Simulation of the Creation of a Single Quantum Gate to Change the Quantum State of a Photon Using Quarter-Wave Plates

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Abstract: The design and simulation of single Pauli gates and Hadamard gates were performed using MATLAB. The Pauli-X gate was constructed using two quarter-wave plates QWP and was evaluated on the quantum states $|0\rangle$ and $|1\rangle$. The Pauli-Y gate design used six quarter-wave plates QWP. When the gate function was tested on the quantum states $|0\rangle$ and $|1\rangle$, it was observed that the state $|0\rangle$ transforms to $i|1\rangle$, while the state $|1\rangle$ transforms to $-i|0\rangle$. The Pauli-Z gate was constructed using two quarter-wave plates in two ways and was tested on the quantum states $|0\rangle$ and $|1\rangle$; it was observed that the quantum state $|0\rangle$ remained unchanged, while the quantum state $|1\rangle$ transforms to $|-1\rangle$ in both ways. Finally, improving the quantum interference by using two quarter-wave plates leads to obtaining a Hadamard gate, which transforms the input state $|0\rangle$ or $|1\rangle$ into a superposition state between $|0\rangle$ and $|1\rangle$. Finally, we emphasize the importance of QWP in developing more efficient and effective quantum systems.

1 INTRODUCTION

A series of operations performed by a conventional computer can be condensed into a single operation using a quantum computer at an extremely high speed [1], [2] as a result of a quantum superposition state that has given quantum computers infinite possibilities resulting from the mixture of states 0 and 1, unlike the conventional computer, which is restricted to these two states only [3], [4].

$$\alpha|0\rangle + \beta|1\rangle. \tag{1}$$

Where quantum state $|0\rangle$ represents horizontal polarization $|H\rangle$, quantum state $|1\rangle$ represents vertical polarization $|V\rangle$, and alpha and beta represent complex parameters that determine the state of the qubit.

The quantum bit is treated mathematically without referring to the physical reality, and it can be projected onto many physical systems, such as atoms, the polarization of a single photon, and others [5].

Operations performed on quantum information that change the quantum state represent the essence of quantum gates [6]. Because they operate at the quantum level, they have the ability to process complex information in an atypical way [7]-[9]. Quantum gates modify quantum states and represent a method for changing and modifying the information stored in quantum bits [10]-[13].

Pauli gates and Hadamard gates are fundamental to quantum computing. Pauli gates perform specific transformations on the state of a qubit, such as a flip around the fundamental axes in quantum space [14]. In contrast, Hadamard gates generate quantum superposition by transforming a qubit from a ground state to an equal mixture of all possible states [15]. We will explain how each gate works:

- Pauli-$\hat{X}$ Gate (quantum NOT gate) [16], [17]

$$|0\rangle \xrightarrow{\hat{x}} |1\rangle \Rightarrow \begin{pmatrix}1\\0\end{pmatrix} \xrightarrow{\hat{x}} \begin{pmatrix}0\\1\end{pmatrix} \tag{2}$$

and

$$|1\rangle \xrightarrow{\hat{x}} |0\rangle \Rightarrow \begin{pmatrix}0\\1\end{pmatrix} \xrightarrow{\hat{x}} \begin{pmatrix}1\\0\end{pmatrix}. \tag{3}$$

Equations (2) and (3) demonstrate the function of the Pauli x-gate and its capacity to transform the quantum state from horizontal polarization $|0\rangle$ to vertical polarization $|1\rangle$ and vice versa [18].

- Pauli-$\hat{Y}$ Gate [16]-[18]:

$$|0\rangle \xrightarrow{\hat{Y}} i|1\rangle \Rightarrow \begin{pmatrix}1\\0\end{pmatrix} \xrightarrow{\hat{Y}} \begin{pmatrix}0\\i\end{pmatrix} \tag{4}$$

and

$$|1\rangle \xrightarrow{\hat{Y}} -i|0\rangle \Rightarrow \begin{pmatrix}0\\1\end{pmatrix} \xrightarrow{\hat{Y}} \begin{pmatrix}-i\\0\end{pmatrix}. \tag{5}$$

Equations (4) and (5) illustrate the influence of the Pauli y gate on the quantum states |H⟩ and |V⟩. This gate produces an effect analogous to the Pauli X gate yet imparts a relative phase to the quantum state. This is seen from the manifestation of the complex number i [17].

- Pauli-$\hat{Z}$ Gate [16], [18]:

$$|0\rangle \xrightarrow{\hat{z}} |0\rangle \Rightarrow \begin{pmatrix}1\\0\end{pmatrix} \xrightarrow{\hat{z}} \begin{pmatrix}1\\0\end{pmatrix} \tag{6}$$

and

$$|1\rangle \xrightarrow{\hat{z}} |-1\rangle \Rightarrow \begin{pmatrix}0\\1\end{pmatrix} \xrightarrow{\hat{z}} \begin{pmatrix}0\\-1\end{pmatrix}. \tag{7}$$

Equations (6) and (7) illustrate that horizontal polarization |0⟩ remains invariant when subjected to a Pauli gate z, whereas vertical polarization |1⟩ experiences a variation in its quantum phase without altering its fundamental state [16].

- Hadamard Gate [15], [16]:

$$|0\rangle \xrightarrow{\hat{H}} \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \tag{8}$$

and

$$|1\rangle \xrightarrow{\hat{H}} \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle). \tag{9}$$

Equations (8) and (9) demonstrate that an equal superposition of |H⟩ and |V⟩ is produced when either quantum state |H⟩ or |V⟩ is inputted. A phase difference is observed in the output of input quantum state 1, indicated by a negative sign preceding the output of quantum state 1 [18].

A Quarter Wave Plate (QWP) is a key optical part of quantum optics [19]. It controls the polarization states of light and is very important for this purpose. For example, the design of quantum gates [20] is built on the ideas of interference and refraction. QWP is used in many quantum applications. The QWP changes the way light is polarized [21]. As seen in Figure. 1, light that goes through the QWP changes from circular to linear polarization and back again [23]. For users who need to control the polarization state precisely, this feature of the QWP makes it a useful tool.

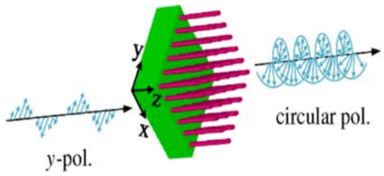

Figure 1: Shows the quarter-wave panel used to convert Y-Linear Polarization of light into circular polarization and vice versa [22].

Projection effect matrix for a plate with four waves [24]

$$\hat{A}_Q(\theta) = \begin{pmatrix}a & c\\ b & d\end{pmatrix}. \tag{10}$$

Where:

$$a = \cos^2(\theta) + i\sin^2(\theta),$$

$$b = (1-i)\sin(\theta)\cos(\theta),$$

$$c = (1-i)\sin(\theta)\cos(\theta),$$

$$d = i\cos^2(\theta) + \sin^2(\theta).$$

Studying QWP and understanding its properties contributes to enhancing the design of quantum gates and improving the overall performance of quantum systems, paving the way for innovative applications in multiple fields.

2 GATE DESIGN SIMULATION

In this section, we explain how to produce quantum gates using quarter-wave plates and test these gates on the two main qubit states, 0 and 1. Equation (10), which simulates the effect of a quarter-wave plate, is simulated in Figure 2.

The main role of quantum gates is to control quantum states by changing the polarization state using quarter-wave plates. The polarization angle is the main factor for changing the quantum state. See Table 1.

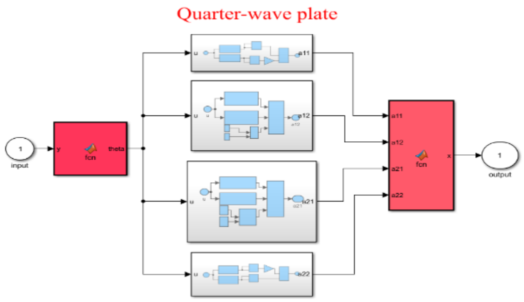

Figure 2: Simulation diagram of a Quarter-wave plate.

Table 1: Showing the design and operation of single quantum gates.

GATE	*QWP*	θ	*INPUT*	*OUTPUT*
Pauli-X	*2*	*π/4*	$\|0\rangle$	$\|1\rangle$
			$\|1\rangle$	$\|0\rangle$
Pauli-Y	*6*	*0, π/4, π/2*	$\|0\rangle$	$i\|1\rangle$
			$\|1\rangle$	$-i\|0\rangle$
Pauli-Z	*2*	*0*	$\|0\rangle$	$\|0\rangle$
	2	*π*	$\|1\rangle$	$\|-1\rangle$
Hadamard	*2*	*π/8*	$\|0\rangle$	$1/\sqrt{2}(\|0\rangle+\|1\rangle)$
			$\|1\rangle$	$1/\sqrt{2}(\|0\rangle-\|1\rangle)$

2.1 Pauli-X Gate

When $\theta = \frac{\pi}{4}$, Projection effect matrix for a quarter-wave plate $\hat{A}_Q(\theta)$ from (10) become:

$$\hat{A}_Q\left(\frac{\pi}{4}\right) = \begin{pmatrix} \frac{1}{2}+\frac{1}{2}i & \frac{1}{2}(1-i) \\ \frac{1}{2}(1-i) & \frac{1}{2}+\frac{1}{2}i \end{pmatrix}. \quad (11)$$

Pauli-X gate equals

$$\hat{X} = \hat{A}_Q\left(\frac{\pi}{4}\right) \cdot \hat{A}_Q\left(\frac{\pi}{4}\right) = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \quad (12)$$

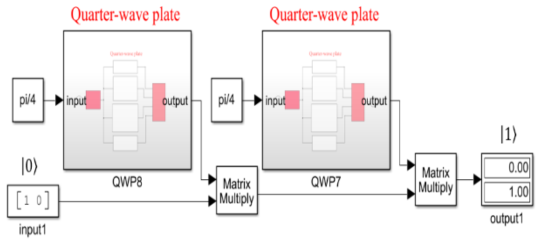

Figure 3: Pauli-X Gate Simulation with input $|\mathbf{0}\rangle$.

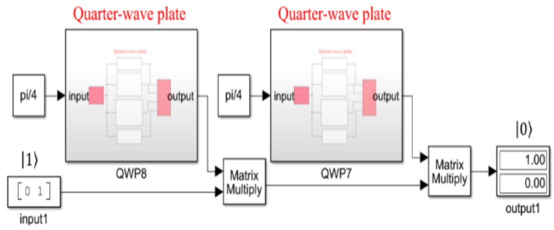

Figure 4: Pauli-X Gate Simulation with input $|\mathbf{1}\rangle$.

2.2 Pauli-Y Gate

When $\theta = 0$, Projection effect matrix for a quarter-wave plate $\hat{A}_Q(\theta)$ become:

$$\hat{A}_Q(0) = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}. \quad (13)$$

When $\theta = \frac{\pi}{4}$, Projection effect matrix for a quarter-wave plate $\hat{A}_Q(\theta)$ become:

$$\hat{A}_Q\left(\frac{\pi}{4}\right) = \begin{pmatrix} \frac{1}{2}+\frac{1}{2}i & \frac{1}{2}(1-i) \\ \frac{1}{2}(1-i) & \frac{1}{2}+\frac{1}{2}i \end{pmatrix}. \quad (14)$$

When $\theta = \frac{\pi}{2}$, Projection effect matrix for a quarter-wave plate $\hat{A}_Q(\theta)$ become:

$$\hat{A}_Q\left(\frac{\pi}{2}\right) = \begin{pmatrix} i & 0 \\ 0 & 1 \end{pmatrix}. \quad (15)$$

Pauli-Y gate equals:

$$\hat{Y} = \hat{A}_Q(0)\hat{A}_Q\left(\frac{\pi}{2}\right)\hat{A}_Q(0)\hat{A}_Q(0)\hat{A}_Q\left(\frac{\pi}{4}\right)\hat{A}_Q\left(\frac{\pi}{4}\right), \quad (16)$$

$$\hat{Y} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}. \quad (17)$$

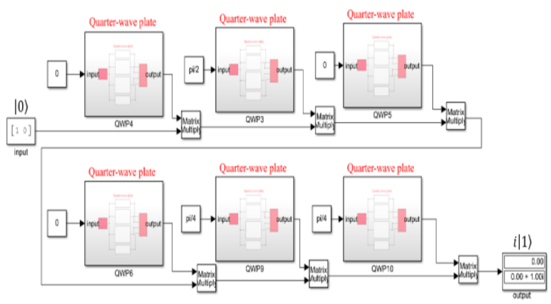

Figure 5: Puli-Y Gate Simulation with input $|\mathbf{0}\rangle$.

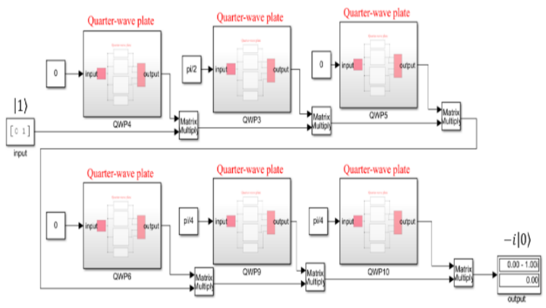

Figure 6: Puli-Y Gate Simulation with input $|\mathbf{1}\rangle$.

2.3 Pauli-Z Gate

When $\theta = 0^{\circ}$, Projection effect matrix for a quarter-wave plate $\hat{A}_Q(\theta)$ become:

$$\hat{A}_Q(0) = \begin{pmatrix} 1 & 0 \\ 0 & i \end{pmatrix}. \quad (18)$$

From (18) it is clear that a quarter-wave plate can be used to obtain the S-gate, which is also known as the square root of the Z-gate.To obtain the Pauli-Z gate, two quarter-wave panels are used, and the angle of each plate is set at 0. Equation (18) represents the matrix for each plate.

$$\hat{Z} = \hat{A}_Q(0) \cdot \hat{A}_Q(0) = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \quad (19)$$

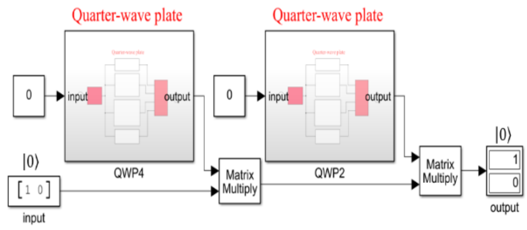

Figure 7: Puli-Z Gate Simulation with input $|\mathbf{0}\rangle$.

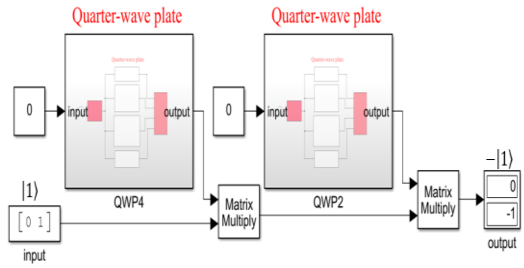

Figure 8: Puli-Z Gate Simulation with input $|\mathbf{1}\rangle$.

To create a Pauli-Z gate in a second way, we use two quarter-wave panels; the angles of the plates are set to π degrees each. The projection effect of the quarter-wave panel is $\hat{A}_Q(\pi) = \hat{A}_Q(0)$, so the Pauli-Z gate matrix remains unchanged, as in (19).

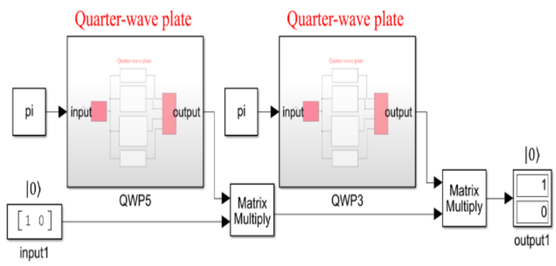

Figure 9: Puli-Z Gate Simulation with input $|\mathbf{0}\rangle$.

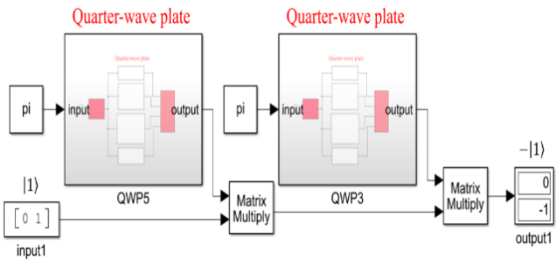

Figure 10: Puli-Z Gate Simulation with input $|\mathbf{1}\rangle$.

2.4 Hadamard Gate

When $\theta = \pi/8$, Projection effect matrix for a quarter-wave plate $\hat{A}_Q(\theta)$ become:

$$\hat{A}_Q\left(\frac{\pi}{8}\right) = \begin{pmatrix} M & Q \\ N & R \end{pmatrix} \quad (20)$$

Where the letters M, N, Q and R represent the following complex numbers:

M=0.8536+1465i,

N=0.3536-0.3536i,

Q=0.3536-0.3536i,

R=0.1465+0.8536i;

To obtain the Hadamard gate, two quarter-wave panels are used, and the angle of each plate is set at $\theta = \pi/8$. (20) represents the matrix for each plate. The Hadamard gate becomes:

$$\widehat{H} = \hat{A}_Q\left(\frac{\pi}{8}\right) \cdot \hat{A}_Q\left(\frac{\pi}{8}\right) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad (21)$$

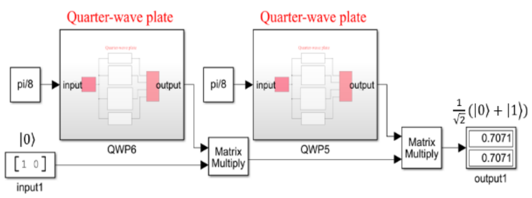

Figure 11: Hadamard Gate Simulation with input $|\mathbf{0}\rangle$.

Determining the optimal QWP angle as a critical factor in improving the performance of a quantum gate has enhanced the understanding of optical transitions and their impact on quantum information.

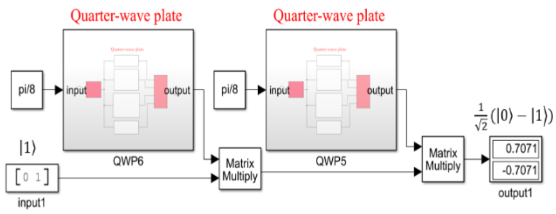

Figure 12: Hadamard Gate Simulation with input $|\mathbf{1}\rangle$.

RESULTS AND DISCUSSION

3.1 Results

In constructing the Pauli-x gate, we used two quarter-wave plates with an angle of $\pi/4$ for each plate. While the Pauli-Y gate was designed using six quarter-wave plates, three of which are at an angle of 0, two at an angle of $\frac{\pi}{4}$ for each plate, and one at an angle of $\pi/2$. As for the Pauli-z gate, we were able to construct it in two ways, using two of quarter-wave plates with an angle of 0 for each plate for the first design, and a pair of quarter-wave plates with an angle of π for each plate for the second design. In the Hadamard gate, two quarter-wave plates with an angle of $\theta = \pi/8$ for each plate were used. All the gates under study were tested in the quantum states 0 and 1.

3.2 Discussion

Quarter-wave panels are a great choice for complex quantum computing applications due to the polarization precision they provide. In the quantum gate X using two quarter-wave plates, their operation is illustrated in Figures 3 and 4. When the angle of each plate is set to $\pi/4$, we find that the input quantum state $|\mathbf{0}\rangle$ has been transformed to $|\mathbf{1}\rangle$and vice versa, thus achieving the desired polarization. The Y-quantum gate, six quarter-wave plates with angles arranged at 0, π/2, 0, 0, π/4, and π/4are used to produce the Pauli-Y gate. We find that introducing the quantum state $|\mathbf{0}\rangle$ onto the six polarization plates one by one will produce a new quantum state $i|\mathbf{1}\rangle$ with a change in direction and phase, as shown in Figure 5. While in Figure 6, we find that introducing the qubit $|\mathbf{1}\rangle$ will produce the qubit $-i|\mathbf{0}\rangle$ which means there is a change in direction and phase. From Figures 7 and 9, we see that the quantum state $|\mathbf{0}\rangle$ is not affected and remains in the same state after passing through the quarter-wave plates that form the Pauli-Z gate, while the quantum state $|\mathbf{1}\rangle$ becomes $|-\mathbf{1}\rangle$, which means that the phase has changed by π, as is clear in Figures 8 and 10. Finally, the Hadamard gate shows an equal superposition between 0 and 1 when state 0 is introduced, as in Figure 11. While the Hadamard gate is applied to state 1, it also creates a quantum superposition, but with a different phase, note Figure 12. The design of single quantum gates using a quarter-wave plate can be said to be an important step towards enhancing the efficiency of quantum systems.

4 CONCLUSIONS

The use of QWP has been shown to improve the fidelity of qubit transitions between 0 and 1, demonstrating the effectiveness of quantum gates in reducing errors and improving the overall performance of quantum systems. Determining the optimal QWP angle is also a critical element in the design of quantum gates.

Quarter-waveplate quantum gates offer a simple and efficient optical design. However, practical constraints such as fabrication precision, optical alignment, and environmental factors such as temperature and pressure affect the performance of the quantum system due to their impact on the refractive index of these panels. Therefore, improving lattice fabrication methods and minimizing optical losses are essential to achieving optimal quantum system performance.

The physicist's understanding of the operation of quarter-wave panels contributes to the design of highly efficient quantum gates to improve innovation in the production of a quantum computer with distinctive capabilities.

REFERENCES

[1] M. S. Teja; P.H.S. Kumar,"Introduction to quantum computing," Quantum Computing and Communications, 61, 2020.

[2] B.H. Ahmad; et al.,"Quantum computing: fundamentals, implementations and applications," IEEE Open Journal of Nanotechnology, 3, 61-77, 2022.

[3] H. Vikas; et al, "Forthcoming applications of quantum computing: peeking into the future," IET Quantum Communication, 1, 35-41, 2020.

[4] B.N.B. Campbell; B.M. Vetleseter," Secondary and University Students' Descriptions of Quantum Superposition," The Physics Teacher, 63, 32-34, 2025.

[5] A. Jürgen," Entangled systems: new directions in quantum physics," John Wiley & Sons, 44,2007.

[6] M. Swathi; R. Bhawana," Fundamentals of quantum computation and basic quantum gates," In: Handbook of Research on Quantum Computing for Smart Environments, IGI Global, 1-24, 2023.

[7] M. Mohammed; M. B. Saleh; I. A. Akhlite, "A General Method for Designing n-Qubits Entanglement Creating Circuits and its Application in Different 4-qubits Entanglement Creating Circuits," Journal of Physics: Conference Series, IOP Publishing,1999, 2021.

[8] H. Dylan, et al. ,"Quantum computing for finance," Nature Reviews Physics, 5, 450-465, 2023.

[9] V. Tamás; A.S. Yaiza; O. Manuel," Quantum types: going beyond qubits and quantum gates," In: Proceedings of the 5th ACM/IEEE International Workshop on Quantum Software Engineering, 49-52, 2024.

[10] T.M. Vahid; M.D. Ali," Quantum states and quantum computing," arXiv preprint arXiv:2409.15285, 2024.

[11] P. Martin; B. Časlav," Quantum-state preparation with universal gate decompositions," Physical Review A—Atomic, Molecular, and Optical Physics, 83, 032302, 2011.

[12] D. Ivan; N. Vijay,"Entanglement based detection, networking, sensing, and radars," IEEE Photonics Journal, 16, 1-10, 2024.

[13] H. Ciaran, et al., "Quantum gates," Quantum Computing for the Quantum Curious, 49-57, 2021.

[14] D. Shuvra; M. Sourangshu," Implementation of all-optical Pauli-Y gate by the integrated phase and polarisation encoding," IET optoelectronics, 12, 176-179, 2018.

[15] J. B. DM; N. Linden; P. Skrzypczyk, "The Hadamard gate cannot be replaced by a resource state in universal quantum computation, " Quantum, 8, 1470, 2024.

[16] D. Paromita; R. Sapana; M. Sourangshu," Intensity and phase encoding for realization of integrated Pauli X, Y and Z gates using 2D photonic crystal," Optics & Laser Technology, 152, 108141, 2022.

[17] B.R. KAZEM; M.B. SALEH,"The effect of pauli gates on the superposition for four-qubit in bloch sphere," journal of kerbala university, 18, 2020.

[18] H. Mahboobeh; et al.," Decomposition of diagonal hermitian quantum gates using multiple-controlled pauli Z gates," ACM Journal on Emerging Technologies in Computing Systems (JETC), 2014, 11, 1-10, 2014.

[19] W. Yunlong; et al.," Experimental demonstration of efficient high-dimensional quantum gates with orbital angular momentum," Quantum Science and Technology, 7, 015016, 2021.

[20] H. Lu; et al.," Topologically Protected Quantum Logic Gates with Valley-Hall Photonic Crystals," Advanced Materials, 36, 2311611, 2024.

[21] R. S. Gangi; et al.," Polarization state transformation using two quarter wave plates: application to Mueller polarimetry," Applied Optics, 55, B14-B19, 2016.

[22] C. Deli; et al.," Single-layer all-dielectric quarter-wave plate and half-wave plate metasurfaces for polarization conversion in the visible light region," Optical Engineering, 61, 025104-025104, 2022.

[23] P. Jhon; H. Cristian; T. Rafael," Tunable birefringence and elliptical polarization eigenmodes in a biplate of two quarter-wave plates," Journal of the Optical Society of America A, 40, 1959-1968, 2023.

[24] S. Kanad; et al.," Experimental realization of universal quantum gates and six-qubit entangled state using photonic quantum walk," arXiv preprint arXiv:2403.06665, 2024.

[25] G. Eason, B. Noble, and I.N. Sneddon, "On certain integrals of Lipschitz-Hankel type involving products of Bessel functions," Phil. Trans. Roy. Soc. London, vol. A247, pp. 529-551, April 1955.

[26] J. Clerk Maxwell, A Treatise on Electricity and Magnetism, 3rd ed., vol. 2. Oxford: Clarendon, 1892, pp.68-73.

[27] I.S. Jacobs and C.P. Bean, "Fine particles, thin films and exchange anisotropy," in Magnetism, vol. III, G.T. Rado and H. Suhl, Eds. New York: Academic, 1963, pp. 271-350.

[28] K. Elissa, "Title of paper if known," unpublished.

[29] R. Nicole, "Title of paper with only first word capitalized," J. Name Stand. Abbrev., in press.

[30] Y. Yorozu, M. Hirano, K. Oka, and Y. Tagawa, "Electron spectroscopy studies on magneto-optical media and plastic substrate interface," IEEE Transl. J. Magn. Japan, vol. 2, pp. 740-741, August 1987 [Digests 9th Annual Conf. Magnetics Japan, p. 301, 1982].