# Optimum Design of Electromagnets for Magnetic Levitation of Transport Systems based on the Inverse Problem Solutions

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Abstract—Article is devoted to design of optimum electromagnets for magnetic levitation of transport systems. The method of electromagnets design based on the inverse problem solution of electrical equipment is offered. The method differs from known by introducing a stage of minimization the target functions providing the stated levitation force and magnetic induction in a gap, and also the mass of an electromagnet. Initial values of parameters are received, using approximate formulas of the theory of electric devices and electrical equipment. The example of realization of a method is given. The received results show its high efficiency at design. It is practical to use the offered method and the computer program realizing it as a part of system of the automated design of electric equipment for transport with a magnetic levitation.

*Keywords:* optimization, inverse problems, finite elements method, electromagnet, magnetic field, design automatization.

## I. INTRODUCTION

Now more and more strict requirements of ecological purity and profitability are imposed to modern transport. In this regard searches of the new principles of transport systems creation are conducted. One of the perspective directions of their development considers applying of a magnetic levitation of trains that allows, eliminating mechanical contact with road structure, to increase movement speed up to 500 km/h [1]–[3].

Intensive works on creation of transport with a magnetic levitation have been conducted from 70th of the XX century in Germany, Japan, Great Britain and in other countries [4–7]. More than 25 modifications of experimentation's trains are created. In China and Japan commercial transport systems with a magnetic levitation are successfully operated [8,9]. The Shanghai express build up speed of 430 km/h, a gap between magnetic levitation and iron rail is 15 mm.

It should be noted that magnetic levitation receives the increasing use in mechanical engineering (magnetic bearings, positioning systems, etc.) [10][11].

Process of design of levitation transport systems divided into a number of stages. At the first stage the electric power necessary for power supply of electromagnets and the linear traction engine is estimated. At the second stage the system of power supply for transfer on train of necessary power is selected (values of voltage, current-collectors, etc.). At the third stage design of traction system and magnetic levitation is carried out. Thus, having information on tension size, AC to DC converters for supply of electromagnets are designed, and then all system of electromagnetic levitation in a complex with the converter, a control system and an electromagnet, and also traction system is optimized.

The materials reported in this article are intended to be used at the first design stage.

In last years the inverse problems solution in design of technical objects [12] and creation on their basis systems of the automatization design of the electrotechnical equipment were widely used. In this regard an actual task is creation of methods of the automatization design of the electromagnets applied in levitation transport systems with the use of such approach.

## II. DESIGN OF ELECTROMAGNET

Designs of the electromagnets fixed on the special console connected with train are presented in fig. 1 [4]. In figure: 1 – iron rail of levitation system; 2 – bearing; 3 – levitation electromagnet core; 4 – coil; 5 – coil; 6 – core of an electromagnet of side stabilization; 7 – iron rail of side stabilization system; 8 – console.

We will consider design of a levitation electromagnet. We consider known the mass of train  $\mathbf{M}$ , quantity of levitation electromagnets  $\mathbf{n}$ , their length  $\mathbf{L}$ . It is required to determine the sizes of an electromagnet section and a magnetic potential difference (MPD) of the coil (fig. 2) so that the demanded levitation force, the minimum mass of an electromagnet and reliable control system were provided.

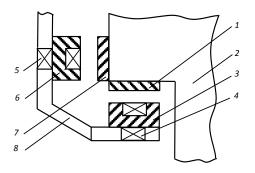


Fig. 1. Designs of electromagnets and their placement on the console, onnected with train.

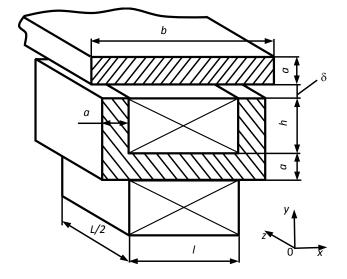


Fig. 2. Central section of an electromagnet. L – electromagnet length on an axis  $0z. \ensuremath{$ 

# III. MATHEMATICAL MODEL

We will determine levitation force on one electromagnet by a formula

$$F_l = Mg/n \tag{1}$$

where g – acceleration of gravity.

Using an approximate formula from the theory of electric devices for electromagnet attraction force [13]

$$F_l = 2 \frac{B_\delta^2 S_p k_b}{2\mu_0} \tag{2}$$

where  $B_{\delta}$  – average magnetic induction in an air gap of  $\delta$ ;  $k_b$  – the coefficient considering a buckling of a magnetic flux;  $k_b = 1.2 \div 1.5$ ;  $\mu_0$  – a magnetic constant,  $\mu_0 = 4\pi \cdot 10^{-7}$  H/m, we will receive initial approach for the pole area

$$S_p^{(0)} = La^{(0)} = \frac{\mu_0 F_l}{B_\delta^2 k_b}$$
(3)

Taken set values for  $B_{\delta}$ , *L*,  $k_b$  we obtain on the basis of (3) formula for initial value of the size  $\langle a \rangle$ 

$$a^{(0)} = \frac{S_{\pi}^{(0)}}{L} = \frac{\mu_0 F_l}{B_{\delta}^2 k_{\nu} L}$$
(4)

Let permeability of ferromagnetics be infinite. Using a known ratio [14]

$$\oint_{l} \vec{H} d\vec{l} = iw$$

We will receive a formula for definition of initial approach of a magnetic potential difference (MPD)  $F_{\rm M} = iw$  of the coil

$$iw^{(0)} = \frac{B_{\delta}}{\mu_0} 2\delta \tag{5}$$

We will determine the area of a port of the coil by a formula

$$S_{\rm c}^{(0)} = l^{(0)} h^{(0)} = \frac{i w^{(0)}}{k_z j}, \qquad (6)$$

where j – the chosen current density in the coil,  $k_z$  – coefficient of filling of a port of the coil on copper,  $k_z = 0.7$  is accepted.

From (6) follows

1

$$l^{(0)} = \frac{S_c^{(0)}}{h^{(0)}} = \frac{iw^{(0)}}{k_z j h^{(0)}}$$
(7)

At calculation of magnetic fields we consider a curve of magnetization of steel is unique, isotropic mediums.

We will determine the mass of an electromagnet by a formula

$$M_{em} = \left[2ah + \left(\frac{S_c}{h} + 2a\right)a\right]L\rho_{st} + 2S_ck_zL\rho_{cu}$$
(8)

where  $\rho_{st}$ ,  $\rho_{cu}$  – density of a ferromagnetic (steel) and copper respectively.

In a formula (8) a variable value is only *h*.

Necessary and sufficient conditions of a minimum of  $M_{em}$  as shown here:

$$\frac{dM_{em}}{dh} = 0; \quad \frac{d^2M_{em}}{dh^2} > 0 \tag{9}$$

On the basis of (9) we will receive

$$\frac{dM_{em}}{dh} = \left[2a - \frac{S_c a}{h^2}\right] L\rho_{st} = 0 \; ; \; \frac{d^2 M_{em}}{dh^2} = \frac{S_c a}{h^3} L\rho_{st} > 0 \tag{10}$$

The value h at which  $M_{em}$  is minimum, is defined from (10)

$$h^{(0)} = \sqrt{\frac{S_c^{(0)}}{2}} = \sqrt{\frac{iw^{(0)}}{2k_z j}}$$
(11)

We will determine width of an iron rail by a formula

$$b^{(0)} = l^{(0)} + 2a^{(0)} + 0.02 \tag{12}$$

#### IV. SOLUTION ALGORITHM

At minimization of mass of  $M_{em}$  with use of expression (8) it is necessary to consider two restrictions providing operability of system of magnetic levitation

$$B_{\delta}^{(k)} \le B_{\delta}^* \tag{13}$$

$$F_l \le F_l^{(k)} \le F_l \left( 1 + \varepsilon_1 \left( F_l \right) \right) \tag{14}$$

where  $B_{\delta}^{(k)}$  and  $F_{l}^{(k)}$  – the values calculated on k's algorithm step.

Usually  $B_{\delta}^* = 0.4 - 0.7$  T, that ensures functioning of system in the nonsaturated mode.

We will transform restrictions to following target functions

$$J_1^{(k)}(iw) = (B_\delta^{(k)} - B_\delta^*)^2$$
(15)

$$J_{2}^{(k)}(a) = \left[ \left( 1 + \varepsilon_{1}(F_{l}) \right) F_{l} - F_{l}^{(k)} \right]^{2}$$
(16)

We have a multicriteria task with three target functions (15), (16) and (8). We will execute the solution of a task by a lexicographic method of ordering [15], taking into account features of a task.

The iterative algorithm of an electromagnet design consists of the following stages:

1. We determine levitation force of one electromagnet Fl by a formula (1). We assign values to sizes of a gap of  $\delta$ , current density in coils *j*, airgap flux density  $B_{\delta}^{*}$ , electromagnet length *L*, coefficients  $k_b$  and  $k_z$ , errors of magnetic induction definition  $\varepsilon_1(B_{\delta}^{*})$  and force  $\varepsilon_2(F_l)$ .

We determine by formulas (4), (5), (7), (11), (12) initial values  $a^{(0)}$ ,  $iw^{(0)}$ ,  $h^{(0)}$ ,  $l^{(0)}$ ,  $b^{(0)}$ .

Further for each k=0,1,2,...

2. We solve a direct problem of a stationary magnetic field calculation and force  $F_l$  by finite elements method, using

$$a = a^{(k)}, iw = iw^{(k)}, h = h^{(k)}, l = l^{(k)}, b = b^{(k)}.$$

3. We minimize functionalities  $J_1^{(k)}$  and  $J_2^{(k)}$  by method of gradient descent, checking performance of conditions

$$J_1^{(k)}(iw) \le \left[\varepsilon_1 \left(B_{\delta}^*\right)\right]^2 \tag{17}$$

$$J_{2}^{(k)}(a) = [\varepsilon_{2}(F_{l})]^{2}$$
(18)

As a result we define values  $iw^{k+1}$  and  $a^{k+1}$  at which conditions (17) and (18) are carried out.

4. We minimize the mass of an electromagnet. We determine parameters *h* and *l* by formulas

$$h^{(k+1)} = \sqrt{\frac{iw^{(k+1)}}{2k_{3}j}} \qquad l^{(k+1)} = \frac{iw^{(k+1)}}{k_{3}jh^{(k+1)}}$$

Further we calculate

$$b^{(k+1)} = l^{(k+1)} + 2a^{(k+1)} + 0.02$$

#### V. COMPUTING EXPERIMENT RESULTS

We will review an example. It is required to determine the sizes and MPD of electromagnets of train levitation system with the mass M = 60 t, including n = 20 electromagnets, located in two ranks; length of each L = 1 m. Train length – 27 m. We choose:  $j = 3 \cdot 10^6$  A/m<sup>2</sup>;  $B_{\delta} = 0.7$  T;  $k_b = 1.5$ ;  $k_z = 0.7$ ;  $\delta = 0.015$  m;  $\varepsilon_1(B_{\delta}^*) = 0.01$ ;  $\varepsilon(F_i) = 0.01$ .

We carry out the first stage of algorithm:  $F_l = 30$  kN;  $a^{(0)} = 0.034$  m;  $iw^{(0)} = 16711$  A;  $h^{(0)} = 0.063$  m;  $l^{(0)} = 0.126$  m;  $b^{(0)} = 0.215$  m.

Go over to the stages 2-4 execution.

As a result we will receive parameters of the electromagnet of the minimum weight on the fourteenth iteration providing magnetic induction in a gap of an electromagnet  $B_{\delta} = 0.7$  T and levitation force  $F_l = 30$  kN with the set error:  $a^{(14)} = 0.054$  m;  $iw^{(14)} = 14180$  A;  $h^{(14)} = 0.058$  m;  $l^{(14)} = 0.116$  m;  $b^{(14)} = 0.245$  m.

The sizes specified above and MPD of an electromagnet are received at motionless train. At the movement vortex currents are induced in a ferromagnetic rail. Distribution of eddy-currents in a rail according to the law of electromagnetic inertia is so that their field is directed opposite to an electromagnet field that leads to reduction of uplift power.

The method of field and forces calculation of the electromagnet moving along a ferromagnetic plate was considered in work [16]. The famous software ANSYS Maxwell allows making such calculations. By means of this software it is determined that for creation of the electromagnet providing levitation force  $F_l$  at the movement of train over iron rail with a speed V = 400 km/h shall be

$$F_{lcalc} = k_{ec} F_l$$

where  $k_{ec} = 1.35$  at specific electrical conduction of rail  $\gamma = 2 \cdot 10^6 \quad (\Omega \cdot m)^{-1}$  and a gap of  $\delta = 0.015 \text{ m}$ . For the considered electromagnet  $F_{lcalc} = 40.5 \cdot 10^3 \text{ N}$ .

Thus, uplift force decreases by 35 % in the considered device at a speed V = 400 km/h because of induced eddycurrents. Value  $k_{ec}$  depends on speed, for example, at V = 200 km/h  $k_{ec} = 1.21$ , at V = 800 km/h  $-k_{ec} = 1.5$ .

Using the algorithm described above, we will determine the sizes and MPD of an electromagnet for  $F_{lcalc} = 40.5 \cdot 10^3$  N,that will provide the required force of a levitation  $F_i = 30 \cdot 10^3$  N.

On the thirteenth iteration parameters of an electromagnet of the minimum mass  $M_{em} = 344$  kg are determined providing magnetic induction in a gap of an electromagnet  $B_{\delta} = 0.7$  T (fig. 3) and levitation force  $F_{\pi} = 40.5$  kN with the set error:  $a^{(13)} = 0.091$  m;  $iw^{(13)} = 16990$  A;  $h^{(13)} = 0.064$  m;  $l^{(13)} = 0.127$  m;  $b^{(13)} = 0.328$  m.

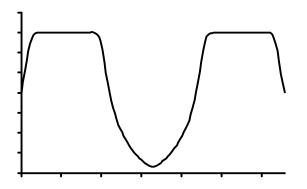


Fig. 3. Distribution of the module of magnetic induction in an electromagnet gap.

## CONCLUSIONS

The method of optimum design of electromagnets of magnetic levitation on the basis of the solution of conditionally correct inverse problems with use of two target functions received on the basis of restriction for magnetic induction in an electromagnet gap on tongue attraction force is offered. Minimization is carried out by method of gradient descent. At the same time magnetic potential difference of the magnetizing coil and the geometrical sizes of a pole of an electromagnet are defined. Minimization of the third target function (mass of an electromagnet) carried out analytically with use of necessary and sufficient conditions allowed to receive formulas for calculation of its optimum sizes. Iterative algorithms of the inverse problems solution of magnetic fields are constructed. The offered method allows reducing considerably time of design of electromagnets in comparison with the known methods, for example, by penalty function method. It is practical to use the offered method and the realizing computer program as a

part of the computer-aided engineering system of levitation transport systems.

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