VSC Adaptive Nonuniform Sampling Approach for Resource Optimization in Networked Controlled System

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Abstract: This paper presents an Adaptive Event-Triggered nonlinear Control method suitable for Networked Control Systems (NCS) with optimized resource exploitation. The work aims to ensure Networked Control Systems performance with lowered network usage and adaptive nonuniform controller execution. The nonlinear control strategy utilizes a predefined sliding variable, defined by system states and a nonlinear switching function, to maintain system stability within a specified boundary. This stability boundary is governed by an adaptive triggering condition, which balances system performance against network data-rate constraints and resource optimization. The adaptive triggering condition dynamically determines triggering law based on the state's ℓ^2 metrics and auxiliary internal dynamics system. Internal dynamic systems guarantee the stability property of the NCS and leverage resource optimization for nonuniform control algorithm execution. The minimum inter-event time for a nonuniform approach is derived to address network limitations and controller computation burden. The effectiveness of the proposed NCS method is validated through experimental results on a real system with UDP communication protocol.

1 INTRODUCTION

Sampled data systems have been a subject of research for many decades. When sampling and controller update occurs periodically, a well-established theory exists for analyzing stability and designing control strategies for such systems [1]-[4]. This mature theoretical foundation and stability analysis of the time-delayed system has significantly influenced the development of Networked Control Systems (NCS) [5]-[7]. However, in practical applications, the intervals between consecutive sampling instants in an NCS are typically time-varying rather than fixed. For instance, when an NCS experiences packet dropouts or denial-of-service (DoS) attacks, it can be modeled as a sampled data system with nonuniform sampling, also referred to as aperiodic or variable sampling. To efficiently manage limited network resources, the update frequency of sensors or controller units should be deliberately reduced. Many NCS components such as sensors, actuators, and embedded systems are battery-powered, and periodic activity of signal sampling with minimal variance from the last acquisition and controller updates can lead to unnecessary energy consumption and network resource usage. In such cases, nonuniform controller paradigms emerge as a promising strategy to optimize resource usage by activating sampling only when significant changes occur. The nonuniform sampling controller design for NCS is a viable solution for reliable operation. In the last two decades, many scholars have dealt with the nonuniform sampling approach for linear systems, where the stability is analyzed based on the time-delayed system [8],[9]. The analysis estimates the NCS's robustness and optimal behavior in network imperfection, where the controller execution remains under time triggering policy. The viable of a nonuniform sampling system is an Event-triggering approach (ET). ET is prevalently employed to act when the controller update policy is fulfilled [10]. The ET does not sample the system at uniform time intervals but instead executes actions based on a predefined triggering rule [11],[12]. While constant periodicity is disregarded, it allows for computational relaxation. However, it must still ensure the closed-loop properties of stability, state convergence, and time performance.

The presented research deals with designing adaptive Variable Structure Control (VSC) in the form of ET paradigms. A well-known approach of VSC is Sliding Mode Control (SMC), which ensures robustness and fast response by forcing system trajectories onto a predefined specified surface [13]. The main future of an SMC is a discontinuous control action concerning the preselected sliding variable, where the control alternates between different structures to maintain the system's trajectory on the sliding surface. Fast switching behavior makes sliding mode control well-suited for complex and uncertain systems [14]. While SMC excels at managing uncertainties and disturbances, its main drawback is the nonlinear output, characterized by high-frequency oscillations around the sliding manifold [15]. These oscillations are undesirable as they can lead to increased wear on the physical system, unwanted vibrations, excessive heat dissipation, and, in NCS, excessive network usage. SMC with ET approach relaxes latter conditions and improves controller performance. The primary benefit of this approach is incorporating an adaptive triggering rule that leverages the characteristics of relative triggering policies [16]. Adaptation of the triggering rule is based on the predefined desired value, where the stability of NCS-SMC systems is ensured with absolute convergence to the sliding surface with an adaptive triggering rule.

In the SMC mode, when the state trajectory is far from the sliding surface, the switching function remains unchanged until the trajectory crosses the sliding surface. In such a case, theoretically, no update is needed. When the trajectory is in the vicinity of the sliding variable, the practical sliding mode is introduced to ensure system stability [17],[18]. The practical sliding mode ensures the absolute boundness of the state trajectory, where the bandwidth is proportional to the triggering condition. With an adaptive triggering rule, the execution and network usage can efficiently be alleviated. The efficiency of the proposed approach is examined in the real NCS with the positioning system with a servo drive.

The structure of the paper is as follows: Section 2 presents the problem formulation and state transformation with error variables introduced into the system. The SMC design with an adaptive triggering approach is presented in Section 3. Section 4 introduces the ET approach for two previously designed SMC controllers. Two different triggering rules are suggested and derived lower nonzero TI values are presented. Section 5 presents the results and comparisons of the TT and ET strategies. Section 6. is the conclusion of the paper.

2 PRELIMINARIES

The single-input, single-output nonlinear system is given in the following class,

$$\begin{aligned} \dot{x} &= f(x)x + g_1(x)u + g_2(x)d \\ y &= h(x)x \end{aligned} \tag{1}$$

The functions $f(x)$, $g_1(x)$, $g_2(x)$, $h(x)$ are Lipschitz with respect to its arguments. $x(t) = \left[x_1(t)\ \ x_2(t)\right]^T \in {}^2$ is a state vector and $u(t) \in$ is the input variable. Due to the nonlinearity of (1), the parameters $f(x)\colon {}^2 \to$ and $g_1(x), g_2(x)\colon {}^2 \to$ depend on the operation point of (1). The matched disturbance is defined as $d\colon \ \to$. All the parameters are assumed to be bounded such as, $|f(x)| \le A$, $B_{i,\min} \le |g_i(x)| \le B_{i,\max}$, $i = 1,2$, and $|d| \le D$, where $A, B_{i,\min}, B_{i,\max}$ and D are known positive constants. The state transformation is made by introducing new error variables,

$$e = \begin{bmatrix} e_1 \\ e_2 \end{bmatrix} = \begin{bmatrix} x_d - x_1 \\ \dot{x}_d - x_2 \end{bmatrix}, \tag{2}$$

where x_d and $\dot{x}_d$ are the desired value and its time derivative, respectively and holds $e = [e_1\ \ e_2]^T \in {}^2$. The new transformed system is,

$$\begin{aligned} \dot{e} &= f(x)e - g_1(x)u + \tilde{d} + m(x)x_d' \\ y &= h(x)e \end{aligned}, \tag{3}$$

where $x_d' = [\dot{x}_d\ \ \ddot{x}_d]^T \in {}^2$ is the time derivative of the desired value x_d. The disturbance $\tilde{d} = -g_2(x)d$ is assumed to be bounded $|\tilde{d}| \le \Delta_d$. The boundary Δ_d is a positive value and holds $D < \Delta_d$.

3 VSC CONTROLLER DESIGN WITH ADAPTIVE TRIGGERING MECHANISM

The VSC controller under SMC paradigms is designed for the given system in (1). The reaching phase of the SMC is developed regarding the preselected sliding function given as,

$$S \quad \left\{ s \in \ {}^{2} : s = ce = 0 \right\}, \tag{4}$$

where is $c = [c_1 \ \ 1]$. The derivative of the function (4) with respect to time is,

$$\begin{aligned} &\dot{s} = c_1 \dot{e}_1 + \dot{e}_2, \\ &\dot{s} = c\dot{e}, \\ &\dot{s} = -f(x)e_2 - g_1(x)u + c_1 e_2(t) + \tilde{d}(t) + \ddot{x}_d + f(x)\dot{x}_d, \end{aligned} \tag{5}$$

The SMC controller is designed to lead the variable s to (4), with respect to disturbance and uncertainty boundary Δ_d.

$$\begin{aligned} &u = g_1(x)^{-1} \begin{pmatrix} k_1 e_1 + (c_1 - a + k_2) e_2 + \rho \operatorname{sgn}(s) \\ + \ddot{x}_d + a\dot{x}_d \end{pmatrix}, \\ &\rho \geq \Delta_d \end{aligned} \tag{6}$$

where $k = [k_1 \ \ k_2]$, $k \in \ {}^{2}$ are linear feedback gains and holds $k > 0$. The gain values will be discussed in relation to ET. The controller u for the system in (3) is designed with an additional feedback loop in term $k_1 e_1$, which does not directly influence the system's stability but has a significant effect on the tracking capabilities of the NCS in ET mode. For the static feedback in (6), the nominal values of the functions are used $f(\tilde{x}) = a,\ g_i(\tilde{x}) = b_i,\ i = 1, 2$.

3.1 Event-Triggered VSC

In the latter section, the VSC-SMC controller was derived for the continuous system, as given in (3). The VSC controller will be implemented in discrete form, where the Backward-Euler and Backward-Differencing methods are mainly used in real-time applications. For the controller (6), instead of classic fixed update time T_t, the ET approach introduces interevent time T_i, which is defined by triggering rules and system dynamics. The T_i is defined with two successive updates defined as $T_i = \{t_{i+1} - t_i\}_{i=0}^{\infty}$. When the controller is updated at t_i, the last output value $u(t_i)$ is held until the new update is required t_{i+1} and holds for all $t \in [t_i, t_{i+1})$. The induced error between the last update and the current value due to the discrete implementation is defined as $\xi(t) = e(t) - e(t_i)$, $\xi(t) = [\xi_1(t) \ \ \xi_2(t)]^T$. At the time of update, the error is $\xi(t) = 0$, where $\xi(t) = e(t_i) - e(t_i) = 0$ holds. The error variable is crucial by determining the triggering condition of ET. The error variable is crucial by determining the triggering condition of ET. The ideal sliding mode is possible only in theory, where the manifold $s = 0$ is ensured with continuous operation of the $sgn(s)$ function. In practice, this cannot be achieved due to the discrete operation of the SMC controller. The variable remains bounded, depending on the selected sampling time T_s. It is similar; in ET, the system trajectory remains bounded by the preselected triggering conditions, which define a practical sliding mode [19]. The practical sliding mode occurs if a finite time $t_1 \in [t_i, \infty)$ exists for any given constant μ when the sliding variable s reaches the vicinity of $s = 0$ and remains there for all time $t > t_1$. The sliding variable is bounded with $|s| \leq \Omega,\ \Omega \in \ {}_{+} / \ {}_{\infty}$. The triggering rule and nonzero, minimum positive inter-event time can be defined throughout the variable evolution $\xi(t)$ between two successive updates. The controller in (6) with nominal parameters at the given update time $t = t_i$ is given as,

$$u(t_i) = b_1^{-1} \begin{pmatrix} k_1 e_1(t_i) + (c_1 - a + k_2) e_2(t_i) + \rho \operatorname{sgn}(s(t_i)) \\ + \ddot{x}_d(t_i) + a\dot{x}_d(t_i) \end{pmatrix}. \tag{7}$$

Theorem 1: *Consider system (3) with the sliding manifold (4) and controller (6). The parameter γ is given such that,*

$$\left\| \left[(c_1 - a) + k_2 \quad k_2 c_1 \right] \right\| \left\| \xi(t) \right\| < \gamma, \tag{8}$$

for all $t > 0$, where $\gamma \in \ $, $\gamma > 0$, and $k_2 > 0$, $k_1 = c_1 k_2$. The event triggering is established if the controller gain is selected as,

$$\rho > \gamma + \Delta_d, \tag{9}$$

Proof: The stability analysis is performed with the Laypunov function $V(t) = \frac{1}{2} s(t)^2$. Substituting (5) to

$$\dot{V} = s\dot{s} = s\left((c_1 - a) e_2 - b_1 u + \tilde{d} + \ddot{x}_d + a\dot{x}_d \right), \tag{10}$$

and respect to the time derivative (10) gives,

$$\begin{aligned} \dot{V} &= s(t)\left((c_1 - a) e_2(t) - g_1 u(t_i) + \tilde{d}(t) + \ddot{x}_d(t) + a\dot{x}_d(t) \right), \\ &= s(t) \begin{pmatrix} (c_1 - a) e_2(t) - b_1 \left(b_1^{-1} \begin{pmatrix} k_1 e_1(t_i) + (c_1 - a + k_2) e_2(t_i) \\ + \rho \operatorname{sgn}(s(t_i)) + \ddot{x}_d(t_i) + a\dot{x}_d(t_i) \end{pmatrix} \right) \\ + \tilde{d}(t) + \ddot{x}_d(t) + a\dot{x}_d(t) \end{pmatrix}, \end{aligned}$$

$$
\begin{aligned}
&= s(t)\Big((c_1-a)\big(e_2(t)-e_2(t_i)\big)-k_1e_1(t_i)-k_2e_2(t_i)-\rho\operatorname{sgn}\big(s(t_i)\big)+\tilde{d}(t)\Big),\\
&= s(t)\begin{pmatrix}(c_1-a)\xi_2(t)-\begin{pmatrix}e_2(t_i)+c_1e_1(t_i)+(k_2-1)\\ \cdot\left(e_2(t_i)+\dfrac{k_1-c_1}{k_2-1}e_1(t_i)\right)\end{pmatrix}\\ -\rho\operatorname{sgn}\big(s(t_i)\big)+\tilde{d}(t)\end{pmatrix},\quad k_1=c_1k_2,\\
&= s(t)\begin{pmatrix}(c_1-a)\xi_2(t)-\begin{pmatrix}e_2(t_i)+c_1e_1(t_i)+(k_2-1)\\ \cdot\big(e_2(t_i)+c_1e_1(t_i)\big)\end{pmatrix}\\ -\rho\operatorname{sgn}\big(s(t_i)\big)+\tilde{d}(t)\end{pmatrix},\\
&= s(t)\Big((c_1-a)\xi_2(t)-k_2\big(e_2(t_i)+c_1e_1(t_i)\big)-\rho\operatorname{sgn}\big(s(t_i)\big)+\tilde{d}(t)\Big),
\end{aligned}
$$

With the introduced sliding variable error $\xi_s(t)=s(t)-s(t_i)$, the term $e_2(t_i)+c_1e_1(t_i)=s(t_i)$ gives $e_2(t_i)+c_1e_1(t_i)=s(t)-\xi_s(t)$. Substituting the relation into a stability analysis gives,

$$
\begin{aligned}
\dot{V} &= s(t)\begin{pmatrix}(c_1-a)\xi_2(t)-k_2\big(s(t)-\xi_s(t)\big)\\ -\rho\operatorname{sgn}\big(s(t_i)\big)+\tilde{d}(t)\end{pmatrix},\\
&= s(t)\begin{pmatrix}(c_1-a)\xi_2(t)-k_2s(t)+k_2\xi_s(t)\\ -\rho\operatorname{sgn}\big(s(t_i)\big)+\tilde{d}(t)\end{pmatrix},\\
&= s(t)\begin{pmatrix}(c_1-a)\xi_2(t)-k_2s(t)+k_2\big(\xi_2(t)+c_1\xi_1(t)\big)\\ -\rho\operatorname{sgn}\big(s(t_i)\big)+\tilde{d}(t)\end{pmatrix},\\
&= s(t)\begin{pmatrix}(c_1-a+k_2)\xi_2(t)+k_2c_1\xi_1(t)-k_2s(t)\\ -\rho\operatorname{sgn}\big(s(t_i)\big)+\tilde{d}(t)\end{pmatrix},\\
&\le |s|\gamma-k_2s^2-|s|\rho+|s|\Delta_d,\\
&\le -|s|(\rho-\gamma-\Delta_d)-k_2s^2<0,
\end{aligned}
$$

where $\rho>\gamma+\Delta_d$ stability s is guaranteed for a long time $t\in[t_i,t_{i+1})$. The stability condition $\dot{V}<0$ is ensured if $\operatorname{sgn}\big(s(t_i)\big)=\operatorname{sgn}\big(s(t)\big)$, otherwise boundary is defined as,

$$
\begin{aligned}
&|s(t)-s(t_i)|=|ce(t)-ce(t_i)|,\\
&\quad<\|c\|\big\|\big[(c_1-a)+k_2 \quad k_2c_1\big]\big\|^{-1}\gamma=\tilde{k}_l\gamma,
\end{aligned}
\qquad (11)
$$

where $\Omega_l=\{e\in\ ,|s|=|ce|<\tilde{k}_l\gamma\}$ and the triggering rule (8) is defined as $\|\xi(t)\|<\big\|\big[(c_1-a)+k_2 \quad k_2c_1\big]\big\|^{-1}\gamma$. The stability in the region Ω_l is defined by the Lyapunov function $V(t)=\frac{1}{2}e_1(t)^2$, where holds $e_2=s-c_1e_1$. The trajectory is confined to the region $\|e_1\|<\|c_1\|^{-1}\tilde{k}_l\gamma$.

3.2 Adaptive ET approach

The stability analysis and the static triggering law are defined in (8). The adaptation algorithm for parameter γ can serve as an internal mechanism to adjust parameters γ in specific regions defined as $\gamma\in[\underline{\gamma},\bar{\gamma}]$, where holds $\gamma>0$, and $\underline{\gamma}<\bar{\gamma}$. Adaptive triggering is introduced with triggering law,

$$
\|e\|\ge\eta+\gamma,\qquad \gamma>0,\eta>0, \qquad (12)
$$

where η is an internal parameter and can be defined as,

$$
\dot{\eta}=-\alpha\eta+\Theta\big(\gamma-\|e\|\big), \qquad (13)
$$

and holds $\eta(0)=\eta_0,\ \eta,\eta_0,\Theta,\alpha\in{}_0^+$. System in (13) is a strictly positive function. The adaptation algorithm acts as a mechanic that converges to zero if the static law γ is employed. Take, for example, the condition when triggering has not occurred.

$$
\|e\|<\eta+\beta, \qquad (14)
$$

and holds $-\eta\le\beta-\|e\|$. If we rewite the system given in (13), we get,

$$
\begin{aligned}
\dot{\eta}&=-\alpha\eta+\Theta\big(\beta-\|e\|\big)\\
&=-\alpha\eta-\Theta\eta\\
&=-(\alpha+\Theta)\eta
\end{aligned}
, \qquad (15)
$$

where it holds that the derivative of the (13) is a strict negative function. The stability analysis of ET with an adaptation system (13) and (10) is given with the Lyapunov function,

$$
V_A=V+\eta, \qquad (16)
$$

and the derivative of V_A is equal to,

$$
\begin{aligned}
\dot{V}_A&=\dot{V}+\dot{\eta}\\
\dot{V}_A&=-|s|(\rho-\gamma-\Delta_d)\\
&\quad-k_2s^2-(\alpha+\Theta)\eta<0
\end{aligned}
. \qquad (17)
$$

The system's stability with the adaptive system (13) is ensured.

4 NETWORKED CONTROL SYSTEM

The structure of the networked control system is illustrated in Figure 1. The controller algorithm runs on a networked computer, while the triggering rule (12) is evaluated on the plant. It is assumed that the plant is equipped with a real-time system that has computational capabilities and communication interfaces based on the LwIP stack. The real-time system handles simple tasks such as evaluating triggering conditions, signal conditioning, and managing communication. The User Datagram Protocol (UDP) is used for the given adaptive ET implementation for data transmission. Data packets are transmitted across multiple network hops, where delays and packet loss may occur. Packet loss in the network is modeled as a loss delay, with the maximum allowable Round Trip Time (RTT) serving as a threshold for packet loss detection. On the plant side, a dedicated packet-loss timer is employed. If the watchdog timer expires, the system requests new data from the server to maintain reliable communication. On the server side, the Python server is employed.

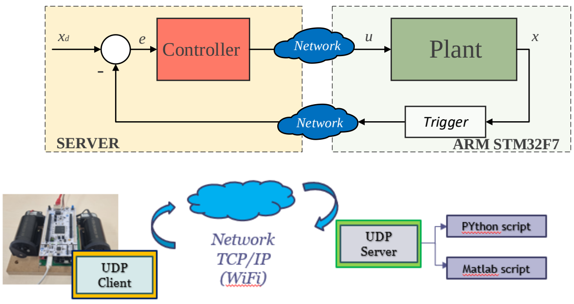

Figure 1: NCS configuration based on adaptive triggering approach.

Regarding the NCS operation and discrete implementation of the ET controller, ensuring that the inter-event time will not tend to the Zeno phenomena is necessary [20],[21]. The inter-event time has to be a lower positive bound, where the lower bound does not violate network performance. The estimated network performance with the RTT parameter is presented in Figure 2. The average RTT is $RTT \approx 2.7ms$, with maximal RTT deviance $\Delta_{RTT} = 0.212ms$.

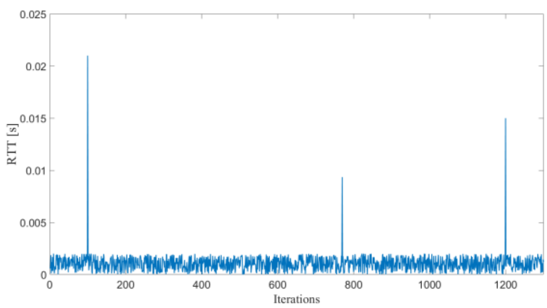

Figure 2: Network performance with RTT parameter.

The inter-event time T_i is determined based on the error analysis between two consecutive sampled states given as,

$$\frac{d}{dt}\|e(t)\| \le \left\|\frac{d}{dt}e(t)\right\| = \left\|\frac{d}{dt}\begin{bmatrix}\xi_1(t)-\xi_1(t_n)\\ \xi_2(t)-\xi_2(t_n)\end{bmatrix}\right\| = \left\|\frac{d}{dt}\begin{bmatrix}\xi_1(t)\\ \xi_2(t)\end{bmatrix}\right\|. \tag{18}$$

where is $\xi(t_n)=0$, according to the last update, which follows,

$$\frac{d}{dt}\|\xi(t)\| \le \left\|\begin{matrix}\begin{bmatrix}0 & 1\\ 0 & -a\end{bmatrix}\begin{bmatrix}e_1(t)\\ e_2(t)\end{bmatrix}+\begin{bmatrix}0 & 0\\ 0 & 1\end{bmatrix}\tilde{d}(t)-\\ \begin{bmatrix}0\\ 1\end{bmatrix}\begin{pmatrix}k_1e_1(t_i)+(c_1-a+k_2)e_2(t_i)\\ +\rho\,\mathrm{sgn}(s(t_i))+\ddot{x}_d(t_i)+a\dot{x}_d(t_i)\end{pmatrix}\end{matrix}\right\|,\quad M_l=-\begin{bmatrix}k_1 & 0\\ 0 & (c_1-a)+k_2\end{bmatrix}$$

$$\le \|f\|\|e(t)\|+\|M_l\|\|e(t_i)\|+\rho+\|P\|x'_d$$

$$=\|f\|\|\xi(t)\|+(\|f\|+\|M_l\|)\|e(t_i)\|+\rho+\|P\|\|x'_d\|.$$

The solution of the differential equation is,

$$\|\xi(t)\| \le \|f\|^{-1}\begin{pmatrix}(\|f\|+\|M_l\|)\|e(t_i)\|+\\ \rho+\|P\|\|x'_d\|\end{pmatrix}\left(e^{\|f\|(T_l)}-1\right).$$

Regarding (8) holds,

$$\left\|\begin{bmatrix}(c_1-a)+k_2 & k_2c_1\end{bmatrix}\right\|^{-1}\eta = \|f\|^{-1}\begin{pmatrix}(\|f\|+\|M_l\|)\|e(t_i)\|+\rho\\ +\|P\|\|x'_d\|\end{pmatrix}\left(e^{\|f\|T_l}-1\right)$$

The inter-event time is equal to,

$$T_l \ge \|f\|^{-1}\ln\left(\frac{\|f\|\eta}{\left\|\begin{bmatrix}(c_1-a)+k_2 & k_2c_1\end{bmatrix}\right\|\begin{pmatrix}(\|f\|+\|M_l\|)\|e(t_i)\|\\ +\rho+\|P\|\|x'_d\|\end{pmatrix}}+1\right). \tag{19}$$

From the derived condition (19), it is obvious that the parameter T_i is positively lower bound and depends on the adaptation system η, triggering condition γ, and controller parameters ρ, k_1, k_2. The selection of the parameters depends on possible network imperfection, DoS, and package drops. All the network uncertainty is modeled as a delayed system [5],[9],[21], where selection of the triggering law can be determined in the way to compensate such delays and preserve NCS performance [21].

5 EXPERIMENTAL RESULTS

For the experimental system, the positioning system with parameters $a = 0.32$, $b_{1,2} = 1.21$ and state variables x_1-angle$[\deg]$, x_2-velocity$[RPM]$ is presented in Figure 3. The real-time experiments were performed on the ARM® Cortex®-M4 based STM32F7xx MCU with Digital-Signal Processing and Floating-Point Unit (DSP and FPU), operating frequency of 180MHz and implemented *LwIP* stack. The motor was driven with an NXP-MC33926 H-controller and pulse-width modulation-PWM technique, with a carrier frequency of 10kHz. The preselected sampling time of the TT implementation is 1ms. System parameters are; $k = [8.56\ 3.35]$ $c_1 = 9.1$, $\rho = 22.5$, $\gamma = 0.15$, $\alpha = 2.93$, $\Theta = 0.89$ and $\eta_0 = \left|\max(x_d) - x_0\right| / 3$.

The NCS performance is evaluated by given performance indices,

$$RMS_n = \sqrt{\frac{1}{n_s}\sum_{k=1}^{n_s} n^2},\ n \in \{x_1, s, u\}, \tag{20}$$

where n_s is the number of evaluated samples. Efficiency is measured by comparing three types of VSC controllers. The first controller is implemented based on the classic time-triggering technique, and its states are marked as TT. The second version is implemented based on a fixed ET technique, and the system states are marked as ET. The third version is implemented with an adaptive ET approach and is marked as AET.

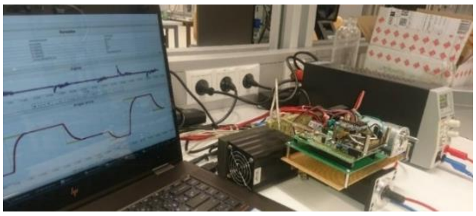

Figure 3: Real-time NCS system.

The results of adaptive ET in NCS structure are presented in the following Figure 4 – Figure 7.

Performance indices are presented in Table 1.

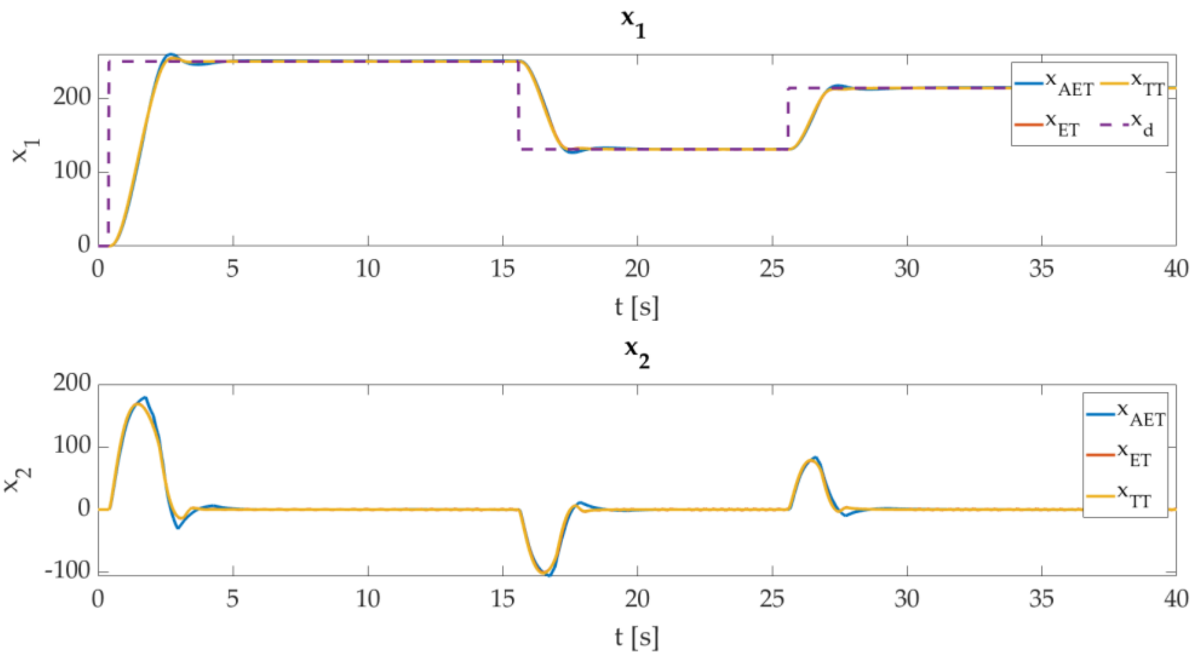

Figure 4: State variables; $x_1[\deg]$, $x_2[RPM]$, $x_d[\deg]$ position, velocity, and desired value, respectively.

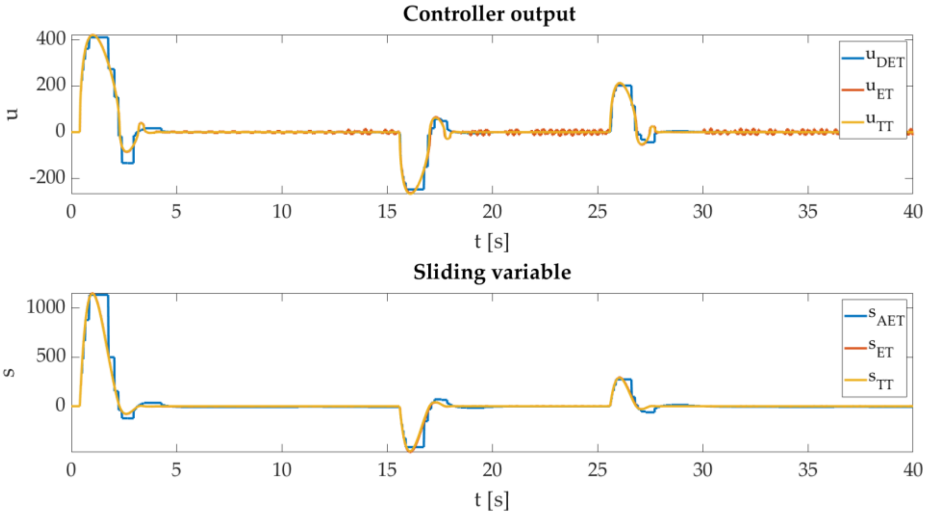

Figure 5: Controller output and sliding variable.

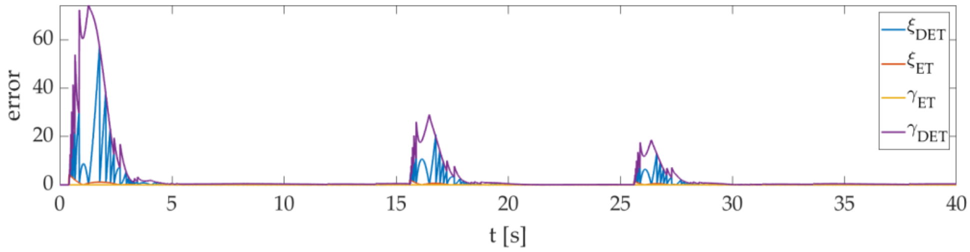

Figure 6: Triggering boundary γ and update error ξ .

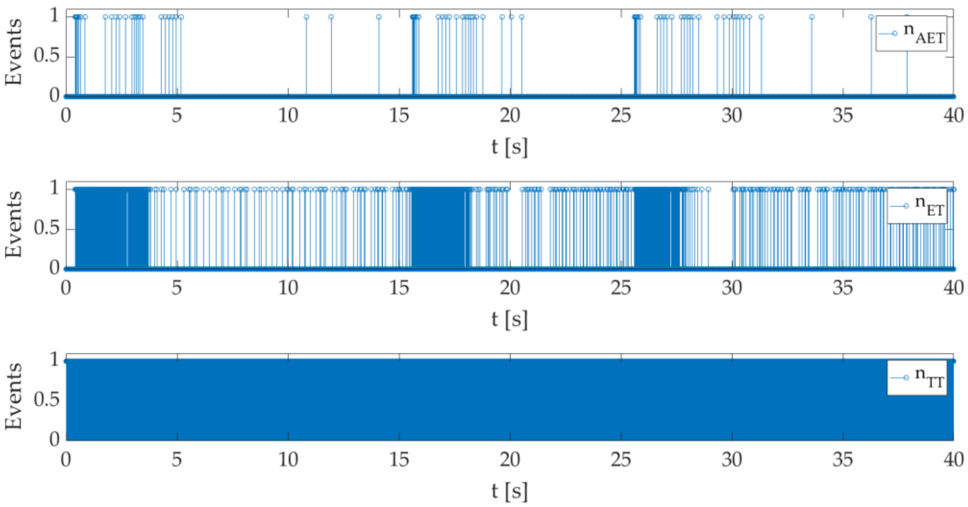

Figure 7: Controller update flags.

Table 1: Performance indices of different controller implementations.

Cont	RMSx	RMSu	RMSs	minT_i	maxT_i	meanT_i	Flag
TT	206.7	88.07	182.9	0.001	0.001	0.001	4e4
ET	206.9	88.23	184.5	0.001	0.84	0.09	993
DET	207.1	90.1	201.3	0.025	5.56	1.17	83

6 CONCLUSIONS

This paper introduces the implementation of an adaptive event-triggered nonlinear controller for networked control systems. The proposed approach offers a viable alternative to time-triggered execution and ET with a fixed triggering bound, making it particularly advantageous for NCS applications with data rate limitations. This method enhances usage efficiency in resource-constrained environments by incorporating network uncertainty directly into the controller design, with a proper selection of the controller parameters, triggering rule, and adaptation system. Furthermore, this work is a valuable foundation for research in different NCS configurations, such as multi-agent systems, distributed control, and task scheduling in embedded systems. The adaptation system can be managed in different scenarios based on the desired values' properties and the state trajectory's vicinity to the sliding manifold. Such an approach will be beneficial for the system with fast varying desired values.

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