Changes Measurement in Biological Signals Based on Dynamic Time Warping Procedure on the Example of Repolarization Part of HeartbeatCycle

Master Thesis

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Annotation

Master thesis discusses the analysis of changes in biological signals on time based on dynamic time warping algorithm (DTW). Special attention is paid to problems of tiny changes analysis in complex nonstationary biological signals.

Electrocardiographic (ECG) signals are used as an example in this study; in particular, repolarization segments of heart beat cycles.

The aim of the research is studyingthe possibility of applying DTW algorithm for the analysis of small changes in the repolarization segments of heart beat cycles.

The research has the following tasks:

- Studying repolarization segments of heart beat cycles, and methods of their analysis;

- Studying DTW algorithm and its modifications, finding the most appropriate modification for analyzing changes in biological signals;

- Development of methods for analyzing the warping path(output parameter of DTW algorithm).

Contents

Introduction

Modern methods of functional diagnostics of the human body based on a studying of different biological signals, described the internal processes of the body. Improving of technologies for recording signals and increase the power of hardware allow record a large amount of signals with high precision. However, obtainedsignals do not always contain necessary for diagnostics information. Often, after recording additional processing of signals is required that reduces speed of analysis and increase its complexity.

Modern technologies allow simplifying signal processing procedures, in particular,on replacing expert and analytical methods of signal processing withintellectual control and diagnostics systems.

One of the modern methods forsignals waveform changes analysis is a dynamic time warping algorithm (DTW).

The aim of this research is to studyan applicability of this algorithm to analysis of small changesin complex non-stationary biological signals such as repolarization segments in electrocardiographic signals (ECG).

The following tasks are solved in this study:

- studying of repolarization segments in ECG signal;

- Definingan applicability of DTW to analysis of changes in repolarization segments;

- Development of a method for warping path curve analyzing (output parameter of DTW algorithm.

1 Features of biological signals and their processing methods

Modern clinical researches are usually based on functional diagnostic procedures. Most of them use biological signals, which describe changes in parameters of some biological process in a human body, for analysis. Sometimes these signals are only available information about functioning of body and destructive changes in it. Therefore, accuracy and timeliness of diagnostics depends on accuracy of signals recording and quality of their analyzing.

Features of physiological processes in human body determine features of biological signals. Despite on variety of biological processes, there are a number of features, typical for all of them.

Many processes in biological objects are diffuse. That means one process can flow into another process and vice versa. In this case, it is not possible to precisely detect boundaries of that processes. To do this, expert estimation based on knowledge and experience of medical specialists, are used [1].

Parameters of biological processes can change their values on time within some boundaries. The speed of these changes might be variable and nondeterministic. It caused quasiperiodicity of measured biological signals, i.e. signals can change their parameters during time due to stretching, shrinking or amplifying within some limits. Parameters changes can be stochastic.

Values of parameters of biological signals and their changes are usually quite small. It cause additional problems of recognition significant changes of signal in the amount of changes caused by noises and measuring inaccuracies appeared when recording signals.

ECG signal and its features

One of the most important processes in human body are processes connected with electrical activity of conductive tissues. Studying of these processes is carried out on electrocardiographic (ECG) and electroencephalographic (EEG) signals. When analyzing these signals the same approaches are used [17] but procedures and aims of studying are different. The focus of this research is in analyzing of ECG signals.

ECG signal describes functioning of conducting system of heart. This system provides contracting and relaxing of heart muscle. Electrical activity change in conducting system is a quasiperiodic diffuse nondeterministic biological process, included two phases: depolarization (contraction) and repolarization (relaxation) [1,19].

Quasiperiodicity of this process is caused by changes in heartbeats depending on human activity and body stage. ECG signal is nondeterministic due to different internal and external factors, which are influenced on functioning of myocardium and leadsduring recording process [1,19].

In general, ECG signal is a set of three-dimensional cycles, made by rotation of so-called heart vector when propagating electrical impulses through myocardium. To simplify analyzing of ECG data this signal is divided into three projections on coordinate axes (Figure 1) [1].

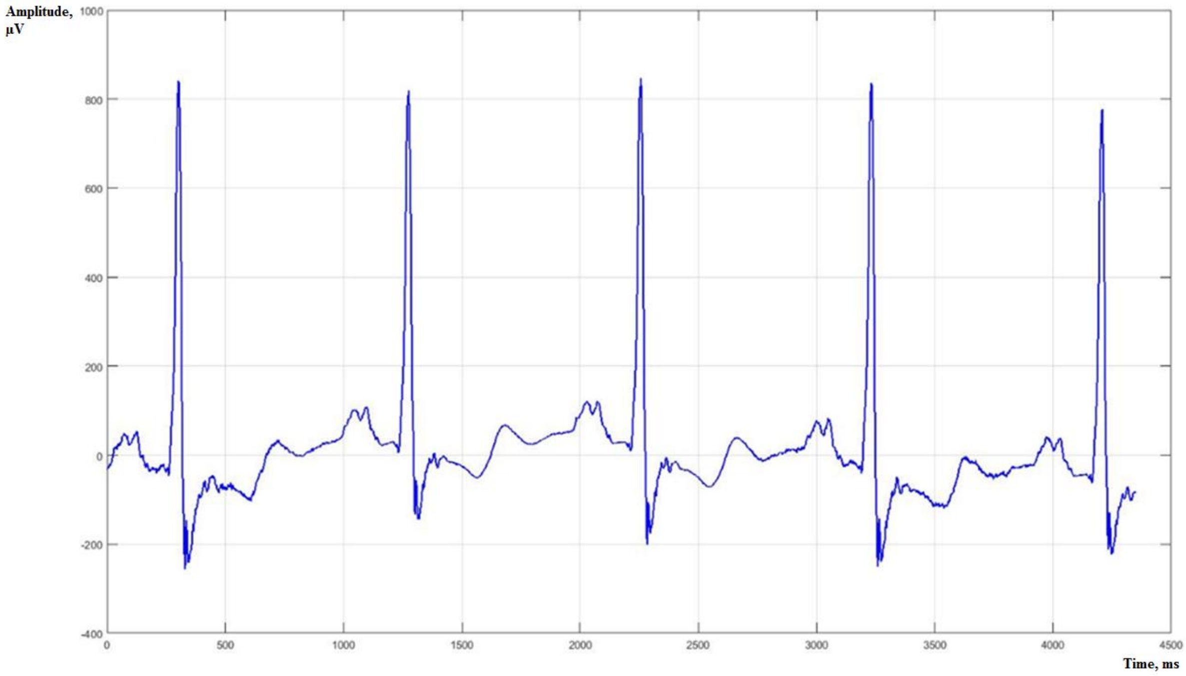

Figure 1 – Fragment of X-projection of ECG signal

For detailed studying of different processes occurring in myocardium, ECG signal is divided into single heartbeats. These beats then divided into different segments, intervals and complexes, conventionally corresponding to various processes.

All features of the whole ECG signal, described in this section are also typical for individual parts of this signal.

Repolarization process of ECG signal and its features

Repolarization is a process of myocardial cells relaxing, occurring after excitation by depolarization wave [1]. Repolarization wave, as well as depolarization one propagates gradually through all cells of myocardium. When this process starts in some areas of heart muscle, in other areas depolarization process is still take part. Thus, the starting point of the repolarization process can not be exactly defined. In the ECG signal, ST-segment conditionally corresponds to the repolarization process. This segment of ECG is less studied then other ones due toproblems with finding boundaries of this segment and recognizing changes in it. Changes occurring in this area are too small and their analyzing was impossible for a long time. Modern technologies and systems allow studying this segment more detailed [18]. Researches of this segment showed some patterns that are indirect signs of several diseases. In particular, it was found that significant changes for diagnosticmight occur in the last ST-segment of normal sinus rhythm before pattern of abnormal beats [18].

Problems of recording and processing biological signals

To represent parameters of biological process in the form of observed signal, recording and processing of parameter changes in time is required. The main low of metrology says that the true value of any physical quantity exists, but it is not possible to get it using any instruments. Since the observed signal is based on a series of values of some physical parameter of biological system, the law is also valid for him. It means that any observed biological signal always represents a physical signal with an error. In addition, when recording any signal, different noises are always exist in measuring channel and its environment.

Changes in ST-complexes of ECG signals are tiny in comparing to a whole signal. Moreover, these changes exist in the high-frequency part of signal spectrum [1, 19]. To find them low-frequency filters are required. Both recording and filtering procedures add inaccuracies in resulting signals.

In this case, significant tiny changes in resulting signal may be comparable in amplitude to noise and inaccuracies of signal. This can lead to a wrong interpretation of the signal changes. To solve this problem, high resolution signals with minimal whole inaccuracy and minimal noise level need to be used for analyzing. Such signals are not always available, thereforea tool for recognition significant tiny changes in a whole amount of changes in signal need to be developed. One method for such recognition is time series data mining. The DTW algorithm can be used as a tool for making a knowledge base forintellectual analyzing system. In particular, DTW allows doingso-called subsequent matching procedure for sufficiently large parts of signal.When using statistically large amount of patterns, this procedure will allow to find regularities in these patterns. After that, clustering of patterns will be done based on these regularities and additional information about test signals. Clustered patterns will be a training data for classification model, which will classify new patterns based on known data.

2 Features of dynamic time warping algorithm and its modifications

Classical dynamic time warping algorithm and its modifications

Dynamic time warping algorithm (DTW) is one of algorithms for measuring of shifting signals from each other using coordinate space warping [4,5]. The name of the algorithm means that it warps not all axes of coordinate space but only time axis. This algorithm was introduced firstly as a tool for speech recognition [4]. Nowadays it is used for similarity measurement of different signals in many fields of knowledge [4-10].

To use this algorithm two signals are needed, one of them will be referencesignal, other will be test one [4].

Algorithm transforms coordinate space using local stretching and shrinking of time axis [5]. This transformation causes corresponding shifting of points in both reference and test signals. The aim of transformation is to provide maximal matching between test and reference signals after warping with minimal amount and volume of changes in time axis. The set of shifting of signals caused by transformation is known as warping path. That is an output parameter of DTW algorithm.

In the following description reference signal is denoted as $R = (R(t_1)\ldots R(t_M))$, test signal is denoted as $T = (T(t_1)\ldots T(t_N))$.

Algorithm includes three steps. At the first step local distance matrix *d* (1) are calculated. Elements of this matrix are pair wise distances between points of reference and test signals.

$$d = \begin{pmatrix} d(1,1) & \cdots & d(1,N) \\ \vdots & \ddots & \vdots \\ d(M,1) & \cdots & d(M,N) \end{pmatrix} \qquad (1)$$

Distances here and in the following description are measured in points. Real distance between signals depends on sampling rate of analyzed signals.

At the second step accumulated distances matrix D (2) are calculated based on matrix (1).

$$D = \begin{pmatrix} D(1,1) & \cdots & D(1,N) \\ \vdots & \ddots & \vdots \\ D(M,1) & \cdots & D(M,N) \end{pmatrix} \quad (2)$$

Each element *D(m,n)* of matrix (2) is calculated as minimal sum of distances between two segments of signals $R_m = (R(t_1)\ldots R(t_m))$ and $T_n = (T(t_1)\ldots T(t_n))$, where $m \in [1 \ldots M]$ and $n \in [1 \ldots N]$ are points indices of reference and test signals correspondingly [4].

Calculation of (2) includes two steps: initialization and internal elements calculation [4]. At the initialization step, the boundaries of matrix arecalculated using formulas (3), (4), (5).

$$D(1,1) = d(1,1), \quad (3)$$

$$D(m,1) = d(m,1) + D(m-1,1), \quad (4)$$

$$D(1,n) = d(1,n) + D(1,n-1) \quad (5)$$

Formulas for calculating internal elements of matrix (2)are definedbyspecial weighting matrix that will be described further. In classical DTW the following formula is used [13]:

$$D(m,n) = d(m,n) + \min(D(m-1,n), D(m,n-1), D(m-1,n-1) \quad (6)$$

Elements of accumulated distance matrix are calculated iteratively. At the formulas (4), (5), (6) $m \in [2 \dots M]$, $n \in [2 \dots N]$are points indices of reference and test signals correspondingly.

After calculating matrix (2) the warping path *w* are calculated on the basis of this matrix. This path includes a set of minimal allowable transitions from *D(M,N)*to*D(1,1)*in matrix (2). Allowable transitions are defined by weighted matrix (7). This matrix provides rules of moving through matrices (1) and (2) and thus determine formulas for calculating matrix (2) and warping path [13].

$$\begin{bmatrix} m & n-1 \\ m-1 & n-1 \\ m-1 & n \end{bmatrix} \tag{7}$$

Values of *m* and *n* in (7) are the same as values in previous formulas. Matrix (7) is used in classical DTW. Modifications of this matrix has significant influence in form of warping path curve.

Weighting matrix has a graphical representation, which is shown on Figure 2.

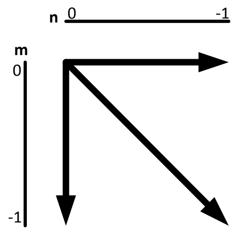

Figure 2 – Graphical representation of weighting matrix for classical DTW

As described above, optimal path is calculated based on accumulated distance matrix with the help of transition rules defined by weighting matrix. Calculation starts from the last element of (2) and goes through all matrix until the first element will be reached. During calculation in each next point of matrix (2) minimal value of allowable points are defined. Point with minimal value of accumulated distance is selected as the next point of warping path. In classical algorithm the following formula is used for this calculation [13]:

$$D(m,n) = d(m,n) + \min(D(m-1,n), D(m,n-1), D(m-1,n-1) \qquad (8)$$

Warping path is a two-dimensional array, which includes numbers of points matched during warping.

Graphical representation of warping path is a warping curve (Figure 8). On this figure, warping curve is shown with a heat map graph of accumulated distance matrix. If the algorithm works correctly, warping path is lying inside parallelogram with vertexes, defined by accumulated distance matrix and constraints of algorithm [13].

Classical DTW algorithm has several constraints, which influence on warping curve form. There are two types of constraints: local and global [14].

Global constraints require warping path to satisfy the following:

$$M1 = w(N1); \qquad (9)$$

$$M2 = w(N2) \quad (10)$$

where M1, M2 – first and end points of reference signal, N1, N2 – first and end points of test signal,w – mapping function corresponding to the warping path.This constraint means that all points of both test and reference signals must be involved in warping procedure.

Local constraints define form of weighting matrix. These constraints mean that during warping path calculation only transitions between neighboring elements are allowed and moving backward is not allowed. These constraints force weighting matrix to have elements like in (7). Other forms of this matrix are not allowed.

Due to high sensitivity of DTW algorithm to types and changes of input signals, classical approach is not appropriate in several cases. In particular, some types of changes in signals can cause imperfect matching between reference and test signals after warping. To solve this task, a large amount of DTW modifications are developed [13].

Modifications of dynamic time warping algorithm

Modifications of DTW algorithm allow relaxing or eliminating local and global constraints. In several cases, this allows to make better matching then in classical way. To describe problems of matching, graphical representation of DTW is useful (Figure 3).

Figure shows warping of two simple test signals. Warping function here is a set of lines with different length and slope. Each line connects pair of points from warping path array.

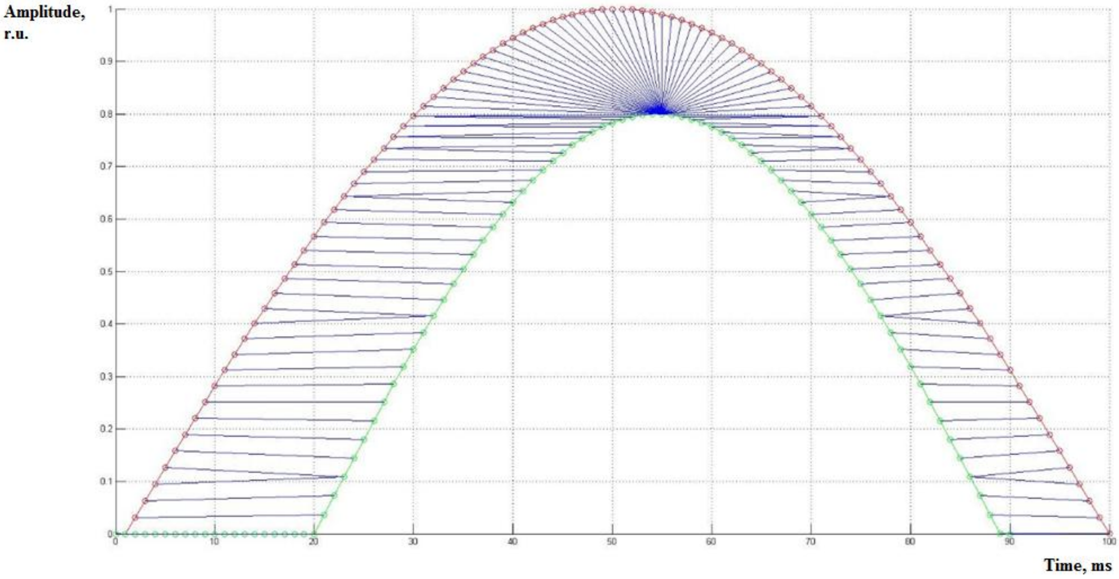

Figure 3 – Matching function based on warping path array

The figure shows multiple matching areas: a group of lines connects multiple points of test signal with single point of reference signal, and vice versa. Experimental study of these areas behavior showed that their appearance is mainly caused by amplitude difference between test and reference signals. Width difference between signals also affect their areas, but less than amplitude. The influence of signals shifting for these areas is negligible. Multiple matching areas causesignals distortion afterwarping. An example of such distortion for the signals shown in Figure 3 is shown on the Figure 4.

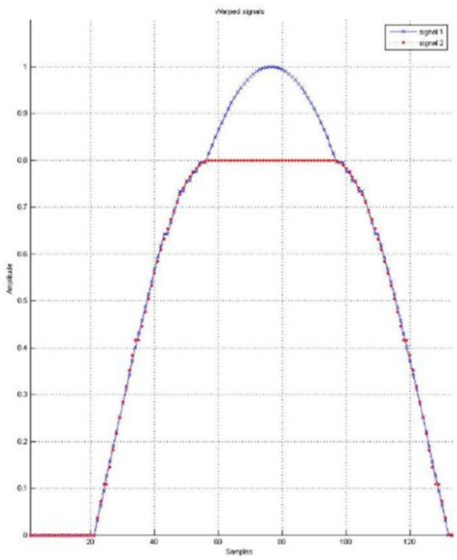

Figure 4 – An example of signal distortion after warping

Distortions in signals have a negative impact on the warping curve and can lead to mistakes in analysis of signals changes, thus modification of the algorithm must reduce distortions.

Global constraints modifications allow exclusion several points of signals from warping. These modifications allow not all points to be used in warping. There are three types of such modifications: open end modification, open beginning modification and modification with complete elimination of global constraints [14]. The “open end” modification relaxes constraint (10) (points at the end of signals are excluded), “open beginning” relaxes constraint (9) (points at the beginning of signals are excluded)and third modification relax all global constraints (points both at the end and at the beginning are excluded).

Amount of excluded points can vary depending on signal type and task of study. This approach is useful when the distribution density of points in signals are different in the middle and at the ends are different. They can also decrease calculation time, if the ends points of signal are irrelevant to the analysis (for example if points of input reference and test signalsmatches).

Local constraints modifications allow changing the form of the weighting matrix (7) by adding new elements or changing existing ones. Such modifications are presented in [14]. It should be noted, that when adding the elements to the weighting matrix amount of columns must be constant.

There are symmetric and asymmetric weighting matrix modifications. For instance, matrix (11) is an asymmetric modification and (12) is symmetric one.

$$\begin{bmatrix} m & n-1 \\ m-1 & n-2 \\ m-1 & n-1 \\ m-1 & n \end{bmatrix} \tag{11}$$

$$\begin{bmatrix} m & n-1 \\ m-1 & n-2 \\ m-1 & n-1 \\ m-2 & n-1 \\ m-1 & n \end{bmatrix} \tag{12}$$

To illustrate the concept of modifications symmetry, graphical representation of different weighting matrices are used (Figures 5 and 6)

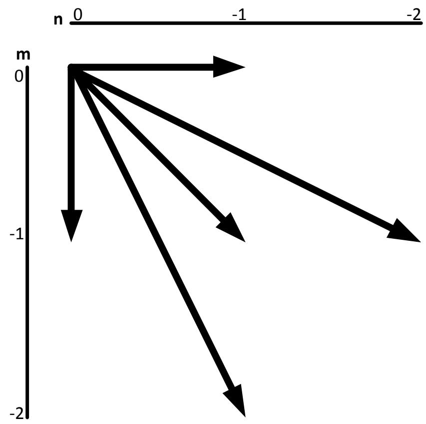

Figure 5 – Symmetric modification of weighting matrix

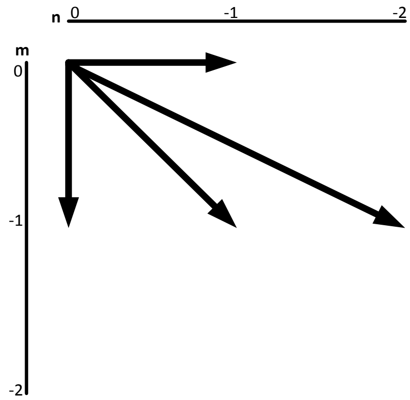

Figure 6 – Asymmetric modification of weighting matrix

On figures, there are one central diagonal transition and several left and right side transitions. Modification will be symmetric if for each side transition the same opposite side transition exists. If for at least one side transition the same opposite transition is not exists, modification will be asymmetric. Thus, it is easy to see that the classical matrix (7) is symmetric modification.

Analysis of the impact of local modifications of DTW to thewarping curve form

Modifications with global constraints relaxing only exclude some points from warping and not significantly impact to the form of warping curve. Therefore, influence of modifications with relaxing of local constraints on the warping curve was studied in this work. A comparative analysis of warping curves in classical DTW algorithm, symmetric (with matrix 12) and asymmetric (with matrix 11) modifications is described in the following. Signals, shown on Figure 7, are used as input signals for algorithm.

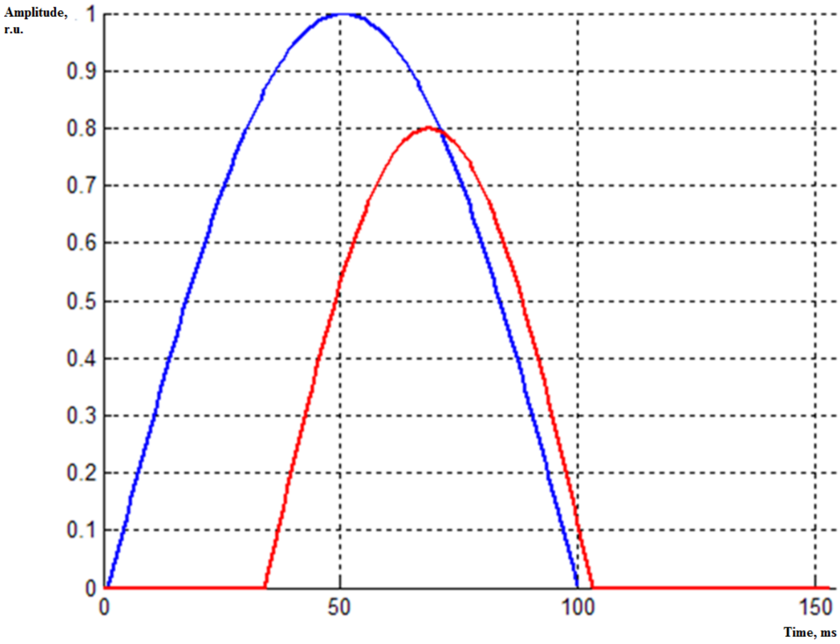

Figure 7 – test (red) and reference (blue) signals

Comparison of warping curve forms, magnitude of signals distortions after warping and amount of multiple matching areas was carried out in the study.

Figures 8 –10 presents matrices of accumulated distances together with warping curves curves for classical and modified algorithms respectively.

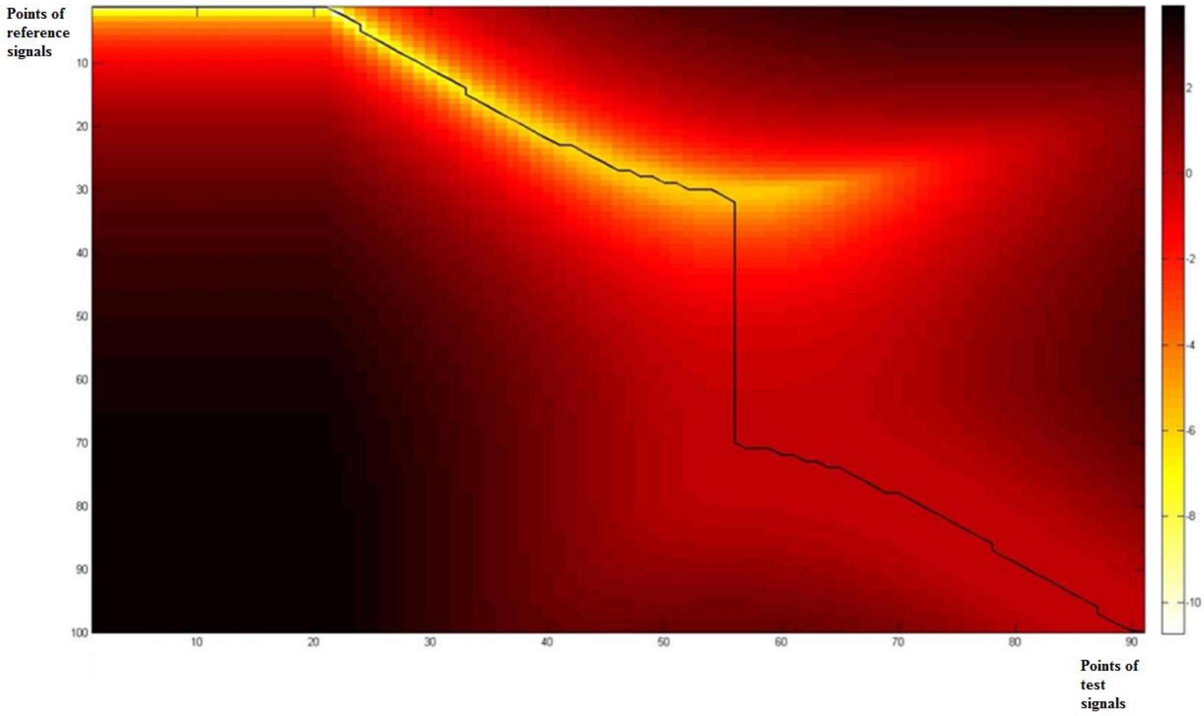

Figure 8 – Accumulated distance matrix together with warping path curve (black line) for classical DTW algorithm (lightest areas correspond to smallest distances, darkest areas correspond to largest distances)

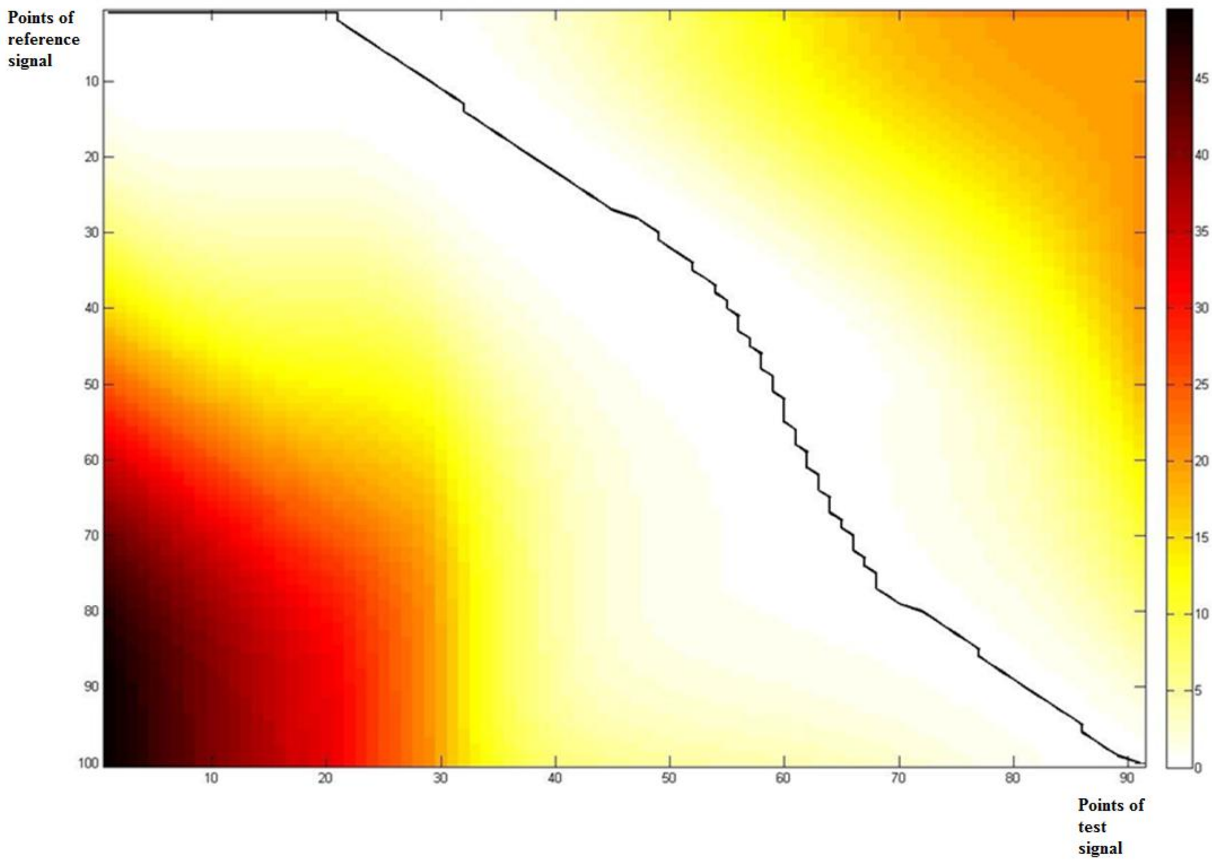

Figure 9 – Accumulated distance matrix together with warping path curve (black line) for asymmetric modification of weighting matrix of DTWalgorithm (lightest areas correspond to smallest distances, darkest areas correspond to largest distances)

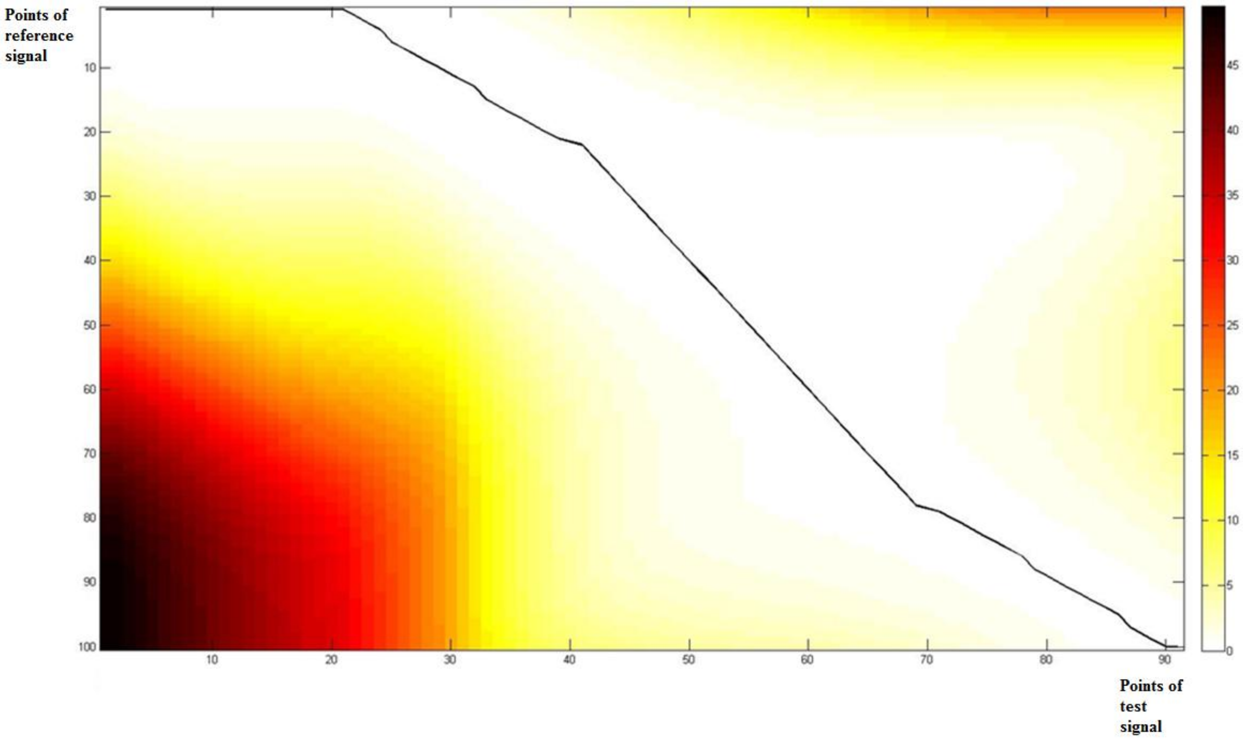

Figure 10 – Accumulated distance matrix together with warping path curve (black line) for symmetric modification of weighting matrix of DTW algorithm (lightest areas correspond to smallest distances, darkest areas correspond to largest distances)

In all three cases, the warping path curve placed in the smallest distances area of accumulated distance matrix.

Warping path curve in a classic algorithm includes a linear (vertical or horizontal) segment corresponding to amplitude difference between test and reference signals. Increase in the amplitude difference cause linear increase in this area.

Segments of simultaneous increase and decrease of the test signals and base makebroken line segments placed close to the diagonal. Experiments shown that when shrinking test signal relative to the reference one, these areas of warping path areshifting to the left with increase in width difference of input signals [15].

Experiments also shown that linear segments at the beginning and at the end (it is not shown on graphs) of the warping path curve correspond to a linear displacement of the test signal relative to the reference one.

Studyingof symmetric modification of DTW algorithm showed that the warping path curve in this case is smoother than in classical variant. Linear segment corresponding to the amplitude difference here has a slope. Segments of warping path corresponding to linear displacements betweensignals is not changed.

The form of warping path curve created by asymmetric modification of DTWalgorithm is generally similar to the form of warping path curve created bysymmetric one. Changes are only in segment corresponding to amplitude difference between signals. This segment has a form of broken line.

Figures 11 – 13 show waveforms of input signals before and after the warping for classic and modified algorithms respectively.

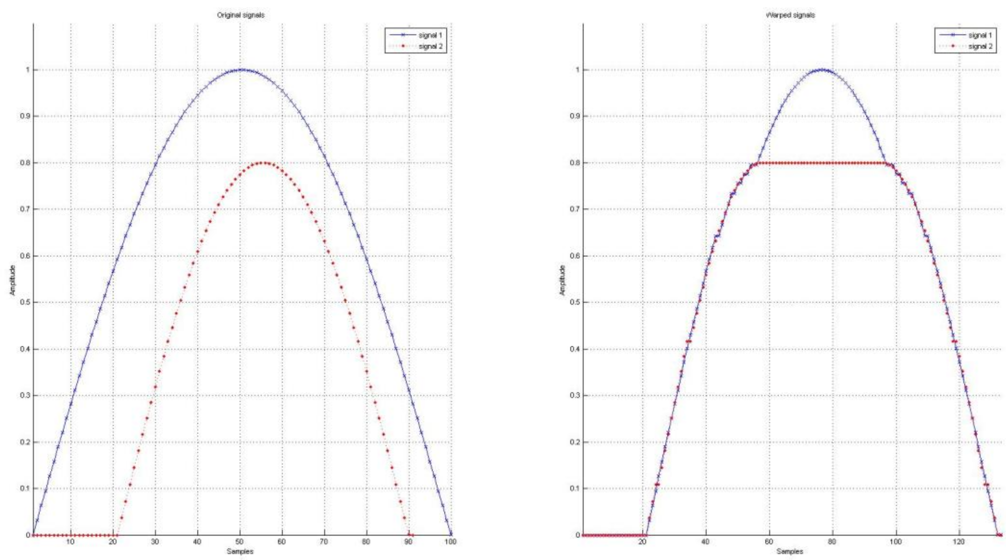

Figure 11 – Signals before (left) and after (right) warping for classical DTW algorithm

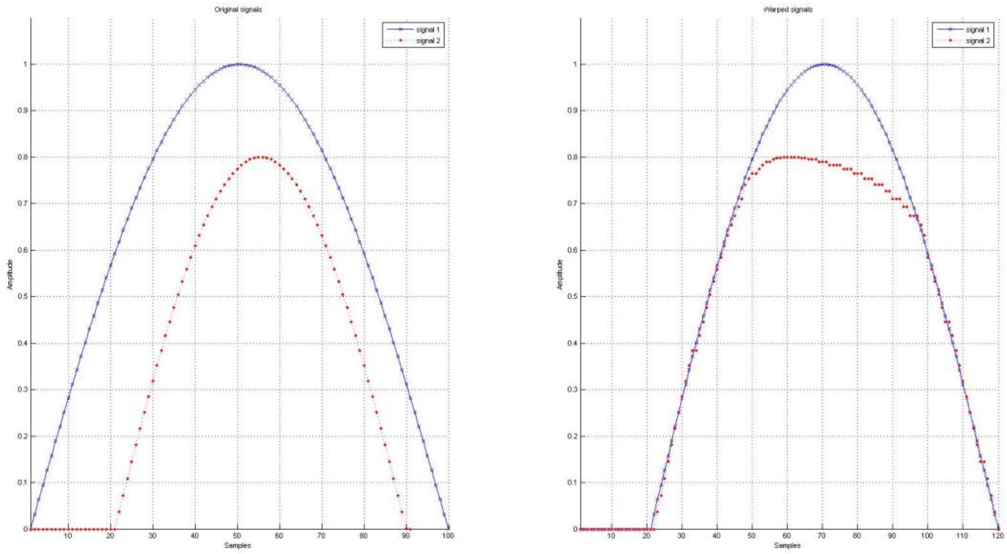

Figure 12 – Signals before (left) and after (right) warping for asymmetric modification of weighting matrix of DTW algorithm

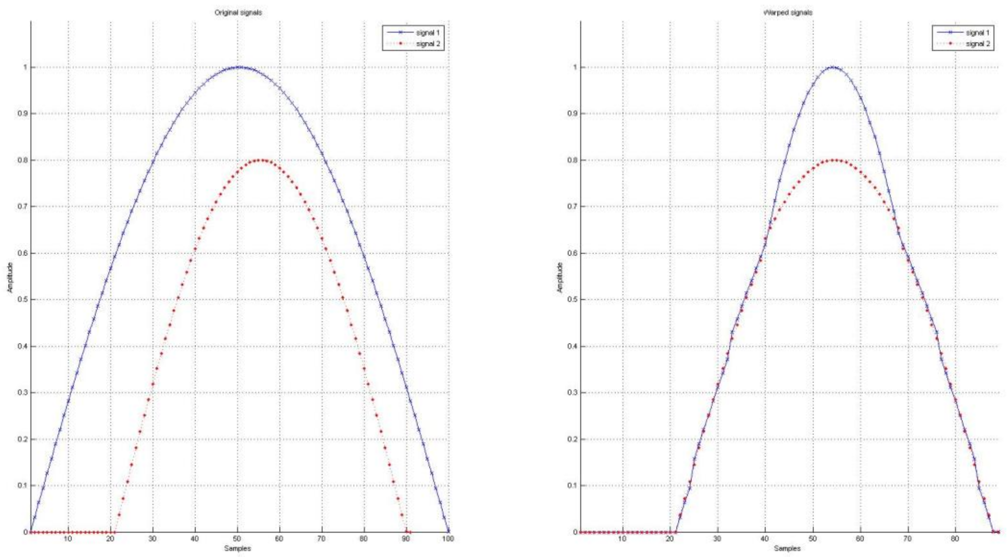

Figure 13 – Signalsbefore (left) andafter (right) warpingforsymmetric modification of weighting matrix of DTW algorithm

Studying of signal distortions after warping shown that the biggest distortion is observed when using a classical DTW algorithm. Asymmetric modification causes smaller distortion, but it is still significant. Symmetric modification provides the smallest distortion of signals and make nearly perfect matching in width of signals. However, none modifications canideally match signals when warping. To solve this problem so-called two-dimensional warping might be used [12], which allows changing both time and amplitude axes, causing non-linear transformation of the entire coordinate plane in four directions.

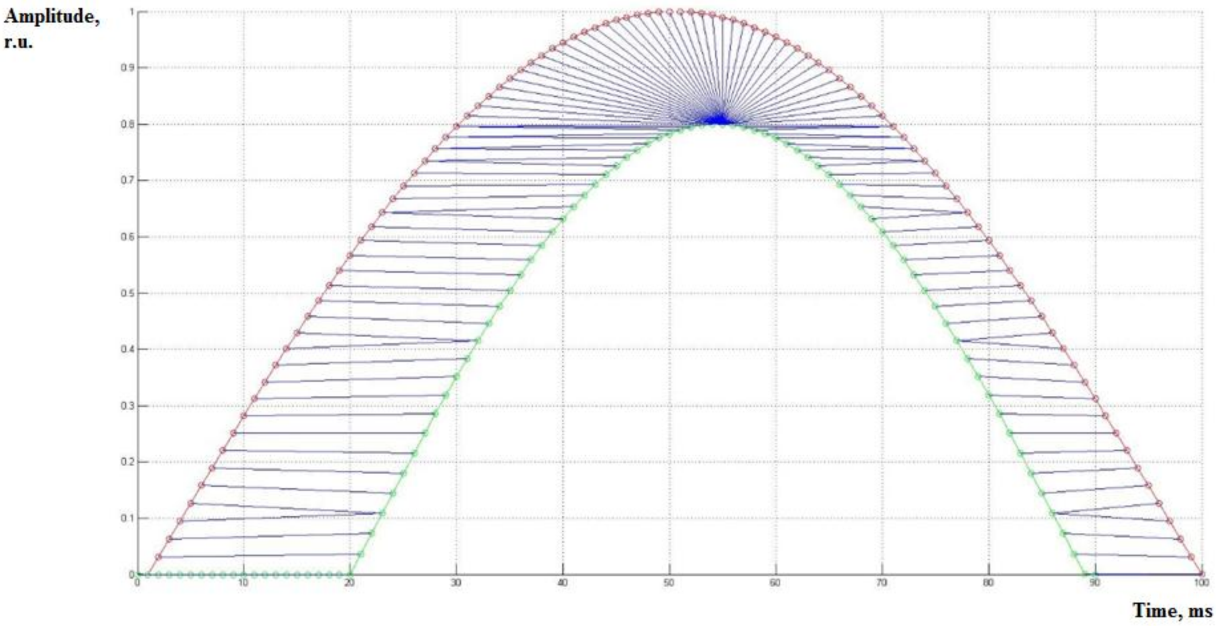

Figure 14 – Matching function for classical DTW algorithm

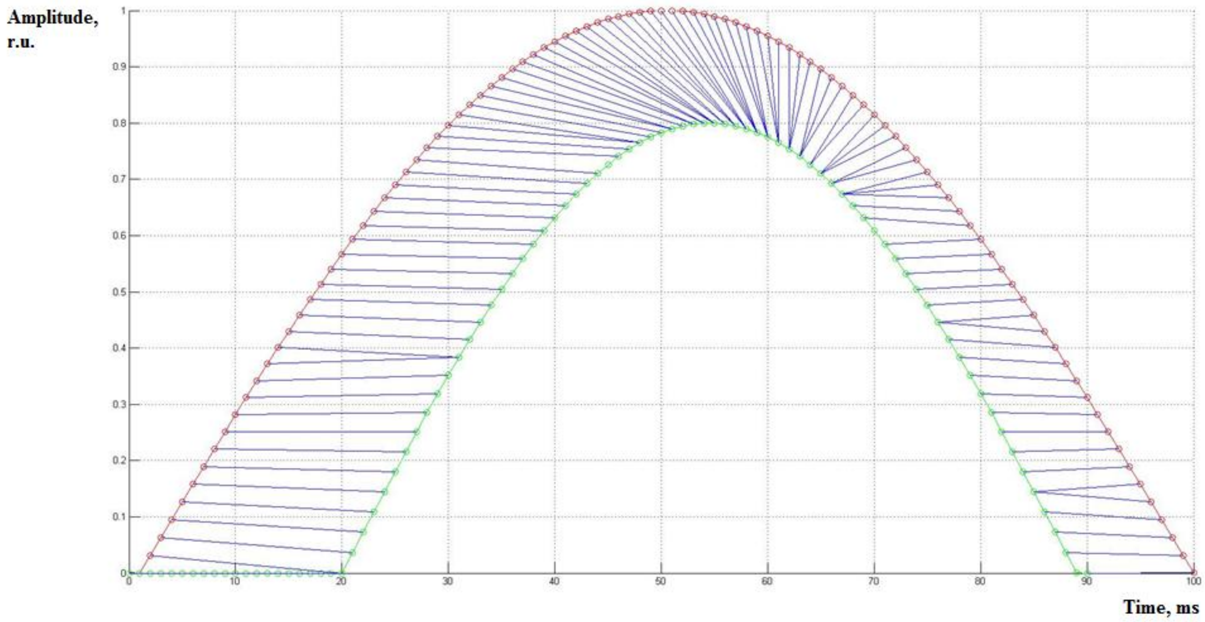

Figure 15 – Matchingfunction for asymmetric modification of weighting matrix of DTW algorithm

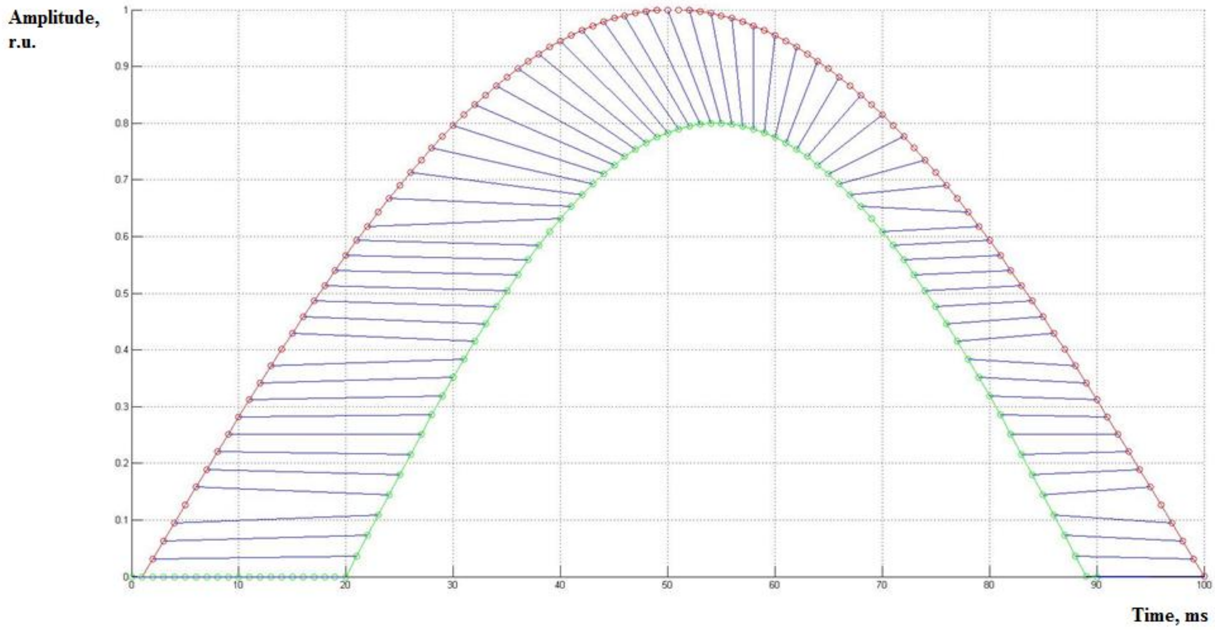

Figure 16 – Matching function for symmetric modification of weighting matrix of DTW algorithm

Figures 14 – 16 shown matching functions for all modifications.

During testing multiple matching areas (areas where multiple points of one signal match single point of other one) in matching function graphs were found. Studying of these areas shown the largest amount of it observed when using classical algorithm. Matching function in this case has large multiple matching segment, corresponding to the amplitude difference between signals. The matching function for asymmetric modificationincludes large number of small multiple matching segments, and the matching function for symmetric modification hasalmost no such areas.

After all studies it was foundthat, classical DTW algorithm deals with signals matching task worse than its modifications. Symmetric modification cause fewer distortions in signals after warping then asymmetric one. Form of warping path curve for symmetric algorithm are smoother than for asymmetric one. In addition, symmetric modification allows avoiding multiple matching areas.

Thus, symmetricmodification of DTW algorithmprovides perfect results for matching signals.

3 Warping path curve and ways of its analyzing

Properties of warping path curve

Warping path is an output parameter of DTW algorithm. Warping path curve is a graphical representation of it. This chapter discusses approaches of analyzing form of warping path curve to findchanges of input test signals waveform.

To find approach of warping path analyzing, clear definition of warping path and its properties description is needed. Then it is needed to determine factors influencing the form of warping path curve.

Warping path is a two-dimensional array, which contains pairs of points, are being matched when warping. Thus, warping path determines form of matching function.

Shapeof warping path curve strictly depends onlocal constraints defined by weighting matrix and global constraints (9) and (10), unlesscorresponding modifications are used. In addition, as shown in previous chapter, significant influenceto the curve shape has additional allowabletransitions in the weighting matrix. Symmetry of weighting matrix modifications is also an important factor affecting the curve shape.

One of the main properties of the warping path curve is the locality of its changes. This means that local changesof the input signals cause local changes in the shape of warping path curve. This feature allows precisely allocate signals changes on the curve and determine their boundaries. This property is always work, regardless of the modification and complexity of signals.

Another important property of warping path curve is the following: when test and reference signals are the same, warping path curveis a diagonal passing through the first and last points of the accumulated distance matrix

(2). This property is useful for the analysis of the warping path curve. Method for analyzing the curve will be developed based on this feature.

The method of analyzing changes insignals waveforms based on the shape of the warping path curve

Warping path curve shows qualitative differences between test and reference signals and allows allocating differences in signals. However, by the warping path curveitself is inconvenient to get quantitative parameters of difference between input signals, because it contains no numerical data about this difference.

One way of obtainingnumeric data about differencesbetween the input signals is the comparison of the real warping path curve with some template. To make this template, diagonal described in second property of warping path might be used.

The warping path curve is a mapping function of the test signal to the reference one. It can be found from the second propertythat diagonal is a mapping function of reference signal to itself. Theoretically, the distances between real warping path and diagonal arenumerical indices of differences between test and reference signals.

To obtaindescribed diagonal, the following equation (13) is used.This diagonal passing through points with coordinates (1, 1) and (N, M), respectively, where N is the length of test signal, the M islength of reference signal.

$$(1 - M)x + (N - 1)y + (M - N) = 0 \tag{13}$$

There are at least three types of distances between points of warping path curve and diagonal defined by equation (13): distance along the X-axis, distance along the Y-axis and the shortest distance definedas a perpendicular going from a given point to the diagonal. All three distances for a single point of the warping path curve are shown in Figure 17.

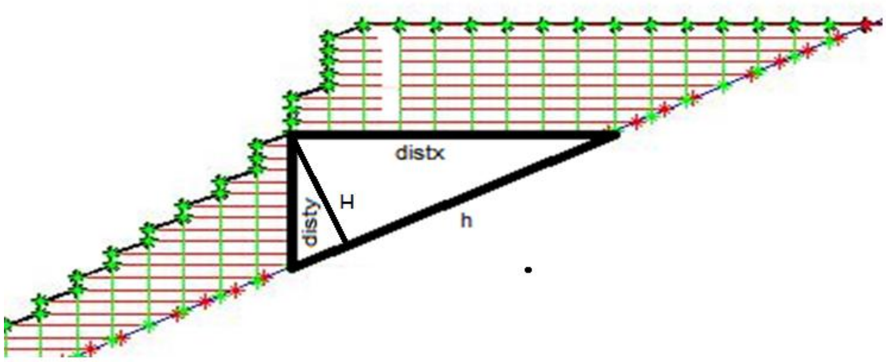

Figure 17 – Geometric interpretation of distances

As shown on figure, the distances along axes together with a segment of diagonal make a right triangle. Minimal distance between point anddiagonal is the height of this triangle. Thus, to calculate all three types of distances, standard formulas might be used.

To determine the points of intersection of distances along axes and diagonal it is needed to find x and $y$values from equation (13) using formulas (14), (15) correspondingly.

$$x = \frac{(N - M) + (1 - N)y}{(1 - M)} \tag{14}$$

$$y = \frac{(N - M) + (M - 1)x}{(N - 1)} \tag{15}$$

Distances along axes is definedusing formulas (16), (17).

$$distx_i = \|wx_i - x_i\| \tag{16}$$

$$disty_i = \|wy_i - y_i\| \tag{17}$$

In formulas (16), (17) wx_i - x-component of optimal path array, wy_i - y-component of optimal path array.

Minimal distance H_i between points of warping path curve and diagonal is defined using formulas (18), (19), (20).

$$h_i = \sqrt{distx_i^2 + disty_i^2} \quad (18)$$

$$p_i = \frac{(distx_i + disty_i + h_i)}{2} \quad (19)$$

$$H_i = \frac{h_i}{2}\sqrt{p_i(p_i - distx_i)(p_i - disty_i)(p_i - h_i)} \quad (20)$$

Figure 18 shows a graph of distances along coordinate axes for all points of the warping path curve.

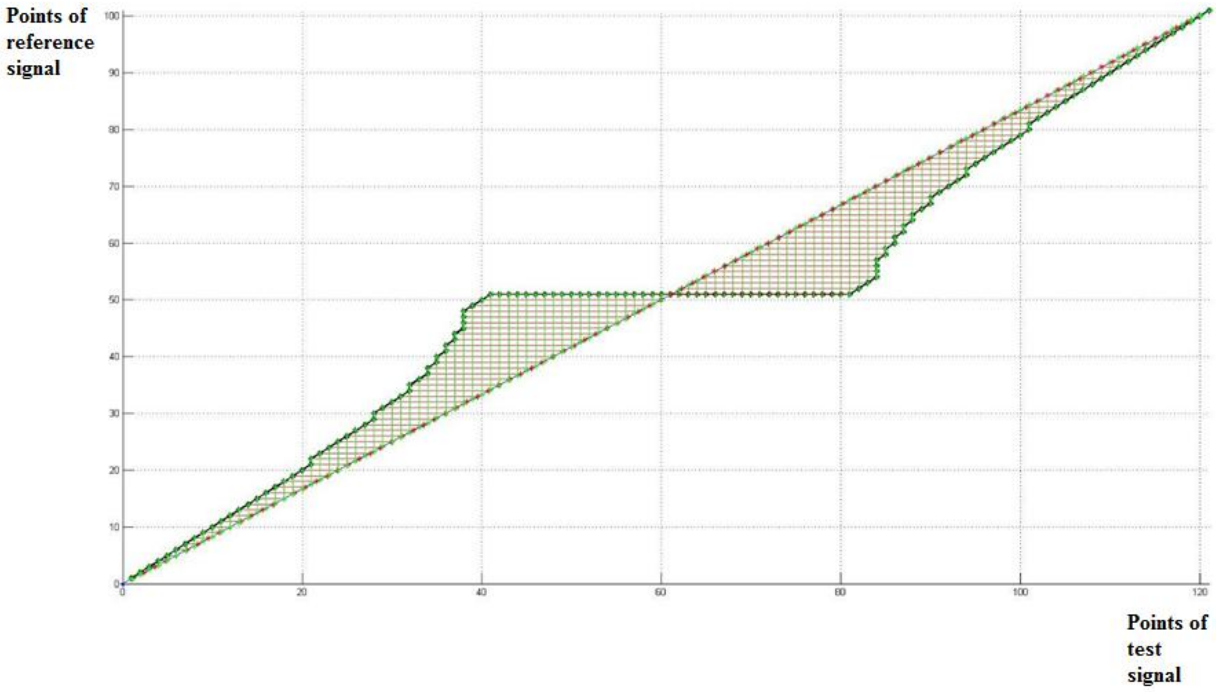

Figure 18 – Graph of distances along coordinate axes far all points of warping path curve

Practical importance has studying of the distances distribution function for all three types of distances (Figure 19)

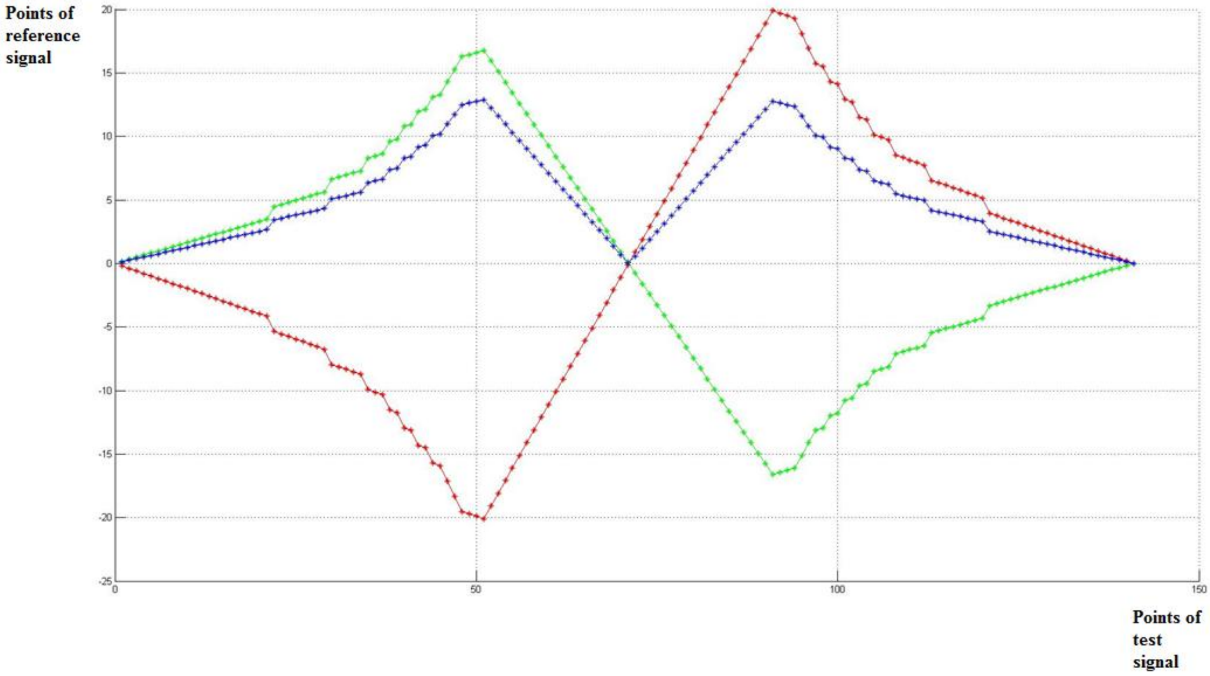

Figure 19 – Distance distribution curves

Analysis of minimal distances between warping path curve points and diagonal allow determining the deviation degree of the test signal points from points of reference one with which it is matched. Analysis of distances along axes allows estimating the horizontal and vertical components of test signal points deviation from the reference signal points.

These statements are based on theoretical researches and the study of the DTW algorithm properties. For practical implementation of described method, experiments on test and real data need to be carried out.

Experimental study of the warping path curve behavior under various conditions

To check applicability of developed method for changes analysis in real signals series of experiments is conducted.

The aim of experiments is to study the impact of input signalschanges to deviation of warping path from the diagonal.

To simplifyprocessing of experiment results at the first stepsimple single-wave sine signals will be used.

As a reference signal, single-wave sine signal with constant parameters will be used. Parameters of test signal will be calculated based on reference signal parameters. There are four parameters (or factors in experiment planning theory) of these signals:

- signal amplitude;
- signal width;
- initial displacement from reference signal;
- the end segment of zero values.

Each parameter will have two values (or levels in experiment planning theory) during experiment. High level will be calculated as 1.2 of base level value. Low level will be calculated as 0.8 of base level value. As a base level, values of reference signal parameters are choosing. As a base level value It will be enough, because described signals are simple and linear changes of warping path are expected when linearly changing input signals. Fractional factorial experiment (FFE) of type 2^{4-1} will be used as a plan of experiment. This plan allows to decrease amount of experiments are needed to find a model of changes in warping path curve. To encode factors the expression (21) will be used [2].

$$x_j = \frac{\tilde{x}_j - \tilde{x}_{j0}}{I_j}, \tag{21}$$

where x_j and $\tilde{x}_j$ are encoded and real value of each factor respectively, $\tilde{x}_{j0}$ is a real value base level of each factor, I_j is varying interval, j - factor number.

At the first step of experiment,a linear model (22) of warping path curve changes is used.

$$y = b_0 + \sum_{j=1}^{n} b_j x_j \ , \tag{22}$$

It should be noted that the use of this model in a standard form (where regression model for each point are found) is not possible, because in this case it is necessary to find a models foreach point of warping path curve.Therefore, corresponding models was found only for several special points. These are intersection points of warping path curve and diagonal and points of maximal and minimal deviation from diagonal.

Experimental study with the plan described above was carried out for the classical DTW algorithm (weighting matrix 7) and for symmetrical modification of the weighting matrix (weighting matrix 11).

The study revealed symmetrical distribution forall three types of distances with relatively simple changes in form of input signals, however when using more complex changes, asymmetry in distances distribution is observed due to the combined effect of two and more factors, which in these conditions can not be eliminated. This effect was observed for both classical algorithm, and its modification.

Figures 20-25 show the results of experiments with opposite values of factor for classical DTW algorithm.

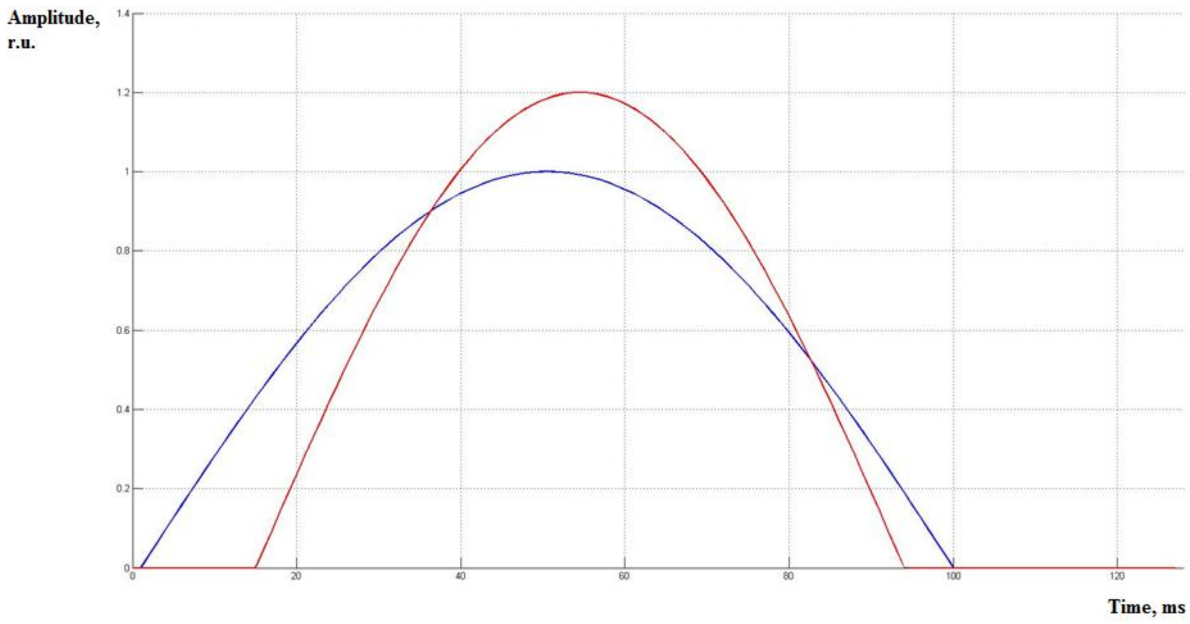

Figure 20 – test (red) and reference (blue) signals used at the first experiment with simple signals

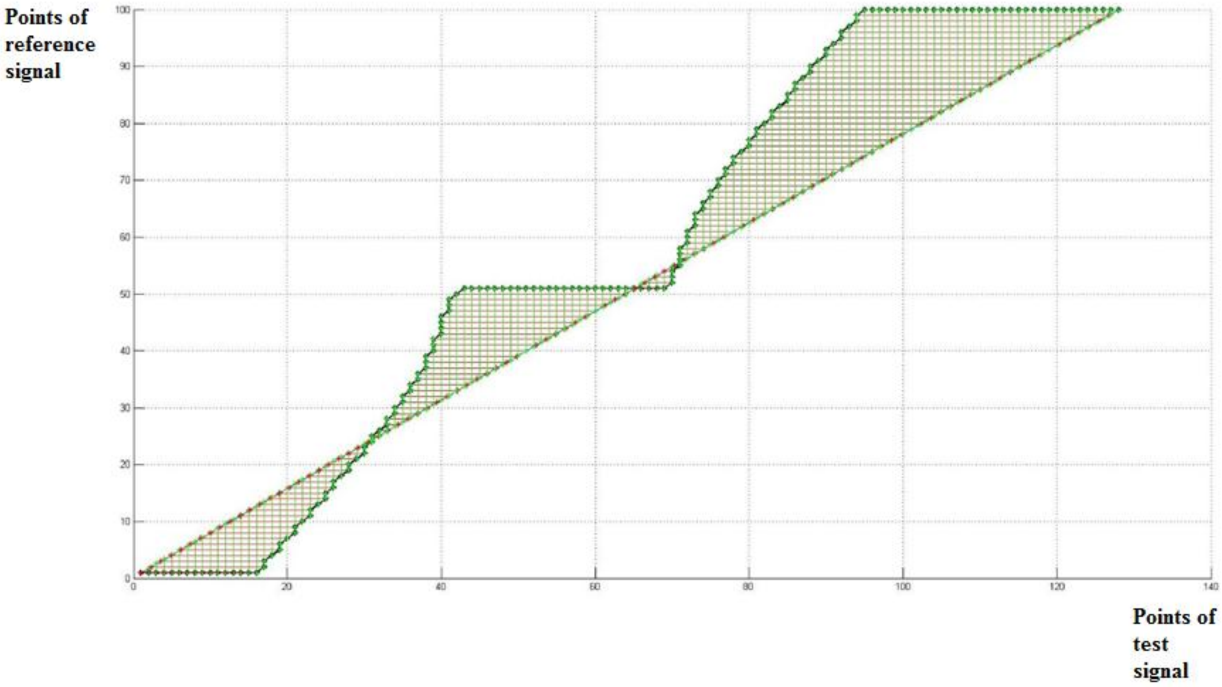

Figure 21 – warping path curve obtained at the first experiment for classical DTW algorithm

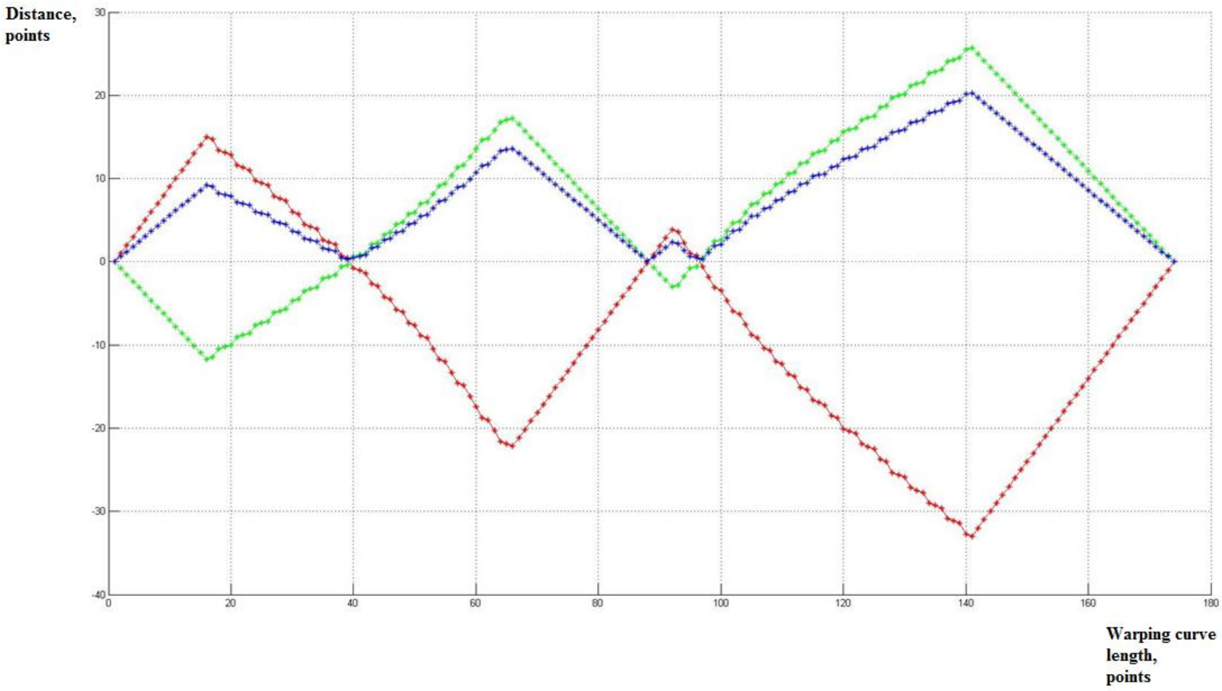

Figure 22 –Distance distribution curves, obtained at the first experiment for classical DTW algorithm (red line – distances on X-axis, green line – distances on Y-axis, blue line – the shortest distance)

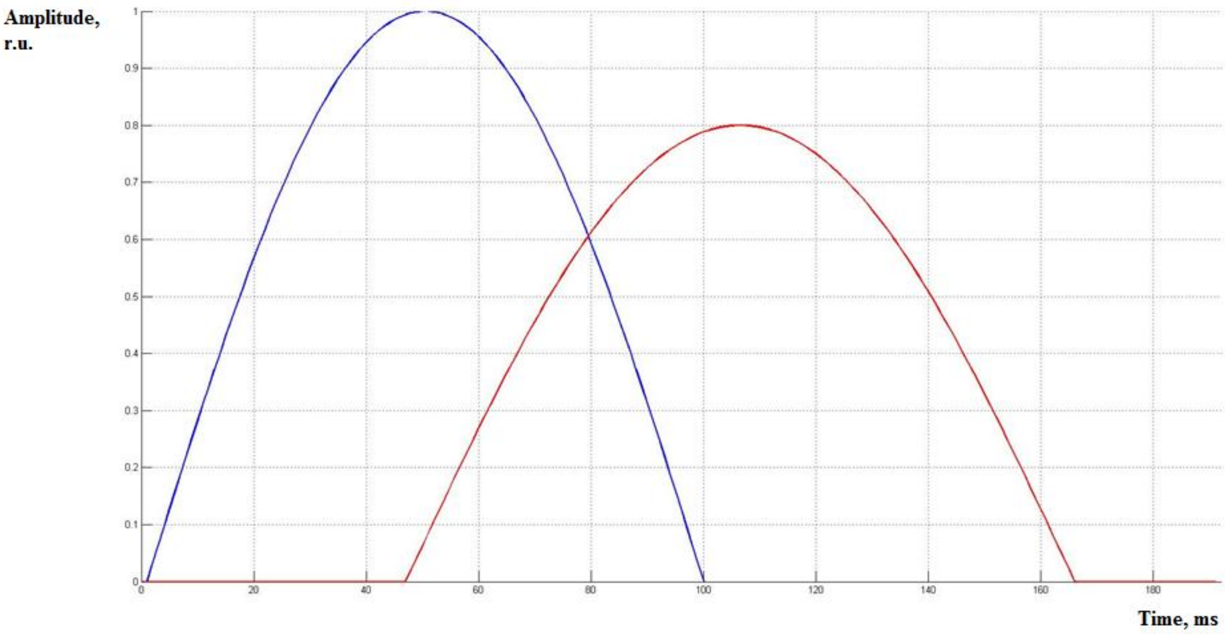

Figure 23– test (red) and reference (blue) signals used at the last experiment with simple signals

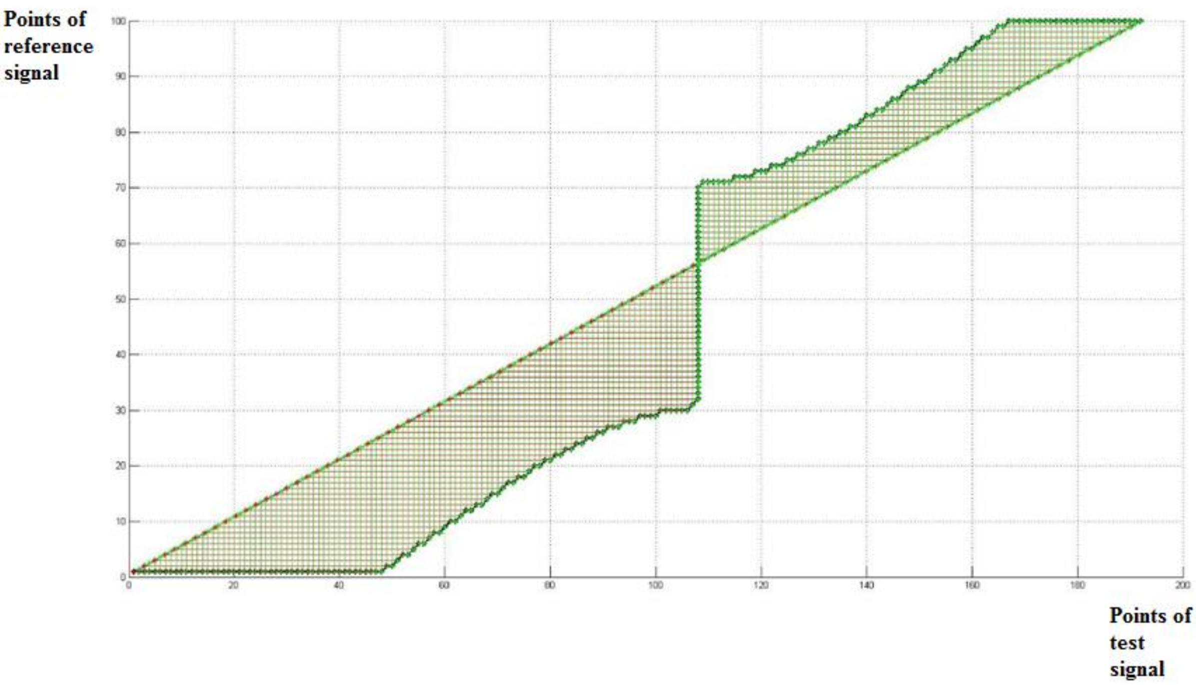

Figure 24 – Warping path curve obtained at the first experiment for classical DTW algorithm

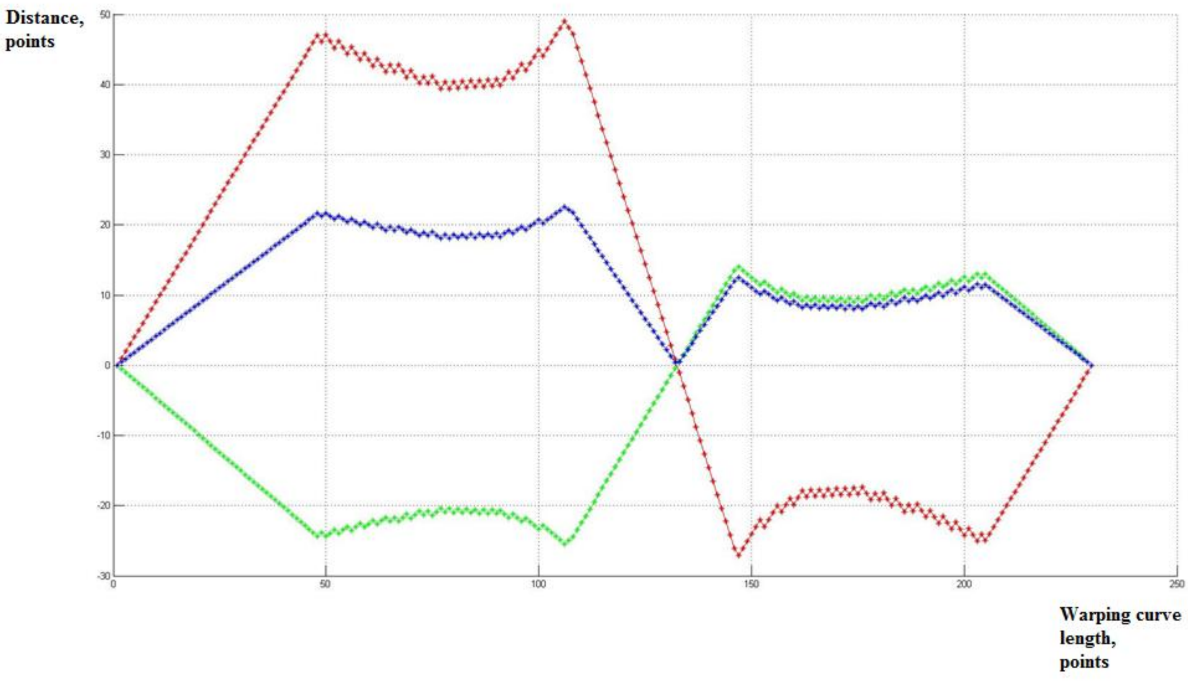

Figure 25 – Distance distribution curves obtained at the last experiment for classical DTW algorithm (red line – distances on X-axis, green line – distances on Y-axis, blue line – the shortest distance)

Experiments showed that changes in displacement values of the test signal relative to the reference one leads to a shifting of the warping path curve from diagonal. Theseshifts correspond to linear deviations from diagonal at the beginning and ending of warping path curve respectively. The length of these segmentscorresponds to the length of real displacement of input signals.

Changes in amplitude and width of input signals together have a more complex effect on the warping path curve. It is not possible to separate the combined influence of these factors with use graphs obtained from the study. In this case, the influence of signals amplitude-to-width ratio on deviation of warping path curve from diagonal was evaluated. The study showed, that the difference in amplitude and width of input signals make linear segment inside warping path curve, which intersects the diagonal. If amplitude of test signal,more than amplitude of reference one and width of test signal less than width of reference one, a linear segment is horizontal; and vice versa, if amplitude of test signal less than amplitude of reference one and width of test signal more than width of reference one, a linear segment is vertical. In the first case, distance distribution curves have more than one zero-valued points and at the second case, these curves have only one zero-valued point.

The initial and final offset affect not only the magnitude of the initial and final linear segmentsin distance distribution curves, they also lead to shifting of non-linear segments corresponding to simultaneously nonlinearly increase or decrease both test and reference signals. Length of these nonlinear segments corresponds to segments of simultaneous nonlinear increase or decrease in input signals.

Similar resultswere obtained by analyzing modification of the algorithm. Figures 26-29 shows the results of two experiments with the same parameters of signals, as in the classic algorithm.

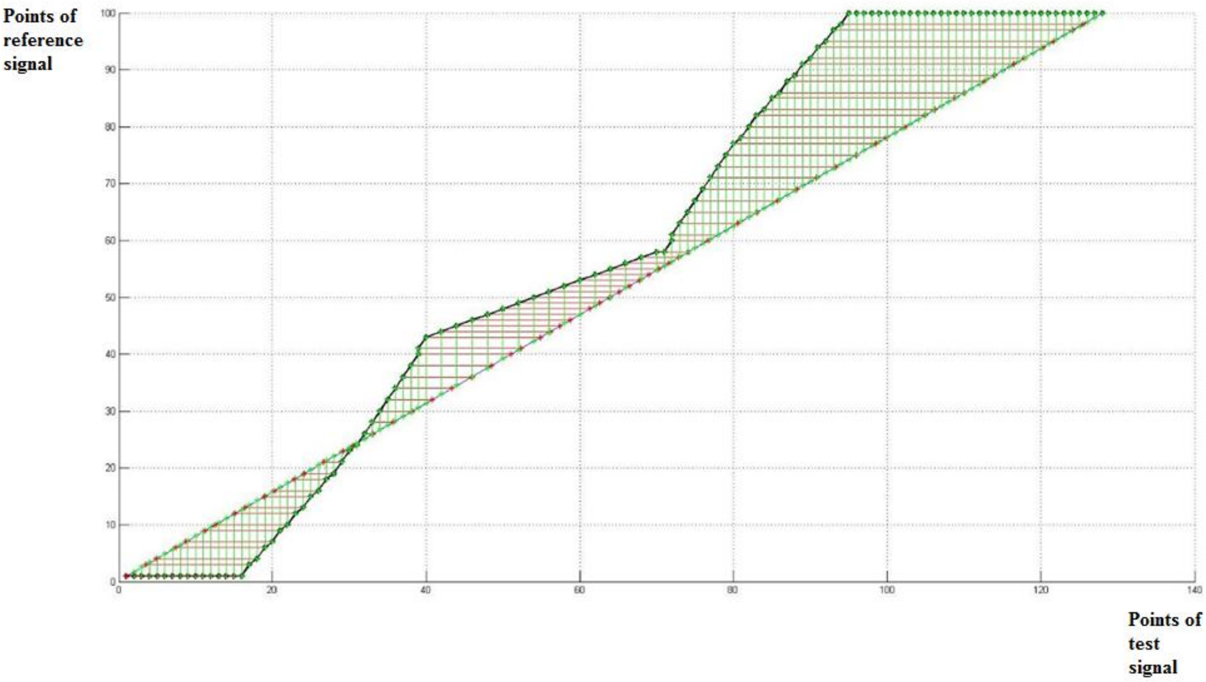

Figure 26 – warping path curve obtained at the first experiment for symmetric modification of weighting matrix

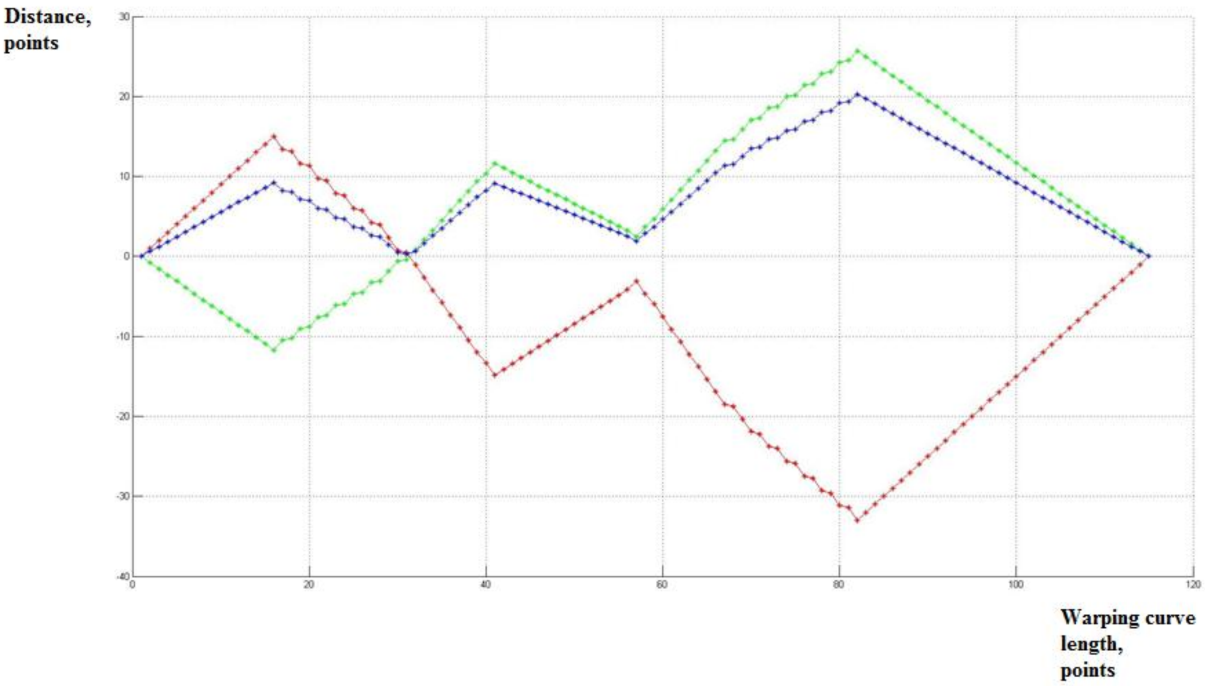

Figure 27 – Distance distribution curves obtained at the first experiment for symmetric modification of weighting matrix (red line – distances on X-axis, green line – distances on Y-axis, blue line – the shortest distance)

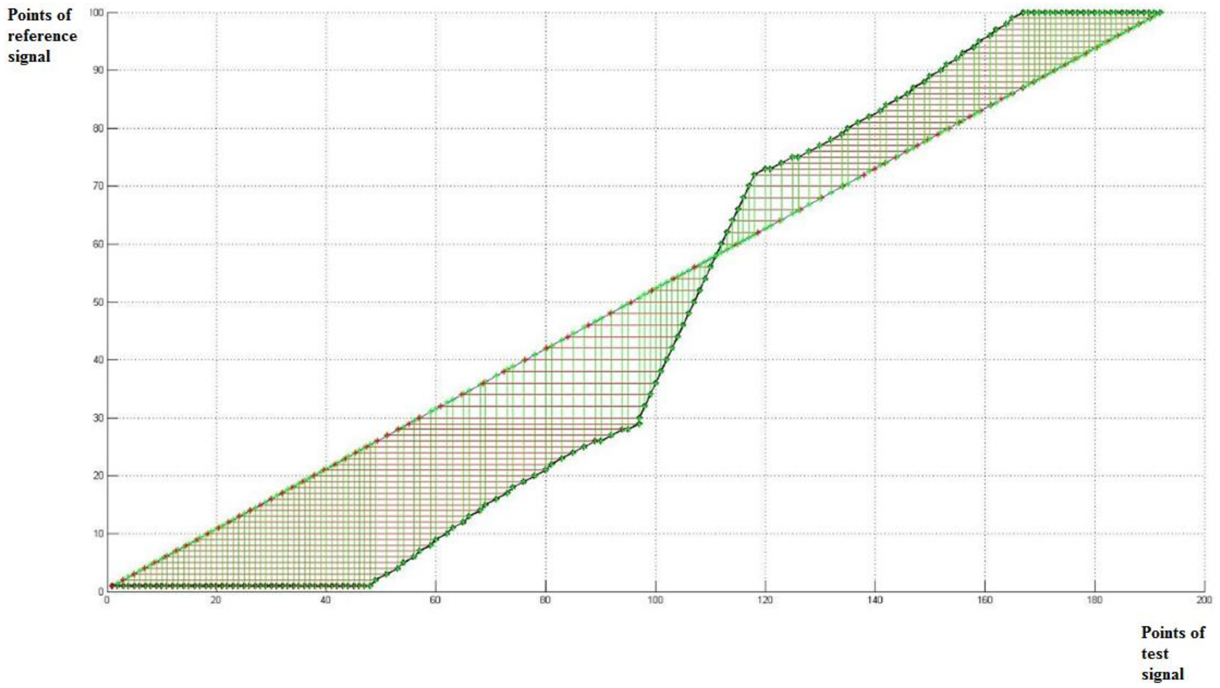

Figure 28 – warping path curve obtained at the last experiment for symmetric modification of weighting matrix

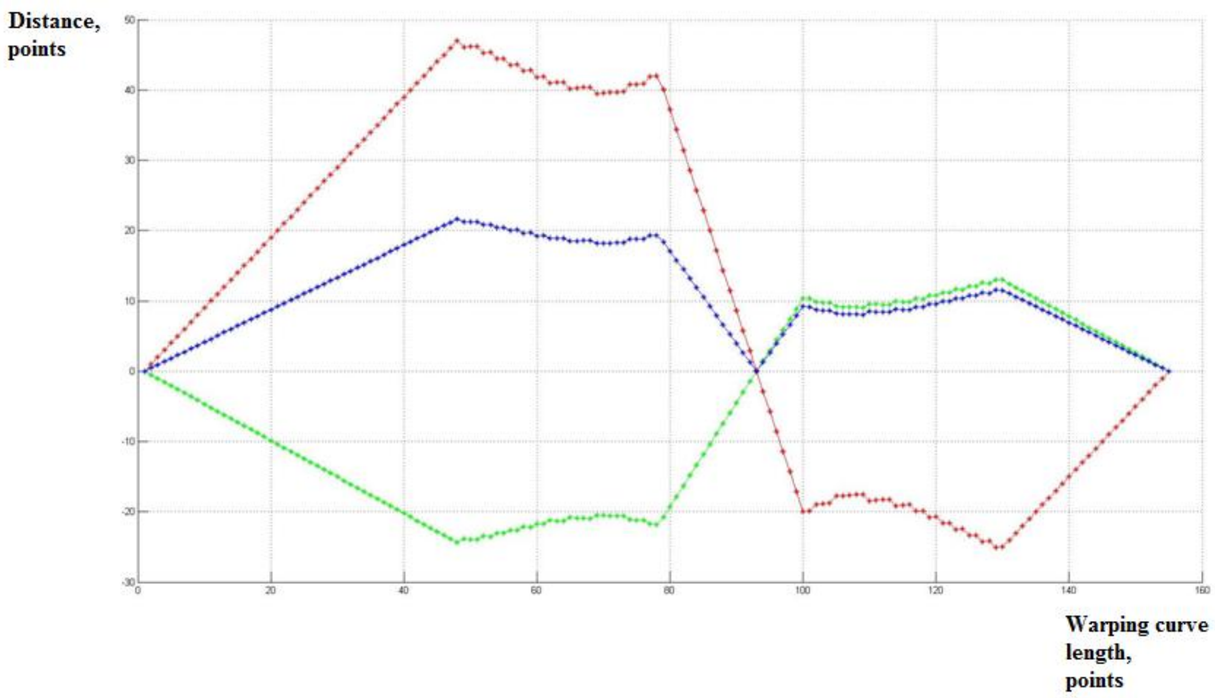

Figure 29 – Distance distribution curves obtained at the last experiment for symmetric modification of weighting matrix (red line – distances on X-axis, green line – distances on Y-axis, blue line – the shortest distance)

Analysis of experiment results showed that modification of the algorithm does not affect the shape of warping path segments corresponding to the displacements between reference and test signals. The same regularities as in the classical algorithmare observed in them.

Main changes are observed in the segment of warping path curve corresponding to the amplitude difference between signals. It was found that as well as in classical algorithm, both amplitude and width of signals simultaneously affects shape of this segment. Influence of these factors can not be separated. However, when using modifications of algorithm, this segment has a slope, by which it is impossible to definitely estimateamplitude-to-width ratio of signals.

It can be concluded that the best results when analyzing simple signals were obtained using classical DTW algorithm, since it allows estimating parameters of input signals more precise, especiallyamplitude-to-width ratio of signals.

Results described above, was obtained only for simple single-wave signals. In the next section more complex signals will be considered.

4 Study of warping path curve behavior when testing complex signals

Developing of complex test signals

Due to high sensitivity of the DTW algorithm to type of input signals and their changes, it is necessary to conduct additional experiments on more complex test signals with known parameters.

The aim of these experiments is to check applicability of developed method of analysis the warping path curve for analyzing tiny changes in repolarization segments of ECG signals. Test signals must fully reflect all features of real biological signals described above.

At the first step, the impact of dynamic changes in input signals(in particular, increasing and decreasingof its frequency) on the shape of warping path curve was studied. For this purpose, signals with variable frequency (Figure 30) and signals basedon chirp function (Figure 31) was developed.

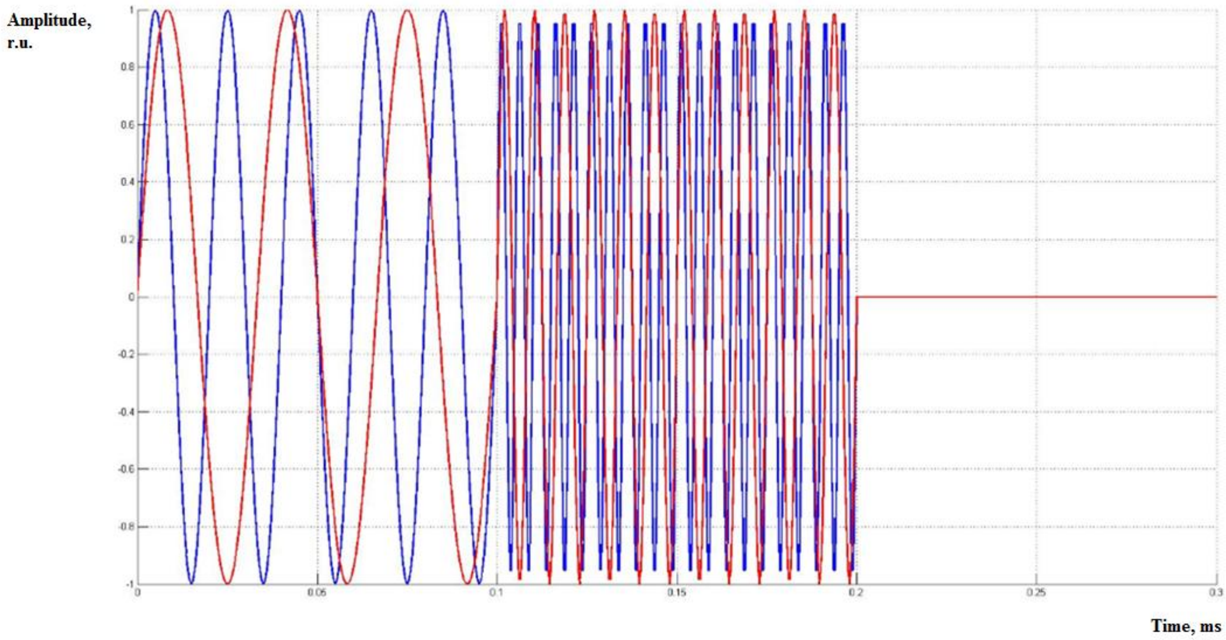

Figure30 – Artificial signals with variable frequency (red signal is test, blue signal is reference)

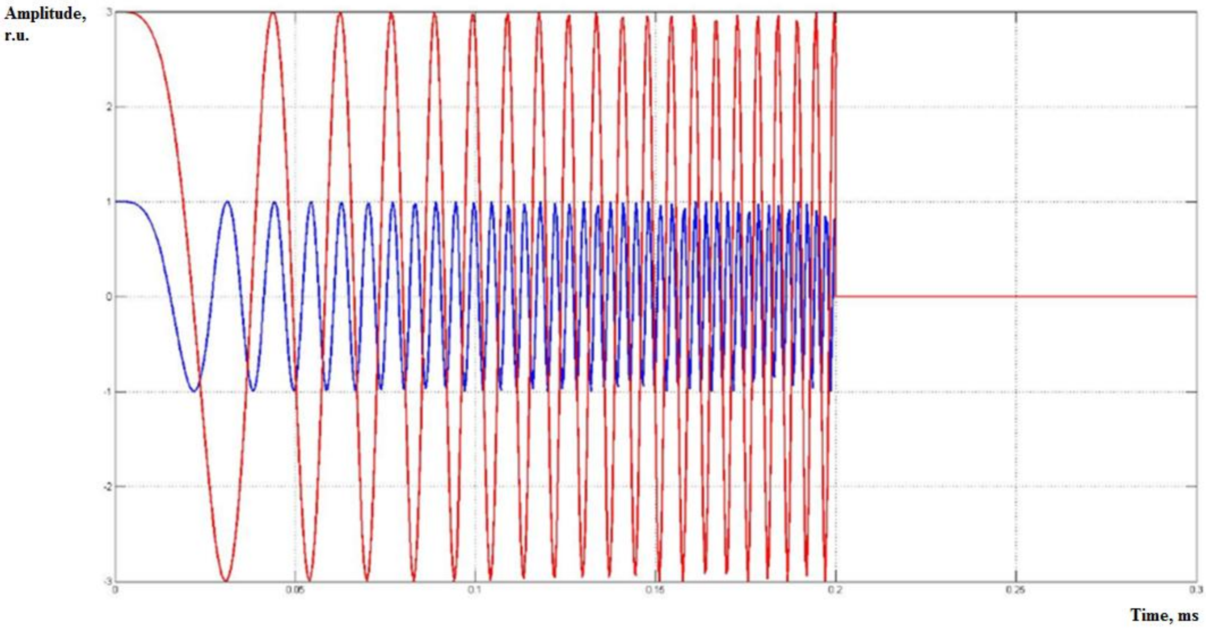

Figure 31– Artificial signals based on chirp function (red signal is test, blue signal is reference)

To test algorithm applicability foranalyzing real repolarization segments, test signals based on impulse sequence filtered second order Butterworth filter (Figure 32) was developed [20].

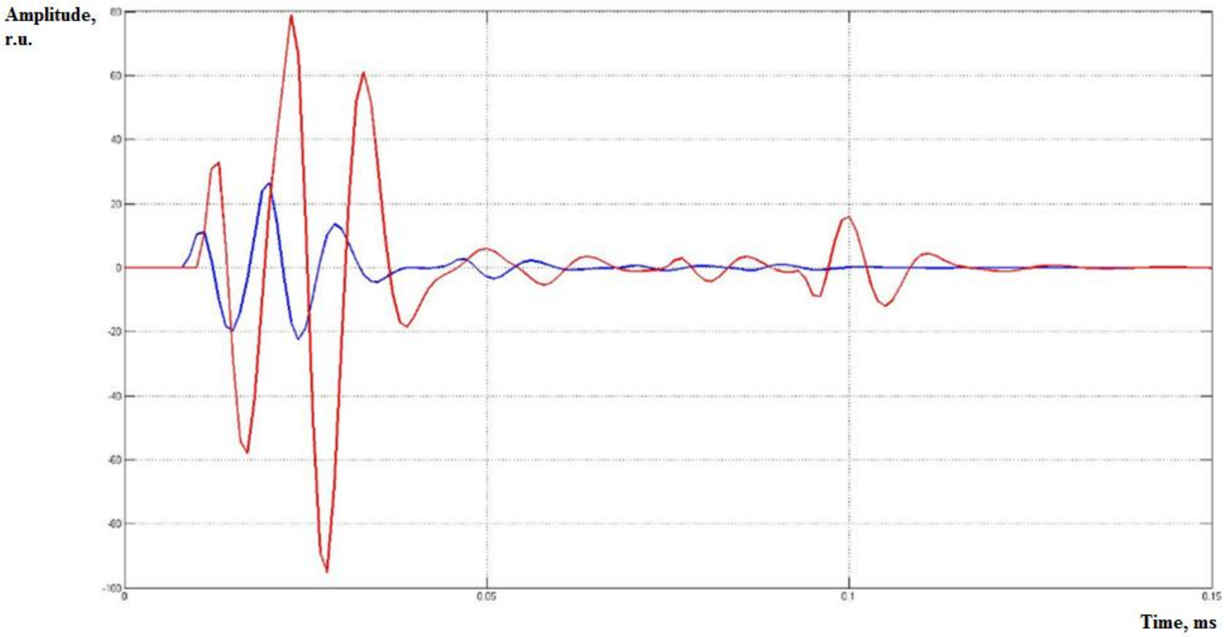

Figure 32 – Artificial signals based on impulse sequence filtered with second order Butterworth filter (red signal is test, blue signal is reference)

This signal perfectly reflects real features of ST-complexesin real ECG.

Results of experiments with three test signals, described above, using classical DTW algorithm are shown on Figures 33-35.

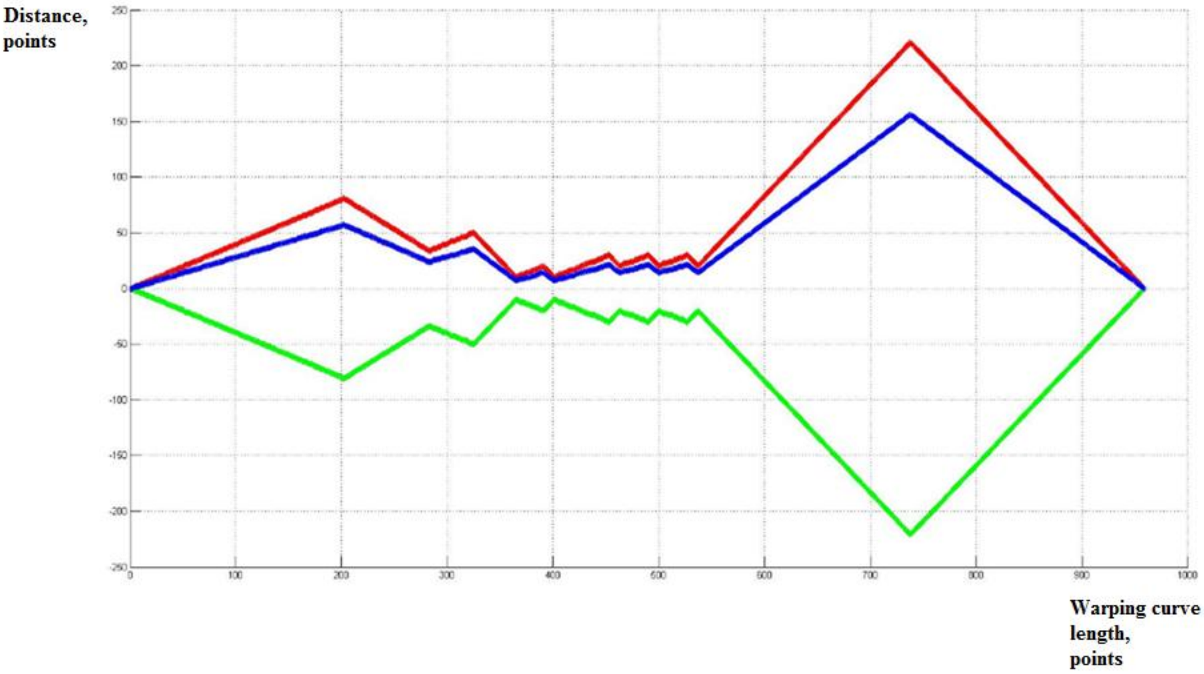

Figure 33 – Distance distribution curves for signals with variable frequency for classical DTW algorithm

(red line – distances on X-axis, green line – distances on Y-axis, blue line – the shortest distance)

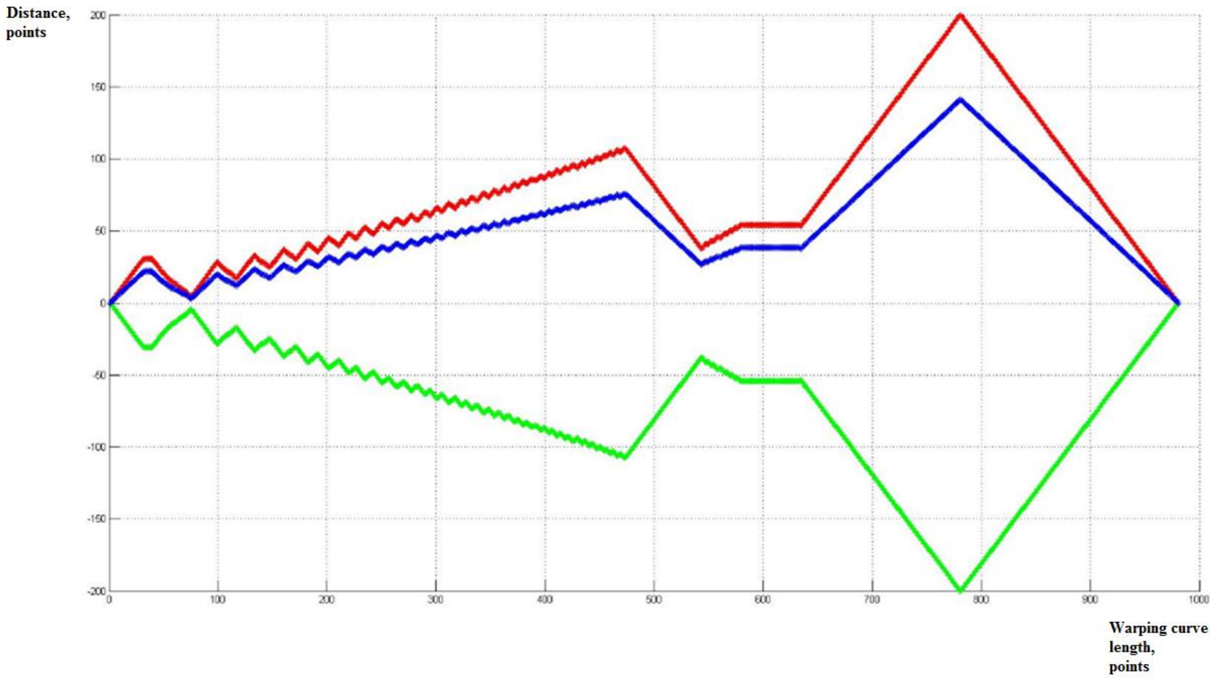

Figure 34 – Distance distribution curves for signals based on chirp function for classical DTW algorithm

(red line – distances on X-axis, green line – distances on Y-axis, blue line – the shortest distance)

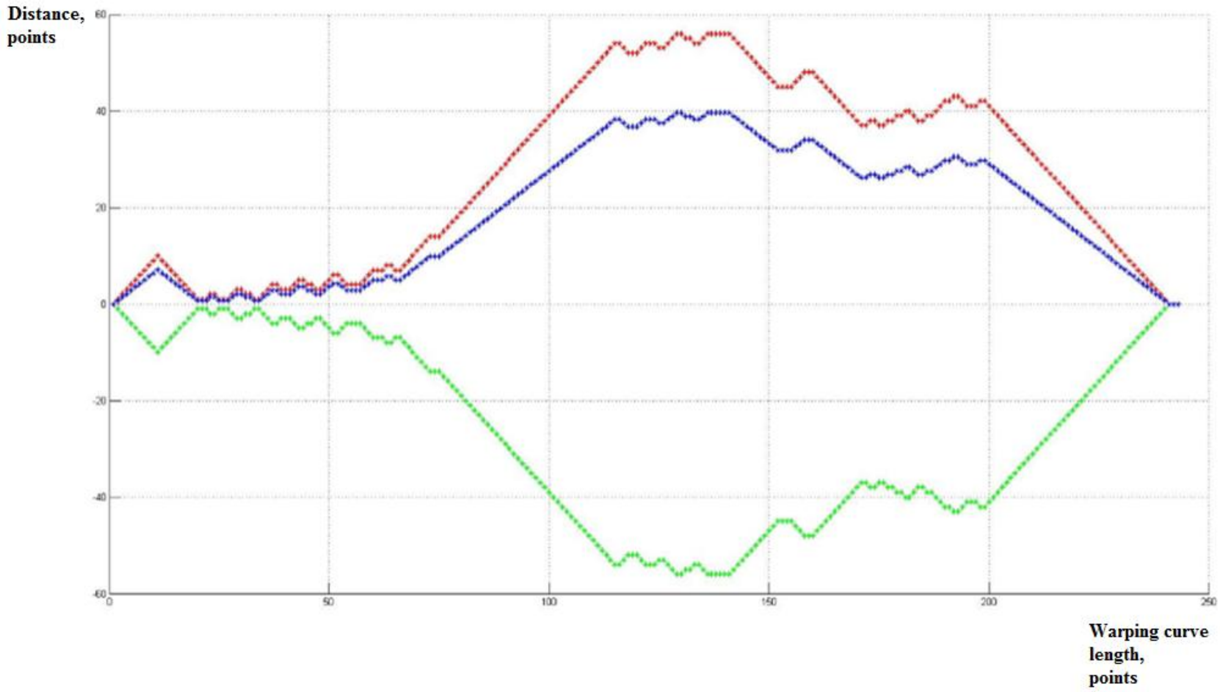

Figure 35 – Distance distribution curves for signals based on impulse sequence filtered with second order Butterworth filter for classical DTW algorithm

(red line – distances on X-axis, green line – distances on Y-axis, blue line – the shortest distance)

Experiments with classical algorithm showed that, when changing parameters of complex test signals, the same regularities in changes of distance distribution curves and warping path curves as in simple signals are observed. However, in case of complex signals there are no zero-valued points in distance distribution curves. Despite this, shifting of signal segments with the same amplitude cause linear segments within the distance distribution curves, but these segmentsobserved not only at the ends of the curves (as in case of simple signals), but also inside curve. That fact make analysis more complicated. Moreover, segments, corresponding to amplitude differencesare also observedinside curves.

Curves obtained with a symmetric modification of DTW algorithm are presented in Figures 36-38.

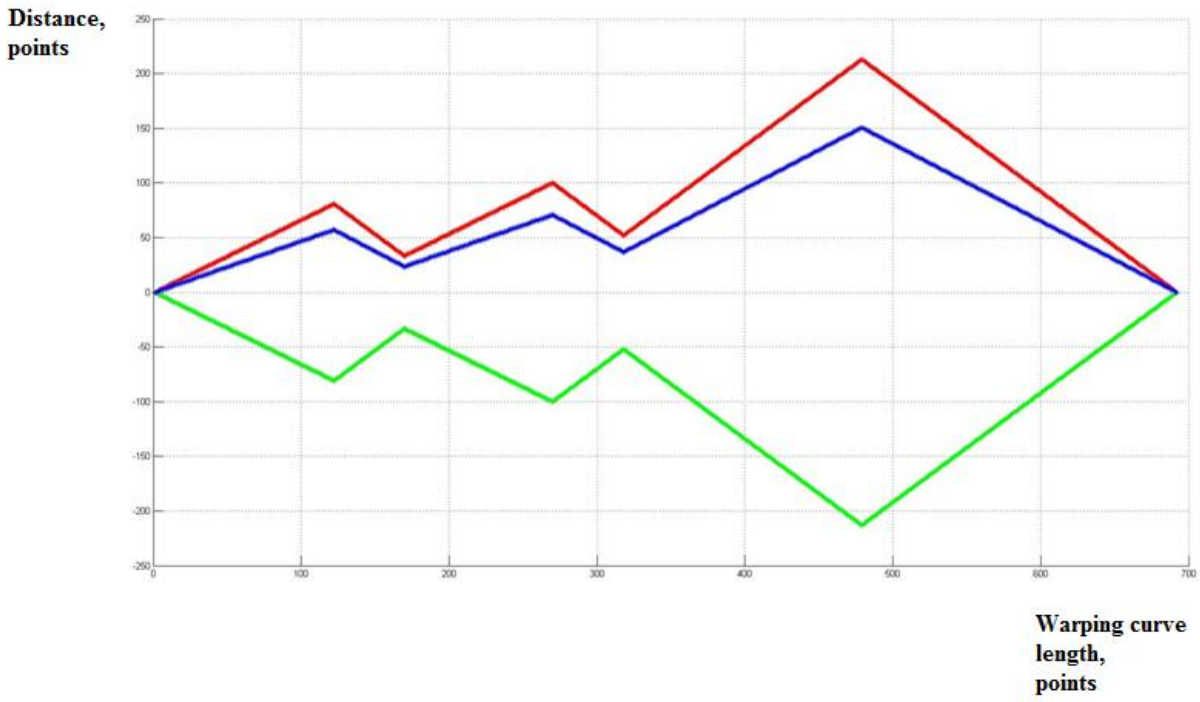

Figure 36 – Distance distribution curves for signals with variable frequency for symmetric modification of weighting matrix (red line – distances on X-axis, green line – distances on Y-axis, blue line – the shortest distance)

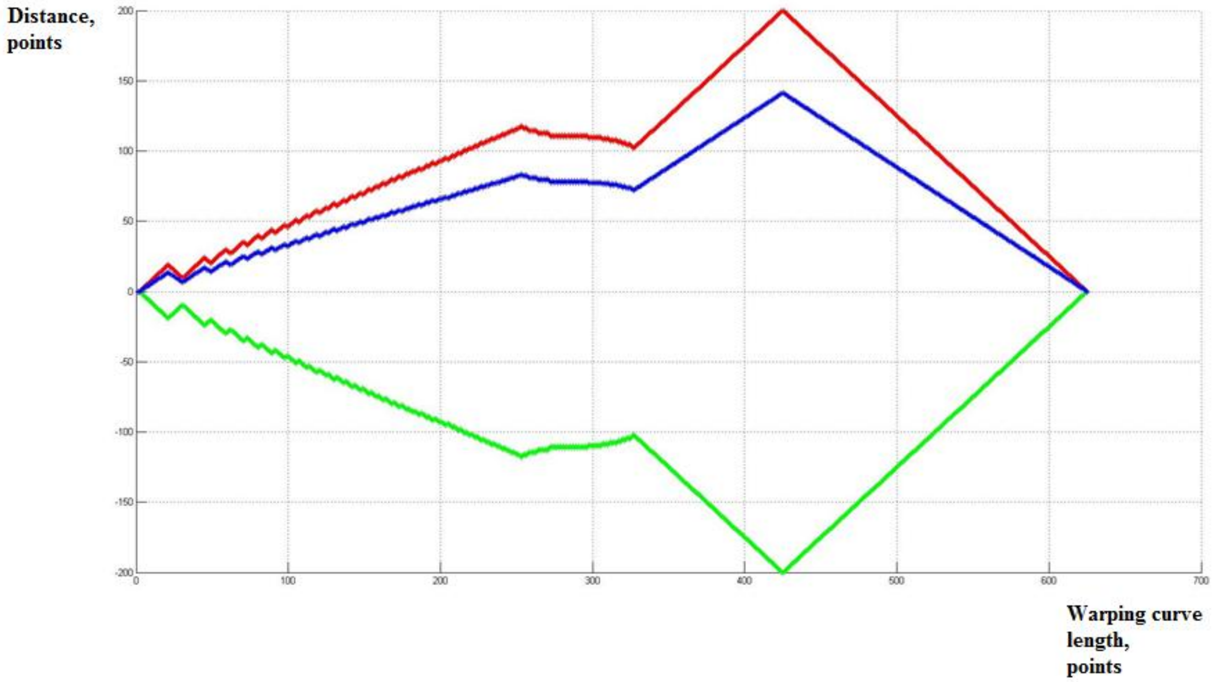

Figure 37 – Distance distribution curves for signals based on chirp function for symmetric modification of weighting matrix (red line – distances on X-axis, green line – distances on Y-axis, blue line – the shortest distance)

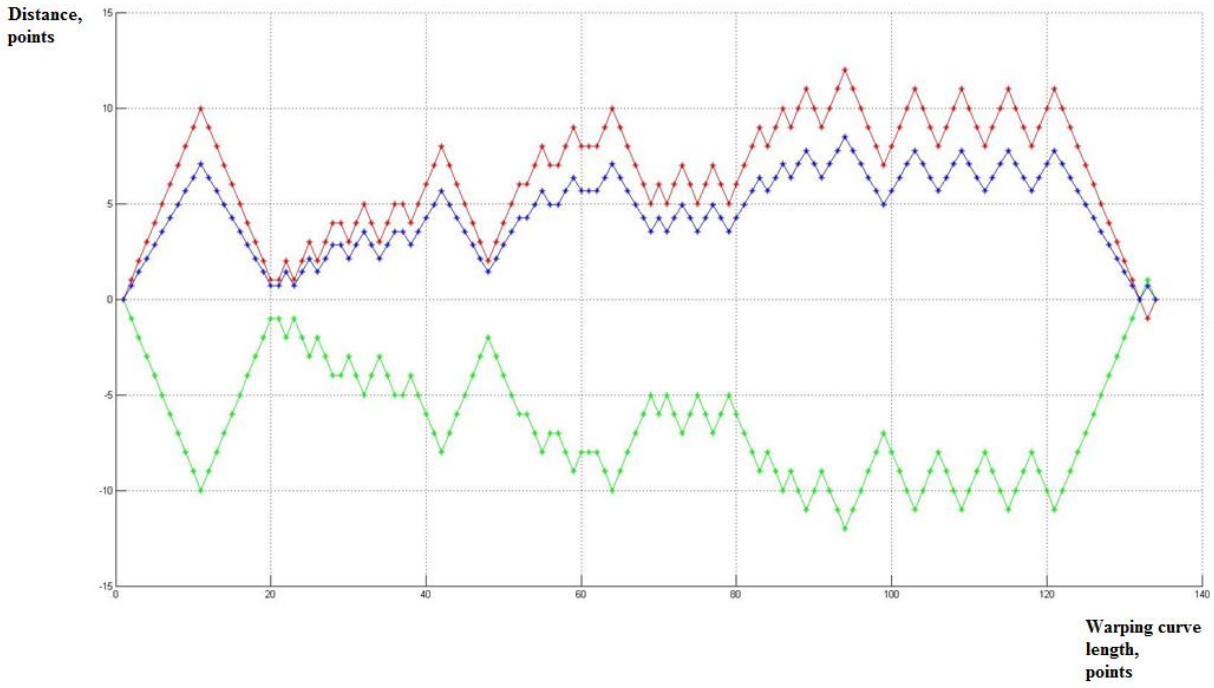

Figure 38– Distance distribution curves for signals based on impulse sequence filtered with second order Butterworth filter for symmetric modification of weighting matrix (red line – distances on X-axis, green line – distances on Y-axis, blue line – the shortest distance)

For curves obtained with symmetric modifications the same regularities as for classical algorithm are observed. However, distance distribution curves are generally smoother than in previous case. Sometimes, it is not possible to determine exactly the type of change, and even its location. For getting additional informationmatching function might be used. Such functions for all types of signals and all studied algorithmsare shown in Figures 39 – 44. Length and slope of matching lines provide additional information about the changes. These functions, in particular, allow distinguishingsegments corresponding to shifts and amplitude difference(it is difficult to distinguish these parameters in complex signals).

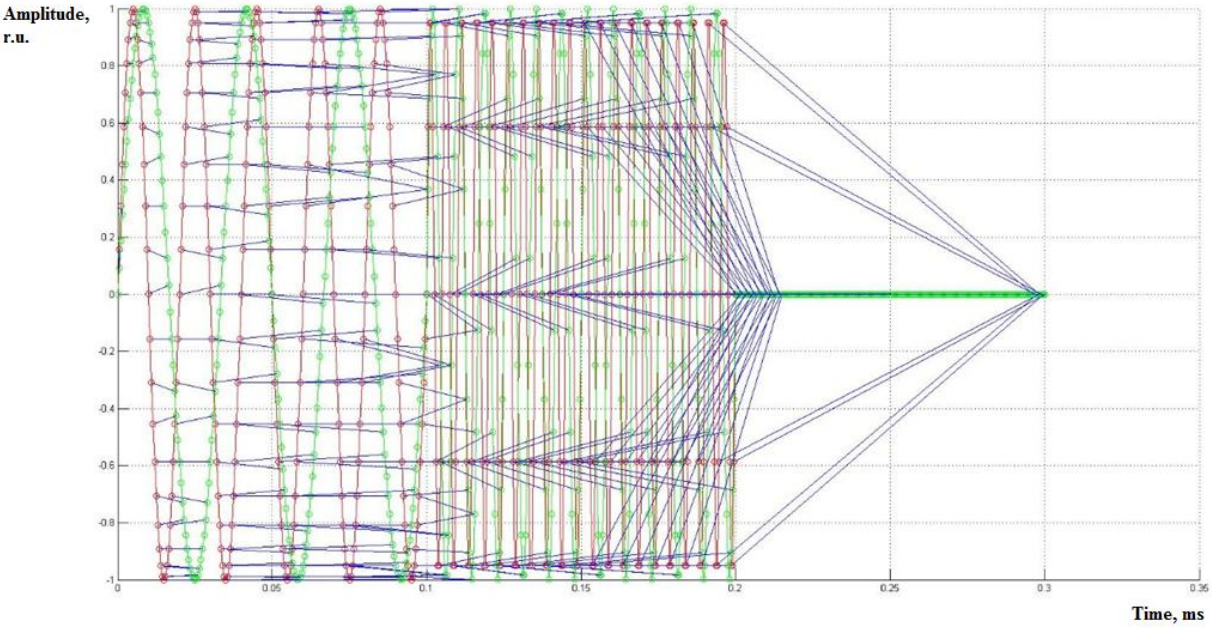

Figure 39 – Matching function for signals with variable frequency for classical DTW algorithm

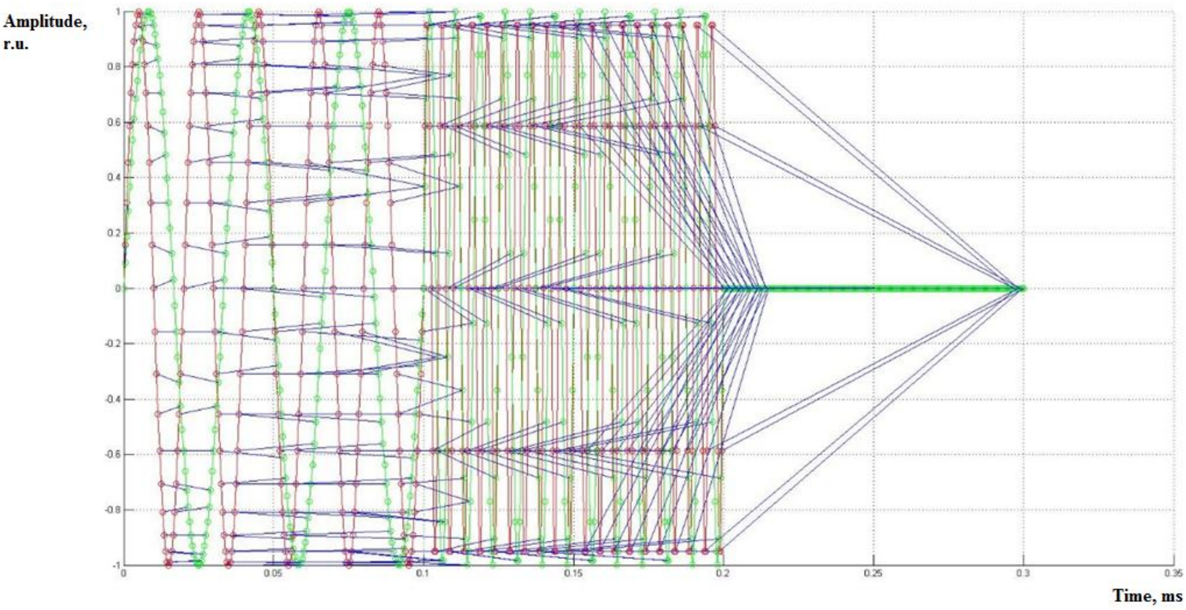

Figure 40 – Matching function for signals with variable frequency for symmetric modification of weighting matrix

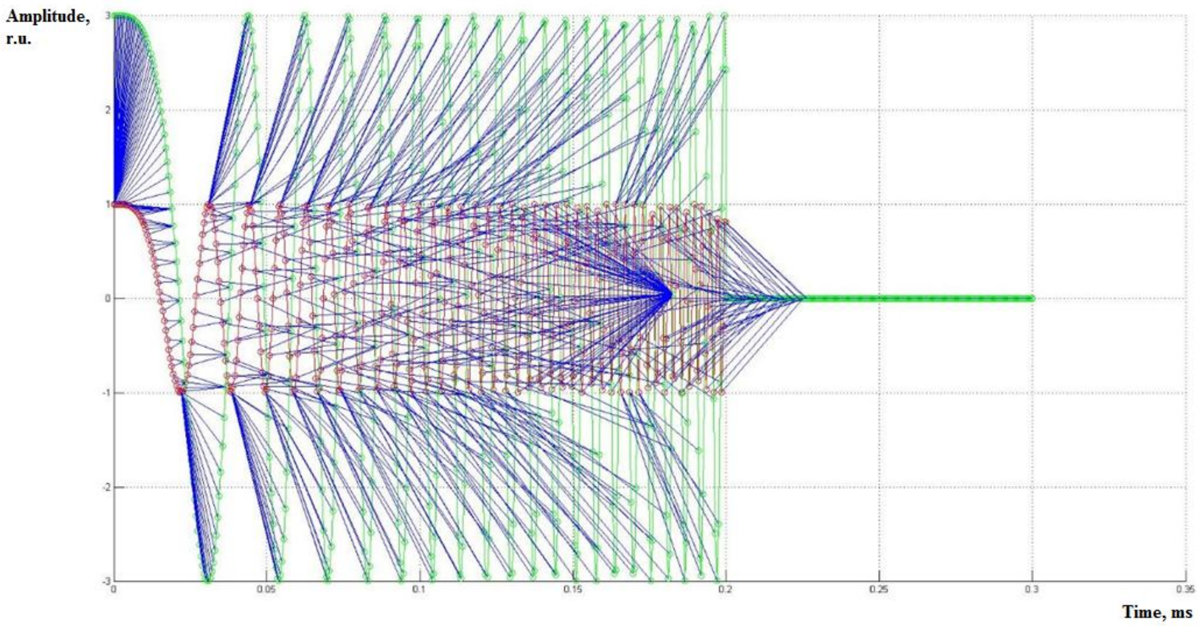

Figure 41 – Matching function for signals based on chirp functionfor classical DTW algorithm

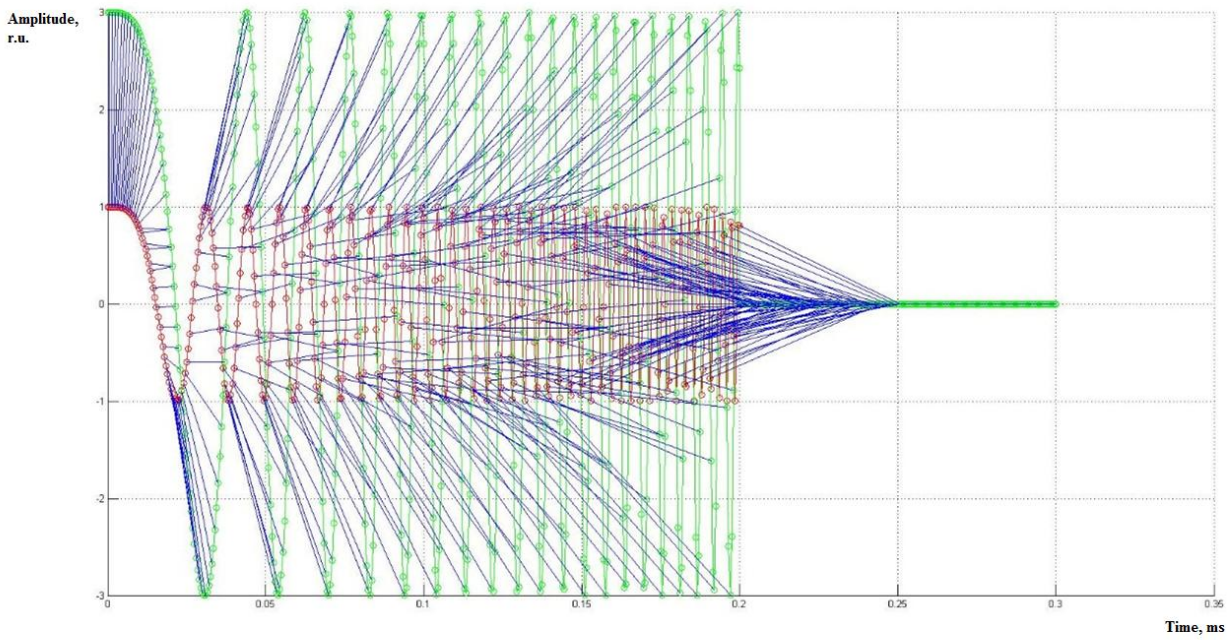

Figure 42 – Matching function for signals based on chirp function for symmetric modification of weighting matrix

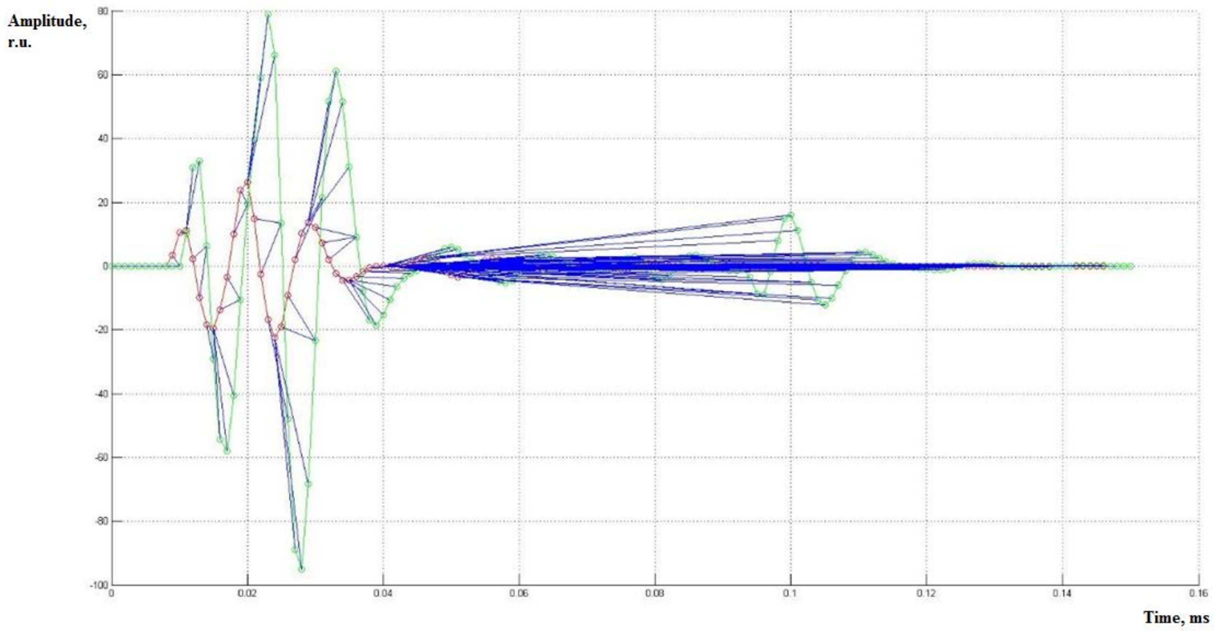

Figure 43 – Matching function for signals based on impulse sequence filtered with second order Butterworth filterfor classical DTW algorithm

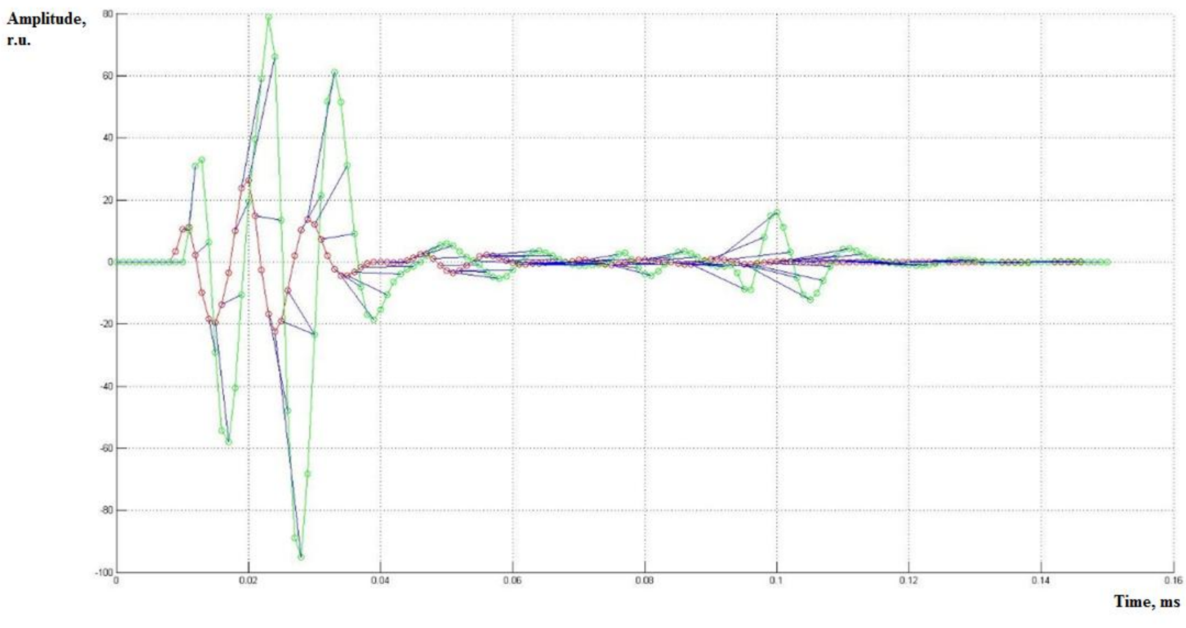

Figure 44 –Matching function for signals based on impulse sequence filtered with second order Butterworth filterfor symmetric modification of weighting matrix

It should be noted that for classical algorithm,when using signals with low frequency, there is the following rule for determining the changes type: displacements in signal correspond to sloped segments of distance distribution curves, amplitude differences correspond to horizontal and vertical segments(depending on amplitude-to-width ratio of the signal). When using signals with high frequency, these changes are superimposed to each other. In this case there are only sloping segments in distance distribution curves and determining the type of changesusing these curves is difficult.

Difference in width of signals in both cases poorly reflected in the distance distribution curves. In general, when studying complex signals, classical algorithm presents better results than symmetric modification. It allows identifying type of changes in input signals and defining their boundaries more precisely. However, determining the type and boundaries of changes in high-frequencysignals is difficult. To obtain more informationabout signal changes, matching function described above can be used. Studying shown that the analysis of length and slopeof matching lines can be additional source of information aboutwaveform differences in input signals, but this matching function have tobe usedtogether with distance distribution curves and warping path curve to make precise analysis.

In general,symmetric modification of DTW algorithm provides better matching than classical one, studying of matching function obtained with this modification, shown that not all points of input signals are involved in the matching. This problem does not allow using this matching function at this step of research. This fact needs additional study.

The study of complex signals revealed that the method of analysis described in the previous chapter, generally applicable to analysis of waveform changes in complex signals, simulated parameters of real biological signals. Classical algorithmreflects changes in input signals better than its modification;however, for obtaining the more precise results, additional study is needed.

Studying of real data

When analyzing standard ECG signal, it is not possible to determine the smallest changes in ST-complex, which reveal a changesin the repolarization process, and can be indicators of abnormalities. These changes are very smalland appears in high frequency part of signal.

Therefore, special preparation of such signals is needed. For this purpose, low pass filters are used. They allow removinglow-frequency part of the signal. Signal after filteringis shown on Figure 45.

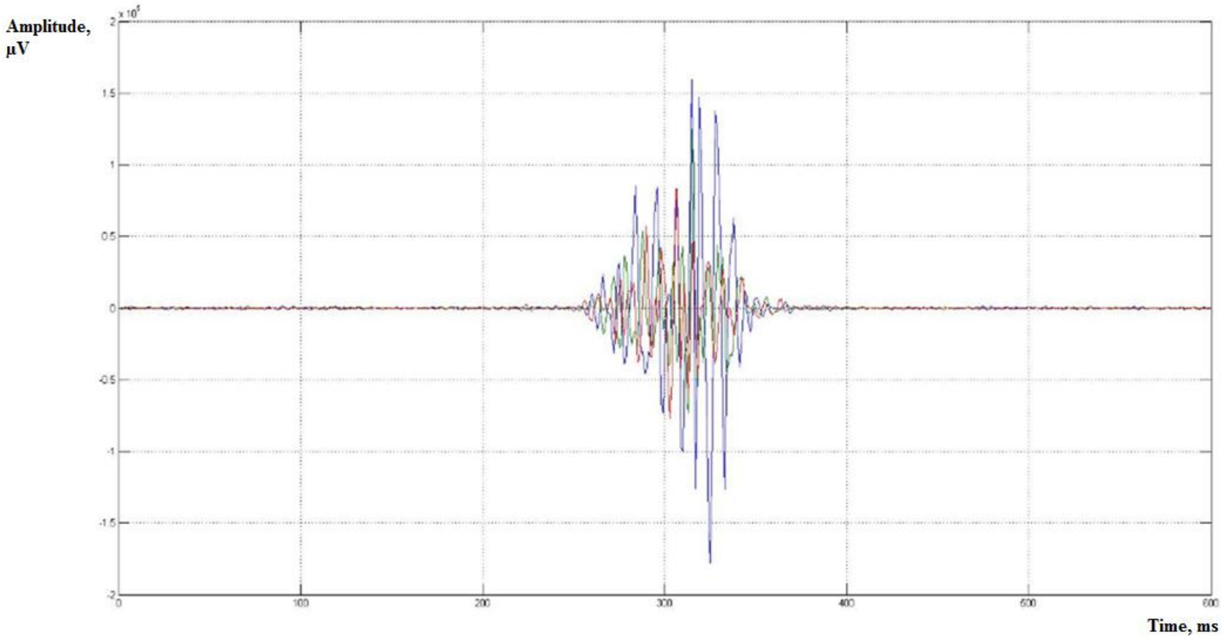

Figure 45 – Heartbeat cycle with filtered low-frequency part

The figure shows all three components of the signal. Only one component of this signalis usedfor analysis. Before analyzing the part of signal corresponding to repolarization process need to be extracted(Figure 46). This extraction is made using expert estimation methods.

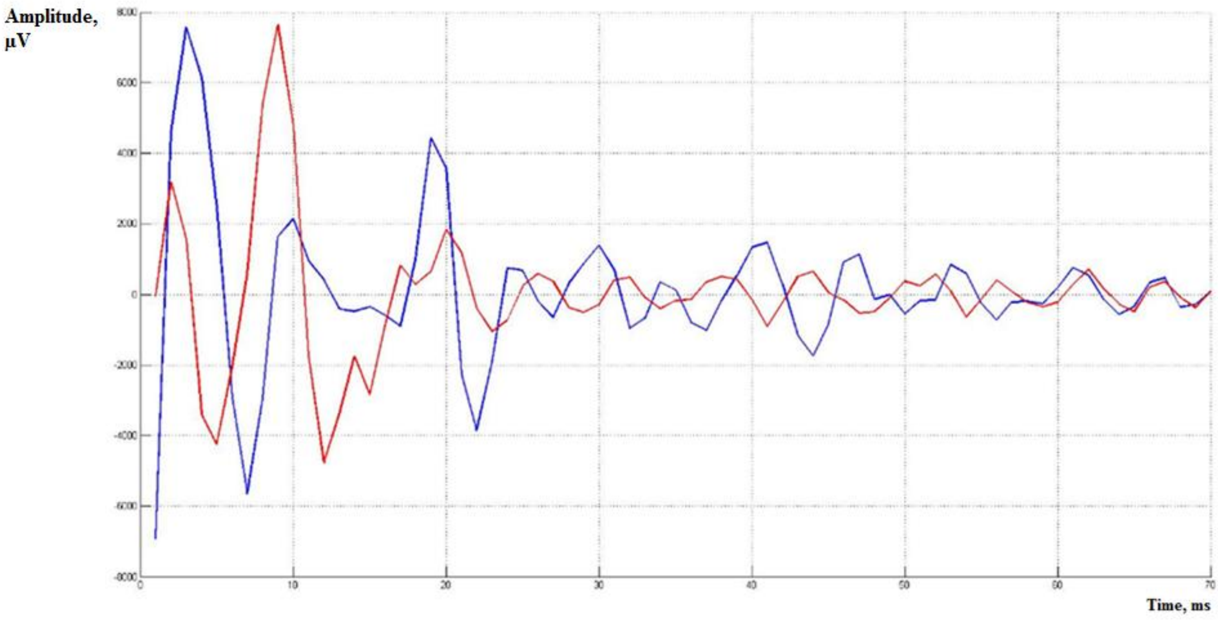

Figure 46– Part of real ST-segment of ECG signal (red signal is test, blue signal is reference)

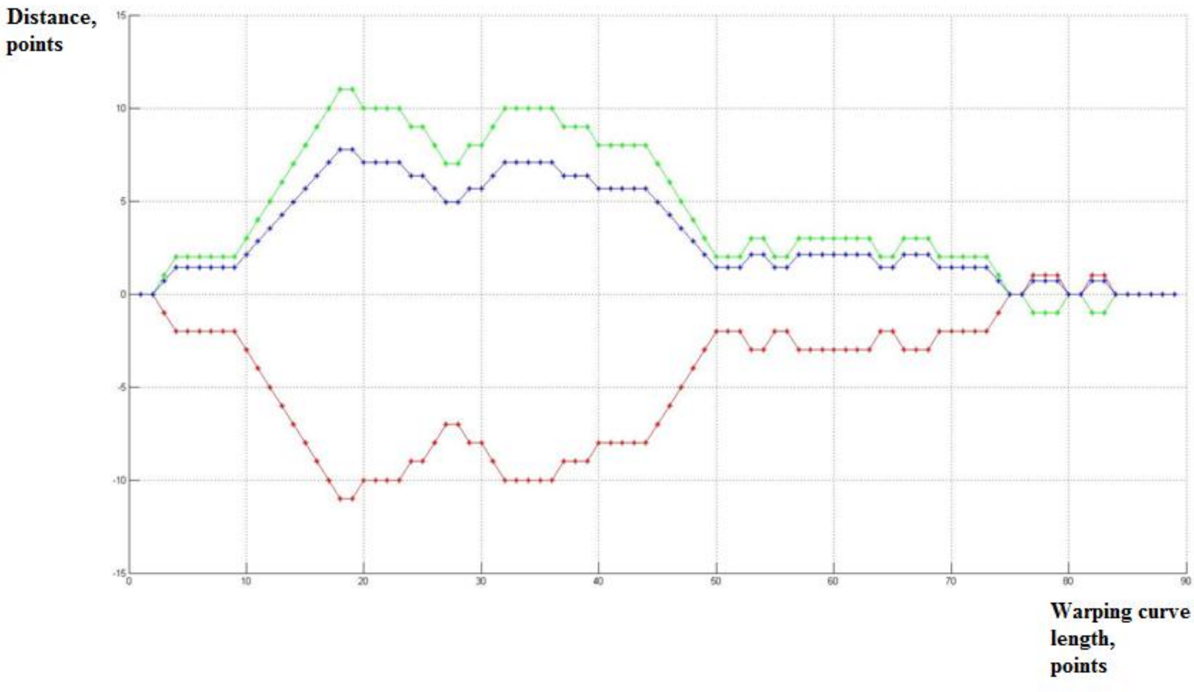

Figure 47 – Distance distribution curves for parts of real ST-segment of ECG signal for classical DTW algorithm (red line – distances on X-axis, green line – distances on Y-axis, blue line – the shortest distance)

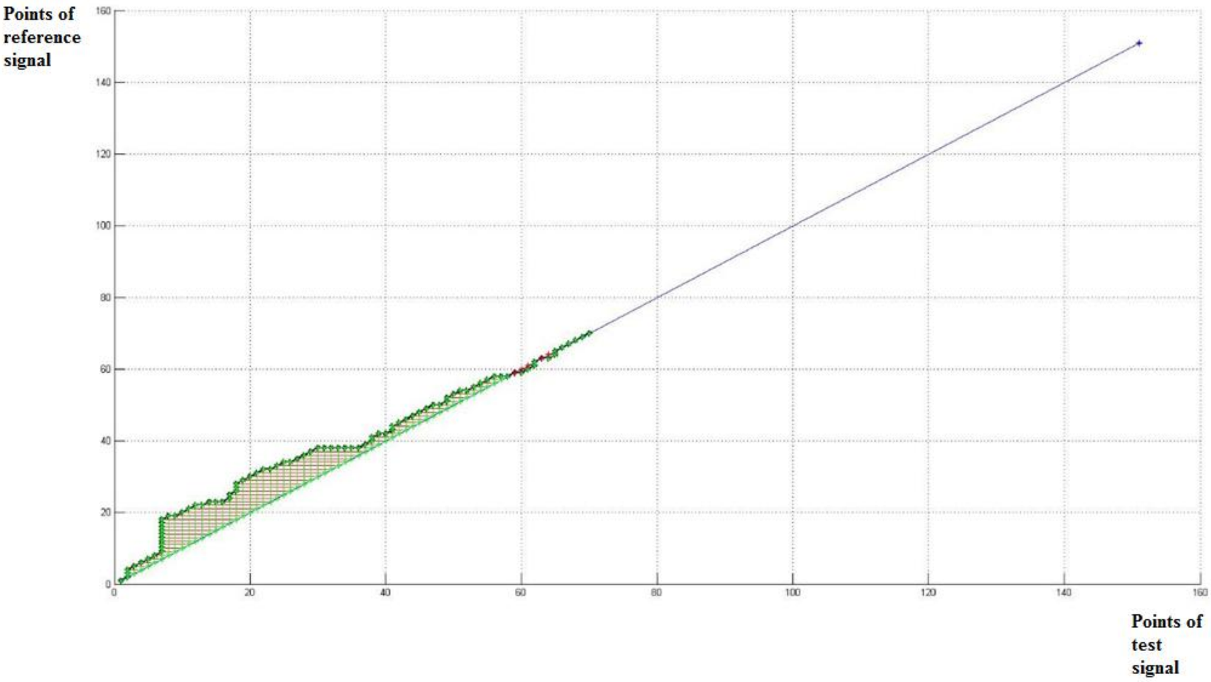

Figure 48 – Warping path curve for parts of real ST-segment of ECG signalfor classical DTW algorithm

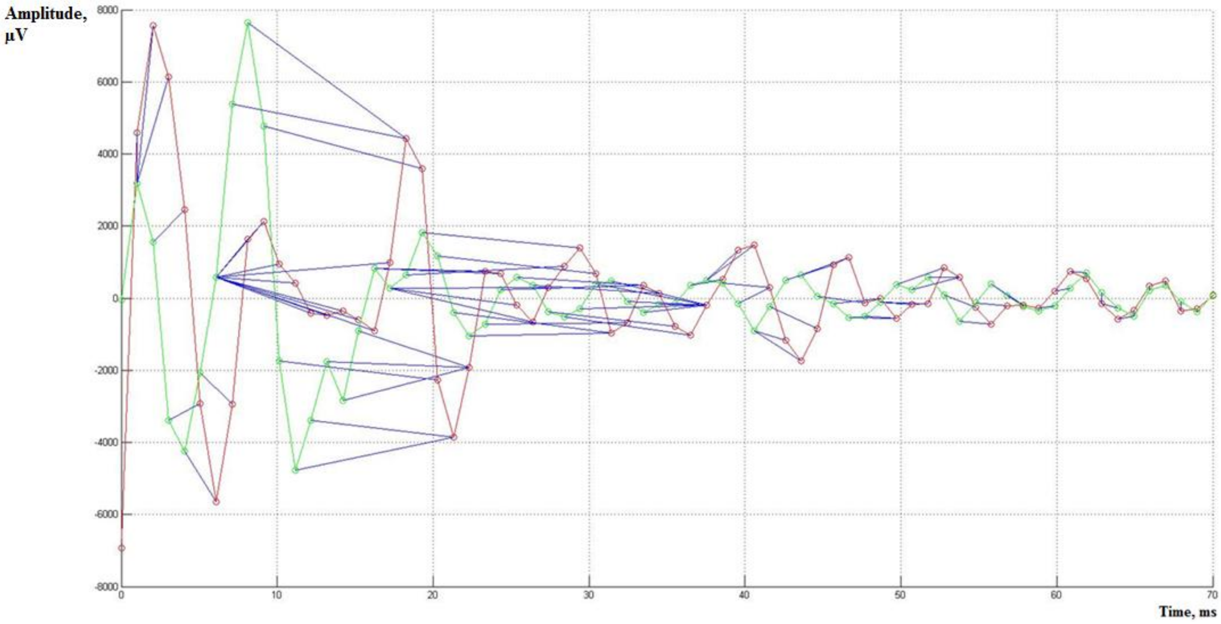

Figure 49 – Matching function for parts of real ST-segment of ECG signalfor classical DTW algorithm

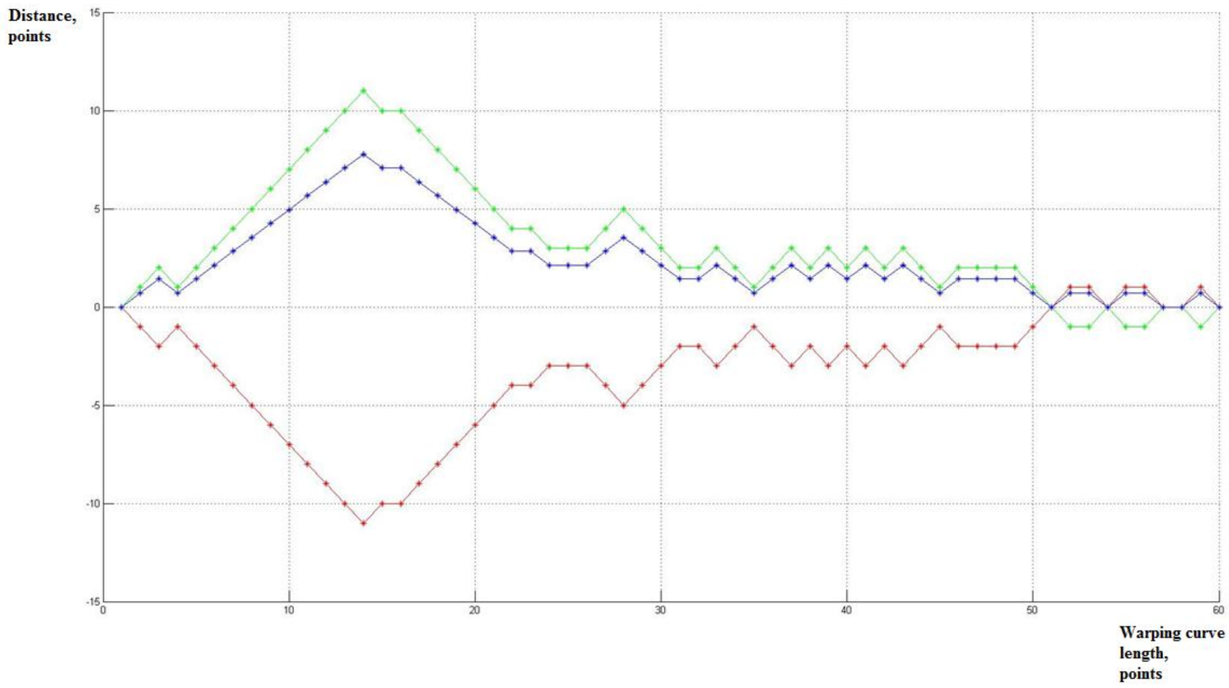

Figure 50 – Distance distribution curvesfor parts of real ST-segment of ECG signalfor symmetric modification of weighting matrix (red line – distances on X-axis, green line – distances on Y-axis, blue line – the shortest distance)

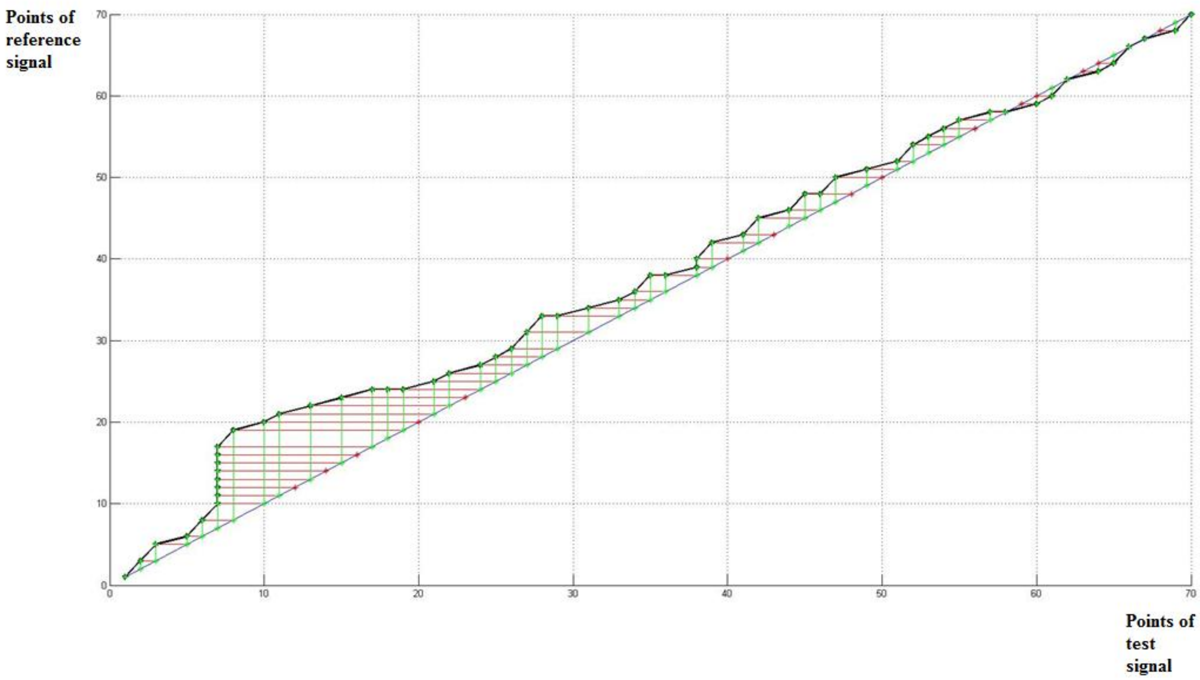

Figure 51 – Warping path curvefor parts of real ST-segment of ECG signalfor symmetric modification of weighting matrix

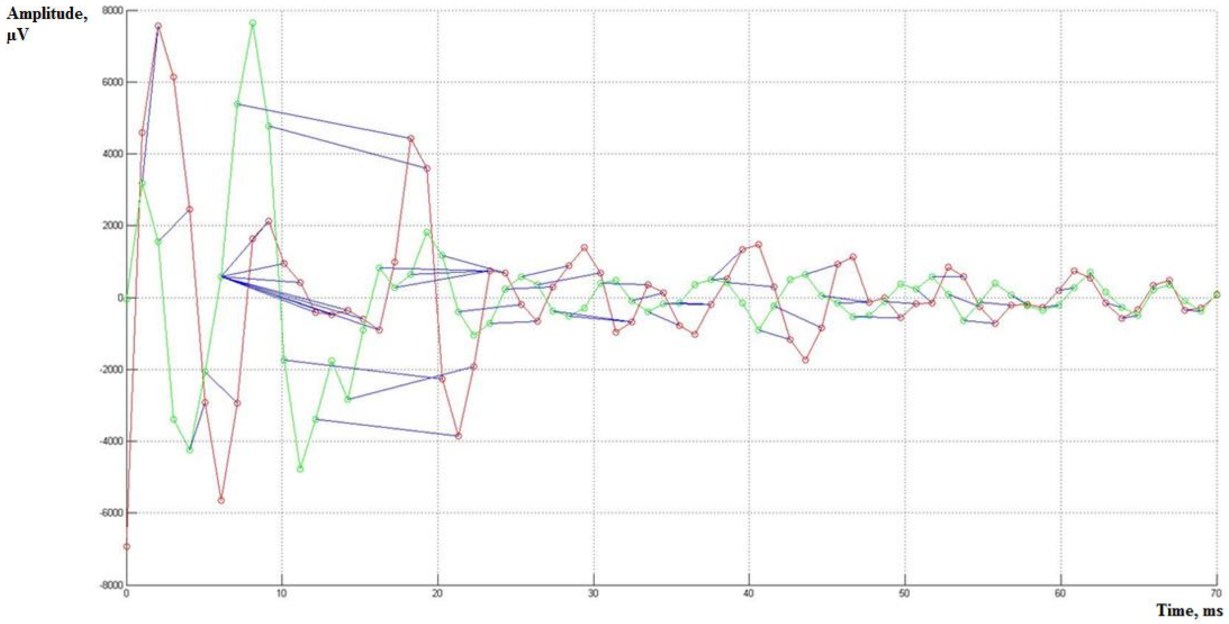

Figure 52 – Matching functionfor parts of real ST-segment of ECG signalfor symmetric modification of weighting matrix

For signals, shown on Figure 46, testing waveform changes analysis with classical and modified DTW algorithm was carried out. Figures 47 - 52 shows distance distribution curves, warping path curves and matching functions for real signals, obtained with use classical DTW algorithm and its symmetric modification.

Studies showed that regularities, described in the previous chapter are also observed when analyzing real signals. Tus, developed methodic for waveform changes analysis in input signals of DTW algorithm is in general appropriate for analyzing changes in ST-complexes of ECG signals.

Conclusion

At the study biological signals and their common features was studied. Features of the ECG signal as an example of biological signals was considered. Problems of ECG signals analysis and their perspective solutions was described.

Method of analyzing changes of input signals of DTW algorithm based on warping path curve, matching function and distance distribution curves was developed.

This method was tested on series of experiments with artificial signals of different complexity and real signals. As a plan of experiment using fractional factorial experiment was used.

The study showed that developed method is applicable for analysis of small changes in complex biological signals. However, it was found that for obtaining morepreciseestimation of location and type of changesin input signals, all curves and functions, described above, must be used together, since each of them provide incomplete information about changes of input signals.

Comparative analysis of classical DTW algorithm and algorithm with symmetric modification of weighting matrix was conducted. It was found that classical algorithm reflects the change of the input signal more precisely than its modification.

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