Financial Structure Choice in Owner-Managed Firms.

Entrepreneurial Characteristics, Utility of Control, and the Competitive Context.

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Abstract

The present thesis crafts a framework to investigate how a risk averse ownermanager's corporate control considerations affect simultaneously conducted investment and financing decisions in awareness of the competitive context on the product market. The model configures the decision maker's objective function as depending on both, monetary income and decision power exerted via her ownership share. To the best of our knowledge, this study provides the first theoretical examination of financial structure choice which (1) simultaneously endogenizes financing and investment decisions, (2) accounts for the implications of the decision maker's corporate control considerations arising on the grounds of her eventual ownership share, and (3) examines the crucial role of firms' individual and environmental characteristics (including the prevailing market conditions such as the competitive context, the firm's operations/cost structure, and the decision maker's risk attitude). The model generates elementary results which relate to and complement pertinent findings within the differing streams of theoretical financial structure literature. Further results clearly contrast the findings of prior theories, but are consistent with existing empirical evidence.

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Symbols

a	potential demand
AC	average costs
b	slope of the (expected) demand function
$B(\cdot)$	binomial distribution operator
С	marginal costs
C	total costs
CE	certainty equivalent
D	the decision maker
e	Euler's number
E	total amount of equity
E^{ex}	amount of outside equity (from external equity providers)
f	maximum marginal costs (upper bound)
F	fixed costs
g	slope of marginal costs
h^n	history of alternating price adjustments up to move n

Н	upper swing bound
k	final move in the sequence of alternating price adjustments
K	the firm's total stock of capital
LI	Lerner Index
L	lower swing bound
n	index of price moves
р	price
$Pr(\cdot)$	probability operator
Q	demand function
<i>r</i> ₁	risk-free interest rate
r_2	interest rate of the corporate debt
s^i,s^j	strategy pair for firms i and j
S	firm surplus before taxes
Т	after-tax fraction of firm surplus defined by $(1 - \tau^c)$
t	time stage
$U(\cdot)$	the decision maker's utility function
$u(\cdot)$	first utility component (representing the decision maker's utility of wealth)
$v(\cdot)$	second utility component (representing the decision maker's utility of control)
V_F	value of the firm after reimbursement of all debt holders
W^D_0	the decision maker's initial wealth
W^D_1	the decision maker's end-of-period wealth
X	arbitrary positive number
y_i	binary variable which indicates "no" or "yes" votes in a ballot

Y	number of "yes" votes in a ballot defined by $Y = \sum_{i=1}^{n} y_i$
Z	standard normally distributed random variable $(z \sim \mathcal{N}(0, 1))$

α	fraction of the decision maker's initial wealth invested into the riskless asset
Г	incurred amount of corporate debt
ε	price elasticity of demand
ε	smallest possible increment in shares
η	Arrow-Pratt measure of absolute risk aversion
λ	Kuhn-Tucker variable
μ	expected value
П	the firm's operational profit over the given period
Π^{full}	profits made by one firm by taking the entire market (full monopo- ly profit)
$\Pi^{ m shared}$	profits made by one firm by sharing the market (shared monopoly profit)
ρ	environmental risk parameter (normally distributed)
σ	standard deviation
σ^2	variance
$ au^c$	corporate tax rate
ϕ	fraction of the firm owned by the decision maker
Φ	number of shares owned by the decision maker
Ω	defined by $\Omega = u'(CE)$

С	degree of corporate control
ł	placeholder value (see Definition 5.1)
L	Lagrangean
$\mathcal{N}(\cdot)$	normal distribution operator
Q	quota required per ballot
1~	placeholder value (see Definition 5.1)
${\mathcal T}$	total number of shares

- \mathbb{E} expectations operator
- \mathbb{S}^* set of undominated strategy pairs

Abbreviations

CAPM	Capital-Asset-Pricing-Model
CARA	constant absolute risk aversion
EV	expected value
FOC	first order condition
IPO	initial public offering
LHS	left-hand side
M&A	mergers & acquisitions
MPNE	Markov perfect Nash equilibrium
pdf	probability density function
RHS	right-hand side
SPNE	subgame perfect Nash equilibrium
$\mathrm{TL}^{\mathrm{Max}}$	maximum target level of corporate investment
$TL^{\rm Min}$	minimum target level of corporate investment
VNM	von Neumann/Morgenstern

CHAPTER ONE INTRODUCTION

1.1 Problem Definition and Purpose of the Study

How do firms' decision makers conduct their financial structure¹ decisions? The present thesis scrutinizes this question by arguing that addressing three basic shortcomings of the pertinent theoretical financial structure research changes several standard results of the literature and leads to further insights. These three shortcomings refer to

- (1) the separation of *investment and financing decisions*,
- (2) the neglect of *corporate control considerations* of the firms' decision makers,
- (3) the isolation of financial structure analysis from the properties of the *competitive context* and of the focal firm's *operations/cost structure*.

By explicitly accounting for these notions, and by additionally emphasizing the role of firms' individual and environmental characteristics (such as *wealth constraints* and *environmental risk*), the present thesis means to contribute to overcome the prevalent research gap concerning the linkage between financial structure choice and these (widely neglected) antecedents.

Based on the irrelevance theorem of Modigliani/Miller in their famous 1958 article, a long-standing debate on the optimal design of financial structure evolved throughout the following decades. Ever since, a variety of different con-

The terms *financial structure* and *capital structure* are used synonymously and only comprise the durable capital endowment of enterprises, i.e., *equity* and *long term debt*. Short-term financing is excluded from the considerations of the present study.

ditional approaches have been developed, each of them illuminating capital structure decisions from an idiosyncratic perspective. Though these differing theoretical approaches have broadly confirmed the general importance of various influencing factors (such as taxes, asymmetric information, and agency costs), their explanatory contribution can hardly be considered satisfying. This assertion is strongly supported by the fact that observable debt-equity-ratios are subject to considerable variations, even within industries that are apparently homogeneous with respect to the identified influencing factors.²

The present thesis argues that the three shortcomings outlined above constitute a prevalent characteristic of the majority of works dedicated to the question under scrutiny. Existing models typically disregard the focal decision maker's corporate control considerations. Moreover, a conceptual separation between investment and financing decisions is practically ubiquitous. Finally, the widespread isolation of financial structure analyses from the properties of the output market needs to be underscored. The model developed in this thesis discloses how results are influenced and even reversed by the deliberate consideration of these notions. To the best of our knowledge, the present study provides the first theoretical examination of financial structure choice which simultaneously accounts for these three aspects briefly outlined in the following.

1.1.1 Simultaneous Investment and Financing Decisions

One of the most astonishing features of the pertinent financial structure literature concerns the fact that practically all significant models strictly separate the financing decision from the investment decision. A firm's financial structure is merely interpreted as the debt-equity-ratio. The determination of this ratio is usually analyzed in total isolation from the determination of the firm's overall investment level. Hence, this overall investment level is usually taken as fixed and purely exogenous. Arguably, this may result in misleading conclusions, especially for entrepreneurial decision makers who of course have to make decisions about both the composition *and* the level of their firm's capital endowment. If the interrelation between level and mixture of the firm's capitalization is nontrivial, simultaneously endogenizing these two features of financial structure may alter or even reverse the results and, thereby, provide a rationale for prevalent contradictions in existing literature.

The present thesis postulates a more holistic view by arguing that the optimal relative proportion between debt and equity (the financing decision) is tightly interwoven with the choice of their absolute levels (the investment deci-

² See MacKay/Phillips (2005), pp. 1433-1435; or Myers (2001), p. 82.

sion). Contrary to the existing literature,³ the model developed in this thesis aims at overcoming this conceptual separation by simultaneously endogenizing both decisions. Thereby, the model does not fall victim to the myopia described above.

1.1.2 Financial Structure and Corporate Control

An important notion underlying the determination of financial structure concerns the collateral properties of equity and debt, i.e., equity carries voting rights whereas debt does not. Surprisingly, a direct question that immediately arises on the grounds of this consideration has been largely ignored by the financial literature: how will a *controlling shareholder* (who owns a considerable block of shares and acts as an owner-manager) use the relative proportions of inside equity, outside equity, and debt in the firm's stock of capital to maintain or defend her *intra-organizational power*? Minimizing the probability of a loss of control might be regarded as a fundamental consideration of controlling shareholders.

The present thesis accounts for this consideration by configuring the focal decision maker's objective function as not only depending on her pecuniary wealth, but also on the degree of corporate control exerted via her ownership share. Our analysis shows that accounting for the *utility of control*, in conjunction with a supposed risk aversion of the decision maker, triggers interesting deviations from the standard results of the pertinent financial literature.

1.1.3 Financial Structure and the Competitive Context

The description of firms' behavior on markets and the description of firms' internal properties, such as financial structure, have been intensely scrutinized by economic literature for several decades. However, for the most part of the contributions dealing with these two major components of the theory of the firm, the respective analytical coverage exhibits a conceptual separation which isolates the examination of the focal aspect. The internal linkage connecting both elements has been discussed rather sparsely. However, the interdependencies of financial markets and output markets are often multifaceted and only partially understood by economic theory.

A stylized illustration of these interdependencies obviously suggests the employment of features stemming from the theory of industrial organization. In contrary to the analytical frame of a general equilibrium context, modern indus-

³ See Chapter 2.

trial organization literature is commonly based on a partial analysis of firms' behavior on product markets. Unfortunately, this theoretical workhorse fundamentally disregards the influence of financing decisions by implicitly presuming that all market competitors exhibit a sufficient endowment with equity capital.⁴ While both demand and supply side of product markets have been examined to a considerable extent, the manifold interweavements with related "preparatory markets" such as financial markets have been largely neglected.⁵ Hence, the interdependencies of corporate financial structure choice and product market characteristics have, hitherto, remained mostly unexplored.

The present thesis aims at contributing to surmount the disclosed research deficit. It investigates the antecedents of corporate financial structure decisions by developing a model which explicitly accounts for the prevalent competitive context on the output market.

1.2 Conceptual and Methodological Foundation

Within the scope of a scientific study, the disclosure and clarification of its underlying conceptual foundation allows for both a coherent demarcation of its general range of significance and an accurate setting of individual research priorities.⁶ Economic literature generally differentiates between two basic conceptual perspectives,⁷ namely the transformation-oriented *evolutionary concept*⁸ and the scarcity-oriented *rationality concept*.

The present study's underlying conceptual foundation is outlined by the rationality perspective. Within the frame of the rationality concept, economic activities essentially serve the purpose of profit maximization. Enterprises are interpreted as production devices, and human beings pursuit the maximization

⁴ See Tirole (1993).

⁵ Stadler (1997), p. 153. This contribution provides a brief discussion of research deficits regarding the unexplored interdependencies of financial markets and product markets.

⁶ See Ulrich (2001), pp. 85-100.

⁷ See, for example, Sachs/Hauser (2002), pp. 18-23; Specht (1997), pp. 24-25; or Thommen (2004), p. 441.

⁸ The evolutionary concept interprets enterprises as non-deterministic and open social systems which interact with their environment. The underlying conception of human nature is analogously characterized by the perception of humans as complex beings. Economic activities are essentially seen as processes that serve the satisfaction of human needs via goods and services. See Nelson/Winter (1982).

of their private utility. All endeavors aimed at this maximization are characterized by two important notions which assume that human beings, firstly, react to *incentives* and, secondly, are subject to *scarcity*. The rationality perspective is characterized by the employment of a simplified, stylized conception of the real world.

Basically, the essential purpose of the consecutive analysis is to derive a definite set of conclusions from a given set of assumptions via a deductive process of reasoning. We develop a positive model which aims at the disclosure of cause-and-effect-chains and emphasizes prevalent causal relations. Such causal relations are most easily expressed by employing logical "if-then" assertions, so that the validity of the "if"-condition (i.e., the set of underlying assumptions) triggers the "then"-result. A mathematical treatment is well-suited for both making explicit these underlying assumptions and conducting the deductive reasoning process to derive the corresponding results. Furthermore, the examination of causal relations within an imperfectly competitive environment includes the elicitation of the reactive relatedness of competing enterprises' decisions. This in turn establishes the necessity of an interactive decision theory to explore their competitive behavior. Within the scope of microeconomic theory, interactive decision problems are referred to as games, and the interactive decision theory to describe, explain, and predict the outcome of interactive decision problems is referred to as game theory. Consequently, game theory, which is deeply intervoven with the conception of the rationality perspective,⁹ provides a suitable and highly developed analytical framework for the examination of imperfectly competitive market interactions.

Hence, given the outlined research purpose, the rationality concept appears to be indeed particularly well-suited for the imminent analytical challenge. This challenge is tackled by employing a mathematical treatment, which allows for a rigorous description of the underlying assumptions, the analytical process, and the derived results. The analytical construct developed in the present thesis is not supposed to provide a comprehensively realistic reflection of the world, but to explicitly disclose coherences between defined influencing factors of an ownermanager's financial structure decisions. Thus, the deliberate employment of stylizations and abstractions, which corresponds to the methodological nature of economic modeling in the spirit of the rationality perspective, establishes and determines the architecture of the chosen research design.

⁹ Game theory achieved its remarkable success mainly because it allows for a rigorous examination of the multifarious implications of rationality, self-interest, and equilibrium within the context of both market and nonmarket interactions. See Gibbons (1997), p. 127.

On a further note, it feels necessary to underscore that the analysis conducted throughout the present thesis is grounded in the *methodological individualism*.¹⁰ Hence, a collectivity is never seen as an autonomous instance of decision making. The modeling is rather based on the presumption of strictly individual choices. Thereby, group behavior is to be explained by the aggregation of decisions by individuals, and these individual decisions constitute the basis for all observable (and unobservable) actions, including interactions with other individuals. The methodological individualism complies with the rationality concept's postulate of *rational players*. A decision maker is found to be rational if she¹¹ exhibits consistent behavior with respect to a given objective, i.e., if she conducts all decisions at her disposal so as to maximize her personal objective function. This objective function is solely based on her individual set of preferences.

1.3 Risk and Expected Utility

When considering the foundations of rational decision making, it is important to distinguish between *actions* and *consequences*. Actions are deliberately chosen and induce corresponding consequences. A rational decision maker has well-defined preferences over these consequences and is meant to choose a feasible set of actions that leads to her most desired set of consequences.¹²

From an ex ante perspective, a definite mapping of actions to consequences (in the sense that each action deterministically leads to a particular consequence) is hardly possible. Hence, we need to shed light on the choices of a decision maker in an uncertain environment in which the actual correspondence between actions and consequences is *stochastic*.

Surprisingly, the notion of risk has a relatively short history within the field of economic research. A rigorous formalization of a decision maker's choice in the presence of uncertainty was not accomplished before the path breaking effort of von Neumann/Morgenstern (VNM) in 1944.¹³ VNM crafted a rigorous axiomatic foundation for rational decision-making under uncertainty which was

¹⁰ See Schumpeter (1908).

¹¹ Throughout this thesis we will refer to the focal decision maker (the controlling shareholder) as female.

¹² Kreps (1988) provides an exhaustive and intuitive exposition of the axiomatic foundations that underlie the basic theory of choice.

¹³ Even though Knight (1921) and Ramsey (1931) provided important antecedents.

tightly connected to the notion of *expected utility*. Once this basic task was completed, VNM's expected utility hypothesis obtained an important generalization by Savage (1954). In the VNM frame, uncertainty is viewed as objective, i.e., there is a definite quantification of how likely various outcomes are, expressed in the form of a probability distribution. Savage's frame views uncertainty as being subjective, i.e., the probability distribution is a numerical representation of the decision maker's *personal expectations* and emerges on the grounds of her *private beliefs*. There are no externally imposed objective probabilities.

The shape of a decision maker's utility function is determined by her attitude towards risk. A decision maker who strictly prefers receiving the expected value (EV) of an uncertain asset for certain rather than the uncertain asset is called *risk averse*; a decision maker who prefers to receive the uncertain asset rather than the EV for certain is called *risk seeking*. A decision maker who is indifferent between the EV for certain and the uncertain asset is called *risk neutral*. A central feature of quantified utility models is that diminishing marginal utility in the sense of Gossen's first law¹⁴ implies risk aversion. If risk aversion holds for all possible outcomes over an arbitrary asset X with support [$\underline{X}; \overline{X}$], then the utility function U is concave in X. Similarly, risk seeking behavior implies that U(X) is convex, and risk neutrality implies that U(X) is linear. Hence, knowing that a decision maker is risk averse substantially restricts the shape of her utility function.¹⁵

The model developed in this thesis assumes risk averse decision makers, i.e., utility functions are strictly *concave* and *increasing* in wealth. In other words, a decision maker will always prefer more to less (U' > 0) and her marginal utility of wealth decreases as wealth increases (U'' < 0). It will be shown how risk aversion, in the interplay with endogenous investment decisions, crucially determines the adopted financial structure and changes some of the standard results of the pertinent literature.

Conform to Savage (1954), we model the space of consequences as the space of possible end-of-period wealths for the focal decision maker. The action space is accordingly defined by the given set of (constrained) decision variables at her disposal. It is important to understand that the chosen actions have neither an impact on the properties of future states of the world nor on their (subjective) probability distribution.

¹⁴ See Gossen (1854).

¹⁵ See also Demange/Laroque (2006), pp. 71-72.

1.4 Structure of the Study

The present thesis is subdivided into eight chapters. The remainder is structured as follows.

Chapter 2 gives a representative overview on the different conditional approaches within the financial structure literature and elaborates on how our setting relates to these existing models. Chapter 3 outlines the analytical framework and sets up the fundamental building blocks of our model. Chapter 4 provides a simplified analysis in order to prepare the ground for the subsequent chapters, which examine the role of fixed costs (Chapter 5), incorporate the firm's full revenue and cost structure (Chapter 6) and scrutinize the product market context (Chapters 6 and 7). To be more specific, Chapter 6 delineates investment and financing decisions in a monopolistic setting, before the influence of a product market competitor in a duopolistic setting is scrutinized in Chapter 7. The thesis concludes with Chapter 8, which summarizes the findings and compares them against the results of prior research, before discussing relevant implications and possible limitations.

$\begin{array}{c} {}^{\rm Chapter} \\ TWO \end{array}$ Literature Review

Corporate financial structure has been explored by many scholars. Their explorations aim at understanding the relative proportions of debt and equity adopted by firms to finance prospective target investments. Typically, these target investments are exogenously given, which constitutes one of our major criticisms of existing models. In the following, the differing streams of literature that have mainly contributed to the theoretical examination of corporate financial structure choice are reviewed.¹⁶

2.1 Financial Literature until the mid-1980s

The pertinent financial literature until the mid-1980s can be roughly categorized into

- (1) Neoclassical study of finance,
- (2) New institutional study of finance.

2.1.1 Neoclassical Study of Finance

In his pioneering examination of decision criteria regarding optimal investment behavior, Fisher (1930) presents a model which, as an elementary attribute, disregards risk and which can be deemed to be the basis for all subsequent theoretical disquisitions on the properties of corporate investment and funding decisions. The presumption of a risk-free environment renders obsolete all discrimination between different investors (shareholders, creditors). The costs of capital correspond to the unique global interest rate prevalent on the financial

¹⁶ Nota bene, this chapter is meant to provide a representative rather than exhaustive review.

market. The optimal level of capitalization is reached once the marginal rentability of the investment equals the marginal costs of capital.

Based on Fisher's considerations, the basic works of Durand (1952) and Solomon (1963) deduced an optimal capital structure by simple capital cost considerations. This optimal capital structure is characterized by a minimization of total capital costs. It stringently depends on the leverage-contingent progression of the costs of both debt and equity. Firm leverage is defined as the proportion of debt vs. equity, i.e., the debt-equity-ratio of an enterprise. The respective progressions of the costs of equity and debt are based on intuitive assumptions regarding the behavior of creditors and equity investors. Since creditors have prior claim on the firm's assets and earnings, equity investors demand a higher rate of return to compensate for their higher risk. Hence, Solomon (1963) presumes the costs of equity (rate of return demanded by equity investors) to be higher than the costs of debt (interest rate), considering both as distinct functions of the adopted debt-equity-ratio. The corresponding visual representation is depicted in Figure 1 (adapted from Bitz, 2000).



Figure 1: Overall capital cost function and optimal firm leverage choice. (Source: adapted from Bitz, 2000)

Increasing firm leverage, i.e., gradually substituting higher-risk equity by lower-risk debt (which, thus, exhibits a lower interest rate compared to the rate of return requested by equity investors) lowers the average costs of capital faced by the firm. As long as the firm's debt-equity-ratio does not exceed a defined critical level of firm leverage, the bankruptcy risk remains negligible and the creditors do not adjust their prices. Hence, the cost curves are assumed constant up to this "threshold level". As firm leverage rises beyond the threshold level, investors react by adjusting their demanded prices to the augmented bankruptcy risk, which is resulting in a subsequent augmentation in the slope of both functions. The resulting total capital cost curve allows for the identification of a leverage-contingent minimum, which simultaneously determines the optimal financial structure choice of the firm.¹⁷

Economics literature concerned with the exploration of capital cost properties received a strong boost in its theoretical foundation from the subsequently emerging general equilibrium approaches. In their famous 1958 article, Modigliani/Miller have shown that, under a set of restrictive assumptions, firm value is independent of the corporate financial structure. Their basic assumptions include the presence of an efficient financial market, risk-neutrality, and the absence of taxes, bankruptcy costs, and asymmetric information. This assertion, often referred to as the *irrelevance theorem*, is reckoned one of the principal results of the neoclassical study of finance. In essence, Modigliani and Miller's celebrated paper is widely regarded as the starting point for modern corporate finance theory. It constitutes a pioneering effort to provide a theory of financial structure by systematically analyzing assets and drawbacks of disposable capital sources.

However, the irrelevance theorem also has been subject to strong criticism. This criticism mainly concentrated around two conceptual aspects of the model, namely the disregard of the impact of bankruptcy risks and, in particular, of tax imposition. A re-consideration by Modigliani/Miller (1963) accounts for the latter aspect.¹⁸ It is shown that a leveraged firm holds a superior value compared to a non-leveraged firm if interest payments shield income from corporate taxation. The value of this *debt tax shield* equals the product of the debt amount and the tax rate. Bearing in mind the results of the Capital-Asset-

¹⁷ Moreover, it has been broadly acknowledged that a gradual substitution of equity by debt causes an increase in the return on equity as long as the internal return exceeds the costs of debt. This relationship between the return on equity and the costs of debt is commonly labeled as the *leverage effect*. See, for example, Perridon/Steiner (2004), pp. 488-489.

¹⁸ Modigliani/Miller (1963) assume a simplified fiscal system, including a corporate income tax and an individual income tax.

Pricing-Model (CAPM), it is easy to show that this result holds even if debt is considered to be risky.¹⁹

The former of the two critical aspects is emphasized by Stiglitz (1969), who stresses that leveraged firms are prone to higher interest rates due to their increased bankruptcy risk. Stiglitz notes that, as long as firms and banks differ with respect to their probability of default, there must be a difference between the interest on deposits and the interest on credits – even when assuming perfect markets.

Thus, within the frame of the theoretical investigation of financial structure decisions, these two factors constitute the first market-related imperfections discussed in the pertinent literature, thus leading to a gradual turn-away from the perfect market premise. By the adoption a static one-period model, Kraus/Litzenberger (1973) subsequently derived an internal optimum of the debt-equity-ratio by trading off leverage-related tax savings against bankruptcy-related losses in firm value. Bankruptcy occurs if the firm cannot meet its debt obligations, and basically implies costs.²⁰ Expected bankruptcy costs are incurred when the (perceived) probability that the enterprise will default on debt is positive.²¹ Since this probability rises with the chosen firm leverage, the expected bankruptcy costs increase in the same direction, inducing an internal cost optimum in the interplay with potential tax-savings. This comparison of taxes and bankruptcy costs constitutes the quintessence of the static *trade-off theory*, a branch of research which presumes that firms aim for a target financial structure by gradually augmenting the level of firm leverage until the mar-

¹⁹ A general CAPM-based proof of Modigliani and Miller's propositions can be found in Sharpe (1964).

²⁰ First of all, bankruptcy leads to a transfer of control rights from the previous shareholders to the creditors. Whether bankruptcy results in an actual liquidation or in a continuation of the business primarily depends on the relative size of the liquidation value compared to the going concern value. See Haugen/Senbet (1978), p. 383 ff.

²¹ Contingent on the new control right holders' (i.e., the debtholders') intention to institute bankruptcy proceedings, potential bankruptcy costs may be direct and/or indirect. Direct bankruptcy costs are essentially all administrative and legal charges associated with the bankruptcy proceedings. Indirect bankruptcy costs are essentially all costs that may emerge as a consequence of the public announcement of the firm's financial distress. For example, these could be profit losses incurred by the firm due to imminent degradations of business relations with suppliers or customers. If an enterprise is known to be close to insolvency, suppliers may be less willing to continue their business relations due to private risk considerations. Analogously, customers may abstain from buying its service or goods in fear of being exposed to a future situation where the firm cannot assure its warranties. However, the general relevance of bankruptcy costs is by no means beyond dispute. In particular, Haugen/Senbet (1978) assert that bankruptcy-related costs are negligible if capital market prices are assumed to be competitively determined.

ginal tax-advantage of additional debt is just offset by the marginal increase in the present value of possible bankruptcy costs.²² In short, equity is substituted with debt until the firm value is maximized.

Empirical literature, at first glance, offers considerable evidence for the validity of the trade-off theory.²³ However, one of the basic corollaries of the trade-off theory is a positive correlation between the locus of optimal firm leverage and profitability. Highly profitable enterprises exhibit a lower bankruptcy risk than less profitable firms and are, thereby, able to exploit the taxadvantage of debt financing to a larger extent. Hence, for highly profitable firms, the internal optimum of the debt-equity-ratio is larger than for less profitable firms. Intriguingly, empirical results disclose the exact opposite relation: highly profitable firms are mainly characterized by a low level of firm leverage. Several studies concordantly confirm the negative relation between firm leverage and profitability.²⁴ Given the trade-off-theoretical implication that highly profitable enterprises are able to and will exploit their valuable debt tax shields, this empirically observable negative relation constitutes a fundamental discrepancy which has been tagged very fittingly as "the capital structure puzzle" (Myers, 1984). Moreover, Frank/Goyal (2008) point out that corporate income taxes do only exist for roughly a century. Debt financing, however, has been common a long time before such corporate income taxes were introduced.²⁵ Therefore, tax considerations can hardly be deemed to completely justify the use of debt,²⁶ which obviously renders all attempt to empirically confirm the general validity of the trade-off-theory quite futile.

Aside from the trade-off-theoretical framework, Miller (1977) arrived at the conclusion that the consideration of different sources of tax imposition may reproduce the initial 1958 result of Modigliani/Miller: the value of the firm is independent of its financial structure. Though, in this case, the independence is of a statistical kind; while claiming the overall amount of equity and debt in the market to be fixed, Miller arrives at no prediction on how these quantities are divided up among firms.

Further articles that follow the same spirit of analysis have shown that accounting for the existence of individual tax imposition leads to fairly heterogeneous results. A particularly interesting paper by De Angelo/Masulis (1980)

²² See, for example, Myers (2001), p. 88 ff.

²³ See, for example, Fama/French (2002), or Rajan/Zingales (1995).

²⁴ See Myers (1984), Titman/Wessels (1988), or Rajan/Zingales (1995).

²⁵ See Frank/Goyal (2008), p. 59 f.

²⁶ This assertion does, not bene, not allow for the reverse conclusion that taxes can be safely disregarded when examining today's enterprises financing decisions.

points out that the firm's objective is the optimization of its periodical surplus after taxes. These authors show that Miller's 1977 irrelevance result is heavily sensitive to simple and realistic modifications in the modeling of the corporate tax code. They demonstrate that the explicit consideration of tax deductible non-cash charges²⁷ such as accounting depreciation allowances²⁸ triggers a unique interior optimum for the firm's leverage choice in the market equilibrium.²⁹ Hence, De Angelo/Masulis show that the value of the firm depends on the deductible amount of the taxable surplus and, thereby, on leverage decisions.

2.1.2 New Institutional Study of Finance

The new institutional study of finance essentially scrutinizes issues beyond the explanatory reach of the neoclassical approaches. Unlike these approaches, new institutional financial theory acts on the assumption of imperfect markets, which are subject to transaction costs and information asymmetries, thus accounting for the existence of institutions like, e.g., financial intermediaries. The new institutional perspective does not merely describe pertinent aspects of the financial sector, but actually analyzes them and seeks for normative recommendations.³⁰

New institutional approaches have early on dealt with the impact of *agency* $considerations^{31}$ on financial contracting and, thereby, on corporate investment behavior. The application of such agency considerations has substantially contributed to the establishment of the new institutional study of finance. Although still neglecting the strategic features and implications of the competitive context,³² the consideration of agency costs and information asymmetries

- ²⁹ See De Angelo/Masulis (1980), p. 27.
- 30 See Perridon/Steiner (2004), p. 24 f.

³² See Hellwig (1989) for a general discussion.

²⁷ Such tax deductible non-cash charges are tagged *tax shield substitutes for debt* by De Angelo/Masulis.

²⁸ As pointed out by, among other, Samuelson (1964), lawmakers' desire to make the net present value of cash revenue streams independent of the tax rate and to avoid the conversion of income taxes into taxes on principal induces them to allow for subtractions from the net operational revenue, i.e., tax deductible accounting depreciation allowances. See Samuelson (1964), p. 604.

³¹ Agency theory explores the behavior of human beings within contractual relations, and how their actions are affected by incentives. These incentives serve as a device to align the interests of the contracting parties.

proved to be particularly prolific with regard to the theoretical examination of firms' financial structure decisions.

Within the frame of these agency theoretical models, financial structure essentially serves as a device to either *minimize agency costs* or to *credibly disclose private information*. The starting point of the first category is the 1976 article by Jensen/Meckling, whose fundamental concept of agency cost minimization via financial structure decisions has been subject to a multitude of subsequent variations. The most significant works related to the second category are Leland/Pyle (1977), Ross (1977), Grossman/Hart (1982), and Myers/Majluf (1984).

Minimization of agency costs. Significant research effort has been devoted to models that determine corporate financial structure by employing agency considerations.³³ This stream of economic literature is mainly based on the examination of conflicts of interest between stakeholders. Applying it to the analysis of firms' financial structure decisions reveals three differing types of stakeholders linked to the enterprise, each of them holding divergent interests: managers, shareholders, and creditors.

Professional managers are delegated by the shareholders who remunerate them in order to act to the best of the shareholders' interests. However, as Jensen/Meckling (1976) highlighted, managers are foremost conducting their economic actions according to their own interest. In other words, they take decisions that rather maximize their own than the shareholders' utility.³⁴ In particular, managers may abstain from devoting their complete effort to the firm or they may play on their discretionary power to extract indirect benefits. Such benefits can be manifold, e.g. the consumption of corporate commodities for personal aims or the choice of projects yielding a certain prestige for the manager. Managerial self-interest may be influenced by incentive devices like compensation schemes or share grants, but such alignment of divergent interests is necessarily imperfect and, primarily, costly. Hence, the emergence of conflicts of interest, which is inherent to the separation of firm ownership and operational control, creates costs - the so-called agency costs. Following the definition of Jensen/Meckling (1976), agency costs can be classified according to three basic categories:

³³ See, for examples, the articles of Williamson (1988) or Harris/Raviv (1991).

³⁴ Essentially, this kind of conflict arises as a natural consequence of less-than-full managerial residual claimancy. Managers, thus, do not gain the entire benefit from their profit-enhancing activities – although internalizing the full costs associated with these actions.

- *Monitoring costs*, which refer to the principal's (shareholder's) costs of supervision of the agent (manager).
- *Bonding costs*, which refer to the agent's (manager's) costs of credibly ruling out activities that contradict the interests of the principal (shareholder).
- *Residual loss*, which reflects the impossibility of setting up complete contracts and refers to the cash value of the principal's welfare loss from the agent's residual self-interested behavior (which is not "captured" by monitoring and bonding).

The inefficiencies caused by managerial opportunism are reduced if the manager owns a larger fraction of the firm.³⁵ Thus, for an exogenously given absolute level of firm capitalization,³⁶ augmenting the debt level increases the manager's share of the firm's equity and, thus, diminishes the loss due to conflicting interests. Another significant aspect is articulated in an illustrious statement by Michael Jensen, who describes the fundamental problem

"...to motivate managers to disgorge the cash rather than investing it below the cost of capital or wasting it on organizational inefficiencies." (Jensen, 1986, p. 323.)

Within this context, corporate debt can serve as a means to discipline managers by reducing the disposable free cash flow: managers are forced towards an optimal provision of effort and are restrained from generating unnecessary agency costs to their personal advantage.

Grossman/Hart (1982) stress another convenience of debt financing: if managers are averse to corporate bankruptcy (maybe because of potential losses in reputation on the market for managerial workforce), then augmenting the debt level can incentivize managers to conduct better decisions. For example, a manager could be incentivized by debt to rather decrease the probability of bankruptcy instead of increasing her private benefits by diverting corporate commodities to personal purposes.

In a nutshell, corporate debt increases managers' fractional residual claim, reduces the free cash flows available to them, and drives them towards efficient

³⁵ Since increasing the manager's fraction of ownership is obviously equivalent to a proportional increase of managerial residual claim.

³⁶ As already indicated, the present thesis argues that keeping the stock of capital fixed lies at the heart of many standard results found in the existing literature. The interrelation between investing and financing exhibits a considerable degree of sophistication, and our analysis will indeed show that the simultaneous endogenization of both decisions changes results and leads to further insights.

activities in order to balance corporate bankruptcy risks. Thus, increasing the level of debt may indeed hold incentivizing power. As summarized by Myers (2001),

"...a high debt ratio can be dangerous, but it can also add value by putting the firm on a diet." (Myers, 2001, p. 19.)

The second form of conflicting interests originates from diverging objectives between debt and equity investors, i.e., creditors and shareholders. Such conflicts are particularly prominent when there is a risk of default. If the probability of default is positive, then shareholders can cause value-transfers to their own benefit (and to the detriment of the debt holders) by investing into more risky projects than initially designated. Controlling shareholders (or managers who are delegated by the shareholders) are presumably more affine towards investment projects which maximize shareholder wealth compared to projects which maximize total firm value. Hence, they are likely to implement risky projects with a negative net present value in which a small increase in equity value is completely eroded by a greater decrease in debt value (overinvestment problem). Moreover, they are likely to avoid safe investments with a positive net present value in which a small decrease in equity value is overcompensated by a greater increase in debt value (underinvestment problem).³⁷ Both overinvestment and underinvestment behavior are generally summarized as the effect of opportunistic asset substitution.³⁸

But the creditors, anticipating the shareholders' behavior, preventatively underrate the firm value by an amount that corresponds to the agency costs of debt. These expected costs are, thus, factored into the price of debt (i.e., the requested interest rates) at the time when it is issued. Moreover, the creditors will be driven to furnish debt contracts with protection clauses safeguarding their investments.³⁹ Evidently, the elaboration and implementation of such contractual safeguards engenders additional costs. Thus, the shareholders' tenure of control rights tends to result in an increase in the costs of debt and, thereby, in less leverage.

³⁷ See Myers (1977), p. 147.

³⁸ By the asset substitution effect, the stock value may increase at the expense of the bond value due to the limited liability of shareholders. Being residual claimants, they gain at the expense of the creditors, even when the firm value remains constant. Diamond (1989) and Hirshleifer/Thakor (1992) show how the problem of asset substitution is dampened by the consideration of managerial reputation concerns.

³⁹ Such protection clauses are commonly tagged *covenants*.

Summing up, agency theory brings forth an optimal financial structure that, in principle, is a compromise solution representing a minimization of total agency costs. As pointed out by Hellwig (1989), the agency-theoretical exploration of corporate financing behavior essentially constitutes a subset of the general theory of incentive-compatible contracting. Its aim is

"...to characterize optimal (second-best) contracts in situations of moral hazard and to interpret and explain the contracts that we actually observe as (Pareto) optimal under given incentive constraints. Similarly, financial institutions such as banks and nonbank intermediaries are to be explained as efficient mechanisms for reducing certain types of moral hazard..." (Hellwig, 1989, p. 278.)

Reduction of information asymmetries. The other major stream of new institutional research on the structure of corporate financing is concerned with informational asymmetries between stakeholders subdivided into *insiders* and *outsiders*. Within the frame of the disclosed context, *insiders* refers to decision makers that participate in the inner operations of the firm, while *outsiders* refers to decision makers who are external to the firm, but who affect and are affected by firm properties. Hence, controlling shareholders or managers are insiders who hold private information about the firm (e.g. the properties of investment options or the characteristics of the return stream). Creditors and non-controlling shareholders are outsiders who only have beliefs that are subject to incomplete and fragmented clusters of information.

Leland/Pyle (1977) have explored the concept of entrepreneurial riskaversion to develop a signaling equilibrium where an entrepreneur's willingness to invest her personal wealth into a firm constitutes a quality signal of this firm to the market. Hence, the existence of information asymmetries triggers the result that, in contrast to the findings of Modigliani/Miller (1958, 1963), the financial structure of the firm is related to firm value even in the absence of tax imposition. Leland/Pyle (1977) is among of the very few financial structure models which consider the impact of risk aversion. However, this model merely scrutinizes the (indirect) signaling effect. Leland/Pyle do not craft a holistic frame where an owner-manager's risk exposition is directly balanced against the level of expected personal returns. Such a frame would necessitate the introduction of an alternative risk-free asset for the decision maker's personal investment strategy. This would allow for an apportionment of her personal wealth between the risky firm project and the safe alternative.⁴⁰

Ross (1977) directly relates the debt-equity-ratio to the financial market's evaluation of the firm quality. More specifically, Ross finds that the firm value

⁴⁰ Our results will show that such a setting creates trade-offs which crucially drive the financial structure choice of an owner-manager. See Chapter 4.

positively depends on the chosen level of this ratio. Good quality firms can incur debt with a much lower bankruptcy risk than bad quality firms, eventually inducing a separating equilibrium in the spirit of Spence (1973), where the level of debt constitutes a signal of the firm quality to the financial market. The same kind of reasoning can be directly transferred to the mode of corporate dividend distribution. This mode indicates the expected level of future cash flows. High levels of dividend distribution increase the bankruptcy risk of low quality firms and are, thus, not imitable by the latter. Hence, in this context the level of dividend distribution also serves as a quality signal to the financial market. On the other hand, the distribution of dividends also has a negative impact on the firm, since it consequentially reduces its possibilities of financing target investments by low-cost capital (i.e., inside equity in the form of undistributed earnings). Thus, signaling firm quality via dividend distribution induces costs that have to be traded-off against the firm's "valuation gain" on the financial market.⁴¹

Myers/Majluf (1984) show that, in the presence of a financial market with imperfect information about the quality of firms or projects, good qualities are possibly underrated to such an extent that they are forced to finance target investments via internal funds. Firms are, thus, forced to abandon profitable projects for which the necessary financing cannot be procured.⁴² Hence, an initial public offering (IPO) of shares transmits two possible kinds of information, i.e., an overrating of the firm value by the financial market or a low gear information asymmetry. For an environment with considerable information asymmetries, this brings about the notion that an IPO constitutes a negative signal with respect to the quality of the target investment project and, thereby, to the firm value. Vice versa, abstaining from raising equity is interpreted by investors as good news with respect to the firm value. As a result, all enterprises will strictly prefer debt to equity.

Upon these insights rests the conception of the *pecking order theory*, which essentially analyzes the interrelation between informational costs (due to information asymmetries) and the respective corresponding properties of disposable capital sources. The pecking order theory, contrary to the trade-off theory, abolishes the presumption of a target capital structure. It rather argues that financing sources are appealed according to an order which is inversely proportional to their respective expected informational costs, thus imputing the existence of a preference hierarchy with respect to the disposable financing op-

⁴¹ Williams (1988) has developed a model that derives a signaling equilibrium which considers these diverging effects.

⁴² In principle, this mechanism constitutes a prime example for the famous *lemons market problem* in the vein of Akerlof (1970).

tions.⁴³ Accordingly, firms preferentially cover their financing needs via internally generated funds, i.e., undistributed earnings, which are evidently not prone to information asymmetries. If the available internal cash flows are not able to fully cover the (exogenously given) financing needs, thus fostering the requirement of additional funds, firms will then raise external capital in the form of debt. Lastly, if the debt capacity has also been exhausted, firms will raise outside equity in order to satisfy any remaining financing requirements.⁴⁴ Consequently, there is no static optimal or target financial structure for liquidityconstrained enterprises. Financial structure decisions are rather resulting from aggregate external funding needs, which, due to their dependence from (exogenous) short-term investment opportunities and the level of disposable internal cash flows, may be very volatile in nature.

Hence, the pecking order theory rationalizes the well-documented preference of listed corporations for internal funding and the small importance of stock issues, which were formerly attributed to the managers' desire to avoid the disciplinary forces of the capital market. According to the pecking order theory, this behavior is due to the asymmetric distribution of information with respect to the firm value.⁴⁵

Since the pecking order theory implies a strict preference for internal financing, profitable enterprises which generate high earnings are expected to exhibit a lower optimal level of debt financing compared to less profitable firms. Thus, the pecking order framework, unlike the trade-off theory, proves to be in line with most of the empirical observations in this regard. However, empirical results do not generally confirm a stringent hierarchy of financing as postulated by the pecking order theory.⁴⁶ Particularly considering young and growing enterprises, Fama/French (2002) observe a systematic raising of equity besides or even before debt.⁴⁷ Consequently, in a later article, these authors categorically deny the validity of the pecking order theory:

"...financing with equity is not a last resort, and asymmetric information problems are not the sole (or perhaps even an important) determinant of capital structure as suggested by the pecking order theory" (Fama/French, 2005, p. 551.)

⁴⁷ See Fama/French (2002), p. 15 ff.

⁴³ See Myers/Majluf (1984), p. 187 ff.

⁴⁴ See Myers/Majluf (1984), p. 187.

⁴⁵ See Myers (2001), p. 93.

⁴⁶ See Frank/Goyal (2003) or Fama/French (2002).
Summing up, in the presence of information asymmetries, corporate financial structure is driven by the controlling shareholders' desire to influence the financial market's valuation of the firm according to their private interests. Approaches scrutinizing such information asymmetries between insiders and outsiders are by now a well-established part of the pertinent theoretical finance literature.

We conclude that the new institutional study of finance certainly provides a valuable basis for the contractual assessment of operating enterprises' financing decisions under different environmental conditions. As illustrated, financial structure is interpreted as a potential means to mitigate intra-organizational incentive problems and/or informational asymmetries among insiders and outsiders. However, this perspective raises some elementary questions. Above all, why should financial structure be utilized to incentivize decision makers despite the easy availability of a rich set of direct compensation-driven incentive-schemes? Moreover, possible trade-offs resulting from the decision maker's risk aversion are widely excluded in these models. As stressed above, Leland/Pyle (1977) is among the few papers which identifies risk aversion as a driving factor of financial structure choice, but the underlying signaling mechanism is rather indirect.

Furthermore, the totality of contractual approaches disclosed above simply suppose that post-contractual actions are conducted in an individually "optimal" way by the contracting parties. Such actions are, however, mostly rather based on strategic interaction than on unilateral optimization.⁴⁸ The contracttheoretic approach as brought forward by the new institutional study of finance totally neglects the strategic importance of the competitive context for both borrowers (competing liquidity-constrained firms) and lenders (competing banks). Thus, the stylization drafted by the new institutional study of finance neither meets the prevalent complexity of market structures nor their strategic interdependencies.

Moreover, the financing decision is persistently examined in total isolation from the investment decision. All the papers discussed above take the firm's stock of capital as exogenously given without accounting for the non-trivial interrelation between investing and financing.

 $^{^{48}}$ See Hellwig (1989) for a more advanced discussion on this notion.

2.2 Financial Literature after the mid-1980s

A branch of financial research that emerged due to the limited explanatory power of the traditional approaches established during the late 1980's and is henceforth referred to as novel approaches. By the deliberate employment of cutting-edge perspectives on financial structure's mode of function within operating enterprises, these novel approaches shed new light on the focal issue. While traditional theories primarily focused on the financing function, these novel approaches constitute an advancement by scrutinizing the role of finance within a broader context including other firm-relevant aspects (like corporate strategy, internal organization, or product policy). This connective approach brings forth a departure from the long-established isolated examination of financial structure and, in principle, allows for a "wide-screen" elucidation of its role within the overall context. At this juncture, one new emphasis of financial structure literature relates to corporate control considerations and resulted from the intensified level of M&A activities in the mid-eighties. The emergence of capital structure models considering the firms' market interactions constitutes the second new research emphasis. Both notions directly relate to cardinal focus areas of the present thesis.

2.2.1 Corporate Control Considerations

External funding is provided by investors on condition that the (expected) returns from corporate investments are shared. While debt contracts typically specify a lump-sum payment on the creditors' invested capital amount which is independent from firm performance, equity contracts allocate residual returns after creditor reimbursements. These returns in turn crucially depend on corporate decisions (like project choice, staffing decisions, choice of distribution channels, marketing strategies etc.) and, thereby, on the choices made by the eventual tenant of the corresponding intra-organizational control rights. Thus, the determination of the corporate financing mix comprises one crucial aspect that had so far been neglected by the financial literature, namely the notion of *corporate control*.

Most of the research concerned with the relationship between corporate control and financial structure addresses Anglo-American markets, where ownership is dispersed. However, recent empirical studies suggest that disposition on corporate decisions is often concentrated in the hands of a blockholder or a small controlling group of shareholders.⁴⁹ Large part of the enterprises in conti-

⁴⁹ See, for example, La Porta et al. (1999), Faccio et al. (2001), or Claessens et al. (2002).

nental Europe and Asia are characterized by large controlling shareholders.⁵⁰ Evidently, personal corporate control considerations play a much larger role for such controlling shareholders than for professional managers, who are hired by (dispersed) shareholders. Hence, modeling the firm's decision maker as an owner-manager (as opposed to a salaried professional manager in a principal-agentsetting) appears to be a promising approach for the model developed in the present study.

Most notably, prior financial research shifted into focus the linkage between the general market for corporate control and financial structure. This linkage has been explicitly emphasized by several scholars, most notably Harris/Raviv (1988) and Stulz (1988), who showed how financial structure decisions influence the outcome of takeover combats. Hence, calibrating the corporate financing mix so as to control the value of the firm, the takeover probability, and the corresponding takeover premium was identified as a new function of financial structure choice. In summary, firms that are prone to takeovers are expected to increase their leverage on average, which induces an increase of the stock price. Thereby, the takeover probability decreases. Thus, the theories brought forward by Harris/Raviv (1988) and Stulz (1988) adopt a strict short-term view to identify financial structure choice as an instrument of response to imminent takeover threats.

The driving force underlying these findings is the fact that equity carries voting rights whereas debt does not. Surprisingly, a far more direct question that immediately arises on the grounds of this consideration has not been addressed by the literature: how will a controlling shareholder (who owns a considerable block of shares) use the relative proportions of inside equity, outside equity, and debt in the firm's stock of capital to maintain or defend her *intra-organizational power*?

Power is an intensively discussed area of research in most social sciences. In economic theory, however, the notion of power is still playing a minor role.⁵¹ The literature on compensating wage differentials⁵² scrutinizes the relation between monetary income associated with a working activity, and other (desirable or undesirable) attributes of that particular activity. Following that literature's line of argument, individuals who derive utility from part of their work are willing to trade off that utility against a lower monetary income. Intra-organizational power or control, in the sense of being able to direct and give orders to sub-

⁵⁰ See de La Bruslerie/Latrous (2007).

⁵¹ Frey/Kucher (2002).

⁵² See, for example, Thaler/Rosen (1976), Brown (1980), or Rosen (1986).

ordinates, may certainly be seen as such a "rewarding" part of a decision maker's working activity. 53

Frey/Kucher (2002) analyze empirical data on manager wages in Switzerland and capital market returns in the United States. The authors find that having more subordinates does not significantly increase a manager's wage. They interpret this as evidence that people are willing to pay for power. Hence, decision makers are supposed to value both wealth (W) and control (C), having utility functions of the form $U \equiv U(W, C)$ with both first derivatives being nonnegative. Exerting a higher degree of control can, thus, compensate a lower monetary income, and more wealth can compensate a lower degree of control. This described trade-off might be regarded as a fundamental consideration of controlling shareholders. Technically, this can be translated via an additional utility component that enters the overall objective function of the focal decision maker (i.e., the controlling shareholder), besides her utility from wealth.

What remains is the question of how to actually measure the degree of control. Following the economic theory of voting power, corporate control can be parameterized in terms of *power indices* for simple voting games. Banzhaf (1965, 1968) has demonstrated that a player's power in a voting body is not necessarily proportional to that player's number of votes.⁵⁴ Subsequent work by Cubbin/Leech (1983) and Leech (1987, 1990, 2002) draws on the theory of voting power to scrutinize the relationship between ownership concentration and corporate control. These papers describe the problem faced by an ownercontrolled firm with profitable investment opportunities requiring the raising of new capital. If the firm is short of inside equity (e.g., due to limitations to the owner's personal wealth), expansion may entail new shareholders and a consequent loss of control.

Interestingly, these considerations are largely absent from the corporate finance literature to date. As opposed to Frey/Kucher (2002), who assess the number of subordinates as a proxy for the manager's power, the present thesis establishes the basic link between financial structure and the utility of corporate control by means of the ownership structure. This ownership structure obtains as a direct result of the financial structure decision. More precisely, it depends on the adopted mix of inside equity (the part of the decision maker's personal wealth that is invested into the firm) and outside equity (from external investors). To the best of our knowledge, this linkage between financial structure

⁵³ Evidence for the importance of power as a motivator for decision makers is given in McClelland/Burnham (2008).

⁵⁴ See also Holler (1985) and Felsenthal/Machover (2004).

decisions and the utility of control constitutes a, hitherto, widely unexplored area within the corporate finance literature.

2.2.2 Market Interactions

All the literature streams described so far do not explore whether there exists a link between firms' financial structure and the conditions imposed by the markets – aside from financial markets. In a survey on corporate financial structure theories, Harris/Raviv (1991) explicitly stressed that the role of market interactions had been very sparsely explored and emphasized it as the most promising field for further research:

"In our view, models which relate capital structure to products and inputs are the most promising. This area is still in its infancy and is short on implications relating capital structure to industrial organization variables such as demand and cost parameters, strategic variables, etc." (Harris/Raviv, 1991, p. 351.)

However, since this academic appeal, few scholars have shown interest in explicitly connecting corporate financial structure decisions with the output market characteristics faced by the corresponding enterprises. Basically, Harris/Raviv (1991) identify two basic types of relation between financial structure and output markets: the relation of financial structure to *industry or product characteristics*, and the relation of financial structure to the *competitive context*.

Concerning the first type (how financial structure relates to industry or product characteristics), Titman (1984) was the first to stress that debt levels are not only affected by equity and debt holders, but also by the firm's nonfinancial stakeholders (e.g., customers, workers, suppliers), who likewise have to be considered as claimants to the firm's cash flows. This notion was elaborated within the frame of several exemplary contexts, most notably scrutinizing customer needs (Titman, 1984), product quality considerations (Maksimovic/Titman, 1991), or supplier/worker bargaining power (Sarig, 1998). The 1991 article by Maksimovic/Titman appears to be particularly mentionable. The authors investigate the impact of financial structure on a firm's choice of product quality and the feasibility of its products' warranties. They arrive at the conclusion that, ceteris paribus, increases in debt are accompanied by a decline in quality.

Concerning the second type (how financial structure relates to competitive strategy), the pioneering article of Brander/Lewis (1986) remains the publication of reference. Essentially, managers or controlling shareholders are assumed to maximize equity value rather than profits or total firm value. As a basic principle, financial structure influences the payoffs to equity and, thereby, affects the equilibrium strategies on the output market. Brander/Lewis (1986) were the first to show how firms can deploy financial structure as a means to commit themselves to certain output strategies vis-à-vis their product market competitors. Various subsequent theoretical models⁵⁵ follow the spirit of Brander/Lewis and investigate differing modes according to which output markets may both influence and be influenced by the choice of corporate financial structure in an oligopolistic context. They concordantly confirm that a firm's (and its competitors') output or pricing decisions are tightly interwoven with its adopted financial structure.

The list of empirical results referable to the connection of financial structure and the product market proves to be fairly short. At this juncture, two particular works of reference can be emphasized, namely the pioneering studies of Chevalier (1995) and Phillips (1995). Both authors explore whether increases in firm leverage (on an aggregate level) intensify or dampen product market competition. Hence, the causal direction examined by these papers is antipodal to the present study.

Chevalier (1995) scrutinizes leveraged buy-out (LBO) activity in the supermarket industry and arrives at the conclusion that increasing debt-levels tend to have a softening impact on product market rivalry. Phillips (1995) analyzes four industries – fiberglass insulation, tractor trailer, polyethylene, and gypsum industries. While arriving at the same conclusion as Chevalier for the first three of them, his results concerning the gypsum industry are antipodal. In order to provide a rationale for these inter-industrial differences, Phillips stresses the fact that the gypsum industry differs from the other three industries in that its entry barriers, are particularly low.

One apparent shortcoming of both works is their obvious inability to endogenously explain the essential reference parameter of their investigation: the structure of corporate financing. For instance, in the LBO study of Chevalier it remains totally unclear why takeovers where actually necessary to conduct the financial restructuring measures. If increased firm leverage actually softens product-market rivalry, incentives to incur debt must already have been present prior to the takeover.⁵⁶

⁵⁵ See Gertner et al. (1988), Maksimovic (1988), Poitevin (1989a, 1989b), Glazer (1989), Bolton/Scharfstein (1990), Showalter (1995), and Wanzenried (2003).

⁵⁶ It could be reckoned that creditors needed a signal due from a target company. This would of course imply that pre-contractual information asymmetries have an important role in the design of such financial contracts.

The results obtained from two additional empirical studies by Kovenock/Philips (1995, 1997) are also worthy of mention. In essence, both papers conclude that high leverage tends to dampen output-market competition. This assertion is supported by an earlier study by Opler/Titman (1994), which essentially indicates that high-grade levered firms tend to lose market shares to their unlevered competitors. More recent studies by Istaitieh/Rodriguez (2003) and MacKay/Phillips (2005) queue up with the above stated findings by showing that financial leverage is higher in concentrated industries where the intensity of product market rivalry is fairly relaxed.

The fundamental contribution of Brander/Lewis (1986) shall now be discussed in greater detail. Their model is concerned with the examination of strategic behavior opportunities of controlling shareholders which stem from their limited liability in case of corporate bankruptcy. Its rationale rests on the basic consideration of Jensen/Meckling (1976) that increasing levels of debt drive the shareholders towards riskier strategies.

Brander/Lewis (1986) develop a two-stage game where the structure of financing is chosen in the first stage and the production level is subsequently determined in the second stage. The rational choices of the firms are characterized by the adoption of a subgame perfect Nash equilibrium concept.⁵⁷ By assuming firms' future profits to be subject to risk, the authors introduce an "environmental" random variable which affects these future profits. In the unfavorable states of the environment, the firm cannot meet its debt obligations, which is ultimately leading to corporate bankruptcy. The shareholders then lose their firm-inherent money holdings to the benefit of the creditors.

The authors employ an expected utility criterion to characterize the behavior of risk-neutral decision makers who only have one investment option with respect to their private wealth, namely investing into the firm.⁵⁸ The authors queue up with the vast majority of the financial literature by still abstracting from the elementary capital investment decision and presupposing an exogenously fixed stock of capital.⁵⁹ Only the *relative proportion* of debt and equity,

⁵⁷ In a subgame perfect Nash equilibrium, firms correctly anticipate the second-period equilibrium choices prior to making their first-period choices.

⁵⁸ As stressed earlier, one crucial pillar of the model developed in the present thesis will be the introduction of a (riskless) alternative investment possibility for the (risk averse) decision maker. This constitutes a major deviation from the setting of Brander/Lewis (1986).

⁵⁹ In another article by Brander/Spencer (1989), the corporate level of capitalization is endogenous and directly influences the production level. However, this article (which is

the debt-equity-ratio, is susceptible to variations. Brander/Lewis neither consider fiscal impacts nor any kind of bankruptcy costs.⁶⁰ In their model, the financial structure choice essentially constitutes a commitment to a particular production strategy. The strategic choice of a particular structure of financing restricts a firm's (and its competitors') subsequent strategic options and, thus, can be taken as a means to commit to particular competitive behavior. Hence, market outcomes are not solely determined by parameters such as cost or time leadership, but also by the mode according to which financial structure influences the competitive interaction.

From this point of view, the study of Brander/Lewis is closely related to the strategic commitment literature. These models typically consider a decision variable which is chosen in the first stage and influences the production level in the second stage. This variable serves as a commitment device which, thus, can be employed as a "strategic weapon" by firms competing in an oligopolistic context. Apart from financial structure properties, it can for example be represented by the level of advertising (Joosten, 2007), the intensity of Research & Development (Zhang/Zhang, 1997), or the design of employee incentive contracts (Fershtman/Judd, 1987; Sklivas, 1987).

One major result of Brander/Lewis is that, for symmetric firms, increases in firm leverage are enhancing the equilibrium production level if marginal profits are higher in better environmental states. The effect is reversed if the inverse relation holds. Defining $z \in [\underline{z}; \overline{z}]$ as the environmental random variable, and \hat{z}_i as the lower threshold of z underneath which the firm cannot meet its debt obligations, the intuition associated with this result is relatively easy to understand. If marginal profits are higher in better environmental states (i.e., when zis large), an increase of debt, which diminishes the domain of favorable states⁶¹ and which is compensated by an increase in the production level, will in turn increase the value of profits realized in every favorable state of the world. Augmenting firm leverage, thus, triggers an endogenous shifting effect which is depicted in Figure 2.

primarily concerned with moral hazard considerations) does not explicitly deal with the duopolistic aspect.

⁶⁰ Bankruptcy costs are included in the analysis of Brander/Lewis (1988).

⁶¹ Brander/Lewis explicitly state that

[&]quot;...an increase in debt causes \hat{z}_i to rise, meaning that the range of states over which the firm becomes bankrupt is expanded." (Brander/Lewis, 1986, p. 961).



Figure 2: Leverage-induced reduction of shareholders' residual claimancy area. (Source: adapted from Brander/Lewis, 1986)

The figure shows that a leverage-induced upward-shift of the lower threshold \hat{z}_i expands the range of states over which the firm faces bankruptcy (area left to the threshold level). Due to the limited liability, levered shareholders extract payoffs only in good states. This means that, after an increase in firm leverage, former low marginal profit states are no longer relevant to the shareholders, since they are moved from the shareholders' residual claimancy area to the bankruptcy region, where residual claimancy is held by the creditors. The shareholders will, thus, seek to compensate this reduction in the equity-relevant area by an increase in the level of output.

With regard to the output effect of the strategic commitment to a particular financial structure, Brander/Lewis likewise demonstrate that, if marginal profits are higher in better environmental states, a unilateral debt level increase of one of the two competing firms causes an augmentation of this firm's equilibrium output. At the same time, its competitor's equilibrium output decreases. Again, the effect is reversed if the reverse condition (i.e., marginal profits are *lower* in better environmental states) holds.

All these results are gathered by adopting a comparative statics approach. The choice of debt is, thus, purely exogenous so far. To endogenize this choice, Brander/Lewis rely on the Nash equilibrium principle. All choices made at the first stage are contingent on the players' rational expectations regarding the choices made at the second stage (where the realizations of the first stage decision variables are taken as given). The intricacy underlying the first stage decision calculus accrues from the fact that controlling shareholders' first stage objective function differs from the second stage, since they maximize *total firm value*, i.e., the sum of equity value and debt value, rather than sole equity value. The basic point stressed by Brander/Lewis is that shareholders account for the reaction of the debtholders, who, given the possibility of bankruptcy, will purchase bonds only for their true value. Hence, the maximization of the controlling shareholders' first stage objective function reveals two basic types of

conflicting interests. Firstly, between creditors and shareholders, and secondly, between the focal firm and its competitor. Since an increase in firm leverage increases the output level and, thereby, alters the equilibrium, a particular debt structure constitutes a commitment to a particular output structure. The leverage-induced augmentation in the level of output exacerbates the conflict of interest between creditors and shareholders and lowers the debt value.

The fundamental postulate developed in this second part of the Brander/Lewis article can be outlined as follows: if marginal profits are higher in better environmental states, the focal industry's debt levels are strictly positive. If the reverse condition holds, no debt is incurred, i.e., all financing needs are entirely covered by equity. This result implies that leveraged firms will generate more output compared to the traditional oligopoly case, where bond issuing is disregarded and firms are implicitly assumed to be totally equity financed. If marginal profits exhibit a positive correlation to the states of the world, leverage confers a strategic advantage, since it causes the competitor's equilibrium output to decrease. Since this consideration is valid for both competitors, taking on debt exhibits a pro-competitive effect, since it intensifies output rivalry on the product market.

Summing up, Brander/Lewis demonstrate that the level of debt may significantly affect the strategic context. Limited liability in an uncertain environment induces oligopolistic enterprises to take riskier decisions, similar to the 1976 article of Jensen/Meckling. Firm leverage, thus, exhibits actual strategic value and functions as a commitment device which allows for an induced decrease of the competitor's equilibrium output level, while the own output level is simultaneously increased. A particular notion which immediately arises from the study of Brander/Lewis concerns the fact that, at equilibrium, the chosen debt levels of the firms will not maximize their aggregate total value. In other terms, the strategic use of debt leads to a prisoner's dilemma where both firms, in their desire to commit to high levels of output, adopt excessively aggressive behavior, eventually leading to reduced profits on firm and industry level. This notion suggests that cooperative financial arrangements might constitute an attractive collusive device⁶² and is explicitly emphasized by the authors:

⁶² Poitevin (1989a) provides an in-depth analysis of oligopolistic collusion induced by highly concentrated credit markets. However, the range of areas where such commitment devices that facilitate collusion could possibly be identified is wide. For example, Kirstein/Kirstein (2009) show that a legal framework like collective wage agreements may be utilized by oligopolists as a stable commitment device to cartelize product markets.

"If credit markets for a particular industry are quite concentrated, then lenders would have incentives to act as facilitating decision makers for collusion." (Brander/Lewis, 1986, p. 968.)

As stressed earlier, Brander/Lewis suppose a fixed stock of capital. Hence, they analyze the determination of the firm's debt-equity-ratio in isolation from the determination of its global investment level. It has been underscored on several occasions that this is one common major premise of the existent financial structure literature. The present thesis argues that departing from that premise of separation by simultaneously endogenizing both *level and structure* of the firm's overall capitalization may considerably change the results of the pertinent literature.

CHAPTER THREE

OUTLINE OF THE ANALYTICAL FRAMEWORK

3.1 Preface

The theoretical framework developed in this study is supposed to investigate how a decision maker's risk aversion, her corporate control considerations, interest rates, tax imposition, and the cost structure affect simultaneously conducted investment and financing decisions in awareness of the competitive context on the product market. The aim of this chapter is to outline the analytical framework, to disclose the time frame, and to set up the fundamental building blocks of the model. The derivation of the results is conducted throughout the subsequent Chapters 4, 5, 6, and 7.

Although economic theory has come a long way towards a better understanding of why and how firms choose their capital structure, there is no current model that addresses the three core notions outlined in the introductory chapter, i.e.,

- (1) the integration of *investment and financing decisions*,
- (2) the consideration of the firms' decision makers' *utility of control*,
- (3) the consideration of the *competitive context* on the product market.

From our literature review we further conclude that several pertinent basic imperfections, be it the characteristics of the decision makers (who exhibit risk-averse behavior, are subject to restricted personal wealth, and value corporate control), the firm's properties (such as the cost structure), or institutional constraints (such as accounting rules and tax imposition), lead beyond the friction-less world of Modigliani/Miller (1958). The different idiosyncratic approaches

that followed Modigliani/Miller do not account for the interplay of these imperfections with the three core notions presented above and do not allow for a sound explanation of corporate investment and financing decisions.

The model developed throughout the present thesis accounts for these imperfections. It simultaneously incorporates major elements from the fields of finance, industrial organization, and corporate governance. This constitutes an appealing approach, since it deliberately connects a study of the *inside of the firm* (financial structure, cost structure) with a study of the *outside of the firm* (market structure) against the backdrop of the decision maker's individual characteristics (risk aversion, control considerations) and exogenous institutional parameters (interest rates, taxation).

The model follows the spirit of the 1986 article by Brander/Lewis inasmuch as it essentially connects financial structure decisions to the output market. However, it starts from the consideration that financing decisions are primarily affiliated with the disposability of corresponding production facilities, the decision maker's (subjective) perception of the future product market conditions, her personal control considerations, and her environmental constraints. The model may be viewed as an expansion of the work of Brander/Lewis (1986) in the sense that some of their most restrictive basic assumptions (e.g., riskneutrality of firms, exogenous firm investment/fixed stock of capital, nonconsideration of corporate control, negligence of the cost and revenue structure, absence of taxation) are dropped and the notion of power enters the analytical frame by means of the focal decision maker's control considerations.

The analytical purpose well complements another recent theoretical examination by Liu/Miao (2007). These authors investigate an entrepreneur's investment and financing decisions in a complete markets environment by adopting a continuous-time model. They show that managerial characteristics, corporate governance, and financial markets influence capital structure and ownership concentration. However, corporate control considerations, the cost structure, and the product market context are completely disregarded within their model. The subsequent analysis is supposed to successively account for these important factors.

3.2 Basic Assumptions

The model frame discloses the link between financial structure choice, individual and environmental properties, and the cost structure. It relates corporate financial structure choice with a von Neumann-Morgenstern utility function tied onto the focal decision maker in an uncertain environment, which allows for the consideration of risk-averse behavior. The decision maker has subjective beliefs regarding the probability distribution of profits in future states of the world.⁶³ The set of basic model assumptions displays as follows:

- A1 The focal decision maker values only two goods, namely *wealth* and *control.* Hence, her welfare is only affected by
 - the amount of end-of-period wealth which she can acquire, and
 - the degree of control which she can exert throughout the given period.
- A2 Wealth and control are achieved through the determination of
 - her personal investment strategy, and
 - the corporate investment and financing strategy.
- A3 Corporate investing and financing decisions are assumed to be conducted simultaneously and to be definite, i.e., they cannot be reversed throughout the given period.
- ${\bf A4} \quad {\rm The \ economy \ lasts \ for \ exactly \ one \ period.}$
- A5 Discounting is disregarded.

The model frame configures the decision maker's objective function as depending on her degree of corporate control and on her total personal end-ofperiod wealth. Ex ante, her initial wealth is apportioned between the firm's stock of capital and an external risk-free asset. The firm may additionally raise debt or outside equity. Since the latter option relocates ownership and, thereby, control,⁶⁴ a non-monetary component enters the decision maker's overall utility function to account for the potential threat of diminished control that she faces when calling for equity from external investors.

Thus, the present model accounts for implications arising from the decision maker's corporate control considerations. In addition, to attain an adequate

⁶³ A similar approach is employed by Sandmo in his seminal 1971 article on the theory of the (risk-averse) firm in an uncertain environment.

⁶⁴ We suppose all stock purchased by equity to be common stock with one vote per share.

approximation of reality, we depart from the widely-spread restriction of a fixed stock of capital: the focal decision maker can not only choose between the *relative proportions* of debt and equity, but is actually able to define their respective *absolute levels*.

The remainder of this chapter is subdivided into two main sections. In a first step, Section 3.3 outlines the general frame of the model. This allows for an accurate depiction of the basic determinants considered within the subsequent analysis. In particular, the set of environmental constraints imposed by the institutional framework is exposed and examined with respect to its influence on the decision maker's final wealth level. In a second step, Section 3.4 operationalizes the notion of intra-organizational power and establishes the basic link between financial structure and the utility of ownership and corporate control.

3.3 General Model Setup

Consider an entrepreneur's investment and financing decisions within the scope of a mono-periodical finite horizon model. A priori, the focal entrepreneur holds full ownership. She, thereby, holds all control rights and acts as a controlling shareholder. The focal entrepreneur who, henceforth, will be tagged as the *decision maker D*, decides on her personal investment strategy and on the design of her firm's financial structure. Corporate funding can be obtained from her own private wealth, from creditors, or from external equity investors. Once the firm's financial structure is set, some time elapses, firm profits and payoffs are realized, and the firm is wound down.

D firstly decides on how to invest her initial private wealth at t=0 (period start). D is characterized by a utility function engendering risk aversion with respect to her personal end-of-period wealth. Let this utility function be depicted by

(1)
$$U(W_1^D, \mathcal{C}) = u(W_1^D) + v(\mathcal{C}),$$

where W_1^D represents D's total end-of-period wealth (at t=1) and \mathcal{C} the degree of corporate control exerted by D over the given period. Too keep things simple, D's overall welfare is additively separable in wealth and control. Hence, the decision maker's degree of control per se does not have a direct impact on her final wealth $W_1^{D.65}$ The model merely considers *D*'s direct utility of control (i.e., her "enjoyment" of being able to direct and give orders to subordinates, see also section 2.2.1), but disregards possible "instrumental" utility that may originate from an impact of control on operative decisions.

The decision maker's monetary risk aversion is governed by the usual concavity assumption regarding u, i.e.,

 $u'(W_1^D) > 0$ and $u''(W_1^D) < 0$.

The explicit form of the decision maker's utility function is given in Section 3.3.4. D maximizes the sum of her end-of-period expected utility of wealth (depicted by $\mathbb{E}\{u(W_1^D)\}$) and of her utility of corporate control (depicted by $v(\mathcal{C})$). Hence, the basic problem which D needs to solve is

$$Max \mathbb{E}\{U(W_1^D, \mathcal{C})\}$$
$$\equiv$$
$$Max \left[\mathbb{E}\{u(W_1^D)\} + v(\mathcal{C})\right]$$

where $\mathbb{E}\{\cdot\}$ represents the expectations operator.⁶⁶ While \mathcal{C} is solely a function of the ownership structure (which D determines by deciding on her ownership share), W_1^D is a function of D's personal investment into the firm, of the global level of investment into the firm (i.e., the size of the firm's stock of capital), and of the adopted financial structure, which are all determined by means of D's disposable decision variables (ϕ , α , Γ). ϕ denotes the share of the firm owned by D, α denotes the fraction of D's wealth invested into a riskless investment alternative, and Γ denotes the amount of debt incurred by the firm.

Figure 3 depicts the general time frame of the model.

⁶⁵ However, W_1^D may indirectly depend on it via the optimal ownership share.

⁶⁶ Evidently, only D's final wealth is subject to risk, but not her degree of control that is determined at the start of the given period and enjoyed over the period. Hence, the expectations operator is only relevant for u, but not v.



Figure 3: The general time frame of the model.

3.3.1 The Wealth Function

As depicted in Figure 4, the decision maker's wealth increase is fed by two basic components. The first accrues from the potential income generated by an assumed opportunity to invest into a risk-free asset which yields a certain rate of return amounting to r_1 over the given period (*bond income*). This bond income is subject to an individual tax imposition at the rate τ_1^I (*tax rate on bond income*). The second component accrues from the potential income generated by the risky investment into the firm project, subject to an individual tax imposi-

tion at the rate τ_2^I (*tax rate on equity income*). This equity income comes in the form of cash dividends.



Figure 4: Components of the decision maker's personal wealth increase.

Let W_0^D represent D's *initial personal wealth* (t=0). She fully invests her initial wealth by assigning a fraction $\alpha \in [0, \overline{\alpha}]$ of W_0^D to the riskless investment alternative, leaving a fraction $1 - \alpha$ for the risky investment into the firm. Hence, α is D's first decision variable. The upper bound $\overline{\alpha}$ is assumed to be close to (but slightly smaller than) unity, since the decision maker needs to keep at least a small part of the firm.⁶⁷ The second decision variable $\phi \in [\phi, 1]$ is defined as the share of the firm owned by D.⁶⁸ The lower bound ϕ is assumed to be close to (but slightly greater than) zero, since the decision maker needs to keep at least a small part of the firm. Further, let $S(1-\tau^c)$ describe the surplus distributed by the firm after corporate tax imposition at the rate τ^c . Therefrom, D's final wealth after taxes consists of



⁶⁷ The present model is concerned with the case of an owner-managed firm, where there is no fundamental separation of ownership and control. The underlying assumption is driven by the idea that the focal decision maker may extract some personal residual benefit from running or managing the firm. Possible sources of personal benefit from running/managing a firm might be seen in the *D*'s intention to keep a family-owned company alive (maybe despite not being very profitable) or *D*'s disutility from displacing longstanding employees. This assumption follows the spirit of pertinent financial contracting literature like Aghion/Bolton (1992) or Hart (1995).

⁶⁸ Retaining a fraction ϕ of the total equity evidently implies the issuing of the fraction $(1 - \phi)$ to external investors, hence reflecting the ownership distribution.

which is simply the sum of her initial wealth (first term) and her wealth increase over the given period (terms two and three). The first part of this wealth increase (second term) accrues from the riskless asset's return, which is subject to individual taxes amounting to $\alpha W_0^D r_1 \tau_1^I$. The second part of D's wealth increase (term three) accrues from the distributed surplus (dividends) after corporate taxes, subject to individual taxation amounting to $\phi S(1-\tau^C)\tau_2^I$. Hence, dividend payouts are obviously subject to double taxation. This is an entirely usual feature of numerous fiscal systems (like, for example, the US tax code) and well-established in the finance literature.⁶⁹

By considering that $W_0^D = \alpha W_0^D + (1-\alpha) W_0^D$ and by slight rearrangement of (2), *D*'s final wealth displays as

(3)
$$W_1^D = \alpha W_0^D (1 + r_1 (1 - \tau_1^I)) + (1 - \alpha) W_0^D + \phi S (1 - \tau^c) (1 - \tau_2^I).$$

While D's bond income is riskless, her equity income depends on the uncertain environment and on her choices regarding the level and structure of the firm's capital endowment.

The respective determinants of the surplus S shall subsequently be made explicit. D invests the amount $(1 - \alpha)W_0^D$ of her private wealth into the firm project. On behalf of the firm, she can additionally call for external funds in two different ways, namely by incurring debt with a fixed reimbursement of the creditors by an interest rate r_2 at period end, or by calling for *outside equity* from other investors,⁷⁰ which leads to a diminished share of the firm held by D. Defining E^{ex} as the amount of the raised outside equity allows for the depiction of D's actual share of the firm as

(4)
$$\phi = \frac{(1-\alpha)W_0^D}{(1-\alpha)W_0^D + E^{\text{ex}}} = \frac{(1-\alpha)W_0^D}{E}$$

which implicitly defines the firm's total equity E as

(5)
$$E = (1-\alpha)W_0^D + E^{\text{ex}} = \frac{(1-\alpha)W_0^D}{\phi}$$
.

⁶⁹ See, for example, Brealey/Myers (2008), p. 504 ff.

⁷⁰ In the frame of the present model, external equity investors are dispersed and (a priori) stay off entrepreneurial decisions. Thus, D always holds more shares than any other single equity investor. However, external equity investors still constitute a threat for D, who may lose her power of decision when holding less than 50 percent of the firm (assuming an absolute majority rule).

Considering D's third decision variable Γ as the amount of the incurred corporate debt, the firm's *total capital endowment* is given by

(6)
$$K = (1-\alpha)W_0^D + E^{ex} + \Gamma = E + \Gamma.$$

It is assumed that the firm's stock of capital K equals the level of total investment into its production facilities. The total end-of-period shareholder wealth generated by the firm, i.e., the equity value, displays as

(7)
$$V_E = (\Pi - A - r_2 \Gamma)(1 - \tau^c) + K - \Gamma,$$

with Π as the firm's operational profit. The parameter A represents an accounting depreciation term which basically serves the purpose to keep accountable the initial firm value.⁷¹ It is assumed that the initial investment fully amortizes over the given period, thus $A = E + \Gamma = K$. Under these assumptions, the first term between parentheses in (7) is equal to the firm's surplus S before corporate taxes, hence fiscal imposition is applied to

(8)
$$S = \Pi - A - r_2 \Gamma = \Pi - (1 + r_2) \Gamma - (A - \Gamma) = \Pi - (1 + r_2) \Gamma - E.$$

Interest payments $r_2\Gamma$ are tax deductible cash charges. Thus, similar to Modigliani/Miller (1963), debt exhibits a tax advantage in that it serves to shield corporate earnings from taxes. Moreover, similar to De Angelo/Masulis (1980), firms also have tax deductible non-cash charges, i.e., the tax code treats accounting depreciation A as deductible.⁷²

Substituting (8) into (3) and considering that, from (4) and (5), $\phi E = ((1-\alpha)W_0^D/E)E = (1-\alpha)W_0^D$, we obtain

$$W_1^D = \alpha W_0^D (1 + r_1 (1 - \tau_1^I)) + \phi (\Pi - (1 + r_2)\Gamma)(1 - \tau^c)(1 - \tau_2^I) + (1 - (1 - \tau^c)(1 - \tau_2^I))(1 - \alpha) W_0^D,$$

⁷¹ Accumulated accounting depreciation primarily constitutes non-cash expenses and is applied to reduce the book value of assets over time as they are consumed or used up during the value-added process of the firm. Meigs/Meigs (1983, p. 90) or Wolk et al. (2004, p. 330 f.), among others, provide comprehensive outlines of this practice.

⁷² As already noted by Samuelson (1964) and taken on by De Angelo/Masulis (1980), the consideration of tax deductible accounting depreciation crucially influences financial structure decisions. Its above representation (in (7) and (8)) basically relies on the modeling of De Angelo/Masulis (1980).

which, by slight rearrangement, allows for the disclosure of D's wealth increase over the given period as the differential between W_1^D and W_0^D :

$$W_1^D - W_0^D = \alpha W_0^D r_1 (1 - \tau_1^I) + (\phi (\Pi - (1 + r_2)\Gamma) - (1 - \alpha) W_0^D) (1 - \tau^c) (1 - \tau_2^I).$$

With regard to D's balancing between the riskless asset and the risky investment into the firm, the above expression clearly shows that it is the *relative* proportion of the respective tax rates, i.e., $(1 - \tau_1^I)/(1 - \tau^c)(1 - \tau_2^I)$, which crucially determines her decision, not their absolute value.⁷³

To ease the further exposure, the following assumptions are presumed to hold henceforward: $(1-\tau^c) = T$, and $\tau_1^I = \tau_2^I = 0$. It is worth highlighting that this abstraction from individual taxes merely serves simplifying purposes and does not materially alter the consecutive analysis or narrow its explanatory power. Moreover, since tax rates are defined as exogenous constants, it is straightforward that in order to return to the initial setting, it suffices to set $T = (1-\tau^C)(1-\tau_2^I)$ and $r_1 = r_1(1-\tau_2^I)$.

D's final wealth, from these assumptions, is given by

(9)
$$W_1^D = W_0^D + \alpha W_0^D r_1 + (\phi (\Pi - (1+r_2)\Gamma) - (1-\alpha)W_0^D)T.$$

At first glance, this expression appears a bit awkward and difficult to interpret. The reason for this is that, as will be shown, the above expression is very well-suited for the subsequent algebraical treatment. Moreover, it is easy to verify (see Appendix A 3.1) that expression (9) is equivalent to $W_1^D = (1+r_1)\alpha W_0^D + \phi(S \cdot T + E)$, which clearly expresses D's final wealth as a function of the riskless asset's return (first term) and of the return from the risky firm investment (second term).

 Π denotes the firm's operational profit over the given period. This firm profit is subject to environmental risk. Thus, D perceives Π as a random variable and estimates its distribution in accordance with her subjective beliefs about the distribution of future environmental states (these beliefs are further specified below). Expression (9) plainly illustrates that D's final wealth simply is a

⁷³ This observation gives support to one of the basic main premises outlined in De Angelo/Masulis (1980), for which the authors provided merely intuitive justification. They analyze three possible "tax brackets", contingent on whether $(1 - \tau_I^I)$ is smaller than, equal to, or greater than $(1 - \tau^c)(1 - \tau_2^I)$. See De Angelo/Masulis (1980), p.6.

linear transformation of Π . Since Π is a random function, its linear transformation W_1^D is likewise a random function. Hence, *D*'s subjective beliefs about the probability distribution of future environmental states directly translate into corresponding subjective beliefs about the distribution of future firm profits and, consequently, of her final personal wealth.

3.3.2 The Distribution of Future States

To proceed forward in the exposure, the distribution according to which D anticipates future states and, thereby, future profits has to be specified. At this juncture, it appears plausible to presume that D's major difficulty resides in her endeavor to accurately estimate the demand characteristics, since an enormous number of influential parameters which overlay with each other has to be taken into account. A partial list of such factors includes consumers' tastes, their income or general budget constraints, their capacities of information processing, their geographical allocation, exogenously induced or erratic trends, crosselasticities between products, business cycles, economic shocks, political framing, national or international legislation, and evidently – competition.

Hence, the firm profit which D anticipates ex ante appears to be a random function which is influenced by a large number of, more or less independent, random factors. Assuming D's beliefs concerning the distribution of future firm profits to be governed by a normal (Gaussian) probability distribution does, thus, appear to be an appropriate representation. As a major virtue, the normal distribution provides the technical convenience to be completely specified by the first two moments of the random variable, which obviously alleviates the consecutive mathematical treatment. Thereby, $\Pi \sim \mathcal{N}(\mu_{\Pi}, \sigma_{\Pi}^2)^{74}$ and

(10)
$$\Pi = \mu_{\Pi} + \sigma_{\Pi} z,$$

where $z \sim \mathcal{N}(0,1)$. Hence, z is a standard normal random variable with probability density $f(z) = \frac{1}{\sqrt{2\pi}} e^{-(\frac{1}{2})z^2}$. Equation (10) obtains by virtue of one wellestablished general property of the normal distribution: given a Gaussian ran-

⁷⁴ For technical reasons, we assume $\mu_{\Pi} > 2\sigma_{\Pi}$. In general, for a Gaussian function, with μ being its mean value and σ its standard deviation, when supposing $\mu > 2 \sigma$, then negative realizations of the corresponding random variable are practically excluded (having an actual probability very close to zero). Hence, we can approximate expectations of functions of this random variable by a truncation in the interval [$\mu - 2\sigma, \mu + 2\sigma$].

dom variable $X \sim \mathcal{N}(\mu, \sigma^2)$, then $z = (X-\mu)/\sigma \sim \mathcal{N}(0,1)$, i.e., z is governed by a standard normal distribution. Hence, profit can be expressed as given by (10).⁷⁵

Thereby, D's end-of-period wealth can also be displayed in terms of its mean and standard deviation, i.e.,

(11)
$$W_1^D = \mu_{W_1^D} + \sigma_{W_1^D Z},$$

with

$$\mu_{W_1^D} = W_0^D + \alpha W_0^D r_1 + (\phi(\mu_{\Pi} - (1 + r_2)\Gamma) - (1 - \alpha)W_0^D)T$$

and

$$\sigma_{W_1^D} = \phi T \sigma_{\Pi}.$$

3.3.3 The Decision Maker's Maximization Problem

The above specifications allow for an outline of the entire maximization problem which D seeks to solve by balancing against each other her three decision variables

- ϕ (share of the firm owned by D),
- α (fraction of D's wealth invested into the riskless asset), and
- Γ (amount of debt incurred by the firm).

The decision maker chooses (ϕ, α, Γ) to solve the following constrained optimization problem:

(12)
$$\max_{\phi, \alpha, \Gamma} \left[\mathbb{E} \{ u(\mu_{W_{1}^{D}} + \sigma_{W_{1}^{D}} Z) \} + v(\mathcal{C}) \right], \qquad \text{s. t.} \qquad \frac{\phi \leq \phi \leq 1,}{0 \leq \alpha \leq \overline{\alpha},} \\ 0 \leq \Gamma \leq \overline{\Gamma}.$$

As stated earlier, both the lower bound of ϕ and the upper bound of α exist due to the decision maker's requirement to keep a minimum share of the firm. Generally, ϕ and $\overline{\alpha}$ must verify the condition $\phi = \frac{(1-\overline{\alpha})W_0^D}{(1-\overline{\alpha})W_0^D + E^{ex}}$. Therefore, if $\overline{\alpha} \approx 1 \Leftrightarrow \phi \approx 0$. The third decision variable Γ is constrained by $\overline{\Gamma}$, which de-

⁷⁵ Hartung et al. (1991, p. 143 ff.) provide a concise outline of the normal distribution's general properties.

notes the maximum level of debt that is available to the firm. This maximum level of debt is assumed to be purely exogenous and will henceforth be tagged the *debt capacity*.

D maximizes the sum of her end-of-period expected utility of wealth (i.e., $\mathbb{E}\{u(\mu_{W_1^D} + \sigma_{W_1^D}z)\})$ and of her utility of corporate control (i.e., $v(\mathcal{C})$). Hence, the model setup reflects the decision maker's considerations regarding the emerging threat of power sharing that comes along with the involvement of external equity investors. The general properties of $v(\mathcal{C})$ and \mathcal{C} are formally derived in section 3.4.

3.3.4 Constant Absolute Risk Aversion (CARA)

To further specify the decision maker's preferences, D is assumed to exhibit *constant absolute risk aversion* (CARA) with respect to her personal end-ofperiod wealth. Therefore, a natural exponential function is adopted to model the decision maker's utility profile. Exponential utility functions model a constant degree of risk sensitivity and allow for a rigorous mathematical treatment. Let

(13)
$$u(W_1^D) = -e^{-\eta W_1^D},$$

which evidently belongs to the class of CARA utility functions. The parameter η represents the Arrow-Pratt measure of absolute risk aversion which, by definition, is equal to $\mathbb{E}\{-u''(W_1^D)\}/\mathbb{E}\{u'(W_1^D)\}$.⁷⁶ This ratio is constant with respect to W_1^D and allows for the disclosure of D's certainty equivalent CE as

(14)
$$CE = \mu_{W_1^D} - \frac{\eta}{2} \sigma_{W_1^D}^2$$
.

It is important to underscore that the results derived throughout the subsequent chapters still hold when abandoning this CARA assumption. As long as the decision maker's risk premium is positive (i.e., she *is* risk averse), the qualitative results are valid. In other words, the presence of risk aversion is crucial, but not its specific form.

⁷⁶ Note that $u' = \eta e^{-\eta W_1^D}$, $u'' = (-\eta)\eta e^{-\eta W_1^D}$ and, thereby, $-u''/u' = \eta$. The greater η , the more concave is u, hence indicating D's increasing risk aversion.

3.4 The Decision Maker's Corporate Control Considerations

Since control is determined by ownership, the parameter \mathcal{C} (which represents D's degree of corporate control) is a direct function of the decision maker's ownership share ϕ . We assume $v(\mathcal{C})$ to be a positive affine transformation of $\mathcal{C}(\phi)$. To keep things simple, we assume that D's utility of control is linear in her actual degree of control. Let

(15) $v(\mathcal{C}) = \mathcal{C}(\phi).$

Hence, before we can approach the analysis of D's maximization problem, we need to shed light on the basic characteristics of $C(\phi)$. By drawing on the economic theory of the measurement of voting power, this section derives the properties of D's power curve $C(\phi)$ from the level of influence exerted by the decision maker vis-à-vis a set of dispersed holders of the remaining common stock.

3.4.1 Swing Probabilities as a Measurement of Voting Power

According to the economic theory of voting power, corporate control can be parameterized in terms of *power indices* for simple voting games.

We assume D to be a large shareholder⁷⁷ who faces a set of dispersed small holders of the remaining common stock. With less than absolute majority ownership, a shareholder's control can never be full.⁷⁸ Accordingly, the corresponding degree of control can be operationalized by the ex-ante *probability* of winning ballots.⁷⁹ To analyze the interrelation between share ownership and this probability, it is necessary to distinguish between a shareholder's *voting weight*, represented by her proportion of the overall shares, and her *voting power*, as the ability to swing a coalition of other players from losing to winning by joining it.⁸⁰

⁷⁷ "Large shareholder" means that D holds a block of shares.

 $^{^{78}}$ See Cubbin/Leech (1983), p. 355 f.

⁷⁹ Contributions by Cubbin/Leech (1983) and Leech (1987, 1990) develop a formal framework to assess the probabilistic voting power of a large voting block in case of minority control.

⁸⁰ Banzhaf (1965, 1968) has shown that actual voting power is not necessarily proportional to the number of votes.

Providers of outside equity are supposed to be highly dispersed and are therefore referred to as "small" shareholders; the decision maker D is the only "large" shareholder. Let $\Phi \in [\underline{\Phi}; \mathcal{T}]$ represent the number of D's shareholdings and n the number of dispersed shares. The total number of shares is fixed and normalized to \mathcal{T} with $\mathcal{T} = n + \Phi$. Considering $\phi \in [\underline{\phi}, 1]$ as D's share of the firm, we have

$$\Phi = \phi \mathcal{T}$$

Let ϵ represent the smallest possible increment in shares. We assume small shareholders to be homogeneous with respect to their share ownership and ϵ shares per small shareholder. A decision by majority vote requires the quota $Q = T/2 + \epsilon$. Assume further that D holds at least 2ϵ shares, i.e., $\Phi = 2\epsilon$.

Since under majority control the votes of the minority are irrelevant, D's voting power is absolute in the domain $Q \leq \Phi \leq \mathcal{T}$, where she enjoys absolute majority ownership and can swing every possible coalition of small shareholders. In formal language,

$$\mathcal{C}(\Phi) > 0, \ \mathcal{C}'(\Phi) = \mathcal{C}''(\Phi) = 0$$
 for $\mathcal{Q} \le \Phi \le \mathcal{T}.$

To assess the decision maker's voting power in the domain $\underline{\Phi} < \Phi < Q$, i.e., below the threshold level Q, we have to formally examine to what extent she can swing coalitions when owning less than Q shares. Assuming binary decisions where each shareholder can vote either with "yes" or "no", there are 2^n different possible coalitions of the small shareholders. Let the decision of each small shareholder *i* be expressed by the dichotomous variable y_i with

$$y_i = \begin{cases} 1, \text{ in case of a "yes"-vote,} \\ 0, \text{ in case of a "no"-vote,} \end{cases}$$

and let $Y = \sum_{i=1}^{n} y_i$. Hence, Y represents the total number of "yes"-votes among the small shareholders per ballot.

Similar to Leech (1990), it is further assumed that every small shareholder votes each way with equal probability independently of the others. i.e., the individual probability p that a small shareholder votes with "yes" equals 0.5. To be more precise, p = 0.5 is a *subjective* perception of the decision maker who is totally *uninformed* with respect to possible future decision situations and the

preferences of the small shareholders.⁸¹ Under this assumption, Y is a *binomial* random variable distributed with parameters n and p = 0.5, i.e., $Y \sim B(n, 0.5)$.⁸²

Therefrom, it is straightforward to compute D's degree of control (according to the absolute Banzhaf Index) as the *probability* that she can swing the ballot. This occurs when Y is at least $\mathcal{Q} - \Phi$ (which will be henceforth tagged as the *lower swing bound* $L(\Phi)$) and at most $\mathcal{Q} - \epsilon$ (which will be henceforth tagged as the *upper swing bound* $H(\Phi)$). Accordingly, D's *swing probability* ($\widehat{=}$ voting power) in the considered domain $\underline{\Phi} \leq \Phi < \mathcal{Q}$ obtains as the binomial probability $Pr(\mathcal{Q} - \Phi \leq Y \leq \mathcal{Q} - \epsilon)$.

As an exemplifying numerical illustration, let us assume that D's shareholding Φ amounts to 45 shares, as opposed to 55 dispersed shareholdings of 1 share per small shareholder (i.e., n = 55). The total number of shares is $\mathcal{T} = \Phi + n = 100$. A decision by majority vote requires absolute majority support and $\epsilon = 1$, hence Q = 51. The binary option space gives rise to 2^{55} different possible coalitions among the small shareholders and $Y \sim B(55, 0.5)$. Accordingly, D's voting power is $Pr(6 \leq Y \leq 50) = 0.9999999999$ (see Table 1 in Appendix A 3.2). Evidently, D is virtually dominant with her swing probability being very close to unity.

3.4.2 The Power Curve

D's voting power is absolute when she holds absolute majority ownership. Consequently, her power curve exhibits constant value unity for all values of Φ equal to and above Q.

To formally derive the progression of D's power curve for values of Φ below Q, consider two distinct reference states A and B. Assume that in state B, the decision maker has one share more than in state A. In formal language, state A is characterized by Φ_A (D's shareholdings in state A) and $n(\Phi_A)$ (number of small shareholders in state A). State B is accordingly characterized by Φ_B and $n(\Phi_B)$. Let $\Phi_B = \Phi_A + \epsilon$ (hence, $\Phi_A < \Phi_B < Q$).

⁸¹ Leech (1990) additionally shows that the assumption p = 0.5 is equivalent to assuming that small shareholders' voting probabilities are randomly and independently drawn from distributions with mean $\frac{1}{2}$, regardless of the actual distribution forms. Given that the index does not require a uniform distribution, its behavioral interpretation in terms of a probability model possesses substantial generality. See Leech (1990), p. 298.

⁸² See Leech (2002), p. 12 f.

The number of small shareholders is $n(\Phi) = \mathcal{T} - \Phi$. For the constant quorum $Q = \mathcal{T}/2 + \epsilon$, the lower and upper swing bounds that obtain for each of the two reference states are given by

$$\begin{split} L(\Phi_A) &= \mathcal{Q} - \Phi_A \qquad \text{and} \qquad H(\Phi_A) = \mathcal{Q} - \epsilon \ \text{ for state } A, \text{ and} \\ L(\Phi_B) &= \mathcal{Q} - (\Phi_A + \epsilon) \quad \text{and} \qquad H(\Phi_B) = \mathcal{Q} - \epsilon \ \text{ for state } B. \end{split}$$

Consequently, D's voting power is given by

$$\begin{split} & \Pr(\mathcal{Q} - \Phi_A \leq Y \leq \mathcal{Q} - \epsilon) & \text{for state } A, \text{ and} \\ & \Pr(\mathcal{Q} - (\Phi_A + \epsilon) \leq Y \leq \mathcal{Q} - \epsilon) & \text{for state } B. \end{split}$$

In order to subsequently disclose the basic properties of D's power curve in the domain below Q, we make use of the following lemma.

Lemma 3.1: The lower swing bound does not exceed the mean value of the corresponding probability density function (pdf) for all possible values of Φ .

Proof: Defining $\mu(\Phi)$ as the mean of the pdf which obtains from the respective value of Φ , we need to show that $L(\Phi) \leq \mu(\Phi)$ for all Φ .

- We know that $\Phi = \mathcal{T} n(\Phi)$. Recalling that $\mathcal{T} = 2(\mathcal{Q} \epsilon)$, and substituting this into the first expression, we directly deduce $\Phi = 2(\mathcal{Q} \epsilon) n(\Phi)$.
- From the symmetry of the binomial distribution (since p = 0.5), we can immediately infer $\mu(\Phi) = n(\Phi)/2$. Hence, the relation $L(\Phi) \leq \mu(\Phi)$ is equivalent to $Q 2(Q \epsilon \mu(\Phi)) \leq n(\Phi)/2$, which, by substituting $Q = T/2 + \epsilon$, can be simplified to $n(\Phi) \leq T 2\epsilon$.
- By substituting $\mathcal{T} = \Phi + n(\Phi)$, the above relation further simplifies to $2\epsilon \leq \Phi$. This inequality is generally fulfilled from our basic assumptions concerning the large shareholder D.

By virtue of Lemma 3.1 we are able to immediately deduce the subsequent proposition.

Proposition 3.1: For $\Phi \in [\underline{\Phi}; Q)$ and $n = \mathcal{T} - \Phi$ dispersed shareholders who vote randomly,

- (1) D's power curve increases monotonically in Φ ,
- (2) D's power curve is concave in Φ .

Proof: It is straightforward to see that the lower bound $L(\Phi)$ is a diminishing function of Φ , while the upper bound $H(\Phi)$ is constant. Hence, a marginal increase of Φ by ϵ decreases the lower swing bound. Given the shape of the binomial pdf (which approaches continuity for $n \to \infty$) and the invariance of $H(\Phi)$, a marginal increase of Φ decreases $L(\Phi)$ and, thereby, increases the area underneath the density function (which is the bounded cumulative distribution function $Pr(L \leq Y \leq H)$), i.e.,

$$\Pr(\mathcal{Q} - (\Phi + \epsilon) \le Y \le \mathcal{Q} - \epsilon) > \Pr(\mathcal{Q} - \Phi \le Y \le \mathcal{Q} - \epsilon),$$

is true for all $\Phi \in [2\epsilon; Q)$, which proves the first part of the claim.

From Lemma 3.1, the lower swing bound always lies left from the mean of the density function. Since the pdf is monotonically increasing up to its mean, the marginal power-increase from a diminishment of the lower swing bound (by an increase of Φ) monoton-ically decreases. Hence, the increase of the power curve in Φ happens at a decreasing rate. In formal language, the relation

$$Pr(\mathcal{Q} - (\Phi + 2\epsilon) \le Y \le \mathcal{Q} - \epsilon) - Pr(\mathcal{Q} - (\Phi + \epsilon) \le Y \le \mathcal{Q} - \epsilon)$$

$$< Pr(\mathcal{Q} - (\Phi + \epsilon) \le Y \le \mathcal{Q} - \epsilon) - Pr(\mathcal{Q} - \Phi \le Y \le \mathcal{Q} - \epsilon)$$

holds for all $\Phi \in [2\epsilon; Q)$, which proves the second part of the claim.

Hence, the decision maker's power curve $\mathcal{C}(\Phi)$ is strictly increasing and concave up to the given threshold level, and flat from there onwards.⁸³

Considering that ϕ is a positive affine transformation of Φ (since $\phi = \Phi/\mathcal{T}$), the properties of $\mathcal{C}(\phi)$ directly bleed off. Merely for simplicity, we henceforth suppose an absolute majority rule with a threshold level at $\phi = \frac{1}{2}$. Therefore, the basic properties of $\mathcal{C}(\phi)$ are given by

$$\begin{aligned} \mathcal{C}(\phi) > 0, \ \ \mathcal{C}'(\phi) > 0, \ \ \mathcal{C}''(\phi) < 0 & \text{for} & \underline{\phi} \le \phi \le \frac{1}{2}, \\ \mathcal{C}(\phi) > 0, \ \ \mathcal{C}'(\phi) = \mathcal{C}''(\phi) = 0 & \text{for} & \frac{1}{2} < \phi \le 1. \end{aligned}$$

While owning at least half of the firm's equity, D enjoys full voting power and, thus, exerts absolute control. Hence, $C(\phi)$ is an increasing, concave function until $\phi = \frac{1}{2}$ and a constant from $\phi = \frac{1}{2}$ on.

Since we assume $v(\mathcal{C}(\phi)) = \mathcal{C}(\phi)$ (see Eq. (15)), we henceforth treat v as directly depending on ϕ , since $v(\phi)$ exhibits the same basic properties as $\mathcal{C}(\phi)$,⁸⁴ i.e.,

(16)	$v(\phi) > 0,$	$v'(\phi) > 0, \ v''(\phi) < 0$	for	$\underline{\phi} \leq \phi \leq \frac{1}{2},$
	$v(\phi) > 0,$	$v'(\phi) = v''(\phi) = 0$	for	$\frac{1}{2} < \phi \le 1.$

⁸³ Table 1 (see Appendix A 3.2) shows D's degree of corporate control as measured by voting power (for $\mathcal{T}=100$, $\epsilon=1$, $\mathcal{Q}=51$) when her share ownership lies below the threshold level \mathcal{Q} . Figure 6 (Appendix A 3.2) gives a graphical illustration of the corresponding power curve over the whole feasible domain of Φ . As D's ownership share approaches \mathcal{Q} from below, her power increases accordingly, but at a decreasing rate.

⁸⁴ In fact, any arbitrary positive affine transformation $v(\mathcal{C}) = a\mathcal{C} + b$ (where a > 0) keeps the basic properties of \mathcal{C} .



Figure 5: General properties of $\mathcal{C}(\phi)$ and $v(\phi)$.

Appendix to Chapter 3

A 3.1 Transformation of D's final wealth function

The decision maker's final wealth as displayed by (9) is given by

$$W_1^D = W_0^D + \alpha W_0^D r_1 + (\phi (\Pi - (1 + r_2)\Gamma) - (1 - \alpha) W_0^D)T.$$

Expanding the first two terms yields

$$W_0^D + \alpha W_0^D r_1 = (1 + r_1) \alpha W_0^D + W_0^D - \alpha W_0^D.$$

Since $W_0^D - \alpha W_0^D = (1 - \alpha) W_0^D = \phi E$, we obtain

$$W_0^D + \alpha W_0^D r_1 = (1 + r_1) \alpha W_0^D + \phi E.$$

From (8), we further know that $S = \Pi - (1 + r_2)\Gamma - E$. By multiplying with ϕT we obtain $\phi ST = \phi T(\Pi - (1 + r_2)\Gamma) - \phi TE$, which can be expressed as

$$\phi ST = (\phi(\Pi - (1 + r_2)\Gamma) - (1 - \alpha)W_0^D)T.$$

This expression exactly corresponds to the third term in (9). Hence, we indeed obtain

$$W_{1}^{D} = W_{0}^{D} + \alpha W_{0}^{D} r_{1} + (\phi(\Pi - (1 + r_{2})\Gamma) - (1 - \alpha)W_{0}^{D})T$$

$$= (1 + r_{1})\alpha W_{0}^{D} + \phi E + \phi ST$$

$$= (1 + r_{1})\alpha W_{0}^{D} + \phi(ST + E).$$

A 3.2 Numerical and graphical illustration of D's voting power

$\mathcal{T}=100, \epsilon=1, Q=51, p=0.5$							
Φ	n	$L(\Phi)$	$H(\Phi)$	D's voting power			
				(absolute Banzhaf Index)			
50	50	1	50	~1			
49	51	2	50	~1			
48	52	3	50	~1			
47	53	4	50	~1			
46	54	5	50	~1			
45	55	6	50	.999999999			
44	56	7	50	.999999996			
43	57	8	50	.999999986			
42	58	9	50	.999999954			
41	59	10	50	.999999860			
40	60	11	50	.999999607			
39	61	12	50	.999998967			
38	62	13	50	.999997452			
37	63	14	50	.999994066			
36	64	15	50	.999986877			
35	65	16	50	.999972339			
34	66	17	50	.999944213			
33	67	18	50	.999892006			
32	68	19	50	.999798755			
31	69	20	50	.999638070			
30	70	21	50	.999370357			
29	71	22	50	.998938216			
28	72	23	50	.998261138			
27	73	24	50	.997229716			
26	74	25	50	.995699762			
$\frac{1}{25}$	75	26	50	.993486827			
24	76	27	50	.990361751			
23	77	28	50	.986047917			
22	78	29	50	.980220895			
21	79	30	50	.972511077			
20	80	31	50	.962509738			
19	81	32	50	949778744			
18	82	33	50	933863824			
10	83	34	50	.914311026			
16	84	35	50	.890685640			
15	85	36	50	.862592633			
14	86	37	50	829697393			
13	87	38	50	791745503			
12	88	39	50	748580225			
11	89	40	50	700156512			
10	90	41	50	646550539			
9	91	42	50	.587964060			
8	92	43	50	.524723216			
7	93	44	50	457271809			
6	94	45	50	.386159405			
5	95	46	50	.312024951			
4	96	47	50	235576849			
3	97	48	50	157570611			
$\tilde{2}$	98	49	50	.078785306			

 $\label{eq:Table 1} {\rm Table \ 1} \\ {\rm Numerical \ illustration \ of \ } D {\rm `s \ voting \ power \ when \ owning \ less \ than \ } Q \ {\rm shares}$



Figure 6: D's power curve (T=100, Q=51).

$\mathbf{FOUR}^{\text{CHAPTER}}$

THE FIRM WITH A SIMPLIFIED REVENUE STRUCTURE

4.1 Preface

This chapter constitutes the first analytical stage of the present thesis. It discloses the fundamental building blocks of the model by distinctly examining how the decision maker's behavior in terms of investing and financing is modified by the interplay of the different model parameters. For now, the analysis is cut-down to the most simplistic form and essentially treats the firm as an unspecified "profit machine" generating random returns which are linear in the aggregate investment level. Thus, firm profits directly and merely depend on the initial investment. Evidently, this rudimentary representation totally disregards both the cost structure and the market context. These will be accounted for in the subsequent Chapters 5, 6, and 7.

4.2 Analysis

The model frame is supposed to examine the behavior of a decision maker who acts as an owner-manager and who evaluates the benefits and costs of alternative investment and financing plans. The analysis endogenizes the corporate investment level and deduces a preference hierarchy which is attributed to the disposable capital sources.

Having elucidated the properties of the decision maker's utility function in Chapter 3, the analysis of her investment and financing choices can now be approached. We start by considering a rudimentary case where neither the firm's revenue structure nor its cost structure are made explicit. The firm simply generates stochastic end-of-period cash flows which are assumed to be linear in the overall firm investment.

4.2.1 The Solution Approach

As disclosed by (12), D faces a constrained optimization problem. We tackle this by setting up a classical Lagrangean (\mathcal{L}) with its corresponding Lagrange multipliers $(\lambda_1, \ldots, \lambda_6)$. By utilizing the certainty equivalent, the expectations operator disappears, i.e., $u(CE) \equiv \mathbb{E}\{u(W_1^D)\}$. The Lagrangean of D's optimization problem is

(17)

$$\mathcal{L} = u(CE) + v(\phi) + \lambda_1(\phi - \underline{\phi}) + \lambda_2(1 - \phi) + \lambda_3\alpha + \lambda_4(\overline{\alpha} - \alpha) + \lambda_5\Gamma + \lambda_6(\overline{\Gamma} - \Gamma).$$

In the optimum, the above Lagrangean must simultaneously satisfy its necessary first order conditions and the below stated set of *Kuhn-Tucker conditions*:

$\lambda_1(\phi - \underline{\phi}) =$	0,	(KT.01)
$\lambda_2(1-\phi) =$	0,	(KT.02)
$\lambda_3 \alpha =$	0,	(KT.03)
$\lambda_4(\overline{\alpha} - \alpha) =$	0,	(KT.04)
$\lambda_5 \Gamma =$	0,	(KT.05)
$\lambda_6(\overline{\Gamma} - \Gamma) =$	0,	(KT.06)
$(\phi - \underline{\phi}) \geq$	0,	(KT.07)
$(1-\phi) \geq$	0,	(KT.08)
$\alpha \geq$	0,	(KT.09)
$(\overline{\alpha} - \alpha) \geq$	0,	(KT.10)
$\Gamma~\geq$	0,	(KT.11)
$(\overline{\Gamma} - \Gamma) \geq$	0,	(KT.12)
$\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6 \geq$	0.	(NNC)

Since the decision maker faces a concave objective function and linear constraints, the solution of the above Lagrangean yields a global maximum of D's objective function $\mathbb{E}\{u(\mu_{W_1^D} + \sigma_{W_1^D} Z)\} + v(\mathcal{C})$. Partially differentiating \mathcal{L} with respect to each of the three decision variables (ϕ, α, Γ) yields the three first order conditions (FOCs):

$$\begin{split} \frac{\partial \mathcal{L}}{\partial \phi} &= u'(CE)(\partial CE/\partial \phi) + v'(\phi) + \lambda_1 - \lambda_2 = 0, \\ \frac{\partial \mathcal{L}}{\partial \alpha} &= u'(CE)(\partial CE/\partial \alpha) + \lambda_3 - \lambda_4 = 0, \\ \frac{\partial \mathcal{L}}{\partial \Gamma} &= u'(CE)(\partial CE/\partial \Gamma) + \lambda_5 - \lambda_6 = 0. \end{split}$$

By the nature of the certainty equivalent (as disclosed by (14)) and by setting $u'(CE) = \Omega$, these FOCs display as

$$\begin{split} \frac{\partial \mathcal{L}}{\partial \phi} &= \Omega (\partial \mu_{W_1^D} / \partial \phi - \eta \sigma_{W_1^D} (\partial \sigma_{W_1^D} / \partial \phi)) + v'(\phi) + \lambda_1 - \lambda_2 = 0, \\ \frac{\partial \mathcal{L}}{\partial \alpha} &= \Omega (\partial \mu_{W_1^D} / \partial \alpha - \eta \sigma_{W_1^D} (\partial \sigma_{W_1^D} / \partial \alpha)) + \lambda_3 - \lambda_4 = 0, \\ \frac{\partial \mathcal{L}}{\partial \Gamma} &= \Omega (\partial \mu_{W_1^D} / \partial \Gamma - \eta \sigma_{W_1^D} (\partial \sigma_{W_1^D} / \partial \Gamma)) + \lambda_5 - \lambda_6 = 0. \end{split}$$

4.2.2 The Profit Function

Since W_1^D is a function of Π , we have to make explicit the profit function in order to proceed further. For the time being, the firm is supposed to generate stochastic returns which are linear in the total investment level K. Thus, since neither the demand nor the cost function are made explicit, the firm's profit directly and merely depends on K, i.e.,

$$\Pi = (1 + \rho)K,$$

where $\rho \sim \mathcal{N}(\mu_{\rho}, \sigma_{\rho}^2)$ is a Gaussian random variable which is stochastically independent from $K^{.85}$. Thereby, analogical to the previous demonstrations, its

 $^{^{85}\,}$ Similar to Brander/Lewis (1986), ρ can be interpreted as an unspecified "environmental" risk parameter.
linear transformation Π is also a Gaussian random variable with $\mu_{\Pi} = (1 + \mu_{\rho})K$ and $\sigma_{\Pi} = \sigma_{\rho}K$. By substituting both into (10), firm profit displays as

(18)
$$\Pi = \mu_{\Pi} + \sigma_{\Pi} z = (1 + \mu_{\rho})K + (\sigma_{\rho}K)z.$$

The differential terms in the FOCs can now be solved. As shown in the appendix to this chapter (see Appendix A 4.1), the FOCs then display as

$$\begin{split} & \underline{FOC.1:} \\ & \frac{\partial \mathcal{L}}{\partial \phi} = (\mu_{\rho} - r_2)\Gamma T - \eta \Gamma \phi K(\sigma_{\rho} T)^2 + (v'(\phi) + \lambda_1 - \lambda_2)/\Omega = 0, \\ & \underline{FOC.2:} \\ & \frac{\partial \mathcal{L}}{\partial \alpha} = -(\mu_{\rho} - r_1/T)W_0^D T - \eta W_0^D \phi K(\sigma_{\rho} T)^2 + (\lambda_3 - \lambda_4)/\Omega = 0 \\ & \underline{FOC.3:} \\ & \frac{\partial \mathcal{L}}{\partial \Gamma} = (\mu_{\rho} - r_2)\phi T - \eta K(\sigma_{\rho} \phi T)^2 + (\lambda_5 - \lambda_6)/\Omega = 0. \end{split}$$

4.2.3 The Decision Maker's Preference Hierarchy of Financing

The question addressed throughout this subsection is as follows. Is there a definite preference hierarchy attached to the firm's disposable funding sources and, if so, what are its preconditions and properties? We approach this question by identifying the possible values which D's decision variables may simultaneously adopt in the optimum.

To ensure the decision maker's participation in the firm, it is assumed that the (expected) returns generated by the firm are higher than the return of the riskless asset. Furthermore, the firm's average returns are assumed to exceed the due interest rate r_2 ,⁸⁶ i.e.,⁸⁷

$$\mu_{\rho} > r_1/T$$
 and $\mu_{\rho} > r_2$.

⁸⁶ Evidently, the validity of $\mu_{\rho} > r_2$ constitutes the precondition for the well-known *leve-rage-effect* to crop up.

⁸⁷ By means of the FOCs and [KT.3 - KT.6], it is straightforward to see that these two relations are indeed necessary conditions for the existence of an optimal interior solution with $0 < \alpha^* < \overline{\alpha}$ and $0 < \Gamma^* < \overline{\Gamma}$, i.e., with $\lambda_3 = \lambda_4 = \lambda_5 = \lambda_6 = 0$.

The right hand sides of these two inequalities are related inasmuch as their relative size crucially determines D's arbitration between her personal investment into the firm (which is controlled by means of the decision variable α) and the incurred level of corporate debt (which is controlled by means of the decision variable Γ). There are three basic possibilities, i.e.,

$$\begin{aligned} r_2 T > r_1, & \text{or} \\ r_2 T < r_1, & \text{or} \\ r_2 T = r_1. \end{aligned}$$

We assume the first case to be the normal case, i.e., a positive after-tax interest spread which ensures $r_2T > r_1$ (the remaining two cases are formally discussed in the appendix to this chapter, see Appendix A 4.2).

To thoroughly explore the decision maker's arbitration between α and Γ , we have to examine the conditions under which FOC.2 and FOC.3 hold simultaneously. Multiplying the former by ϕ and the latter by W_0^D , and subsequently adding up these new expressions yields the reduced condition

(19)
$$-(r_2 - r_1/T)\phi W_0^D T + \phi(\lambda_3 - \lambda_4)/\Omega + W_0^D(\lambda_5 - \lambda_6)/\Omega = 0.$$

By means of this reduced condition, the below-stated results can easily be demonstrated. We make use of the following Lemma to subsequently prove Proposition 4.1.

Lemma 4.1: For $r_2 - r_1/T > 0$, optimal solutions for α and Γ must fulfill

i)	$0 < \alpha^* < \overline{\alpha}$	Λ	$\Gamma^* = 0 ,$	or
ii)	$\alpha^* = 0$	٨	$\Gamma^* = 0 ,$	or
iii)	$\alpha^* = 0$	٨	$0 < \Gamma^* < \overline{\Gamma}$, or
iv)	$\alpha^* = 0$	Λ	$\Gamma^* = \overline{\Gamma}.$	

Proof: The four cases of Lemma 4.1 are proved consecutively.

- i) From KT.3 and KT.4, $0 < \alpha < \overline{\alpha}$ implies $\lambda_3 = \lambda_4 = 0$. Hence, the reduced condition (19) can only hold if $\lambda_5 = (r_2 r_1/T)\Omega\phi T > 0$ and $\lambda_6 = 0$. From KT.5, a positive value of λ_5 necessitates $\Gamma = 0$, which proves the first part of Lemma 4.1.
- *ii)* From KT.5 and KT.6, $\Gamma=0$ implies $\lambda_5 \ge 0$ and $\lambda_6=0$. Hence, the reduced condition (19) can only hold in two cases, namely if either
 - $\lambda_5 = (r_2 r_1/T)\Omega\phi T > 0$ and $\lambda_3 = \lambda_4 = 0$ (which constitutes the case covered by the first part of Lemma 4.1), or if
 - $\lambda_3 + \lambda_5 W_0^D / \phi = (r_2 r_1 / T) \Omega W_0^D T > 0$ and $\lambda_4 = 0$ (which, from *KT.3*, implies $\alpha = 0$, thus proving the second part of Lemma 4.1).

- iii) From KT.5 and KT.6, $0 < \Gamma < \overline{\Gamma}$ implies $\lambda_5 = \lambda_6 = 0$. Hence, the reduced condition (19) can only hold if $\lambda_3 = (r_2 r_1/T)\Omega W_0^D T > 0$ and $\lambda_4 = 0$. From KT.3, a positive value of λ_3 necessitates $\alpha = 0$, which proves the third part of Lemma 4.1.
- *iv)* From *KT.5* and *KT.6*, $\Gamma = \overline{\Gamma}$ implies $\lambda_5 = 0$ and $\lambda_6 \ge 0$. Hence, the reduced condition (19) can only hold if $\lambda_3 \lambda_6 W_0^D / \phi = (r_2 r_1/T)\Omega W_0^D T > 0$. Thus, λ_3 has to be positive, which, from *KT.3*, implies $\alpha = 0$. This proves the last part of Lemma 4.1.

Figure 7 gives a graphical representation of the result outlined by Lemma 4.1. It shows that, given our basic assumptions $\mu_{\rho}T > r_2T > r_1$, simultaneous interior solutions for both α and Γ are impossible. Appendix A 4.1 provides a formal proof that such simultaneous interior solutions for α and Γ are possible if and only if $r_2T = r_1$ is assumed.



Figure 7: Relationship between Γ^* and α^* .

The below-stated Proposition constitutes one of the main results of this chapter.

Proposition 4.1: Suppose $\mu_{\rho}T > r_2T > r_1$. In the optimum, the decision maker firstly invests her personal wealth into the firm before calling for corporate debt.

Proof: Proposition 4.1 immediately follows from Lemma 4.1.

For $r_2 - r_1/T > 0$ (Lemma 4.1), D only invests into the risk-free asset if no corporate debt is incurred (positive α^* necessitates $\Gamma^*=0$). Furthermore, corporate debt is only incurred if D's personal investment into the firm is maximized (positive Γ^* necessitates $\alpha^*=0$). Hence, the decision maker firstly invests her personal wealth into the firm before calling for debt (which is only incurred if D's personal wealth is not sufficient to meet her corporate investment objectives).

Proposition 4.1 conveys that the decision maker is driven towards a definite *preference hierarchy* regarding the two focal funding options. The intuition behind this result becomes quite evident when considering the following. Given $r_2T > r_1$, the effective costs of debt are *higher* than D's opportunity costs of personally investing into the firm, since the return of the risk-free asset is relatively low. Hence, D's personal wealth takes precedence over debt to feed the firm's stock of capital.⁸⁸ In fact, the decision maker's wealth constraint is not binding (i.e., her initial wealth is sufficient to meet her corporate investment objectives by itself) in cases *i*) and *ii*), contrary to cases *iii*) and *iv*), where the resulting "financing gap" is (as good as possible) filled by external debt.

It is easy to show that the reverse hierarchy is adopted if the reverse basic condition $r_2T < r_1$ holds. The special case $r_2T = r_1$ induces total ambiguity regarding the adopted hierarchy between α and Γ (formal proofs are given in the appendix to this chapter, see Appendix A 4.1). Hence, a fundamental dichotomy regarding the decision maker's financing behavior becomes apparent, which is contingent upon whether r_2T is smaller or greater than r_1 . Recalling that Tdecreases in the corporate tax rate,⁸⁹ high levels of corporate taxation will drive firms towards a stronger preference of external debt. This observation confirms the crucial role of tax imposition as described in the pertinent literature.

In what follows, we continue supposing $r_2T > r_1$.⁹⁰ Having elucidated the decision maker's preference hierarchy concerning the investment of her personal wealth (i.e., "inside equity") and the raising of debt, we still need to shed light on the role of the third funding source which can potentially feed the firm's stock of capital, i.e., "outside equity" from external investors. For this purpose, we examine D's corresponding behavior by means of delineating the possible

⁸⁸ Adopting a totally different approach, this result gives strong support to similar findings attained by Brander/Spencer (1989).

⁸⁹ $T = (1 - \tau^c).$

⁹⁰ As demonstrated in the appendix to this chapter, the reverse case follows easily in a symmetric manner.

values of the decision variable ϕ in the optimum. Multiplying FOC.1 by ϕ and FOC.3 by Γ and subsequently subtracting these new expressions yields the reduced condition

(20)
$$\phi(v'(\phi) + \lambda_1 - \lambda_2) - \Gamma(\lambda_5 - \lambda_6) = 0.$$

By means of the reduced condition (20) (for which both ϕ and Γ are optimized), the second main result is presented.

Proposition 4.2: Suppose $\mu_{\rho}T > r_2T > r_1$. In the optimum, a continuum of feasible solutions for the value of ϕ exist, which satisfy

$$\frac{1}{2} < \phi^* \le 1.$$

Proof: In the optimum, as demonstrated by the four cases of Lemma 4.1, there are three general possibilities for the solution value of Γ . Therefrom, the feasible solutions of ϕ are deduced.

- *i)* + *ii)* For $\Gamma=0$, the reduced condition (20) simplifies to $\phi(v'(\phi) + \lambda_1 \lambda_2) = 0$, which, from *KT.1* and *KT.2*, can only hold if $v'(\phi) = \lambda_1 = \lambda_2 = 0$. From the nature of the power curve, $v'(\phi)$ can only be zero for $\frac{1}{2} < \phi \le 1$.
- *iii)* For $0 < \Gamma < \overline{\Gamma}$, KT.5 and KT.6 imply $\lambda_5 = \lambda_6 = 0$. Hence, the reduced condition (20) can only hold if $v'(\phi) = \lambda_1 = \lambda_2 = 0$. Thereby, $\frac{1}{2} < \phi \leq 1$.
- iv) For $\Gamma = \overline{\Gamma}$, KT.5 and KT.6 imply $\lambda_5 = 0$ and $\lambda_6 \ge 0$. Hence, the reduced condition (20) displays as $\phi(v'(\phi) + \lambda_1 \lambda_2) + \overline{\Gamma} \lambda_6 = 0$. Since, from KT.2, a positive λ_2 implies $\phi = 1$, this equation can only hold if either $v'(\phi = 1) = \lambda_1 = 0 \land \lambda_2 = \overline{\Gamma} \lambda_6 / \phi \ge 0$, or $v'(\phi) = \lambda_1 = \lambda_2 = \lambda_6 = 0$. Thereby, feasible solutions of ϕ must necessarily satisfy $\frac{1}{2} < \phi \le 1$.

Proposition 4.2 conveys a substantial result. As long as her corporate investment objectives can be attained without debt and as long as her disposition on corporate decisions is assured, the decision maker is indifferent regarding the amount of outside equity brought in by external investors. The intuition behind this observation gets very understandable when becoming aware that in cases i) and ii) D's personal return of an additional unit of outside equity is zero. Despite the fact that an augmentation of K by additional outside equity induces firm profits to increase, the additional surplus is exactly redistributed among the new equity providers. Hence, the decision maker cannot raise additional personal revenue from her investment by raising outside equity. In fact, as can be verified in the appendix to this chapter, her marginal revenue (relating to ϕ) is zero as long as D's corporate investment objectives can be attained without calling for debt ($\Gamma=0$), i.e.,

$$\partial \mu_{W_1^D} / \partial \phi = (\mu_{\rho} - r_2) \Gamma T = 0$$

holds in case i) and ii). Thereby, and by considering (1) and (16), we directly infer that

$U'(\phi) > 0,$	for	$\underline{\phi} \le \phi \le \frac{1}{2},$	and
$U'\!(\phi)=0$	for	$\frac{1}{2} < \phi \leq 1.$	

Hence, ceter is paribus, D's utility is maximized for all values of ϕ that exceed $\frac{1}{2}.$

4.2.4 The Decision Maker's Corporate Investment Objectives

By proving Propositions 4.1 and 4.2, we deduced D's preference structure concerning her decision variables. However, her actual corporate investment objectives still remain to be made explicit. This investment decision is absent from most of the existing models concerned with financial structure choice, despite the fact that both decisions are thoroughly associated with each other. The decision maker's investment objectives can be endogenized and quantified in terms of *target levels* of corporate investment, namely a *maximum target level* and a *minimum target level*.

These target levels constitute optimal trade-off points where D's (expected) *personal wealth-gain* and her *risk exposure* are balanced. A more detailed discussion is provided at the end of this section. For now, it suffices to understand that

- the maximum target level constitutes an upper bound up to which the decision maker reckons an increase of the firm's stock of capital *by her private wealth* to be personally beneficial;
- the minimum target level constitutes an upper bound up to which she reckons an increase of the firm's stock of capital *by debt* to be beneficial.

Given D's strict preference hierarchy regarding her disposable funding sources, debt is actually only incurred if her initial wealth level lies below this minimum target level.

Cases i) and ii)

We begin by treating the first two cases, where $\Gamma^*=0$. As shown in these parts of Lemma 4.1, W_0^D is sufficiently large to meet *D*'s corporate investment objectives without calling for any debt. The proof of the first two cases in Lemma 4.1 has further disclosed that with

 $\begin{aligned} i) \quad \lambda_5 &= (r_2 - r_1/T)\Omega\phi T & \wedge & \lambda_3 &= \lambda_4 = \lambda_6 = 0, \quad \text{or with} \\ ii) \quad \lambda_3 &+ \lambda_5 W_0^D / \phi = (r_2 - r_1/T)\Omega W_0^D T & \wedge & \lambda_4 = \lambda_6 = 0, \end{aligned}$

FOC.2 and FOC.3 simultaneously hold. Thus, in both cases, we are facing a system of only two equations.

Scrutinizing case i), where $0 < \alpha^* < \overline{\alpha}$ and $\Gamma^* = 0$, we have

$$\begin{split} & \underline{\text{FOC.1:}} \\ & (v'(\phi^*) + \lambda_1 - \lambda_2)/\Omega = 0, \text{ and} \\ & \underline{\text{FOC.2} \equiv \text{FOC.3:}} \\ & -(\mu_{\rho} - r_1/T) W_0^D T - \eta W_0^D \phi^* T^2 \sigma_{\rho}^2 K = 0. \\ & \text{With } \phi^* K = (1 - \alpha^*) W_0^D + \phi^* \Gamma^* \text{ and } \Gamma^* = 0, \text{ FOC.2/3 displays as} \end{split}$$

(21)
$$(1-\alpha^*)W_0^D = (\mu_\rho - r_1/T)/\eta T\sigma_\rho^2$$
,

which discloses the optimal amount of D's personal wealth invested into the firm (LHS of the equation) and her maximum target level of corporate investment (RHS of the equation). D's initial wealth W_0^D is largely sufficient to reach this maximum target and allows for an investment of the excess amount $\alpha^* W_0^D$ into the riskless alternative.

Scrutinizing case *ii*), where $\alpha^* = \Gamma^* = 0$, we have

$$\begin{split} & \underline{\text{FOC.1:}} \\ & (v'(\phi^*) + \lambda_1 - \lambda_2)/\Omega = 0, \text{ and} \\ & \underline{\text{FOC.2} \equiv \text{FOC.3:}} \\ & -(\mu_{\rho} - r_1/T) W_0^D T - \eta W_0^D \phi^* T^2 \sigma_{\rho}^2 K - \lambda_3/\Omega = 0. \end{split}$$

With $\phi^* K = (1 - \alpha^*) W_0^D + \phi^* \Gamma^*$ and $\alpha^* = \Gamma^* = 0$, FOC.2/3 displays as

(22)
$$W_0^D = (\mu_\rho - r_1/T)/\eta T \sigma_\rho^2 - \lambda_3/\Omega \eta W_0^D T \sigma_\rho^2,$$

which discloses that D, despite investing her full wealth into the firm, does not reach her maximum target of corporate investment. The remaining difference between the maximum target level and D's wealth endowment is given by $\lambda_3/\Omega\eta W_0^D T \sigma_\rho^2$, which evidently decreases if W_0^D increases to approach the maximum target level from below.

In both case *i*) and *ii*), FOC.1 displays as $(v'(\phi^*) + \lambda_1 - \lambda_2)/\Omega = 0$, which, as demonstrated by Proposition 4.2, implies

$$\frac{1}{2} < \phi^* \le 1$$

for both cases. Hence, the decision maker is indifferent with respect to raising outside equity as long as her corporate control remains absolute.

Cases *iii*) and *iv*)

We now treat the last two cases, where $\Gamma^* > 0$. As shown in these parts of Lemma 4.1, W_0^D is not sufficiently large to meet D's corporate investment objectives without calling for additional debt. The proof of the last two cases in Lemma 4.1 has further disclosed that with

 $\begin{array}{lll} iii) & \lambda_3 = (\mathbf{r}_2 - \mathbf{r}_1/T) \Omega W_0^D T & & \wedge & \lambda_4 = \lambda_5 = \lambda_6 = 0, \quad \text{or with} \\ iv) & \lambda_3 - \lambda_6 W_0^D / \phi = (\mathbf{r}_2 - \mathbf{r}_1/T) \Omega W_0^D T & & \wedge & \lambda_4 = \lambda_5 = 0, \end{array}$

FOC.2 and FOC.3 simultaneously hold. Thus, we are again facing a system of only two equations.

Scrutinizing case *iii*), where $\alpha^* = 0$ and $\Gamma^* > 0$, we have

$$\begin{aligned} & \underline{FOC.1:} \\ (\mu_{\rho} - r_2) \Gamma^* T - \eta \Gamma^* \phi^* T^2 \sigma_{\rho}^2 K + (v'(\phi^*) + \lambda_1 - \lambda_2) / \Omega = 0 \\ & \underline{FOC.2 \equiv FOC.3:} \\ (\mu_{\rho} - r_2) \phi^* T - \eta \phi^{*2} T^2 \sigma_{\rho}^2 K = 0. \end{aligned}$$

Knowing that $\phi^* K = (1 - \alpha^*) W_0^D + \phi^* \Gamma^*$ and $\alpha^* = 0$, FOC.2/3 displays as

(23)
$$W_0^D + \phi^* \Gamma^* = (\mu_\rho - r_2)/\eta T \sigma_\rho^2$$
,

which, considering that W_0^D is fully invested into the firm, discloses the incurred level of corporate debt (Γ^*) in order to reach the *minimum target level* of corporate investment (RHS of the equation).

Substituting FOC.2/3 into FOC.1 yields the well-known expression $(v'(\phi^*) + \lambda_1 - \lambda_2)/\Omega = 0$, which, again, implies

$$1/2 < \phi^* \le 1.$$

Hence, the decision maker is indifferent with respect to raising outside equity as long as her corporate control is absolute and as long as her minimum target level of corporate investment can be attained.

However, since Γ^* now has a positive value, the size of the feasible interval of ϕ^* depends on the given parameter values. Evidently, with a positive level of corporate debt, Equation (23) can only hold if $W_0^D < (\mu_{\rho} - r_2)/\eta T \sigma_{\rho}^2$. The resulting financing gap is filled by the debt. More precisely, by the *fraction* of debt that induces a *personal wealth-gain* for the decision maker at period end, i.e., the fraction $\phi^* \Gamma^*$. Hence, as long as enough debt can be incurred to fulfill the condition $\phi^* \Gamma^* = (\mu_{\rho} - r_2)/\eta T \sigma_{\rho}^2 - W_0^D$ for even the smallest possible value of ϕ^* , the full interval (¹/₂; 1] is feasible for ϕ^* . Since the smallest possible value of ϕ^* is marginally greater than ¹/₂, the inequality

$$\begin{split} ^{1\!\!/2}\,\overline{\Gamma} > ((\mu_{\rho} \text{-} r_2)/\eta T \sigma_{\rho}^2 - W_0^D) \\ \Longleftrightarrow \\ \overline{\Gamma} > 2((\mu_{\rho} \text{-} r_2)/\eta T \sigma_{\rho}^2 - W_0^D) \end{split}$$

must be fulfilled, i.e., the debt capacity must be greater than two times the financing gap.

If the exogenous variable $\overline{\Gamma}$ is gradually decreased below this level, the decision maker's effective degrees of freedom concerning the filling of the financing gap are likewise gradually diminished. In fact, the feasible interval for ϕ^* becomes increasingly contracted from below to finally reduce to the unique possible value $\phi^*=1$ as soon as the debt capacity is (smaller or) equal to the financing gap, i.e., as soon as $\overline{\Gamma} \leq (\mu_{\rho} - r_2)/\eta T \sigma_{\rho}^2 - W_0^D$, which means that the boundary to case iv) is attained.

Scrutinizing this case *iv*), where $\alpha^* = 0$ and $\Gamma^* = \overline{\Gamma}$, we have

$$\begin{split} & \underline{\text{FOC.1:}} \\ & (\mu_{\rho} - r_2)\Gamma^*T - \eta\Gamma^*\phi^*T^2\sigma_{\rho}^2K + (v'(\phi^*) + \lambda_1 - \lambda_2)/\Omega = 0, \\ & \underline{\text{FOC.2} \equiv \text{FOC.3:}} \\ & (\mu_{\rho} - r_2)\phi^*T - \eta\phi^{*2}T^2\sigma_{\rho}^2K - \lambda_6/\Omega = 0. \end{split}$$

Knowing that $\phi^* K = (1-\alpha) W_0^D + \phi^* \Gamma^*$, $\alpha^* = 0$, and $\Gamma^* = \overline{\Gamma}$, FOC.2/3 displays as

(24)
$$W_0^D + \phi^* \overline{\Gamma} = (\mu_\rho - r_2) / \eta T \sigma_\rho^2 - \lambda_6 / \Omega \eta T^2 \sigma_\rho^2 \phi^*.$$

which discloses that D invests her full wealth into the firm and uses the full debt capacity in order to approach her minimum target as near as possible. The remaining difference between the minimum target level and $W_0^D + \phi^* \overline{\Gamma}$ is given by the term $\lambda_6 / \Omega \eta T^2 \sigma_\rho^2 \phi^*$, which evidently compensates for the fact that $W_0^D + \overline{\Gamma} < (\mu_\rho - r_2) / \eta T \sigma_\rho^2$ and implies a positive λ_6 . As shown in the proof of Proposition 4.2, a positive value of λ_6 necessitates a positive λ_2 , which, from KT.2, implies $\phi^*=1$.

Generally encompassing all four cases, the optimality conditions given by the equations (21) - (24) all exhibit an identical structure. The left-hand sides denote D's pecuniary involvement in the firm (i.e., "her" part of the firm's asset value $\phi^* K^*$), which she determines so as to meet (or approach) her target level (right-hand sides of the equations). We always have

$$\overbrace{(1-\alpha^*)W_0^D + \phi^*\Gamma^*}^{\phi^*E^* + \phi^*\Gamma^*} \longrightarrow Maximum/Minimum Target Level.$$

This discloses a very important notion. The decision maker's target levels of corporate investment do *not* reflect the total investment intensity of the firm, but merely the part of the firm's capital stock that translates into cash flows for D. This is the sum of her part of all equity-induced returns and her part of all

debt-induced returns. Recalling the intuition behind Proposition 4.2, it becomes now even clearer that, as long as no debt-induced returns exist (cases *i*) and *ii*), where $\Gamma^*=0$), outside equity has absolutely no effect in terms of reaching *D*'s (maximum) target level of corporate investment. Thus, the decision maker is completely indifferent regarding the amount of outside equity brought in by external investors as long as her power-related utility component $v(\phi)$ is maximized (i.e., as long as ϕ is greater than $\frac{1}{2}$).

The right-hand sides of equations (21) and (23) plainly reveal how the decision maker's target levels vary as functions of her risk aversion and the underlying environment of the firm. The target levels can be directly tied to the stochastic structure of the firm's cash flow, increasing in μ_{ρ} and decreasing in both σ_{ρ}^2 and η . In fact, the actual existence of these target levels is ultimately due to the decision maker's risk-aversion. They constitute reference points at which an optimal trade-off between D's (expected) personal wealth-gain and her risk exposure is attained.⁹¹ Concerning the maximum target level which is aimed at in cases *i*) and *ii*), the decision maker trades off the wealth effect of personally investing into the firm against the corresponding cash flow risk of stock ownership. Concerning the minimum target level which is aimed at in cases *iii*) and *iv*), the decision maker trades off the wealth effect of debt (leverage effect) against the corresponding cash flow risk of due corporate interest payments.⁹²

Hence, the results show that the decision maker's investment objectives become *more ambitious* with improving average environmental states, decreasing environmental volatility, and decreasing risk aversion of the decision maker.

The impact of corporate taxation on the decision maker's investment and financing decisions is particularly interesting. Recall our basic assumption $r_2T > r_1$. For increasing rates of corporate taxation ($\tau^c \uparrow$ or $T \downarrow$), the minimum target level increases, while the maximum target level decreases. Hence, the domain of the optimality case *ii*) gets smaller and smaller and finally vanishes at the limit of our basic assumption, i.e., when $r_2T = r_1$. This is because, with growing corporate taxation, the difference between decision maker's costs of debt (r_2T) and her opportunity costs of investing into the firm (r_1) gets smaller

⁹¹ Recall that W_1^D is a linear transformation of the environmental risk parameter ρ . With respect to *D*'s personal revenue, it is easy to verify that not only the expectation value $\mu_{W_1^D}$, but also the risk measure $\sigma_{W_1^D}^{2^2}$ is an increasing function of both inside equity and debt. Hence, increasing *D*'s expected personal wealth-gains by means of augmenting either the level of her personal firm investment or the level of corporate debt imperatively comes along with a greater *personal risk exposure* of the focal entrepreneur.

⁹² This observation is perfectly in line with the well-established view that debt finance ("financial leverage") pushes up the financial risk of the common stock. See Brealey/Myers (2008, p. 229).

and smaller to finally vanish. Increasing τ^c even further would then reverse our basic assumption to $r_2T < r_1$, and consequently the preference hierarchy between inside equity and corporate debt would also be reversed compared to Proposition 4.1 (see Appendix A 4.2 for a formal proof).

4.2.5 The summarized Pattern of Investing and Financing

In order to summarize the decision maker's fundamental behavioral pattern in terms of investing and financing, we denote the minimum and maximum target level of corporate investment as TL^{Min} and TL^{Max} respectively, i.e.,

$$\begin{split} \mathrm{TL}^{\mathrm{Max}} &= (\mu_{\rho} - r_1/T)/\eta T \sigma_{\rho}^2, \quad \text{and} \\ \mathrm{TL}^{\mathrm{Min}} &= (\mu_{\rho} - r_2)/\eta T \sigma_{\rho}^2. \end{split}$$

As demonstrated above, our basic assumptions give rise to four possible cases.

$$\begin{split} i) \quad \text{if} \qquad & W_0^D > \mathrm{TL}^{\mathrm{Max}}, \\ \Rightarrow \qquad & \frac{1}{2} < \phi^* \le 1, \\ & \alpha^* > 0, \, \text{satisfying} \, \, (1 - \alpha^*) W_0^D = \mathrm{TL}^{\mathrm{Max}}, \\ & \Gamma^* = 0, \\ & \mathrm{TL}^{\mathrm{Max}} \le K^* \le 2 \mathrm{TL}^{\mathrm{Max}}. \end{split}$$

D's private wealth is sufficiently large to meet her corporate investment objectives, i.e., it exceeds her maximum target level of corporate investment. Hence, she incurs no corporate debt and divides her personal investment between the firm's stock of capital and the riskless alternative. As long as she retains full corporate control, the decision maker is indifferent regarding the amount of outside equity brought in by external investors.

$$\begin{array}{ll} ii) \quad \mathrm{if} \qquad \mathrm{TL}^{\mathrm{Min}} < W^D_0 \leq \mathrm{TL}^{\mathrm{Max}}, \\ \Rightarrow & \frac{1/2}{2} < \phi^* \leq 1, \\ & \alpha^* = 0, \\ & \Gamma^* = 0, \\ & W^D_0 \leq K^* \leq 2W^D_0. \end{array}$$

D's private wealth is sufficiently large to exceed her minimum target level of corporate investment, but smaller than her maximum target level. Hence, she incurs no corporate debt and invests her full personal wealth into the firm in order to reach the best possible rapprochement towards her maximum target level. As long as she retains full corporate control, the decision maker is indifferent regarding the amount of outside equity brought in by external investors.

$$\begin{split} &iii) \quad \text{if} \qquad W_0^D \leq \mathrm{TL}^{\mathrm{Min}} \leq W_0^D + \overline{\Gamma}, \\ &\Rightarrow \qquad {}^{l\!\!\!/ 2} < \phi^* \leq 1, \, \text{satisfying} \ W_0^D + \phi^* \Gamma^* = \mathrm{TL}^{\mathrm{Min}}, \\ &\alpha^* = 0, \\ &\overline{\Gamma} \geq \Gamma^* \geq (\mu_\rho - r_2) / \eta \, T \sigma_\rho^2 - W_0^D, \, \text{satisfying} \ W_0^D + \phi^* \Gamma^* = \mathrm{TL}^{\mathrm{Min}}, \\ &K^* = \mathrm{TL}^{\mathrm{Min}} / \phi^*. \end{split}$$

D's private wealth is too small to meet her minimum target level of corporate investment. Hence, she invests her full personal wealth into the firm's stock of capital and incurs supplementary corporate debt in order to reach this minimum target level (filling the financing gap). Inasmuch as this financing gap can be filled by the relevant fraction of debt $\phi^*\Gamma^*$ and as long as she retains full corporate control, the decision maker is indifferent regarding the amount of outside equity brought in by external investors.

$$\begin{array}{ll} vi) \quad \mathrm{if} \qquad W^D_0 + \overline{\Gamma} < \mathrm{TL}^{\mathrm{Min}}, \\ \Rightarrow \qquad \phi^* = 1, \\ \alpha^* = 0, \\ \Gamma^* = \overline{\Gamma}, \\ K^* = W^D_0 + \overline{\Gamma} < \mathrm{TL}^{\mathrm{Min}}. \end{array}$$

The sum of D's private wealth and the maximum amount of debt is too small to meet her minimum target level of corporate investment. Hence, she invests her full personal wealth into the firm's stock of capital and incurs as much corporate debt as possible in order to reach the best possible rapprochement towards her minimum target level. In order to maximize her claims over the return stream generated by the firm, the decision maker does not incur any outside equity. Thereby, she fully benefits from the corporate debt and its leverage effect, hence maximizing her utility function.

Figure 8 depicts the range of values for $(\phi^*, \alpha^*, \Gamma^*)$ throughout the four cases. It gives a graphical representation of the three decision variables' possible values in the optimum, contingent on the level of the decision maker's initial wealth W_{ρ}^D .



Figure 8: Wealth-dependent optimality space of (ϕ, α, Γ) .

The right solid vertical line which separates the cases *i*) and *ii*) represents the decision maker's maximum target level $\text{TL}^{\text{Max}} = (\mu_{\rho} - r_1/T)/\eta T \sigma_{\rho}^2$. For all values of W_0^D which exceed this level, *D*'s private wealth is sufficient to meet her corporate investment objectives. Hence, case *i*) holds.

The central solid vertical line which separates the cases *ii*) and *iii*) represents the minimum target level $\text{TL}^{\text{Min}} = (\mu_{\rho} - r_2)/\eta T \sigma_{\rho}^2$. For all values of W_0^D which lie between both target levels, D's private wealth is only sufficient to meet her minimum, but not her maximum target level of corporate investment. Hence, case *ii*) holds.

The left solid vertical line which separates the cases iii) and iv) represents the wealth level $\text{TL}^{\text{Min}} - \overline{\Gamma}$, from where on the given debt capacity $\overline{\Gamma}$ is sufficient to close the financing gap. For all values of W_0^D which exceed this level, her wealth allows for a complete filling of the financing gap by balancing debt and outside equity to satisfy Equation (23). Hence, case iii) holds. For all values of W_0^D which fall short of this level, D keeps full ownership to personally benefit from the full debt capacity in order to approach her minimum target level as good as possible. Hence, case iv) holds. The dashed vertical line within the case *iii*)-section indicates the level from where on ϕ^* can take its lowest possible value with the given debt capacity. For all values of W_0^D which exceed this level, the decision maker has full freedom to choose the level of ϕ^* in the interval (1/2, 1] to satisfy the optimality condition (23). For all values of W_0^D which fall short of this level, the financing gap is greater than half of the debt capacity, which means that the lower optimality bound of ϕ must rise further beyond 1/2 to satisfy (23). When gradually decreasing W_0^D below this level, the feasible interval of ϕ^* becomes increasingly contracted from below to finally reduce to the unique value $\phi=1$ as soon as $W_0^D = \text{TL}^{\text{Min}} = (\mu_{\rho} - r_2)/\eta T \sigma_{\rho}^2 - \overline{\Gamma}$, meaning that the boundary to case *iv*) is attained.

The shaded areas indicate the range of possible values of Γ^* and ϕ^* . It is important to understand that these optimal values are interdependent. However, since the value of Γ^* is only ambiguous in case *iii*), this interdependence only becomes evident when inspecting the third case. In fact, the relative vertical distance of Γ^* to its corresponding *upper optimality bound* (blue dashed curve) exactly equals the relative vertical distance of ϕ^* to its corresponding *lower optimality bound* (red solid curve), and vice versa. If we suppose that case *iii*) holds and D chooses an extreme value for Γ^* which lies on its upper optimality bound, the corresponding value of ϕ^* must lie on its lower optimality bound. The same is true in the opposite case.

This linear relationship is illustrated by Figure 9 (the lower and upper optimality bounds are represented by $(\underline{\Gamma^*}, \underline{\phi^*})$ and $(\overline{\Gamma^*}, \overline{\phi^*})$ respectively).



Figure 9: Linear relationship between Γ^* and ϕ^* .

4.2.6 Comparative Statics

From the disclosed pattern of investing and financing, a set of immediate comparative statics results emerges as subsequently summarized.

Comparative Statics Result 1: The lower the risk aversion, the cash flow volatility or the initial wealth and the higher the expected firm-related returns or the corporate tax rate,

- the fewer will be invested into the riskless alternative by trend;
- the more probable becomes a high fraction of debt in the firm's stock of capital;
- the higher will be the ownership concentration by trend.

These observations directly bleed off from the nature of D's investment objectives (as disclosed in (21) and (23)). However, it is particularly worth examining the limiting case. If the decision maker approaches risk neutrality $(\eta \rightarrow 0)$, her investment targets approach infinity. The same is true if the environmental risk tends towards zero $(\sigma_{\rho}^2 \rightarrow 0)^{.93}$ As a consequence, the decision maker is induced to massively engage into that option which offers the best expected after-tax deal. In other words, she simply pumps the maximum possible amount of both her own wealth and debt into the firm. Accordingly, *iv* is the only remaining case, where no outside equity is incurred and ownership concentration is at its maximum.

Comparative Statics Result 2: The higher the corporate tax rate,

- the more will be invested into the riskless alternative by trend;
- the more probable becomes a high fraction of debt in the firm's stock of capital.

We have already elaborated on the impact of corporate taxation. It was shown that the minimum target level increases in the corporate tax rate, while the maximum target level decreases. Hence, the domain of the optimality case ii (see Figure 8) gets smaller and smaller, while the domains of case i) and iii) are growing. Thereby, scenarios in which the decision maker invests into the riskless asset (case i) and scenarios in which the decision maker incurs corporate debt (case iii) become more probable.

⁹³ Evidently, a lower environmental volatility σ_{ρ}^2 directly translates into a lower variability of the decision maker's personal return on investment, i.e., a lower cash flow volatility.

Comparative Statics Result 3: A lower debt capacity will, ceteris paribus, lead to a higher ownership concentration by trend.

This result confirms an observation which has already been discussed when examining the cases *iii*) and *iv*) (equations (23) and (24)). It discloses an interesting notion concerning possible (real) economic consequences of banking regulation. If corporate debt is needed to close the financing gap, an exogenous lowering of $\overline{\Gamma}$ (like credit rationing induced by, say, newly imposed banking restrictions like the Basel III accord) will, ceteris paribus, contract the feasible interval of ϕ^* from below (i.e., the lower bound of the interval tends against its upper bound 1) as soon as the debt capacity becomes smaller than two times the financing gap.

Comparative Statics Result 4: The smaller the difference $(\mu_{\rho} - r_2)$,

- the lower the minimum target level;
- the fewer corporate debt will be incurred by trend.

As we have seen while examining equations (21) and (23), it is the interplay of risk aversion and interest payments which induces an inner optimum for the level of corporate debt incurred by the decision maker. D trades off the leverage effect of debt against the cash flow risk of corresponding interest payments. As the intensity of the leverage effect declines (the difference $(\mu_{\rho} - r_2)$ becomes smaller), the level up to which it outweighs the interest related cash flow risk decreases accordingly. Again, the result is easily rendered comprehensible by examining the limiting case. If $(\mu_{\rho} - r_2)$ approaches zero, the minimum target level (as disclosed in (23) totally vanishes and only cases *i*) and *ii*) are left.

Comparative Statics Result 5: The higher D's initial wealth endowment, the higher will be the overall investment volume of the firm by trend.

An increase of W_0^D increases the overall investment level in case *ii*) and *iv*), while it has no effect in case *i*) and *iii*). Thereby, the corporate investment level is a (weakly) increasing function of the decision maker's personal wealth. This result compellingly highlights that not only funding decisions, but also actual investing decisions are crucially depending on the severeness of the decision maker's personal liquidity constraint.

In fact, the actual feasibility of external equity financing directly depends on both, the debt capacity (as indicated by Comparative Statics Result 3) and the level of the disposable internal funds: for sufficiently low levels of $\overline{\Gamma}$ and W_0^D , no outside equity will be raised at all. In general, slow-growing and highly profitable firms in mature markets are likely to generate high cash amounts from retained earnings, contrary to fast-growing but less profitable firms in emerging markets. Hence, such young enterprises will abstain from external equity financing as long as the sum of their internally available funds and the debt capacity falls short of the minimum target level of corporate investment.

As a further remark, the debt capacity is treated as an exogenous constant in the present model. A refinement of the model which assumes $\overline{\Gamma}$ to be positively related to W_0^D could be expected to foster an endogenous amplification of this effect: the lower W_0^D , the lower $\overline{\Gamma}$, and the lower the feasibility of outside equity. It could be worthwhile to examine to conditions under which this mechanism yields a separating equilibrium in which small emerging firms invest low and large mature firms invest heavily.

4.3 Conclusion

This chapter examined the behavior of a decision maker who acts as an ownermanager and who evaluates the benefits and costs of alternative investment and financing plans. Case-contingent interior solutions were obtained so that marginal costs and marginal benefits are balanced.

The analysis firstly allowed for the endogenous derivation of precise corporate investment objectives which are contingent on the decision maker's subjective beliefs (via μ_{ρ} and σ_{ρ}^2), her risk sensitivity (via η), and exogenous institutional parameters (via r_1 , r_2 , and T). Secondly, a hierarchy according to which the disposable funding sources are ranked arises on the grounds of these investment objectives. Inasmuch as the decision maker can attain her corporate investment objectives, she exclusively relies on equity for the firm's stock of capital. Otherwise, debt is incurred to fill the financing gap. Since the decision maker's personal return on investment is unaffected outside equity, she is indifferent regarding the involvement of external equity providers as long as her investment objectives can be attained and her intra-organizational power remains absolute.

Concerning the impact of D's of risk-sensitivity on the corporate financing mix (Comparative Statics Result 1), our findings fundamentally deviate from the results of Jensen/Meckling (1976). According to their logic (which is essentially based on agency considerations), decreasing risk-aversion gradually shifts down the relative proportion of debt in the firm's stock of capital. In fact, our model would generate exactly the same result if the stock of capital had been assumed to be exogenously fixed. Since decreasing risk-aversion drives the decision maker towards a higher involvement into the (risky) firm project, she would decrease α in order to invest a larger fraction of her personal wealth into the firm, hence substituting debt with equity. However, by endogenizing the decision maker's actual investment decision, our analysis shows that decreasing risk-aversion *additionally* pushes her towards an absolute augmentation of the firm's stock of capital. Hence, she increases both, her personal investment into the firm *and* the amount of corporate debt. At the same time, she reduces the amount of outside equity in order to reap more of the financial leverage effect of debt. Hence, our analysis shows how the simultaneous endogenization of investing and financing decisions changes the results and leads to further insights.

Further concerning the derived funding hierarchy, our findings obviously resemble the compelling results delivered by the pecking order theory (POT) of Myers/Majluf (1984). Similar to the POT, our results provide a clear explanation for why most small firms raise outside funds primarily in the form of debt.⁹⁴ Moreover, likewise similar to the POT, our model implies a negative relation between profitability and firm leverage when interpreting the decision maker's initial wealth endowment as retained earnings from previous periods. This prediction proves to be in line with most of the available empirical data. However, empirical studies have also observed a systematic raising of outside equity besides or before debt (as stressed by Fama/French, 2002), which strongly contradicts the POT's predictions. Thereby, our model does allow for a substantial enrichment of the POT's hierarchy postulate, since the preceding analvsis provides a clear rationale for suchlike behavior. In fact, the hierarchy proposed by the present model can be considered to be less rigid compared to the POT. This circumstance is mainly driven by the adoption of a non-linear utility function, since otherwise results naturally tend towards corner solutions. In addition, it feels necessary to stress the fact that the disclosed results are tapped by using an approach which completely differs from the pecking order framework. While the insights of Myers/Majluf (1984) crucially rest upon the interrelation between informational costs and the corresponding respective features of the disposable capital sources, the corporate financing mix as derived from our analysis directly depends on the interplay of individual characteristics of the decision maker and (perceived) environmental properties. Costs of informational asymmetries play no role in our setting.

⁹⁴ This assertion is especially true for continental European markets like Germany, which is confirmed by an empirical study by Hermanns (2006).

Appendix to Chapter 4

A 4.1 Solving the differential terms in the three FOCs

The FOCs are given by

$$\begin{split} &\frac{\partial \mathcal{L}}{\partial \phi} = \Omega(\partial \mu_{W_1^D} / \partial \phi - \eta \sigma_{W_1^D} (\partial \sigma_{W_1^D} / \partial \phi)) + v'(\phi) + \lambda_1 - \lambda_2 = 0, \\ &\frac{\partial \mathcal{L}}{\partial \alpha} = \Omega(\partial \mu_{W_1^D} / \partial \alpha - \eta \sigma_{W_1^D} (\partial \sigma_{W_1^D} / \partial \alpha)) + \lambda_3 - \lambda_4 = 0, \\ &\frac{\partial \mathcal{L}}{\partial \Gamma} = \Omega(\partial \mu_{W_1^D} / \partial \Gamma - \eta \sigma_{W_1^D} (\partial \sigma_{W_1^D} / \partial \Gamma)) + \lambda_5 - \lambda_6 = 0. \end{split}$$

Since $\sigma_{W_1^D} = \phi T \sigma_{\Pi}$, $\sigma_{\Pi} \equiv \sigma_{\Pi}(K)$, and $K \equiv K(\phi, \alpha, \Gamma)$, we can rewrite FOC.1 as

$$\begin{split} \Omega(\partial\mu_{W_{1}^{D}}/\partial\phi - \eta(\phi T\sigma_{\Pi})(\partial\sigma_{W_{1}^{D}}/\partial\phi)) + v'(\phi) + \lambda_{1} - \lambda_{2} &= 0 \\ \Leftrightarrow \ \partial\mu_{W_{1}^{D}}/\partial\phi - \eta(\phi T\sigma_{\Pi})(T\sigma_{\Pi} + \phi T(\partial\sigma_{\Pi}/\partial K)(\partial K/\partial\phi)) + (v'(\phi) + \lambda_{1} - \lambda_{2})/\Omega &= 0 \\ \Leftrightarrow \ \partial\mu_{W_{1}^{D}}/\partial\phi - \eta\phi T^{2}\sigma_{\Pi}^{2} - \eta\phi^{2}T^{2}\sigma_{\Pi}(\partial\sigma_{\Pi}/\partial K)(\partial K/\partial\phi) + (v'(\phi) + \lambda_{1} - \lambda_{2})/\Omega &= 0 \end{split}$$

Treating FOC.2 and FOC.3 analogously yields

$$\begin{split} \Omega(\partial\mu_{W_{1}^{D}}/\partial\alpha - \eta(\phi T\sigma_{\Pi})(\partial\sigma_{W_{1}^{D}}/\partial\alpha)) + \lambda_{3} - \lambda_{4} &= 0 \\ \Leftrightarrow \ \partial\mu_{W_{1}^{D}}/\partial\alpha - \eta(\phi T\sigma_{\Pi})(\phi T(\partial\sigma_{\Pi}/\partial K)(\partial K/\partial\alpha)) + (\lambda_{3} - \lambda_{4})/\Omega &= 0 \\ \Leftrightarrow \ \partial\mu_{W_{1}^{D}}/\partial\alpha - \eta\phi^{2}T^{2}\sigma_{\Pi}(\partial\sigma_{\Pi}/\partial K)(\partial K/\partial\alpha) + (\lambda_{3} - \lambda_{4})/\Omega &= 0 \end{split}$$

$$\begin{split} \Omega(\partial\mu_{W_{1}^{D}}/\partial\Gamma - \eta(\phi T\sigma_{\Pi})(\partial\sigma_{W_{1}^{D}}/\partial\Gamma)) + \lambda_{5} - \lambda_{6} &= 0 \\ \Leftrightarrow \ \partial\mu_{W_{1}^{D}}/\partial\Gamma - \eta(\phi T\sigma_{\Pi})(\phi T(\partial\sigma_{\Pi}/\partial K)(\partial K/\partial\Gamma)) + (\lambda_{5} - \lambda_{6})/\Omega &= 0 \\ \Leftrightarrow \ \partial\mu_{W_{1}^{D}}/\partial\Gamma - \eta\phi^{2}T^{2}\sigma_{\Pi}(\partial\sigma_{\Pi}/\partial K)(\partial K/\partial\Gamma)) + (\lambda_{5} - \lambda_{6})/\Omega &= 0 \end{split}$$

Thereby, we now have

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial \phi} &= \partial \mu_{W_1^D} / \partial \phi - \eta \phi T^2 \sigma_{\Pi}^2 - \eta \phi^2 T^2 \sigma_{\Pi} (\partial \sigma_{\Pi} / \partial K) (\partial K / \partial \phi) \\ &+ (v'(\phi) + \lambda_1 - \lambda_2) / \Omega = 0, \end{aligned}$$

$$\begin{split} \frac{\partial \mathcal{L}}{\partial \alpha} &= \partial \mu_{W_1^D} / \partial \alpha - \eta \phi^2 T^2 \sigma_{\Pi} (\partial \sigma_{\Pi} / \partial K) (\partial K / \partial \alpha) + (\lambda_3 - \lambda_4) / \Omega = 0, \\ \frac{\partial \mathcal{L}}{\partial \Gamma} &= \partial \mu_{W_1^D} / \partial \Gamma - \eta \phi^2 T^2 \sigma_{\Pi} (\partial \sigma_{\Pi} / \partial K) (\partial K / \partial \Gamma) + (\lambda_5 - \lambda_6) / \Omega = 0. \end{split}$$

To rewrite these three equations as explicit functions of the decision variables, we need to solve the differential terms. We tackle this problem by first computing the three derivatives $\partial \mu_{W_1^D} / \partial \phi$, $\partial \mu_{W_1^D} / \partial \alpha$, and $\partial \mu_{W_1^D} / \partial \Gamma$.

From (11), we have $\mu_{W_1^D} = W_0^D + \alpha W_0^D r_1 + (\phi(\mu_{\Pi} - (1+r_2)\Gamma) - (1-\alpha)W_0^D)T$ and $\sigma_{W_1^D} = \phi T \sigma_{\Pi}$. Hence,

From (6), we can infer $K = (1-\alpha)W_0^D + E^{ex} + \Gamma = E + \Gamma = (1-\alpha)W_0^D / \phi + \Gamma$. Hence,

$$\begin{array}{lll} \partial K / \partial \phi & = & (\alpha - 1) W_0^D / \phi^2 = -(1 - \alpha) W_0^D / \phi^2 = -E / \phi \ , \\ \partial K / \partial \alpha & = & -W_0^D / \phi \ , \\ \partial K / \partial \Gamma & = & 1. \end{array}$$

Further recalling the functional forms of μ_{Π} and σ_{Π} as given by (18), i.e., $\mu_{\Pi} = (1 + \mu_{\rho})K$ and $\sigma_{\Pi} = \sigma_{\rho}K$, the three focal derivatives display as

$$\begin{split} \partial \mu_{W_1^D} / \partial \phi &= ((1+\mu_{\rho})K - (1+r_2)\Gamma + \phi(-E/\phi)(1+\mu_{\rho}))T \\ &= ((\mu_{\rho} - r_2)\Gamma + (1+\mu_{\rho})E - (1+\mu_{\rho})E)T \\ &= (\mu_{\rho} - r_2)\Gamma T. \\ \partial \mu_{W_1^D} / \partial \alpha &= (r_1 + T)W_0^D + (1+\mu_{\rho})(-W_0^D/\phi)\phi T \\ &= -(\mu_{\rho} - r_1/T)W_0^D T. \end{split}$$

$$\begin{split} \partial \mu_{W^D_1} / \partial \Gamma &= (-(1{+}r_2) + (1{+}\mu_\rho)) \phi T \\ &= (\mu_\rho {-}r_2) \phi T. \end{split}$$

Solving the remaining differential terms in the FOCs then yields

$$\begin{split} \frac{\partial \mathcal{L}}{\partial \phi} &= (\mu_{\rho} - r_2)\Gamma T - \eta \phi T^2 (\sigma_{\rho} K)^2 - \eta \phi^2 T^2 \sigma_{\rho}^2 K (-E/\phi) + (v'(\phi) + \lambda_1 - \lambda_2)/\Omega = 0 \\ &= (\mu_{\rho} - r_2)\Gamma T - \eta \phi T^2 (\sigma_{\rho} K)^2 + \eta \phi T^2 \sigma_{\rho}^2 K E) + (v'(\phi) + \lambda_1 - \lambda_2)/\Omega = 0 \\ &= (\mu_{\rho} - r_2)\Gamma T - \eta \phi T^2 (\sigma_{\rho} K)^2 + \eta \phi T^2 \sigma_{\rho}^2 K (K - \Gamma)) + (v'(\phi) + \lambda_1 - \lambda_2)/\Omega = 0 \\ &= (\mu_{\rho} - r_2)\Gamma T - \eta \Gamma \phi T^2 \sigma_{\rho}^2 K + (v'(\phi) + \lambda_1 - \lambda_2)/\Omega = 0. \\ \\ \frac{\partial \mathcal{L}}{\partial \alpha} &= -(\mu_{\rho} - r_1/T) W_0^D T - \eta \phi^2 T^2 \sigma_{\rho}^2 K (-W_0^D/\phi) + (\lambda_3 - \lambda_4)/\Omega = 0 \\ &= -(\mu_{\rho} - r_1/T) W_0^D T + \eta W_0^D \phi T^2 \sigma_{\rho}^2 K + (\lambda_3 - \lambda_4)/\Omega = 0. \\ \\ \frac{\partial \mathcal{L}}{\partial \Gamma} &= (\mu_{\rho} - r_2) \phi T - \eta \phi^2 T^2 (\sigma_{\rho} K) \sigma_{\rho} + (\lambda_5 - \lambda_6)/\Omega = 0 \\ &= (\mu_{\rho} - r_2) \phi T - \eta \phi^2 T^2 \sigma_{\rho}^2 K + (\lambda_5 - \lambda_6)/\Omega = 0. \end{split}$$

Hence, from the nature of $\mu_{W^D_1},\;\sigma_{W^D_1},\;\mu_{I\!I},$ and $\sigma_{I\!I},$ the three FOCs indeed display as

$$\begin{split} & \frac{\text{FOC.1:}}{\partial \phi} = (\mu_{\rho} - r_2)\Gamma T - \eta\Gamma\phi T^2 \sigma_{\rho}^2 K + (v'(\phi) + \lambda_1 - \lambda_2)/\Omega = 0, \\ & \frac{\text{FOC.2:}}{\partial \Delta} \\ & \frac{\partial \mathcal{L}}{\partial \alpha} = -(\mu_{\rho} - r_1/T)W_0^D T - \eta W_0^D \phi T^2 \sigma_{\rho}^2 K + (\lambda_3 - \lambda_4)/\Omega = 0, \\ & \frac{\text{FOC.3:}}{\partial \Gamma} \\ & \frac{\partial \mathcal{L}}{\partial \Gamma} = (\mu_{\rho} - r_2)\phi T - \eta\phi^2 T^2 \sigma_{\rho}^2 K + (\lambda_5 - \lambda_6)/\Omega = 0. \end{split}$$

A 4.2 Formal discussion of the cases where $r_2T < r_1$ and $r_2T = r_1$

Proposition A4.1: Given our two basic assumptions $\mu_{\rho} > r_1/T$ and $\mu_{\rho} > r_2$, simultaneous interior solutions for both α and Γ are possible if and only if

$$r_2 T = r_1$$

Proof: Considering [KT.3, ..., KT.6], simultaneous interior solutions for both α and Γ , (i.e., the simultaneous validity of $0 < \alpha < \overline{\alpha}$ and $0 < \Gamma < \overline{\Gamma}$) necessitate that the corresponding condition $\lambda_3 = \lambda_4 = \lambda_5 = \lambda_6 = 0$ holds. Thereby, the reduced condition (19) can only be valid in one very particular case, namely if $r_2 - r_1/T = 0$ holds.

In this particular case, the decision maker assigns no definite preference hierarchy to the two focal options. In other words, she is indifferent between investing her personal wealth into the firm or calling for corporate debt and personally investing into the risk-free asset. Analogous to the normal case, the intuition behind this observation is perfectly evident, since with $r_2T = r_1$, the effective costs of debt do exactly outweigh D's opportunity costs of personally investing into the firm. These opportunity costs are given by the return of the risk-free asset.

Proposition A4.2: For $r_2T < r_1$, optimal solutions for α and Γ must fulfill

i)	$\alpha^* = \overline{\alpha}$	٨	$0 < \Gamma^* < \Gamma$	or
ii)	$\alpha^* = \overline{\alpha}$	٨	$\Gamma^* = \overline{\Gamma}$	or
iii)	$0 < \alpha^* < \overline{\alpha}$	٨	$\Gamma^* = \overline{\Gamma},$	or
iv)	$\alpha^* = 0$	٨	$\Gamma^* = \overline{\Gamma}.$	

Proof: Analogous to the proof of Lemma 4.1, the four cases are proved consecutively.

- i) From KT.5 and KT.6, $0 < \Gamma < \overline{\Gamma}$ implies $\lambda_5 = \lambda_6 = 0$. Hence, the reduced condition (19) can only hold if $\lambda_3 = 0$ and $\lambda_4 = -(r_2 r_1/T)\Omega W_0^D T > 0$. From KT.4, a positive value of λ_4 necessitates $\alpha = \overline{\alpha}$, which proves the first part.
- ii) From KT.3 and KT.4, $\alpha = \overline{\alpha}$ implies $\lambda_3 = 0$ and $\lambda_4 \ge 0$. Hence, the reduced condition (19) can only hold in two cases, namely if either
 - $\lambda_4 = -(r_2 r_1/T)\Omega W_0^D T > 0$ and $\lambda_5 = \lambda_6 = 0$ (which constitutes the case covered by the first part), or if
 - $\lambda_4 + \lambda_6 W_0^D / \phi = -(r_2 r_1 / T) \Omega W_0^D T > 0$ and $\lambda_5 = 0$ (which, from KT.6, implies $\Gamma = \overline{\Gamma}$, thus proving the second part).
- iii) From KT.3 and KT.4, $0 < \alpha < \overline{\alpha}$ implies $\lambda_3 = \lambda_4 = 0$. Hence, the reduced condition (19) can only hold if $\lambda_5 = 0$ and $\lambda_6 = -(r_2 r_1/T)\Omega\phi T > 0$. From KT.6, a positive value of λ_6 necessitates $\Gamma = \overline{\Gamma}$, which proves the third part.
- iv) From KT.3 and KT.4, $\alpha=0$ implies $\lambda_3 \geq 0$ and $\lambda_4=0$. Hence, the reduced condition (19) can only hold if $\lambda_3 \lambda_6 W_0^D / \phi = (r_2 r_1/T) \Omega W_0^D T < 0$. Since $(r_2 r_1/T) \Omega W_0^D T < 0$, λ_6 has to be positive, which, from KT.6, implies $\Gamma=\overline{\Gamma}$. This proves the last part.

Proposition A4.3: Suppose $r_2T < r_1$. In the optimum, the decision maker firstly calls for corporate debt before investing her personal wealth into the firm.

Proof: Proposition A4.3 immediately follows from Proposition A4.2.

For $r_2T < r_1$, D only invests into the firm if corporate debt is maximized (non-maximal α^* necessitates $\Gamma^* = \overline{\Gamma}$). As long as the corporate debt capacity is not attained, D will abstain from investing her personal wealth into the firm (non-maximal Γ^* necessitates $\alpha^* = \overline{\alpha}$). Hence, the decision maker firstly calls for corporate debt before investing her personal wealth into the firm (which is only done if the debt capacity (i.e., the maximum possible amount of debt) is not sufficient to meet her corporate investment objectives).

It is easy to see that, for $r_2T < r_1$, the exact reverse hierarchy compared to the normal case (as conveyed by Proposition 4.1) is adopted. Since the effective costs of debt are *lower* than D's opportunity costs of personally investing into the firm (the return of the risk-free asset is relatively high), debt takes precedence over D's personal wealth to feed the firm's stock of capital. In fact, the disposable amount of debt is sufficient to meet her corporate investment objective in cases *i*) and *ii*), contrary to cases *iii*) and *iv*), where the resulting "financing gap" is (as good as possible) filled by investing part her personal wealth. Hence, the analysis of the reverse case to our normal case indeed follows in a symmetric manner.

CHAPTER FIVE THE ROLE OF FIXED COSTS

5.1 Preface

So far, a heavily simplified profit function has been assumed where neither the demand nor the cost structure is made explicit. In particular, the adopted profit function implicitly supposes that there are no operative fixed costs. The results show that the leverage effect of fixed interest payments on debt heavily influences the adopted investing and financing behavior of the decision maker. Hence, it can be suspected that fixed payments that are not financial but operative may play an important role that has hitherto been neglected in the analysis.

5.2 Analysis

The purpose of this chapter is to highlight the role of fixed costs for the decision maker's financial structure decision. In order to accurately isolate the pure effect, we still abstract from the demand properties and the according pricing decision. Thereby, the analysis can be conducted without either facing the additional complexity of a fourth decision variable (the price) or crafting a very narrow setting where the market price is exogenously fixed. A more sophisticated definition of the firm's profit as a function of costs, demand, and price is provided in the next chapter.

Facing the same utility function and certainty equivalent as disclosed by (13) and (14), the Lagrangean of D's optimization problem remains unchanged and still displays as

$$\mathcal{L} = u(CE) + v(\phi) + \lambda_1(\phi - \phi) + \lambda_2(1 - \phi) + \lambda_3\alpha + \lambda_4(\overline{\alpha} - \alpha) + \lambda_5\Gamma + \lambda_6(\overline{\Gamma} - \Gamma)$$

with the corresponding set of Kuhn-Tucker conditions

$\lambda_1(\phi - \underline{\phi}) = 0,$	(KT.01)
$\lambda_2(1-\phi) = 0,$	(KT.02)
$\lambda_3 \alpha = 0,$	(KT.03)
$\lambda_4(\overline{\alpha} - \alpha) = 0,$	(KT.04)
$\lambda_5 \Gamma = 0,$	(KT.05)
$\lambda_6(\overline{\Gamma}-\Gamma) = 0,$	(KT.06)
$(\phi - \underline{\phi}) \ge 0,$	(KT.07)
$(1-\phi) \geq 0,$	(KT.08)
$\alpha \geq 0,$	(KT.09)
$(\overline{\alpha} - \alpha) \geq 0,$	(KT.10)
$\Gamma \geq 0,$	(KT.11)
$(\overline{\Gamma} - \Gamma) \geq 0,$	(KT.12)
$\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6 \geq 0.$	(NNC)

Given that the above Lagrangean is unchanged compared to the previous chapter, the general appearance of the three FOCs likewise remains identical, i.e.,

$$\begin{split} \frac{\partial \mathcal{L}}{\partial \phi} &= \Omega(\partial \mu_{W_1^D} / \partial \phi - \eta \sigma_{W_1^D} (\partial \sigma_{W_1^D} / \partial \phi)) + v'(\phi) + \lambda_1 - \lambda_2 = 0, \\ \frac{\partial \mathcal{L}}{\partial \alpha} &= \Omega(\partial \mu_{W_1^D} / \partial \alpha - \eta \sigma_{W_1^D} (\partial \sigma_{W_1^D} / \partial \alpha)) + \lambda_3 - \lambda_4 = 0, \\ \frac{\partial \mathcal{L}}{\partial \Gamma} &= \Omega(\partial \mu_{W_1^D} / \partial \Gamma - \eta \sigma_{W_1^D} (\partial \sigma_{W_1^D} / \partial \Gamma)) + \lambda_5 - \lambda_6 = 0. \end{split}$$

5.2.1 The Profit Function

As indicated above, the firm's profit function still merely depends on K, but now fixed costs have to be considered, i.e.,

$$\Pi = (1+\rho)K - F.$$

 Π is a Gaussian random variable with $\mu_{\Pi} = (1+\mu_{\rho})K - F$ and $\sigma_{\Pi} = \sigma_{\rho}K$. By substituting both into (10), firm profit displays as

(25)
$$\Pi = \mu_{\Pi} + \sigma_{\Pi} z = (1 + \mu_{\rho}) K - F + (\sigma_{\rho} K) z \,.$$

Similar to the preceding chapter, we can now solve the differential terms in the three FOCs. Considering the new profit function, we now obtain⁹⁵

0,

$$\begin{split} & \frac{\text{FOC.1:}}{\partial \phi} = (\mu_{\rho} - r_2)\Gamma T - FT - \eta \Gamma \phi K(\sigma_{\rho} T)^2 + (v'(\phi) + \lambda_1 - \lambda_2)/\Omega = \\ & \frac{\text{FOC.2:}}{\partial \alpha} \\ & \frac{\partial \mathcal{L}}{\partial \alpha} = -(\mu_{\rho} - r_1/T)W_0^D T - \eta W_0^D \phi K(\sigma_{\rho} T)^2 + (\lambda_3 - \lambda_4)/\Omega = 0, \\ & \frac{\text{FOC.3:}}{\partial \Gamma} \\ & \frac{\partial \mathcal{L}}{\partial \Gamma} = (\mu_{\rho} - r_2)\phi T - \eta K(\sigma_{\rho} \phi T)^2 + (\lambda_5 - \lambda_6)/\Omega = 0. \end{split}$$

5.2.2 The Decision Maker's Preference Structure of Financing

Similar to the preceding chapter, this subsection aims at examining the properties of the decision maker's preference structure regarding her disposable funding sources by identifying the possible values of (ϕ, α, Γ) in the optimum. We continue supposing $r_2T > r_1$ to be the normal case, hence the effective costs of debt are higher than D's opportunity costs of personally investing into the firm.

Comparing the three FOCs with the last chapter where fixed costs where neglected, only FOC.1 has changed and comprises the additional term -FT. Hence, since FOC.2 and FOC.3 remain unchanged, the presence of fixed costs

 $^{^{95}}$ See the appendix to this chapter (A 5.1) for the detailed calculus.

leaves unaffected the decision maker's arbitration between α and Γ (which is governed by FOC.2 and FOC.3). The reduced condition (19) still holds, and the results outlined by Lemma 4.1 and Proposition 4.1 (Chapter 4) remain valid, i.e., D firstly invests her personal wealth into the firm before calling for corporate debt. As before, this is understandable, since with $r_2T > r_1$, the effective costs of debt are *higher* than D's opportunity costs of personally investing her private wealth into the firm.

However, the decision maker's behavior concerning the raising of outside equity (which is governed by the changed FOC.1) is indeed affected. In order to subsequently examine this decision, we again delineate the possible values of ϕ in the optimum. Multiplying FOC.1 by ϕ and FOC.3 by Γ and then subsequently subtracting these new expressions and dividing by T yields the reduced condition

(26)
$$-F + \phi(v'(\phi) + \lambda_1 - \lambda_2)/T - \Gamma(\lambda_5 - \lambda_6)/T = 0.$$

By means of the reduced condition (26) (for which both ϕ and Γ are optimized), the Proposition 5.1 is presented.

Proposition 5.1: Suppose $\mu_{\rho}T > r_2T > r_1$. In the optimum, there exists a solution for ϕ , which satisfies the following properties:

i)	$0 < \alpha^* < \overline{\alpha}$	٨	$\Gamma^* = 0$	٨	$\underline{\phi} \le \phi^* \le \frac{1}{2},$	or
ii)	$\alpha^* = 0$	Λ	$\Gamma^* = 0$	٨	$\underline{\phi} \leq \phi^* \leq \frac{1}{2},$	or
iii)	$\alpha^* = 0$	٨	$0 < \Gamma^* < \overline{\Gamma}$	٨	$\underline{\phi} \leq \phi^* \leq \frac{1}{2},$	or
iv)	$\alpha^* = 0$	٨	$\Gamma^* = \overline{\Gamma}$	٨	$\underline{\phi} \le \phi^* \le 1.$	

Proof: Similar to the proof of Proposition 4.2, the feasible solutions of ϕ are deduced from the four cases as delineated by Lemma 4.1 in Chapter 4.

- i) + ii) For $\Gamma=0$, the condition (26) simplifies to $-F+(v'(\phi) + \lambda_1 \lambda_2)/T = 0$. From KT.2, this expression can only hold if $v'(\phi) > 0$ and $\lambda_2 = 0$. Hence, ϕ can only take the unique value that satisfies $(v'(\phi) + \lambda_1)/T = F$. This unique value of ϕ must ensure that the expression $(v'(\phi) + \lambda_1)$ is positive, which, from KT.1 and from the properties of $v(\phi)$, implies $\phi \leq \phi \leq \frac{1}{2}$.
- iii) For $0 < \Gamma < \overline{\Gamma}$, KT.5 and KT.6 imply $\lambda_5 = \lambda_6 = 0$. Hence, the reduced condition (26) simplifies to $-F + (v'(\phi) + \lambda_1 \lambda_2)/T = 0$. From KT.2, this expression can only hold if $v'(\phi) > 0$ and $\lambda_2 = 0$. Thereby, ϕ can only take the unique value that satisfies $(v'(\phi) + \lambda_1)/T = F$, which, from KT.1 and from the properties of $v(\phi)$, implies $\phi \le \phi \le \frac{1}{2}$.
- iv) For $\Gamma = \overline{\Gamma}$, KT.5 and KT.6 imply $\lambda_5 = 0$ and $\lambda_6 \ge 0$. Hence, (26) simplifies to $-F + (v'(\phi) + \lambda_1 \lambda_2)/T + \Gamma \lambda_6/T = 0$. With a possibly positive λ_6 , the third term on the LHS of this expression ensures that the solution value of ϕ can be both, lower or higher than $\frac{1}{2}$, hence $\phi \le \phi \le 1$.

It needs to be stressed that, unlike in the previous chapter, ϕ^* now is perfectly identical throughout the first three cases, i.e., ϕ^* always satisfies $(v'(\phi^*) + \lambda_1)/T = F$. Since this solution is strictly unique, Proposition 5.1 reveals that, in the presence of fixed costs, the decision maker's indifference regarding the amount of outside equity brought in by external investors in cases *i*) and *ii*) totally disappears. Moreover, the feasible domain of ϕ^* is expanded below $\frac{1}{2}$. These deviations are tightly interwoven with each other and result from a basic trade-off that the decision maker faces. This trade-off is rendered understandable by means of Proposition 5.2, which will be presented after the below-stated Lemma 5.1.

Lemma 5.1: If fixed costs are

($low (F < \Gamma(\mu_{\rho} - r_2)) $		then	$\left(\partial \mu_{W_1^D} / \partial \phi > 0 \right)$
ł	balanced ($F = \Gamma(\mu_{\rho} - r_2)$)	,	then ·	$\left\{ \partial \mu_{W_{1}^{D}}^{D} / \partial \phi = 0 \right\}.$
(high $(F > \Gamma(\mu_{\rho} - r_2))$		then	$\left(\frac{\partial \mu_{W_1^D}}{\partial \phi} < 0 \right)$

Proof: From Appendix A 5.1, we know that $\partial \mu_{W_1^D} / \partial \phi = (\mu_{\Pi} - (1+r_2)\Gamma - E(\partial \mu_{\Pi} / \partial K))T$. From $\mu_{\Pi} = (1+\mu_o)K - F$, we derive $\partial \mu_{\Pi} / \partial K = 1+\mu_o$. Hence, we obtain

The RHS of this equation is positive (negative, zero) if F is smaller than (greater than, equal to) $\Gamma(\mu_o - r_2)$, which proves the statement.

Lemma 5.1 paves the way for Proposition 5.2.

Proposition 5.2: Consider cases i) and ii), where $\Gamma^* = 0$. An additional unit of outside equity then strictly increases the decision maker's expected income.

Proof: With $\Gamma^*=0$, Lemma 5.1 shows that a positive F implies $\partial \mu_{W_1^D} / \partial \phi < 0$. Recalling that $\phi = W_0^D / (E+\Gamma)$, we know that ϕ strictly decreases in the amount of outside equity, Hence, a negative $\partial \mu_{W_1^D} / \partial \phi$ implies that D's marginal personal return of outside equity $\partial \mu_{W_2^D} / \partial E^{ex}$ is indeed positive, which proves the claim.

As we can now see, the reason for the observed deviation from the previous chapter's indifference result (as indicated by Proposition 4.2) are indeed the fixed costs, since, with a positive F, D's marginal personal return of outside equity is positive as long as $F > \Gamma(\mu_{\rho} - r_2)$. Hence, increasing the amount of outside equity (by reducing ϕ) induces a positive wealth effect. With a "traditional" model approach, i.e., if no utility of corporate control had been considered, this operating leverage effect would yield a corner solution where ownership concentration is minimized by incurring the maximum amount of outside equity $(\phi^* = \phi)$. However, by considering the power-related utility component $v(\phi)$, reducing ϕ induces a negative power sharing effect on the decision maker's utility function. In the optimum, D trades off the marginal utility gain of the wealth effect of outside equity against the marginal utility loss of its power sharing effect to attain a unique interior solution ϕ^* for her optimal share of the firm's total equity.

From these considerations, a closer inspection of the optimality case iv) shall be conducted by means of Proposition 5.3.

Proposition 5.3: Consider case iv), where $\Gamma^* = \overline{\Gamma}$ and $\phi \leq \phi^* \leq 1$. If

- $\begin{array}{ll} \bullet & \overline{\Gamma} < F/(\mu_{\rho} r_2), & \mbox{then} & \underline{\phi} \le \phi^* \le \frac{1}{2} ; \\ \bullet & \overline{\Gamma} > F/(\mu_{\rho} r_2), & \mbox{then} & \phi^* = 1; \\ \bullet & \overline{\Gamma} = F/(\mu_{\rho} r_2), & \mbox{then} & \frac{1}{2} < \phi^* \le 1. \end{array}$

Proof: We proceed with the proof by focusing the outlined three cases. From Proposition 5.1, we know that case iv) implies $\Gamma^* = \overline{\Gamma}$. From Lemma 5.1, we further know that $\partial \mu_{W_1^D} / \partial \phi = ((\mu_o - r_2)\Gamma - F)T.$

- For $\overline{\Gamma} < F/(\mu_{\rho} r_2)$, $\mu_{W_1^D}(\phi)$ strictly decreases in ϕ (and has a maximum at $\phi = \phi$). The power curve $\mathcal{C}(\phi)$ is strictly concave and increasing in ϕ for $\phi \leq \phi \leq \frac{1}{2}$, and flat for $\frac{1}{2} < \phi \leq 1$. Hence, the aggregate curve $\mu_{W^{D}}(\phi) + \mathcal{C}(\phi)$ has a unique maximum in the interval $(\frac{1}{2},1]$ (see Figure 10).
- For $\overline{\Gamma} > F/(\mu_{o} r_{2})$, $\mu_{W_{c}^{D}}(\phi)$ has a maximum at $\phi = 1$. The power curve $\mathcal{C}(\phi)$ has a maximum throughout the whole interval (1/2, 1]. Hence, the aggregate curve $\mu_{W_{v}}(\phi) + \mathcal{C}(\phi)$ has a unique maximum at $\phi = 1$ (see Figure 12).
- For $\overline{\Gamma} = F/(\mu_{\rho} r_2)$, $\mu_{W_1^D}(\phi)$ is flat. The power curve $\mathcal{C}(\phi)$ has a maximum throughout the whole interval (1/2, 1]. Hence, the aggregate curve $\mu_{W^{\rm P}_{\rm c}}(\phi) + {\cal C}(\phi)$ is also maximized throughout the whole interval (1/2, 1] (see Figure 11).

Figure 10 gives a graphical representation of this trade-off and discloses ϕ^* as the point where the decision maker's aggregate benefit from wealth and control is maximized.



Figure 10: Trade-off wealth vs. power sharing for $\overline{\Gamma} < F/(\mu_o - r_2)$.

D's personal marginal return of outside equity decreases in Γ and increases in F (since $\partial \mu_{W_1^D} / \partial \phi$ increases in Γ and decreases in F). With regard to Figure 10, this means that a higher debt capacity ($\overline{\Gamma} \uparrow$) rotates the wealth curve (since the slope $\partial \mu_{W_1^D} / \partial \phi$ becomes less negative, i.e., tends towards zero). Consequently, the solution ϕ^* (i.e., the maximum of the aggregate curve) is gradually shifted to the right, i.e., the optimal amount of outside equity is gradually reduced.

As soon as the wealth curve becomes horizontal (if the available amount of debt is high enough to guarantee that $\partial \mu_{W_1^D} / \partial \phi = 0$), the indifference result concerning ϕ^* is restored. This is depicted in Figure 11.



Figure 11: Trade-off wealth vs. power sharing for $\overline{\Gamma} = F/(\mu_{\rho} - r_2)$.

It is worth examining the limiting case of no fixed costs (Chapter 4) by means of the above figure. If F=0, the wealth curve is a constant of value zero, and the aggregate curve is equal to the power curve. Consequently, it is also maximized throughout the whole interval (1/2, 1], and the decision maker is completely indifferent concerning ϕ^* in this interval – exactly as shown in Chapter 4.

If the available debt capacity $\overline{\Gamma}$ gets even higher, the slope of the wealth curve becomes positive and the optimal solution of ϕ jumps to 1, which is now the unique maximum of the aggregate curve. This is depicted in Figure 12.



Figure 12: Trade-off wealth vs. power sharing for $\overline{\Gamma} > F/(\mu_{\rho} - r_2)$.

5.2.3 The Decision Maker's Investment Objectives

Conform to the previous chapter, we now examine D's actual corporate investment objectives, i.e., her target levels of corporate investment.

Cases i) and ii)

We begin by treating the first two cases, where $\Gamma^*=0$. As shown in these parts of Lemma 4.1 (Chapter 4), W_0^D is sufficiently large to meet *D*'s corporate investment objectives without calling for any corporate debt. The proof of the first two cases in Lemma 4.1 has further disclosed that with

 $i) \quad \lambda_5 = (r_2 - r_1/T)\Omega\phi T \qquad \qquad \wedge \qquad \lambda_3 = \lambda_4 = \lambda_6 = 0, \text{ or with}$

ii)
$$\lambda_3 + \lambda_5 W_0^D / \phi = (\mathbf{r}_2 - \mathbf{r}_1 / T) \Omega W_0^D T \quad \wedge \quad \lambda_4 = \lambda_6 = 0,$$

FOC.2 and FOC.3 simultaneously hold. Thus, in both cases, we are facing a system of only two equations.

Scrutinizing case i), where $0 < \alpha^* < \overline{\alpha}$ and $\Gamma^* = 0$, we have

$$\frac{\text{FOC.1:}}{-F + (v'(\phi^*) + \lambda_1 - \lambda_2)/\Omega T = 0, \text{ and}}$$

$$\frac{\text{FOC.2} \equiv \text{FOC.3:}}{-(\mu_{\rho} - r_1/T) W_0^D T - \eta W_0^D \phi^* T^2 \sigma_{\rho}^2 K = 0.}$$
With $\phi^* K = (1 - \alpha^*) W_0^D + \phi^* \Gamma^*$ and $\Gamma^* = 0$, FOC.2/3 displays as
$$(27) \quad (1 - \alpha^*) W_0^D = (\mu_{\rho} - r_1/T)/\eta T \sigma_{\rho}^2,$$

which discloses the optimal amount of D's personal wealth invested into the firm (LHS of the equation) and her maximum target level of corporate investment (RHS of the equation). D's initial wealth W_0^D is largely sufficient to reach this maximum target and allows for an investment of the excess amount $\alpha^* W_0^D$ into the riskless alternative.

Scrutinizing case *ii*), where $\alpha^* = \Gamma^* = 0$, we have

$$\frac{\text{FOC.1:}}{-F + (v'(\phi) + \lambda_1 - \lambda_2)/\Omega T} = 0,$$

$$\frac{\text{FOC.2} \equiv \text{FOC.3:}}{-(\mu_{\rho} - r_1/T)W_0^D T - \eta W_0^D \phi^* T^2 \sigma_{\rho}^2 K - \lambda_3/\Omega} = 0.$$
With $\phi^* K = (1 - \alpha^*)W_0^D + \phi^* \Gamma^*$ and $\alpha^* = \Gamma^* = 0$, FOC.2/3 displays as
$$(28) \qquad W_0^D = (\mu_{\rho} - r_1/T)/\eta T \sigma_{\rho}^2 - \lambda_3/\Omega \eta W_0^D T \sigma_{\rho}^2,$$

which discloses that D does not reach her target level, despite investing her full wealth into the firm in order to approach the maximum target as near as possible. The remaining difference between the maximum target level and D's wealth endowment is given by $\lambda_3/\Omega W_0^D \eta T^2 g^2 \sigma_Q^2$, which evidently decreases if W_0^D increases to approach the maximum target level from below. In both case *i*) and *ii*), FOC.1 displays as $-F+(v'(\phi^*)+\lambda_1-\lambda_2)/\Omega T=0$, which, as demonstrated by Proposition 5.1, implies

$$\phi \le \phi^* \le \frac{1}{2}$$

for both cases. Hence, the decision maker raises outside equity until her utility gain from the corresponding wealth effect is exactly outweighed by her utility loss from the corresponding power sharing effect.

Cases iii) and iv)

We now treat the last two cases, where $\Gamma^* > 0$. As shown in these parts of Lemma 4.1, W_0^D is *not* sufficiently large to meet D's corporate investment objectives without calling for additional debt. The proof of the last two cases in Lemma 4.1 has further disclosed that with

 $\begin{array}{lll} iii) \ \lambda_3 = (r_2 - r_1/T)\Omega W_0^D & \wedge & \lambda_4 = \lambda_5 = \lambda_6 = 0, & \text{or with} \\ iv) \ \lambda_3 - \lambda_6 W_0^D / \phi T = (r_2 - r_1/T)\Omega W_0^D & \wedge & \lambda_4 = \lambda_5 = 0, \end{array}$

FOC.2 and FOC.3 simultaneously hold. Thus, we are again facing a system of only two equations.

Scrutinizing case *iii*), where $\alpha^* = 0$ and $\overline{\Gamma} > \Gamma^* > 0$, we have

$$\begin{split} & \underline{\text{FOC.1:}} \\ & -FT + (\mu_{\rho} - r_2)\Gamma^*T - \eta\Gamma^*\phi^*T^2\sigma_{\rho}^2K + (v'(\phi^*) + \lambda_1 - \lambda_2)/\Omega = 0 \\ & \underline{\text{FOC.2} \equiv \text{FOC.3:}} \\ & (\mu_{\rho} - r_2)\phi^*T - \eta\phi^{*2}T^2\sigma_{\rho}^2K = 0. \end{split}$$

Knowing that $\phi^* K = (1 - \alpha^*) W_0^D + \phi^* \Gamma^*$ and $\alpha^* = 0$, FOC.2/3 displays as

(29)
$$W_0^D + \phi^* \Gamma^* = (\mu_\rho - r_2)/\eta T \sigma_\rho^2,$$

which, considering that W_0^D is fully invested into the firm, discloses the incurred level of corporate debt (Γ) in order to reach the *minimum target level* of corporate investment (RHS of the equation). Substituting FOC.2/3 into FOC.1 yields the well-known expression $-F + (v'(\phi^*) + \lambda_1 - \lambda_2)/\Omega T = 0$, which implies that the unique solution value of ϕ must satisfy

$$\phi \le \phi^* \le \frac{1}{2}.$$

Contrary to the previous simplified analysis, the presence of fixed costs ensures that there now is a *unique* pair of values (ϕ^*, Γ^*) which satisfies the two optimality conditions. Similar to the former results, the financing gap is accordingly filled by $\phi^*\Gamma^*$. Hence, as long as the debt capacity $\overline{\Gamma}$ is sufficiently great to guarantee $\overline{\Gamma} > \Gamma^* > 0$, the disclosed result holds.

Scrutinizing case *iv*), where $\alpha^* = 0$ and $\Gamma^* = \overline{\Gamma}$, we have

FOC.1:

$$-FT + (\mu_{\rho} - \mathbf{r}_2)\Gamma^*T - \eta\Gamma^*\phi^*T^2\sigma_{\rho}^2K + (v'(\phi^*) + \lambda_1 - \lambda_2)/\Omega = 0,$$

 $\underline{FOC.2} \equiv FOC.3$:

 $(\mu_{\rho} - r_2)\phi^* T - \eta \phi^{*2} T^2 \sigma_{\rho}^2 K - \lambda_6 / \Omega = 0.$

Knowing that $\phi^* K = (1-\alpha) W_0^D + \phi^* \Gamma^*$, $\alpha^* = 0$, and $\Gamma^* = \overline{\Gamma}$, FOC.2/3 displays as

(30)
$$W_0^D + \phi^* \overline{\Gamma} = (\mu_\rho - r_2) / \eta T \sigma_\rho^2 - \lambda_6 / \Omega \phi^* \eta T^2 \sigma_\rho^2.$$

which discloses that D invests her full wealth into the firm and uses the full debt capacity in order to either reach her minimum target or to approach it as near as possible. However, for $\Gamma^* = \overline{\Gamma}$, Proposition 5.1 has shown that ϕ^* can be both lower or higher than $\frac{1}{2}$ in the optimum. It has further been shown that the value of ϕ^* crucially depends on the relative size of the debt capacity compared to the fixed costs. If the debt capacity is relatively low compared to the fixed costs (i.e., if $\overline{\Gamma} < F/(\mu_{\rho} - r_2)$ holds), the decision maker raises considerable amounts of outside equity ($\phi \le \phi^* \le \frac{1}{2}$) in order to fill the financing gap and to balance her utility gains from its corresponding positive wealth effect against her utility losses from its corresponding negative power sharing effect (see Figure 10). If the debt capacity is relatively high compared to the fixed costs ($\overline{\Gamma} > F/(\mu_{\rho} - r_2)$), she raises no outside equity ($\phi^* = 1$) in order to avoid its negative wealth effect (see Figure 12). Values of ϕ^* between $\frac{1}{2}$ and 1 are only possible in the very special case where $\overline{\Gamma} = F/(\mu_{\rho} - r_2)$ holds (see Figure 11).
Generally encompassing all four basic cases, the decision maker's target levels (as given by the right-hand sides of the equations (27) and (29)) are increasing in μ_{ρ} and decreasing in both σ_Q^2 and η . Hence, the comparative statics results as derived from the simplified profit function still hold when assuming fixed costs. The decision maker's investment objectives still become *more ambitious* with improving average environmental states, decreasing demand volatility, and decreasing risk aversion.

5.2.4 The summarized Pattern of Investing and Financing

As has been shown, in the presence of fixed costs the minimum and maximum target level of corporate investment are still given by

$$\begin{split} \mathrm{TL}^{\mathrm{Max}} &= (\mu_{\rho} - r_{1}/T)/\eta T \sigma_{\rho}^{2} , \qquad \text{and} \\ \mathrm{TL}^{\mathrm{Min}} &= (\mu_{\rho} - r_{2})/\eta T \sigma_{\rho}^{2} . \end{split}$$

There are again four possible cases.

$$\begin{split} i) \quad \text{if} \qquad W_0^D > \mathrm{TL}^{\mathrm{Max}}, \\ \Rightarrow \qquad \underline{\phi} \leq \phi^* \leq \frac{1}{2}, \, \text{satisfying } -F + (v'(\phi^*) + \lambda_1 - \lambda_2) / \Omega T = 0, \\ \alpha^* > 0, \, \text{satisfying } (1 - \alpha^*) W_0^D = \mathrm{TL}^{\mathrm{Max}}, \\ \Gamma^* = 0, \\ K^* = \mathrm{TL}^{\mathrm{Max}} / \phi^*. \end{split}$$

D's private wealth is sufficiently large to meet her corporate investment objectives, i.e., it exceeds her maximum target level of corporate investment. Hence, she incurs no corporate debt and divides her personal investment between the firm's stock of capital and the riskless alternative. The decision maker raises outside equity in order to balance her utility gains from the corresponding wealth effect against her utility losses from the corresponding power sharing effect.

$$\begin{split} ii) \quad & \text{if} \qquad \text{TL}^{\text{Min}} < W_0^D \leq \text{TL}^{\text{Max}}, \\ \Rightarrow \quad & \underline{\phi} \leq \phi^* \leq \frac{1}{2}, \text{ satisfying } -F + (v'(\phi^*) + \lambda_1 - \lambda_2) / \Omega T = 0, \\ & \alpha^* = 0, \end{split}$$

$$\begin{split} &\Gamma^* = 0 \,, \\ &K^* = (\mathrm{TL}^{\mathrm{Max}} - \lambda_3 / \Omega \eta W_0^D T \sigma_\rho^2) / \phi^* \,. \end{split}$$

D's private wealth is sufficiently large to meet her minimum target level of corporate investment, but smaller than her maximum target level. Hence, she incurs no corporate debt and invests her full personal wealth into the firm in order to reach the best possible rapprochement towards her maximum target level. The decision maker raises outside equity in order to balance her utility gains from the corresponding wealth effect against her utility losses from the corresponding power sharing effect.

$$\begin{array}{ll} iii) \mbox{ if } & W_0^D \leq {\rm TL}^{\rm Min} \leq W_0^D + \overline{\Gamma} \ , \\ \Rightarrow & \underline{\phi} \leq \phi^* \leq \frac{1}{2}, \mbox{ satisfying } -F + (v'(\phi^*) + \lambda_1 - \lambda_2) / \Omega T = 0, \\ & \alpha^* = 0, \\ & \overline{\Gamma} \geq \Gamma^* = ({\rm TL}^{\rm Min} - W_0^D) / \phi^*, \\ & K^* = {\rm TL}^{\rm Min} / \phi^*. \end{array}$$

D's private wealth is too small to meet her minimum target level of corporate investment. Hence, she invests her full personal wealth into the firm's stock of capital and incurs supplementary corporate debt in order to reach this minimum target level (filling the financing gap). The decision maker raises outside equity in order to balance her utility gains from the corresponding wealth effect against her utility losses from the corresponding power sharing effect.

$$\begin{split} & iv) \quad \mathrm{if} \qquad W_0^D + \overline{\Gamma} < \mathrm{TL}^{\mathrm{Min}}, \\ & \Rightarrow \qquad \underline{\phi} \leq \phi^* \leq \frac{1}{2} \quad \mathrm{if} \qquad \overline{\Gamma} < F/(\mu_\rho - r_2), \\ & \phi^* = 1 \qquad \mathrm{if} \qquad \overline{\Gamma} > F/(\mu_\rho - r_2), \\ & \frac{1}{2} < \phi^* \leq 1 \quad \mathrm{if} \qquad \overline{\Gamma} = F/(\mu_\rho - r_2), \\ & \mathrm{satisfying} - F + (\mu_\rho - r_2)\Gamma^* - \eta\Gamma^*\phi^* T\sigma_\rho^2 K + (v'(\phi^*) + \lambda_1 - \lambda_2)/\Omega T = 0. \\ & \alpha^* = 0, \\ & \Gamma^* = \overline{\Gamma}, \\ & K^* = (\mathrm{TL}^{\mathrm{Min}} - \lambda_6/\Omega\phi^*\eta T^2\sigma_\rho^2)/\phi^*. \end{split}$$

The sum of D's private wealth and the debt capacity is not sufficient to meet her minimum target level of corporate investment. Hence, the decision maker invests her full personal wealth into the firm's stock of capital and incurs the maximum amount of corporate debt in order to approach the minimum target as near as possible. If the debt capacity is relatively low compared to the fixed costs, D raises considerable amounts of outside equity ($\phi \leq \phi^* \leq \frac{1}{2}$) in order to balance her utility gains from its (correspondingly *positive*) wealth effect against her utility losses from its negative power sharing effect. If the debt capacity is relatively high compared to the fixed costs, she raises no outside equity ($\phi^* = 1$) in order to avoid its (correspondingly *negative*) wealth effect.

Similar to the previous chapter, Figure 13 depicts the properties of $(\phi^*, \alpha^*, \Gamma^*)$ throughout the four cases. It gives a graphical representation of the three decision variables' possible values in the optimum, contingent on the level of the decision maker's initial wealth W_0^D .



Figure 13: Properties of $(\phi^*, \alpha^*, \Gamma^*)$ in the presence of fixed costs.

The right solid vertical line which separates the cases *i*) and *ii*) represents the decision maker's maximum target level $\text{TL}^{\text{Max}} = (\mu_{\rho} - r_1/T)/\eta T \sigma_{\rho}^2$. For all values of W_0^D which exceed this level, *D*'s private wealth is sufficient to meet her corporate investment objectives. Hence, case *i*) holds. The central solid vertical line which separates the cases *ii*) and *iii*) represents the minimum target level $\text{TL}^{\text{Min}} = (\mu_{\rho} - r_2)/\eta T \sigma_{\rho}^2$. For all values of W_0^D which lie between both target levels, D's private wealth is only sufficient to meet her minimum, but not her maximum target level of corporate investment. Hence, case *ii*) holds.

The left solid vertical line which separates the cases *iii*) and *iv*) represents the wealth level $\text{TL}^{\text{Min}} - \overline{\Gamma}$, from where on the given debt capacity $\overline{\Gamma}$ is sufficient to close the financing gap. For all values of W_0^D which exceed this level, her wealth allows for a complete filling of the financing gap by taking on debt to satisfy Equation (29). Hence, case *iii*) holds. For all values of W_0^D which fall short of this level, D exhausts the full debt capacity in order to approach her minimum target level as good as possible. Hence, case *iv*) holds.

As a major deviation from our findings in the previous Chapter, the value of ϕ^* is now independent from the decision maker's initial wealth. Propositions 5.1 and 5.3, have disclosed that ϕ^* has an identical value somewhere between ϕ and $\frac{1}{2}$ in the cases *i*), *ii*), and *iii*), while it depends on the size of available debt capacity in case *iv*). This dependence is quite sophisticated and does not allow for a comprehensive depiction of ϕ^* in case *iv*) in the above Figure 13. It is therefore illustrated by means of the separate Figure 14.



Figure 14: Relationship between ϕ^* and $\overline{\Gamma}$ in case *iv*).

Figure 14 illustrates that ϕ^* gradually approaches $\frac{1}{2}$ as long as the debt capacity is relatively low (i.e., below $F/(\mu_{\rho} - r_2)$). For a higher debt capacity, ϕ^* is strictly equal to unity and D holds full ownership. The very special case where the debt capacity exactly equals $F/(\mu_{\rho} - r_2)$ induces an ambiguous value of ϕ^* between $\frac{1}{2}$ and unity.

5.3 Conclusion

In order to account for the effects of fixed charges besides the due interest payments on the corporate debt, this chapter introduced operative fixed costs into the profit function. The disclosed pattern of investing and financing is largely conform to the simplified scenario examined in the previous chapter. Again, case-contingent interior solutions were obtained so that marginal costs and marginal benefits are balanced.

The main deviation induced by the introduction of fixed costs is the fact that, except for the very special case $\overline{\Gamma} = F/(\mu_{\rho} - r_2)$, ϕ^* now has a strictly unique value throughout the four cases. The interior solution ϕ^* (where the positive wealth effect and the negative power sharing effect of outside equity are balanced) has a unique value somewhere in the domain $[\phi, \frac{1}{2}]$ for cases *i*), *ii*), and *iii*). For case *iv*), ϕ^* lies either in the interval $(\phi, \frac{1}{2})$ or is strictly equal to unity, contingent on whether the debt capacity is lower or higher than $F/(\mu_{\rho} - r_2)$, In the absence of fixed costs (F=0), the indifference result of the previous chapter is restored.

Compared to the previous chapter, the corporate investment objectives are not only contingent on the decision maker's subjective beliefs (via μ_{ρ} and σ_{ρ}^2), her risk sensitivity (via η), and exogenous institutional parameters (via r_1 , r_2 , and T), but also on the actual characteristics of her utility component $v(\phi)$ (since, $v'(\phi^*)$ can now become different from zero). The stronger the curvature of $v(\phi)$ (i.e, the lower $v''(\phi)$), the higher will be the ownership concentration and the lower will be the amount of outside equity in the firm's stock of capital.

Given that the decision maker's investment objectives remain substantially unchanged, the comparative statics results as deduced in Chapter 4 also remain valid. The general results of this chapter reinforce the claim that our findings differ from the POT's assertions in that they allow for an explanation of systematic incurring of outside equity besides or even before debt, which is largely supported by empirical observations. In this respect, the incremental contribution of the present chapter consists in the precise outline of the conditions under which outside equity is incurred (or not), and the disclosure of the nontrivial driving mechanism that underlies the resulting ownership concentration in the optimum.

Appendix to Chapter 5

A 5.1: Solving the differential terms in the three FOCs

The FOCs are given by

$$\begin{split} \frac{\partial \mathcal{L}}{\partial \phi} &= \Omega (\partial \mu_{W_1^D} / \partial \phi - \eta \sigma_{W_1^D} (\partial \sigma_{W_1^D} / \partial \phi)) + v'(\phi) + \lambda_1 - \lambda_2 = 0, \\ \frac{\partial \mathcal{L}}{\partial \alpha} &= \Omega (\partial \mu_{W_1^D} / \partial \alpha - \eta \sigma_{W_1^D} (\partial \sigma_{W_1^D} / \partial \alpha)) + \lambda_3 - \lambda_4 = 0, \\ \frac{\partial \mathcal{L}}{\partial \Gamma} &= \Omega (\partial \mu_{W_1^D} / \partial \Gamma - \eta \sigma_{W_1^D} (\partial \sigma_{W_1^D} / \partial \Gamma)) + \lambda_5 - \lambda_6 = 0. \end{split}$$

Since $\sigma_{W_I^D} = \phi T \sigma_{\Pi}$, $\sigma_{\Pi} \equiv \sigma_{\Pi}(K)$, and $K \equiv K(\phi, \alpha, \Gamma)$, we can rewrite FOC.1 as

$$\begin{split} \Omega(\partial\mu_{W_{1}^{D}}/\partial\phi - \eta(\phi T\sigma_{\Pi})(\partial\sigma_{W_{1}^{D}}/\partial\phi)) + v'(\phi) + \lambda_{1} - \lambda_{2} &= 0 \\ \Leftrightarrow \ \partial\mu_{W_{1}^{D}}/\partial\phi - \eta(\phi T\sigma_{\Pi})(T\sigma_{\Pi} + \phi T(\partial\sigma_{\Pi}/\partial K)(\partial K/\partial\phi)) + (v'(\phi) + \lambda_{1} - \lambda_{2})/\Omega &= 0 \\ \Leftrightarrow \ \partial\mu_{W_{1}^{D}}/\partial\phi - \eta\phi T^{2}\sigma_{\Pi}^{2} - \eta\phi^{2}T^{2}\sigma_{\Pi}(\partial\sigma_{\Pi}/\partial K)(\partial K/\partial\phi) + (v'(\phi) + \lambda_{1} - \lambda_{2})/\Omega &= 0 \end{split}$$

Treating FOC.2 and FOC.3 analogously yields

$$\begin{split} \Omega(\partial\mu_{W_{1}^{D}}/\partial\alpha - \eta(\phi T\sigma_{\Pi})(\partial\sigma_{W_{1}^{D}}/\partial\alpha)) + \lambda_{3} - \lambda_{4} &= 0 \\ \Leftrightarrow \ \partial\mu_{W_{1}^{D}}/\partial\alpha - \eta(\phi T\sigma_{\Pi})(\phi T(\partial\sigma_{\Pi}/\partial K)(\partial K/\partial\alpha)) + (\lambda_{3} - \lambda_{4})/\Omega &= 0 \\ \Leftrightarrow \ \partial\mu_{W_{1}^{D}}/\partial\alpha - \eta\phi^{2}T^{2}\sigma_{\Pi}(\partial\sigma_{\Pi}/\partial K)(\partial K/\partial\alpha) + (\lambda_{3} - \lambda_{4})/\Omega &= 0 \end{split}$$

$$\begin{split} \Omega(\partial\mu_{W_{1}^{D}}/\partial\Gamma - \eta(\phi T\sigma_{\Pi})(\partial\sigma_{W_{1}^{D}}/\partial\Gamma)) + \lambda_{5} - \lambda_{6} &= 0 \\ \Leftrightarrow \ \partial\mu_{W_{1}^{D}}/\partial\Gamma - \eta(\phi T\sigma_{\Pi})(\phi T(\partial\sigma_{\Pi}/\partial K)(\partial K/\partial\Gamma)) + (\lambda_{5} - \lambda_{6})/\Omega &= 0 \\ \Leftrightarrow \ \partial\mu_{W_{1}^{D}}/\partial\Gamma - \eta\phi^{2}T^{2}\sigma_{\Pi}(\partial\sigma_{\Pi}/\partial K)(\partial K/\partial\Gamma)) + (\lambda_{5} - \lambda_{6})/\Omega &= 0 \end{split}$$

Thereby, we now have

$$\frac{\partial \mathcal{L}}{\partial \phi} = \partial \mu_{W_1^D} / \partial \phi - \eta \phi T^2 \sigma_{\Pi}^2 - \eta \phi^2 T^2 \sigma_{\Pi} (\partial \sigma_{\Pi} / \partial K) (\partial K / \partial \phi) + (v'(\phi) + \lambda_1 - \lambda_2) / \Omega = 0,$$

$$\begin{split} \frac{\partial \mathcal{L}}{\partial \alpha} &= \partial \mu_{W_1^D} / \partial \alpha - \eta \phi^2 T^2 \sigma_{\Pi} (\partial \sigma_{\Pi} / \partial K) (\partial K / \partial \alpha) + (\lambda_3 - \lambda_4) / \Omega = 0, \\ \frac{\partial \mathcal{L}}{\partial \Gamma} &= \partial \mu_{W_1^D} / \partial \Gamma - \eta \phi^2 T^2 \sigma_{\Pi} (\partial \sigma_{\Pi} / \partial K) (\partial K / \partial \Gamma) + (\lambda_5 - \lambda_6) / \Omega = 0. \end{split}$$

To rewrite these three equations as explicit functions of the decision variables, we need to solve the differential terms. We tackle this problem by first computing the three derivatives $\partial \mu_{W_{T}^{D}} / \partial \phi$, $\partial \mu_{W_{T}^{D}} / \partial \alpha$, and $\partial \mu_{W_{T}^{D}} / \partial \Gamma$.

From (11), we have $\mu_{W_1^D} = W_0^D + \alpha W_0^D r_1 + (\phi(\mu_{\Pi} - (1+r_2)\Gamma) - (1-\alpha)W_0^D)T$ and $\sigma_{W_1^D} = \phi T \sigma_{\Pi}$. Hence,

From (6), we can infer $K = (1-\alpha)W_0^D + E^{ex} + \Gamma = E + \Gamma = (1-\alpha)W_0^D/\phi + \Gamma$. Hence,

Further recalling the functional forms of μ_{Π} and σ_{Π} as given by (25), i.e., $\mu_{\Pi} = (1 + \mu_{\rho})K - F$ and $\sigma_{\Pi} = \sigma_{\rho}K$, the three focal derivatives display as

$$\begin{split} \partial \mu_{W_1^D} / \partial \phi &= ((1+\mu_{\rho})K - F - (1+r_2)\Gamma + \phi(-E/\phi)(1+\mu_{\rho}))T \\ &= ((\mu_{\rho} - r_2)\Gamma - F + (1+\mu_{\rho})E - (1+\mu_{\rho})E)T \\ &= ((\mu_{\rho} - r_2)\Gamma - F)T. \\ \partial \mu_{W_1^D} / \partial \alpha &= (r_1 + T)W_0^D + (1+\mu_{\rho})(-W_0^D/\phi)\phiT \\ &= -(\mu_{\rho} - r_1/T)W_0^DT. \end{split}$$

$$\begin{split} \partial \mu_{W^D_1} / \partial \Gamma &= (-(1{+}r_2) + (1{+}\mu_\rho)) \phi T \\ &= (\mu_\rho {-}r_2) \phi T. \end{split}$$

Solving the remaining differential terms in the FOCs then yields

$$\begin{split} \frac{\partial \mathcal{L}}{\partial \phi} &= ((\mu_{\rho} - r_2)\Gamma - F)T - \eta\phi T^2(\sigma_{\rho}K)^2 - \eta\phi^2 T^2 \sigma_{\rho}^2 K(-E/\phi) + (v'(\phi) + \lambda_1 - \lambda_2)/\Omega = 0 \\ &= -FT + (\mu_{\rho} - r_2)\Gamma T - \eta\phi T^2(\sigma_{\rho}K)^2 + \eta\phi T^2 \sigma_{\rho}^2 KE) + (v'(\phi) + \lambda_1 - \lambda_2)/\Omega = 0 \\ &= -FT + (\mu_{\rho} - r_2)\Gamma T - \eta\phi T^2(\sigma_{\rho}K)^2 + \eta\phi T^2 \sigma_{\rho}^2 K(K-\Gamma)) + (v'(\phi) + \lambda_1 - \lambda_2)/\Omega = 0 \\ &= -FT + (\mu_{\rho} - r_2)\Gamma T - \eta\Gamma\phi T^2 \sigma_{\rho}^2 K + (v'(\phi) + \lambda_1 - \lambda_2)/\Omega = 0. \\ \\ \frac{\partial \mathcal{L}}{\partial \alpha} &= -(\mu_{\rho} - r_1/T)W_0^D T - \eta\phi^2 T^2 \sigma_{\rho}^2 K (-W_0^D/\phi) + (\lambda_3 - \lambda_4)/\Omega = 0 \\ &= -(\mu_{\rho} - r_1/T)W_0^D T + \eta W_0^D \phi T^2 \sigma_{\rho}^2 K + (\lambda_3 - \lambda_4)/\Omega = 0. \\ \\ \frac{\partial \mathcal{L}}{\partial \Gamma} &= (\mu_{\rho} - r_2)\phi T - \eta\phi^2 T^2(\sigma_{\rho}K)\sigma_{\rho} + (\lambda_5 - \lambda_6)/\Omega = 0 \\ &= (\mu_{\rho} - r_2)\phi T - \eta\phi^2 T^2 \sigma_{\rho}^2 K + (\lambda_5 - \lambda_6)/\Omega = 0. \end{split}$$

Hence, from the nature of $\mu_{W^D_1},~\sigma_{W^D_1},~\mu_{\Pi},$ and $\sigma_{\Pi},$ the three FOCs indeed display as

$$\begin{split} &\frac{\text{FOC.1:}}{\partial \mathcal{L}} \\ &\frac{\partial \mathcal{L}}{\partial \phi} = -FT + (\mu_{\rho} - r_2)\Gamma T - \eta \Gamma \phi T^2 \sigma_{\rho}^2 K + (v'(\phi) + \lambda_1 - \lambda_2)/\Omega = 0, \\ &\frac{\text{FOC.2:}}{\partial \alpha} \\ &\frac{\partial \mathcal{L}}{\partial \alpha} = -(\mu_{\rho} - r_1/T) W_0^D T - \eta W_0^D \phi T^2 \sigma_{\rho}^2 K + (\lambda_3 - \lambda_4)/\Omega = 0, \\ &\frac{\text{FOC.3:}}{\partial \Gamma} \\ &\frac{\partial \mathcal{L}}{\partial \Gamma} = (\mu_{\rho} - r_2) \phi T - \eta \phi^2 T^2 \sigma_{\rho}^2 K + (\lambda_5 - \lambda_6)/\Omega = 0. \end{split}$$

CHAPTER SIX The Firm as a Monopolist

6.1 Preface

This chapter aims at transferring the analytical frame into the market context, and to delineate the focal firm's investment and financing decisions in the monopoly case. This aim necessitates a more sophisticated definition of the firm's profit as a function of *costs*, *demand* and *price*. Our basic model premises (A1) - (A5) still hold.

An augmentation in the firm's stock of capital is supposed to diminish marginal costs. Hence, the adopted financial structure exhibits a direct relation with the firm's capacities of producing and operating on the market.

We continue supposing that D has subjective beliefs regarding the probability distribution of future firm profits. In this respect, we presume that D's major difficulty resides in her endeavor to accurately estimate the demand characteristics. Hence, rather than being interpreted as an unspecified environmental parameter, the stochastic variable which enters the model is now explicitly defined as the demand.

The firm has monopolistic price-setting power. Consequently, the decision maker is able to influence the demand by virtue of her pricing decision. Hence, our model is extended by means of an additional decision variable that D has at her disposal. End-of-period cash flows are now not only governed by the well-known vector (ϕ , α , Γ), but additionally by the market price, denoted by the new decision variable p.

6.2 Analysis

We consider the case where firm profit is given as an explicit function of the corresponding revenue and cost structure. The firm conducts monopolistic pricing decisions. Hence, price is now transformed into an endogenous variable which enters the profit function.

We adopt a classic model of uniform pricing, where the firm faces a standard downward-sloping linear demand curve. The firm charges one identical price to all customers and allows them to order any amounts at this price. Production then takes place according to the actual demand induced by the announced price. From our general model setup, D's final wealth and utility function are still given by (9) and (13) respectively.

6.2.1 The Solution Approach

Having price setting power, the decision maker's optimization problem is expanded by one additional decision variable, namely the price p. Hence, the new Lagrangean exhibits an additional set of constraints concerning p and now displays as

$$\mathcal{L} = U(CE) + v(\phi) + \lambda_1(\phi - \phi) + \lambda_2(1 - \phi) + \lambda_3\alpha + \lambda_4(\overline{\alpha} - \alpha) + \lambda_5\Gamma + \lambda_6(\overline{\Gamma} - \Gamma) + \lambda_7(p - p),$$

subject to the Kuhn-Tucker conditions

$\lambda_1(\phi - \underline{\phi}) = 0,$	(KT.01)
$\lambda_2(1-\phi) = 0,$	(KT.02)
$\lambda_3 \alpha = 0,$	(KT.03)
$\lambda_4(\overline{\alpha} - \alpha) = 0,$	(KT.04)
$\lambda_5 \Gamma = 0,$	(KT.05)
$\lambda_6(\overline{\Gamma}-\Gamma) = 0,$	(KT.06)
$\lambda_7(p-\underline{p}) = 0,$	(KT.07)
$(\phi - \underline{\phi}) \ge 0,$	(KT.08)
$(1-\phi) \geq 0,$	(KT.09)

$\alpha \geq 0,$	(KT.10)
$(\overline{\alpha} - \alpha) \geq 0,$	(KT.11)
$\Gamma \geq 0,$	(KT.12)
$(\overline{\Gamma} - \Gamma) \geq 0,$	(KT.13)
$(p - \underline{p}) \ge 0,$	(KT.14)

$$\lambda_1, \lambda_2, \lambda_3, \lambda_4, \lambda_5, \lambda_6, \lambda_7 \geq 0.$$
 (NNC)

We suppose $\underline{p} = \underline{c}$ to exclude negative marginal returns, i.e., to make sure that $(p-c) \ge 0 \ \forall K$.

Partially differentiating \mathcal{L} with respect to $(\phi, \alpha, \Gamma, p)$ yields the same three FOCs as in the previous chapter, but supplemented by a fourth one:

$$\begin{split} \frac{\partial \mathcal{L}}{\partial \phi} &= \Omega(\partial \mu_{W_1^D} / \partial \phi - \eta \sigma_{W_1^D} (\partial \sigma_{W_1^D} / \partial \phi)) + v'(\phi) + \lambda_1 - \lambda_2 = 0 \\ \frac{\partial \mathcal{L}}{\partial \alpha} &= \Omega(\partial \mu_{W_1^D} / \partial \alpha - \eta \sigma_{W_1^D} (\partial \sigma_{W_1^D} / \partial \alpha)) + \lambda_3 - \lambda_4 = 0, \\ \frac{\partial \mathcal{L}}{\partial \Gamma} &= \Omega(\partial \mu_{W_1^D} / \partial \Gamma - \eta \sigma_{W_1^D} (\partial \sigma_{W_1^D} / \partial \Gamma)) + \lambda_5 - \lambda_6 = 0, \\ \frac{\partial \mathcal{L}}{\partial p} &= \Omega(\partial \mu_{W_1^D} / \partial p - \eta \sigma_{W_1^D} (\partial \sigma_{W_1^D} / \partial p)) + \lambda_7 = 0. \end{split}$$

6.2.2 The Profit Function

Since W_1^D is a function of Π , we need to make explicit the new profit function in order to proceed further. As stressed above, firm profit now depends on the exogenous market price p, the stochastic demand Q, and on the cost function C. Costs are in turn depending on Q and on the investment level K, i.e.,

$$C(Q, K) = F + c(K)Q,$$

where F and c(K) represent fixed costs and marginal costs respectively. Hence, the firm's profit function is given by

$$\Pi = (p - c)Q - F,$$

where the demand $Q \sim \mathcal{N}(\mu_Q, \sigma_Q^2)$ is a Gaussian random variable which can accordingly be expressed as

$$Q = \mu_O + \sigma_Q z.$$

Thereby, its linear transformation Π is also a Gaussian random variable with $\mu_{\Pi} = (p-c)\mu_Q - F$ and $\sigma_{\Pi} = (p-c)\sigma_Q$. By substituting both into (10), firm profit displays as

(31)
$$\Pi = \mu_{\Pi} + \sigma_{\Pi} z = (p-c)\mu_Q - F + ((p-c)\sigma_Q)z.$$

6.2.3 The Marginal Cost Function

With c merely depending on the investment level K and constant F, the cost function C(Q, K) = F + c(K)Q discloses that marginal costs are constant in quantity. Hence, marginal costs constitute a limit which the unit costs AC = C/Q asymptotically approach from above, i.e.,

$$\lim_{Q \to \infty} AC = c(K).$$

By adjusting the investment level K via (ϕ, α, Γ) , the decision maker is able to determine the actual value of this limit, i.e., an augmentation in the firm's stock of capital diminishes marginal costs.

Considering that $K \in [\underline{K}, \overline{K}]$,⁹⁶ and supposing that the corresponding upper bound \overline{K} is sufficiently small,⁹⁷ the marginal cost function can be linearly approximated by

$$c(K) = f - gK.$$

6.2.4 The Demand Function

 Π is a function of the demand Q, which in turn is a function of the market price p. Since p is an endogenous variable, we need to make explicit Q(p) in order to proceed further. Q is a Gaussian random variable which can be expressed as

⁹⁶ We know that $K = (1-\alpha)W_0^D/\phi + \Gamma$. Hence, $K \in [\underline{K}, \overline{K}] \equiv [(1-\overline{\alpha})W_0^D, W_0^D/\underline{\phi} + \overline{\Gamma}]$.

⁹⁷ To exclude negative marginal costs, we assume the relation $\overline{K} < f/g$ to be fulfilled.

$$Q = \mu_O + \sigma_Q z.$$

As it is standard in the pertinent literature, we assume μ_Q to linearly decrease in the market price, i.e.,

 $(32) \qquad \mu_Q = \mathbf{a} - bp\,,$

with a, b > 0. The dispersion parameter σ_Q in contrast is supposed to be independent from p. Hence, the demand function displays as

(33)
$$Q(p) = a - bp + \sigma_Q z.$$

It is easy to see that the stochastic nature of the market demand is modeled by means of $(a + \sigma_Q z)$, which indicates the demand that realizes for p=0 and which will be henceforth tagged as the *potential demand*. Hence, the random component within the demand function is additive and merely affects the potential demand (which deviates from its mean value a by the normally distributed offset $\sigma_Q z$). In contrast, the price-sensitivity of the demand function (as measured by the slope b) is unaffected by risk.



Figure 15: Properties of the demand curve.

Similar to the preceding two chapters, we can now solve 98 the differential terms in the four FOCs to obtain

$$\frac{FOC.1:}{\partial \phi} = (p-c)\mu_Q - F - \eta\phi T((p-c)\sigma_Q)^2 - (1+r_2)\Gamma - E((g\mu_Q) - \eta\phi T\sigma_Q^2(p-c)g) + (v'(\phi) + \lambda_1 - \lambda_2)/\Omega T = 0,$$

$$FOC.2:$$

$$\begin{split} &\frac{\partial \mathcal{L}}{\partial \alpha} = (1 + r_1/T) - (g\mu_Q) + \eta \phi T \sigma_Q^2(p - c)g + (\lambda_3 - \lambda_4)/\Omega W_0^D T = 0, \\ &\frac{\text{FOC.3:}}{\partial \Gamma} \\ &\frac{\partial \mathcal{L}}{\partial \Gamma} = -(1 + r_2) + (g\mu_Q) - \eta \phi T \sigma_Q^2(p - c)g + (\lambda_5 - \lambda_6)/\Omega \phi T = 0, \\ &\frac{\text{FOC.4:}}{\partial \rho} \\ &\frac{\partial \mathcal{L}}{\partial \rho} = -b(p - c) + \mu_Q - \eta \phi T \sigma_Q^2(p - c) + \lambda_7/\Omega \phi T = 0. \end{split}$$

6.2.5 The Decision Maker's Preference Structure of Financing

Similar to the preceding chapters, this subsection aims at examining the decision maker's arbitration between inside equity (i.e., her personal investment into the firm) and the incurred level of corporate debt.

We continue supposing $r_2T > r_1$, hence the effective costs of debt are higher than D's opportunity costs of personally investing into the firm. The expected profit per unit of capital employed is also assumed to exceed the due reimbursement per unit of debt employed. Thereby, our basic assumptions display as

$$\mu_{\Pi}/K > 1 + r_2 > 1 + r_1/T.$$

Most notably, the satisfaction of these basic conditions assures the decision maker's participation in the firm (since returns generated by the firm exceed the riskless asset's return) and gives rise to the leverage effect (since the firm's return on capital exceeds the due coupon payment).

 $^{^{98}}$ See the appendix to this chapter (Appendix A 6.1) for the calculus.

To explore D's arbitration between inside equity (governed by α) and the incurred level of corporate debt (governed by Γ), we have to examine the conditions under which FOC.2 and FOC.3 hold simultaneously. Adding up both equations yields the reduced condition

(34)
$$-(r_2 - r_1/T) + (\lambda_3 - \lambda_4)/\Omega W_0^D + (\lambda_5 - \lambda_6)/\Omega \phi T = 0.$$

By means of this reduced condition (for which both α and Γ are optimized), Lemma 6.1 is presented in order to facilitate the subsequent proof of Proposition 6.1.

Lemma 6.1: For $\mu_{\Pi}/K > 1 + r_2 > 1 + r_1/T$, optimal solutions for α and Γ must fulfill

i)	$0 < \alpha^* < \overline{\alpha}$	٨	$\Gamma^* = 0 ,$	or
ii)	$\alpha^* = 0$	٨	$\Gamma^* = 0 ,$	or
iii)	$\alpha^* = 0$	٨	$0 < \Gamma^* < \overline{\Gamma}$,	or
iv)	$\alpha^* = 0$	٨	$\Gamma^* = \overline{\Gamma}.$	

Proof: The four cases are proved consecutively.

- i) From KT.3 and KT.4, $0 < \alpha < \overline{\alpha}$ implies $\lambda_3 = \lambda_4 = 0$. Hence, the reduced condition (19) can only hold if $\lambda_5 = (r_2 r_1/T)\Omega\phi T > 0$ and $\lambda_6 = 0$. From KT.5, a positive value of λ_5 necessitates $\Gamma = 0$, which proves case *i*).
- ii) From KT.5 and KT.6, $\Gamma=0$ implies $\lambda_5 \ge 0$ and $\lambda_6=0$. Hence, the reduced condition (19) can only hold in two cases, namely if either
 - $\lambda_5 = (r_2 r_1/T)\Omega\phi T > 0$ and $\lambda_3 = \lambda_4 = 0$ (which constitutes the case covered by the first part of Lemma 6.1), or if
 - $\lambda_3 + \lambda_5 W_0^D / \phi T = (r_2 r_1 / T) \Omega W_0^D > 0$ and $\lambda_4 = 0$ (which, from KT.3, implies $\alpha = 0$, thus proving case *ii*).
- iii) From KT.5 and KT.6, $0 < \Gamma < \overline{\Gamma}$ implies $\lambda_5 = \lambda_6 = 0$. Hence, the reduced condition (19) can only hold if $\lambda_3 = (r_2 r_1/T)\Omega W_0^D > 0$ and $\lambda_4 = 0$. From KT.3, a positive value of λ_3 necessitates $\alpha = 0$, which proves case *iii*).
- *iv)* From *KT.5* and *KT.6*, $\Gamma = \overline{\Gamma}$ implies $\lambda_5 = 0$ and $\lambda_6 \ge 0$. Hence, the reduced condition (19) can only hold if $\lambda_3 \lambda_6 W_0^D / \phi T = (r_2 r_1/T) \Omega W_0^D > 0$. Thus, λ_3 can only be positive, which, from *KT.3*, implies $\alpha = 0$. This proves case *iv*).

The decision maker's preference structure regarding inside equity and debt directly follows from Lemma 6.1.

Proposition 6.1: For $\mu_{\Pi}/K > 1+r_2 > 1+r_1/T$, the decision maker firstly invests her personal wealth into the firm before calling for corporate debt.

Proof: D only invests into the risk-free asset if no corporate debt is incurred (positive α^* necessitates $\Gamma^*=0$). Furthermore, corporate debt is only incurred if D's personal investment into the firm is maximized (positive Γ^* necessitates $\alpha^*=0$). This structure can only exist if the decision maker firstly invests her personal wealth into the firm before calling for debt (which is only incurred if D's personal wealth is not sufficient to meet her corporate investment objectives).

Hence, the new profit function has no impact on D's preference hierarchy concerning inside equity and corporate debt. With $r_2T > r_1$, the effective costs of debt are *higher* than D's opportunity costs of personally investing her private wealth into the firm.

Since the new decision variable p has to be considered in this chapter, we proceed by first examining D's behavior in terms of pricing (section 6.2.6). We then scrutinize the raising of outside equity and the resulting ownership concentration (section 6.2.7). The decision maker's corporate investment objectives are examined in section 6.2.8.

6.2.6 The Decision Maker's Pricing Behavior

To explore D's behavior in terms of pricing, we scrutinize the new FOC.4, which displays as

FOC.4:

$$-b(p-c) + \mu_{\Omega} - \eta\phi T\sigma_{Q}^{2}(p-c) + \lambda_{7}/\Omega\phi T = 0.$$

By inspecting the case-contingent basic shape of the FOCs,⁹⁹ we infer that, in the optimum,

$$(35) \quad \mu_Q - \eta \phi T \sigma_Q^2(p-c) = \begin{cases} (1+r_1/T)/g, & \text{if case } i) \text{ holds} \\ ((1+r_1/T) + \lambda_3/\Omega W_0^D T)/g, & \text{if case } ii) \text{ holds} \\ (1+r_2)/g, & \text{if case } iii) \text{ holds} \\ ((1+r_2) + \lambda_6/\Omega \phi T)/g, & \text{if case } iv) \text{ holds} \end{cases}$$

In order to condense the subsequent analytical procedure and to ease the algebraical exposure, we make use of the following definition.

⁹⁹ See subsection 6.2.8 with the respective derivation of FOC.2/3 throughout the corresponding appendices (A 6.3 and A 6.4).

Definition 6.1: Denote by $(\mathbf{r}, \mathbf{\ell})$ the two values which satisfy

$$\mathbf{r} = \begin{cases} r_1/T & \text{in cases } i \text{) and } ii \text{)} \\ r_2 & \text{in cases } iii \text{) and } iv \text{)} \end{cases} \quad and \quad \mathbf{\ell} = \begin{cases} 0 & \text{in case } i \text{) and } iii \text{)} \\ \lambda_3/\Omega W_0^D T & \text{in case } ii \text{)} \\ \lambda_6/\Omega \phi T & \text{in case } iv \text{)} \end{cases}.$$

By virtue of (35) and Definition 6.1, we can generally state that, in the optimum,

(36)
$$\mu_Q - \eta \phi T \sigma_Q^2 (p-c) = ((1+r)+\ell)/g,$$

and we can further express FOC.4 in its general form which covers all four cases, i.e.,

(37)
$$-b(p-c) + ((1+r)+\ell)/g + \lambda_7/\Omega\phi T = 0.$$

Lemma 6.1: In the optimum, the price strictly exceeds its lower bound, i.e.,

$$p^* > \underline{p} = \underline{c}.$$

Proof: In order to fulfill FOC.4, the first term in (37) must be negative. With $p = \underline{p}$, Equation (37) displays as

$$-b(p-c) + ((1+r)+\ell)/g + \lambda_7/\Omega\phi T = 0.$$

Since $(\underline{p}-c) \leq 0$, the first term of the above equation can only be positive or zero. Hence, (37) cannot be fulfilled for p = p but instead requires p > p.

It is easy to understand that, since there are fixed costs to be recovered, the contribution margin has to be strictly positive in order to avoid negative profits. As a consequence, given that $p^* > \underline{p}$, KT.7 implies $\lambda_7 = 0$ in the optimum. Hence, the general form of FOC.4 simplifies to

(38)
$$-b(p-c) + ((1+r)+\ell)/g = 0.$$

By means of (38), the actual price in the optimum expressed as the sum of the marginal costs and the contribution margin can now be easily determined.

Proposition 6.2: In the optimum, the price and marginal costs must fulfill

(39)
$$p^* = c^* + (1 + r + \ell) / bg$$

Proof: Algebraically solving (38) for p yields the above expression.

The contribution margin is given by $(1+r+\ell)/bg$. It decreases in both the price-sensitivity of the demand (measured by the parameter b) and the capitalsensitivity of the marginal costs (measured by the parameter g). The significance of b proves to be particularly interesting. As b increases, consumers react more sensitive to price changes and the optimal price accordingly decreases to finally converge against the marginal cost level for $b \to \infty$.¹⁰⁰ This result yields a valuable insight into the decision maker's pricing decision, since it shows how the mark-up chosen in the optimum is crucially depending on the exogenous parameter b. Further inspection of (35) and Definition 6.1 reveals that the optimal price negatively depends on the demand volatility and on the degree of D's risk aversion. Evidently, this effect of σ_Q^2 and η strictly decreases in b.

Moreover, Equation (38) also allows the deduction of the demand as expected by the decision maker in the optimum.

Proposition 6.3: The decision maker's expected demand in the optimum is given by

(40)
$$\mu_{Q}^{*} = (1 + \eta \phi^{*} T \sigma_{Q}^{2} / b) \left((1 + r + \ell) / g \right).$$

Proof: Consider the decision maker's choices in the optimum. From Equation (36), we know that $((1+r)+\ell)/g = \mu_Q - \eta\phi T\sigma_Q^2(p-c)$. From Equation (38), we further infer that $(p-c) = -((1+r)+\ell)/bg$. Substituting these two equations into (38) and subsequently solving for μ_Q then yields $\mu_Q = (1 + \eta\phi T\sigma_Q^2/b) ((1+r+\ell)/g)$.

Evidently, the expected demand as disclosed by (40) is induced by D's optimal choices concerning her decision variables (which are of course interdependent). Hence, since the expected demand in the optimum depends on the chosen investment level K^* ,¹⁰¹ the decision maker adjusts her decision variables to calibrate her expected demand according to Equation (40).

Moreover, similar to Proposition 6.2, the above-stated Proposition 6.3 plainly reveals the significance of the price-sensitivity of the consumer demand. If the demand becomes steeper (the slope *b* increases), the expected demand in the optimum accordingly decreases and converges against $(1+r+\ell)/g$ for $b \to \infty$.

Propositions 6.2 and 6.3 give rise to an interesting corollary:

¹⁰⁰ Correspondingly, the optimality condition (39) would change to $p^* \simeq c^*$.

¹⁰¹ The higher the overall firm investment K, the lower the marginal cost level. The lower marginal costs, the lower the optimal monopoly price (for a given corresponding mark-up). Consequently, given that demand is downward sloping in p, the expected demand in the optimum positively depends on the chosen investment level.

Corollary 6.1: The greater the price-sensitivity of the consumers, the lower will be the general impact of

- the decision maker's risk aversion,
- the demand volatility that the decision maker anticipates ex ante.

Hence, a high price sensitivity of demand interestingly reduces the impact of risk on D's optimal choices. In the limit, the impact of risk is totally neutralized, i.e., $b \to \infty$ has the same effect as if D would approach risk-neutrality $(\eta \to 0)$.¹⁰²

This observation constitutes an important deviation from the results derived in Chapter 5. It obtains as a direct consequence of monopoly case, i.e., the decision maker's ability to freely determine the market price. It is a well established finding, that the size of a monopolist's optimal contribution margin negatively depends on the price-sensitivity of the demand curve.¹⁰³ The more sensitive demand is reacting to price changes, the greater is the relative size of the volume effect compared to the price effect.

The decision maker faces the following trade-off. By decreasing price and, thereby, increasing the quantity sold, firm revenue is augmented due to additional sales (positive volume effect), but also diminished due to the lower price paid by already existing buyers (negative price effect). This trade-off induces the decision maker to choose a price level that maximizes the sum of both effects. This maximum is crucially depending on the slope of the demand curve, i.e., it strictly decreases in b. The classical way to formally operationalize this relationship is to examine the *Lerner Index* as a measure of the contribution margin in the optimum (relative to the price). In fact, it is straightforward to show that the Lerner Index indeed *negatively* depends on the slope of the demand curve.¹⁰⁴ Since, contrary to the scenarios examined in the previous chapters, the decision maker can now adapt the firm's contribution margin to her expected demand curve, the announced importance of b emerges.

6.2.7 Outside Equity and Ownership Concentration

Similar to the preceding chapters, the decision maker's behavior with respect to the raising of outside equity remains to be scrutinized. In order to examine out-

¹⁰² Correspondingly, the optimality condition (40) would change to $\mu_Q^* \simeq (1 + r + \ell)/g$.

 $^{^{103}}$ See Pfähler/Wiese (1998).

 $^{^{104}}$ A formal derivation of the *Lerner Index* is given in the appendix to this chapter (see Appendix A 6.2).

side equity and, thereby, ownership concentration, we again explore the possible values of ϕ in the optimum. By substituting (39) and (40) into FOC.1 we obtain

(41)
$$(1+r+\ell) \left[-(bf-a + ((1+r+\ell)/g)(1+\eta\phi T\sigma_Q^2/b)) \right] - bgF$$
$$+ (v'(\phi) + \lambda_1 - \lambda_2) bg/\Omega T = 0.$$

By means of the reduced optimality condition (41), Proposition 6.4 is presented.

Proposition 6.4: In the optimum, there exists a unique solution for ϕ , which satisfies:

$$\phi \le \phi^* \le \frac{1}{2}.$$

Proof: Consider the reduced condition (41). The term in square brackets is clearly negative. Thus, the term $(v'(\phi)+\lambda_1-\lambda_2)bg/\Omega T$ has to be positive to satisfy the condition. Moreover, ϕ can evidently only take the unique value that satisfies

$$(v'(\phi)+\lambda_1-\lambda_2)bg/\Omega T = -(1+r+\ell)\left[-(bf-a+((1+r+\ell)/g)(1+\eta\phi T\sigma_Q^2/b))\right] + bgF.$$

This unique value of ϕ must ensure that the expression $(v'(\phi) + \lambda_1 - \lambda_2)$ is positive, which, from KT.1, KT.2, and the properties of $v(\phi)$, implies $\phi \le \phi^* \le \frac{1}{2}$.

Like in the previous scenario (Chapter 5), the decision maker is not indifferent regarding outside equity. In the optimum, D trades off the marginal utility gain of the wealth effect of outside equity against the marginal utility loss of its power sharing effect to attain a unique interior solution ϕ^* for her optimal share of the firm's total equity. But unlike our previous findings, ϕ^* can never exceed $\frac{1}{2}$. In contrary, exogenously decreasing D's initial wealth $(W_0^D\downarrow)$ shifts downwards the solution ϕ^* , i.e., the optimal amount of outside equity is gradually increased (the mechanism which underlies this effect is discussed in the next subsection). Hence, the pricing power of the monopolist causes ϕ^* to actually tend towards zero, rather than unity in case iv).

6.2.8 The Decision Maker's Corporate Investment Objectives

Similar to the previous chapters, this subsection examines D's target levels of corporate investment.

Cases i) and ii)

We begin by treating the first two cases, where $\Gamma^* = 0$. As shown in these parts of Lemma 6.1, W_0^D is sufficiently large to meet *D*'s corporate investment objectives without calling for any debt. The proof of the first two cases in Lemma 6.1 has further disclosed that with

 $\begin{aligned} i) \quad \lambda_5 &= (r_2 - r_1/T)\Omega\phi T & \wedge & \lambda_3 &= \lambda_4 &= \lambda_6 &= 0, \\ ii) \quad \lambda_3 &+ \lambda_5 W_0^D / \phi T &= (r_2 - r_1/T)\Omega W_0^D & \wedge & \lambda_4 &= \lambda_6 &= 0, \end{aligned}$ or with

FOC.2 and FOC.3 simultaneously hold. Thus, in both cases, we are facing a system of only two equations.

Scrutinizing case *i*), where $0 < \alpha^* < \overline{\alpha}$ and $\Gamma^* = 0$, we have

FOC.1:

$$\begin{aligned} (p-c)\mu_Q - F - \eta\phi^* T((p-c)\sigma_Q)^2 - K((g\mu_Q) - \eta\phi^* T\sigma_Q^2 g(p-c)) \\ + (v'(\phi^*) + \lambda_1 - \lambda_2)/\Omega T &= 0, \end{aligned}$$

$$\underbrace{\text{FOC.2} \equiv \text{FOC.3:}}_{(1+r_1/T) - g\mu_Q} + \eta\phi^* T\sigma_Q^2 g(p-c) &= 0. \end{aligned}$$

By adequate substitution, 105 we can rewrite this system in its equivalent form

$$\begin{split} & \underline{\text{FOC.1:}} \\ & -F - ((f\text{-}p)(1+r_1/T))/g + (v'(\phi^*) + \lambda_1 - \lambda_2)/\Omega T = 0, \\ & \underline{\text{FOC.2} \equiv \text{FOC.3:}} \\ & \phi^* K = (\mu_Q - (1+r_1/T)/g)/\eta Tg\sigma_Q^2 + \phi^*(f\text{-}p)/g. \end{split}$$

With $\phi^* K = (1 - \alpha^*) W_0^D + \phi^* \Gamma^*$ and $\Gamma^* = 0$, FOC.2/3 displays as

(42)
$$(1-\alpha^*)W_0^D = (\mu_Q - (1+r_1/T)/g)/\eta Tg\sigma_Q^2 + \phi^*(f-p)/g.$$

which discloses the amount of D's personal wealth invested into the firm (LHS of the equation) and her *maximum target level* of corporate investment

 $^{^{105}}$ See the appendix to this chapter (A 6.3) for the detailed calculus.

(RHS of the equation). D's initial wealth is largely sufficient to reach this maximum target and allows for an investment of the excess amount into the riskless alternative.

Scrutinizing case *ii*), where $\alpha^* = \Gamma^* = 0$, we have

FOC.1:

$$\begin{split} (p-c)\mu_Q - F &- \eta\phi T((p-c)\sigma_Q)^2 - K((g\mu_Q) - \eta\phi T\sigma_Q^2 g(p-c))) \\ &+ (v'(\phi^*) + \lambda_1 - \lambda_2)/\Omega T = 0, \\ \\ \underline{FOC.2 \equiv FOC.3:} \\ (1+r_1/T) - g\mu_Q + \eta\phi^* T\sigma_Q^2 g(p-c) + \lambda_3/\Omega W_0^D T = 0. \end{split}$$

This system differs from case i) only in that the second equation exhibits an additional term, i.e., $\lambda_3/\Omega W_0^D T$. Hence, solving in a similar manner as in case i) yields the equivalent form

$$\begin{split} & \underline{\text{FOC.1:}} \\ & -F - ((f\text{-}p)(1 + r_1/T))/g + (v'(\phi^*) + \lambda_1 - \lambda_2)/\Omega T = 0, \\ & \underline{\text{FOC.2} \equiv \text{FOC.3:}} \\ & \phi^* K = (\mu_Q - ((1 + r_1/T) + \lambda_3/\Omega W_0^D T)/g)/\eta Tg\sigma_Q^2 + \phi^*(f\text{-}p)/g. \end{split}$$

With $\phi^* K = (1 - \alpha^*) W_0^D + \phi^* \Gamma^*$ and $\alpha^* = \Gamma^* = 0$, FOC.2/3 displays as

(43)
$$W_0^D = ((\mu_Q - (1 + r_1/T)/g)/\eta T\sigma_Q^2 + \phi^*(f-p))/g - \lambda_3/\Omega W_0^D \eta T^2 g^2 \sigma_Q^2,$$

which discloses that D invests her full wealth into the firm in order to approach her maximum target as near as possible. The remaining difference between the maximum target level and D's wealth endowment is given by $\lambda_3/\Omega W_0^D \eta T^2 g^2 \sigma_Q^2$, which evidently decreases if W_0^D increases to approach the maximum target level from below.

In both case *i*) and *ii*), the respective form of FOC.1 is identical and displays as $-F - ((f-p)(1+r_1/T))/g + (v'(\phi^*) + \lambda_1 - \lambda_2)/\Omega T = 0$, which, from the properties of $v'(\phi^*)$, implies

$$\underline{\phi} \le \phi^* \le \frac{1}{2}$$

for both cases. Hence, the decision maker raises outside equity until her utility gain from the corresponding income effect is exactly outweighed by her utility loss from the corresponding power sharing effect.

Cases iii) and iv)

We now treat the last two cases, where $\Gamma^* > 0$. As shown in these parts of Lemma 6.1, W_0^D is *not* sufficiently large to meet D's corporate investment objectives without calling for additional debt. The proof of the last two cases in Lemma 6.1 has further disclosed that with

iii)
$$\lambda_3 = (r_2 - r_1/T)\Omega W_0^D$$

 $\wedge \qquad \lambda_4 = \lambda_5 = \lambda_6 = 0,$ or with
 iv) $\lambda_3 - \lambda_6 W_0^D / \phi T = (r_2 - r_1/T)\Omega W_0^D$
 $\wedge \qquad \lambda_4 = \lambda_5 = 0,$

FOC.2 and FOC.3 simultaneously hold. Thus, we are again facing a system of only two equations.

Scrutinizing case *iii*), where $\alpha^* = 0$ and $\overline{\Gamma} > \Gamma^* > 0$, we have

FOC.1:

$$\begin{split} (p-c)\mu_Q - F - \eta \phi^* T((p-c)\sigma_Q)^2 - (1+r_2)\Gamma^* &- E((g\mu_Q) - \eta \phi^* T\sigma_Q^2(p-c)g) \\ &+ (v'(\phi^*) + \lambda_1 - \lambda_2)/\Omega T = 0, \end{split}$$

 $\underline{\text{FOC.2} \equiv \text{FOC.3:}}$ $-(1+r_2) + g\mu_Q - \eta \phi^* T \sigma_Q^2 (p-c)g = 0.$

By adequate substitution, 106 we can again rewrite this system in its equivalent form

$$\begin{split} & \underline{\text{FOC.1:}} \\ & -F - ((f\text{-}p)(1+r_2))/g + (v'(\phi^*) + \lambda_1 - \lambda_2)/\Omega T = 0, \\ & \underline{\text{FOC.2} \equiv \text{FOC.3:}} \\ & \phi^* K = (\mu_Q - (1+r_2)/g)/\eta Tg\sigma_Q^2 + \phi^*(f\text{-}p)/g. \end{split}$$

With $\phi^* K = (1 - \alpha^*) W_0^D + \phi^* \Gamma^*$ and $\alpha^* = 0$, FOC.2/3 displays as

 $^{^{106}}$ See the appendix to this chapter (A 6.4) for the detailed calculus.

(44)
$$W_0^D + \phi^* \Gamma^* = (\mu_O - (1+r_2)/g)/\eta Tg\sigma_Q^2 + \phi^* (f-p)/g,$$

which, considering that W_0^D is fully invested into the firm, discloses the incurred level of corporate debt (Γ) in order to reach the *minimum target level* of corporate investment (RHS of the equation).

FOC.1 discloses again that the unique solution value of ϕ satisfies

$$\underline{\phi} \le \phi^* \le \frac{1}{2}.$$

There is a *unique* pair of values (ϕ^*, Γ^*) which satisfies the two optimality conditions. The financing gap is accordingly filled by $\phi^*\Gamma^*$. Hence, as long as the debt capacity $\overline{\Gamma}$ is sufficiently large to guarantee $\overline{\Gamma} > \Gamma^* > 0$, the disclosed result holds.

Scrutinizing case *iv*), where $\alpha^* = 0$ and $\Gamma^* = \overline{\Gamma}$, we have

FOC.1:

$$\begin{split} (p-c)\mu_Q - F - \eta \phi^* T((p-c)\sigma_Q)^2 - (1+r_2)\Gamma - E((g\mu_Q) - \eta \phi^* T \sigma_Q^2(p-c)g) \\ + (v'(\phi^*) + \lambda_1 - \lambda_2)/\Omega T &= 0, \end{split}$$

This system differs from case *iii*) only in that the second equation exhibits an additional term, i.e., $-\lambda_6/\Omega \phi^* T$. Hence, solving in a similar manner as in case *iii*) yields the equivalent form

$$\begin{split} & \underline{FOC.1:} \\ & -F - ((f - p)(1 + r_2)/g + \lambda_6/\Omega\phi^*T) + (v'(\phi^*) + \lambda_1 - \lambda_2)/\Omega T = 0, \\ & \underline{FOC.2 \equiv FOC.3:} \\ & \phi^* K = (\mu_Q - ((1 + r_2) + \lambda_6/\Omega\phi^*T)/g)/\eta Tg\sigma_Q^2 + \phi^*(f - p)/g. \end{split}$$

Knowing that $\phi^* K = (1-\alpha^*) W_0^D + \phi^* \Gamma^*$, $\alpha^* = 0$, and $\Gamma^* = \overline{\Gamma}$, FOC.2/3 displays as

(45)
$$W_0^D + \phi^* \overline{\Gamma} = ((\mu_Q - (1+r_2)/g)/\eta T \sigma_Q^2 + \phi^*(f-p))/g - \lambda_6/\Omega \phi^* \eta T^2 g^2 \sigma_Q^2.$$

which discloses that D invests her full wealth into the firm and uses the full debt capacity in order to either reach her minimum target or to approach it as near as possible. The remaining difference between the minimum target level and D's wealth endowment is given by $\lambda_6/\Omega \phi^* \eta T^2 g^2 \sigma_Q^2$, which evidently depends on ϕ^* . If W_0^D exogenously decreases (i.e., the decision maker's liquidity constraint is sharpened and the financing gap becomes larger), ϕ^* decreases to close the gap. Hence, contrary to our previous findings, ownership becomes *less* concentrated when moving deeper into case *iv*).

Generally encompassing all four basic cases, the decision maker's target levels (as given by the right-hand sides of equations (42) and (44)) are increasing in μ_Q and decreasing in both σ_Q^2 and η . Hence, similar to the simplified "profit machine" setting, the decision maker's corporate investment objectives become more ambitious with improving average environmental states (operationalized via the stochastic demand), decreasing demand volatility, and decreasing risk aversion of the decision maker. However, with the new profit function, the parameter g now additionally enters the investment objectives. It is straightforward to see that the decision maker's target levels are decreasing in g. The intuition behind this result is that g can be interpreted as a direct measurement for the marginal effectiveness of capital in terms of reducing marginal costs. Thereby, the immediate intuition bleeds off is as follows: the stronger the cost-reducing effect of capital, the lower is the actual capital requirement of the firm.

6.2.9 The summarized Pattern of Investing and Financing

The decision maker's behavioral pattern in terms of investing and financing can be summarized similar to the previous chapters. The respective minimum and maximum target level of corporate investment are given by

(46)
$$\begin{aligned} \mathrm{TL}^{\mathrm{Max}} &= ((\mu_Q - (1 + r_1/T)/g)/\eta T \sigma_Q^2 + \phi^*(f\text{-}p))/g, \quad \text{and} \\ \mathrm{TL}^{\mathrm{Min}} &= ((\mu_Q - (1 + r_2)/g)/\eta T \sigma_Q^2 + \phi^*(f\text{-}p))/g. \end{aligned}$$

The disclosure makes use of the two below-stated definitions.

Definition 6.2: Denote by $\hat{\phi} < \frac{1}{2}$ the largest value of ϕ for which the optimality condition (41) is fulfilled if $\alpha=0$.

Definition 6.3: Denote by $\widehat{\phi}$ (with $\underline{\phi} < \widehat{\phi} < \widehat{\phi}$) the largest value of ϕ for which the optimality condition (41) is fulfilled if $\Gamma = \overline{\Gamma}$.

Further considering the case contingent shape of $(1+\mathcal{F}+\ell)$ as given in Definition 6.1, we are once again taken back to our acquainted four cases.

$$\begin{split} i) \quad \text{if} \qquad W_0^D > \mathrm{TL}^{\mathrm{Max}}, \\ \Rightarrow \quad \underline{\phi} \leq \phi^* = \widehat{\phi} < \frac{1}{2}, \qquad \text{satisfying} \quad -F - ((f - p)(1 + r_1/T))/g \\ & + (v'(\phi^*) + \lambda_1 - \lambda_2)/\Omega T = 0, \\ \alpha^* > 0, \text{ satisfying } (1 - \alpha^*) W_0^D = \mathrm{TL}^{\mathrm{Max}}, \\ \Gamma^* = 0, \\ K^* = \mathrm{TL}^{\mathrm{Max}}/\phi^*, \\ p^* = c^* + (1 + r_1/T)/bg. \end{split}$$

D's private wealth is sufficiently large to meet her corporate investment objectives, i.e., it exceeds her maximum target level of corporate investment. Hence, she incurs no corporate debt and divides her personal investment between the firm's capital stock and the riskless alternative. The decision maker raises outside equity in order to balance her utility gains from the corresponding income effect against her utility losses from the corresponding power sharing effect.

$$\begin{split} ii) \quad \mathrm{if} \qquad \mathrm{TL}^{\mathrm{Min}} < W_0^D \leq \mathrm{TL}^{\mathrm{Max}}, \\ \Rightarrow \quad \underline{\phi} \leq \phi^* < \widehat{\phi} < \frac{1}{2}, \qquad \mathrm{satisfying} \quad -F - ((f - p)(1 + r_1/T))/g \\ & + (v'(\phi^*) + \lambda_1 - \lambda_2)/\Omega T = 0, \\ \alpha^* = 0, \\ \Gamma^* = 0, \\ K^* = (\mathrm{TL}^{\mathrm{Max}} - \lambda_3/\Omega W_0^D \eta T^2 g^2 \sigma_Q^2)/\phi^*, \\ p^* = c^* + ((1 + r_1/T) + \lambda_3/\Omega W_0^D T)/bg. \end{split}$$

D's private wealth is sufficiently large to meet her minimum target level of corporate investment, but not her maximum target level. Hence, she incurs no corporate debt and invests her full personal wealth into the firm in order to reach the best possible rapprochement towards her maximum target level. The decision maker raises outside equity in order to balance her utility gains from the corresponding wealth effect against her utility losses from the corresponding power sharing effect.

$$\begin{array}{ll} iii) \mbox{ if } & W_0^D \leq {\rm TL}^{\rm Min} < W_0^D + \widehat{\phi} \ \overline{\Gamma} \ , \\ \Rightarrow & \underline{\phi} \leq \widehat{\phi} < \phi^* < \widehat{\phi} < \frac{1}{2} \ , & \mbox{ satisfying } -F - ((f - p)(1 + r_2))/g \\ & + (v'(\phi^*) + \lambda_1 - \lambda_2)/\Omega T = 0, \\ & \alpha^* = 0, \\ & \overline{\Gamma} > \Gamma^* = ({\rm TL}^{\rm Min} - W_0^D)/\phi^*, \\ & K^* = {\rm TL}^{\rm Min}/\phi^*, \\ & p^* = c^* + (1 + r_2)/bg. \end{array}$$

D's private wealth is too small to single-handedly meet her minimum target level of corporate investment. Hence, she invests her full personal wealth into the firm's stock of capital and incurs supplementary corporate debt in order to reach this minimum target level (filling the financing gap). The decision maker raises outside equity in order to fill the financing gap and to balance her utility gains from the corresponding wealth effect against her utility losses from the corresponding power sharing effect.

$$\begin{split} iv) \quad \mathrm{if} \qquad & W_0^D + \widehat{\phi} \ \overline{\Gamma} \leq \mathrm{TL}^{\mathrm{Min}}, \\ \Rightarrow \qquad & \underline{\phi} \leq \phi^* \leq \widehat{\phi} < \frac{1}{2}, \qquad \mathrm{satisfying} \quad -F - ((f - p)(1 + r_2)/g + \lambda_6/\Omega \phi^* T) \\ & + (v'(\phi^*) + \lambda_1 - \lambda_2)/\Omega T = 0, \\ & \alpha^* = 0, \\ & \Gamma^* = \overline{\Gamma}, \\ & K^* = (\mathrm{TL}^{\mathrm{Min}} - \lambda_6/\Omega \phi^* \eta T^2 g^2 \sigma_Q^2)/\phi^* \\ & p^* = c^* + ((1 + r_2) + \lambda_6/\Omega \phi^* T)/\mathrm{bg}. \end{split}$$

D invests her full personal wealth into the firm's stock of capital and incurs as much corporate debt as possible in order to approach her minimum target of corporate investment as near as possible. She raises outside equity in order to balance her utility gains from its corresponding wealth effect against her utility losses from its corresponding power sharing effect.

Figure 16 depicts the properties of $(\phi^*, \alpha^*, \Gamma^*)$ throughout the four cases, contingent on the level of the decision maker's initial wealth W_0^D .



Figure 16: Properties of $(\phi^*, \alpha^*, \Gamma^*)$ in the monopoly case.

As in the corresponding figures in the previous chapters, the vertical lines in Figure 16 separate our acquainted four cases. The above figure shows that the decision maker's behavior in terms of investing and financing remains similar to the previous chapter, except for one important deviation. Compared to the previous chapter, the pricing power of the monopolist causes ϕ^* to actually tend towards zero when exogenously decreasing W_0^D in case iv) to widen the financing gap. Thereby, a lower ownership concentration results in the optimum, and the monopolistic firm's stock of capital is larger compared to the previous chapter.

6.3 Conclusion

This chapter examined the scenario with monopolistic pricing power. The decision maker's investment and financing decisions were connected to both the firm's cost and revenue structure and its pricing behavior. Again, casecontingent interior solutions were obtained so that marginal costs and marginal benefits are balanced. D's pattern of investing and financing is still largely conform to our previous findings (Chapter 5). Similar to Chapter 5, the decision maker's corporate investment objectives are again contingent on her subjective beliefs (via μ_Q and σ_Q^2), her risk sensitivity (via η), exogenous institutional parameters (via r_1 , r_2 , and T), and on the characteristics of the power-related utility component $v(\phi)$. But with the newly introduced profit function of this chapter, D's corporate investment objectives now also crucially depend on the parameter g. More precisely, they are decreasing in g, since an increase in the effectiveness of capital in terms of reducing marginal costs obviously diminishes the firm's actual capital requirements.

Moreover, the decision maker's ability to single-handedly determine the market price yields some interesting additional insight. Despite the fact that her general preference structure of financing remains substantially unchanged, the optimal value of ϕ now tends towards zero, rather than unity when gradually sharpening the decision maker's personal liquidity constraint (i.e., exogenously increasing the financing gap to get deeper into case iv). Hence, compared to the previous chapter, our analysis postulates a larger fraction of outside equity in the firm's stock of capital and, thereby, a lower ownership concentration in the monopoly case. Interestingly, this seems to be indeed a typical feature of large global corporations which enjoy massive market power and often exhibit a massive stock exchange capitalization.¹⁰⁷ Since ϕ^* tends towards zero, the optimal overall firm investment K^* is accordingly greater than in the previous chapters.

¹⁰⁷ See, for example, Kale/Yee (2010) and MacKay/Phillips (2005).

Appendix to Chapter 6

A 6.1: Solving the differential terms in the four FOCs

The four FOCs are given by

$$\begin{split} \frac{\partial \mathcal{L}}{\partial \phi} &= \Omega (\partial \mu_{W_1^D} / \partial \phi - \eta \sigma_{W_1^D} (\partial \sigma_{W_1^D} / \partial \phi)) + v'(\phi) + \lambda_1 - \lambda_2 = 0, \\ \frac{\partial \mathcal{L}}{\partial \alpha} &= \Omega (\partial \mu_{W_1^D} / \partial \alpha - \eta \sigma_{W_1^D} (\partial \sigma_{W_1^D} / \partial \alpha)) + \lambda_3 - \lambda_4 = 0, \\ \frac{\partial \mathcal{L}}{\partial \Gamma} &= \Omega (\partial \mu_{W_1^D} / \partial \Gamma - \eta \sigma_{W_1^D} (\partial \sigma_{W_1^D} / \partial \Gamma)) + \lambda_5 - \lambda_6 = 0, \\ \frac{\partial \mathcal{L}}{\partial p} &= \Omega (\partial \mu_{W_1^D} / \partial p - \eta \sigma_{W_1^D} (\partial \sigma_{W_1^D} / \partial p)) + \lambda_7 = 0. \end{split}$$

Since the first three FOCs are identical to the previous chapter, the corresponding derivation takes place by the same token as in Appendix A 5.1 and yields

$$\begin{split} & \frac{\text{FOC.1:}}{\partial \mathcal{L}} \\ & \frac{\partial \mathcal{L}}{\partial \phi} = (p - c)\mu_Q - F - \eta\phi T ((p - c)\sigma_Q)^2 - (1 + r_2)\Gamma - E((g\mu_Q) - \eta\phi T\sigma_Q^2(p - c)g) \\ & + (v'(\phi) + \lambda_1 - \lambda_2)/\Omega T = 0, \end{split}$$

$$\begin{aligned} & \frac{\text{FOC.2:}}{\partial \alpha} = (1 + r_1/T) - g\mu_Q + \eta\phi T\sigma_Q^2(p - c)g + (\lambda_3 - \lambda_4)/\Omega W_0^D T = 0, \end{aligned}$$

$$\begin{aligned} & \frac{\text{FOC.3:}}{\partial \Gamma} = -(1 + r_2) + g\mu_Q - \eta\phi T\sigma_Q^2(p - c)g + (\lambda_5 - \lambda_6)/\Omega\phi T = 0. \end{aligned}$$

FOC.4 is given by

$$\Omega(\partial \mu_{W_1^D} / \partial p - \eta \sigma_{W_1^D} (\partial \sigma_{W_1^D} / \partial p)) + \lambda_7 = 0$$

which, since $\sigma_{W_I^D} = \phi T \sigma_{\Pi}$ and $\sigma_{\Pi} \equiv \sigma_{\Pi}(p)$, can be expressed as

$$\partial \mu_{W_1^D} / \partial p - \eta \phi^2 T^2 \sigma_{\Pi} (\partial \sigma_{\Pi} / \partial p) + \lambda_7 / \Omega = 0.$$

From the functional form of $\mu_{W_I^D}$ as disclosed by Equation (11), we can infer $\partial \mu_{W_I^D} / \partial p = (\partial \mu_{\Pi} / \partial p) \phi T$. By substituting this expression into FOC.4 and subsequently dividing by ϕT we obtain

$$\partial \mu_{\Pi} / \partial p - \eta \phi T \sigma_{\Pi} (\partial \sigma_{\Pi} / \partial p) + \lambda_7 / \Omega \phi T = 0.$$

From (31), we know that $\mu_{\Pi}=(p-c)\mu_Q-F$ and $\sigma_{\Pi}=(p-c)\sigma_Q$. From (32), we know that $\mu_Q=a-bp$. Hence

$$\partial \mu_{\Pi} / \partial p = -b(p-c) + \mu_{O}$$
 and $\partial \sigma_{\Pi} / \partial p = \sigma_{Q}$.

Substituting both into FOC.4 then yields

$$\frac{\text{FOC.4:}}{\partial p} = -b(p-c) + \mu_Q - \eta\phi T\sigma_Q^2(p-c) + \lambda_7/\Omega\phi T = 0.$$

A 6.2: Formal derivation of the Lerner Index

Consider the Lerner Index as a measure for the size of contribution margin (relative to the price) in the optimum. Denote it by

(47)
$$LI = (p-c)/p.$$

Considering that, from (38),

(48)
$$(p-c) = ((1+r)+\ell)/bg$$
,

and substituting (48) into (47), we obtain

(49)
$$LI = ((1+r)+\ell)/bgp.$$

Further denote by ε the price elasticity of the demand. It is defined by

$$(50) \qquad \varepsilon = \mu_Q' p \ / \ \mu_Q < 0.$$

Solving (50) for p and subsequently substituting into (49) then yields

(51)
$$LI = ((1+r+\ell)/g\mu_Q)(-1/\varepsilon) = (p-c)/p.$$

Hence, we obtain the classical postulate of a negative relation of the Lerner Index with the inverse price elasticity of demand. However, interestingly, this effect is weighted by the term $(1+r+\ell)/g\mu_Q$, which is reflecting *D*'s investment and financing choices and her environmental constraints. From (50), we can further infer that, since $\mu_Q' = -b$, *LI* indeed *negatively* depends on the slope of the demand curve, i.e., the greater the slope *b*, the smaller *LI* and, consequently, the smaller the markup that *D* chooses in the optimum.

A 6.3: Simplifying the system of equations for case i)

We have

FOC.1:

$$\begin{split} (p-c)\mu_Q - F - \eta\phi T((p-c)\sigma_Q)^2 - K((g\mu_Q) - \eta\phi T\sigma_Q^2 g(p-c)) \\ + (v'(\phi) + \lambda_1 - \lambda_2)/\Omega T = 0, \end{split}$$

FOC.2/3:

$$(1+r_1/T) - (g\mu_O) + \eta\phi T\sigma_Q^2 g(p-c) = 0,$$

Substituting FOC.2/3 into FOC.1 yields

FOC.1: $(p-c)(\mu_Q - \eta\phi T\sigma_Q^2(p-c)) - F - K(1+r_1/T) + (v'(\phi) + \lambda_1 - \lambda_2)/\Omega T = 0,$ FOC.2/3:

$$\mu_Q - \eta \phi T \sigma_Q^2(p-c) = (1+r_1/T)/g,$$

which, by repeating the previous substitution and slightly rearranging, displays as

$$\begin{split} & \underline{\text{FOC.1:}} \\ & ((p-c)/g - K)(1 + r_1/T) - F + (v'(\phi) + \lambda_1 - \lambda_2)/\Omega T = 0, \\ & \underline{\text{FOC.2/3:}} \\ & \phi(p-c) = (\mu_Q - (1 + r_1/T)/g)/\eta T \sigma_Q^2. \end{split}$$

Considering that c = f - gK, we obtain

$$\begin{split} & \underline{\rm FOC.1:} \\ & ((p-f)/g)(1+r_1/T) - F + (v'(\phi) + \lambda_1 - \lambda_2)/\Omega T = 0, \\ & \underline{\rm FOC.2/3:} \\ & \phi(p-f+gK) = (\mu_Q - (1+r_1/T)/g)/\eta T \sigma_Q^2. \end{split}$$

By slight rearrangement, these two expressions indeed display as

$$\begin{split} & \underline{\text{FOC.1:}} \\ & -F - ((f\text{-}p)(1 + r_1/T))/g + (v'(\phi) + \lambda_1 - \lambda_2)/\Omega T = 0, \\ & \underline{\text{FOC.2} \equiv \text{FOC.3:}} \\ & \phi K = (\mu_Q - (1 + r_1/T)/g)/\eta Tg\sigma_Q^2 + \phi(f\text{-}p)/g. \end{split}$$

A 6.4: Simplifying the system of equations for case *iii*)

We have

<u>FOC.1:</u>

$$\begin{split} (\mathbf{p}-c)\boldsymbol{\mu}_Q - F - \eta\phi T((\mathbf{p}-c)\sigma_Q)^2 - (1+r_2)\Gamma - E((g\boldsymbol{\mu}_Q) - \eta\phi T\sigma_Q^2(\mathbf{p}-c)g) \\ + (v'(\phi) + \lambda_1 - \lambda_2)/\Omega T &= 0, \end{split}$$

 $FOC.2 \equiv FOC.3:$ $-(1+r_2) + (g\mu_Q) - \eta \phi T \sigma_Q^2 (p-c)g = 0.$

Substituting FOC.2/3 into FOC.1 yields

$$\begin{split} & \underline{\text{FOC.1:}} \\ & (p-c)(\mu_Q - \eta\phi T\sigma_Q^2(p-c)) - F - (1+r_2)K + (v'(\phi) + \lambda_1 - \lambda_2)/\Omega T = 0, \\ & \underline{\text{FOC.2} \equiv \text{FOC.3:}} \\ & \mu_Q - \eta\phi T\sigma_Q^2(p-c) = (1+r_2)/g, \end{split}$$

which differs from case i) only in that the term $(1+r_1/T)$ has been replaced by $(1+r_2)$ (see previous derivation in A 6.3). Hence, these two expressions are indeed equivalent to

$$\begin{split} & \underline{\text{FOC.1:}} \\ & -F - ((f\text{-}p)(1 + \text{r}_2))/g + (v'(\phi) + \lambda_1 - \lambda_2)/\Omega T = 0, \\ & \underline{\text{FOC.2/3:}} \\ & \phi K = (\mu_Q - (1 + \text{r}_2)/g)/\eta Tg\sigma_Q^2 + \phi(f\text{-}p)/g. \end{split}$$

SEVEN

STRATEGIC INTERACTION: THE DUOPOLY CASE

7.1 Preface

Contrary to the preceding chapters, strategic choices in this final analytical stage are not solely influenced by the interplay of market structure and consumer demand properties. This chapter endogenizes the influence of competitors, which allows for a disclosure of the interrelations between the firms' strategic investment, financing, and pricing choices¹⁰⁸ and the resulting competitive outcome.

In a context of imperfect competition, strategic interaction brings about the notion that the outcome depends on the decisions of more than one single player. Hence, the decision maker no longer faces a passive market environment. The possible choices of one firm's decision maker have a direct influence on the behavior of the rival firm's decision maker. Presuming that both are fully aware of this interdependency, an elicitation of the reactive relatedness of competing enterprises' strategic choices necessitates a game theoretical analysis to describe, explain, and predict the outcome of this interactive decision problem. Given the fact that firms' decision makers only have subjective beliefs about the demand properties, the price appears to be the feasible parameter of interaction in this market game.

The investigation of strategic firm interaction in imperfect product market competition constitutes a major domain covered by the academic field of industrial organization. The first conclusive analysis of such oligopolistic market interaction was provided by Cournot (1838) and taken on by Bertrand (1883). In

¹⁰⁸ In a slight abuse of formal accurateness and in order to ease the verbal exposition, this chapter will sporadically refer to "firms" when speaking of their respective decision makers.
the standard Bertrand framework, two symmetric firms produce a homogeneous good at constant marginal costs and are competing in prices. Each firm is getting half of the market if it matches the price of the rival firm, and the whole market if it undercuts its rival's price. As a result, both firms eventually set the competitive price which is equal to marginal costs, hence leading to zero profits. Therefrom originates the so-called *Bertrand 'paradox'*: two market suppliers are sufficient to ensure the perfectly competitive outcome. It may well be argued that the word "paradox" is particularly inappropriate in this context. Given that the Bertrand frame yields a result which blatantly contradicts real world observations. Contrarily to the Bertrand prediction, firms are typically competing in prices and still make profits. Moreover, the Bertrand result renders obscure why firms should even bother to enter a non-monopolistic market at all, given that no profit can be made. This is especially true when supposing fixed costs of entry or production (even if supposed extremely small). Hence, even for minimal costs of entry or production, (imperfectly) competitive markets shouldn't even exist. These considerations rather suggest that the Bertrand result is not a "paradox", but simply the logical consequence of a wrong model that inadequately represents the real world.

A first (non-formalized) approach to iron out the dissatisfying result of the original Bertrand frame was given by Edgeworth (1897, 1925), who introduced capacity constraints exogenously imposed onto the firms. If production capacity is lower than the demand at the competitive price (Q(p=c)), the Bertrand equilibrium can no longer sustain. This result obtains because if one firm charges a price above the competitive level, the rival firm cannot meet all the demand. So the first firm can cover the residual demand and enjoys positive profit. A rigorously formalized advancement of this idea was developed in the capacity precommitting to a certain production capacity before competing in prices, firms are able to extract Cournot profits.

A number of further well-established studies have proposed various alternative refinements: product differentiation (Hotelling, 1929; d'Aspremont et al., 1979; Shaked/Sutton, 1982), tacit collusion (Chamberlin, 1929, 1933), repeated interaction (Friedman, 1971, 1977), consumer switching costs (Klemperer, 1987), consumer neglect of small price changes (Allen/Thisse, 1992), or market segmentation (Roy, 2000; Galeotti/Moraga-González, 2008). These models introduce substantial refinements to the original Bertrand framework, which crucially rests upon the notions of symmetry between competitors and the homogeneous nature of their products.

A particularly interesting class of pricing games was developed by Stahl (1986), Farm/Weibull (1987), and Maskin/Tirole (1988). These authors keep the symmetric Bertrand-Nash concept, but introduce the possibility of imme-

diate price-responses for competing firms. By doing so, they show that equilibrium prices above the Bertrand outcome are possible without explicit collusion or commitment to trigger strategies.

Hence, a wide variety of alternative approaches has been accumulated in the pertinent literature, correspondingly leading to a similarly wide variety of equilibrium solutions of the pricing game between the scrutinized market competitors. Therefore, it feels inadequate to single out one definitive model variant in order to connect it to our financial structure model. Exhaustive examination and rigorous evaluation of price competition would by itself constitute a considerable research program and lies well outside the scope of the present thesis. We will rather rely on the field of industrial organization and outline the properties (and preconditions) of several prominent equilibrium solutions of the pricing game which can be inferred from the pertinent publications in this literature (Section 7.2). We will then subsequently examine how these price equilibria respectively influences and change the decision makers' financial structure choices (Section 7.3).

We consider a two-stage setting as depicted in Figure 17. In Stage 1, the firms simultaneously choose their financial structure according to the model developed in the preceding chapters. The pricing game then follows in Stage 2. In order to determine the optimal behavior of the firms in the first and second stage, the notion of the subgame perfect Nash equilibrium (SPNE) is applied. Hence, the game is solved via backward induction, starting with the derivation of the optimal firm behavior at Stage 2. Firstly, the equilibrium in terms of pricing is determined for given, but arbitrary financial structure choices. Subsequently, the first-stage financial structure equilibrium is derived by assuming that firms correctly anticipate the second-stage pricing-behavior.



Figure 17: Time line and stages of the game.

The remainder of this chapter is organized as follows. Section 7.2 discusses major refinements to the original Bertrand frame and outlines the respective possible equilibrium outcomes of the second-stage pricing game. Section 7.3 then connects these outcomes with the firms' first-stage financial structure decisions. The chapter concludes with Section 7.4, where the obtained results are discussed.

7.2 Stage Two: Optimal Pricing

In the second stage of the game, financial structure is taken as given. For a given financial structure, the decision maker's utility function $U(W_1^D)$ strictly increases in her final wealth W_1^D , which in turn (being a linear transformation of the profit function) strictly increases in Π (see Chapter 3). Hence, we are able to directly base our argument on the firm's profit function.

As indicated above, a vast number of model frames exists in the pertinent literature, each of them relaxing or refining one or several crucial assumptions of the original Bertrand setting. These refined models typically circumvent the problematic Bertrand result of zero firm profits and, hence, yield predictions which appear to be far closer to what can be empirically observed. We will distinctly discuss the properties and outcomes of three major classes of such refinements in order to craft a solid basis for the subsequent examination of the first-stage financial structure decision, which depends on the actual equilibrium that decision makers expect in the second-stage pricing game.

7.2.1 Capacity Constraints

Edgeworth (1897, 1925) identified capacity constraints as a potential property of oligopolistic firms which would allow them to eventually sustain equilibrium prices above the marginal cost level and, hence, make positive profits. If a firm has a production capacity smaller than Q(p=c), it cannot meet all the market demand. The price pair

$$(p_1^*, p_2^*) = (c, c)$$

can no longer be an equilibrium, because an upward deviation by one of the firms would endow this firm with a residual demand of those customers, which cannot be served by the other firm. Hence, the deviating firm faces a positive demand at a positive markup price and, thus, makes a profit. Consequently, the Bertrand solution can no longer be an equilibrium.

The actual equilibrium solution requires an explicit assumption concerning the rationing scheme. Two rationing schemes are particularly pertinent in the literature, namely *efficient rationing* and *proportional rationing*. Efficient rationing supposes that the most "eager" consumers (i.e., those with the highest willingness to pay) buy from firm 1. Firm 2 then faces the residual demand of the remaining customers. Suchlike ordering of customers according to their willingness to pay leads to a maximized consumer surplus, hence the name "efficient rationing". In contrast, proportional rationing rather supposes that customers are uniformly distributed. Hence, all customers have the same probability of being served by a given firm.

For small capacities, Beckman (1967) assumed proportional rationing and showed that firms' reduced-form profit functions¹⁰⁹ have the exact Cournot shape. The same result was obtained by Levitan/Shubik (1972) for the case of efficient rationing. For larger capacities, only a mixed-strategy equilibrium exists. This mixed-strategy equilibrium was identified by Kreps/Scheinkman (1983) for the case of efficient rationing and symmetric capacities. Allen/Hellwig (1986) then showed that as the number of firms increases, equilibria converge in distribution to a perfectly competitive price, while the monopoly prices persist with decreasing but positive probability.

To sum up, all these authors showed that equilibrium price pairs between the marginal cost level and the monopoly price may hold for capacity constrained firms competing in prices.

7.2.2 Product Differentiation

The second crucial assumption in the original Bertrand frame is the perfect substitutability of the firm's products. Consumers are indifferent between goods at an equal price and they buy from the lowest priced producer.

Against the backdrop of real world observations, this assumption is of course very questionable. At least part of the customers are arguably willing to pay a premium in order to purchase a product with a certain attribute, like for example brand name or provenience. Such attributes are differentiators among competing firms' products and may substantiate in a multitude of further di-

 $^{^{109}\,}$ Reduced-form profit functions are

[&]quot;(...) profit functions that would obtain if the firms produced quantities (...) and an auctioneer picked the price so as to clear the market." (Tirole, 1993, p. 216.)

mensions, such as sales point location, delivery time, subjective quality perceptions, availability, consumers' information etc.

If the assumption of perfectly homogeneous products is relaxed, the demand for each firm's product positively depends on the price charged for the competitor's product. Technically speaking, the cross-elasticity of the demand is finite. The outcome of such price competition with differentiated products differs from the Bertrand conclusions. The firms charge prices above the marginal cost level and, consequently, reap positive profits.

To exemplify this, one may think of two symmetric firms located at different places.¹¹⁰ Customers are geographically dispersed and uniformly distributed. Travelling to the firms' sales points entails positive costs for the customers. If Firm 1 charges $p_1 = c$, Firm 2 can still keep at least a few customers who are located nearby if it charges a higher price $p_2 = c + x$ (where x is a small markup). For these customers, the travelling costs are lower than the price differential x. Hence, the competitive price pair $(p_1, p_2) = (c, c)$ is no equilibrium anymore.

Hotelling (1929) showed that the equilibrium price pair of such a game indeed lies above marginal costs. The actual value of the equilibrium price positively directly depends on the degree of differentiation.¹¹¹ Hence, the two extremes cases of

- *no differentiation* (both firms are located in the same spot), or
- total differentiation (firms serve mutually exclusive geographical areas)

yield exactly the two polar equilibrium solutions: the original Bertrand outcome for the case of no differentiation, and the monopoly price pair for the case of total differentiation). The basic relation between the degree of differentiation and the resulting equilibrium market price p^* is depicted in Figure 18.

 $^{^{110}\,}$ Example adapted from Tirole (1993), p. 212.

¹¹¹ See Tirole (1993), p.212. A general outline on how to measure the degree of product differentiation is provided by, for example, Pfähler/Wiese (1998).



Figure 18: Equilibrium Price with Product Differentiation

7.2.3 Dynamic Price Competition and Flexibilized Pricing

The Bertrand frame assumes irrevocable price announcements. Firms are rigidly committed to their announced price with no possibility for later changes. This may be criticized on two major grounds.

Firstly, competing firms are likely to interact more than once. Consequently, pricing decisions should be at least possible before each iteration. Chamberlin (1929) was the first to suggest that such repeated interaction may annul the Bertrand outcome, since the threat of a vigorous price war deters the firms' seduction to undercut their respective rival. Hence, the competitors may tacitly collude in a completely non-cooperative manner.

Secondly, one may wonder why firms should not be able to instantly react to the price announcement of their competitor. Admittedly, detection lags such as sealed-bid proposals in markets with a monopsonist buyer may justify the original Bertrand assumptions.¹¹² However, such detection lags are far from frequent. Major industries, especially markets for commodities like gasoline or markets for capital equipment, are characterized by very open and flexible pricing behavior. Stahl (1986) and Farm/Weibull (1987) crafted model frames where firms can immediately respond to prices announced by their competitors. These authors assumed (small) costs of such immediate price changes and ar-

¹¹² Professional services industries such as top management consulting may serve as an example here.

rive at the conclusion that a positive price-cost margin obtains as equilibrium result.

Both notions share the common property that the threat of retaliatory price cuts triggers a implicitly collusive equilibrium situation. However, the underlying motives for "retaliation" differ in the two approaches. Given our basic assumptions concerning the time frame of the model, repeated interaction does not seem to fit well into the overall frame set up throughout the present thesis. Hence, we will rather rely on the notion of flexibilized pricing in one-shot games in order to present a consistent argument for the decision maker's eventual price expectations.

Stahl (1986) studies price competition for a setting where prices are revocable, but at the cost of delaying sales. He shows that any price between the marginal cost level and the monopoly price can be sustained as an equilibrium. Admittedly, the resulting large set of possible equilibrium price pairs may be seen as an embarrassment of riches. Yet, Stahl (1986) argues that the monopoly price pair obtains as what he calls the "natural focal equilibrium" (Stahl, 1986, p.89). This means that the monopoly price pair exhibits the property that the resulting firm returns

- are symmetric, and
- are Pareto optimal from the viewpoint of the two competing firms.

Hence, again, a multitude of possible equilibrium market prices can potentially be expected by the competing firms' decision makers. The following section (7.2.4) takes on the thread of Stahl (1986) and crafts a refined model of revocable pricing in order to arrive at a stronger equilibrium prediction.

7.2.4 A Special Case: Equilibrium Analysis for Costlessly Revocable Prices

The previous section has shown that flexibilized pricing games circumvent the problematic Bertrand result of zero firm profits. Stahl (1986) shows that monopoly price pair obtains as a focal equilibrium among a rich set of candidates. However, this shared monopoly outcome may also be derived as a truly unique equilibrium if assuming costlessly revocable prices. The present subsection models this special case in order to enrich the insight with respect to such flexibilized one-shot pricing games and to supplement the findings of Stahl (1986). The hurried reader is, however, welcome to skip the present subsection in order to proceed to the first-stage analysis.

Suppose that there are two identical firms, indexed $i, j = 1, 2, i \neq j$, producing a homogeneous good. Sales are shared equally if prices p_i and p_j are equal, resulting in an equal profit of Π^{shared} for each of the two firms. Otherwise the whole market is captured by the firm which has set the lower price and makes a profit of Π^{full} , leaving zero demand for the other firm.

Firms publicly announce their prices and can *immediately* and *costlessly* revise these prices in a sequence of alternating moves, hence mutually responding to their rival's previous price announcement.¹¹³ In the resulting adjustment process, each firm's chosen price is a best response to the price the other firm chose the move before. This second stage lasts until the adjustment process settles down to a "steady state" where the prices are fixed, i.e., none of the firms changes its previously announced price anymore. Production and trade only start after the prices are fixed. Hence, firms are merely concerned with the situation at the end of the second stage, after all price responses have been completed and the steady state is reached. Therefore, we are *not* considering a supergame. No payoffs occur before the sequence of price changes ends, and these payoffs only and merely depend on the final price pair.¹¹⁴ In this respect, the model frame is similar to the approach of Marschak/Selten (1978).

The firms play a game of complete information. It is assumed that a firm's investment, financing, and pricing choices can be observed by the rival firm's decision maker. Empirical evidence here obviously depends on the specific industrial context. However, Katz (1991) argues that strategic variables may have commitment value despite being unobservable. Hence, our results may still hold with incomplete information.

Since demand is downward sloping, profit Π is strictly concave in price with a global interior maximum at the monopoly price p^M . Due to the positive fixed costs which have to be borne by both firms, the profit made by one single firm by taking the entire market at a given price exceeds the sum of the profits made if the market is shared at that price. With two firms, the difference is exactly amounting to F, i.e., $\Pi^{full}(p) = 2\Pi^{shared}(p) + F$, $\forall p$ (see Figure 19).

¹¹³ Hence, the second stage of the model can be interpreted as a sequential subgame with an indefinite number of moves and where each move (i.e., price response) has a negligible duration.

¹¹⁴ A supergame would induce payoffs for each "round" of moves, contingent on the strategies played in that round. Similar to Stahl (1986), the above outlined game absolutely has a one-shot nature, hence ruling out the possibility to attribute the attainability of a collusive outcome to an application of the Folk theorem.



Figure 19: Properties of the firm profits.

We make use of the following definition in order to alleviate the subsequent analysis.

Definition 7.1: Denote by \tilde{p} the unique price which satisfies

$$\tilde{p} < p^M$$
 \wedge $\Pi^{full}(\tilde{p}) = \Pi^{shared}(p^M).$

As can easily be verified by looking at Figure 19, either firm makes a greater profit by sharing the market at the monopoly price p^M than by taking the whole market at a price below \tilde{p} .

In Stage 2, the firms make their sequential price announcements, alternating at each move $n \in (1, ..., k)$. Thereby, each price announcement is welldefined and denoted by $p_n^{i/j}$. Without loss of generality, assume that Firm 1 moves first and announces a price p_1^1 . Firm 2 then responds at move two by announcing a price p_2^2 . Firm 1 can now change its price at move three to p_3^1 , and in this case, Firm 2 has the option to change at move four to p_4^2 and so forth. This sequential adjustment process ends as soon as any of the two firms decides to leave its previously announced price unchanged, yielding the final price vector (p_{k-2}^i, p_{k-1}^j) where both prices are mutual best responses.

move 1 2 3 4 ... k-2 k-1 k
price
$$p_1^1 \quad p_2^2 \quad p_3^1 \quad p_4^2 \quad \dots \quad p_{k-2}^i \quad p_{k-1}^j \quad p_k^i$$

 $\stackrel{!}{=}$

Figure 20: Sequence of alternating price adjustments until move k.

Since all price announcements are observable, all past actions are common knowledge, hence yielding a commonly known sequence of previously announced prices $(p_1^i, p_2^j, ..., p_{n-1}^i)$ at each move n. Such a sequence up to that move is called the *history* h^n . A strategy specifies a price announcement p_n^i for each move n, contingent on the history up to that move. Any strategy pair (s^i, s^j) induces a well-defined sequence of alternating price announcements which lasts until, for any firm, $p_n^i = p_{n-2}^i$, meaning that this firm keeps its previously announced price unchanged. Since trade only starts after the steady state is reached, profits only depend on the final price vector (p_{k-2}^i, p_{k-1}^j) that is induced by a strategy pair.

By means of the following two propositions, we proceed by first identifying the equilibrium outcome of the described pricing game and then checking for uniqueness.

Proposition 7.1: The shared monopoly outcome (p^M, p^M) with both firms charging the monopoly price is an equilibrium.

Proof: Consider the following strategy s^{i*} :

$$p_{n}^{i} = \left\{ \begin{array}{cccc} p^{M}, & \mbox{if} & n = 1 \\ & \mbox{or} & p_{n-1}^{j} \geq p^{M} \\ & \mbox{or} & \mbox{min} \, \{p_{n-2}^{i}, p_{n-1}^{j}\} \leq \tilde{p} \\ & \mbox{p,} & \mbox{otherwise} \end{array} \right. .$$

Basically, the strategy profile s^{i*} prescribes either firm to announce p^M if the lowest currently announced price has fallen to \tilde{p} , and to match the monopoly price if announced (or exceeded) by the rival. Otherwise, \tilde{p} is announced, which then in turn initiates a consecutive joint move to the monopoly price. If a focal firm's rival complies with s^{i*} , the rival's only possible price announcements are p^M and \tilde{p} , for which s^{i*} prescribes p^M as the focal firm's reaction in both cases. Hence, only the monopoly price can be repeated by a firm in accordance with s^{i*} . Since p^M maximizes either firm's payoff, no firm can do better by deviating from this strategy profile, given that the other complies with it. Thereby, we obtain $s^{i*} \rightarrow p_1^i = p^M$ and $s^{2*} \rightarrow p_2^2 = p^M$.

Considering the above strategy profile renders evident the fact that actual price movements occur only off the equilibrium path. No price adjustment process actually takes place in equilibrium, since neither firm has an incentive to deviate from the monopoly price, given the pricing strategy of the rival firm. Announcing the critical price \tilde{p} can be interpreted as a signal for the intention to return to the monopoly price, since repeating \tilde{p} can never be rational, given the rival's compliance with s^{i*} .

Proposition 7.2: Assume that

- (i) firms do not play (weakly) dominated strategies,
- (ii) profits are strictly concave in price with an inner maximum at the monopoly price.

Then, the equilibrium outcome (p^M, p^M) is unique.

Proof: Denote by S^* the set of undominated strategy pairs (s^i, s^j) . We need to show that all strategy pairs in S^* induce the shared monopoly outcome. This is done in three consecutive steps:

- (1) Any arbitrary strategy which drives the corresponding firm towards leaving the entire market to its rival yields zero profits. Hence, any other strategy that prescribes a matching of the rival's price does at least as good (i.e., it yields zero profits if the rival firm in turn undercuts the price until marginal cost level, or positive profits otherwise). Consequently, any strategy which implies leaving the market to the rival firm is dominated and, thus, cannot be part of a strategy pair in S^* . Inversely argued, all strategy pairs in S^* induce a shared market with identical prices.
- (2) Given that all equilibrium outcomes imply a shared market, the maximal equilibrium profit that a firm can make is the shared monopoly profit $\Pi^{shared}(p^M)$.
- (3) A strategy pair such as (s^{1*}, s^{2*}) (see proof to Proposition 7.1) which gives rise to the outcome (p^M, p^M) ensures the maximal possible equilibrium profit for each firm. Since no firm can do better, any strategy pair in S^* must induce the outcome (p^M, p^M) .

Consequently, any subgame perfect equilibrium with undominated strategies must yield the price pair (p^M, p^M) as the unique outcome.

By explicitly allowing for costlessly revocable price announcements before the advent of customers, the above analysis shows that, in the second-stage pricing subgame, firms share the market equally and charge the monopoly price. At first glance, this is an astonishing result in the light of the standard Bertrand predictions. However, it only reinforces the well established view that the theoretical analysis of market games is heavily sensitive to the commitment of players – no matter if voluntary or involuntary. Irrevocable pricing (such as in the standard Bertrand framework) can be interpreted as an implicit form of involuntary commitment. Hence, it is not very surprising that removing this commitment leads to drastically altered results. Given that only the final price pair determines the payoffs to the firms, the second stage of the overall game yields a unique, Markov perfect Nash equilibrium (MPNE) where both firms eventually charge the monopoly price. As Fudenberg/Tirole (1991) outline, Markov strategies are generally characterized by the common property that

"(...) the past influences current play only through its effect on a state variable that summarizes the direct effect of the past on the current environment. A Markov perfect (Nash) equilibrium (...) is a profile of Markov strategies that yields a Nash equilibrium in every proper subgame." (Fudenberg/Tirole, 1991, p. 501.)

Obviously, price (more precise: each current pair of prices) is the state variable in the present context.

The shared monopoly outcome does not obtain as a consequence of explicit collusion, since firms are acting independently and do not cooperate in a direct sense. It is also not a consequence of mutual punishment strategies of the players. The collusive outcome is rather driven by the important assumption that firms consider the mutual possibility to respond to each price announcement of the rival before the advent of customers.

It is important to note that, ex ante, the number of potential price reactions has to be unlimited. If such a limit would exist, then there would be a last round of price revision in which the players would try to outmaneuver each other, just as in a finitely iterated prisoners' dilemma. Consequently, this end game effect would bring forth the same outcome as the standard Bertrand pricing game.

As outlined in the previous subsection (see 7.2.3), flexibilized pricing games have firstly been examined by Stahl (1986), Farm/Weibull (1987), and Maskin/Tirole (1988). The main difference between Maskin/Tirole (1988) and our setting is that the present game is truly one-shot and firms have total freedom in adjusting their prices. In Maskin/Tirole (1988), firms sell repeatedly to customers, but prices cannot be adjusted at every moment. Sales are hence realized according to the current price vector in every period. Thereby, the authors derive conditions under which Edgeworth price cycles¹¹⁵ are a MPNE. We have

¹¹⁵ Edgeworth price cycles are asymmetric price variations and work as follows. If one firm undercuts its rival, it will capture all or a very large portion of the market. Beginning from an above-marginal-cost price-level, one firm initiates a round of undercutting by pricing below the current level. The rival in turn responds with a match or a slight undercut. Undercutting continues until the price equals the marginal cost level. One competitor then restores prices. The rival follows as quickly as possible, and the cycle reiterates. See Edgeworth (1925) and Maskin/Tirole (1988).

presented a much simpler pricing game. Our analysis underscores that the shared monopoly outcome may sustain even if firms only sell once.

Our setting yields the shared monopoly outcome as the unique equilibrium. This result contrasts the findings of Stahl (1986), who considers a revocable pricing game with fixed stocks and indirect costs of delayed sales. His setting induces a continuum of equilibria, in which the shared monopoly outcome only constitutes one possibility. In contrast, with neither fixed stocks nor costs of delayed sales being relevant in the present model, we arrive at the clear prediction of p^M as the only price that can prevail in equilibrium.

The stability of the equilibrium solution obtains as a direct consequence of the scrutinized pricing game: all price cuts can be immediately met with retaliatory price cuts; hence undercutting the rival's price would be a myopic strategy. It is, however, important to understand that the firms are by no means engaged in a repeated game. The Bertrand result is circumvented despite keeping the one-shot nature of the original game. As pointed out earlier, symmetry of competitors and homogeneity of products is likewise maintained.

One interesting feature of our setting is that it exhibits a compelling connection to the field of bargaining theory. It may be interpreted as a transfer of alternating-offers-bargaining (Rubinstein, 1982; Ståhl, 1972) into a context of price competition. In such settings, players bargain over a "pie" of a certain size. Players make alternating offers on how to divide the pie, and the respective other player may accept or reject. For each rejection, the pie "melts" by a certain discount rate, which represents rejection costs or probability of breakup. With regard to our setting, the maximum pie may be interpreted as the maximum industry profit $Q(p^M)$, and the resulting collusive shared monopoly outcome resembles a cooperative equilibrium with no discounting. However, the solution of our setting is more robust since, as repeatedly stressed, the aboveoutlined game requires no explicit cooperation in order to attain the "maximum pie"-solution.

7.2.5 Interim Conclusion

This section has presented three major classes of pricing games that deviate from the original Bertrand frame in one respective crucial characteristic. Be it the consideration of capacity constraints, different kinds of product differentiation, or flexibilized pricing – all of these refinements considerably enlarge the set of possible equilibrium solutions of the pricing game up to the whole interval between the monopoly price and competitive price (i.e., at marginal cost level). Subsection 7.2.4 crafted an exemplifying setting of costlessly revocable pricing decisions which yields the shared monopoly outcome as a unique equilibrium. Similar to Stahl (1986), we feel the need to underscore that the scarce evidence of such alternating price fluctuations should not be seen as a severe indication against the relevance of our result. The equilibrium analysis has shown that actual price movements occur only off the equilibrium path. Thereby, the general result obtains: allowing for revocable price announcements may enable firms to tacitly collude and, hence, reach the shared monopoly outcome – despite a formally non-cooperative setting.

It needs to be stressed that the pertinent literature harbors a multitude of further refinements of the original Bertrand pricing game, each of them yielding equilibrium prices above marginal costs. Other assumptions that can be relaxed include the neglect of possible switching costs (Klemperer 1987), the perfect pricing information by consumers (Diamond, 1971), or the absence of institutional facilitators for collusion (Kirstein/Kirstein 2009).

Hence, we conclude that, in general, the second-stage pricing game may yield a multitude of equilibrium price pairs, contingent on the exact properties of the firms involved and on the prevalent rules of the game. It was shown that, depending on the exact circumstances, price pairs above marginal costs up to the shared monopoly outcome are attainable as unique equilibrium solutions of the second stage. The analysis of the first stage must therefore disclose, whether and how these different potential pricing games (and their respective equilibrium solutions) influence the decision makers' financial structure decisions if she rationally anticipates the outcome of the prevalent competitive context.

7.3 Stage One: Optimal Financial Structure Choice

In the first stage of the overall game, both firms simultaneously conduct their financial structure choice. Hence, each firm chooses the vector (ϕ^* , α^* , Γ^*) while anticipating the equilibrium outcome of the second-stage pricing subgame.

As shown in the preceding subsection, a multitude of price pairs may obtain as the equilibrium outcome in stage two, contingent on the existence of capacity constraints, on the nature of the produced good, and on the exact rules of the prevalent pricing game. Therefore, and in order to keep things general, we will subsequently examine the firms' first-stage investment and financing decisions for two distinct scenarios, i.e.,

- (1) the equilibrium price expected by the decision makers is equal to the monopoly price (Section 7.3.1);
- (2) the equilibrium price expected by the decision makers is lower than the monopoly price (Section 7.3.2).

The prevalent competitive context is known in advance. Hence, since the competing firms are symmetric, we can suppose homogeneous expectations of the decision makers with respect to the outcome of the second-stage pricing game.

7.3.1 Financial Structure Choice for Expected Equilibrium Prices at the Monopoly Level

We have shown that the monopoly price pair obtains as the unique equilibrium outcome if prices are costlessly revocable before the advent of customers. Even when supposing that these price changes entail costs of delaying sales, the monopoly price pair still is the natural focal equilibrium in the sense of Stahl (1986).

Thereby, we consider the first scenario where each firm chooses $(\phi^*, \alpha^*, \Gamma^*)$ while predicting that, eventually,

$$p^{i*} = p^M$$
 and $Q^{Market} = Q(p^M)$

Hence, since the second-stage equilibrium predicts an equally shared market, each firm expects to eventually serve only half of the total market demand (see Figure 1), i.e.,

$$Q^{i*} = \frac{Q(p^M)}{2} = (\mu_Q + \sigma_Q z)/2$$

Hence, the decision maker of a duopolistic firm considers that

$$\mu_Q^i = \mu_Q/2$$
, and $\sigma_Q^i = \sigma_Q/2$.



Figure 21: The anticipated demand.

In order to disclose the consequences of a shared market, we examine the inverse expected market demand (i.e., the inverse expected demand that a monopolist would face), which is given by

(52)
$$p(\mu_Q) = a/b - (1/b)\mu_Q$$
.

The ratio a/b depicts the prohibitive price from which on no positive demand exists in the market. To accordingly obtain the inverse expected demand of the duopolist *i* (who considers that $\mu_{Q^i} = \mu_Q/2 = a/2 - (b/2)p$), we have to substitute *a* by a/2 and *b* by b/2. Thereby,

(53)
$$p(\mu_{Q^i}) = a/b - (2/b)\mu_{Q^i}$$

Interestingly, this inverse expected demand curve of the duopolist is exactly identical to the expected marginal revenue curve (MR) of the monopolist. Figure 22 gives a graphical representation of the focal situation.



Figure 22: The duopoly case with the monopoly price pair as equilibrium.

Since the (duopolistic) decision maker now takes the price as given by the expected second stage equilibrium outcome, her only remaining possibility to calibrate the contribution margin according to her liking is to determine the marginal cost level by means increasing or decreasing the corporate investment level K. Figure 22 shows that the monopoly price p^M induces the same optimal marginal cost level for the duopolist as for the monopolist, since in both cases, the marginal revenue equals marginal costs. Hence, as a duopolist who expects that, eventually, the market price will be p^M , the decision maker sets the same K as if she was a monopolist. Accordingly, D's target levels of corporate investment are completely unaffected by the entrance of a rival firm if the shared monopoly outcome is expected in the first stage.¹¹⁶ Thereby, her financial structure decisions are exactly identical to her choices as disclosed throughout the previous chapter. However, due to the shared market demand, firm profits (and

¹¹⁶ See Appendix A 7.1 for a formal algebraic proof.

D's return on the firm investment) have accordingly dropped compared to the monopoly case.

Thereby, we are taken back to our well-known four cases which summarize the decision maker's behavioral pattern in terms of investing and financing in the optimum. If the decision maker anticipates the monopoly price as the second-stage equilibrium outcome, her optimal financial structure choice in Stage 1 completely corresponds to the monopoly scenario (see Chapter 6).

$$\begin{split} i) \quad \text{if} \qquad W_0^D > \mathrm{TL}^{\mathrm{Max}}, \\ \Rightarrow \qquad \underline{\phi} \leq \phi^* = \widehat{\phi} < \frac{1}{2}, \\ \alpha^* > 0, \\ \Gamma^* = 0, \\ K^* = \mathrm{TL}^{\mathrm{Max}} / \phi^*. \end{split}$$

$$\begin{array}{ll} ii) \quad \mathrm{if} & \mathrm{TL}^{\mathrm{Min}} < W^D_0 \leq \mathrm{TL}^{\mathrm{Max}}, \\ \Rightarrow & \underline{\phi} \leq \phi^* < \widehat{\phi} < \frac{1}{2}, \\ & \alpha^* = 0, \\ & \Gamma^* = 0, \\ & K^* = (\mathrm{TL}^{\mathrm{Max}} - \lambda_3 / \Omega W^D_0 \eta T^2 g^2 \sigma_Q^2) / \phi^*. \end{array}$$

$$\begin{array}{ll} iii) \mbox{ if } & W^D_0 \leq {\rm TL}^{\rm Min} < W^D_0 + \widehat{\phi} \ \overline{\Gamma} \ , \\ \\ \Rightarrow & \underline{\phi} \leq \widehat{\phi} < \phi^* < \widehat{\phi} < \frac{1}{2} \ , \\ \\ & \alpha^* = 0 , \\ \\ & \overline{\Gamma} > \Gamma^* = ({\rm TL}^{\rm Min} - W^D_0) / \phi^* , \\ & K^* = {\rm TL}^{\rm Min} / \phi^* . \end{array}$$

$$\begin{split} iv) \quad \text{if} \qquad & W_0^D + \widehat{\phi} \ \overline{\Gamma} \leq \text{TL}^{\text{Min}}, \\ \Rightarrow \qquad & \underline{\phi} \leq \phi^* \leq \widehat{\phi} < \frac{1}{2}, \\ & \alpha^* = 0, \\ & \Gamma^* = \overline{\Gamma}, \\ & K^* = (\text{TL}^{\text{Min}} - \lambda_6 / \Omega \phi^* \eta T^2 g^2 \sigma_Q^2) / \phi^*. \end{split}$$

7.3.2 Financial Structure Choice for Expected Equilibrium Prices below the Monopoly Level

Depending on the rules of the second-stage pricing game, a multitude of different equilibrium outcomes below the monopoly price level may obtain (see section 7.2). Therefore, it feels necessary to examine the impact of such "lower" equilibrium outcomes on the decision maker's first-stage financial structure choices.

From (31) and (32), we know that $\mu_{\Pi}{}^{i} = (p^{i}-c)\mu_{Q} - F = (p^{i}-c^{i})(a-bp^{i}) - F$. By setting the first derivative $\partial \mu_{\Pi}{}^{i} / \partial p^{i}$ equal to zero and solving for p^{i} , the optimal price p^{i*} must satisfy

(54)
$$p^{i*} = (a/b + c^{i*})/2.$$

Equation (54) is in line with the well-established optimal pricing rule which characterizes a market with a linear demand function.¹¹⁷ However, since the eventual market price is taken as "exogenously" given by the expected secondstage equilibrium, the marginal cost level has to be adjusted in order to satisfy the above optimality condition for different expectations regarding the equilibrium market price. By solving (54) for c^{i*} , we obtain

(55)
$$c^{i*} = 2p^{i*} - a/b$$
.

The above expression immediately confirms our conclusion from Figure 22: the mere entrance of a rival firm without expecting a change in the eventual price level (i.e., if $p^{i*} = p^M$) leaves the marginal cost level unchanged (since the prohibitive price remains constant, i.e., a/b = (a/2)/(a/2)). Taking the first derivative $\partial c^{i*}/\partial p^{i*}$ accordingly yields

$$(56) \qquad \partial c^{i*} / \partial p^{i*} = 2 > 0,$$

which discloses that each change of the expected equilibrium price leads to a change of the optimal marginal cost level into the same direction. Marginal costs do not only follow the price change, but actually amplify it by the factor two. Hence, if the decision maker expects the equilibrium price $p^{i*} = p^M - x$ (where x is an arbitrary positive number), then $c^{i*} = c^M - 2x$. This is illustrated by Figure 23.

¹¹⁷ See Pfähler/Wiese (1998), p. 66.



Figure 23: The duopoly case with a lower equilibrium price pair.

If the decision makers expect a second stage equilibrium price level below the monopoly price, the marginal costs are lowered (by increasing K) in order to keep the contribution margin great enough. Figure 23 and Equation (56) show that the contribution margin (p-c) is not just kept constant. It is actually increased, in order to compensate for the increased level of risk due to the greater investment intensity.

As soon as the expected equilibrium price falls below the level where c is pushed to zero,¹¹⁸ the marginal costs cannot be lowered anymore and the contribution margin gradually diminishes again. The same is true for the expected returns of the decision maker. If the expected equilibrium price further decreases, the contribution margin further shrinks until the critical point is reached where D's expected return on the firm investment $\phi \mu_{II} T / \phi K - 1$ is no more ex-

¹¹⁸ For simplicity, we assume that $\underline{c}=0$. This is in line with our prior assumption $\overline{K} < f/g$ (see Section 6.2.3 and Footnote 97).

ceeding the alternative riskless asset's return r_1 . In other words, we are drawn towards the limit of our basic assumption, i.e.,

$$(\phi \mu_{\Pi} / \phi K - 1)T \rightarrow r_1$$

 \Leftrightarrow
 $(\mu_{\Pi} / K - 1)T \rightarrow r_1.$

If the decision maker would expect an equilibrium market price below this critical level, she would minimize her investment into the firm and accordingly invest as much as possible into the alternative asset, i.e. α^* would jump to $\overline{\alpha}$. Figure 24 gives a graphical representation of the relationship between the expected equilibrium price p^{i*} and α^* for the optimality case iv).



Figure 24: Personal Firm Investment and Equilibrium Price for case iv).

 \hat{p} is the critical price for which $(\mu_{\Pi}(\hat{p})/K-1)T = r_1$. Hence, the dashed vertical line depicts the limit of our basic assumption. The decision maker has an incentive to invest her private wealth into the firm if (and only if) the expected equilibrium price exceeds this level. Lower expected equilibrium prices would induce D to reduce her personal investment into the firm to the minimum amount $(1-\overline{\alpha})W_0^D$. Simultaneously, she would exploit the full debt capacity in order to opportunistically maximize her personal returns from this minimum personal firm investment.

Concerning the decision maker's summarized investment and financing behavior, she augments the firm's stock of capital K compared to the monopoly scenario if she expects a lower equilibrium market price in the second stage. Accordingly, her target levels of corporate investment are increasing if the expected equilibrium market price decreases.¹¹⁹ Thereby, the vertical lines in Figure 16 are shifted to the right, as depicted by Figure 25.



Figure 25: Properties of $(\phi^*, \alpha^*, \Gamma^*)$ in the duopoly case.

7.4 Conclusion

This chapter integrated the model frame developed in the preceding chapters into a strategic duopoly context by outlining a two-stage game with initial investment and financing choices and subsequent pricing decisions. By considering substantial refinements to the original Bertrand frame, we circumvent the corresponding zero-profits-result despite considering price competition.

It has been shown that the resulting equilibrium market price in Stage 2 crucially depends on the prevalent competitive context (including firm properties like production capacity, or the nature of the offered product, or the exact rules of the pricing game). Therefore, we examined how the respective competitive context (with the decision makers' corresponding price expectations) influ-

¹¹⁹ See Appendix A 7.2 for a formal algebraic proof.

ence the investment and financing behavior in Stage 1. We distinguished the case where the prevalent competitive context gives rise to an expected shared monopoly outcome (e.g. if prices are costlessly revocable or if products are to-tally differentiated), and the case where a lower price pair is expected (e.g., if firms are capacity constrained or if products exhibit an intermediate degree of differentiation).

If the decision makers expect the equilibrium of the pricing-subgame to be characterized by $p^{i*} = p^M$, each firm's optimal financial structure decision in the initial stage corresponds to the monopoly case, as long as the duopolist's demand is sufficient to satisfy our basic assumption $(\mu_{\Pi}/K - 1)T > r_1$. If, however, a lower equilibrium price pair is expected, the decision makers' target levels of corporate investment augment and the firms investment more intensely in order to reduce the marginal cost level. At first glance, this may sound astonishing, since, apparently, less favorable circumstances (which lead to lower-thanmaximal equilibrium outcomes) lead to a *higher* monetary investment into the firm. The reason for that is the decision maker's monetary risk aversion. Risk aversion implies that the decision maker's utility function is concave in money. Hence, compared to the monopolist, the duopolist faces a far higher marginal utility of wealth. In other words, the slope of the utility function at the locus of the duopolist's expected final wealth is higher, since the shared market implies in any case

$$W_1^{D^{Duopolist}} < W_1^{D^{Monopolist}}.$$

Now recall that the target levels are reference points where an optimal trade-off between D's (expected) personal wealth-gain and her risk exposure is attained (see Chapter 4). Since each monetary unit invested into the firm exhibits a higher marginal utility for the duopolistic decision maker compared to the monopolist, and since D's risk-exposure rises with the intensity of the firm investment, this optimal trade-off point has to be higher for a risk-averse duopolist.

Appendix to Chapter 7

A 7.1: Target levels for $p^{i*} = p^M$

From (45), the target levels are given by

(57)
$$\mathrm{TL}^{\mathrm{Max/Min}} = (\mu_Q - (1 + \mathcal{r} + \boldsymbol{\ell})/g)/\eta T \sigma_Q^2 g + \phi(\mathbf{f} - p)/g.$$

From Equation (36) we further infer that

(58)
$$\phi(p-c) = (\mu_Q - (1+r+\ell)/g)/\eta T \sigma_Q^2$$

Substituting (57) into (36) yields

(59)
$$\phi(p-c) = gTL^{Max/Min} - \phi(f-p)$$

which, by solving for $\mathrm{TL}^{\mathrm{Max}/\mathrm{Min}}$ displays as

 $\mathrm{TL}^{\mathrm{Max/Min}} = \phi(f-c)/g.$

Since c = 2p - a/b, (see Equation (55)), we accordingly obtain

(60)
$$TL^{Max/Min} = \phi(f + a/b - 2p)/g.$$

Since the prohibitive price remains constant if a rival enters the market (a/b = (a/2)/(a/2)), the target levels remain unchanged as long as the expected price corresponds to the monopoly scenario.

A 7.2: Target levels for $p^{i*} < p^M$

In order to show how the target levels respond to a change in the market price, we take the partial derivative from Equation (60) with respect to p.

(61)
$$\partial \mathrm{TL}^{\mathrm{Max/Min}}/\partial p = -(2\phi/g)p < 0$$
.

Hence, since the target levels are decreasing in p, a lower expected market price leads to higher target levels of corporate investment.

$\mathbf{EIGHT}^{CHAPTER}$

SUMMARY AND DISCUSSION OF RESULTS

8.1 Summary of Results

Explaining firms' financial structure decisions remains intensely debated in the corporate finance literature. Throughout the literature review in Chapter 2, we highlighted that in more than fifty years of research since Modigliani/Miller (1958), a surprisingly far-ranging disaccord between numerous conditional approaches has been established. Moreover, fundamental contradictions between the predictions made by established theories and empirical observations exist. Apart from these disaccords and contradictions, we identified three general problems in the pertinent theoretical literature:

Problem 1: A conceptual separation which isolates the respective analyses of *financing* and *investment* decisions from each other.

- Problem 2: A widespread disregard of corporate control considerations.
- **Problem 3:** A conceptual separation which isolates financial structure analysis from
 - (1) the properties of the *competitive context*, and
 - (2) the properties of the focal firm's *operations/cost structure*.

The central purpose of the present thesis has been to account for these three shortcomings in order to explain an owner-managed firm's financial structure choices. To the best of our knowledge, it provides the first theoretical study of financial structure choice which

- simultaneously endogenizes financing and investment decisions,
- accounts for the implications of the decision maker's corporate control considerations arising on the grounds of her eventual ownership share, and
- examines the crucial role of firms' individual and environmental characteristics (including the decision maker's risk attitude, the prevailing output market conditions, and the cost structure).

The present study has examined the interplay between these notions with respect to their overall impact on corporate investment and financing decisions. Thereby, this thesis, firstly, means to be a contribution towards overcoming the existing research gap concerning the linkage between the antecedents of financial structure choice and the output market conditions encountered by firms. It further means to contribute to an analytical consolidation of corporate financing and investment behavior. Lastly, by supposing the focal decision maker's utility function to depend on both, monetary income and decision power, our model combines these considerations with the implications arising from D's corporate control affinity.

Chapter 3 outlines the general time frame of the model and discloses the building blocks of the model, which serves as the basis for the subsequent derivation of results throughout Chapters 4, 5, 6, and 7. Chapter 4 provides a simplified analysis which, for the time being, passes on the firm's actual revenue and cost structure by merely assuming that profits are linearly increasing in the firm investment. This simplified analysis distinctly examines the decision maker's behavior in terms of investing and financing, contingent on the interplay of the different model parameters. Our setting adopts a holistic view by considering that the optimal relative proportions of debt and equity (the financing decision) are tightly interwoven with the determination of their absolute levels (the investment decision).

A central result obtained throughout Chapter 4 is the endogenous derivation of precise corporate investment objectives which are contingent on the decision maker's subjective beliefs, her risk sensitivity, and exogenous institutional parameters. On the grounds of these investment objectives, a blatant resemblance arises to the findings of the pecking order theory (POT) of Myers (1984) and Myers/Majluf (1984). The POT, which is one of the most influential contributions in the corporate financial structure literature to date, is based on the argument that asymmetric information problems drive the capital structure of firms and predicts that companies recur to outside equity only as a last resort, after cheaper options such as internal funds and debt have been exhausted. In other words, the POT postulates a hierarchy of financing with a preference for internal over external funding and for debt over equity. Our results likewise suggest such a funding hierarchy and provide a clear explanation for the widely observed preference of small firms for debt as a primary external funding source. However, unlike the POT, the hierarchy proposed by our model does not contradict empirical studies which observe a systematic raising of outside equity besides or even before debt (as particularly stressed by Fama/French, 2002). Interestingly, our results are not prone to the same antagonism, but instead provide a clear rationale for such empirical observations. The decision maker is indifferent regarding the amount of outside equity as long as her corporate investment objectives can be attained without debt and as long as her disposition on corporate decisions is assured (Proposition 4.2). Moreover, our model's funding hierarchy results as a direct consequence of the interplay of the decision maker's individual characteristics (such as risk aversion and profitability expectations) and environmental properties (such as interest spreads and tax imposition). Our results are, thus, grounded on a completely different approach, since costs of informational asymmetries between firm insiders and firm outsiders play no role in our setting. Hence, the present study shows that "pecking orders" of financing may even arise in total absence of informational costs.

Another interesting result concerns the fact that the feasibility of external equity financing negatively depends on both, the debt capacity and the amount of disposable internal funds. In the limit, if $\overline{\Gamma}$ and W_0^D are sufficiently low, no outside equity will be raised at all (see Figure 8 on p. 70). This has an interesting implication. We have already stressed that slow-growing, highly profitable firms in mature markets are likely to have high cash amounts from retained earnings, contrary to fast-growing but less profitable firms in emerging markets. Moreover, established firms also tend to have a considerably larger debt capacity. Hence, according to our model, young enterprises can be expected to exhibit considerably lower levels of external equity financing compared to established firms.

Chapter 5 expands the view and shows how the existence of operative fixed costs influences the decision maker's investment and financing choices. As long as fixed costs are positive and $\overline{\Gamma} \neq F/(\mu_{\rho} - r_2)$, a unique optimal amount of outside equity exists. This optimal amount balances against each other two antipodal effects of outside equity, i.e., the positive wealth effect and the negative power sharing effect. We have shown that the strength of this power sharing effect (and, thereby, the locus of the optimal trade-off point) directly depends on the characteristics (i.e., the curvature) of the power-related utility component $v(\phi)$. The stronger its curvature, the lower will be the amount of outside equity raised in the optimum and, consequently, the higher will be the optimal ownership concentration.

The simplified analysis of Chapter 4 and 5 paves the way for the subsequent chapters 6 and 7. These chapters successively incorporate the focal firm's actual revenue and cost structure and the product market context. Chapter 6 scrutinizes the case where the firm has monopolistic price-setting power. In contrast to the previous simplified approach, the firm's profit is defined as a function of costs, demand and price. The decision maker can now influence the expected firm revenue by virtue of her pricing decision. In fact, the price becomes D's fourth decision variable in her endeavor to optimize end-of-period cash flows. Our results have shown that the price-sensitivity of the consumer demand has a dampening effect on the impact of risk on the decision maker's choices. Moreover, it has been shown that monopolistic pricing power causes ϕ^* to actually tend towards zero, rather than unity in case *iv*). Thereby, the firm's stock of capital exhibits a larger fraction of outside equity, hence inducing a lower ownership concentration in the monopoly case.

Chapter 7 then scrutinizes a two-stage duopoly setting with initial investment and financing choices and subsequent pricing decisions. For the secondstage pricing subgame, a variety of competitive contexts which circumvent the original Bertrand prediction of zero profits is outlined. However, no judgment on the "right" model of price competition is given. The scrutinized competitive contexts (which focused on capacity constraints, product differentiation, or the exact rules of the pricing game) do by no means constitute an exhaustive, but rather a representative selection. The subject of the present thesis is not the examination of such differing modes of price competition, but the impact of the corresponding equilibrium outcomes on investment and financing decisions. This impact is explored throughout the analysis of the decision maker's first-stage financial structure choice, where differing expected equilibrium price pairs serve as a backdrop. It has been shown that competitive contexts which imply an expected shared monopoly outcome accordingly yield financial structure decisions which correspond to a monopolist's behavior. Competitive contexts which imply lower expected equilibrium price pairs lead to intensified levels of firm investment. Thereby, the contribution margin is widened and the optimal trade-off point between the decision maker's personal wealth gain and her risk exposure is attained.

8.2 Relation to Prior Literature

Throughout Chapter 2, we reviewed the differing streams of literature that have mainly contributed to the theoretical examination of corporate financial structure choice. In many respects, our analysis relates to, complements, or contrasts the findings of these prior theories. We already stressed the similarity of our results to the POT and that, unlike the POT, they do not contradict common empirical observations of early outside equity raising. In addition, our setting can be contrasted against several further pertinent approaches.

By connecting the decision maker's financing decisions with the competitive context faced by the firm, our model relates to the seminal work of Brander/Lewis (1986). However, we adopted a wider perspective by abandoning their most restrictive basic assumptions (i.e., risk-neutrality of the decision maker, an exogenously fixed investment level, non-consideration of control considerations, and negligence of the cost and revenue structure). In Brander/Lewis (1986), debt merely serves as an incentive mechanism to credibly commit the firm to a larger output level. This incentive mechanism crucially depends on the direction of the correlation between the firm's marginal profits and the future states of the world. Our approach is completely different, since debt directly enters the decision maker's personal investment problem. It serves as a lever to her personal investment into the firm and, thereby, also augments her personal risk exposition. The decision maker raises corporate debt in order to maximize her objective function, given her risk attitude, her personal beliefs, the environmental properties, and the market characteristics.

As already stressed, according to our model, young and less profitable enterprises can be expected to exhibit considerably lower levels of external equity financing compared to established and highly profitable firms. This contrasts the tradeoff-theoretical findings of Kraus/Litzenberger (1973). As indicated in the literature review (Chapter 2), the trade-off theory postulates a strictly positive relation between profitability and firm leverage. However, empirical results disclose the exact opposite relation: highly profitable firms are mainly characterized by a low level of firm leverage. This is concordantly confirmed by various studies¹²⁰ and in line with the findings of the present thesis.

Our comparative statics show that decreasing uncertainty (measured by the demand volatility) leads to higher fractions of debt in the firm's stock of capital by trend. This contradicts the findings of Wanzenried (2003) and Showalter (2005), who arrive at the strict opposite conclusion. The reason for this contradiction lies in a difference concerning the basic "function" of debt in the corresponding models. Wanzenried (2003) and Showalter (1995) queue up with Brander/Lewis (1986), i.e., debt serves merely strategic purposes. Taking on debt gives a strategic advantage, since the presence of uncertainty induces liability-constrained decision makers to ignore low states of demand. As a result, debt leads to price increases for the firms. In our model, investment and financ-

¹²⁰ See Myers (1984), Titman/Wessels (1988), or Rajan/Zingales (1995).

ing decisions are simultaneously endogenized, and debt serves as a direct means to attain the decision maker's investment objectives. Since these investment objectives negatively depend on the degree of uncertainty, a lower uncertainty induces a higher stock of firm capital. Thereby, depending on which of the four optimality cases is the starting point, debt can only increase or remain constant.

In the pertinent literature, debt has also been shown to be useful for offsetting the problem of over-investment and reducing perk consumption when the firm is controlled by a professional manager who owns little or no equity of the firm (Jensen, 1986; Stulz, 1990; Hart, 1995). These authors, however, typically assume that investment decisions are controlled by a professional manager. Hence, they ignore any active role of large shareholders in corporate control. In contrast, the present study does not consider any kind of managerial moral hazard problems – be it asset substitution or consumption of perquisites. In fact, the decision maker herself is considered to be a *controlling shareholder* who acts as an owner manager.

Novel theoretical approaches scrutinizing financial structure decisions emphasized not only the contractual problems arising from the divergent interests of "direct" stakeholders (e.g. managers, shareholders, and creditors), but also the relevance of financing decisions for "external" stakeholders (competing firms, suppliers, consumers, etc.) These novel approaches elaborated that firms' choices made with respect to financial structure influence both their own behavior in the output market and the behavior of other market participants, thus affecting competitive equilibrium outcomes. However, these novel approaches (like the totality of prior approaches) still largely exhibit a conceptual separation isolating the respective analyses of financing and investment decisions from each other. Hence, they still fail to provide a holistic examination of corporate financial structure choice in the sense of an *interconnected* analysis of investment and financing. Our setting accounts for this prevailing conceptual deficit of the pertinent literature and shows how the simultaneous endogenization of investing and financing decisions influences and changes the results obtained by earlier investigations, hence leading to further insights.

It has been shown that, in contrast to Jensen/Meckling (1976), our setting predicts that decreasing risk-aversion gradually shifts down the relative proportion of debt in the firm's stock of capital (see Chapter 4). It has also been stressed that this result would have been exactly inverted, had we queued up with Jensen/Meckling (and the bulk of the existing financial literature) by assuming a fixed stock of firm capital. The same is true for our above-outlined findings concerning firm leverage and the degree of uncertainty, which contrasts prior results in the literature (Wanzenried, 2003; Showalter, 1995). As emphasized by Harris/Raviv (1988, 1991), controlling shareholders want to retain their power, i.e., their disposition on corporate decisions. In our setting, it is straightforward to understand that the decision maker faces a tradeoff: raising outside equity to finance corporate target investments and possibly diluting (or losing) control, or keeping control and, in case of a binding liquidity constraint, passing on prospective cash flows. Contrary to prior approaches, our study considers the properties of this trade-off and explores its effects on the financial structure choice. The power-related utility component $v(\phi)$ functions like a "brake" for the raised amount of outside equity, since the (wealthinduced) marginal utility of outside equity is traded off against its (power-lossinduced) marginal disutility. This trade-off is additionally influenced by the level of debt taken on in the optimum, which in turn results from another trade-off (wealth vs. risk). Hence, the decision maker's optimal investment level and financing mix obtains from a non-trivial interplay between these two tradeoffs.

Existing models typically assume shareholder risk neutrality.¹²¹ However, it can be argued that investors of all types generally exhibit a certain degree of aversion towards risk. Corporations and institutions may well behave less risk averse (compared to individuals) due to their large asset bases and greater ability to diversify. Nonetheless, diversification does not lead to risk neutrality in the strict sense, and Gossen's law of diminishing marginal utility¹²² suggests at least some risk aversion for every decision maker. Therefore, our model relates corporate financial structure choice with a von Neumann-Morgenstern utility function in order to explore how risk aversion influences a decision maker's financial structure decisions in an uncertain environment. In order to craft a coherent setting for this exploration, we introduced a riskless investment alternative that the focal decision maker has at her disposal in addition to the risky firm investment. It has already been stressed several times that the assumption of risk aversion crucially drives large part of our findings. With a risk neutral decision maker, the volatility of cash flows would play no role, and the utility function would be linear in wealth. As a consequence, the trade-off between wealth and risk would vanish and the decision maker would simply exploit the full debt capacity and invest her total wealth into the firm (see Comparative Statics Result 1 in Chapter 4). By the consideration of risk aversion, we adopted a concave utility function and, thereby, attained potential interior solutions for both α^* and Γ^* .

¹²¹ A rare exception is provided by Leland and Pyle (1977). The difference resides in the fact that these authors scrutinize the signaling role of financial structure. In our model, financial structure decisions have no signaling effect.

¹²² See Gossen (1854).

On a further note, it feels particularly noteworthy that the chosen investment levels and the corresponding financing choices are tightly depending on exogenous factors. Our analysis has disclosed that an increased (expected) profitability of projected firm projects contributes to a higher corporate debt level. Hence, if we acknowledge that such increases in profitability are often the result of industrial innovations and technological breakthroughs, our results exhibit a compelling connection to the pertinent Schumpeterian theory of business cycles.¹²³

8.3 Limitations and Outlook

Of course, this study has made a set of specific assumptions concerning the internal and external properties faced by the focal decision maker. It goes without saying that a multitude of alternative and/or additional assumptions could be made to explore the generalizability of our findings.

For example, one limitation concerns the static character of the model. In reality, enterprises are typically able to adapt and restructure their financials over time; but our model disregards such considerations.

Another critical feature of our model setting concerns the assumption of normally distributed expectations of the decision maker. This implies that, in principle, negative firm profits, bankruptcy, and, thereby, negative personal cash flows for the decision maker can occur with positive probability. Accounting for limited liability of the decision maker would then substantially complicate the technical treatment of the objective function. We have circumvented this pitfall by introducing the technical assumption $\mu > 2\sigma$, which allowed us to approximate the decision maker's expectations by a truncation in the interval of the corresponding random variable, i.e., $[\mu - 2\sigma, \mu + 2\sigma]$. Abandoning this assumption certainly constitutes a possible further avenue to explore. However, this would require a substantial refinement and sophistication of D's objective function, since the interplay of taxation and the limited liability clause would induce discontinuities in the first derivative of W_1^D .¹²⁴ Hence, our model does not head towards an exhaustive theory of bankruptcy. However, since bank-

¹²³ See Schumpeter (1939) and, for a thorough discussion, Andersen (2009).

¹²⁴ With a negative surplus, our mode of taxation would effectively turn the tax into a subsidy (see Eq. (2)). Hence, a rigorous modeling would require the distinction of several sub-cases contingent on, firstly, whether S is positive or negative and, secondly, whether the firm goes bankrupt or not.

ruptcy exhibits the important feature that it transfers control and does not necessarily lead to liquidation, a deep exploration of the corresponding considerations of an entrepreneurial decision maker would certainly enrich and complement our findings.

One of the crucial pillars of the model is the underlying granularity assumption regarding external equity investors. We have shown that for a very high number of small shareholders, the large blockholder D's power curve obtains as depicted in Figure 6 (see Appendix A 3.2). However, this leaves unconsidered the possibility of the emergence of a second block of votes and the corresponding non-trivial effects on D's voting power. A thorough discussion which also includes the impact of voting caps on the power curve is conducted in Kirstein/Koné (2010), from where it is easy to understand that the actual shape of the power curve (and, thereby, of the power-related utility component $v(\phi)$) directly depends on the focal context concerning the structure of votes. However, our setting provides a resilient analytical frame which, from a technical standpoint, could be smoothly adjusted to account for higher complexity in this respect.

Possibilities to extend our model frame are manifold. For example, we have implicitly assumed that firms have easy access to financial markets, with no costs of raising debt or outside equity. One could, however, argue that access to capital markets entails costs of implementing specific information standards such as adequate financial statements or business plans. Including such costs would certainly influence the decision maker's optimal choices.

Another area of potential expansion concerns the central notion of corporate control. In our setting, the decision maker's degree of control does not *per* se exert a direct influence on her personal income. Our model interprets utility of control merely as an "intrinsic" enjoyment. Technically speaking, D's overall utility function is additively separable in wealth and control. A promising way of further sophistication may concern the additional consideration of "instrumental" power. Managing operations and giving orders to subordinates may, besides being intrinsically rewarding, well result in the deliberate diversion of tangible and/or intangible resources to the decision maker's benefit. By such distorted operative decisions, further distortions of the financial structure decisions taken in the optimum can be expected.

Regarding the competitive analysis, an obvious avenue for further generalization of the model is the consideration of asymmetry between the competing firms and their respective decision makers. For example, one may let the firms' cost structures differ from each other or the decision makers may exhibit different attitudes towards risk. Further scrutinized asymmetries could encompass actual expectations, wealth and capacity constraints, and access to external capital.

Moreover, the adoption of an expected utility model can obviously never be free from fundamental critique, and the application of alternative concepts (like, e.g., a prospect theoretical frame in conjunction with substantial bankruptcy risk) certainly constitutes an interesting avenue for future research. However, as already pointed out in 1982 by Schoemaker,

"...until richer models of rationality emerge, (expected utility) maximization may well remain a worthwhile benchmark against which to compare, and toward which to direct, behavior." (Schoemaker, 1982, p. 556.)

Certainly, a large amount of work still remains to be done in this field. Besides the above-outlined limitations of the theoretical frame, a vast void of empirical studies scrutinizing a systematic comparison of continental European and Anglo-American industries exists. Against the backdrop of considerable differences in the respective institutional frameworks regarding corporate governance, such systematic investigation appears to be a worthwhile avenue towards a better understanding of how firms' decision makers conduct their financial structure decisions.

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