (2004). They show that a policy with non-identical lot sizes can outperform a policy with identical lot sizes. However, the structure of an efficient repair cycle prevails also when the assumption of equal lot sizes is lifted.

Next to the analysis of the basic model context several extensions have been proposed that relax some of the assumptions made so far. Teunter (2004), for instance, relaxes the assumption of an instantaneous manufacturing and repair process in order to derive more general expressions for the number of manufacturing and repair lots and their corresponding lot sizes. Since only a heuristic procedure is introduced on how to determine these values, Konstantaras and Papachristos (2008) extend Teunter's work by developing an algorithm that leads to the optimal policy for certain parameter classes. By incorporating stochastic lead times and thereby including the possibility of backorders, Tang and Grubbstrom (2005) extend the basic model. Two general options are recommended on how such a system can be dealt with, a cycle ordering model and a dual sourcing ordering policy. Both approaches are compared in a numerical study that indicates certain parameter specifications under which one approach outperforms the other. Furthermore, several papers have been published by Richter and Dobos (e.g. Richter, 1996a,b; Richter and Dobos, 1999) that relax the assumption of a constant rate of return. In their contributions, several situations are presented in which a so called pure strategy is always optimal. In this context, a pure strategy means that either every returned product is repaired or everything is disposed of immediately. Therefore, a mixed strategy in which a part of the returned products is repaired and the rest is disposed of is always dominated by one of the pure strategies. Finally, the assumption of continuous demand and return rates has been relaxed by several authors. Consequently, the former EOQ-type model becomes a dynamic lot sizing problem. The contribution of Teunter et al. (2006) extends well-known dynamic lot sizing heuristics such as the Silver Meal or the Part Period algorithm in order to test their performance in a remanufacturing environment. In their work, the adapted Silver Meal approach revealed an average percentage deviation of around 8 % compared to the optimal solution. Schulz (2011b) improves among other things their approach by incorporating ideas known from the static environment and reduces the average error to around 2 %.

Common to all contributions is that they do not consider the remanufacturing process explicitly. Although some authors speak of remanufacturing, they analyze a remanufacturing system in the same way as a repair system. This may lead to wrong conclusions as it is not regarded that the remanufacturing process itself consists of two different subprocesses, a disassembly process in which the returned products are disassembled and a rework process in which the obtained components are brought to an as-good-as-new quality (for possible definitions see Thierry et al., 1995; Atasu et al., 2010). By explicitly incorporating both subprocesses in this contribution, the decisions that need to be made regarding disassembly and rework are decoupled which generalizes the basic models used so far.

Next to this generalization, this Chapter will further relax the assumption of a deterministic yield, i.e. the number of components obtained by disassembly is not known with certainty beforehand. To present a practical application of this problem, the remanufacturing process for a car engine can be analyzed. When a batch of returned engines is disassembled, the remanufacturer does not know in advance how many remanufacturable components can be obtained. This is because the quality of the returns cannot always be assessed before disassembly. Hence, such a process can only be analyzed thoroughly when both processes disassembly and rework are evaluated separately. Considering stochastic yields has attained significant interest in the scientific literature as the basic work of Yano and Lee (1995) as well as the overview of Grosfeld-Nir and Gerchak (2004) indicate. However, most of the contributions presented by Grosfeld-Nir and Gerchak (2004) describe pure manufacturing environments which cannot be entirely translated to a remanufacturing system as such a system inherits greater risks to be dealt with (for details, see Toffel, 2004). Nevertheless, stochastic yields have also been studied in a remanufacturing environment. Inderfurth and Langella (2006), for instance, have concentrated their analysis specifically on the yield risk within the disassembly process. Yet, they focus on a multi-product multi-component problem setting in which a given discrete demand for components has to be satisfied by either disassembling used products or manufacturing new components. The authors develop in their contribution heuristic methods on how to deal with such a problem in which they neglect the presence of setup costs for the disassembly and the remanufacturing process. In another work, Ferrer (2003) evaluates four different scenarios in a single period remanufacturing environment that differ in their process capabilities. For each scenario, the optimal policy has been derived. In a numerical study, all four scenarios have been tested and compared.

After this short introduction, the problem assumptions and the nomenclature used in the remainder of the paper are illustrated in Section 3.2. Section 3.3 presents thereafter two solution procedures to find the optimal solution in the deterministic yield scenario for a given predefined policy structure before Section 3.4 widens the scope to a stochastic yield problem and presents three heuristic approaches that facilitate the decision making process in such an environment. The fifth Section of this Chapter (3.5) conducts a numerical experiment in order to test the heuristics' performance. Finally, a conclusion and an outlook are given in the last Section.

3.2 Problem setting and model formulation

A company engaged in the area of remanufacturing that remanufactures several used products (e.g. engines) coming back from their customers is the background for the problem setting. To keep the analysis simple, the focus shall be restricted to only one specific remanufactured product named A. Figure 3.1 presents the general structure of this simplified system which is modeled as a multi-level inventory system containing three stages. Further simplifications are made regarding the fact that there are neither lead nor processing times. Furthermore, no disposal option is included in the problem setting.

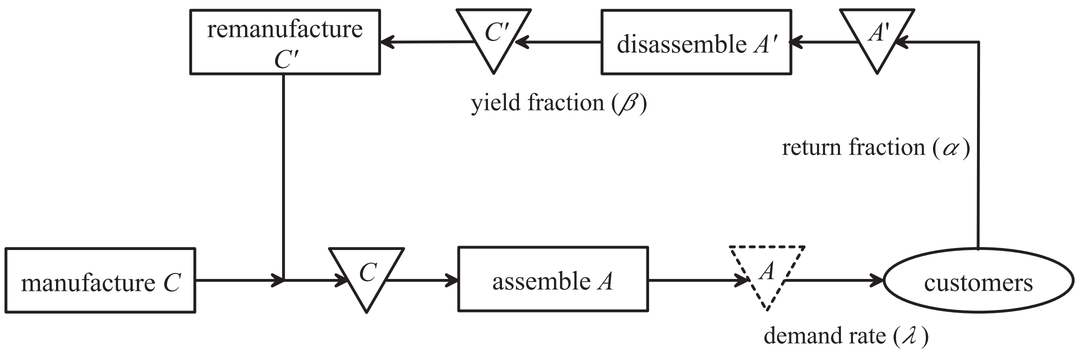

Fig. 3.1: Inventory system in a remanufacturing environment

The customers' demand for the final product A is assumed to be constant and depletes the finished goods inventory continuously by a constant rate of λ units per time unit. In order to satisfy that demand, the company manufactures the final product by using component C which represents the most important component of the product. For the sake of simplicity, only the most important component C is included in the analysis. However, the proposed model could be easily extended to a multi-component setting. The assembly process is supposed to be a flow line process at which the final product is assembled continuously and immediately delivered to the customers, i.e. there is no stocking point for the final product A. When the customers have no further use for their product A (e.g. it is broken or its leasing contract ends) they have the opportunity to return the product to the company. Yet, only a fraction (named α) of those products in the market returns to the producer. For the subsequent analysis, the return flow of used products (which are denoted A') fills the used product inventory by the constant and continuous rate of $\lambda\alpha$. By disassembling A' the worn component C' is obtained. Although the process of disassembly typically consists of manual work, a setup cost prevails for adjusting the required disassembly tools and/or measuring devices that allow an improved assessment of the reusability of components before disassembly. Within this model K_D represents the setup costs for a disassembly batch while h_D is the holding cost incurred for storing one unit of A' for one time unit. Due to different stages of wear, not all returned products contain a reworkable component C'. The ratio of the number of reworkable items obtained from the disassembly of A' to the rate of product returns $\lambda\alpha$ is denoted by β. Assuming that at most one reworkable component C' can be obtained by disassembling one unit of A' the ratio β must not exceed one while being non-negative. As the released components C' cannot be used directly for the assembly of the final product A since they usually do not meet the designated quality standards, these components have to be remanufactured. Since the remanufacturing process incurs a cost of K_R for setting up the cleaning and mechanical rework tools, a batching of reworkable components takes place as well. Hence, some reworkable components need to be stored before the next remanufacturing batch is started resulting in costs of h_R per unit and time unit. It is furthermore assumed that each component that is remanufactured is brought to an as-good-as-new condition. All successfully reworked components are held in a serviceables inventory at a cost of h_M per unit and time unit. In order to secure the final product assembly of A, some components of C have to be manufactured in addition (as α and β are usually smaller than one). The relevant costs are denoted by K_M representing the cost for setting up a manufacturing lot for component C. Newly manufactured components are held in the same serviceables inventory as remanufactured ones and it is supposed that the holding costs do not differ between both sourcing options. A detailed discussion on the topic on how to set the holding cost parameters can be found in Teunter et al. (2000). In general, the holding costs (when interpreted as costs for capital lockup) of all levels are connected by the following inequality since more value is added to the component on each level, i.e. $h_D < h_R < h_M$.

Balancing setup and holding costs shall be achieved by applying an average cost approach to this model. This is commonly done for one-level inventory systems by using the well known EOQ-model formulation but can be easily extended to a multi-level environment by respecting the stipulated assumptions of the EOQ-model (e.g. infinite planning horizon with constant costs over time). As a result, an optimal cyclic pattern is obtained by minimizing the average cost per time unit. In order to control the entire system, three decision variables are required. Firstly, the length of the disassembly cycle T determines the lot size of each disassembly batch ($\lambda\alpha T$) under the assumption that there is only one disassembly lot per cycle. This assumption is made for the sake of simplicity as an additional decision variable (number of disassembly lots per cycle) would complicate the analysis significantly. However, if we consider high setup costs of disassembly, we conjecture that this assumption of one disassembly lot per cycle assures the optimality of the introduced deterministic policy. Furthermore, by fixing the number of remanufacturing lots R per disassembly cycle, their equal lot size can be computed by $\lambda\alpha\beta T/R$. Finally, the number of manufacturing lots M per disassembly cycle determines the lot sizes of the manufacturing lots to be $\lambda(1-\alpha\beta)T/M$. The subsequent section presents the optimal solution of a completely deterministic setting in which all parameters are known with certainty.

3.3 Deterministic yields

In this section, a model is introduced that permits the evaluation of the optimal number of manufacturing and remanufacturing lots in a disassembly cycle with one disassembly batch. Before expanding the scope to stochastic yields from disassembly which represents the core issue of this Chapter, the deterministic setting is studied in order to gain insight into the interrelations of the whole system. Figure 3.2 illustrates the behavior of the relevant inventory levels for three consecutive disassembly cycles. As a matter of fact, the optimal decision variables (T, R, and M) remain constant over time in a deterministic environment. As shown in Figure 3.2 below, the manufacturing lots are always positioned after the remanufacturing lots in the serviceables inventory. This is obvious as this strategy strictly dominates the strategy of starting a cycle on the serviceables level with a manufacturing lot due to the increased holding costs on the remanufacturables level.

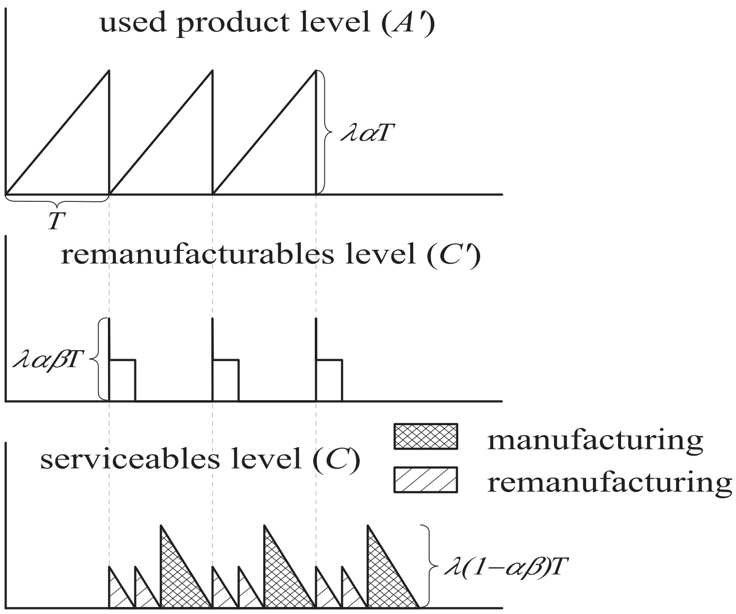

Fig. 3.2: Used product, remanufacturables, and serviceables inventory in a deterministic yield environment (with $D = 1$, R=2, and M=1)

By minimizing the total average cost per time unit, this specific example shows the optimal cycle length T for two remanufacturing lots ($R = 2$) which split the remanufacturables inventory inflow equally and one manufacturing lot ($M = 1$) which satisfies the remaining demand of the assembly process for product A. To analyze the total cost function (TC^D) only two main types of costs have to be considered, the setup costs SC^D and the holding cost HC^D in which the index D indicates the deterministic setting. The total cost function in the deterministic setting can be formulated as follows[2]:

$$TC^D = \frac{SC^D}{T} + \frac{\lambda T}{2} \cdot HC^D \tag{3.1}$$

with $\quad SC^D = K_D + RK_R + MK_M \quad$ and

$HC^D = \alpha h_D + \frac{R-1}{R}\alpha^2\beta^2 h_R + (\frac{\alpha^2\beta^2}{R} + \frac{(1-\alpha\beta)^2}{M})h_M.$

In order to minimize the total cost function which is a mixed-integer non-linear optimization problem two procedures can be applied. The first procedure is a simple enumerative procedure. Since R and M have to be integer, only a finite number of calculations (in which R and M are set to integer values) have to be compared if R and M are restricted to certain intervals. The original objective function simplifies for given values of R and M to a non-linear convex function that only depends on T. Such a problem can be solved easily by using the subsequent equations

$$T^{D^*} = \sqrt{\frac{2 \cdot SC^D}{\lambda \cdot HC^D}} \tag{3.2}$$

$$TC^{D^*} = \sqrt{2\lambda \cdot SC^D \cdot HC^D}. \tag{3.3}$$

There, formula 3.2 is comparable to the determination of the economic order interval. However, the optimality of this solution approach can only be guaranteed if the optimal total cost TC^{D^*} is determined for all possible (R, M) combinations which leads to a large number of calculations. Nevertheless, a good solution can be obtained in a fast manner by restricting the number of possible realizations.

After introducing an enumerative procedure another promising approach is presented next. By relaxing the original objective function (3.1) such that R and M need not to take on integer values, one can prove that the total cost function has only a single local

[2] For details, please refer to the Appendix, page 100. Moreover, a mathematical proof is elaborated in the Appendix (from page 103.) which shows that it is optimal to schedule equal remanufacturing and equal manufacturing lots in a disassembly cycle.

minimum in the relevant area (for T, R, $M > 0$)[3]. Yet, by evaluating the Hessian matrix in this area, it can be shown that the total cost function is not convex in all variables[4]. This leads to the significant problem that a simple rounding procedure cannot be used to obtain the optimal solution for the integer number of remanufacturing and manufacturing lots. Therefore, a solution algorithm could be implemented that is able to globally determine the minimum cost of this mixed-integer non-linear optimization problem. The BARON algorithm, as implemented in the GAMS software package, proved to be a valuable tool for this problem setting. In general, BARON implements deterministic global optimization algorithms of the branch-and-reduce type in order to determine the optimal solution for a mixed-integer non-linear optimization problem. For a detailed description of the algorithm, please refer to Sahinidis and Tawarmalani (2005).

The subsequent section extends the deterministic model of this section to incorporate stochastic yields.

3.4 Stochastic yields

One of the main problems for many practical applications in the area of remanufacturing is that they have to deal with stochastic yields which means that the amount of remanufacturable components obtained from disassembling returned products is not known with certainty (see also Inderfurth and Langella, 2006). Due to the significance of that problem in a remanufacturing planning environment, we will now put forth the extension of the deterministic model introduced in the last section to incorporate stochastic yield fractions resulting from the disassembly process. Although being uncertain, it can be assured that the lowest possible yield fraction β_l cannot be smaller than zero as negative yields would not be reasonable. The largest possible yield fraction β_u, however, cannot exceed the value of one since this describes the situation that from every disassembled used product more than one remanufacturable component is obtained which is ruled out by the assumptions made. Within the range from β_l to β_u a specific

[3] For details, please refer to the Appendix, page 108.

[4] For details, please refer to the Appendix, page 109.

distribution function can be defined which will be denoted in the following analysis by $\varphi(\beta)$. As the number of returned products disassembled per cycle corresponds to $\lambda\alpha T$, the independence of $\varphi(\beta)$ with respect to T reflects the fact that the subsequent analysis assumes stochastic proportional yields (for a definition see Yano and Lee, 1995). Therefore, the formerly used total cost function for a deterministic yield scenario (3.1) has to be extended to incorporate any possible yield outcome. Hence, the total cost of a given stochastic yield scenario can only be formulated as an expected total cost (which will be further on denoted as TC^S) that is presented in the following equation:

$$TC^S = \frac{SC^S}{T} + \frac{\lambda T}{2} \cdot HC^S \tag{3.4}$$

with $\quad SC^S = K_D + \int_{\beta_l}^{\beta_u} (R(\beta)K_R + M(\beta)K_M) \cdot \varphi(\beta)d\beta \quad$ and

$$\begin{aligned} HC^S = {} & \alpha h_D + h_M \int_{\beta_l}^{\beta_u} \tfrac{1}{M(\beta)} \cdot \varphi(\beta)d\beta - 2\alpha h_M \int_{\beta_l}^{\beta_u} \tfrac{1}{M(\beta)} \cdot \beta \cdot \varphi(\beta)d\beta + \\ & \alpha^2 \int_{\beta_l}^{\beta_u} (h_R - \tfrac{h_R}{R(\beta)} + \tfrac{h_M}{R(\beta)} + \tfrac{h_M}{M(\beta)}) \cdot \beta^2 \cdot \varphi(\beta)d\beta. \end{aligned}$$

The fact that for any possible yield realization β an integer number of R and M has to be defined complicates the analysis of the total cost function TC^S significantly. In this setting, $R(\beta)$ describes the optimal number of remanufacturing lots for a given yield fraction β. Likewise, $M(\beta)$ represents the optimal number of manufacturing lots if the yield fraction β is fixed. Due to the fact that β is not known with certainty, the total cost per time unit can only be formulated as an expectation over all different yield realizations. In contrast to the total cost function of the deterministic case (3.1), SC^S and HC^S can be regarded as an expectation of their corresponding deterministic equivalents SC^D and HC^D. As finding the optimal solution for any problem setting cannot be guaranteed, which will be shown later in this Chapter, three different heuristic policies will be presented in the succeeding paragraphs that differ in their degree of sophistication. The first and least complex policy is introduced in the following:

Policy I

The easiest option on how to handle a stochastic problem is to neglect the underlying stochastics in order to derive a deterministic equivalent of the stochastic problem. The first policy introduced proceeds exactly in this manner as it neglects the fact that R and M depend on the yield realization β. Thus, only one value for R and M needs to be derived that is valid for every yield realization between β_l and β_u. To obtain these values, one can insert a specific yield fraction into the deterministic total cost function of the last section (3.1) and apply the recommended solution procedures to obtain R and M. As any yield fraction can be inserted that lies in the range of possible yield realizations and therefore many different combinations of R and M may prevail, we limit the focus of policy I on inserting only the mean yield fraction into the deterministic model since the mean yield is one of the most important characteristics of the underlying yield distribution. As a result, we obtain the values of R^D and M^D that replace $R(\beta)$ and $M(\beta)$ for every possible yield realization β in formula (3.4). The expected total cost of the first policy (TC_I) can therefore be easily calculated by the subsequent equation:

$$TC_I = TC^S(T, R^D, M^D). \tag{3.5}$$

Since policy I is a very simple approach, the decision maker can improve the expected total cost by incorporating the underlying stochastics in the decision making process which is introduced in policy II.

Policy II

Contrary to the first policy, the second policy does not neglect the dependence of R and M on the realization of the random yield fraction β. Nevertheless, in order to keep this policy simple, the disassembly cycle length T is kept constant which reduces the complexity of this policy significantly. For the sake of simplicity, the length of the disassembly cycle T will be set to the optimal deterministic cycle length T^{D^*} obtained by formula (3.2) assuming that the mean yield fraction has been inserted as deterministic equivalent for the underlying yield distribution. The assumption of fixing the cycle length to a specific value can be further used to draw some basic conclusions that can only be drawn for a given cycle length. The stochastic yield realization β determines for each disassembly cycle the number of remanufacturable items. As the number of remanufacturable and manufactured components per cycle always adds up to the value of λT, the number of manufactured items depends as well on the yield realization. However, for both options of demand fulfillment it can be observed that if more items are processed (either by manufacturing or remanufacturing), the number of respective lots in a cycle does not decrease. Therefore, when comparing two different yield realizations with all other parameters remaining constant it can be said: The larger the yield realization is, the more components have to be remanufactured which means that the number of remanufacturing lots per cycle does not decrease. On the other hand, the number of newly manufactured components decreases as larger the yield realization becomes which means that the number of manufacturing lots per cycle does not increase. Figures 3.3 and 3.4 compare both heuristic policies introduced so far for three consecutive disassembly cycles. On the left hand side (Figure 3.3), it can be observed for policy I that regardless of the realized yield fraction the same number of R and M is applied in every cycle (R=2 and M=1). Figure 3.4 on the right hand side, however, shows policy II that reacts for the same cycle length T on the different realizations of β which is supported by the fact that for a small yield realization the number of remanufacturing lots is smaller than for a large yield realization (R=1 in the first cycle compared to R=3 in the third cycle). An opposing behavior can be observed for the number of manufacturing lots per cycle that does not increase the smaller the yield realization is.

These general conclusions cannot only be formulated verbally but also in a mathematical form by introducing so-called transition yield fractions which have the property that either the number of remanufacturing lots or the number of manufacturing lots changes when optimizing the deterministic equivalent problem. For the calculation of the specific yield fraction that is characterized by a switch of the optimal policy from R to R+1 remanufacturing lots, one needs to equate the deterministic total cost functions for R and R+1 as presented in the following equation

$$\frac{SC^D(R)}{T} + \frac{\lambda T}{2} \cdot HC^D(R) = \frac{SC^D(R+1)}{T} + \frac{\lambda T}{2} \cdot HC^D(R+1)$$

with $SC^D(R) = K_D + RK_R + MK_M$ and

$HC^D(R) = \alpha h_D + \frac{R-1}{R}\alpha^2\beta^2 h_R + (\frac{\alpha^2\beta^2}{R} + \frac{(1-\alpha\beta)^2}{M})h_M.$

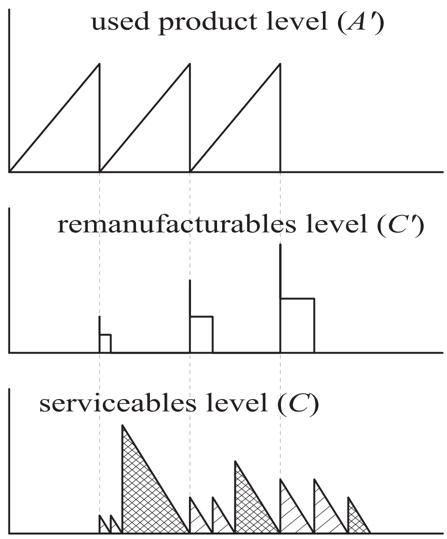

Fig. 3.3: Inventory system in a stochastic yield environment applying policy I (R=2 and M=1)

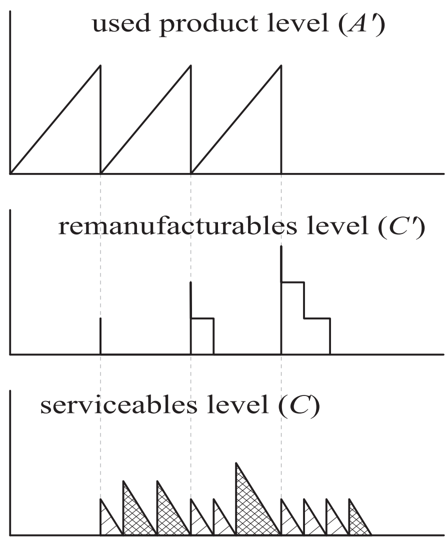

Fig. 3.4: Inventory system in a stochastic yield environment applying policy II

This equation can be solved with respect to β in order to obtain the transition yield fraction $\beta(R)$ at which the optimal decision in the deterministic case switches from R to R+1 for a given cycle length[5]:

$$\beta(R) = \frac{1}{\alpha T} \cdot \sqrt{\frac{2K_R R(R+1)}{\lambda(h_M - h_R)}}. \tag{3.6}$$

Not only is this function monotonously increasing in R which corresponds to the findings above that the number of remanufacturing lots does not decrease for larger values of β but it also does not depend on the number of manufacturing lots per cycle M. Thus, the same analysis can be carried out independently for the transition from M to M-1 manufacturing lots per cycle by equating both total cost functions in order to obtain the transition yield fraction $\beta(M)$[6]:

$$\beta(M) = \frac{1}{\alpha} - \frac{1}{\alpha T} \cdot \sqrt{\frac{2K_M M(M-1)}{\lambda h_M}}. \tag{3.7}$$

Because this function is monotonously decreasing in M, the insight that a larger yield fraction does not lead to less manufacturing lots in a cycle is approved. Consequently,

[5] For details, please refer to the Appendix, page 110.

[6] For details, please refer to the Appendix, page 110.

the lowest and highest values for R and M can be determined by exploiting the two formulae given above. Thus, for the lowest possible yield fraction β_l R_{min} and M_{max} can be computed by the following procedure (analogously R_{max} and M_{min} can be computed for the highest possible yield fraction β_u):

$$R_{min} = \min_R\{\beta(R) \geq \beta_l\} \qquad\qquad M_{max} = \max_M\{\beta(M) \geq \beta_l\}. \tag{3.8}$$

As the disassembly cycle length is fixed to a given value, the distribution function of the stochastic yield fraction can be subdivided into several intervals. Each interval j contains all yield realizations between its lower bound l_j and its upper bound u_j. Such an interval is characterized by the fact that within this interval only one R and M induce the optimal solution for any possible yield fraction. The optimal number of remanufacturing and manufacturing lots in a certain interval j are furthermore denoted by R_j and M_j, respectively. For the identification of the respective interval bounds the following pseudocode can be used:

start $j = 1,\ l_j = \beta_l,\ R_j = R_{min},\ M_j = M_{max},\ \beta(0) = \infty$

while $\min\{\beta(R_j + 1), \beta(M_j - 1)\} < \beta_u$ **do**

  if $\beta(R_j + 1) < \beta(M_j - 1)$ **then**

    $u_j = \beta(R_j + 1)$

    $j = j + 1,\ l_j = u_{j-1},\ R_j = R_{j-1} + 1,\ M_j = M_{j-1}$

  else

    $u_j = \beta(M_j - 1)$

    $j = j + 1,\ l_j = u_{j-1},\ R_j = R_{j-1},\ M_j = M_{j-1} - 1$

  end if

end do

$u_j = \beta_u,\ J = j$

end

After the initialization in which the first interval j=1 is opened (l_1=β_l) and given the values R_{min} and M_{max} the procedure evaluates if the transition to R_{min}+1 or M_{max}-1 is closer to β_l. For the lower of these two values, the upper bound of the first interval u_1 is fixed to the transition rate and the next interval is opened ($l_2 = u_1$).

This procedure stops when both next transitions to R+1 and M-1 are larger than the highest possible yield fraction β_u. At this point, the total number of intervals into which the yield distribution can be separated is determined by the index j which is set to the number of intervals J. As a result, the total yield distribution is separated into several intervals which is depicted for an example in Figure 3.5. In this example (with $\beta_l = 0$ and $\beta_u = 1$) it can be observed that the solution of policy I would have been R=3 and M=4 as this would solve the deterministic equivalent to optimality for β=0.5 .

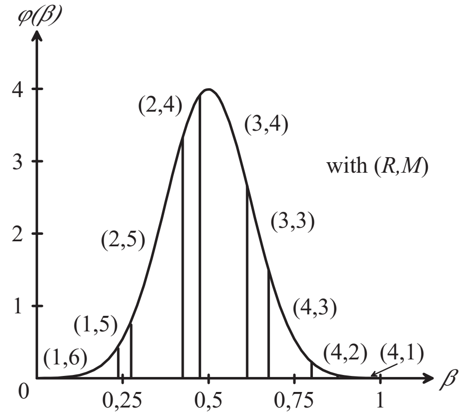

Fig. 3.5: Exemplary separation of a yield distribution according to policy II

As the interval bounds vary with a changing disassembly cycle length T, the expected total cost function for policy II can be formulated as follows using the optimal disassembly cycle length T^{D*} obtained by inserting the mean yield fraction into equation (3.3):

$$TC_{II} = \frac{SC^S}{T^{D^*}} + \frac{\lambda T^{D^*}}{2} \cdot HC^S \tag{3.9}$$

with $\quad SC^S = K_D + \sum_{j \in J} (R_j K_R + M_j K_M) \cdot \int_{l_j}^{u_j} \varphi(\beta) d\beta \quad$ and

$$\begin{aligned} HC^S = {} & \alpha h_D + h_M \sum_{j \in J} \tfrac{1}{M_j} \cdot \int_{l_j}^{u_j} \varphi(\beta) d\beta - 2\alpha h_M \sum_{j \in J} \tfrac{1}{M_j} \cdot \int_{l_j}^{u_j} \beta \cdot \varphi(\beta) d\beta + \\ & \alpha^2 \sum_{j \in J} (h_R - \tfrac{h_R}{R_j} + \tfrac{h_M}{R_j} + \tfrac{h_M}{M_j}) \cdot \int_{l_j}^{u_j} \beta^2 \cdot \varphi(\beta) d\beta. \end{aligned}$$

In comparison to formula (3.4) only a finite number of R and M has to be considered in order to determine the solution using policy II. The formerly required $R(\beta)$ which represents the optimal number of remanufacturing lots for any given yield fraction β has been replaced by R_j after separating the yield distribution into intervals in which only one R is optimal for each yield realization. Consequently, the same simplification holds for the number of manufacturing lots M. However, this solution can be further improved by varying the disassembly cycle length T which is done in the most sophisticated heuristic approach of this contribution.

Policy III

As the convexity of the expected total cost function of policy II (3.9) regarding the only remaining variable T cannot be proven for any possible yield distribution we face the fact that obtaining the optimal solution for this system cannot be guaranteed by using a simple local search algorithm. Nevertheless, we implemented such an algorithm that alters the disassembly cycle length T from its initial value of policy II in order to check whether the expected total cost increases or decreases. The expected total cost function is evaluated by applying the procedure of policy II for any chosen parameter T. The local search procedure stops when both an increase or a decrease of T results in an increasing expected total cost meaning that at least a local minimum has been found that improves the solution of policy II at the expense of an increased complexity.

The following section elaborates a numerical experiment in which all three introduced heuristic policies are tested in order to evaluate their performance in a stochastic yield environment.

3.5 Numerical experiment

The main objective of the numerical experiment conducted in this section is to evaluate the error that can be made when the simplest approach (policy I) is used compared to the more complex ones (policies II and III). In order to estimate the error, several numerical tests have been conducted using randomly generated instances. To our knowledge, no scientific contribution contains reliable and complete real life data for this specific problem setting. As the number of adequate test instances cannot be guaranteed in this case, Rardin and Uzsoy (2001) recommend to create an experimental design based on random test instances. Although they discuss the pitfalls of random test instances in detail, we have applied this procedure to provide a first insight into each policy's performance. All parameters required for the test instances are drawn from a discrete uniform distribution $DU(a, b)$ with a as the lower and b as the upper bound of the distribution. Some parameters are multiplied after the random draw with a constant term in order to obtain reasonable values. Table 3.1 lists all parameters that are randomly drawn in this experiment:

Tab. 3.1: Parameters generated randomly in numerical experiment

Parameter	Generation method
Demand rate	$\lambda \sim DU(1, 10) \cdot 100$
Return fraction	$\alpha \sim DU(6, 18) \cdot 0.05$
Setup cost for disassembly	$K_D \sim DU(0, 50)$
Setup cost for remanufacturing	$K_R \sim DU(1, 100)$
Setup cost for manufacturing	$K_M \sim DU(1, 100)$
Holding cost for used product	$h_D \sim DU(1, 10) \cdot 0.01$
Holding cost for remanufacturable component	$h_R \sim DU(5, 15) \cdot 0.01$
Holding cost for serviceable component	$h_M \sim DU(10, 20) \cdot 0.01$

The return fraction α, for instance, can take on values between 30 % and 90 %, only limited by the fact that the percentage must be an integer multiple of 5 %. Regarding the setup costs, we restricted the possible region on integer values between 0 and 50 for the disassembly process and 1 to 100 for setting up a remanufacturing or a manufacturing lot. For the disassembly lot, we established smaller values as these processes are done manually in some industrial applications and do not necessarily require a specific setup. With respect to the holding costs we implicitly assumed that the holding cost increase from level to level as more effort has been put into the components. This means that each randomly generated instance has to fulfill the presumed inequality $h_D < h_R < h_M$. From these probability distributions, 1,000 instances are drawn and tested for different yield distributions. In general, the yield distribution followed a symmetric beta-distribution within the limits β_l=0 and β_u=1. The parameter that altered the yield distribution is the coefficient of variation ρ that is changed in the limits between 0.05 and 0.55 which is motivated by our experience with an automotive remanufacturer regarding its yield fractions. While a ρ of 0.05 indicates that almost the entire probability mass is centered around the distribution's mean, a coefficient of variation of 0.55 indicates for a beta-distribution within the interval 0 to 1 an approximately uniform yield distribution.

All three introduced heuristic approaches have been tested for all instances. Figure 3.6 illustrates, for instance, the percentage deviation of the expected total costs of policies I and II. $\Delta_{I->II}$ denotes this percentage deviation and is calculated by $\Delta_{I->II}$=TC_I/TC_{II}-1. In detail, this deviation shows the expected percentage loss in performance if policy I (at which only the mean yield fraction is considered to represent the entire yield distribution) is applied instead of policy II. The deviation with respect to the coefficient of variation of the underlying yield distribution is presented with the aid of box plots that do not only show the maximum and minimum deviations but also where half of the deviations are located inside the shaded area around the median.

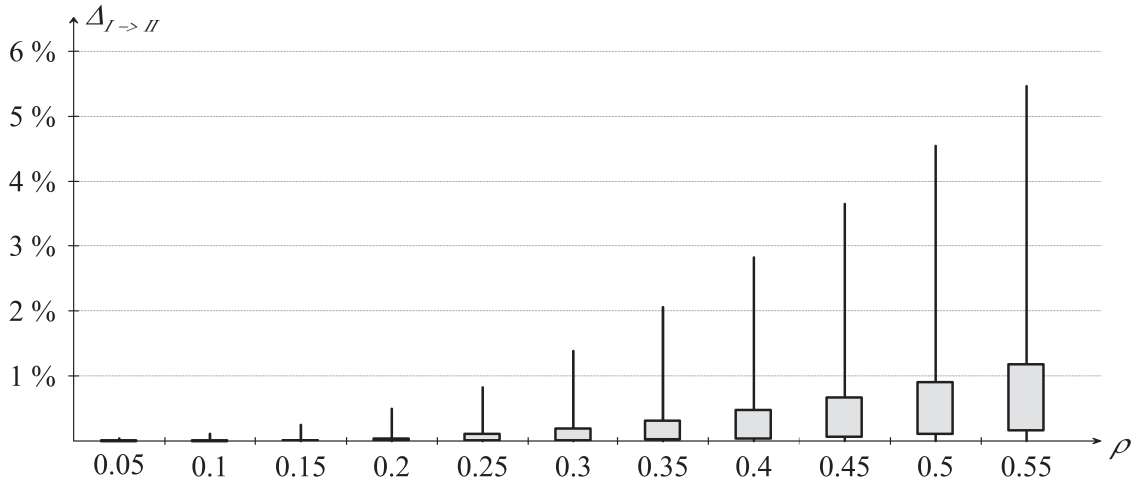

Fig. 3.6: Percentage deviation of policy I compared to policy II

For very small coefficients of variation that are characterized by the fact that almost the entire probability mass is centered around the mean yield, the deviation between policy I and policy II is almost negligible. The reason for that is easy to be found.

Although the yield distribution is defined in the interval between 0 and 1, the range of realizations that have a significant probability is very small. If the optimal number of remanufacturing and manufacturing lots per cycle that is determined by policy I is also optimal for a wide range of yield fractions around the distribution's mean both policies arrive at the same result. However, if the coefficient of variation grows larger the deviations increase as well. For an approximately uniformly distributed yield, for instance, the maximum deviation between policy I and II is around 5.4 %. On the other hand, the minimum deviation is 0 % which means that the optimal cycle pattern of policy I can be still optimal for every yield realization between 0 and 1 even for such a widespread distribution. Although many instances have been tested, the effect of every parameter on the deviation cannot be interpreted without doubt. Yet, some general trends can be derived from the experiments. For instance, it seems to be the case that the percentage gap between policy I and II increases as larger the return rate α becomes. Additionally, the different setup costs seem to influence this gap as well. For high setup costs for disassembly and remanufacturing (K_D and K_R) as well as for small setup costs for manufacturing (K_M) the observed percentage gap increases for a large coefficient of variation of the yield distribution. The same analysis can be conducted for the different holding cost parameters, too. The percentage gap between policy I and II increases if the holding costs h_D, h_R, and h_M deviate significantly. Furthermore, it can be said as larger the difference between R_{min} and R_{max} as well as the difference between M_{min} and M_{max} is as larger is the percentage gap. Finally, no considerable influence on the percentage gap can be observed for the demand per time unit λ.

Figure 3.7 presents the expected deviation of policy II from policy III which means that the cycle length T is varied in order to decrease the total cost function even further. By $\Delta_{II->III}$ this deviation is represented. Regarding the coefficients of variation the same can be observed as for the first examined deviation. For small coefficients of variation there is almost no improvement possible by changing the cycle length. On the other hand, for larger coefficients the percentage gap grows larger which means that an adaptation of T can improve the total cost function. However, these improvements are relatively small (in 97.4 % of all cases smaller than 1 % for ρ=0.55). Regarding the cost deviation between policy II and III, it is even more difficult (in comparison to

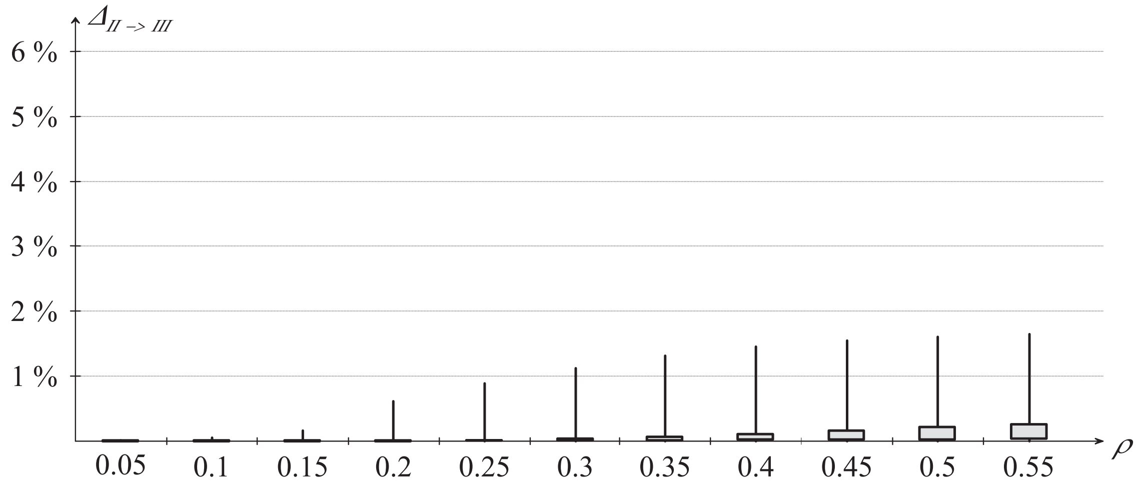

Fig. 3.7: Percentage deviation of policy II compared to policy III

the deviation between policy I and II) to define parameter areas at which the expected deviation is typically high or low. Yet, a general trend can be noticed. The largest deviations can be observed for instances with a large α and a wide spread of the holding cost levels. However, this observation cannot be generalized for all instances with this parameter constellation.

Another interesting question that can be analyzed with this numerical experiment is whether the adapted cycle length (obtained by local search) increases or decreases in comparison to the cycle length of policy I and II that remains constant for all coefficients of variation. In 69.1% of all instances the cycle length decreases while it increases in the remaining 31.9%. Therefore, no general conclusion can be drawn regarding this aspect as no specific parameter constellation can be identified that increases or decreases the cycle length in general[7].

3.6 Conclusion and outlook

This Chapter outlines an approach on how to handle deterministic and stochastic yield fractions within a multi-level remanufacturing system that considers the disassembly process explicitly. While being restricted to a single disassembly lot per cycle, simple

[7] Please refer to the Appendix (page 111.) for a detailled discussion on how to determine a lower and an upper bound for the disassembly cycle length T.

derivations are made with respect to the three necessary parameters, the optimal disassembly cycle length as well as the optimal number of remanufacturing and manufacturing lots per disassembly cycle. By examining both the stochastic and the deterministic case, the error that can be made by neglecting the underlying stochastics is evaluated. The numerical experiment in section 3.5 has confirmed a quite straightforward assumption. The less variability the random yield fraction has, the smaller is the error that is made by using the mean yield policy I instead of the more sophisticated ones. However, there are situations in which using the simple policy I results in performance losses of more than 5 %. Nevertheless, in most cases the decision maker will obtain fairly good results if he neglects the underlying yield distribution and follows the deterministic mean yield fraction approach of policy I. In this sense, a problem setting has been identified in which the influence of stochastic yields does not complicate the decisions to be made significantly.

Next, an outlook regarding future research efforts shall be given. The proposed model can be extended in several ways. For both the stochastic and the deterministic one, the option of allowing more than one disassembly lot per disassembly cycle is a promising extension of the model presented in this contribution. Especially for instances showing a small setup cost of disassembly this might provide a valuable option to decrease the average costs per time unit. Furthermore, it can be studied how a multi-product multi-component setting affects the decision making process in both environments since aspects like multiplicity (one component can be obtained by the disassembly of different product types) have to be incorporated. Another interesting topic that can be included in the analysis is a disposal option. This might be a worthwhile option if the setup costs of remanufacturing are quite high and the yield realization is very small. In the proposed model context, at least one remanufacturing process has to be set up in such a disassembly cycle. However, if there is a disposal option, the obtained components can be disposed of and total customer demand will be satisfied by newly manufactured components, i.e. the optimal number of remanufacturing lots R can be 0. As a last possible extension, all heuristic approaches can be tested not only for stochastically proportional yields but also for non-proportional yields. In order to achieve this objective, the yield fraction distribution cannot be modeled as a beta-distribution any more but has to be modeled for instance with a binomial distribution.

As discussed above, a number of different uncertainties can be found in a real life remanufacturing system. This Chapter has revealed that a possible yield uncertainty can be neglected in a multitude of problem instances when considering joint remanufacturing and manufacturing lot sizing decisions in a recovery system. This is a rather untypical result when real-life industrial applications such as the remanufacturing of car engines face stochastic yields in their process. To obtain good solutions for the lot sizing problem presented above, the only required information regarding the yield distribution is its mean value. By planning the lot sizes with this mean value, the error of neglecting stochastic yields can be reduced in most cases to less than 2 %.

Until now, the demand and return processes have been modeled as constant and continuous flows. Yet, in a real world recovery system this is typically not the case. The following Chapter addresses this issue by adapting the demand and return process to be dynamic and discrete. Moreover, we omit to include the stochastic yield process in the analysis of Chapter 4 for the sake of simplicity. The main objective of the subsequent Chapter is to improve a proposed heuristic solution to this problem setting.

3.7 Appendix

Derivation of the total cost function TC^D (3.1):

The total cost function TC^D minimizes the sum of all relevant setup and holding costs per time unit which is optimal in the model setting presented in Section 3.2. The decisions that need to be made in order to calculate the cost minimum consist of determining the disassembly cycle length T as well as the number of remanufacturing and manufacturing lots per disassembly cycle R and M, respectively. All three decision variables depend on both the setup and the holding costs. The total cost function TC^D is presented subsequently as it has been formulated in Section 3.3:

$$TC^D = \frac{SC^D}{T} + \frac{\lambda T}{2} \cdot HC^D$$

with $\quad SC^D = K_D + RK_R + MK_M \quad$ and
$HC^D = \alpha h_D + \frac{R-1}{R}\alpha^2\beta^2 h_R + (\frac{\alpha^2\beta^2}{R} + \frac{(1-\alpha\beta)^2}{M})h_M.$

The setup cost term SC^D contains all relevant setup costs multiplied with the number of respective lots that are set up in a disassembly cycle. As defined in the model setting, only one disassembly lot is allowed per cycle which leads to $1 \cdot K_D$. However, the number of remanufacturing lots R and manufacturing lots M has to be determined. Consequently, the setup costs for remanufacturing in a disassembly cycle are represented by $R \cdot K_R$ and the setup costs for manufacturing in a disassembly cycle by $M \cdot K_M$. Afterwards, the sum of all setup costs SC^D needs to be divided by T in order to determine the setup costs per time unit.

Regarding the holding costs, the three different stock levels are analyzed separately. Beginning with the used product level, one disassembly cycle is presented in Figure 3.8. As only one disassembly lot is allowed per cycle and the disassembly cycle length is a decision variable, this lot has the size of $\lambda\alpha T$.

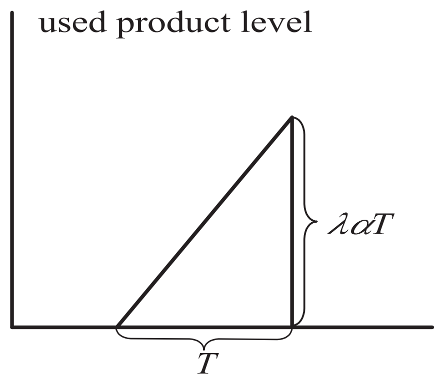

Fig. 3.8: Used product level for one disassembly cycle

The holding costs at the used product level per disassembly cycle can be determined as:

$$\frac{1}{2}T \cdot \lambda\alpha T \cdot h_D.$$

As the holding cost at the used product level need to be calculated per time unit, the formula simplifies to the following equation which represents the first cost component of HC^D:

$$\frac{T \cdot \lambda \alpha T \cdot h_D}{2 \cdot T} = \frac{\lambda T}{2} \cdot \alpha h_D.$$

After analyzing the holding cost at the used product level, the holding cost at the remanufacturables level are considered in the following. Assuming equal remanufacturing lot sizes (which is proven to be optimal on page 103), each remanufacturing lot has the size of $\frac{\lambda\alpha\beta T}{R}$. By initiating a remanufacturing lot one obtains $\frac{\lambda\alpha\beta T}{R}$ serviceable components which satisfy customer demand for $\frac{\alpha\beta T}{R}$ periods. Adjacent, another remanufacturing lot or a manufacturing lot has to be set up since no backlogs are allowed in the model. As the number of remanufacturing lots per cycle R is a decision variable, the remanufacturables inventory depends on the value of R. This dependency is visualized in Figures 3.9 and 3.10. While on the left hand side the remanufacturables inventory is displayed for $R = 2$, the right hand side presents the corresponding inventory for $R = 3$.

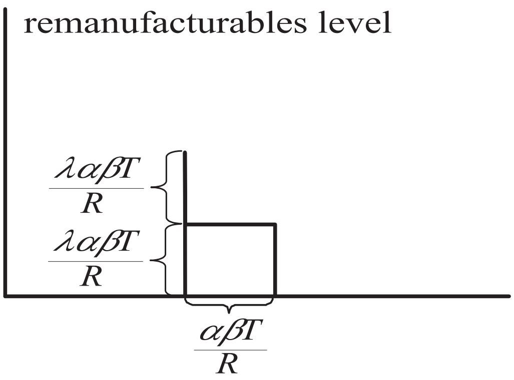

Fig. 3.9: Remanufacturables inventory for R=2

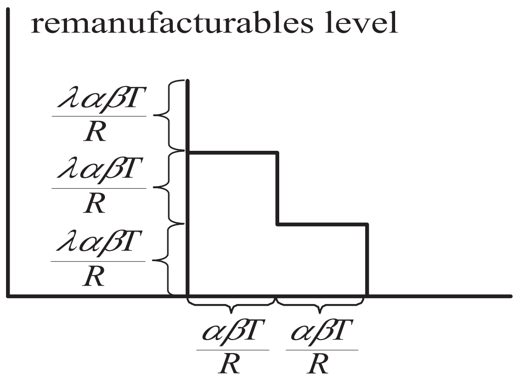

Fig. 3.10: Remanufacturables inventory for R=3

Depending on the number of remanufacturing lots R, the remanufacturables inventory area that is used to calculate the holding cost can be subdivided into several equally sized rectangles. For $R = 2$, as presented in Figure 3.9, only one rectangle of the size $\frac{\lambda\alpha^2\beta^2T^2}{R^2}$ needs to be evaluated. If $R = 3$ on the other hand, three rectangles of this size have to be considered. In general, the number of equally sized rectangles that has to be evaluated can be formulated as $\frac{R(R-1)}{2}$. Thus, the holding costs for the remanufacturables level per time unit can be formulated as:

$$\frac{R(R-1)}{2} \cdot \frac{\lambda\alpha^2\beta^2T^2}{R^2} \cdot h_R \cdot \frac{1}{T} = \frac{\lambda T}{2} \cdot \frac{R-1}{R}\alpha^2\beta^2 h_R.$$

Finally, the holding cost at the serviceables level has to be evaluated. Following the R remanufacturing lots, M equal manufacturing lots are set up at this level. While satisfying the fraction $\alpha\beta$ of customer demand by remanufacturing, $1-\alpha\beta$ of this demand has to be satisfied by manufacturing new components. Hence, each manufacturing lot has the size of $\frac{\lambda(1-\alpha\beta)T}{M}$ items and lasts for $\frac{(1-\alpha\beta)T}{M}$ time units. Figure 3.11 presents the serviceables inventory for a disassembly cycle with two remanufacturing and two manufacturing lots.

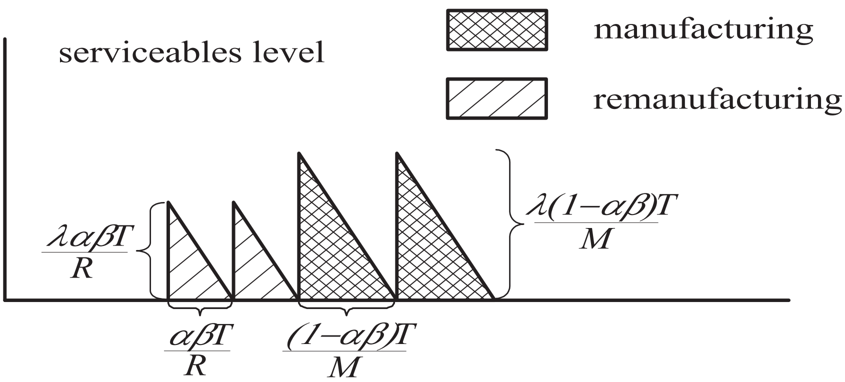

Fig. 3.11: Serviceables level for $R = 2$ and $M = 2$

The holding cost for the serviceables level per time unit can be formulated as follows:

$$(R \cdot \frac{1}{2} \cdot \frac{\lambda\alpha^2\beta^2T^2}{R^2} \cdot h_M + M \cdot \frac{1}{2} \cdot \frac{\lambda(1-\alpha\beta)^2T^2}{M^2} \cdot h_M) \cdot \frac{1}{T} = \frac{\lambda T}{2} \cdot (\frac{\alpha^2\beta^2}{R} + \frac{(1-\alpha\beta)^2}{M})h_M.$$

By determining the holding cost of the serviceables level the holding cost term HC^D is completed and can be used to formulate the total cost function TC^D.

Optimality of equal remanufacturing lots:

For a given cycle length T the total number of remanufactured components is given in the deterministic setting by $\lambda\alpha\beta T$. As a decision variable, the number of remanufacturing lots needs to be determined. Let $Q_{R,i}$ denote the lot size of remanufacturing lot i with $i = 1,..,R$. If all lot sizes are equal, each $Q_{R,i}$ contains exactly $\frac{\lambda\alpha\beta T}{R}$ units. This analysis proves that unequal remanufacturing lot sizes result in higher total cost than equal ones. Therefore, the remanufacturing lot sizes in a more general setting are described as:

$$Q_{R,i} = \frac{\lambda\alpha\beta T}{R} + \Delta_i \qquad \forall i = 1,..,R-1 \tag{3.10}$$

$$Q_{R,R} = \frac{\lambda\alpha\beta T}{R} - \sum_{i=1}^{R-1} \Delta_i. \tag{3.11}$$

The distortion from the equal lot sizes which is denoted for each remanufacturing lot by Δ_i lies within the range of $-\frac{\lambda\alpha\beta T}{R} \leq \Delta_i \leq \frac{(R-1)\cdot\lambda\alpha\beta T}{R}$ as all lot sizes have to be non-negative and cannot exceed the value of $\lambda\alpha\beta T$. In formula (3.11), the lot size $Q_{R,R}$ has been simplified using the fact that the sum of all distortions has to be zero, i.e. $\sum_{i=1}^{R} \Delta_i = 0$. As an illustration, Figure 3.12 presents the remanufacturables and the serviceables inventory for three equally sized remanufacturing lots.

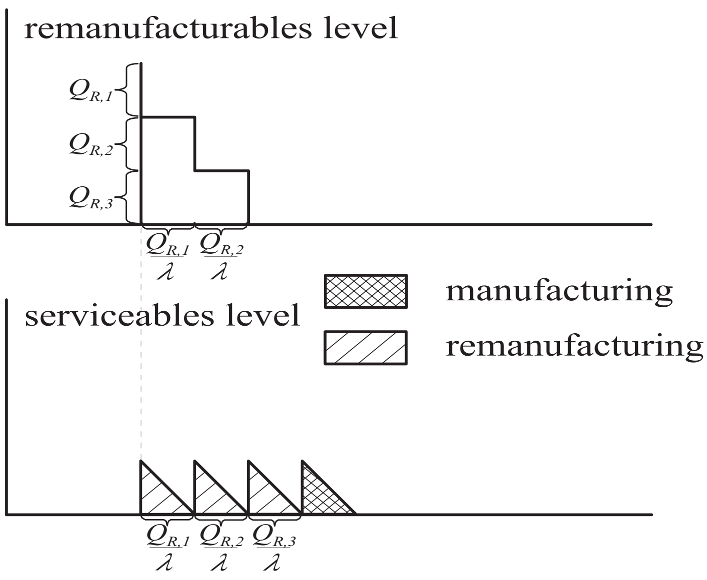

Fig. 3.12: Remanufacturables and serviceables inventory with three equally sized remanufacturing lots

As the remanufacturing lot sizes affect the holding cost of the remanufacturables and the serviceables level, both holding costs have to be analyzed. Starting with the remanufacturables level, let HC_{rem} denote the holding costs per time unit for the remanufacturables inventory. It can be determined by:

$$HC_{rem} = \frac{h_R}{\lambda T} \cdot (\sum_{i=1}^{R-1} Q_{R,i} \cdot \sum_{j=i+1}^{R} Q_{R,j}).$$

By $(\sum_{i=1}^{R-1} Q_{R,i} \cdot \sum_{j=i+1}^{R} Q_{R,j})/\lambda$ the total remanufacturables inventory per disassembly cycle is determined. While the first sum in this formula represents the width of the rectangles the second sum represents the corresponding heights. Using this formula for the example in Figure 3.12 leads to $Q_{R,1} \cdot (Q_{R,2} + Q_{R,3}) + Q_{R,2} \cdot Q_{R,3}$. This expression can be simplified using equations (3.10) and (3.11) and replacing the term $\lambda\alpha\beta T/R$ by X:

$$\begin{aligned}
HC_{rem} &= \frac{h_R}{\lambda T} \cdot \sum_{i=1}^{R-1} Q_{R,i} \cdot \left[\sum_{j=i+1}^{R-1} Q_{R,j} + Q_{R,R} \right] \\
&= \frac{h_R}{\lambda T} \cdot \sum_{i=1}^{R-1} (X + \Delta_i) \cdot \left[\sum_{j=i+1}^{R-1} (X + \Delta_j) + X - \sum_{j=1}^{R-1} \Delta_j \right] \\
&= \frac{h_R}{\lambda T} \cdot \sum_{i=1}^{R-1} (X + \Delta_i) \cdot \left[(R - i) \cdot X - \sum_{j=1}^{i} \Delta_j \right] \\
&= \frac{h_R}{\lambda T} \cdot \left[X^2 \cdot \sum_{i=1}^{R-1} (R - i) + X \cdot \sum_{i=1}^{R-1} \Delta_i \cdot (R - i) - X \cdot \sum_{i=1}^{R-1} \sum_{j=1}^{i} \Delta_j - \sum_{i=1}^{R-1} \Delta_i \cdot \sum_{j=1}^{i} \Delta_j \right].
\end{aligned}$$

It can be shown that $X \cdot \sum_{i=1}^{R-1} \Delta_i \cdot (R-i) - X \cdot \sum_{i=1}^{R-1} \sum_{j=1}^{i} \Delta_j$ is equal to 0. Furthermore, the sum $\sum_{i=1}^{R-1} (R - i)$ can be simplified to $\sum_{i=1}^{R-1} i$. Consequently, the holding cost in the remanufacturables inventory can be formulated as:

$$HC_{rem} = \frac{h_R}{\lambda T} \cdot \left[X^2 \cdot \sum_{i=1}^{R-1} i - \sum_{i=1}^{R-1} \Delta_i \cdot \sum_{j=1}^{i} \Delta_j \right]. \tag{3.12}$$

Let HC_{rem}^0 denote the holding cost of the remanufacturables level with equal remanufacturing lots, i.e. $\Delta_i = 0 \quad \forall i = 1, .., R$. Then, the difference in holding cost for the remanufacturables level if the remanufacturing lots are not equal can be expressed by the difference between HC_{rem} and HC_{rem}^0. This difference can be formulated depending on the distortions Δ_i:

$$\begin{aligned} HC_{rem} - HC^0_{rem} &= \frac{h_R}{\lambda T} \cdot \left[X^2 \cdot \sum_{i=1}^{R-1} i - \sum_{i=1}^{R-1} \Delta_i \cdot \sum_{j=1}^{i} \Delta_j - X^2 \cdot \sum_{i=1}^{R-1} i \right] \\ &= -\frac{h_R}{\lambda T} \cdot \sum_{i=1}^{R-1} \Delta_i \cdot \sum_{j=1}^{i} \Delta_j. \end{aligned} \tag{3.13}$$

It can be shown that this term is strictly negative if at least one Δ_i is not zero. Thus, the holding cost for the remanufacturables level always decrease whenever non-equal remanufacturing lots are initiated. The analysis is now put forth for the serviceables level. Let HC_{serv} denote the holding cost for the serviceables inventory without considering the holding cost for manufacturing lots as it does not depend on changes of the remanufacturing lot sizes.

$$\begin{aligned} HC_{serv} &= \frac{h_M}{2\lambda T} \cdot \left[Q^2_{R,R} + \sum_{i=1}^{R-1} Q^2_{R,i} \right] \\ &= \frac{h_M}{2\lambda T} \cdot \left[(X - \sum_{i=1}^{R-1} \Delta_i)^2 + \sum_{i=1}^{R-1} (X + \Delta_i)^2 \right] \\ &= \frac{h_M}{2\lambda T} \cdot \left[X^2 - 2X \sum_{i=1}^{R-1} \Delta_i + 2 \sum_{i=1}^{R-1} \Delta_i \sum_{j=1}^{i} \Delta_j - \sum_{i=1}^{R-1} \Delta_i^2 + \sum_{i=1}^{R-1} (X^2 + 2X\Delta_i + \Delta_i^2) \right] \\ &= \frac{h_M}{2\lambda T} \cdot \left[R \cdot X^2 + 2 \sum_{i=1}^{R-1} \Delta_i \sum_{j=1}^{i} \Delta_j \right]. \end{aligned}$$

In correspondence to the analysis of the remanufacturables level, HC^0_{serv} denotes the holding cost of the serviceables inventory if all remanufacturing lots are equally sized. Hence, the cost effect of a possible distortion can be calculated by the difference of $HC_{serv} - HC^0_{serv}$ which is presented in the following formula:

$$\begin{aligned} HC_{serv} - HC^0_{serv} &= \frac{h_M}{2\lambda T} \cdot \left[R \cdot X^2 + 2 \sum_{i=1}^{R-1} \Delta_i \sum_{j=1}^{i} \Delta_j - R \cdot X^2 \right] \\ &= \frac{h_M}{\lambda T} \cdot \sum_{i=1}^{R-1} \Delta_i \sum_{j=1}^{i} \Delta_j. \end{aligned} \tag{3.14}$$

This term proves that whenever the remanufacturing lots are not equal (which means that at least one Δ_i is not zero), the holding cost of the serviceables inventory increases. As the increase of holding cost at the serviceables level is always larger than the decrease of the holding cost at the remanufacturables level because of $h_M > h_R$, the total cost increase in any situation that is characterized by the fact that all remanufacturing lots in a disassembly cycle are not of equal size. This means as a conclusion that it is optimal to choose equal remanufacturing lot sizes in a disassembly cycle in this problem setting.

The differences to the results derived in the preceding Chapter 2 can be explained twofold. On the one hand, the problem setting presented there does not consider that the recovery process can be separated into a disassembly and a remanufacturing process. There, a two stage inventory system is analyzed with a continuous inflow of old products to the first stage and a continuous outflow from the second one. In this Chapter, the remanufacturing level with discrete inflows is compared to the serviceables inventory with a continuous outflow. This fact can be used to explain the different results because the basic flow pattern of the analysis has changed. On the other hand, we restrict this Chapter's problem setting to allow only one disassembly batch per cycle on the first stage of the system (the one with the continuous inflow of returns). The results derived above depend on this assumption. In this problem context, the option of initiating different remanufacturing lots only needs to be analyzed when more than one disassembly lot per cycle can be scheduled.

Optimality of equal manufacturing lots:

The analysis of manufacturing lots that are not equally sized is similar to the analysis of non-equal remanufacturing lots on the serviceables inventory level. Thus, a distortion of the manufacturing lot sizes leads always to higher holding costs on the serviceables level. We omit the presentation of the mathematics behind that conclusion as the analysis that led to the derivation of formula (3.14) can be applied.

Optimizing the relaxed total cost function TC^D:

If the number of remanufacturing and manufacturing lots needs not to be integer, the relaxed total cost function (3.1) can be solved to optimality by simple calculations. The partial derivatives of the total cost function with respect to all decision variables have to be obtained at first:

$$\begin{aligned}
\frac{\partial TC^D}{\partial T} &= \frac{\lambda}{2}(\alpha h_D+\alpha^2\beta^2(h_R+\frac{h_M-h_R}{R})+\frac{h_M}{M}(1-\alpha\beta)^2)-\frac{K_D+RK_R+MK_M}{T^2}=0\\
\frac{\partial TC^D}{\partial R} &= \frac{K_R}{T}-\lambda\alpha^2\beta^2 T\cdot\frac{h_M-h_R}{2R^2}=0\\
\frac{\partial TC^D}{\partial M} &= \frac{K_M}{T}-\lambda T h_M\cdot\frac{(1-\alpha\beta)^2}{2M^2}=0.
\end{aligned}$$

The solution of this equation system with respect to all decision variables and assuming that these decision variables have to be positive results in the optimal values for T, R, and M. We obtain

$$R^* = \alpha\beta T^*\cdot\sqrt{\frac{\lambda\cdot(h_M-h_R)}{2K_R}} \tag{3.15}$$

$$M^* = (1-\alpha\beta)T^*\cdot\sqrt{\frac{\lambda h_M}{2K_M}} \tag{3.16}$$

$$T^* = \sqrt{\frac{2(K_D+R^*K_R+M^*K_M)}{\lambda(\alpha h_D+\alpha^2\beta^2(h_R+\frac{h_M-h_R}{R^*})+\frac{h_M}{M^*}(1-\alpha\beta)^2)}}. \tag{3.17}$$

As equation (3.17) contains both optimal values of the remanufacturing and manufacturing lot sizes, this equation can be further simplified by inserting equations (3.15) and (3.16) into (3.17):

$$T^* = \sqrt{\frac{2(K_D+R^*K_R+M^*K_M)}{\lambda\left(\alpha h_D+\alpha^2\beta^2\left(h_R+\frac{h_M-h_R}{R^*}\right)+\frac{h_M}{M^*}(1-\alpha\beta)^2\right)}}$$

$$T^* = \sqrt{\frac{2\left(K_D+\alpha\beta T^*\sqrt{\frac{\lambda(h_M-h_R)K_R}{2}}+(1-\alpha\beta)\,T^*\sqrt{\frac{\lambda h_M K_M}{2}}\right)}{\lambda\left(\alpha h_D+\alpha^2\beta^2\left(h_R+\frac{1}{\alpha\beta T^*}\sqrt{\frac{2K_R(h_M-h_R)}{\lambda}}\right)+\frac{(1-\alpha\beta)}{T^*}\sqrt{\frac{2K_M h_M}{\lambda}}\right)}}$$

$$T^* = \sqrt{\frac{2K_D+T^*\sqrt{2\lambda}\left(\alpha\beta\left(\sqrt{K_R(h_M-h_R)}-\sqrt{K_M h_M}\right)+\sqrt{K_M h_M}\right)}{\lambda\alpha\left(h_D+\alpha\beta^2 h_R\right)+\frac{\sqrt{2\lambda}}{T^*}\left(\alpha\beta\left(\sqrt{K_R(h_M-h_R)}-\sqrt{K_M h_M}\right)+\sqrt{K_M h_M}\right)}}$$

$$
\begin{aligned}
(T^*)^2 &= \frac{2K_D + T^*\sqrt{2\lambda}\Theta}{\lambda\alpha\left(h_D + \alpha\beta^2 h_R\right) + \frac{\sqrt{2\lambda}}{T^*}\Theta} \text{ with } \Theta = \alpha\beta\left(\sqrt{K_R\left(h_M - h_R\right)} - \sqrt{K_M h_M}\right) + \sqrt{K_M h_M} \\
T^* &= \sqrt{\frac{2K_D}{\lambda\alpha\left(h_D + \alpha\beta^2 h_R\right)}}. \qquad (3.18)
\end{aligned}
$$

Equation (3.18) can be used to calculate the optimal number of remanufacturing and manufacturing lot sizes by inserting it in equations (3.15) and (3.16):

$$R^* = \beta \cdot \sqrt{\frac{\alpha K_D(h_M - h_R)}{K_R(h_D + \alpha\beta^2 h_R)}} \qquad (3.19)$$

$$M^* = (1 - \alpha\beta) \cdot \sqrt{\frac{h_M K_D}{K_M \alpha(h_D + \alpha\beta^2 h_R)}}. \qquad (3.20)$$

By inserting the optimal values of T^*, R^*, and M^* into the respective second derivatives it can be proven that the total cost function is in a minimum at this point.

Convexity of the total cost function TC^D:

In order to analyze the convexity of the total cost function it is necessary to show that the Hessian matrix of the total cost function TC^D is positive semidefinite. Therefore, the Hessian matrix H has to be set up and analyzed:

$$
H = \begin{bmatrix}
\frac{\partial^2 TC^D}{\partial T^2} & \frac{\partial^2 TC^D}{\partial T \partial M} & \frac{\partial^2 TC^D}{\partial T \partial R} \\
\frac{\partial^2 TC^D}{\partial M \partial T} & \frac{\partial^2 TC^D}{\partial M^2} & \frac{\partial^2 TC^D}{\partial M \partial R} \\
\frac{\partial^2 TC^D}{\partial R \partial T} & \frac{\partial^2 TC^D}{\partial R \partial M} & \frac{\partial^2 TC^D}{\partial R^2}
\end{bmatrix}.
$$

By calculating the three eigenvalues of the Hessian matrix one can see that two of them are always positive for positive values of T, R, and M. However, the third eigenvalue becomes negative for certain parameter values. We omit the presentation of the eigenvalues as they are of a very complex nature. Therefore, the total cost function TC^D is not entirely convex in all variables in all cases.

Derivation of equations (3.6) and (3.7):

In order to determine the transition yield rate for a switch from R to $R+1$ remanufacturing lots in a cycle the following manipulations have to be done

$$\frac{K_D+(R+1)K_R+MK_M}{T}+\frac{\lambda T}{2}\left(\alpha h_D+\frac{R}{R+1}\alpha^2\beta^2 h_R+\left(\frac{\alpha^2\beta^2}{R+1}+\frac{(1-\alpha\beta)^2}{M}\right)h_M\right)$$
$$=\frac{K_D+RK_R+MK_M}{T}+\frac{\lambda T}{2}\left(\alpha h_D+\frac{R-1}{R}\alpha^2\beta^2 h_R+\left(\frac{\alpha^2\beta^2}{R}+\frac{(1-\alpha\beta)^2}{M}\right)h_M\right)$$

$$\frac{\lambda T}{2}\left(\frac{R}{R+1}\alpha^2\beta^2 h_R+\frac{\alpha^2\beta^2}{R+1}h_M-\frac{R-1}{R}\alpha^2\beta^2 h_R-\frac{\alpha^2\beta^2}{R}h_M\right)$$
$$=\frac{K_D+RK_R+MK_M}{T}-\frac{K_D+(R+1)K_R+MK_M}{T}$$

$$\frac{\lambda T}{2}\alpha^2\beta^2\left(\frac{Rh_R+h_M}{R+1}-\frac{(R-1)h_R+h_M}{R}\right)=-\frac{K_R}{T}$$

$$\frac{\lambda T}{2}\alpha^2\beta^2\left(\frac{h_M-h_R}{R\cdot(R+1)}\right)=\frac{K_R}{T}$$

$$\beta(R)=\frac{1}{\alpha T}\cdot\sqrt{\frac{2K_R R(R+1)}{\lambda(h_M-h_R)}}$$

In the same manner, the transition yield rate $\beta(M)$ can be determined that represents a switch from M to $M+1$ manufacturing batches for a given cycle length T

$$\frac{K_D+RK_R+(M+1)K_M}{T}+\frac{\lambda T}{2}\left(\alpha h_D+\frac{R-1}{R}\alpha^2\beta^2 h_R+\left(\frac{\alpha^2\beta^2}{R}+\frac{(1-\alpha\beta)^2}{M+1}\right)h_M\right)$$
$$=\frac{K_D+RK_R+MK_M}{T}+\frac{\lambda T}{2}\left(\alpha h_D+\frac{R-1}{R}\alpha^2\beta^2 h_R+\left(\frac{\alpha^2\beta^2}{R}+\frac{(1-\alpha\beta)^2}{M}\right)h_M\right)$$

$$\frac{\lambda T}{2}\left(\frac{(1-\alpha\beta)^2}{M+1}h_M-\frac{(1-\alpha\beta)^2}{M}h_M\right)=\frac{MK_M-(M+1)K_M}{T}$$

$$-\frac{\lambda T}{2}\left(\frac{(1-\alpha\beta)^2 h_M}{M(M+1)}\right)=-\frac{K_M}{T}$$

$$1-2\alpha\beta+\alpha^2\beta^2=\frac{2K_M M(M+1)}{\lambda T^2 h_M}$$

$$\beta(M)=\frac{1}{\alpha}\pm\frac{1}{\alpha T}\cdot\sqrt{\frac{2K_M M(M-1)}{\lambda h_M}}$$

As α lies between zero and one, $1/\alpha$ is always larger than one. This means that only one of the possible $\beta(M)$ values is relevant. It is

$$\beta(M) = \frac{1}{\alpha} - \frac{1}{\alpha T} \cdot \sqrt{\frac{2K_M M(M-1)}{\lambda h_M}}.$$

Determining an upper and lower bound for the disassembly cycle length T in the presence of random yields:

This analysis focuses on determining an upper and lower bound for the disassembly cycle length T in a stochastic yield scenario which shall be denoted by T_{min} and T_{max}, respectively. As the question on how to define these bounds appears to be very complex, two different heuristic approaches are presented subsequently. Common to both approaches is the presumption that a stochastic yield distribution can be regarded as a combination of several equivalent deterministic problems. Thus, the approaches assume that the solution to a stochastic yield problem could also be obtained by combining the solutions of a number of deterministic problems. Hence, the lower and upper bound for the stochastic problem T_{min} and T_{max}, respectively, can be obtained by calculating the highest and lowest disassembly cycle lengths of all possible deterministic yield scenarios. Two approaches are introduced subsequently to determine these bounds. At first, the bounds are computed by using the relaxed deterministic total cost function TC^D from Section 3.3. Thereafter, the mixed-integer non-linear optimization approach of this Section is analyzed to propose an alternative approach. At the end, both approaches are evaluated using the data from the numerical study of section 3.5.

Analyzing the relaxed total cost function TC^D

In order to determine the upper and lower bound for the disassembly cycle length using the relaxed total cost function one needs to analyze the optimal disassembly cycle length T^* (3.18) for each possible deterministic yield fraction β. One can see that the first derivative of T^* with respect to β is strictly negative for $0 < \beta < 1$, i.e. the optimal disassembly cycle length decreases as larger the deterministic yield is. This can be mathematically proven as follows

$$\begin{aligned}\frac{dT^*}{d\beta} &= \frac{-\frac{1}{2}\cdot\sqrt{\frac{2\lambda\alpha K_D}{h_D+\alpha\beta^2 h_R}}\cdot 2\alpha\beta h_R}{\lambda\alpha(h_D+\alpha\beta^2 h_R)} \\ &= -\sqrt{\frac{2\alpha K_D}{\lambda(h_D+\alpha\beta^2 h_R)}}\cdot\frac{\beta h_R}{h_D+\alpha\beta^2 h_R} \quad < 0. \end{aligned} \tag{3.21}$$

Therefore, the upper bound of the disassembly cycle length for the relaxed total cost function T^{rel}_{max} (which is indicated by ^{rel}) can be observed for the lowest possible yield fraction, i.e. $\beta = 0$. On the other hand, the lower bound for the disassembly cycle length T^{rel}_{min} can be observed for the largest possible yield fraction, i.e. $\beta = 1$. By inserting $\beta = 0$ and $\beta = 1$ into equation (3.18) the values of T^{rel}_{min} and T^{rel}_{max} can be calculated by the following formulae:

$$T^{rel}_{min} = \sqrt{\frac{2K_D}{\lambda\alpha h_D}} \tag{3.22}$$

$$T^{rel}_{max} = \sqrt{\frac{2K_D}{\lambda\alpha(h_D+\alpha h_R)}}. \tag{3.23}$$

However, these results are derived under the assumption that the number of remanufacturing and manufacturing lots in a disassembly cycle needs not to be integer. This fact shows the heuristic character of this procedure as for example, if $\beta = \epsilon$ (with ϵ being a very small positive number) the optimal number of remanufacturing lots reveals only a very small but positive amount. This is of course not possible in the model context presented above as the number of remanufacturing lots must be a positive integer number. Hence, the smallest possible value for R is one which leads to the fact that the bounds T^{int}_{min} and T^{int}_{max} (where ^{int} indicates the exact mixed-integer approach) must be calculated in a different way. The next analysis focuses on this topic.

Analyzing the mixed-integer non-linear problem

If the number of remanufacturing and manufacturing lots has to be integer, the simple procedure presented above needs to be adjusted in order to cope with this change in the problem setting. Yet, the general approach of the first heuristic is applied to the second heuristic as well. This means that the disassembly cycle length is evaluated for a certain number of possible deterministic yield fractions and the minimum and maximum value become the lower and the upper bound for the cycle length. However, if the number of remanufacturing and manufacturing lots needs to be integer valued, the cycle length T cannot be formulated as a continuous function with respect to the yield β as a switch in either R or M results in a discontinuity of the total cost function. By analyzing these discontinuities as well as the function $T(\beta)$ between these discontinuities, heuristic values for the lower and upper bound of the disassembly cycle length can be determined.

However, the formulation of an algorithm that can handle this specific problem in an efficient manner is very complex. Therefore, we present a simpler approach on how to deal with this problem setting. This approach solves q deterministic problems between the smallest and largest yield fraction using formula (3.2). It shall be mentioned that the interval between two consecutively examined yield fractions is always $1/(q-1)$, as a yield distribution is generally defined between 0 and 1. The following pseudocode can be applied in order to obtain the upper and lower bound for the disassembly cycle length:

For i **= 1 to** q

$\beta = (i-1)/(q-1)$

calculate T_i by using equation (3.2) from the deterministic model of Section 3.3

Next i

$$T_{min}^{int} = \min_i(T_i), \quad T_{max}^{int} = \max_i(T_i)$$

The numerical study conducted in section 3.5 provides a data set of 1000 instances. Both heuristic approaches, the relaxed TC (total cost) approach as well as the integer TC approach are tested for these instances in order to evaluate their performance. Therefore, the actual minimum and maximum cycle lengths (denoted by T^*_{min} and T^*_{max}) for each instance are obtained by applying policy III to all tested yield distributions for all instances. The actually observed minimum values are afterwards compared to the lower bounds T^{rel}_{min} and T^{int}_{min} that are calculated by both heuristic approaches. The left hand side of Table 3.2 summarizes the results of these experiments. The right hand side of this Table illustrates the comparison of the calculated upper bounds T^{rel}_{max} and T^{int}_{max} with the actually observed ones.

Tab. 3.2: Performance of the relaxed and integer total cost approach regarding their estimations of the minimum and maximum disassembly cycle length

	percentage of instances with $T^{rel}_{min} < T^{*}_{min}$	percentage of instances with $T^{rel}_{max} > T^{*}_{max}$
relaxed TC approach	100 %	13.6 %
	percentage of instances with $T^{int}_{min} < T^{*}_{min}$	percentage of instances with $T^{int}_{max} > T^{*}_{max}$
integer TC approach (q=10)	90.7 %	100 %
integer TC approach (q=20)	96.9 %	100 %
integer TC approach (q=50)	98.4 %	100 %
integer TC approach (q=100)	98.8 %	100 %
integer TC approach (q=10000)	99.1 %	100 %

It can be seen that the performance of the relaxed TC approach is ambivalent. While the minimum cycle length is always estimated correctly by formula (3.22), the maximum cycle length T_{max} is frequently underestimated by formula (3.23). By incorporating the fact that the number of remanufacturing and manufacturing lots must be integer, the performance of the integer TC approach can be described as very good. The actually observed upper bounds T_{max} have never been underestimated even for a very small number of calculations. The lower bounds T_{min}, on the other hand, seem to benefit from an increasing number of calculations q. However, the performance of only 10 calculations has already been very good (90.7 % of all estimations are correct). Although the general performance of the integer TC heuristic seems to improve with an increasing number of calculations, the general heuristic approach can be observed by the fact that even if q is very large not all lower bounds are estimated correctly. Yet, the percentage error that is made by the false estimation of the integer TC approach is rather small. The average error over the 9 instances for which the bounds could not be calculated correctly is around 0.463% with a maximum deviation of 0.6 % (for q=10000). To conclude this section, it can be said, that the best method for calculating the lower bound of the disassembly cycle length is to apply the relaxed TC approach using formula (3.22). For estimating the upper bound of the disassembly cycle length, on the other hand, the integer TC approach should be used as it (even for low q values) always estimates the actually observed upper bound of all instances correctly.

4. A new Silver-Meal based heuristic for the single-item dynamic lot sizing problem with returns and remanufacturing

4.1 Introduction

Due to the increasing environmental awareness of firms and the public, the research field of reverse logistics has grown steadily over the past decades[1]. By analyzing not only the forward flow of products from a firm to its customers but also including the corresponding backward flow from the customers to the firm, this research area provides valuable insights on how these flows can be managed efficiently. Among many options (see, e.g., Thierry et al., 1995, for an overview on different alternatives), remanufacturing has been well established in several industries (as has been reported in Kumar and Putnam, 2008). When including remanufactured products in their product portfolio, firms take back products from their customers, rework them to a sufficient condition in order to resell them afterwards. This saves not only a part of the value embedded in the original product but also reduces the demand for natural resources and landfill space substantially (see, for instance, de Brito and Dekker, 2004). In industry, the process of remanufacturing is affected by many stochastic influences as has been depicted, for instance, by Guide (2000) as well as Inderfurth and Langella (2006). As these influences complicate the underlying problem significantly, this Chapter neglects any uncertainty and presents an entirely deterministic system.

[1] This Chapter is based on the work titled 'A new Silver-Meal based heuristic for the single-item dynamic lot sizing problem with returns and remanufacturing' that is accepted for publication in the *International Journal of Production Research* (see Schulz, 2011b).

By assuming setup costs for replenishment orders and holding costs for carrying products in different conditions, a lot sizing problem arises. Such a problem has been analyzed thoroughly for the case of static and continuous demand and return rates. Please refer to Minner and Lindner (2004) or to Chapter 2 for a brief literature review. However, the case of dynamic and discrete demands and returns has not achieved that much attention in the recent literature. Teunter et al. (2006) introduce a dynamic lot sizing model with returns and remanufacturing and distinguish between the case of joint and separate setup cost for the replenishment sources remanufacturing and manufacturing. They test in a large numerical experiment three well-known heuristic approaches that are adapted from the single-item dynamic lot sizing problem without returns. In both model settings, the Silver-Meal based heuristic has been shown to be the best heuristic resulting in an average deviation of 3% from the optimal solution in the joint and 8.3% in the separate setup cost setting. Using heuristics to handle these problems has been motivated by the fact that the authors conjecture the underlying problem of the separate setup cost setting to be NP-hard.

Several other contributions have been made to this specific research field whereas only two shall be mentioned exemplarily. Richter and Sombrutzki (2000) discuss the dynamic lot sizing problem with returns and remanufacturing and analyze a situation in which a sufficiently large number of returned products is available, i.e. the entire demand could be met by solely remanufacturing returned products. They proof that the zero-inventory property known from the dynamic lot sizing problem without returns and remanufacturing must hold in such an environment. Furthermore, they apply a Silver-Meal based algorithm to illustrate the stability of its solution. This Chapter employs a Silver-Meal based algorithm to the situation when the entire demand can only be met by a mix of remanufacturing and manufacturing. As illustrated by Teunter et al. (2006), the zero-inventory property needs not to be valid in such a setting. Pan et al. (2009) extend the analysis of Teunter et al. (2006) by including a disposal option for returned products and by restricting production, remanufacturing, and disposal capacities. They illustrate different problem formulations and elaborate dynamic programming algorithms to solve some of these problems to optimality.

This Chapter proposes a generalization of the Silver-Meal based heuristic introduced by Teunter et al. (2006) for the separate setup cost setting (without disposal option and restricted capacities) by applying methods known from the corresponding static problem. Furthermore, a simple improvement heuristic is applied to the solution obtained to enhance the heuristic's performance. The remainder of this Chapter is organized as follows. Section 4.2 presents the basic assumptions of the model analyzed in this Chapter and describes some solution methods for the underlying problem context. Next to a mixed-integer linear program the Silver-Meal based heuristic introduced in Teunter et al. (2006) and our extension are depicted in this section. Both heuristics are tested extensively in a numerical study in the subsequent Section 4.3. Afterwards, Section 4.4 points out an improvement heuristic and tests its ideas in a numerical experiment. Finally, the last section concludes this Chapter and gives a short outlook on future research opportunities.

4.2 Model formulation and proposed solution methods

4.2.1 Basic assumptions and mixed-integer linear program

In their contribution, Teunter et al. (2006) introduce a dynamic lot sizing model with separate setup costs for remanufacturing and manufacturing as an extension of the well-known Wagner/Whitin model (see Wagner and Whitin, 1958). The basic assumptions of this modeling approach are as follows. As depicted in Figure 4.1, we consider an original equipment manufacturer (OEM) that sells one product over a planning horizon of T periods. In each period $t = 1, .., T$ customers demand a discrete and known amount of this product. The demand in each period is further on denoted by d_t. The OEM provides each customer the opportunity to return her product if it is broken or when she has no further use for it. Whenever a product is returned to the OEM it is inspected whether it can be sufficiently remanufactured. All returns that pass the inspection (which will be denoted by r_t) are brought to the used product inventory. Per time unit a recoverable product incurs a holding cost of h_R while disposing it of preliminarily is assumed to be prohibitively expensive. If required, the OEM can (by paying the setup cost K_R) remanufacture $Q_{R,t}$ returned products in period t in order to bring them to an as-good-as-new condition. Recovery is always successful. After remanufacturing, the recovered products are brought to the final product inventory from which customer demand is satisfied. Yet, as it is not possible to serve the entire demand from remanufacturing returned products, the OEM can replenish his final product inventory alternatively by manufacturing $Q_{M,t}$ products in period t. Setting up a manufacturing lot in period t incurs a setup cost of K_M while holding a final product for one period in the respective inventory costs h_M. Finally, the inventory level at the end of period t is denoted by $y_{R,t}$ for the used product and $y_{M,t}$ for the final product inventory.

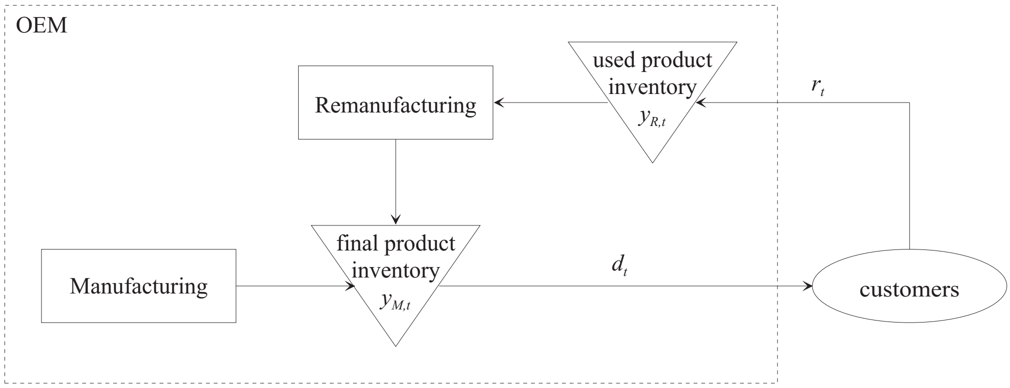

Fig. 4.1: Dynamic lot sizing model with returns and remanufacturing

By means of mixed-integer linear programming this model can be solved to optimality. Next to the notation introduced above two more decision variables are required. If a remanufacturing lot is initiated in period t (i.e. $Q_{R,t} > 0$), the binary decision variable $\gamma_{R,t}$ becomes one. However, if $Q_{R,t} = 0$ the decision variable $\gamma_{R,t}$ remains zero. Likewise, $\gamma_{M,t}$ is set to one when a manufacturing lot is produced in period t and to zero if no product needs to be manufactured. The optimization model can be formulated as:

$$\min C = \sum_{t=1}^{T} (K_R \cdot \gamma_{R,t} + K_M \cdot \gamma_{M,t} + h_R \cdot y_{R,t} + h_M \cdot y_{M,t}) \tag{4.1}$$

s.t.:

$$y_{R,t} = y_{R,t-1} + r_t - Q_{R,t} \qquad \forall t = 1, .., T \tag{4.2}$$

$$y_{M,t} = y_{M,t-1} + Q_{R,t} + Q_{M,t} - d_t \qquad \forall t = 1, .., T \tag{4.3}$$

$$
\begin{aligned}
Q_{R,t} \leq\ & \zeta \cdot \gamma_{R,t} && \forall t = 1,..,T && (4.4)\\
Q_{M,t} \leq\ & \zeta \cdot \gamma_{M,t} && \forall t = 1,..,T && (4.5)\\
y_{R,0} =\ & y_{M,0} = 0 && && (4.6)\\
\gamma_{R,t}, \gamma_{M,t} \in\ & \{0,1\} && \forall i = 1,..,T &&\\
y_{R,t}, y_{M,t}, Q_{R,t}, Q_{M,t} \geq\ & 0 && \forall i = 1,..,T &&
\end{aligned}
$$

The objective function (4.1) minimizes the sum of all relevant setup and holding costs. Constraints (4.2) and (4.3) represent inventory balance equations that describe the inventory at the end of period t as the inventory at the beginning of this period plus its inflows and minus its outflows. In order to ensure that setup costs have to be paid whenever a lot is scheduled, restrictions (4.4) and (4.5) have to be established whereas ζ needs to be a sufficiently large number (e.g. the sum of all demands during the planning horizon). Without loss of generality, the initial inventories in both stocks are set to zero by imposing constraint (4.6). Finally, non-negativity and binary constraints have to be defined as well to assure validity of the decisions made. Interestingly, the zero-inventory property that holds for a dynamic lot sizing model without returns and remanufacturing does not have to be valid in this model setting (as has been discussed by Teunter et al., 2006), i.e. it can be optimal to schedule a (re)manufacturing lot in period t even when the final product inventory at the beginning of t is not depleted. This extends the results of Richter and Sombrutzki (2000) who proved the zero-inventory property to hold when there is a sufficiently large number of returned products in the used product stock at the beginning of the planning horizon. Moreover, Teunter et al. (2006) conjecture that the underlying optimization problem is NP-hard, i.e. it becomes very difficult to obtain the optimal solution for a long planning horizon. Hence, they propose several heuristic algorithms on how to handle this problem. In a large numerical study, the Silver-Meal based heuristic which will be introduced subsequently revealed the best average performance when compared to the optimal solution.

4.2.2 The adapted Silver-Meal heuristic by Teunter et al. (2006)

Unfortunately, the original Silver-Meal heuristic (please refer to Silver and Meal, 1973, for details) cannot be applied to the model context presented above as the final product inventory from which all customer demands are satisfied can be replenished from two sources: manufacturing and remanufacturing. Thus, Teunter et al. (2006) adapted the original Silver-Meal heuristic to include both sources in form of manufacturing (option 1) as well as remanufacturing and manufacturing (option 2) in the decision-making process. The basic idea of clustering the entire planning horizon into smaller time windows (starting in period τ and ending in period z) and choosing those time windows with the smallest cost per period is kept. However, both options that will be described subsequently assume the zero-inventory property to hold.

Option 1: Manufacture only

When applying this option the entire demand in a time window is satisfied by initiating a manufacturing run in period τ. Its lot size would be

$$Q_{M,\tau} = \sum_{i=\tau}^{z} d_i. \tag{4.7}$$

The associated cost per period for the entire time window (which will be denoted as $C^1_{\tau,z}$) contains the setup cost for scheduling a manufacturing lot in τ as well as the cost for carrying products in the final product inventory. Furthermore, the cost for holding the returned products in stock need to be taken into account as well. The total cost per period for option 1 can, hence, be formulated by using equations (4.2) for determining $y_{R,t}$ and equations (4.3) for $y_{M,t}$ as

$$C^1_{\tau,z} = \frac{K_M + h_M \cdot \sum\limits_{t=\tau}^{z} y_{M,t} + h_R \cdot \sum\limits_{t=\tau}^{z} y_{R,t}}{z - \tau + 1}. \tag{4.8}$$

Option 2: Remanufacture (and manufacture if necessary)

The second option introduced by Teunter et al. (2006) seeks to remanufacture in period τ. Yet, as the amount of used products might not be sufficient to cover the entire demand up to period z, a manufacturing lot can be initiated in τ if necessary. Thus, both lot sizes depend on the available number of used products in τ which is by definition $y_{R,\tau-1} + r_\tau$. Both lot sizes for this period are presented in the following formulae:

$$Q_{M,\tau} = \max\left(\sum_{t=\tau}^{z} d_t - y_{R,\tau-1} - r_\tau, 0\right), \qquad Q_{R,\tau} = \min\left(y_{R,\tau-1} + r_\tau, \sum_{t=\tau}^{z} d_t\right). \quad (4.9)$$

Forcing the possibly required manufacturing lot to be scheduled in τ can result in an inefficient solution when there is no immediate demand for at least one of the manufactured products. Hence, all products manufactured in τ will be held in the final product inventory unnecessarily until they are needed. An opportunity to overcome this deficiency will be presented later in this section.

Next to the holding cost for the used and final product stock, the cost per period for the second option $C^2_{\tau,z}$ can contain both setup costs. As a manufacturing lot is only needed when the number of recoverable products is not sufficient, the binary variable $\gamma_{M,\tau}$ represents this fact by being one if a manufacturing run is required and zero else. Therefore, the cost per period for the second option can be formulated as

$$C^2_{\tau,z} = \frac{K_R + K_M \cdot \gamma_{M,\tau} + h_M \cdot \sum_{t=\tau}^{z} y_{M,t} + h_R \cdot \sum_{t=\tau}^{z} y_{R,t}}{z - \tau + 1}. \quad (4.10)$$

For each time window, $C^1_{\tau,z}$ is compared to $C^2_{\tau,z}$ and the smaller one is chosen. Moreover, the basic idea of the Silver-Meal heuristic is applied which means that a time window is extended as long as the smaller cost of both options does not increase. Further on, the heuristic approach introduced by Teunter et al. (2006) is referred to as the SM_2 heuristic since two distinct options are evaluated. Teunter et al. (2006) have tested this heuristic extensively in their contribution. As a result of their numerical study a mean deviation to the optimal solution of 8.3% has been observed over all instances. By generalizing the SM_2 heuristic with two additional options derived from the results of the corresponding static model, we enhance the heuristic's performance. This approach (which will be further on denoted as the SM_4 heuristic) is presented in the following.

4.2.3 The SM_4 heuristic

Although the dynamic lot sizing model with returns and remanufacturing has not been analyzed extensively in literature so far, the corresponding static model (with constant demand and return rates) has received much more attention. Among many contributions, two shall be mentioned explicitly. In his work, Schrady (1967) is the first author who examines this model context. His option on how to handle this problem effectively is to create a cyclic pattern that is repeated over the entire infinite planning horizon. This cyclic pattern begins with a manufacturing lot and is always followed by a constant number of remanufacturing lots R. Teunter (2001) generalizes these findings by introducing cycles that commence with one remanufacturing lot which is always succeeded by a constant number of manufacturing lots M. He argues as well that in order to be efficient each cycle should have either one remanufacturing or one manufacturing lot. As this provides very good solutions to the static problem, both cyclic patterns can be incorporated into the dynamic lot sizing model with returns and remanufacturing which is presented subsequently. While the third option analyzes time windows with a manufacturing lot in τ that is followed by remanufacturing lots in later periods, a time window in the fourth option commences with a remanufacturing lot in τ that is succeeded by a number of manufacturing lots. Two promising effects can be observed when applying both additional options. At first, by considering more than one lot in each time window the used product inventory which is a critical cost factor can be controlled more accurately. Furthermore, contrary to the first two options the zero-inventory property is only presumed to hold for the first period of a time window but not within each time window any more. Hence, a (re)manufacturing lot can be scheduled although the initial final product inventory of the period under consideration is not zero.

Option 3: Manufacture first, remanufacture (in multiple lots) later

When applying this option, a manufacturing lot is scheduled in τ that is followed by one or more remanufacturing lots in the consecutive periods $\tau + 1$ to z. As the amount of products available in the used product stock needs not to be sufficient, the manufacturing lot in period τ must replenish the unavailable products. The number of unavailable products in each period t ranging from $\tau + 1$ to z (which will be referred to as the net requirement NR_t) can be determined as

$$NR_t = \sum_{i=\tau}^{t} (d_i - r_i) - y_{R,\tau-1} \qquad \forall t = \tau+1,..,z. \tag{4.11}$$

As a manufacturing lot has to be scheduled in period τ and no remanufacturing takes place in that period, the lot size $Q_{M,\tau}$ cannot be smaller than d_τ as the entire demand in the first period of the time window has to be met. On the other hand, this lot must be able to complement all unavailable products and corresponds therefore at least to the maximum of all net requirements. Calculating the manufacturing lot size for period τ differs from equation (4.9) as the timing of all returns and demands has to be taken into account for this option. Thus,

$$Q_{M,\tau} = \max\left(d_\tau, \max_{t=\tau+1,..,z} (NR_t)\right), \qquad Q_{R,\tau} = 0. \tag{4.12}$$

In all consecutive periods of the time window under consideration no manufacturing lot will be set up. However, as the amount manufactured in period τ can be sufficient to satisfy the customer demand at least partly between period $\tau+1$ and z, only the actually required products are remanufactured in these periods in order to avoid unnecessary holding cost for the final product inventory. By computing $Q_{M,\tau}$ using equation (4.12) it is ensured that in every period between $\tau+1$ and z enough products are available in the used product stock to be remanufactured. The resulting lot sizes for the remaining time windows can be visualized as in equation (4.13). After determining the relevant inventories $y_{R,t}$ and $y_{M,t}$ using equations (4.2) and (4.3), the total cost per period can be calculated as in (4.14). We obtain

$$Q_{M,t} = 0, \quad Q_{R,t} = \max\left(\sum_{i=\tau}^{t} d_i - \sum_{i=\tau}^{t-1} Q_{R,i} - Q_{M,\tau}, 0\right) \quad \forall t = \tau+1,..,z \tag{4.13}$$

$$C^3_{\tau,z} = \frac{\sum_{t=\tau}^{z} \gamma_{R,t} \cdot K_R + K_M + h_M \cdot \sum_{t=\tau}^{z} y_{M,t} + h_R \cdot \sum_{t=\tau}^{z} y_{R,t}}{z - \tau + 1}. \tag{4.14}$$

After creating a first initial solution for option 3 using formulae (4.12) and (4.13) it must be noticed that the total cost per period of this option (denoted by C_{ini}) can be very high. This is especially the case when a long time window is examined and the setup cost K_R is large. Therefore, a greedy algorithm has been formulated in addition that commences in $\tau+1$ and checks two possible improvement opportunities for each remanufacturing lot. Common to both opportunities I and II is that all products obtained in the remanufacturing lot under consideration (which has been originally scheduled in period k and contains $Q_{R,k}$ products) are replenished alternatively. Therefore, no remanufacturing lot is scheduled in period k in order to save the setup costs incurred. First, the potential cost saving is evaluated if the manufacturing lot in τ is increased by $Q_{R,k}$. Since this decision affects all remanufacturing lots between $\tau + 1$ and k, formula (4.13) is applied to update the corresponding lot sizes. The second opportunity comprises the option to increase the last remanufacturing lot before period k (which is scheduled in period l) by $Q_{R,k}$ products. In order to do that, at least $Q_{R,k}$ products need to be available in the used product stock in period l. On the other hand, if the used product stock does not contain enough recoverable products the difference is manufactured additionally in period τ and again all remanufacturing lots that are affected by this decision are determined using formula (4.13). Obviously, this option cannot be evaluated for the first remanufacturing lot in a time window. Both improvement opportunities are examined for each remanufacturing lot between $\tau + 1$ and z, i.e. at most $2 \cdot (z - \tau)$ different schedules are analyzed. For each schedule, the total cost is calculated by using formula (4.14) and afterwards compared to C_{ini}. The schedule yielding the largest cost saving is chosen and the entire proceeding is repeated until no further improvement can be achieved. Finally, after the greedy local search has been applied, a number of remanufacturing lots R succeed one manufacturing lot. To illustrate the third option in greater detail, the following pseudocode can be implemented as in Figure 4.2.

Option 4: Remanufacture first, manufacture (in multiple lots) later

This option seeks to establish a time window in which a remanufacturing run is started in period τ which is followed by at least one manufacturing lot in the consecutive periods. By assumption, the entire used product inventory is remanufactured in the first period of the time window τ and no manufacturing batch is set up. Obviously, if the number of available recoverable products in period τ is not sufficient to meet the demand of that period d_τ, option 4 cannot be applied and option 2 provides the only solution incorporating a remanufacturing lot in τ. On the other hand, whenever

Fig. 4.2: Pseudocode for Option 3: Manufacture first, remanufacture later

Step 1: Find initial schedule

Determine net requirements using equation (4.11)

Determine $Q_{M,\tau}$ and $Q_{R,\tau}$ using equation (4.12)

Determine $Q_{M,t}$ and $Q_{R,t}$ using equation (4.13)

Determine $y_{M,t}$ and $y_{R,t}$ using equations (4.2) and (4.3)

$C_{ini} = C^3_{\tau,z}$

Step 2: Improve the initial schedule for periods $\boldsymbol{\tau}$ to $\boldsymbol{z}$

For $k = \tau + 1$ to z

    If $Q_{R,k} > 0$ then

        $Q_{M,\tau} = Q_{M,\tau} + Q_{R,k}, \quad Q_{R,k} = 0$

        For $i = \tau + 1$ to k

            Update $Q_{R,i}$ using equation (4.13)

            Update $y_{M,t}$ and $y_{R,t}$ using equations (4.2) and (4.3)

        Next i

        Determine $\Delta C_I(k) = C^3_{\tau,z} - C_{ini}$

        Reset initial schedule

        Find period l (period of the last remanufacturing lot before period k)

        If $y_{R,l} \geq Q_{R,k}$ then

            $Q_{R,l} = Q_{R,l} + Q_{R,k}, \quad Q_{R,k} = 0$

        Else

            $Q_{M,\tau} = Q_{M,\tau} + (Q_{R,k} - y_{R,l}), \quad Q_{R,l} = Q_{R,l} + y_{R,l}, \quad Q_{R,k} = 0$

        End If

        Update $y_{M,t}$ and $y_{R,t}$ using equations (4.2) and (4.3)

        Determine $\Delta C_{II}(k) = C^3_{\tau,z} - C_{ini}$

    End If

Next k

Step 3: Implement the best option

If $\min_{k \in \{\tau+1,\ldots,z\}} (\Delta C_I(k), \Delta C_{II}(k)) < 0$ then

    Implement the best schedule which becomes the updated initial schedule

    $C_{ini} = C_{ini} + \min_{k \in \{\tau+1,\ldots,z\}} (\Delta C_I(k), \Delta C_{II}(k)), \quad$ Goto Step 2:

End If

at least one manufacturing lot is required to satisfy the demand up to period z and $Q_{R,\tau} > d_\tau$, option 2 will be always dominated by option 4 because the holding cost for the final product inventory is smaller. This gives for period τ

$$Q_{M,\tau} = 0, \qquad Q_{R,\tau} = y_{R,\tau} + r_\tau. \tag{4.15}$$

In order to create an initial solution to this option, the lot sizes of the remaining periods have to be determined as well. In each period from $\tau + 1$ to z all missing parts are manufactured as there are no further remanufacturing lots allowed in this time window. The respective formulae are:

$$Q_{M,t} = \max\left(\sum_{i=\tau}^{t} d_i - \sum_{i=\tau}^{t-1} Q_{M,i} - Q_{R,\tau}, 0\right), \qquad Q_{R,t} = 0 \qquad \forall t = \tau+1, .., z. \tag{4.16}$$

Similar to option 3, the initial solution can be quite expensive if a large setup cost K_M prevails. Therefore, a greedy algorithm can be used again to search for possible cost reductions. In contrast to the third option, this algorithm reviews all manufacturing lots. It begins by checking whether it would be less expensive to combine the second and the third manufacturing lot of the time window and proceeds in this manner (merging the third and the fourth manufacturing lot, ...) to the end of the corresponding time window. The alternative revealing the largest cost reduction is implemented and the proceeding is restarted until no further cost reductions are possible. We omit the presentation of this algorithm as its general structure is similar to the one presented for option 3. After applying this algorithm, one remanufacturing lot is followed by a number of manufacturing lots M which can be used to determine the associated cost per period of the fourth option:

$$C^4_{\tau,z} = \frac{K_R + \sum_{t=\tau}^{z} \gamma_{R,M} \cdot K_M + h_M \cdot \sum_{t=\tau}^{z} y_{M,t} + h_R \cdot \sum_{t=\tau}^{z} y_{R,t}}{z - \tau + 1}. \tag{4.17}$$

Including options 3 and 4 into the decision-making process extends the original Silver-Meal based heuristic introduced by Teunter et al. (2006). We will refer to this heuristic as the SM_4 heuristic as the decision to extend the time window will be made by comparing the resulting costs per period of all four options. The following section tests both heuristics extensively in a numerical experiment to assess their performance.

4.3 Numerical experiments

In order to guarantee a fair comparison to the original heuristic of Teunter et al. (2006), the experimental design that has been used to conduct the numerical study presented in this section corresponds mostly to their design. A full factorial study has been chosen in which all instances examined have a planning horizon T of twelve periods in common. Both setup cost parameters K_M and K_R can take on values of 200, 500, and 2000. While the rate of keeping a final product for one period in stock (h_M) is set to one holding a recoverable product for one period (h_R) can cost 0.2, 0.5, and 0.8. All customer demands d_t have been drawn randomly from a normal distribution with a mean of 100 units per period. Likewise, the amount of returned products per period r_t has been drawn from a normal distribution with a mean of 30 (i.e. a return ratio α of 30% prevails), 50, and 70. Both normal distributions were further distinguished into a small and a large variance setting. While the coefficient of variation in the small variance setting has always been set to 10% it takes on the value of 20% in the large variance setting. Contrary to the experiment conducted in Teunter et al. (2006), we omit the use of different demand and return patterns such as positive/negative trends and seasonal patterns. For each demand and return setting 20 instances (instead of 4 in their study) were drawn randomly. Therefore, the full factorial study considers in total $3^4 \cdot 2^2 \cdot 20 = 6480$ different examples.

For all examples both heuristic results have been calculated whereas CPLEX 11 has been used to determine the optimal solution. Both heuristics are evaluated by using the percentage gap to the optimal solution as a performance measure. The results of the numerical experiments are presented in Table 4.1.

By including two additional options in the decision-making process, the average performance of the SM_2 heuristic improves slightly from 7.5% to 6.1% over all instances. Comparing the performance of the SM_2 heuristic to the original numerical study in Teunter et al. (2006) it must be noticed that the performance in our study is slightly better which can be attributed to the differences in the experimental design. Although the SM_4 heuristic reduces the average percentage gap in almost all settings, an improvement of more than 2% can only be observed for a small setup cost for remanufacturing

Tab. 4.1: Performance of the SM_2 and SM_4 heuristic

	Percentage cost error to the optimal solution					
	Average		Standard deviation		Maximum	
	SM_2	SM_4	SM_2	SM_4	SM_2	SM_4
All instances	7.5%	6.1%	7.9%	7.6%	49.2%	47.3%
Demand						
Small variance	7.2%	6.0%	7.9%	7.6%	43.6%	47.3%
Large variance	7.8%	6.1%	8.0%	7.5%	49.2%	43.9%
Returns						
Small Variance	7.3%	6.1%	7.8%	7.6%	47.2%	47.3%
Large Variance	7.7%	6.1%	8.0%	7.5%	49.2%	46.3%
Return ratio α						
30%	5.5%	3.7%	5.5%	4.5%	31.3%	28.5%
50%	8.5%	7.3%	9.4%	8.2%	40.1%	41.8%
70%	8.4%	7.2%	8.0%	8.7%	49.2%	47.3%
K_M						
200	4.3%	3.4%	4.5%	3.6%	20.2%	17.6%
500	5.4%	3.9%	5.2%	3.9%	25.1%	19.3%
2000	12.8%	10.9%	9.9%	10.4%	49.2%	47.3%
K_R						
200	10.9%	6.6%	9.1%	7.8%	49.2%	40.2%
500	7.9%	8.1%	6.6%	8.2%	34.7%	47.3%
2000	3.7%	3.5%	6.0%	5.7%	29.4%	25.7%
h_R						
0.2	5.9%	5.3%	8.0%	8.0%	42.9%	47.3%
0.5	7.5%	6.5%	7.7%	7.6%	49.2%	42.4%
0.8	9.1%	6.3%	7.7%	7.0%	44.4%	40.3%

($K_R = 200$) and a large holding cost for the used product inventory ($h_R = 0.8$). Both heuristics seem to perform well when the return ratio α or the setup cost for manufacturing K_M is low and when the setup cost for remanufacturing K_R is high. Contrary, for the opposite directions the performance of both heuristics is not sufficient with average errors of more than 7%. In the next section the heuristic solutions are examined whether small modifications can be made to the initially obtained solution in order to reduce the total cost significantly.

4.4 Improvement phase

A commonly applied methodology to improve the performance of lot sizing heuristics is to use metaheuristics (see, for instance, Jans and Degraeve, 2007, for an overview). However, metaheuristics rely on an appropriate selection of parameter values which itself might be hard to determine. Therefore, this Chapter omits the use of metaheuristics and tries to enhance the solutions found by the SM_2 and SM_4 heuristic by examining two possible improvement opportunities.

Improvement 1: Check whether two consecutive time windows can be combined

A first improvement to the initial solution can be found by checking whether a cost reduction can be achieved if two consecutive time windows are combined. Hence, it is examined whether one of the four (two) options introduced in Section 4.2 for the SM_4 (SM_2) heuristic could improve the solution for an integrated time window that comprises both initial time windows.

Improvement 2: Check whether a remanufacturing lot can be increased

Being a myopic heuristic approach, the SM_2 and SM_4 heuristics neglect all decisions beyond the time window currently examined. Thus, some solutions revealed that recoverable products are held in stock until the end of the planning horizon although they could have been used instead of manufacturing them later. Starting in the first period of the planning horizon, the algorithm checks for the current period i whether a remanufacturing lot has been initiated. The basic idea of the second improvement is to examine if the total cost can be reduced by enlarging the remanufacturing lot in period i and simultaneously decreasing the first manufacturing lot scheduled after period i by the same amount. In order to that, the algorithm needs to determine at first period n which represents the period of the first manufacturing lot scheduled after period i. Without changing the number and sequence of lots determined initially by the SM_2 or SM_4 heuristic, the maximum number of returned products that can be remanufactured additionally in period i is restricted by two values. First, the number of recoverable products available needs to be taken into account. This number can be determined by the minimum of all subsequent used product inventory levels, i.e. $\min_{k\in\{i,\ldots,T\}}(y_{R,k})$. On the other hand, since the manufacturing lot in period n must be non-negative, the number of additionally remanufactured products must not exceed $Q_{M,n}$. After adapting the corresponding lot sizes $Q_{R,i}$ and $Q_{M,n}$, the algorithm checks whether these changes increase or decrease the total cost determined by equation (4.1). If the total cost can be decreased, the modified lot sizes are kept and the algorithm proceeds with the next period. Contrary, if changing both lot sizes leads to an increase in the total cost, all changes made are reversed and the next period is analyzed.

When approaching the end of the planning horizon, it might be the case that no manufacturing lot is scheduled after period i. In this case, the algorithm examines whether it is possible to reduce the preceding manufacturing lot set up in period l. However, a positive final product stock in period $i-1$ ($y_{M,i-1}$) must prevail in order to follow this idea. This value restricts the possible change of the remanufacturing lot in period i as well as the manufacturing lot $Q_{M,l}$. Furthermore, $Q_{R,i}$ is constrained by the maximum number of recoverable products available for remanufacturing in period i which has been depicted above. Again, when the change in lot sizes increases the total cost, the responsible changes are reversed. Only when it leads to a decrease in costs, the changes are kept and the algorithm proceeds with the next period. To clarify the second improvement in greater detail, the following pseudocode has been elaborated in Figure 4.3.

Fig. 4.3: Pseudocode for Improvement 2

For $i = 1$ to T

    If $Q_{R,i} > 0$ then

        Find period n (period of the next manufacturing lot after period i)

        If $i + 1 \leq n \leq T$ then

            $Q_{R,i} = Q_{R,i} + \min\left(Q_{M,n}, \min_{k\in\{i,\ldots,T\}}\left(y_{R,k}\right)\right)$

            $Q_{M,n} = \max\left(Q_{M,n} - \min_{k\in\{i,\ldots,T\}}\left(y_{R,k}\right), 0\right)$

            Update $y_{M,t}$ and $y_{R,t}$ using equations (4.2) and (4.3)

            If total cost determined by equation (4.1) cannot be reduced then

                Reverse decisions made regarding $Q_{R,i}$ and $Q_{M,n}$

                Update $y_{M,t}$ and $y_{R,t}$ using equations (4.2) and (4.3)

            End If

        Else If $y_{M,i} > 0$ then

            Find period l (period of the last manufacturing lot before

            period i)

            $Q_{R,i} = Q_{R,i} + \min\left(y_{M,i-1}, Q_{M,l}, \min_{j\in\{i,\ldots,T\}}\left(y_{R,j}\right)\right)$

            $Q_{M,l} = \max\left(Q_{M,l} - \min\left(y_{M,i-1}, \min_{j\in\{i,\ldots,T\}}\left(y_{R,j}\right)\right), 0\right)$

            Update $y_{M,t}$ and $y_{R,t}$ using equations (4.2) and (4.3)

            If total cost determined by equation (4.1) cannot be reduced then

                Reverse decisions made regarding $Q_{R,i}$ and $Q_{M,n}$

                Update $y_{M,t}$ and $y_{R,t}$ using equations (4.2) and (4.3)

            End If

        End If

    End If

Next i

As mentioned above, both improvements can be applied to the solutions obtained by the SM_2 and SM_4 heuristic. Table 4.2 summarizes the results of the numerical study in which the superscript $^+$ indicates that the initial solution has been examined for both improvements.

It can be seen that the performance of the SM_4 heuristic could be enhanced substantially from 6.1% to 2.2% by applying both improvements. The larger influence on the solution improvement can be credited to improvement 1 which was able to affect the SM_4 heuristic especially (around 85% of the improvement). That is because by analyzing all four options introduced in Section 4.2 a larger flexibility in satisfying the customer demand is established in comparison to the SM_2 heuristic. Regarding the zero-inventory property, 61.4% of all heuristic solutions obtained by the SM_4 heuristic revealed at least one period in which the zero-inventory property did not hold. In contrast to the original results of the SM_2 heuristic, the SM_4^+ heuristic could reduce the percentage gap to less than half of its original value in almost all settings examined. When comparing the median of all instances the improvement is even more noticeable. While the median of all instances has been 5.6% for the SM_2 heuristic the SM_4^+ heuristic could reduce it to around 1.0%. Interestingly, the SM_4^+ heuristic is able to stabilize the average performance of all settings to lie between 1.2% and 3.4%. Although the SM_4^+ heuristic has reduced the maximum deviation from the optimal solution considerably (as can be observed in the right hand side of both Tables 4.1 and 4.2) there are still instances which perform poorly. Nevertheless, the SM_4^+ heuristic was able to achieve that in only 2% of all instances the percentage gap was larger than 10%. On the contrary, 18% of all instances exhibited a percentage gap of more than 10% when using the original SM_2 heuristic.

As depicted above, the performance of the original SM_2 heuristic has been enhanced substantially. In their work, Teunter et al. (2006) examined not only a Silver-Meal based criterion but also an adapted Least-Unit-Cost and Part-Period approach. These heuristics have been adapted as well by including the additional options and improvement steps. Unfortunately, their performance improved only slightly in comparison to the original work and was not able to outperform the performance gain of the Silver-Meal based approach. However, it must be mentioned that this improvement in

Tab. 4.2: Performance of the SM_2^+ and SM_4^+ heuristic

	Percentage cost error to the optimal solution					
	Average		Standard deviation		Maximum	
	SM_2^+	SM_4^+	SM_2^+	SM_4^+	SM_2^+	SM_4^+
All instances	6.9%	2.2%	7.9%	2.9%	49.2%	24.3%
Demand						
Small variance	6.6%	2.1%	7.9%	2.8%	43.5%	18.9%
Large variance	7.2%	2.4%	8.0%	3.0%	49.2%	24.3%
Returns						
Small Variance	6.8%	2.2%	7.8%	2.9%	47.2%	21.1%
Large Variance	7.1%	2.3%	8.0%	2.9%	49.2%	24.3%
Return ratio α						
30%	4.9%	1.2%	5.4%	1.8%	31.3%	12.1%
50%	8.0%	2.3%	9.3%	2.7%	39.8%	16.2%
70%	8.0%	3.3%	8.0%	3.5%	49.2%	24.3%
K_M						
200	3.5%	2.3%	4.0%	2.6%	20.2%	13.5%
500	4.8%	2.1%	4.9%	2.5%	23.7%	12.8%
2000	12.6%	2.3%	9.9%	3.4%	49.2%	24.3%
K_R						
200	10.0%	1.9%	9.4%	2.1%	49.2%	11.8%
500	7.3%	3.4%	6.6%	3.2%	34.7%	19.1%
2000	3.6%	1.4%	5.9%	2.9%	29.4%	24.3%
h_R						
0.2	5.8%	1.7%	8.0%	2.5%	42.9%	21.1%
0.5	7.0%	2.3%	7.7%	3.0%	49.2%	24.3%
0.8	8.1%	2.8%	7.8%	3.0%	44.4%	20.6%

performance is accompanied by an increase in computational complexity. Although the complexity of the proposed algorithms increased, only a small rise in the computation time for each instance could be observed. Therefore, implementing both the additional options and the improvement steps provides a fast and well performing heuristic algorithm for the dynamic lot sizing problem with returns and remanufacturing.

4.5 Conclusion and Outlook

This Chapter extends the seminal work of Teunter et al. (2006) in the area of simple heuristics for the dynamic lot sizing problem with returns and remanufacturing. In their work, the authors introduce a Silver-Meal based heuristic that analyzes two options to meet customer demand. This Chapter includes two more options to be analyzed that are well-known from the corresponding static lot sizing problem. By doing this, the percentage gap to the optimal solution that has been used as a performance measure could be reduced slightly from 7.5% to 6.1% (mean over all instances). Afterwards, two simple procedures are applied to the initial solutions found by the SM_2 and SM_4 heuristic to improve the results they created. The average percentage gap to the optimal solution has been reduced over all instances to 2.2% when using the SM_4 heuristic's solution as initial one. Comparing this result to the heuristic introduced by Teunter et al. (2006), the average percentage gap has thus been reduced to less than half of its original value.

Future research efforts can be directed to a more detailed modeling of the remanufacturing process. While in this Chapter all remanufacturing operations have been subsumed to a single stage, in industry the process of remanufacturing contains next to the disassembly of returned products also the cleaning and rework of the parts obtained, and finally the re-assembly into as-good-as-new products. Furthermore, including the option to dispose of recoverable products when they are not required and variable unit cost for remanufacturing and manufacturing alter the decision-making process. Another promising research opportunity would be to test the heuristics in a rolling planning horizon environment. As has been shown by Blackburn and Millen (1980) the heuristic might outperform even the optimal solution because of its schedule stability. Another interesting aspect of rolling planning horizon environments that can be analyzed in this context is the uncertainty of demand and return realizations at the end of each planning roll which becomes more accurate as closer one gets to this period.

All model settings presented in the preceding Chapters impose the assumption that the OEM wants to satisfy customer demand any time without exception. The following Chapter 5 discusses this assumption in greater detail by analyzing a long-term planning approach which allows the OEM to endogenously control both his customer demand and his product returns in order to optimize the total profits.

5. Dynamic buy-back for product recovery in end-of-life spare parts procurement

5.1 Introduction

In recent years, original equipment manufacturers (OEMs) of durable goods identified the after-sales market as one of their key business segments[1]. For instance, Cohen et al. (2006) provide results of a 1999 AMR Research report stating that by being active in the aftermarket businesses could generate about 45% of their gross profits. Furthermore, by efficiently handling the supply of spare parts, a competitive advantage can be established if the OEM provides a superior service to his customers, e.g. by guaranteeing the availability of spare parts during a comparably long service period. Thus, the length of the service period becomes an important strategic parameter for management. This period is subdivided into two distinct phases, namely the normal phase and the final phase. During the normal phase the primary product is manufactured and sold to the customers. The final phase starts when serial production ceases and lasts as long as spare parts availability is guaranteed. Therefore, it is often considerably longer than the production period. In the automotive sector, for instance, the final phase usually lasts for 10-15 years. However, several OEMs provide a significantly longer availability for their spare parts as the example of a 30 years service period for Mercedes-Benz cars indicates.

In a recent paper, Kim and Park (2008) propose a model that allows to determine the optimal length of the final phase. They argue that the marketing department seeks

[1] This Chapter is based on the work titled 'Dynamic buy-back for product recovery in end-of-life spare parts procurement' that is accepted for publication in the *International Journal of Production Research* (see Kleber et al., 2011)

to stimulate demand by offering a long period with guaranteed spare parts availability as this signals a high quality of the product (see, e.g., Spence, 1977; Gal-Or, 1989). Obviously, if the final phase would be determined without such considerations by only accounting for the operational costs and revenues of service, it would often be chosen considerably shorter. Our research basically focuses on situations in which both perspectives (marketing and operations) yield large differences in the length of the final period and we propose an efficient method for spare parts management under those circumstances.

From the OEM's perspective, inventory management for spare parts differs considerably from inventory management applied to manufacturing processes, mainly because demand for spare parts is less predictable and highly dynamic on a comparably low level (see, e.g., Kennedy et al., 2002; Huiskonen, 2001). In addition, options for resupply become increasingly rare during the final phase. While in the normal phase production facilities of the primary product can be used, this efficient sourcing option is often no longer recommendable in the final phase due to high fixed costs incurred for a relatively small output. Thus, a frequently adopted strategy is to place a final order at the time when regular production comes to an end. However, this is connected with high stock levels resulting in large holding cost and a high obsolescence risk as all demands occurring in the final phase need to be estimated beforehand. Extra production represents an additional option in the final phase which in contrast to regular production is typically performed in small lots. Nonetheless, this option is under certain circumstances prohibitively expensive or technically infeasible (see Hesselbach et al., 2002, for a comprehensive overview on available options).

There is a one-to-one correspondence between a spare part and the broken component. This creates the opportunity to recover the broken part for later use as a spare part. Part recovery, hence, can complement other sources of spare parts supply. An overview on different recovery processing options is provided by Thierry et al. (1995) including repair, refurbishing, and remanufacturing. Although all of these options can be applied in principle to satisfy an existing spare part demand, this Chapter focuses solely on remanufacturing processes. Remanufactured parts are considered to be as-good-as-new and OEMs frequently offer the same warranty as for new parts. Compared to extra production, remanufacturing is relatively cheap, but since not all broken parts might be remanufacturable it should be accompanied by other options to avoid shortages (see, e.g., Inderfurth and Mukherjee, 2008; Inderfurth and Kleber, 2009).

In case of not being able to fulfill occurring spare part demands and in order to avoid a penalty or a goodwill loss, further options the OEM can offer to his customers range from swapping to buy-back. Swapping refers to a replacement of the dysfunctional product with a new generation product free of charge for the customer (as has been reported by Pourakbar et al., 2008). This option is favorable for high tech products experiencing a considerable price deterioration between successive product generations but is less beneficial for durables. Buy-back of products is typically performed in practice in form of trade-in campaigns. These campaigns, though, foremost intend to increase the sales of new products and thus both functional and broken products are accepted. Although there are many examples from industry (see, e.g., Ray et al., 2005), an acquisition of recoverable parts for satisfying spare part demands is (at best) seen as a side effect and is hence not explicitly stated as motivation for such a campaign.

In this Chapter, however, we emphasize the use of more focused trade-in campaigns which explicitly aim to control the OEM's supply of recoverable parts. By doing so, we abstract from the above mentioned sales promoting effects for other products and isolate the sole effect of buying back broken products on spare parts management. In particular, we are interested in those conditions under which buying back broken products for obtaining spare parts profitably complements the traditional sourcing options final order and remanufacturing. This could for instance be accomplished by using the already existing service network which provides the OEM with a direct access to his customers demanding spare parts.

An active integration of buying back used products into a generic product recovery system has been examined by Minner and Kiesmüller (2002) in a deterministic setting with a stationary price-response function. There, the effects of the acquisition decision on current and future demands are neglected. In our case, however, buying back broken products would on the one hand decrease current and future demands for spare parts since no future spare part demand is generated from a bought back product. On the other hand, customers with a dysfunctional product might accept a comparably low compensation yielding a cheap supply of recoverable parts for the OEM. Therefore, the trade-off between cannibalizing current and future demands to release oneself from the obligation to provide spare parts and creating an additional source of supply for satisfying the remaining demand represents the main focus of this Chapter.

The profitability of the buy-back option depends on constraints on price and quantity decisions but also on the availability of required information. First, the OEM might be able to approach different customers in a specific way. In the marketing literature, a number of market-segmentation approaches are discussed (see, e.g., Kotler and Keller, 2008; Wedel and Kamakura, 2000). Especially, it is argued that one can segment the market by observable and unobservable characteristics. Observable criteria for segmenting customers are mostly geographic or demographic data. Here, one might additionally segment on type of relationship, for instance B2B (car rental enterprise) or B2C (private customer). Criteria that are unobservable typically contain psychographic or behavioral characteristics.

Furthermore, the OEM might be restricted in his flexibility to price-discriminate between customers because of legislation like the Robinson-Patman act in the US. We refer to Bernstein et al. (2006) for a more comprehensive motivation for simple pricing schemes. Finally, the OEM might have no control over the buy-back quantity once he offers a price. This might be the case because he communicates a buy-back campaign in the mass media. Additionally, a quantity restriction of buy-backs for the decentralized repair shops might not be realizable as the demand at each facility is unknown or uncertain in advance.

The remainder of this Chapter is organized as follows. In Section 5.2 we introduce a basic mathematical model on how to incorporate buy-backs in the decision-making process and state its main assumptions. Afterwards, Section 5.3 analyzes a base case scenario and elaborates possible benefits from segmenting the OEM's customers into distinct groups. The fourth section elaborates the critical assumptions made in the basic model and shows how to adapt it to be able to deal with additional constraints and limited information availability as described above. Furthermore, the base case parameters set in Section 5.3 are critically reviewed in a sensitivity analysis. Finally, Section 5.5 summarizes the main conclusions and gives some directions for future research.

5.2 A basic model with buy-back

We consider a single product for which the OEM guarantees the availability of spare parts during the final phase of service. The planning horizon of length T starts at the end of regular production, i.e. at the time when no further products are manufactured to be sold. Thus, at this point in time the number of products with the customers (which we will refer to as the install base) no longer increases. For the sake of simplicity, the considered product includes only one vital component that can fail and needs to be replaced by a spare part to restore its functionality. Otherwise, the product's value would reduce considerably. Failures occur deterministically with rate f, i.e. each period a fraction of the install base requires spare parts to replace the broken components. This is accomplished by the existing service network operated by the OEM which is also used to return broken components to a remanufacturing facility.

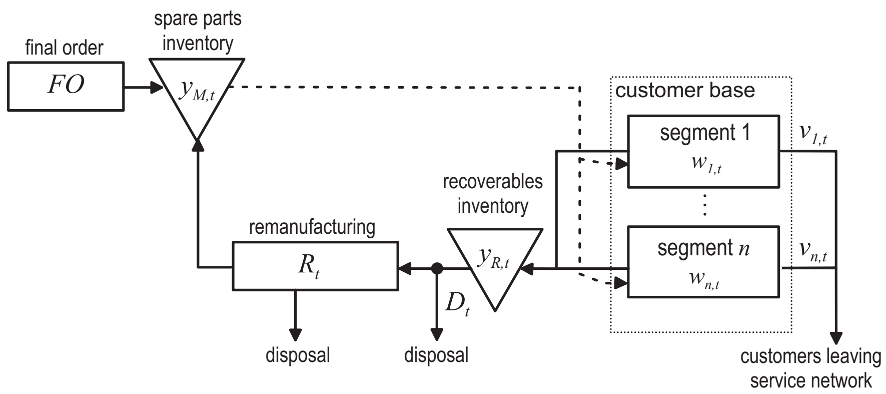

Fig. 5.1: Spare parts supply system

In this Chapter, we focus on the spare parts supply system depicted in Figure 5.1. The notation used is summarized in Table 5.1. Demand for spare parts is satisfied from a central stocking point. Let $y_{M,t}$ denote the OEM's spare parts stock at the end of period t (which corresponds to the final product stock of the preceding Chapters). The OEM can replenish this inventory using two different options. At the beginning of the planning period, he places a final order FO at unit cost c_f. Afterwards, regular production ceases, i.e. manufacturing new parts is only possible at the beginning of the planning horizon. When this happens, remanufacturing broken components from the stock of recoverables $y_{R,t}$ (which corresponds to the used product inventory of the

Tab. 5.1: Notation used

Parameters	
n	Number of customer segments
T	Planning horizon
c_r	Unit cost of remanufacturing
c_f	Final order unit cost
h_M	Spare parts holding costs per unit and period
h_R	Returned products holding costs per unit and period
p_i	Reservation price in customer segment i
p_s	Revenue per spare part sold
β	Remanufacturing yield rate
f	Components failure rate
r	Interest rate
$\bar{w}_i$	Initial product stock in customer segment i
$\bar{y}_{R,0}$	Initial stock of broken products
$\nu_{i,t}$	Percentage of products leaving the OEM's access of segment i in period t
Decision and state variables	
$y_{M,t}$	Spare parts inventory at the beginning of period t
$y_{R,t}$	Recoverables inventory at the beginning of period t
FO	Size of the final order
R_t	Number of remanufactured parts in period t
D_t	Number of broken products disposed of in period t
E_t	Fulfilled spare part demand in period t
$x_{i,j,t}$	Number of broken products bought back from segment i at price p_j in period t
$w_{i,t}$	Number of customers in segment i in period t
$\Theta_{i,t}$	Binary pricing variable for customer segment i in period t

preceding Chapters) becomes the only sourcing option. The parameters h_R and h_M represent the unit holding cost for broken parts and spare parts per period, respectively. In each period t, the OEM must decide on the amount of broken components that he would like to remanufacture R_t at unit cost c_r and on the quantity of broken components to be disposed of D_t. As it is commonly presumed for practical applications, we suppose that revenues for recovering material and costs of extracting materials are about the same size which means that the disposal costs are negligible. Due to an imperfect remanufacturing process only the fraction β of the remanufactured products fulfill the designated quality standards to be sold as spare parts. All costs and revenues are discounted by the interest rate r. Since we analyze a long-term planning environment, possible setup cost for initiating a remanufacturing run (or for scheduling the final order at the beginning of the planning horizon) are neglected.

Both replenishment options exhibit considerable disadvantages. The final order bears the burden of holding spare parts over a long period of time and the option of remanufacturing broken parts cannot provide all spare parts demanded due to the imperfect remanufacturing process. An appropriate way to overcome these deficiencies would be to include the buy-back of broken products as another option. If the OEM decides to buy back, he loses a revenue of p_s per spare part that would be sold otherwise but he also increases the recoverables stock since an additional broken component (included in the product bought back) is returned to the OEM. The compensation paid to the customer to persuade her to sell her broken product depends on her valuation of the product. For this, we assume that all customers value their product differently, but this valuation does not change over time. Different buy-back prices, thus, yield different quantities and decisions upon both must be made simultaneously. In contrast to other approaches (see, e.g., Minner and Kiesmüller, 2002) where a given functional relationship does not change over time, in our long range approach buy-back decisions impact the composition of the install base and therefore, change conditions relevant for later decisions.

For the OEM, individual information upon each customer's valuation for a broken product might hardly be obtainable. He therefore segments his customers into groups $i = 1, ..., n$ in which all customers value their product similarly. The number of functioning products in each customer segment i at the end of period t is denoted by $w_{i,t}$. It is assumed that the initial size of each segment $\bar{w}_i$ is known in advance, and independent of any of the OEM's decisions a fraction $\nu_{i,t}$ of all products in a customer segment i leaves the service network as they are, for example, salvaged at a breakers yard. Let p_i denote the reservation price of all customers in segment i representing their valuation of a defective product. Without loss of generality, the customer segments are arranged such that the inequality $p_1 < ... < p_n$ is satisfied. It is easy to see that only these prices are relevant for the buy-back decision. If the OEM would propose a price to a segment that lies between two adjacent reservation prices, he could easily reduce this price to the lower of the two reservation prices while still being able to acquire the same quantity.

In an idealized setting (denoted by M1) the OEM can decide for each segment separately on which quantities he wishes to buy back for which price. This requires, that the OEM can assign each customer to her segment, i.e. that individual information is available on all customers. Buy-back quantities are denoted by $x_{i,j,t}$ representing the number of broken products bought back from customer segment i at price p_j in period t. Consequently, the amount of broken products that is bought back reduces the number of spare parts sold in period t which will be denoted by E_t. Additionally, the OEM needs to determine the size of the final order FO and in each period t he decides on the number of remanufactured R_t and disposed of parts D_t. Problem M1 can be formulated as follows:

$$
\begin{aligned}
\max \Pi_1 &= \sum_{t=1}^{T} (1+r)^{-t} \left[p_s \cdot E_t - c_r \cdot R_t - h_R \cdot y_{R,t} - h_M \cdot y_{M,t} - \sum_{i=1}^{n} \sum_{j=1}^{n} x_{i,j,t} \cdot p_j \right] \\
&\quad - c_f \cdot FO \qquad (5.1)
\end{aligned}
$$

s.t.

$$E_t = f \sum_{i=1}^{n} w_{i,t-1} - \sum_{i=1}^{n} \sum_{j=1}^{n} x_{i,j,t} \quad t = 1, ..., T \qquad (5.2)$$

$$y_{M,t} = y_{M,t-1} - E_t + \beta \cdot R_t \quad t = 1, ..., T \qquad (5.3)$$

$$y_{M,0} = FO \qquad (5.4)$$

$$y_{R,t} = y_{R,t-1} - R_t - D_t + f \cdot \sum_{i=1}^{n} w_{i,t-1} \quad t = 1, ..., T \qquad (5.5)$$

$$y_{R,0} = \bar{y}_{R,0} \qquad (5.6)$$

$$w_{i,t} = w_{i,t-1} \cdot (1 - \nu_{i,t}) - \sum_{j=1}^{n} x_{i,j,t} \quad i = 1, ..., n \quad t = 1, ..., T \tag{5.7}$$

$$w_{i,0} = \bar{w}_i \quad i = 1, ..., n \tag{5.8}$$

$$\sum_{j=1}^{n} x_{i,j,t} \leq f \cdot w_{i,t-1} \quad i = 1, ..., n \quad t = 1, ..., T \tag{5.9}$$

$$x_{i,j,t} = 0 \quad i, j = 1, ..., n \quad j < i \tag{5.10}$$

$$y_{M,t}, y_{R,t}, E_t, R_t, x_{i,j,t}, w_{i,t} \geq 0 \quad i, j = 1, ..., n \quad t = 1, ..., T \tag{5.11}$$

The OEM maximizes his discounted profit Π_1 which includes each period's net cash flow consisting of the revenue of selling E_t spare parts minus the cost incurred for remanufacturing, stock-keeping, and buy-back as well as the expenses for producing the final order. Constraints (5.2)-(5.11) are interpreted as follows. The number of spare parts sold to the customers E_t is determined in (5.2) by the number of products that break down in t reduced by the amount of broken products the OEM buys back. Constraints (5.3) and (5.5) are inventory balance equations for the spare parts and recoverables inventory with initial levels set in (5.4) and (5.6). The initial spare parts stock equals the size of the final order. The stock of spare parts at the end of period t $y_{M,t}$ is determined by the stock at the end of the previous period $y_{M,t-1}$ reduced by the fulfilled spare parts demand E_t plus the yield from the remanufacturing process $\beta \cdot R_t$. Starting from an initial value $\bar{y}_{R,0}$, the stock of recoverables is reduced in each period by the number of remanufactured R_t and disposed of parts D_t and increases by the number of broken products that return to the OEM.

The development of the number of products in each customer segment is given in balance equation (5.7) while (5.8) represents the initial size of each segment. The segment size reduces by the exogenous drain of leaving customers $w_{i,t-1} \cdot (1 - \nu_{i,t})$ and the total number of bought-back products from that segment. Constraint (5.9) ensures that the number of bought-back products from customer segment i must not exceed the number of broken products in the respective period. By establishing the logical constraint (5.10) it is guaranteed that no buy-back occurs for a lower price than the segment's reservation price. For instance, the OEM cannot acquire any broken product from segment 2 for the price p_1 since this would not be sufficient. The non-negativity restrictions (5.11) assure validity of decisions.

In the idealized setting it can be easily seen that it is not optimal to buy back products for a different price than the segment's specific reservation price. Thus, an optimal solution of M1 will always show $x_{i,j,t} = 0$ for $i \neq j$.

5.3 The value of buy-back under idealistic conditions

5.3.1 Base case parameters

In this section, an example is used to illustrate the potential benefit of buying back broken products and to elaborate the gains of a more detailed customer segmentation. The respective parameter values of the base case scenario are summarized in Table 5.2.

Tab. 5.2: Base case parameter values

n	T	$\bar{w}_1$	ν_1	f	p_s	c_f	c_r	β	r	h_M	h_R	p_1	$\hat{p}_1$
1	80	400	1.5%	10%	10	3	1.5	50%	2.5%	0.2	0.1	20	30

We start our analysis with a single customer segment ($n = 1$) for which all spare part demands must be satisfied for the next 80 periods. A period is hereby defined to be a quarter of a year, i.e. the OEM faces a 20 year planning horizon. The OEM estimates the initial number of products in the install base to be $\bar{w}_1$=400 out of which a fraction of $\nu_1 = 1.5\%$ are leaving the service network each period. The main component fails at a rate $f = 10\%$, i.e. each product has to be repaired on average once in two and a half years yielding a revenue of p_s=10. The OEM estimates that a broken product can be acquired at a price of $p_1 = 20$ being twice the revenue from selling a spare part. Hence, the trade-in price for a functional product is given by $\hat{p}_1 = p_1 + p_s = 30$.

Spare parts are procured by placing a final order at unit cost c_f=3 yielding an initial profit margin of 70%. All products returning to the OEM will be remanufactured at unit cost $c_r = 1.5$. It is assumed that remanufacturing is successful in $\beta = 50\%$ of all cases, i.e. only one of two broken parts can be brought to an as-good-as-new condition. Thus, there is no direct cost advantage for neither parts procured in the final order nor for parts succesfully remanufactured. The discount rate is set to $r = 2.5\%$ per quarter or about 10% per year. Out of pocket holding cost are $h_M = 0.2$ and $h_R = 0.1$ per unit and period for spare parts and recoverable parts, respectively. Taking both discounting and holding cost into account, it would be economically beneficial to satisfy demand from parts procured in the final order for at most 20 periods (5 years) and then to switch to remanufacturing. Hence, the base case parameters depict the situation motivated in the introduction, i.e. the operations manager is confronted with a much longer final phase than he would choose individually.

5.3.2 The value of buy-back without segmentation

Initially, a problem setting is analyzed in which the OEM is not able to segment his customer base. By inserting the parameter values from the preceding subsection into model M1 outlined in Section 5.2, the optimal solution is obtained by using the optimization software CPLEX 11. This solution is henceforth compared with both a benchmark solution that does not allow for buying back broken products and a trade-in solution in which the OEM is obliged to take back all products (functioning and broken) the customers are willing to return. The main results are shown in Table 5.3.

Tab. 5.3: Optimal final order FO, discounted profit Π, relative profit surplus Δ and first period in which buy-back takes place z in the benchmark and the trade-in solution as well as for M1

Benchmark		Optimal Trade-in				Optimal buy-back in M1			
FO_{BM}	Π_{BM}	FO_{TI}	z_{TI}	Π_{TI}	Δ_{TI}	FO_1	z_1	Π_1	Δ_1
935	2390	935	/	2390	0%	658	46	3127	+30.8%

The benchmark solution has been obtained by forcing all buy-back quantities $x_{i,j,t}$ to zero. The solution shows a structure where (as has been examined in a related approach by Kleber and Inderfurth, 2007) there are two phases to be distinguished. In a first phase (periods 1 to 29) the demand for spare parts is satisfied from the final order of size $FO_{BM} = 935$. All broken parts that return are held in the recoverables inventory and none is disposed of. In the second phase (periods 30 to 80) the strategic stock of returned products built up in the first phase is used to serve the entire demand by remanufacturing broken parts from the recoverables inventory. Thus, the size of the final order equals that part of total demand over the planning horizon which cannot be satisfied by remanufacturing. The benchmark solution to the base case scenario yields a total discounted profit of 2390.

When including the buy-back option using model M1, the final order reduces to 658 implying a substantial reduction in holding cost. Although considerably shorter (the first phase ends in period 19), both of the above phases are found as well in the optimal solution. In an adjacent third phase (starting in period $z_1 = 46$), the OEM buys back as many products as are needed to satisfy demand. Interestingly, no stock is build up in the recoverables inventory during that phase since all returns are instantly remanufactured. Hence, each period's buy-back quantity is set to just compensate the yield loss. The discounted profit of the base case scenario increases by about 31% to 3127 when buy-backs are included[2].

A third option for the OEM would be to give all customers the opportunity to sell their products in use. For modeling this situation, an additional binary decision variable $\Theta_{i,t}$ is introduced which represents the OEM's decision to offer a trade-in at price $\hat{p}_i$ in period t. In the objective, p_i is substituted by $\hat{p}_i$, and restrictions (5.2) and (5.9) have to be replaced by

$$
\begin{aligned}
E_t &= f\sum_{i=1}^{n} w_{i,t-1} \quad t = 1, ..., T && (5.12)\\
x_{i,j,t} &\leq \zeta\cdot\Theta_{j,t} \quad i,j = 1, ..., n \quad t = 1, ..., T && (5.13)\\
w_{i,t-1}(1-\nu_{i,t}) - x_{i,j,t} &\leq \zeta\cdot(1-\Theta_{j,t}) \quad i,j = 1, ..., n \wedge i \leq j \quad t = 1, ..., T && (5.14)\\
\sum_{i=1}^{n}\Theta_{i,t} &\leq 1 \quad t = 1, ..., T && (5.15)\\
\Theta_{i,t} &\in \{0,1\} \quad i = 1, ..., n \quad t = 1, ..., T && (5.16)
\end{aligned}
$$

Restriction (5.12) ensures that only functioning products are bought back (after repair if required). Constraint (5.14) ensures that all customers return their products to the OEM when $\Theta_{i,t}$ is set to one. In this case, all remaining products in segment i (the initial inventory minus the drain at the beginning of this period) are procured by the

[2] For a detailed description of the M1 policy structure see the Appendix, page 162.

OEM. If, otherwise, $\Theta_{i,t}$ is set to 0, restriction (5.13) forces the buy-back quantity to 0. Finally, restriction (5.15) guarantees that only one buy-back price can be set at most by the OEM. Not surprisingly, the solution for a trade-in campaign does not deviate from the benchmark solution if the OEM is not able to segment his customer base. This is because selling spare parts remains a profitable business opportunity for the OEM over the entire planning horizon. Hence, running a trade-in campaign before the end of the planning horizon means that the OEM will not be able to sell any spare part for this product any more since his entire customer base would be depleted.

5.3.3 The value of customer segmentation

This subsection broadens the above analysis by allowing the OEM to segment the install base w.r.t. differences in the customers' valuation of the product. The analysis might provide managers with valuable insights on how much effort they should invest in segmenting the install base more thoroughly.

In order to keep the results consistent, the only difference between customer segments is the buy-back price. All other parameters remain the same as in the base case, e.g. the fraction of customers leaving the service network ν_i is kept at 1.5 % for all segments i. For determining the segment specific buy-back prices it is assumed that the willingness to accept a buy-back, i.e. the reservation price, is uniformly distributed among the 400 customers within an interval between 0 and 20. Given n segments, $400/n$ customers with the lowest reservation price are assigned to the first segment, the next $400/n$ customers to segment 2, and so on. Each buy-back price, thus, indicates the value for which all customers of a respective segment would sell their broken products. In the first segment it would be $p_1^n = 20/n$, in the second one $p_2^n = 2 \cdot 20/n$, and so on. The segmentation of customers is sketched in Figure 5.2. The corresponding trade-in prices $\hat{p}_j$ can be determined consequently by adding p_s to the respective segments' buy-back prices.

Table 5.4 depicts the results of the experiments which can be interpreted as follows. As the solution to M1 can react more flexible, the profitability of the buy-back option increases as more different segments have been identified. That is because a more

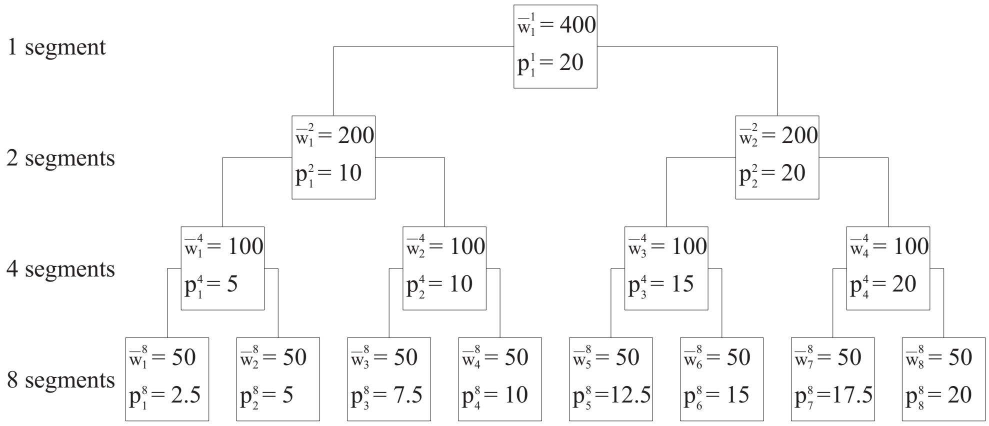

Fig. 5.2: Initial segment sizes $\bar{w}_i^n$ and corresponding buy-back prices p_i^n for different numbers of segments n

Tab. 5.4: Influence of the number of segments n on the final order and discounted profit

	Optimal Trade-in				Optimal buy-back in M1			
n	FO_{TI}	z_{TI}	Π_{TI}	Δ_{TI}	FO_1	z_1	Π_1	Δ_1
1	935	/	2390	0%	658	46	3127	+30.8%
2	848	56	2416	+1.1%	621	41	3383	+41.5%
4	840	52	2474	+3.5%	592	38	3514	+47.0%
8	834	49	2505	+4.8%	582	36	3578	+49.7%
16	831	48	2520	+5.4%	576	35	3610	+51.0%
32	832	45	2526	+5.7%	573	35	3626	+51.7%

precise fragmentation of the install base allows the OEM to approach each customer's actual reservation price. If there is only a rough segmentation of the install base, the OEM offers some customers too high prices since they would have also sold their broken product for a much lower price. However, it can be seen that the additional benefit of a more detailed segmentation does decrease as more different segments are established.

A similar result can be observed for initiating trade-in campaigns. In contrast to the situation in which the OEM cannot differentiate his customers, the solution improves when he is able to fragment his install base. Yet, the possible gain is (due to the lower flexibility compared to M1) much smaller lying between 1 and around 6 %. Thus, the remainder of this Chapter focuses solely on the opportunity to buy back broken products as this has been shown to be the more profitable business option.

5.4 Robustness with respect to critical assumptions and parameters

This section deals with more realistic conditions than those required for the basic model. First, we delineate potential problems when approaching customer segments individually, and assess secondly the impact of such deficiencies on the profitability of the buy-back option.

5.4.1 Critical assumptions

While analyzing the model context, a subset of problems can be established that arises due to possibly existing exogenous constraints, such as communication, information, and pricing constraints. Communication constraints will be analyzed both from an external as well as an internal point of view. The internal view refers to an internal communication within the OEM's service network. Thus, the OEM is able to approach each customer individually to offer her a buy-back and has therefore the flexibility to decide on the quantity he buys back in each planning period. The external view, on the other hand, corresponds to a setting in which the OEM communicates the buy-back offer to all customers simultaneously via a mass media marketing campaign. As the OEM cannot withdraw his offer, he has to accept all broken products his customers intend to sell. Whether the communication focuses on his service network or his customers, thus, determines the OEM's buy-back *quantity flexibility*.

The OEM can face furthermore *information constraints*, if he cannot assign a customer to her corresponding segment and does hence not know from which segment he bought back a broken product. This will typically be the case when the segmentation is based on unobservable criteria, such as psychological or behavioral characteristics (see Kotler and Keller, 2008, Chapter 8).

Tab. 5.5: Three dimensions of flexibility and information availability

<table>
<tr><td colspan="2"></td><td colspan="4">quantity flexibility</td></tr>
<tr><td colspan="2"></td><td colspan="2">yes</td><td colspan="2">no</td></tr>
<tr><td></td><td>individual information</td><td>available</td><td>not avail.</td><td>available</td><td>not avail.</td></tr>
<tr><td rowspan="2">pricing flexibility</td><td>yes</td><td>full pricing and quantity flexibility, full information availability (M1)</td><td colspan="3"></td></tr>
<tr><td>no</td><td>limited pricing and full quantity flexibility, full information availability (M2)</td><td>limited pricing and full quantity flexibility, limited information availability (M3)</td><td colspan="2">limited pricing and quantity flexibility, limited information availability (M4)</td></tr>
</table>

Finally, the OEM can face limited *pricing flexibility*. Pricing constraints describe the OEM's restriction to address each segment independently, i.e. his pricing flexibility is restricted. Therefore, the OEM might be limited to set only one price per period. In this case, he is not able to buy back products from different segments for different prices in a given period. Bernstein et al. (2006) give an overview on reasons why the OEM might be restricted in his pricing format.

These three dimensions, namely pricing and quantity flexibility as well as individual information availability result in eight subclasses of problems (see Table 5.5). However, it can be shown that several subclasses are redundant (empty cells). First, the OEM cannot exploit any pricing and quantity flexibility if he cannot assign his customers to the respective segments (as every customer will apparently claim that she has a high reservation price). Second, we argue that the OEM is only able to communicate one buy-back price per period directly to all his customers if the external view of communication prevails. Hence, if the pricing flexibility acts as an additional constraint only one case needs to be analyzed regardless the information availability. Because of the limited buy-back quantity flexibility, the OEM is required to buy back all products the customers intend to sell. As the OEM is, by assumption, able to estimate the total size of each customer segment, he has no advantage of assigning the customers to the segments since he cannot utilize this information satisfactorily. Yet, there is the possibility to advertize 'up-to' prices. This case, however, could be treated in the same way as the internal view.

5.4.2 The economic impact of critical assumptions

In order to assess the economic impact of the assumptions made, the model M1 needs to be adapted. Subsequently, we describe the required changes. The second setting M2 is characterized by less flexibility than M1 due to its restricted pricing flexibility. This is supported by the fact that only a single buy-back price can be set in each period. In this setting, however, it is still possible to assign each customer to her segment and to choose which quantity to buy from which customer segment. The OEM's pricing decision is described, as before, by the binary decision variable $\Theta_{i,t}$ that determines which buy-back price the OEM sets in period t. It is 1 if the buy-back price p_i is offered and 0 else. In order to implement the second setting M2, constraints (5.13), (5.15), and (5.16) have to be added to the original setting M1. Obviously, due to the additional restrictions imposed the profit of M2 (denoted by Π_2) cannot exceed the profit of M1.

In the third setting M3, the OEM can again only set a single price in each period and he can also choose upon the quantity to take back. However, the absence of available information regarding each customer's assignment results in the problem that it cannot be easily determined how many items were bought back from which customer segment. Hence, further assumptions are required to keep track of the number of customers in each segment. Yet, it is easy to conclude that the profit of this setting must lie between the profits of the less restricted setting M2 and the even more restricted setting M4. A more detailed analysis of this setting will be left for future research.

Setting M4 provides us with the least flexible environment that still allows for customer segmentation. Due to its limited pricing flexibility only a single price can be selected per period, but since this price is externally communicated, all customers for which the offered price exceeds their reservation price return their dysfunctional product to the OEM. Constraint (5.14) must hence be replaced by (5.17).

$$f \cdot w_{i,t-1} - x_{i,j,t} \leq M \cdot (1 - \Theta_{j,t}) \quad i,j = 1,...,n \quad i \leq j \quad t = 1,...,T \qquad (5.17)$$

Constraint (5.17) captures the fact that for a given buy-back price p_j (i.e. $\Theta_{j,t} = 1$) all customers from segments $i = 1, ..., j-1$ are going to sell their broken products.

By solving the respective optimization problems M2 and M4 for n=2 segments the economic impact of the assumptions regarding pricing and quantity flexibility as well as information availability can be evaluated. In Table 5.6, the total discounted profits and the final order sizes are presented for each setting. Interestingly, while showing in general the same solution structure with three phases as M1, the third (buy-back) phase of both settings M2 and M4 is characterized by switching price decisions. While in most periods the low price p_1 is set and broken products from the first customer segment are bought back only, sporadically the price p_2 is set. In those 'campaign' periods a stock of broken products is build up, i.e. more broken products are bought back than are actually needed to satisfy the current period's demand[3].

Tab. 5.6: Total discounted profit, relative deviation from M1 and corresponding final order sizes.

	Benchmark	M1	M2	M4
Total discounted profit Π	2390	3383	3358	3343
Relative deviation from Benchmark Δ	–	+41.6%	+40.5%	+39.9%
Final order size FO	935	621	622	626

The comparably small gap between M1, M2, and M4 can be explained by the similarity of the optimal solution structures. First, it can be observed that the different assumptions do not influence the size of the final order substantially. Second, a variation in the model assumptions results in changes in the optimal solution structure that occur quite late in the planning horizon. As all cash flows are discounted, a deviation in one

[3] For a detailed description of the M2 policy structure see the Appendix, page 163. For the corresponding description of the M4 policy structure, please refer to the Appendix, page 164.

of the later periods does therefore only have a limited effect on the total discounted profit. Although M3 is not explicitly treated, interpreting the optimal objective values of M2 as a lower and of M4 as an upper bound of the optimal objective value, analogous results are to be expected for the not explicitly modeled setting M3. Thus, a main insight from this example is that the OEM can significantly enhance his performance by including buy-backs into his decision-making process even with only limited pricing and quantity flexibility and information availability.

Since we only dealt with a single example so far, the following subsection conducts a sensitivity analysis to provide insights into the robustness of our findings.

5.4.3 Sensitivity to changing parameters

Taking the base case from Section 3 with two segments as starting point, a sensitivity analysis is performed that focuses on the question under which parameter combinations the buy-back option appears to be especially valuable. To achieve this, all relevant parameters are modified to a considerably higher and lower value while keeping all other parameters constant. Since we did not find a substantial difference for the settings M2 and M4, we restrict our discussion to a comparison of M1 and the benchmark solution without buy-back. The corresponding results for M2 and M4 can be found in the Appendix, page 165f. Table 5.7 presents those parameters that seem to have a substantial impact on the profitability of the buy-back option, i.e. the remanufacturing yield rate β, the interest rate r, the final lot unit cost c_f, the length of the planning horizon T as well as both holding cost parameters h_R and h_M.

These findings can be explained as follows. In the benchmark setting, spare part demand can only be satisfied by two options, either by manufacturing spare parts in the final order or by remanufacturing. As serving customers close to the end of the planning horizon becomes more and more expensive, the benchmark solution worsens as the final order becomes larger compared to setting M1. For instance, this is the case if the remanufacturing yield rate β is low and if the interest rate r, the final order unit cost c_f or one of both holding cost parameters become larger. A larger h_R, for instance, means that the remanufacturing operations could have started earlier which

Tab. 5.7: Optimal final order FO, discounted profit Π, first buy-back period z and relative profit change Δ in the benchmark solution and M1 for parameters with significant impact.

		Benchmark		Optimal buy-back in M1			
		FO_{BM}	Π_{BM}	FO_1	z_1	Π_1	Δ_1
base case		**935**	**2390**	**621**	**42**	**3383**	**+41.6%**
	40%	1122	836	689	36	2415	+188.7%
β	**50%**	**935**	**2390**	**621**	**42**	**3383**	**+41.6%**
	60%	748	3821	541	47	4396	+15.1%
	1.25%	935	4142	758	55	4513	+8.9%
r	**2.5%**	**935**	**2390**	**621**	**42**	**3383**	**+41.6%**
	5%	935	567	462	29	2287	+303%
	1.5	935	3793	724	48	4369	+15.2%
c_f	**3**	**935**	**2390**	**621**	**42**	**3383**	**+41.6%**
	4.5	935	986	553	36	2510	+154.4%
	60	795	3156	628	42	3454	+9.4%
T	**80**	**935**	**2390**	**621**	**42**	**3383**	**+41.6%**
	100	1039	1644	610	41	3371	+105%
	0.15	935	2868	652	45	3604	+25.7%
h_M	**0.2**	**935**	**2390**	**621**	**42**	**3383**	**+41.6%**
	0.25	935	1912	587	39	3185	+66.6%
	0.05	935	3210	674	47	3816	+18.9%
h_R	**0.1**	**935**	**2390**	**621**	**42**	**3383**	**+41.6%**
	0.15	935	1789	576	38	3108	+73.7%

reduces the number of spare parts procured in the final order. However, due to its limited flexibility the benchmark solution cannot react appropriately and is therefore less profitable than setting M1. Regarding the length of the planning horizon, it can be said that a longer planning horizon reduces the total profits of the OEM if he does not account for the buy-back option. In turn, incorporating the buy-back option into the spare parts fulfillment strategy allows the OEM to offer even longer service periods while keeping the costs for this additional service at an adequate level.

Table 5.8 presents those parameters that change the advantageousness of the buy-back option only slightly, i.e. the outflow rates ν_1 and ν_2, the buy-back prices p_1 and p_2, and the initial segment sizes $\bar{w}_1$ and $\bar{w}_2$. It can be seen that a decreasing outflow from one of the customer segments improves the relative performance of M1 slightly. This is because the less flexible benchmark solution needs to increase the final order while M1 can react by buying back more broken products. The influence of both buy-back prices appears to be relatively small as well. The larger one of these prices is, the smaller the possible gain becomes. The buy-back price effect shows its impact also when the initial assignment of customers to segments is changed while keeping the total number of customers constant at 400. If, for instance, the initial install base in segment one comprises 300 customers while it contains only 100 in the second segment, the average buy-back price will decrease as p_1 and p_2 remain at 10 and 20, respectively. Interestingly, the deviation Δ_1 remains constant if the number of customers in each segment is multiplied by the same factor.

Finally, other parameters that do not influence the outcome significantly need to be mentioned as well. Among these parameters, the failure rate f can be found. The numerical investigation has revealed that a change in the failure rate does not have a large impact on the profitability of the buy-back option as all decisions are increased or decreased approximately proportionally. This means that for $f = 5\%$ the size of the final order and all subsequent decisions decrease to about half of their initial base case values. Furthermore, the variable cost of remanufacturing a broken product c_r has no substantial influence. This can be explained by the fact that all broken products have to be remanufactured if they are not disposed of beforehand. Thus, no important

Tab. 5.8: Optimal final order FO, discounted profit Π, first buy-back period z and relative profit change Δ in the benchmark solution and M1 for parameters with relatively small impact.

		Benchmark		Optimal buy-back in M1			
		FO_{BM}	Π_{BM}	FO_1	z_1	Π_1	Δ_1
base case		**935**	**2390**	**621**	**42**	**3383**	**+41.6%**
	1%	1020	2222	643	41	3467	+56.1%
ν_1	**1.5%**	**935**	**2390**	**621**	**42**	**3383**	**+41.6%**
	2%	868	2492	598	42	3305	+32.6%
	1%	1020	2222	645	41	3385	+52.4%
ν_2	**1.5%**	**935**	**2390**	**621**	**42**	**3383**	**+41.6%**
	2%	868	2492	590	42	3368	+35.1%
	5	935	2390	595	39	3534	+47.9%
p_1	**10**	**935**	**2390**	**621**	**42**	**3383**	**+41.6%**
	15	935	2390	638	44	3249	+36%
	15	935	2390	605	41	3456	+44.6%
p_2	**20**	**935**	**2390**	**621**	**42**	**3383**	**+41.6%**
	25	935	2390	628	42	3317	+38.8%
	(300/100)	935	2390	594	40	3495	+46.2%
$(\bar{w}_1/\bar{w}_2)$	**(200/200)**	**935**	**2390**	**621**	**42**	**3383**	**+41.6%**
	(100/300)	935	2390	635	41	3256	+36.2%

influence on the buy-back decision can be derived from this parameter. We would like to refer the reader to the Appendix, page 165f, for the corresponding results.

Regarding the other model settings (M2 and M4), the examined numerical examples reveal that the profit loss from restricted information availability and/or quantity and pricing flexibility only reacts slightly when one of the parameter values is altered. The largest loss in total profit that has been observed is 2.7% between settings M4 and M1 in a situation with a large remanufacturing yield rate β=60%. However, tendencies can be identified. The relative deviation between the profits of M4 and M1 seems to increase for a small failure rate f, for a high per unit final lot cost c_f, and for comparably large holding costs h_R and h_M, respectively. These tendencies could also be observed when comparing M2 and M1 but on a less prominent scale. For details, we refer again to the Appendix, page 165f.

5.5 Conclusions

Due to a high profitability, after-sales management has received an ever increasing attention in the recent past. This study was particularly motivated by the automotive industry which continues to give long lasting mobility warranties for their cars. These warranties are obviously an attractive instrument for the marketing and sales department while they impose a challenge for the management of spare parts. This study highlights that buying back broken products is, under certain circumstances which can be found in practice, an attractive instrument to manage the end-of-life service period, especially in situations in which options to resupply are limited to placing a final order and later remanufacturing broken parts.

For different settings regarding the availability of information required for buy-back as well as limited pricing and quantity flexibility we propose simple MILP models that are able to find optimal strategies. After evaluating these strategies in a numerical study we find, that buying back defective products is a beneficial substitute for building up a large inventory of spare parts at the beginning of the planning horizon by procuring parts in a final order. It seems that the availability of detailed information and limitations of pricing and quantity flexibility do not affect the profitability of a product recovery system with buy-back option substantially. A main reason for this result can be found in the structural similarities of the optimal policies that could be observed by numerically examining a representative base case. Interestingly though, the buy-back is performed in form of campaigns in situations where the pricing flexibility is limited, i.e. a regular low price buy-back interval is interrupted by single periods in which a high price is offered to the customers.

It was shown which parameters especially influence the profitability of the buy-back option. Here it seems that those parameters determining the profit impact of the final order size (like unit production and holding cost) seem to be of highest importance, while the influence of buy-back related parameters like prices show only limited impact. This is because our benchmark solution without buy-back only shows small flexibility to react on parameter changes while in the buy-back case, a trade-off is struck between the final order size and later buying back more or less products. In case of a high remanufacturing yield rate, the system can be handled like a repair system (see, e.g., Sherbrooke, 2004) where the buy-back option is less favorable. If the cost of the employed capital is high, it becomes more and more attractive to reduce the final order and instead compensating the customers for not fulfilling the spare parts availability guarantee.

This study is to our knowledge the first attempt to investigate the value of a buy-back option in a closed-loop supply chain for spare parts. There are certainly some limitations to this study which can be overcome by further research. We limit our analysis to a MILP formulation which is numerically solved. Even though the numerical study is restricted to parameters that do not change over time (like e.g. the failure rate or customer valuation of their product), time dependent parameters can be addressed as well. General structural properties of optimal solutions could be obtained by using optimal control methods, as have been successfully applied in product recovery systems (see Kiesmüller et al., 2004; Kleber, 2006). Complementing our deterministic approach, a stochastic simulation could be used to evaluate more realistic models involving uncertainty. Here, due to the high flexibility, buying back broken products becomes an even more attractive option. Finally, another extension would include the case where multiple parts are included in a product and thus, the buy-back would yield inflows of several remanufacturable parts.

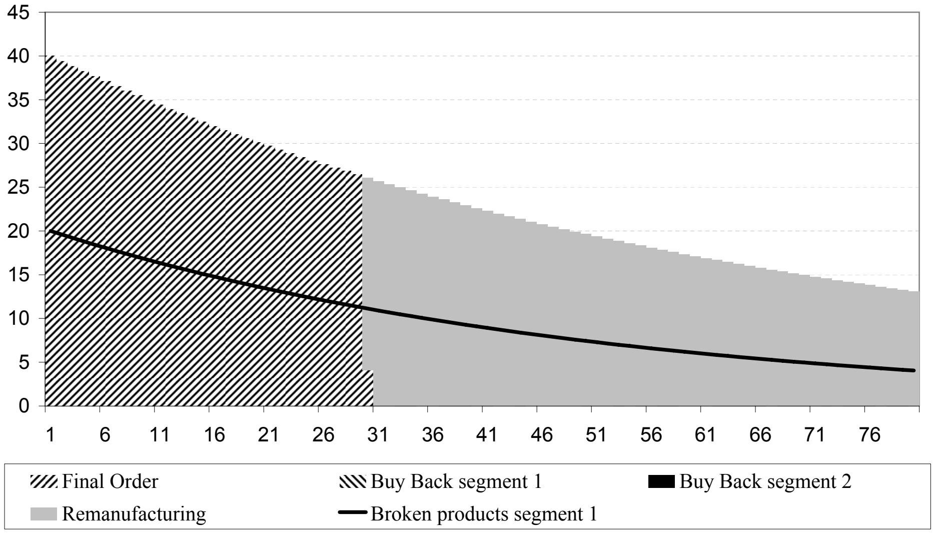

Fig. 5.3: Fulfillment of spare parts demand the benchmark solution

5.6 Appendix

5.6.1 Detailed discussion of policy structures

In the following, we discuss structural properties of optimal solutions. Since the main effects are already observable when two customer segments are distinguished, we restrict ourselves to that case.

Benchmark without buy-back. Figure 5.3 depicts the optimal solution structure. Here, total demand for spare parts (the height of each bar) is presented over the entire planning horizon. Thus, it can be seen that the total demand for spare parts is equal to 40 units in the first period. As customers leave the service network, the demand for spare parts declines over the planning horizon, reaching 31.8% of its first period's value in the last period. Since total demand for spare parts consists of the demand of two different customer segments, the black line in Figure 5.3 indicates the first segment's demand for spare parts, and the distance between the height of each bar and the black line depicts the second segment's demand. Additionally, the color-coded bars in this figure present the respective source from which the demand for spare parts has been satisfied. Two phases can be distinguished without buy-back option. In the first phase (periods 1 to 29) the demand for spare parts is satisfied completely from the final order.

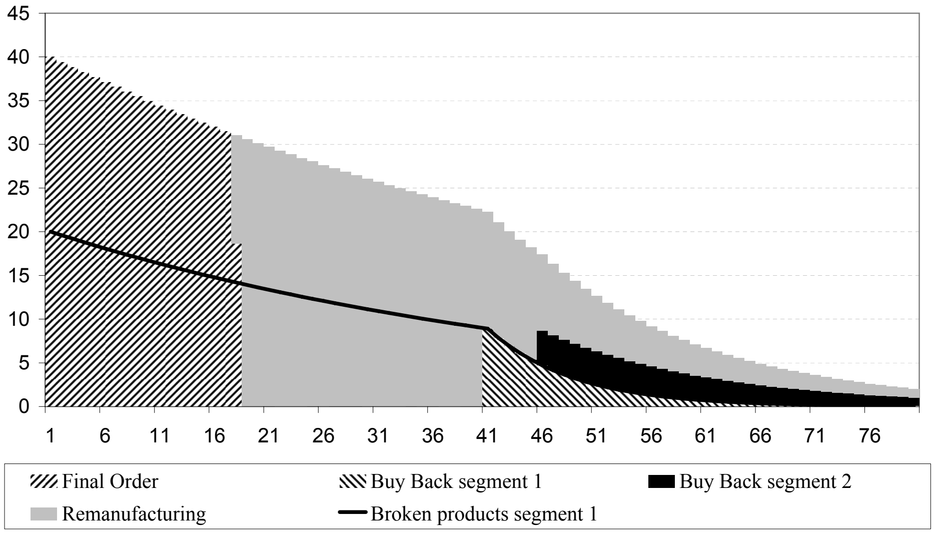

Fig. 5.4: Fulfillment of spare parts demand in case M1

All broken products returning to the OEM in this phase are immediately disassembled and the parts obtained by this procedure are held in the recoverables inventory and none is disposed of. In the second phase (periods 30 to 80) the strategic stock of returned products built up in the first phase is used to serve the entire demand by remanufacturing the recoverables inventory.

M1. In the following, we analyze how this structure changes when including the buy-back option. Figure 5.4 depicts the development of spare part demand in setting M1 in the case of two customer segments. Although considerably shorter, both phases of the benchmark solution can be found in the optimal solution of setting M1 as well. While the first phase consists of 17 periods (from period 1 to 17) in which the entire spare part demand is satisfied by acquisitions made in the final lot, the second phase covers 23 periods (from period 18 to 40). In this phase, all demands are fulfilled by remanufacturing broken products that have been brought to the recoverables inventory in the first phase. In contrast to the benchmark solution, the recoverables inventory is not depleted at the end of the second phase. In a third phase (starting in period 41), the OEM starts to buy back from the first customer segment. These products are remanufactured instantly and are used to satisfy the current demand. The remaining demand which cannot be satisfied by remanufacturing bought-back products from the first segment is fulfilled by remanufacturing broken parts left in stock from the second

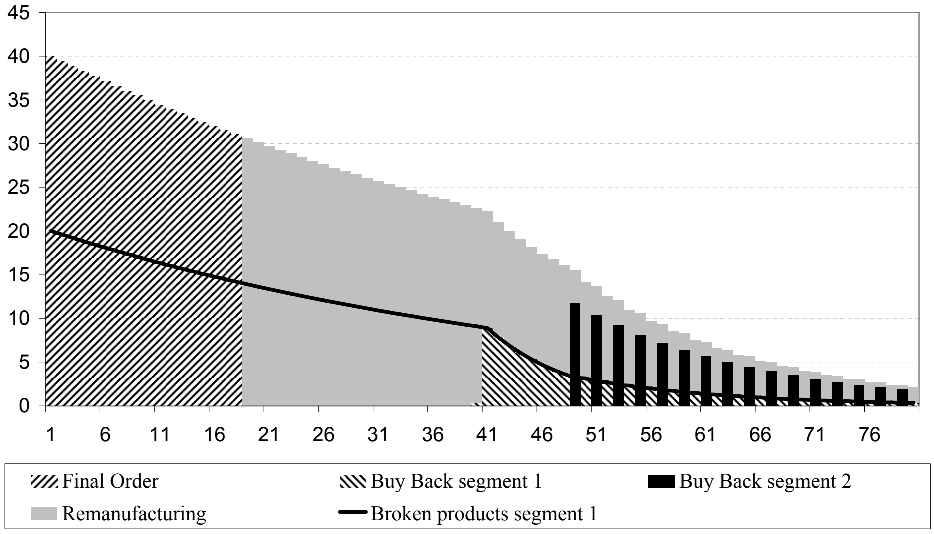

Fig. 5.5: Fulfillment of spare parts demand in case M2

phase. This strategy is followed until the recoverables stock is depleted. Then, the OEM buys back from both segments. Interestingly, no further stock is build up in the recoverables inventory. Hence, the total buy-back quantity is set to just compensate the yield loss. As the OEM has perfect knowledge of his customers and can offer each customer an individual buy-back price, he will only procure broken products from the first segment for the buy-back price p_1.

M2. Figure 5.5 depicts the structure of the optimal solution if the presumption of full pricing flexibility is lifted. One can still find the three phases already explained for setting M1. However, quantity and pricing decisions change in the third phase due to the limited pricing flexibility. In contrast to setting M1, the third phase of setting M2 is characterized by switching price decisions. While in most periods the low price p_1 is set and broken products from the first customer segment are bought back only, sporadically the second price p_2 is set. In those 'campaign' periods a stock of broken products is build up, i.e. more broken products are bought back than are actually needed to satisfy the current period's demand. This strategy is driven by the fact that the OEM wants to set the price p_2 as seldom as possible. Yet, the entire demand of the second segment cannot be satisfied by only using bought-back products from the first segment. Thus, without stock-keeping the OEM would be forced to set p_2 in every period, a strategy that cannot be optimal.

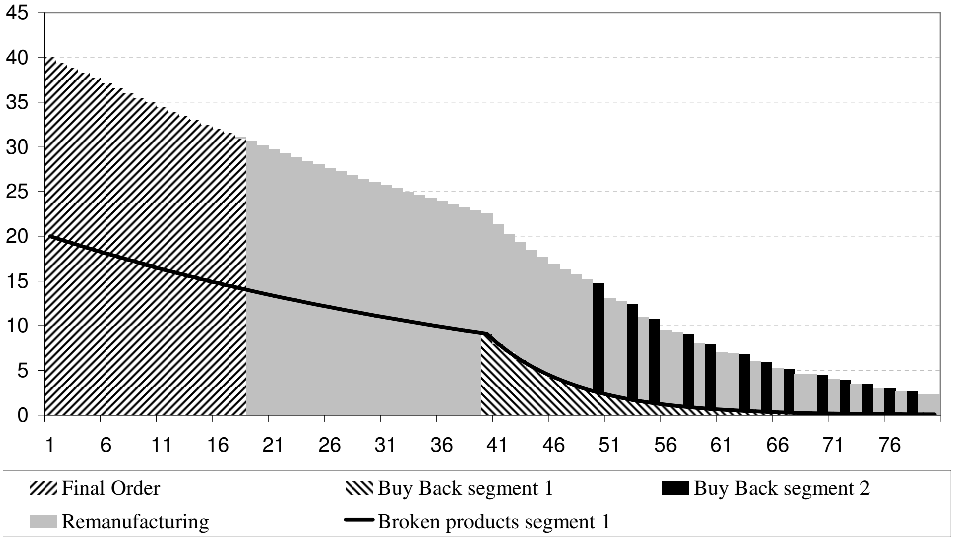

Fig. 5.6: Fulfillment of spare parts demand in case M4

M4. The structure of the optimal solution of setting M4 is illustrated in Figure 5.6. As there is no quantity flexibility in this setting, the OEM has to buy back all broken products from both segments if the price p_2 is set, i.e. he pays all customers in segment 1 a higher price than their reservation price. The third phase exhibits the same switching pattern as in setting M2, except the fact that fewer periods can be observed during which the higher price p_2 is set. This can be explained by the missing quantity flexibility. If the price p_2 is set, the OEM has to buy back all products from the first and the second customer segment. Thus, a higher temporary stock is build up in the recoverables inventory which lasts longer to fulfill future spare part demands than in M2.

5.6.2 Results of the sensitivity analysis

The following Tables 5.9 and 5.10 present the results of the sensitivity analysis of all settings examined (Benchmark, M1, M2, and M4).

altered parameter		Benchmark		M4				M2				M1			
		FO_{BM}	Π_{BM}	FO_4	z_4	Π_4	Δ_4	FO_2	z_2	Π_2	Δ_2	FO_1	z_1	Π_1	Δ_1
base case		**935**	**2390**	**626**	**41**	**3343**	**+39.9%**	**622**	**41**	**3358**	**+40.5%**	**621**	**42**	**3383**	**+41.6%**
	40%	1122	836	689	35	2352	+181.2%	685	35	2366	+182.9%	689	36	2415	+188.7%
β	**50%**	**935**	**2390**	**626**	**41**	**3343**	**+39.9%**	**622**	**41**	**3358**	**+40.5%**	**621**	**42**	**3383**	**+41.6%**
	60%	748	3821	545	48	4372	+14.4%	541	47	4383	+14.7%	541	47	4396	+15.1%
	1.25%	935	4142	781	55	4493	+8.5%	777	55	4496	+8.5%	758	55	4513	+8.9%
r	**2.5%**	**935**	**2390**	**626**	**41**	**3343**	**+39.9%**	**622**	**41**	**3358**	**+40.5%**	**621**	**42**	**3383**	**+41.6%**
	5%	935	567	471	29	2249	+296.3%	461	28	2263	+298.7%	462	29	2287	+303%
	1.5	935	3793	700	48	4332	+14.2%	694	48	4341	+14.5%	724	48	4369	+15.2%
c_f	**3**	**935**	**2390**	**626**	**41**	**3343**	**+39.9%**	**622**	**41**	**3358**	**+40.5%**	**621**	**42**	**3383**	**+41.6%**
	4.5	935	986	559	35	2457	+149.1%	552	35	2478	+151.2%	553	36	2510	+154.4%
	60	795	3156	644	42	3436	+8.9%	639	42	3437	+8.9%	628	42	3454	+9.4%
T	**80**	**935**	**2390**	**626**	**41**	**3343**	**+39.9%**	**622**	**41**	**3358**	**+40.5%**	**621**	**42**	**3383**	**+41.6%**
	100	1039	1644	626	41	3327	+102.3%	620	41	3346	+103.5%	610	41	3371	+105%
	(0.1/0.15)	935	2868	657	44	3569	+24.5%	654	44	3581	+24.9%	652	45	3604	+25.7%
	(0.1/0.2)	**935**	**2390**	**626**	**41**	**3343**	**+39.9%**	**622**	**41**	**3358**	**+40.5%**	**621**	**42**	**3383**	**+41.6%**
	(0.1/0.25)	935	1912	604	39	3138	+64.2%	590	38	3157	+65.1%	587	39	3185	+66.6%
	(0.05/0.2)	935	3210	677	46	3789	+18%	677	46	3798	+18.3%	674	47	3816	+18.9%
(h_R/h_M)	**(0.1/0.2)**	**935**	**2390**	**626**	**41**	**3343**	**+39.9%**	**622**	**41**	**3358**	**+40.5%**	**621**	**42**	**3383**	**+41.6%**
	(0.15/0.2)	935	1789	582	37	3056	+70.8%	586	38	3076	+71.9%	576	38	3108	+73.7%
	(0.05/0.15)	935	3688	725	51	4061	+10.1%	718	50	4067	+10.3%	715	51	4082	+10.7%
	(0.1/0.2)	**935**	**2390**	**626**	**41**	**3343**	**+39.9%**	**622**	**41**	**3358**	**+40.5%**	**621**	**42**	**3383**	**+41.6%**
	(0.15/0.25)	935	1091	555	35	2785	+155.2%	549	35	2809	+157.5%	539	35	2846	+160.8%

Tab. 5.9: Impact of selected parameters I

Tab. 5.10: Impact of selected parameters II

altered parameter		Benchmark		M4				M2				M1			
		FO_{BM}	Π_{BM}	FO_4	z_4	Π_4	Δ_4	FO_2	z_2	Π_2	Δ_2	FO_1	z_1	Π_1	Δ_1
	(1.5%/1%)	1020	2222	664	41	3335	+50.1%	649	40	3356	+51.1%	645	41	3385	+52.4%
	(1.5%/1.5%)	**935**	**2390**	**626**	**41**	**3343**	**+39.9%**	**622**	**41**	**3358**	**+40.5%**	**621**	**42**	**3383**	**+41.6%**
	(1.5%/2%)	868	2492	593	41	3335	+33.8%	589	41	3346	+34.3%	590	42	3368	+35.1%
	(1%/1.5%)	1020	2222	651	41	3422	+54%	643	41	3438	+54.7%	643	41	3467	+56.1%
(ν_1/ν_2)	**(1.5%/1.5%)**	**935**	**2390**	**626**	**41**	**3343**	**+39.9%**	**622**	**41**	**3358**	**+40.5%**	**621**	**42**	**3383**	**+41.6%**
	(2%/1.5%)	868	2492	604	41	3269	+31.2%	601	41	3283	+31.8%	598	42	3305	+32.6%
	(1%/1%)	1105	2051	671	40	3412	+66.3%	680	41	3434	+67.4%	668	41	3468	+69.1%
	(1.5%/1.5%)	**935**	**2390**	**626**	**41**	**3343**	**+39.9%**	**622**	**41**	**3358**	**+40.5%**	**621**	**42**	**3383**	**+41.6%**
	(2%/2%)	801	2592	571	41	3259	+25.8%	568	41	3269	+26.1%	567	42	3289	+26.9%
	(10/15)	935	2390	614	41	3423	+43.2%	603	40	3437	+43.8%	605	41	3456	+44.6%
	(10/20)	**935**	**2390**	**626**	**41**	**3343**	**+39.9%**	**622**	**41**	**3358**	**+40.5%**	**621**	**42**	**3383**	**+41.6%**
	(10/25)	935	2390	640	41	3276	+37.1%	635	41	3291	+37.7%	628	42	3317	+38.8%
	(5/20)	935	2390	597	37	3488	+46%	600	38	3505	+46.7%	595	39	3534	+47.9%
(p_1/p_2)	**(10/20)**	**935**	**2390**	**626**	**41**	**3343**	**+39.9%**	**622**	**41**	**3358**	**+40.5%**	**621**	**42**	**3383**	**+41.6%**
	(15/20)	935	2390	646	44	3220	+34.7%	644	44	3232	+35.2%	638	44	3249	+36%
	(5/15)	935	2390	582	37	3567	+49.3%	578	37	3584	+50%	576	38	3612	+51.1%
	(10/20)	**935**	**2390**	**626**	**41**	**3343**	**+39.9%**	**622**	**41**	**3358**	**+40.5%**	**621**	**42**	**3383**	**+41.6%**
	(15/25)	935	2390	657	44	3151	+31.9%	661	45	3164	+32.4%	652	45	3187	+33.4%
	(300/100)	935	2390	605	41	3473	+45.3%	595	40	3484	+45.8%	594	40	3495	+46.2%
$(\bar{w}_1/\bar{w}_2)$	**(200/200)**	**935**	**2390**	**626**	**41**	**3343**	**+39.9%**	**622**	**41**	**3358**	**+40.5%**	**621**	**42**	**3383**	**+41.6%**
	(100/300)	935	2390	638	39	3217	+34.6%	642	40	3235	+35.4%	635	41	3256	+36.2%
	0.75	935	2808	612	40	3859	+37.4%	620	41	3877	+38%	610	41	3905	+39.1%
c_r	**1.5**	**935**	**2390**	**626**	**41**	**3343**	**+39.9%**	**622**	**41**	**3358**	**+40.5%**	**621**	**42**	**3383**	**+41.6%**
	2.25	935	1971	629	41	2827	+43.5%	626	41	2840	+44.1%	623	42	2863	+45.3%
	5%	468	1195	316	42	1652	+38.3%	318	42	1656	+38.6%	316	43	1679	+40.5%
f	**10%**	**935**	**2390**	**626**	**41**	**3343**	**+39.9%**	**622**	**41**	**3358**	**+40.5%**	**621**	**42**	**3383**	**+41.6%**
	15%	1403	3584	916	40	5087	+41.9%	910	40	5112	+42.6%	911	41	5134	+43.2%

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Erklärung zur Eigenleistung des Verfassers

Die Kapitel 1, 2 und 4 wurden selbständig vom Verfasser der Dissertation erarbeitet.

Kapitel 3 basiert auf einer gemeinsamen Arbeit mit Ivan Ferretti von der Universität Brescia. Die Eigenleistung des Verfassers bestand bei dieser Arbeit in der strategischen Einbettung der Problemstellung, der Modellierung der einzelnen Politiken sowie in der Durchführung und Interpretation der numerischen Untersuchungen.

Kapitel 5 basiert auf einer gemeinsamen Arbeit mit Rainer Kleber und Guido Voigt (beide Otto-von-Guericke Universität Magdeburg). Der Autor der vorliegenden Dissertation war hauptverantwortlich für die Durchführung und Auswertung der numerischen Studie. Zudem arbeitete er maßgeblich an der Formulierung der daraus entstandenen Publikation mit.

Tobias Schulz

Eidesstattliche Erklärung

Hiermit versichere ich an Eides statt, dass diese Dissertation selbstständig und ohne Benutzung anderer als der angegebenen Hilfsmittel angefertigt und alle Stellen, die wörtlich oder sinngemäß aus veröffentlichten oder unveröffentlichten Schriften entnommen wurden, als solche kenntlich gemacht wurden. Zudem erkläre ich, dass dies mein erster Promotionsversuch ist.

Tobias Schulz