The Role of Energy Markets and Expectations in Dynamic General Equilibrium Models

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List of Original Papers

This thesis consists of the following three papers:

1. Oliver Holtemöller and Christoph Schult (2019), 'Expectation Formation, Financial Frictions and Forecasting Performance of Dynamic Stochastic General Equilibrium Models', *Historical Social Research*, **44**(2), 313-339.

2. Katja Heinisch, Oliver Holtemöller and Christoph Schult (2021), 'Power Generation and Structural Change: Quantifying Economic Effects of the Coal Phase-Out in Germany', *Energy Economics*, **95**, **doi: 105008**.

3. Christoph Schult (2020),'Is Risk the Fuel of the Business Cycle?', not published.

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Chapter 1

Overview

Economics is a choice between alternatives all the time. Those are the trade-offs.

Samuelson (2009)

1.1 Motivation

The fifth assessment report of the International Panel on Climate Change (IPCC) states that the global average temperature increased by 0.85 degree Celsius since 1880.[1] In particular, greenhouse gas (GHG) emissions contribute to global warming, where CO_2 is the most important GHG.[2] Previous climate change has already modified natural and human systems. Politicians are aware of the problem and have made agreements through the Kyoto Protocol and the Paris Agreement to reduce GHG emissions voluntarily.

Climate change projections in Pachauri et al. (2014) show that the average global temperature will very likely increase further. However, the magnitude depends on the respective cumulative emissions. Stringent mitigation measures can reduce the global average temperature increase by 3°C by the end of the century.[3] Climate projections are uncertain even if all anthropogenic influences are held constant. The current understanding of natural systems is not perfect.

However, the earth's primary source of energy supply is solar radiation. The climate has been stable for the last centuries, which indicates that energy inflow is almost identical to the energy outflow. Climate scientists report that 50% of the sunlight is in the visible

[1] The global average temperature increase since 1880 lies with a probability of 90% between 0.65 to 1.06°C (see Pachauri et al. 2014).

[2] About two-third of the total cumulative CO_2 emissions between 1750 to 2011 are from fossil fuels and industrial processes.

[3] The reference concentration paths (RCPs) 8.5 and 2.6 simulate the impact of no stringent mitigation effort and relentless mitigation effort, respectively.

spectrum and enters the atmosphere as solar shortwave radiation. Gases, aerosols and the earth's surface albedo directly reflect about 30% to space. The earth's atmosphere absorbs the remaining 20%. Earth's surface absorbs half of the solar shortwave radiation and emits it back as longwave radiation. The transformation of shortwave radiation to longwave radiation releases heat. GHGs reflect longwave radiation to the surface of the earth. An increase in the concentration in GHG increases the duration of trapped longwave radiation or heat in the lower atmosphere. The atmospheric concentration of GHGs have increased since 1950 (see Pachauri et al. 2014) and the increase in CO_2 accelerates. Based on the finding that GHG particles let ultraviolet light pass through and absorb infrared light, policymakers face a typical trade-off. They can either start to implement costly mitigation measures or follow a business as usual path and accept the economic, social, and ecological costs of climate change.[4]

Policymakers need tools to support their decision-making process. This thesis contributes to the decision making process in three ways. First, dynamic general equilibrium models (DGEs) are evaluated.[5] Second, new dynamic general equilibrium models quantify the implied economic effects associated with the reduction in fossil fuel consumption. Third, the historical and potential interaction between market frictions and energy markets is analysed. Appropriate model modifications of the standard DGE model are considered, such as financial frictions or hiring costs.

Nobel laureate William D. Nordhaus used the Dynamic Integrated Climate-Economy model to investigate potential policy measures for reducing GHG emissions. Nordhaus (1992, 1993) show that an optimal mitigation policy is to implement a carbon tax. It is common to discuss the implementation of mitigation measures through the implementation of a CO_2 tax. The main question posed in the economic literature is the magnitude of an optimal tax. Further, postponing the timing of mitigation into the future might accumulate wealth to ease the implementation and transition towards a more sustainable economy.

A reduction of GHG emissions requires transforming the energy market currently based on fossil fuels to an energy market based on energy carriers with low CO_2 emissions. Therefore, national governments need to implement mitigation policies to reduce GHG emissions caused by burning fossil fuels. Coal is the main energy source in the electricity sector and oil in the transportation sector. Electricity generation using coal emits more CO_2 than any other energy source. Renewable energy technologies that can transform the electricity sector are already accessible. Nevertheless, the feasibility and efficiency of a coal phase-out in the electricity market depends crucially on the respective region considered. One main reason is the accessibility to renewable energy sources, which depends on geographical aspects.[6] Nordhaus & Yang (1996) consider the different mitigation paths for multiple regions in the world. The model includes the cost of reducing CO_2 emissions, based on other detailed

[4] Figure 1.1 in Pachauri et al. (2014) depicts this process.

[5] One can solve DGE models for a deterministic or stochastic path of exogenous variables. For a deterministic path, rational agents have perfect foresight about the realisation of the shocks and for a stochastic path, agents form rational expectations about the realisation of the shocks.

[6] The European Environment Agency (EEA) states that in 2017 the share of renewable energy was slightly above 10% in Germany, but more than 30% in Austria (see European Environment Agency 2018).

studies investigating mitigation policies. Further, the authors consider different modes of international cooperation to reduce GHG emissions. Cooperative solutions like the Paris Agreement and other international treaties are more efficient than non-cooperative solutions. The approaches introduced in Nordhaus (1992, 1993) and Golosov et al. (2014) model economic activity by one representative sector and rely on calibration techniques.

A stream in the literature uses integrated assessment models (IAM) with a more detailed disaggregation of sectors and regions compared to Nordhaus (1992). Ciscar et al. (2011) compute the impact of climate change for different geographical regions in Europe. The results indicate that the potential consequences of climate change in this century differ in Europe. Southern Europe will belong to the losers of climate change, while Northern Europe might benefit from increasing temperatures. Impacts also differ by sector. Mercure et al. (2018) look at global macroeconomic consequences of a fossil fuel phase-out. A primary result of the analysis is that a phase-out of fossil fuel has negative impacts on fossil fuel-producing countries (OPEC, US) and slightly positive effects for fossil fuel-importing countries (Europe, China). The model is based on optimising agents and uses calibrated parameters informed by the existing literature. IAM models are very rich in their economic structure, but fail to account for inevitable market frictions. Market frictions determine short- and medium-run consequences of mitigation policies. Consequences of mitigation policies can depend on price rigidities and labour market frictions. Further, IAM is too big to estimate all structural parameters using one standard econometric procedure entirely. Parameter values can depend on each other, and classic regression methods can not take this into account.

A class of models able to capture nominal rigidities and labour market frictions are dynamic stochastic general equilibrium (DSGE) models. Golosov et al. (2014) use a DSGE to analyse optimal fossil fuel taxes to reduce GHG emissions. The model considers coal, oil, and natural gas as fossil fuels. They show that the optimal mitigation strategy should focus on the reduction of coal rather than oil. DSGE models are also commonly estimated using Bayesian techniques described in Herbst & Schorfheide (2015). After the Great Recession, the public discussed the usage of DSGE models based on optimising agents. Stiglitz (2018) provides a rigorous critique of DSGE models. The workhorse DSGE model introduced by Smets & Wouters (2003) and Christiano et al. (2005) derives the equations from representative agents. One representative agent represents all households and another all firms. The production process is simplistic and does not account for different goods explicitly. The primary purpose of the workhorse model was its easy estimation and the ability to guide monetary policy. One main reason for its widespread use, despite its simplistic structure, is the forecasting performance of the model. Smets & Wouters (2007) show that the estimated model produces forecasts comparable to vector autoregressive (VAR) models in the short run and better forecasts in the medium run. However, the micro-foundations of the models are not sufficient to meet the demand of policymakers. Current research proposes several improvements to these shortcomings. DSGE models have only a few contenders to replace them (see Linde 2018), because DSGE models fulfil specific requirements that make them useful for evaluating mitigation policies. First, DSGE models are general equilibrium models able to capture the impact of policies on demand as well as on the supply side. Second, policy decisions will trigger dynamics with path dependencies. Increasing fossil fuel taxes or decommissioning power plants will affect the evolution of the capital stock. It implies that current policies have effects on future production capacities and consumption opportunities. The standard DSGE model primarily analyses the consequences of monetary policy on a business cycle frequency. Analysing mitigation policies requires modifications of the standard DSGE model. One necessary modification is the combination of business cycle analysis and long-run developments in one model, as stated by Ghironi (2018). In order to include the long-run implications of mitigation policies on the respective structure of the economy, it is not possible to solve models around a deterministic steady state. The long-run equilibrium itself is affected by the policies considered. It is also necessary to include the heterogeneity of economic agents affected by specific policy decisions. National and subnational regions might be affected differently by climate change. Northern Europe might experience different climate change effects than Southern Europe. Coastal areas are more vulnerable than non-coastal areas to sea-level rise. Regional heterogeneity of climate change effects can lead to migration on a national and subnational level. Long-run investment decisions into infrastructure and other projects must also consider the potential of migration. Models to guide policy should include migration. A coal phase-out in Germany, e.g., will have different implications across different regions. Therefore, a useful model to guide policy decisions needs to model the heterogeneity of regions explicitly. Increasing heterogeneity also implies more demand for data in order to estimate and calibrate additional structural parameters. Computational power demand increases with more heterogeneity. Therefore, the researcher needs to choose additional heterogeneous features carefully.

For the development of suitable models to guide mitigation policies, it is essential to assess model modifications and their impact of the model ability to describe reality. The first chapter assesses modifications to the workhorse DSGE model for the euro area, the United Kingdom and the United States (US). The chapter evaluates learning to alternate the expectation formation process, financial market frictions and nominal rigidities. In the second chapter, a DGE model quantifies the economic effects of a coal phase-out in Germany. It introduces hiring costs to a model with multiple regions and sectors and reports a univariate and multivariate sensitivity analysis. The last chapter develops and estimates a DSGE model with oil as a production factor and financial frictions for the US. It also stimulates the impact of mitigation policy to reduce oil consumption through increases in oil taxes.

1.2 Chapter Overview

Chapter 2, *"Expectation Formation, Financial Frictions and Forecasting Performance of Dynamic Stochastic General Equilibrium Models"* investigates how well rational expectations, adaptive learning, financial frictions and nominal rigidities in DSGE models describe economic relationships. Researchers modified DSGE models after the financial crisis from 2007 to 2009, because of their inability to forecast the crisis. Researchers have introduced different features to the classical DSGE model by Smets & Wouters (2003). One stream of literature introduces learning by agents to replace the rational expectation hypothesis (see Slobodyan & Wouters 2012). Another stream of literature incorporates financial markets such as Merola

(2015). The chapter analyses various model modifications and evaluates the forecasting performance of the models by different metrics. The main conclusion is that the standard DSGE model developed by Smets & Wouters (2003) is not systemically outperformed either when using adaptive learning or including financial frictions.

Most studies evaluating the forecasting performance of DSGE models focus on the US. The chapter will also consider the ability of the standard DSGE model to forecast critical macroeconomic indicators for the euro area and the United Kingdom. A prerequisite of a model applied to evaluate mitigation policies is also its applicability to different geographical regions. The analysis shows that the forecasting performance of the model for all considered geographical regions is similar. Therefore, the core model ingredients are useful not only for the US but also for other geographical regions. A modification of the assumed expectation formation does not improve the forecasts significantly. Adaptive learning might even decrease the forecasting performance of the model. It seems that adaptive learning is not a very promising modification of the model proposed by Slobodyan & Wouters (2012). It is important to note here that the modification of the expectation formation process is still very close to rational expectations. For forecast horizons exceeding eight quarters, it is not necessary to include nominal rigidities to forecast macroeconomic aggregates. Evaluating mitigation policies in the medium to long run does not require a model with nominal rigidities for wages and prices. However, to evaluate mitigation policies on a business cycle frequency nominal rigidities are essential.

Chapter 3, *"Power Generation and Structural Change: Quantifying Economic Effects of a Coal Phase-Out in Germany"* analyses the potential economic effects of a coal phase-out in Germany. Germany is a signatory of the Paris Agreement in 2015 and commits to reducing its GHG emissions by more than 50% by 2030. Germany uses its natural stocks of lignite to generate roughly 25% of its electricity each year. The greatest lignite stocks are in the region of Rhineland, Lusatia, and Central Germany. A coal phase-out would affect those regions the most. A calibrated multi-sector and multi-region model quantifies the medium-term regional economic consequences. Previous studies investigating potential economic consequences only use descriptive statistics (see, e.g., Markwardt & Zundel 2017) or static input-output models (see, e.g., Buttermann & Baten 2011). However, these models are not able to capture price and wage adjustments. Furthermore, they neglect potential recovery processes through migration. The reduction of lignite coal leads to different regional relative productivity profiles in the long run. The simulated path is the transition from an initial steady-state with lignite as an energy carrier to a terminal steady-state with a lower share of electricity generated by lignite. It is not necessary to include nominal rigidities to evaluate the medium-run consequences of a coal phase-out in Germany, according to the results of the first chapter. In order to evaluate the impact of a coal phase-out on unemployment, the proposed model includes labour market frictions through hiring costs. The approach follows Blanchard & Gali (2010) introducing hiring costs for firms. Further hiring costs depend on a cyclical and non-cyclical component, as discussed in Christiano et al. (2016). Hiring costs introduce an inter-temporal decision problem for the firm. Increasing hiring activity today reduces hiring costs in the future. Hiring costs reduce incentives today to hire recruits. Another important feature of the model is the introduction of migration between regions.

Migration is one of the main determinants for achieving convergence in the labour market. Especially after the German reunification, migration flows from East to West Germany had macroeconomic consequences (see Grossmann et al. 2017). Here, migration ensures recovery of the regional unemployment rates. It is motivated by a random utility maximisation framework (see Beine et al. 2016, for a comprehensive introduction). The model parameters are calibrated and not estimated. It is not possible to evaluate the model uncertainty in a probabilistic framework as one could do using Bayesian techniques. Instead, a univariate and multivariate sensitivity analysis is applied. It allows for quantifying the sensitivity of the results to calibrated parameter values.

Chapter 4, *"Is Risk the Fuel of the Business Cycle?"* develops and estimates a medium-sized DSGE model for the US. Policymakers need to apply appropriate measures to reduce oil consumption. Further, it is necessary to have adequate tools to assess the potential impact of mitigation measures on the economy. One tool to assess the impact of mitigation measures on the economy is DSGE models. Golosov et al. (2014) use a DSGE model to compute optimal taxes to reduce fossil fuel consumption. The structural parameters for the model are not estimated but calibrated. Further, the model does not feature financial markets.

Christiano et al. (2014) (henceforth **CMR**) include financial frictions as described in Bernanke et al. (1999). Through the inclusion of entrepreneurs producing capital services with idiosyncratic productivity, the production process of capital becomes risky. Risk is the variation of idiosyncratic productivity to produce effective capital services for the production process using raw capital. Estimating the model with financial variables shows that risk according to the variance decomposition at the posterior mean is the most important source of the business cycle. Another stream of the existing DSGE literature incorporates oil into DSGE models, such as Balke & Brown (2018), Bergholt et al. (2017), Dhawan & Jeske (2008), Milani (2009).

The chapter combines the two streams of the literature by incorporating financial markets and the market for oil in one model. My baseline model is the one introduced by Christiano et al. (2005) (henceforth **CEE**). I will extend the model to include oil as production factor (henceforth **CEE–Oil**). Therefore, I switch from a Cobb-Douglas production function of the representative firm with two inputs (labour and capital services) to a nested constant elasticity of substitution (henceforth **CES**) production function with three inputs (labour, capital services, and oil). The nested CES production function has three layers, combining at the top layer labour and a composite production factor of oil and capital. The next layer combines oil and capital services to the composite production factor. In each layer, the production factors may be either complements or substitutes. Each layer includes the special case of a Cobb-Douglas production function. Bayesian techniques provide estimates of the structural parameters of the model.

Oil market disturbances have not been a significant driver of the business cycle in the US, according to the results. The comparison between a model with and without financial accelerator shows that oil market disturbances are more critical for explaining investment when not controlling for financial market frictions. A model with financial accelerator emphasises the importance of financial market disturbances for the variation in investment. Oil market disturbances are very persistent and therefore do not alter growth rates of the primary aggregates, but they do lead to prolonged periods of lower economic activity. Mitigation policies in the US to reduce oil consumption by 10%, in the long run, can reduce GDP by 1 to 2 % and initiate a weak recession. The last time oil consumption dropped by this amount was in 1990 caused by the first Gulf War.

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Chapter 2

Expectation Formation, Financial Frictions and Forecasting Performance of Dynamic Stochastic General Equilibrium Models *

Abstract

In this paper, we document the forecasting performance of standardly estimated macroeconomic models and compare it to extended versions which consider alternative expectation formation assumptions and financial frictions. We also show how standard model features like price and wage rigidities contribute to forecasting performance. It turns out that neither alternative expectation formation behaviour nor financial frictions can systematically increase the forecasting performance of standardly estimated macroeconomic models. Only during periods of financial crises, financial frictions improve forecasts. Traditional price and wage rigidities, to the contrary, systematically help to increase the forecasting performance.

JEL Codes: C32, C53, E37
Keywords: Business cycles, economic forecasting, macroeconomic modelling

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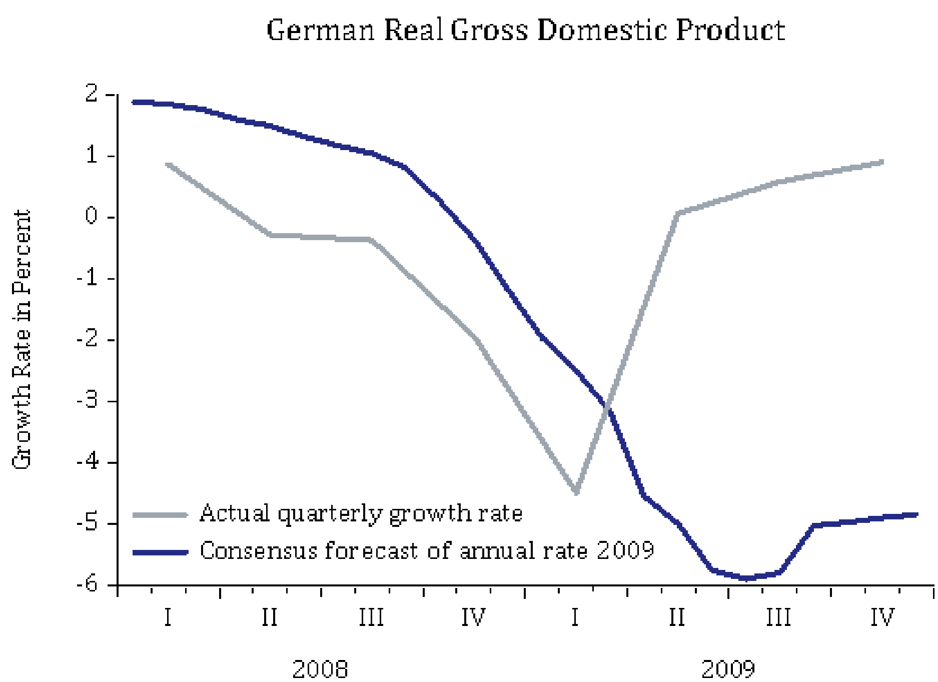
2.1 Introduction

Quantitative macroeconomic models are an important tool for economic policy analysis. Such models are employed to simulate the effects of policy actions on macroeconomic variables and to forecast future macroeconomic development. Since the worldwide financial crisis state-of-the-art macroeconomic modelling has been heavily criticized.[1] A major reason for critique is that state-of-the-art macroeconomic models have not predicted the financial crisis and, in some cases, also not the pace of the recovery.[2] Professional forecasters started to predict a downturn for the year 2009 when economic activity already started to slowdown during the year 2008. Figure 2.1 shows that the mean of forecasts for German GDP growth from professional forecasters considered in the Consensus sample have been positive until October 2008. Consensus forecasts reached their minimum in July 2009 while the actual second quarter growth rate of GDP has already been positive again. Overall it looks like forecasts are following the actual development but not anticipating it.

Figure 2.1: Monthly Consensus forecasts of annual German GDP growth in 2009 and actual quarterly GDP growth rates



Soruces: Consensus Economics, Federal Statistical Office of Germany, own exhibition.

Why are forecasts of future economic developments by macroeconomic models biased?

[1] See Buch et al. (2014) for a discussion of shortcomings of pre-crisis macroeconomic models.

[2] The pre-crisis state-of-the-art features of macroeconomic models are described in Blanchard (2009), for example.

We are not able to reduce the uncertainty about the future to zero, even if we use the correct model. Economic activity today depends on how agents think about the future. Especially investment decisions depend on expectations, but the success and return of investments is always to some extent not predictable. Therefore one source of forecast errors can not be overcome by improving our models. Nevertheless, if we have the correct model our forecast errors should not be biased anymore. But as Figure 2.1 shows this is not the case. Omitted variables is a standard explanation for biased forecasts. An omitted variable bias occurs if the model does not include all relevant variables necessary to describe the economic system. Before the last financial crisis standard macroeconomic models did not include financial market variables such as interest rates spreads, but they seem to be an important determinant of future economic developments. Another potential source of biased forecasts is the assumption how agents behave and form believes about the future. In economics a standard and convenient assumption are rational agents forming rational expectations about the future, i.e. model consistent and using all relevant and available information. A further potential cause for biased forecasts by macroeconomic models is how financial markets and their interaction with the real economy is modeled. Most models before the financial crisis did not attribute a special role to financial markets for the development of the real economy.

In the center of model criticism are the way individual economic behaviour, in particular expectation formation, is captured and the fact that the financial systems and its frictions have been ignored in standard models for a long time. Standard models before the financial crisis usually relied on the rational expectations (RE) hypothesis (Lucas 1976, Muth 1961) and have not included money or credit aggregates. In the aftermath of the financial crisis various model extensions both in the area of expectation formation and in the area of financial frictions have been developed. However, a new standard model has not been established yet.

Furthermore, from an empirical perspective, it is not clear what kind of model features are important to improve the forecasting performance of standard models. In this paper, we document the forecasting performance of an estimated standard pre-crisis macroeconomic model and compare it to extended versions which consider alternative expectation formation assumptions and financial frictions. We also show how standard model features like price and wage rigidities contribute to forecasting performance. Our results suggest that neither alternative expectation formation behaviour nor financial frictions can systematically increase the forecasting performance of simple estimated macroeconomic models. Only during periods of financial crises, financial frictions improve forecasts. Traditional price and wage rigidities, to the contrary, systematically help to increase the forecasting performance.

The paper is structured as follows. In Section 4.2, we describe how the pre-crisis standard macroeconomic model has evolved from earlier approaches to macroeconomic modelling. In Section 2.3, we explain extensions to the simple standard model that have become prominent after the financial crisis, namely financial frictions and adaptive learning. Then we conduct pseudo-out-of-sample forecasts and document forecast performance of the various models in Section 2.4. Finally, Section 2.5 concludes.

2.2 The Pre-Crisis Standard Macroeconomic Model

2.2.1 Short review of empirical macroeconomic modelling

The pre-crisis standard macroeconomic model has been a small or medium-sized dynamic stochastic general equilibrium (DSGE) model. The predecessors of DSGE models have been traditional structural models, models following the London School of Economics (LSE) approach and vector autoregressive (VAR) models.[3] In the sixties and seventies of the twentieth century, structural models have been the dominating technique. These traditional structural models are sometimes designated as Cowles commission approach. A typical empirical analysis within this paradigm consists of three steps (Favero 2001, p. 103): (1) specification of the theoretical models, (2) estimation of parameters, and (3) simulation of the effects of policy actions. The economic model is formulated in terms of behavioural equations and definitional identities and is summarized in the following econometric model:

$$A_0 x_t = A_1^* x_{t-1} + Q^* z_t + e_t \tag{2.1}$$

In this equation, $x_t = (x_{1,t}, x_{2,t}, \ldots, x_{p,t})'$ is a $(p \times 1)$-vector of endogenous variables, z_t is a vector of exogenous variables (especially policy instruments), A_0 and A_1^* are (pxp) coefficient matrices, the matrix Q^* stores the coefficients of the exogenous variables and e_t is a $(p \times 1)$-vector of error terms that is normally distributed with mean zero and covariance matrix Σ_e, $e \sim N(0, \Sigma_e)$. Deterministic terms as well as further lags of endogenous and exogenous variables can be added but are ignored in the following. This set of p equations describes the simultaneous relationships between the variables. The impact of exogenous and lagged endogenous variables on the actual endogenous variables is expressed by the reduced form:

$$x_t = \underbrace{A_0^{-1} A_1^*}_{A_1} x_{t-1} + \underbrace{A_0^{-1} Q^*}_{Q} z_t + \underbrace{A_0^{-1} e_t}_{u_t} \tag{2.2}$$

Equation (2.2) can also be used to forecast x_t (and also the future time path $\{x_{t+h}\}, h \geq 0$) conditional on lagged realizations and exogenous variables. The Cowles commission approach has been criticized extensively: (1) the a priori exogeneity assumptions are controversial. (2) The aggregated behavioural equations in traditional structural models have usually been ad-hoc equations without microeconomic foundations. (3) The coefficient estimates of non-structural models might depend on policy rules and might change over time. Therefore those estimates are not useful to evaluate policy changes (Lucas-Critique). (4) The statistical performance of the estimated model has not been considered seriously. Especially, static regressions of non-stationary variables have led to spurious regressions. A partial response to these criticisms is the LSE approach that focuses especially on the statistical properties of the estimated model but does not question the paradigm of simulating policy effects on the basis of structural forms in principle.[4] The first step of the LSE procedure is the estimation

[3] The modelling review is based on Holtemoeller (2002).

[4] The econometric issues of this approach are for example discussed in Hendry (1995).

of a general dynamic reduced form model that has to pass a sequence of diagnostic tests. Equations for variables that are confirmed to be statistically exogenous can be omitted. Non-stationary variables can also be modeled appropriately (error correction models). A reduction technique is applied to impose non-rejected restrictions on the parameters of the model. The resulting structural form is used for the simulation of policy effects. While the LSE approach is mainly a response to the statistical problems of traditional structural models, the VAR approach, which has achieved enormous popularity after seminal works of Sims (1972, 1980), abandons the a priori exogeneity assumptions by including all relevant variables into the vector of endogenous variables and estimating the reduced form

$$x_t = \sum_{i=1}^{k} A_i x_{t-i} + u_t \tag{2.3}$$

where $u_t \sim N(0, \Sigma_u)$. Deterministic terms are again neglected. The lag length k is determined by statistical criteria. In this framework it can be tested whether a variable is exogenous or not. Different exogeneity concepts have been developed for this purpose, see for example Engle & Granger (1987) and Dufour & Renault (1998). One of these concepts is Granger causality (Granger 1969) that is based on the chronological asymmetry of cause and effect.[5] The main purpose of VAR models is not to simulate the effects of policy actions but to analyze the impact of policy shocks on the variables of interest and to forecast economic variables. This empirical evidence is used to build theoretical models based on microeconomic foundations that are able to produce the empirically observed responses. If a theoretical model is able to reproduce the observed response patterns it is used to derive policy implications. The VAR approach has been extended over time. The observation that many macroeconomic time series exhibit stochastic trends (Plosser & Nelson 1982) has led to the development of cointegration models which were introduced by Engle & Granger (1987). The second main extension of the VAR model is the development of structural VAR (SVAR) models; one of the first contributions to the literature on SVAR models is Bernanke (1986). Following Amisano & Giannini (1997), SVAR models can be characterized by the so-called AB-model:

$$A_0\, x_t = \sum_{i=1}^{k} \underbrace{A_0\, A_i}_{A_i^*}\, x_{t-i} + A_0\, u_t,\ A_0 u_t = B e_t,\ e_t \sim N(0, I_p). \tag{2.4}$$

where I_p denotes a p-dimensional identity matrix. The notion AB-model is based on the definition $A = A_0$, such that the matrices A and B characterize the contemporaneous relationships between endogenous variables and exogenous structural shocks e_t.[6] While VAR models usually have a very good fit and can provide a reasonable characterization of statistical properties of macroeconomic data they ignore theoretical restrictions that stem from

[5] The econometric analysis of VAR models is discussed for example in Hamilton (1994) and Lütkepohl (2005).

[6] See Kilian & Lütkepohl (2017) for a detailed discussion of SVARs and the identification of structural shocks.

general-equilibrium considerations or forward-looking behaviour. Kydland & Prescott (1982) have developed an empirical characterization of macroeconomic time series that is completely derived from the optimizing behaviour of economic agents and that takes general-equilibrium restriction into account. This type of model is today known as dynamic stochastic general equilibrium (DSGE) model and can be represented as follows:[7]

$$\Gamma(\mathrm{E}_t x_{t+1}, x_t, e_{t+1}) = 0, \tag{2.5}$$

where E_t denotes the expectation operator. Expectations are rational in this framework in the sense that they are compatible with the mathematical structure of the model. Often, these models are log-linearized:

$$Ax_{t+1} = Bx_t + Ce_t + Df_{t+1}, \tag{2.6}$$

where f_{t+1} denotes the difference between expectation and actual realization (expectational error). The solution of this model is a recursive law of motion:

$$x_{t+1} = Fx_t + Ge_t, \tag{2.7}$$

which is again a VAR representation of the data, but with theory-based cross-equation restrictions imposed. While early small-scale DSGE models have not performed as well as reduced-form VAR models in terms of statistical fit and forecast performance, models that are used today usually have a very good statistical fit and can even outperform reduced-form models without restrictions in terms of forecasting for certain forecasting horizons (Cai et al. 2018, Del Negro et al. 2006).

2.2.2 The New-Keynesian standard DSGE model

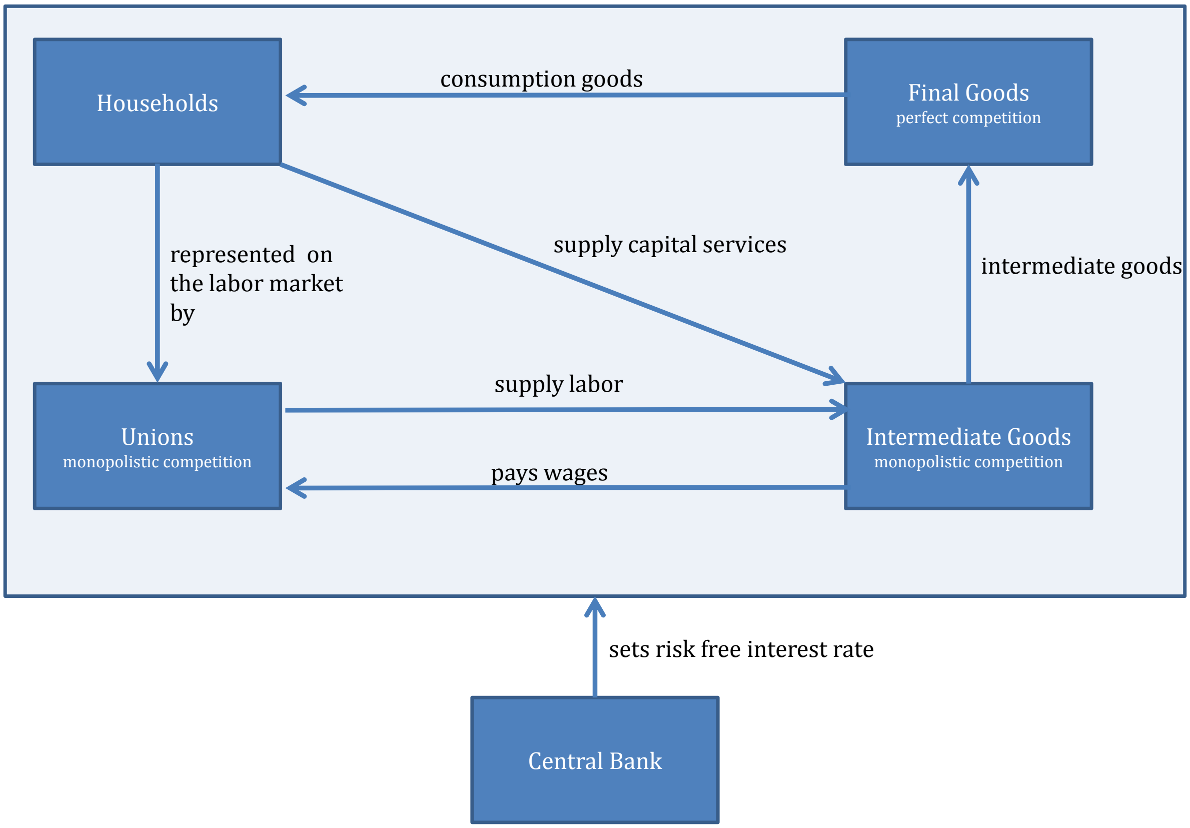
The pre-crisis standard DSGE model has been developed from the framework introduced by Kydland & Prescott (1982) by adding (New-)Keynesian elements like price and wage rigidities. Gali (1999) showed that a small-scale New-Keynesian model can explain important dynamic correlations in macroeconomic data. Methods for estimating DSGE models have been developed, and Smets & Wouters (2003, 2007) provided an estimated New-Keynesian DSGE model that has been intensively used in applied work.[8]

The structure of the Smets and Wouters (SW) model is depicted in Figure 2.2. Five types of agents are considered: households, unions, final goods producers, intermediate goods producers and a central bank. The model represents a closed economy without considering international trade or capital flows.

[7] See DeJong & Dave (2011) for an introduction into DSGE models.

[8] An overview of DSGE models and their usage in policy institutions is given by Christiano & Trabandt (2017).

Figure 2.2: Main structure of the standard New-Keynesian model



Source: own exhibition.

There is a continuum of households modeled by one representative household. Potential implications of heterogeneous behaviour by households on aggregate development are not considered.[9] The representative household maximizes inter-temporal discounted utility over time. The household has multiple income sources: labor, capital services and interest paying securities. Unions negotiate wages, households supply the amount of labor demanded at the negotiated wage. To introduce the empirically observed sluggishness of wages only a fraction of unions is able to reset the wage in the current period. Therefore, unions re-optimizing today take future developments into account. Consequently, future expected developments will affect wages today more than under flexible wages. There are two types of firms: intermediate goods producing firms and final goods producing firms (retailers). Retailers have no market power and are price takers. However, intermediate goods producers can set prices above marginal costs because retailers produce final goods from differentiated

[9] Imposing homogenous behaviour on all agents is a further main critique against the standard model. However, the discussion of heterogeneity in DSGE models is beyond the scope of this paper.

products. Intermediate goods producers use labor and capital services from households to produce intermediate goods. They first choose the amount of labor and capital services as input according to their marginal products and costs. In a further step, they maximize inter-temporal profits by setting prices given the retailer demand for their products. Only a fraction of intermediate goods producers is able to set prices according to current marginal costs and desired mark-ups, similar to unions. Price-setting behaviour is forward-looking and future increases in marginal costs lead to higher inflation today than under flexible prices. The central bank sets the short-term risk-free interest rate for securities. It follows a monetary policy reaction function and varies the interest rate in response to deviations of the inflation rate from target inflation and of the output from potential output. To complete and solve the model it is necessary to specify how agents form expectations on consumption, investment, labor, price of capital services, wages and inflation. In the Smets and Wouters model it is assumed like in most other pre-crisis macroeconomic general equilibrium models that expectations are rational and fully model consistent. Rational expectations require that agents do not only know their own behavioural equations, but the complete structure of the economy. In addition, agents use all information at a specific time point to form their expectations. Systematic expectation errors are excluded.

2.3 Model Extensions

2.3.1 Financial frictions

The pre-crisis standard New-Keynesian model abstracts from financial markets and does not consider financial frictions as a potential source of business cycle fluctuations. In recent years, various extensions of the New-Keynesian standard model have been developed that include financial frictions. Gertler & Kiyotaki (2010), for example, incorporate financial frictions based on earlier work by Bernanke et al. (1999) as a propagation mechanism into the model framework (financial accelerator). In this type of model, creditors must pay a risk-premium in addition to the risk-free rate due to monitoring costs. Christiano et al. (2014) show that financial frictions can account for a significant proportion of business cycle fluctuations in a standard medium-scale DSGE model. Only the external finance premium enters the standard Smets and Wouters model as additional observable variable. In our forecasting exercise, we use a log-linearized version of the model of Merola (2015). Financial frictions are shocks to the spread between the risk-free interest rate and the return to capital. These shocks trigger a decrease in borrowing activities by firms and therefore reduce capital services. This in turn has a negative effect on output and consumption. Since agents know the structure of the economy (rational expectations), the amount of borrowing today is affected by expectations about future developments.

2.3.2 Adaptive learning

Figure 2.1 indicates that even professional forecasters only slowly adapt to new information. This poses serious doubts on the rational expectation hypothesis and models which rely on alternative expectation formation assumptions have evolved. In particular, models with adaptive learning (AL) have been developed, for example Evans (2001), Bullard & Mitra (2002), Evans & Honkapohja (2003, 2006), Carceles-Poveda & Giannitsarou (2007) and Slobodyan & Wouters (2012). Adaptive learning assumes that agents use forecasting models to form beliefs. They update the parameters of their forecasting model in real time. That is, the most recent information is utilized to form beliefs, respectively. While the VAR law of motion implied by rational expectation DSGE models (2.7) exhibits time-invariant coefficient matrices F and G, models with adaptive learning imply time-varying coefficients. The source of the variation in the parameters originates from updating beliefs. In our forecasting exercise, we use an adaptive learning model in which agents adjust their forecasting model and update the coefficients of the model each period.[10] Over time, agents learn from their expectation errors and adjust decision rules and beliefs. In contrast to rational expectation models, this approach allows for systematic expectation errors by agents, but requires that agents learn from their mistakes. In rational expectations models persistence is to a large extent captured by price and wage rigidities. Adaptive learning introduces an additional source of persistence. Consequently, estimated parameters in wage and price setting equations, for example, depend on the way in which expectation formation is specified. Milani (2007) shows that a small-scale New-Keynesian DSGE model estimated with adaptive learning has a better in-sample fit than the same model with rational expectations. Furthermore,Milani & Rajbhandari (2012) find that adaptive learning models have a better in-sample fit than models estimated with news shocks or nearly rational expectations. However, the in-sample fit measures only the performance of the model evaluated with data used for estimation. This measure alone is not appropriate to determine how useful models are as indicators for future developments. Slobodyan & Wouters (2012) and Milani & Rajbhandari (2012) show that adaptive learning performs better for short-run forecasts, but rational expectations are better in producing long-run forecasts. In our forecasting exercise we will evaluate the relative importance of various model features for the forecast performance.

2.4 Forecast Performance

2.4.1 Estimation and data

To investigate whether the extensions to the baseline model are useful for forecasting purposes we estimate various models and compute pseudo-out-of-sample forecasts. We start from the standard New-Keynesian model characterized in Section 2.2 but without price and wage rigidities. We exclude price rigidities by allowing all firms to reset their prices and unions to negotiate wages in each period. The special case without price and wage rigidities collapses to a pure real business cycle model. The full set of models is: baseline model without nominal rigidities (SW-NRI), without price rigidities (SW-NPR), without wage rigidities (SW-NWR), baseline model with price and wage rigidities (SW-RI) and the baseline model with financial accelerator (SW-FA). All models are estimated for rational expectations and

[10] This is based on a Kalman filter approach Hamilton (1994).

adaptive learning. We further estimate a vector autoregressive (VAR) model with and without the external finance premium as endogenous variable model with lag order one as a restriction-free naive benchmark model. The model is estimated for the United States of America (US), the United Kingdom (UK) and the Euro area (EA). For the US, the full sample covers the period from 1954-Q3 to 2017-Q3 at a quarterly frequency. The samples for the UK and the Euro area cover the period between 1999-Q1 and 2017-Q3. So far, most studies estimating DSGE models with adaptive learning have only used US data. We use quarterly seasonally adjusted national accounts data for gross domestic product (GDP), consumption, investment, wages and salaries and total hours worked.[11] Inflation is measured with the GDP deflator and the short-term interest rate set by the central bank is the Federal Funds rate for the US and the money market rate in the UK[12] and Euro area. For the financial accelerator version an additional variable to measure the external finance premium is necessary. For the US, we use the spread between AAA and BAA rated corporate bonds yields. For the Euro area and the UK , we use the spreads implied by the inverse price index of AAA and BAA corporate bonds yields in the Euro area.[13] Bayesian techniques are used to estimate the structural parameters. The prior distribution or the starting distribution for all model specifications for each parameter are the same. The prior distributions are also identical across regions, except for trend parameters for inflation, output growth and interest rate. For each region those parameters are set to the mean of the current sample. The models are then estimated by drawing parameter values from the prior distribution and evaluating the likelihood of the model given the vector of parameters. The posterior distribution of the parameters is the update of the prior distribution given the observed data according to the theorem of Bayes; the posterior distribution is used in the next step as prior to draw parameters from. This procedure is repeated until the likelihood of the model does not improve anymore or a predefined number of iterations is exceeded. A detailed description of this method is provided by Schorfheide (2000). We estimate the model initially using data up to 2006-Q3 and expand the estimation window by one year to re-estimate the model. We use the same procedure for the VAR model. Forecast errors for horizons 1, 4 and 8 quarters at the posterior mode of the estimated parameters are computed. In total we get 44 forecast errors of horizon 1, 41 of horizon 4 and 37 of horizon 8. Based on these forecast errors root mean squared percentage errors (RMSPE) are calculated.

2.4.2 Results

Figure 2.3 shows one-quarter ahead forecasts for the US from 2007-Q4 onwards together with ex-post observed data. The model without nominal rigidities in most cases delivers poor forecasts, independently from the expectation formation specification. However, the

[11] For the US, the data is provided by the Federal Reserve Bank of St. Louis (`https://fred.stlouisfed.org`). For the Euro area and the UK the data source is Eurostat (`http://ec.europa.eu/eurostat/de/data/database`).

[12] Unfortunately no comparable variables are available for the UK. Therefore, we follow the work by Hall (2001) and use Euro area spreads.

[13] Corporate bonds yields are published by IBOXX (`https://ihsmarkit.com/products/iboxx.html`).

pre-crisis standard New-Keynesian model (SW-RI) with rational expectations seem to produce reasonable forecasts. Adding features like financial frictions (FA) or adaptive learning seem not to lead to substantial improvements on average but during the year 2009 financial frictions help to capture the deepness of the recession. This is compatible with the finding by Del Negro et al. (2016) that the predictive power of DSGE models with financial frictions for output growth and inflation is only better during periods of financial distress. For inflation the SW-RI model and the financial accelerator model produce very similar forecasts. Under adaptive learning the forecast for output growth of the SW-RI model and the model with financial accelerator do not track the downturn as well as under rational expectations. Agents update their expectations given the current information not as quick as under rational expectations. Figure 2.4 shows the same forecasts but estimated on a shorter sample to make the results comparable to results for the UK and the Euro area which rely on shorter samples due to data limitations.

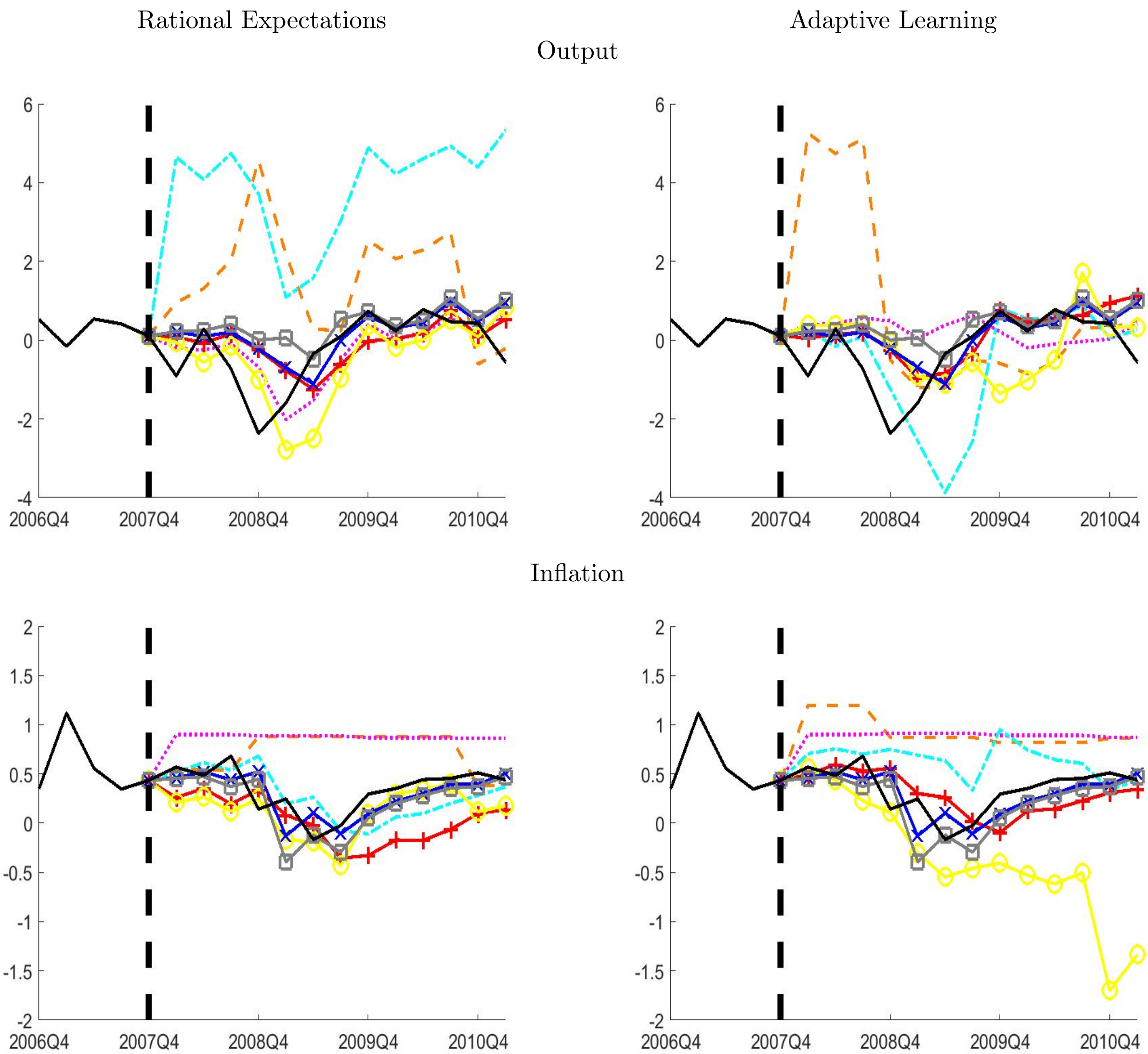
The one-quarter ahead forecasts for the UK are depicted in Figure 2.5. Models with rational expectations predicted no downturn after 2007-Q4 for output growth. Under adaptive learning only the financial accelerator model predicts a downturn, but only with a severe lag. In contrast to the US the GDP deflator in the UK is more volatile in the respective period. Therefore, all models have difficulties tracking this volatile behaviour. The financial accelerator model under adaptive learning does the best job in tracing this behaviour.

Output growth in the Euro area during the crisis is also better predicted one-quarter ahead under adaptive learning with financial accelerator (see Figure 2.6). The same is true for the GDP deflator in the Euro area. All models can predict the downturn only with a lag. In line with the literature, our estimation results regarding the in-sample fit of models with adaptive learning compared to models with rational expectations show that adaptive learning improves the likelihood of the SW-RI model. Table 2.1 reports the log-likelihood for the US, the UK and the Euro area models, respectively. The SW-RI model with adaptive learning has for all countries a larger likelihood than the SW-FA model with financial accelerator. This is also true for rational expectations for the US and Euro area, but not for the UK. In general, the inclusion of wage and price rigidities increase the log-likelihood. It is not clear what nominal friction or model feature is in particular useful to forecast inflation or output by looking at the log-likelihood. Table 2.2 to Table 2.5 report the root mean squared percentage errors for the different regions. Root mean squared percentage errors for all structural models and the VAR with external finance premium are reported relative to the root mean squared percentage error of the VAR without external finance premium for the respective region and horizon. Table 2.2 and Table 2.3 report the root mean squared percentage errors for the full and short US sample, respectively. The RMSPE for Output and the log-determinant is smaller using only more recent information to estimate the VAR. Including the external finance premium into the VAR as endogenous variable will not improve the forecasts for output, but for inflation. The SW-RI model and the SW-FA model are the best models to make one-quarter ahead predictions according to the log-determinant. They perform slightly better as the unrestricted VAR model. As expected, the forecast accuracy decreases almost monotonically with the horizon. Compared to a VAR with one lag the forecast performance of the structural models for output growth is pretty

Table 2.1: Likelihoods

Model	Adaptive Learning	Rational Expectations
	US (full sample)	
SW-NRI	-3449.74	-8411.14
SW-NWR	-2201.14	-8215.23
SW-NPR	-2588.75	-1844.63
SW-RI	-1628.28	-1686.84
SW-FA	-1736.63	-1369.70
	UK	
SW-NRI	-793.89	-810.49
SW-NWR	-807.49	-819.72
SW-NPR	-783.48	-797.32
SW-RI	-747.64	-788.56
SW-FA	-982.40	-906.31
	Euro area	
SW-NRI	-643.11	-659.35
SW-NWR	-641.62	-664.30
SW-NPR	-667.35	-647.83
SW-RI	-575.83	-614.06
SW-FA	-904.96	-762.56
	US (short sample)	
SW-NRI	-450.81	-455.99
SW-NWR	-451.60	-466.44
SW-NPR	-374.68	-386.46
SW-RI	-375.58	-393.38
SW-FA	-822.81	-369.38

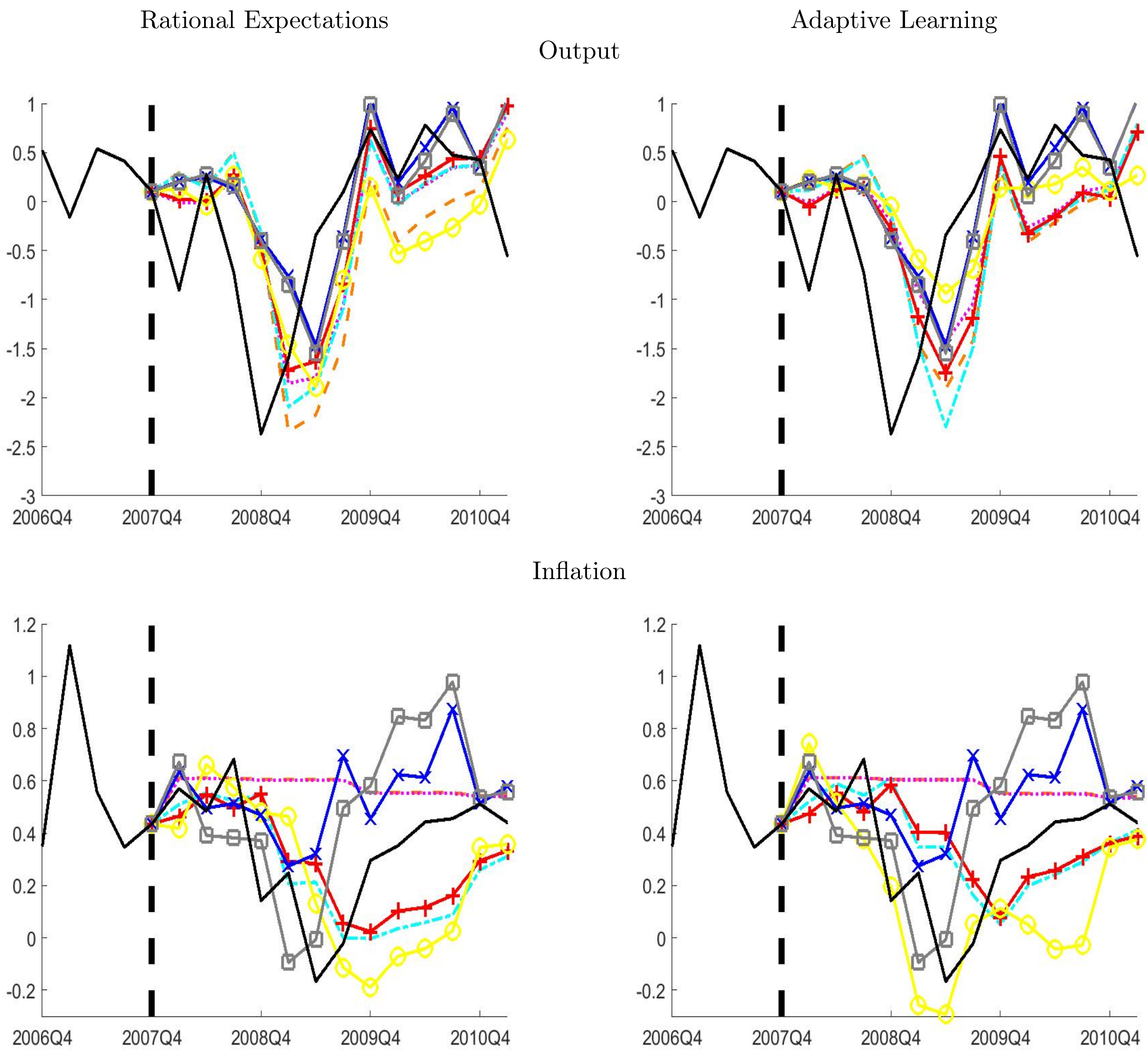
Figure 2.3: One-quarter ahead forecast performance during the crisis US (full sample)



Notes: The lines depict actual data (black) and forecasts: SW without nominal rigidities (orange), SW without price rigidities (magenta), SW without wage rigidities (cyan), SW with nominal rigidities (red), SW with financial accelerator (yellow) and VAR without external finance premium (blue) and with external finance premium (grey).

close regardless of the underlying expectation formation process. The accuracy of inflation forecasts can be improved substantially by using restricted models compared to unrestricted models. Adaptive Learning improves the forecast accuracy for output and inflation in the short sample but not in the full sample. The exclusion of nominal rigidities deteriorates the

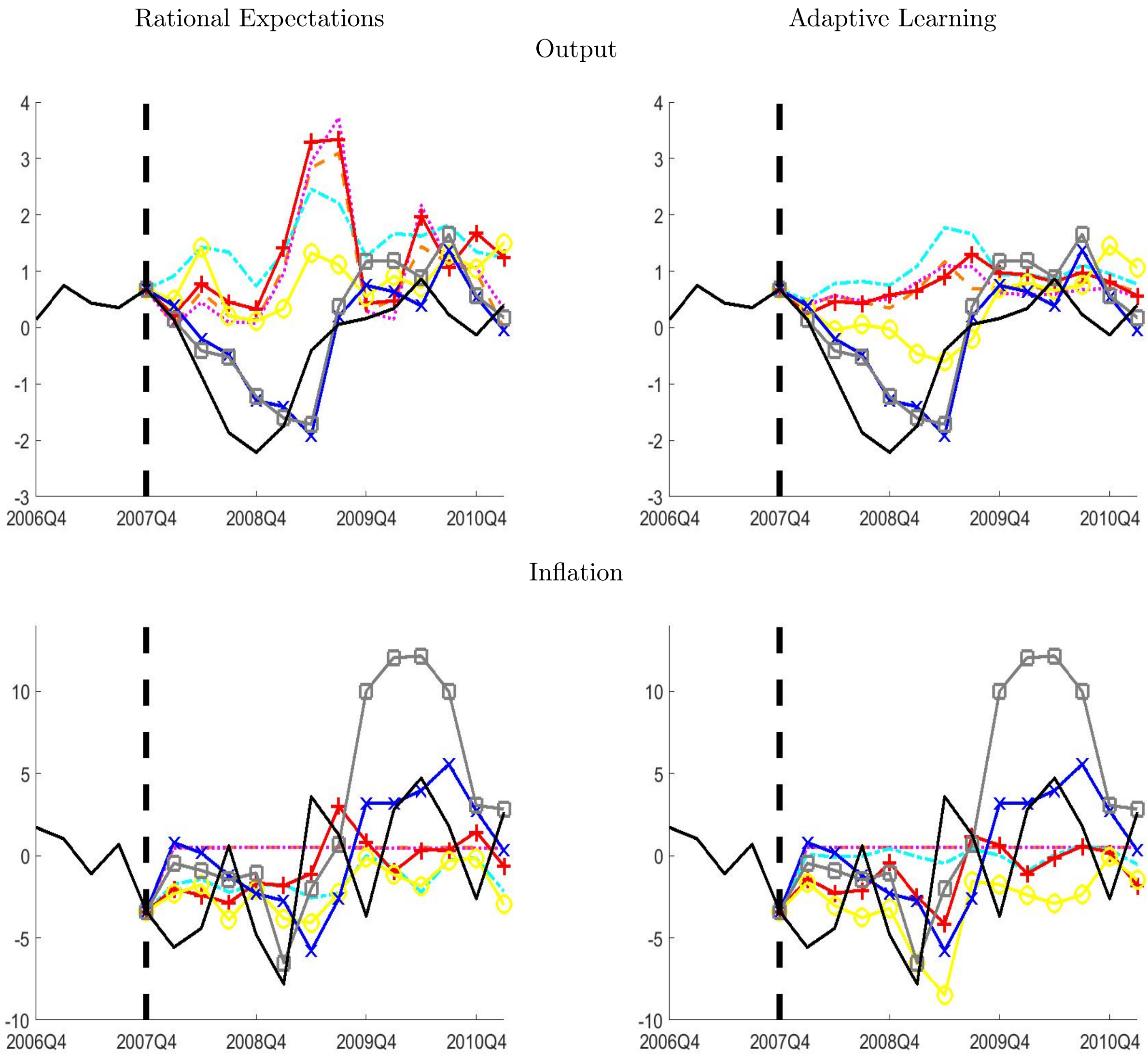
Figure 2.4: One-quarter ahead forecast performance during the crisis US short sample



Notes: The lines depict actual data (black) and forecasts: SW without nominal rigidities (orange), SW without price rigidities (magenta), SW without wage rigidities (cyan), SW with nominal rigidities (red), SW with financial accelerator (yellow) and VAR without external finance premium (blue) and with external finance premium (grey).

forecast accuracy for output growth and inflation. For increasing forecast horizons nominal rigidities become less important. Rigid wages help to improve forecasts for output growth and lead to worse forecasts for inflation. Rigid prices are helpful to forecast inflation but not to forecast output growth. Including both rigidities leads to worse forecasts for inflation

Figure 2.5: One-quarter ahead forecast performance during the crisis United Kingdom

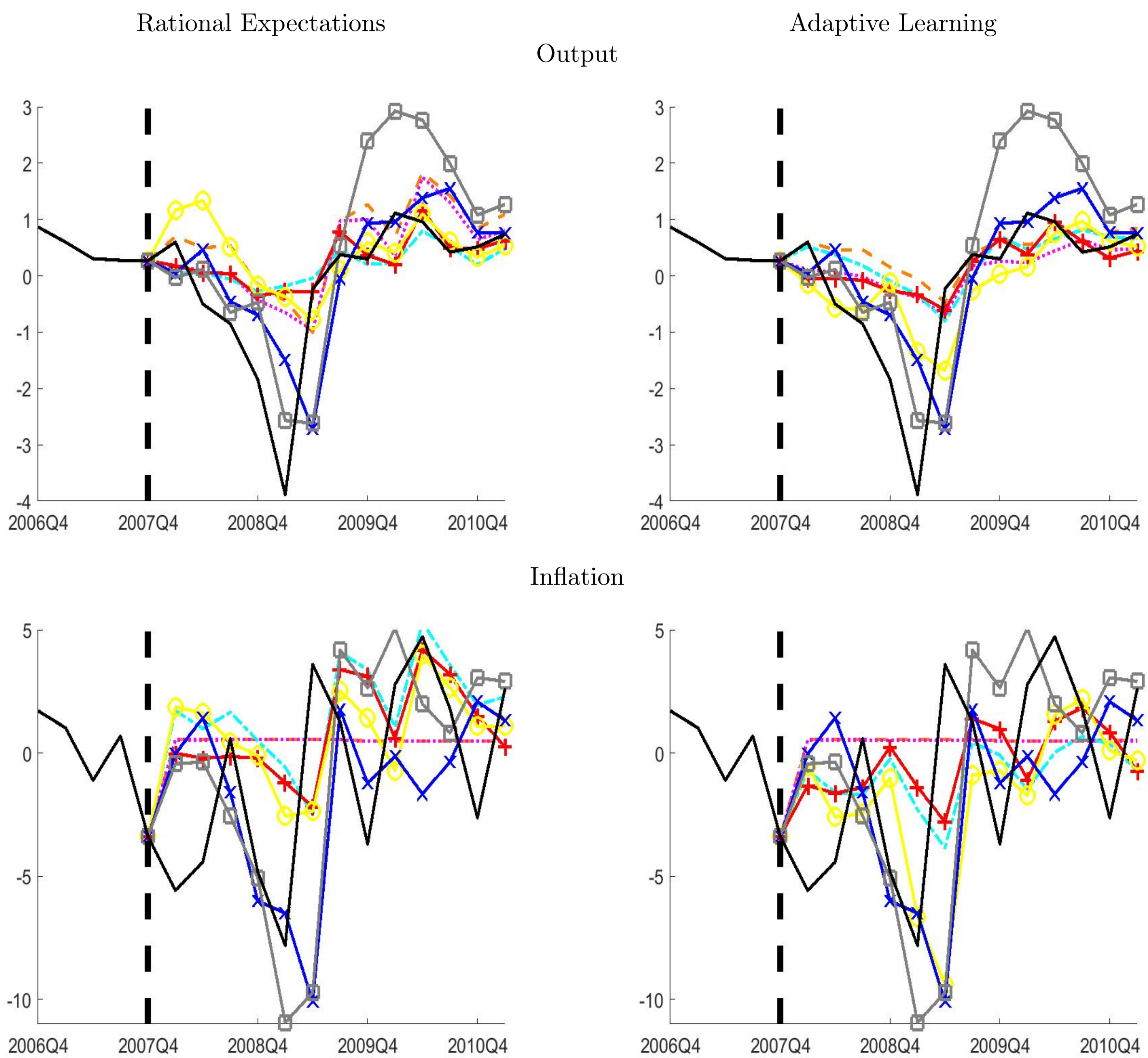


Notes: The lines depict actual data (black) and forecasts: SW without nominal rigidities (orange), SW without price rigidities (magenta), SW without wage rigidities (cyan), SW with nominal rigidities (red), SW with financial accelerator (yellow) and VAR without external finance premium (blue) and with external finance premium (grey).

and output growth compared to the models with only one rigidity.

Table 2.4 reports the results for the United Kingdom. As for the US the inclusion of the external finance premium into the VAR will not lead to better forecasts. Including financial frictions into the structural model improves the forecasts up to one year for output growth compared to a VAR without external finance premium and compared to the SW-RI model.

Figure 2.6: One-quarter ahead forecast performance during the crisis Euro area



Notes: The lines depict actual data (black) and forecasts: SW without nominal rigidities (orange), SW without price rigidities (magenta), SW without wage rigidities (cyan), SW with nominal rigidities (red), SW with financial accelerator (yellow) and VAR without external finance premium (blue) and with external finance premium (grey).

The root mean squared percentage errors do not improve contrary to the US by including nominal rigidities. Adaptive learning improves one-quarter ahead output growth forecasts for the SW-RI, SW-PR and SW-WR model compared to rational expectations. For one-year ahead forecasts this statement remains valid and is also true for the SW-FA model. For two-year ahead forecasts only the SW-WR produces better forecasts with rational expectations

Table 2.2: Root mean squared percentage errors US (full sample)

Horizon	Output	Inflation	Log Determinant	Output	Inflation	Log Determinant
		Adaptive Learning			Rational Expectations	
			Smets and Wouters without nominal rigidities			
1	2.19	3.34	2.21	4.67	4.12	2.42
4	1.04	2.05	1.24	1.11	3.41	1.28
8	1.30	2.19	1.26	1.27	1.21	1.05
			Smets and Wouters without price rigidities			
1	0.66	4.21	1.06	0.65	4.14	1.07
4	0.20	2.19	0.91	0.14	2.17	0.83
8	0.10	1.92	0.76	0.14	1.88	0.89
			Smets and Wouters without wage rigidities			
1	0.79	1.47	1.49	8.06	1.09	2.34
4	0.88	4.86	1.34	1.59	1.57	1.22
8	2.51	2.10	1.41	1.81	1.56	1.19
			Smets and Wouters with nominal rigidities			
1	1.54	0.98	0.92	0.75	1.05	0.95
4	1.07	0.95	0.95	0.46	0.32	0.78
8	0.70	0.92	1.04	0.38	0.54	0.78
			Smets and Woutres with financial accelerator			
1	0.44	2.11	0.95	0.79	1.21	0.65
4	1.61	0.80	0.97	0.37	0.73	0.76
8	2.64	1.20	1.17	0.44	0.67	0.64
			Vector autoregressive model			
	without external finance premium			with external finance premium		
1	10.82	2.44	15.25	1.07	1.23	0.85
4	14.87	4.83	22.72	1.07	0.90	0.86
8	15.60	5.94	25.64	1.00	1.00	0.84

Notes: For the VAR without external finance premium the root mean squared percentage errors for the out-of-sample forecast errors for the respective horizons are reported. Root mean squared percentage errors relative to the VAR model without external finance premium are reported for the different models. Parameters are set to their posterior mode to compute the forecast errors. Posterior distributions of the parameters are estimated every year.

compared to adaptive learning. The forecasts are not systematically better or worse than the

Table 2.3: Root mean squared percentage errors US (short sample)

Horizon	Output	Inflation	Log Determinant	Output	Inflation	Log Determinant
	Adaptive Learning			Rational Expectations		
Smets and Wouters without nominal rigidities						
1	1.86	1.03	1.23	1.85	1.03	1.23
4	2.42	1.02	1.23	2.24	1.03	1.15
8	3.60	0.98	1.46	4.17	0.98	1.44
Smets and Wouters without price rigidities						
1	1.65	1.02	0.91	1.91	1.02	1.08
4	2.10	1.02	1.05	2.86	1.02	1.19
8	3.44	0.98	1.35	4.51	0.98	1.48
Smets and Wouters without wage rigidities						
1	1.90	0.53	1.13	2.17	0.51	1.25
4	2.69	0.96	1.22	3.19	0.84	1.24
8	4.15	0.95	1.49	4.82	0.88	1.51
Smets and Wouters with nominal rigidities						
1	1.65	0.50	0.78	2.02	0.50	1.02
4	1.89	0.82	1.02	3.28	0.83	1.22
8	3.65	0.80	1.33	4.78	0.85	1.51
Smets and Woutres with financial accelerator						
1	0.64	0.33	0.55	0.93	0.60	0.69
4	0.88	0.43	0.77	0.99	0.88	0.85
8	1.72	1.44	1.17	3.09	0.77	1.16
Vector autoregressive model						
	without external finance premium			with external finance premium		
1	8.53	6.35	13.53	0.99	0.91	0.71
4	7.29	6.62	19.00	1.09	0.93	0.92
8	4.59	7.37	16.77	1.19	0.98	0.83

Notes: For the VAR without external finance premium the root mean squared percentage errors for the out-of-sample forecast errors for the respective horizons are reported. Root mean squared percentage errors relative to the VAR model without external finance premium are reported for the different models. Parameters are set to their posterior mode to compute the forecast errors. Posterior distributions of the parameters are estimated every year.

VAR forecasts. The log-determinant for the SW-RI model is smaller than the unrestricted

VAR model at all horizons. The results for the Euro area are shown in Table 2.5. Wage

Table 2.4: Root mean squared percentage errors UK

Horizon	Output	Inflation	Log Determinant	Output	Inflation	Log Determinant
	Adaptive Learning			Rational Expectations		
	Smets and Wouters without nominal rigidities					
1	1.10	0.33	1.06	2.72	0.33	1.12
4	0.77	0.45	0.88	1.10	0.43	0.92
8	1.29	0.31	0.91	1.29	0.30	1.02
	Smets and Wouters without price rigidities					
1	1.20	0.34	0.90	3.24	0.32	1.09
4	0.53	0.45	0.78	1.14	0.44	1.09
8	1.10	0.32	0.78	1.40	0.30	1.09
	Smets and Wouters without wage rigidities					
1	1.86	0.62	1.09	2.60	1.55	1.27
4	0.94	0.34	0.73	1.42	0.96	0.82
8	1.58	0.30	0.82	1.12	0.72	0.79
	Smets and Wouters with nominal rigidities					
1	1.47	1.04	0.96	3.19	0.71	1.28
4	0.70	0.91	0.86	1.37	2.91	1.02
8	1.09	0.43	0.92	1.11	1.01	0.98
	Smets and Woutres with financial accelerator					
1	2.00	0.72	1.07	1.36	1.16	1.30
4	0.72	1.20	0.94	1.50	0.84	1.12
8	0.90	0.92	1.14	0.97	0.30	1.06
	Vector autoregressive model					
	without external finance premium			with external finance premium		
1	3.26	6.66	25.57	1.20	0.93	1.02
4	6.36	5.06	30.22	1.61	1.21	1.13
8	3.88	7.64	30.45	0.74	1.00	1.09

Notes: For the VAR without external finance premium the root mean squared percentage errors for the out-of-sample forecast errors for the respective horizons are reported. Root mean squared percentage errors relative to the VAR model without external finance premium are reported for the different models. Parameters are set to their posterior mode to compute the forecast errors. Posterior distributions of the parameters are estimated every year.

rigidities improve the forecasts for output growth and price rigidities are not helpful to forecast inflation. Therefore it is important to account for rigid wages in the Euro area and not for rigid prices. Adaptive learning does not crucially improve the forecast accuracy of the models in the Euro area. Structural models forecast output growth and inflation better than unrestricted VAR models. It seems that the imposed structure on the implicit reduced form VAR is helpful to improve inflation and output growth forecasts. To compare the performance between adaptive learning and rational expectations more rigorously we report the mean differentials between absolute forecast errors of models with AL and RE. We further test whether the mean is significantly different from zero with the help of a two samples t-test (Härdle et al. 2017, p. 160). The mean deviation in absolute forecast errors is used under the assumption that forecasters are in general only interested how far away they are from the actual value. The implicit assumption here is that over and under prediction result in the same costs to the forecaster. For the US the results are tabulated in Table 2.6. Output growth is better predicted one-quarter ahead by adaptive learning than rational expectations for the model without wage rigidities using the full sample. The SW-WI with rational expectations predicts an initial increase in output growth after 2007-Q4 and under adaptive learning tracks the actual behaviour very well. Otherwise the mean differential of absolute forecast errors is not significantly different from zero at the five percent level. For the baseline SW-RI model and the extension with financial frictions it does not matter to use AL or RE. Inflation is not better predicted under RE or AL for models excluding nominal rigidities. The two-year ahead forecast by the baseline model with financial accelerator for inflation is significantly better under rational expectations than under adaptive learning. If we consider all variables an RBC model performs better under adaptive learning for one-quarter ahead forecasts. The baseline model and the extension with financial accelerator perform better under rational expectations for one-year and two-year ahead forecasts. The results for the short sample reveal that neither adaptive learning nor rational expectations are significantly better for any horizon or variable. The results for the UK and the Euro area are tabulated in Table 2.7. The mean absolute differentials for the UK and the Euro area are negative for the baseline model. This implies that absolute percentage errors have been lower using adaptive learning. Nevertheless, the difference between adaptive learning and rational expectations is not statistically significant. Neither adaptive learning nor rational expectations are preferable in all models to improve forecasts for all variables in the UK and the Euro area. The one-quarter ahead forecasts by the financial accelerator model as depicted in Figures 5 and 6 with adaptive learning and rational expectations always follows the actual development of output growth in the UK and the Euro area with a lag.. Therefore it is not possible to reduce the bias significantly by using adaptive learning rather than rational expectations.

2.5 Conclusion

Standard macroeconomic models have been heavily criticized after the financial crisis because they have not well predicted the great recession 2009. Main points of critique have been the rational expectation hypothesis and the absence of financial variables. In recent years, model alternatives which include financial variables, and which employ other expectation formation specifications than rational expectations have been developed. Empirical research has shown that these extensions can improve the in-sample fit of macroeconomic models. However, in general they do not substantially improve forecasting performance. The critique that standard macroeconomic models neglect the role of financial variables seems a plausible explanation for the poor performance during the last financial crisis. Including the financial accelerator to the Smets and Wouters model improve forecasts after the crisis for output growth. This improvement can not be explained by the inclusion of the external finance premium itself. The inclusion into an unrestricted vector autoregressive model does not improve the forecast performance of the model. But the inclusion of the variable with the restrictions on the coefficients improves the forecasting performance. Therefore, macroeconomists did not neglect an important variable but rather the role of financial markets for the real economy. This paper only considers the external finance premium as additional variable as in Merola (2015). Other papers like Christiano et al. (2014) use also stock returns as measure for net worth to estimate the Smets and Wouters model with financial frictions. Nevertheless, as stated by Del Negro et al. (2016) financial frictions are very helpful to predict the output contraction after the financial crisis but in normal times are not as important. The inclusion of the financial market is important, if we want to investigate potential spill overs to the real economy. A special limitation of micro founded models is the theory based selection of variables to estimate the model. To include additional variables to the model requires a theoretical foundation. This restriction is a drawback compared to models solely driven by data mining procedures. But this limitation allows to have specific explanations for the observed reactions. The use of rational expectations in DSGE models has been heavily criticized. An alternative are expectations formed by adaptive learning like in Slobodyan & Wouters (2012). This alternative framework does perform slightly better, but is not able to statistical significantly beat rational expectations with regard to forecasting output and inflation. Adaptive Learning implies time varying coefficients of the reduced form VAR. The forecasting performance of all models using the short sample have been better than using the full sample for the US. This implies that more distant data might be less informative to predict the present. A usual way to account for less informative data is using moving windows to estimate models or to assign data points further in the past a lower weight. Time varying reduced form parameters per se as introduced by adaptive learning in the full sample for the US do not necessarily improve the forecasting behaviour. The fundamental reason for the change in parameters is probably not alternating forecast models of economic agents. Macroeconomists should spend more attention on how to select appropriate estimation windows for their models. A major source of economic fluctuations are unpredictable structural shocks. These can be understood with the benefit of hindsight within macroeconomic models, and applied macroeconomic analysis should take the model extensions seriously. However, there is not a best unique model to predict the future. Perhaps the most important lesson is that applied macroeconomists should not rely on one single model but have several models in their toolkits. Macroeconometric models are summaries of the empirical behaviour of important macroeconomic time series. What should be included in the summary depends on the question at hand (including the forecasting horizon) and the specific economic conditions in the period and region under investigation. The pre-crisis standard New-Keynesian DSGE model still is an important tool in the toolkit and is still a good starting point for forecasts during normal times.

Table 2.5: Root mean squared percentage errors Euro area

Horizon	Output	Inflation	Log Determinant	Output	Inflation	Log Determinant
	Adaptive Learning			Rational Expectations		
Smets and Wouters without nominal rigidities						
1	0.92	0.24	0.72	0.89	0.22	0.72
4	0.49	0.42	0.62	0.80	0.40	0.89
8	1.27	0.49	0.76	2.73	0.46	1.09
Smets and Wouters without price rigidities						
1	0.58	0.24	0.51	0.64	0.22	0.54
4	0.44	0.43	0.63	0.77	0.39	0.87
8	1.17	0.48	0.78	2.89	0.45	1.09
Smets and Wouters without wage rigidities						
1	0.84	0.37	0.81	0.51	0.52	1.00
4	0.47	0.26	0.64	0.33	0.73	0.70
8	1.47	0.42	0.87	1.65	0.75	0.77
Smets and Wouters with nominal rigidities						
1	0.42	0.50	0.46	0.73	0.75	0.91
4	0.34	0.19	0.47	0.47	0.75	0.80
8	1.56	0.21	0.60	1.43	1.13	0.89
Smets and Woutres with financial accelerator						
1	0.50	0.57	0.81	0.48	0.76	0.71
4	0.68	0.53	0.86	0.34	0.54	0.86
8	1.55	0.79	1.16	1.17	1.16	1.36
Vector autoregressive model						
	without external finance premium			with external finance premium		
1	4.46	9.73	13.33	1.71	0.82	1.17
4	6.73	5.52	17.77	1.13	0.96	1.21
8	1.90	5.10	15.69	2.52	0.96	1.30

Notes: For the VAR without external finance premium the root mean squared percentage errors for the out-of-sample forecast errors for the respective horizons are reported. Root mean squared percentage errors relative to the VAR model without external finance premium are reported for the different models. Parameters are set to their posterior mode to compute the forecast errors. Posterior distributions of the parameters are estimated every year.

Table 2.6: Mean deviations between AL and RE absolute forecast percentage errors for the US

Horizon	US (full sample)			US (short sample)		
	Output	Inflation	All	Output	Inflation	All
	Smets and Wouters without nominal rigidities					
1	-13.99 (0.07)	-0.52 (0.77)	-12.07 (0.01)	0.14 (0.97)	-0.01 (1.00)	0.06 (0.93)
4	-0.98 (0.76)	-1.20 (0.67)	-1.40 (0.34)	0.49 (0.89)	-0.01 (1.00)	0.43 (0.52)
8	0.32 (0.94)	1.52 (0.50)	0.17 (0.94)	-0.54 (0.89)	-0.01 (1.00)	0.59 (0.48)
	Smets and Wouters without price rigidities					
1	0.48 (0.73)	0.05 (0.98)	-0.06 (0.88)	-0.50 (0.87)	-0.00 (1.00)	-0.29 (0.57)
4	0.42 (0.34)	0.04 (0.99)	0.21 (0.76)	-1.60 (0.67)	-0.00 (1.00)	-0.65 (0.32)
8	-0.41 (0.17)	0.08 (0.97)	0.57 (0.62)	-1.67 (0.68)	-0.00 (1.00)	-0.87 (0.29)
	Smets and Wouters without wage rigidities					
1	-27.53 (0.03)	0.41 (0.50)	-17.31 (0.00)	-0.38 (0.91)	0.17 (0.79)	-0.12 (0.88)
4	-1.01 (0.80)	3.61 (0.33)	0.71 (0.52)	-1.11 (0.80)	0.23 (0.85)	-0.19 (0.82)
8	1.80 (0.81)	1.83 (0.44)	3.21 (0.13)	-1.12 (0.80)	0.25 (0.87)	-0.03 (0.97)
	Smets and Wouters with nominal rigidities					
1	1.89 (0.48)	-0.08 (0.86)	0.15 (0.75)	-0.71 (0.82)	0.11 (0.87)	-0.35 (0.48)
4	1.16 (0.65)	0.61 (0.36)	1.32 (0.04)	-2.91 (0.47)	-0.04 (0.98)	-1.02 (0.13)
8	1.67 (0.37)	0.25 (0.79)	3.13 (0.00)	-2.08 (0.63)	-0.08 (0.95)	-1.23 (0.13)
	Smets and Woutres with financial accelerator					
1	-0.47 (0.72)	1.39 (0.09)	0.51 (0.12)	-0.49 (0.71)	-0.21 (0.72)	0.01 (0.96)
4	4.90 (0.18)	0.51 (0.44)	1.45 (0.02)	0.03 (0.98)	-0.57 (0.53)	-0.07 (0.86)
8	6.15 (0.36)	2.21 (0.04)	3.67 (0.00)	-1.57 (0.54)	1.68 (0.36)	-0.35 (0.60)

Note: Values in parentheses denote p-values of two samples t-tests for zero mean of absolute forecast differentials. *Notes*: Values in parentheses denote p-values of two samples t-tests for zero mean of absolute forecast differentials.

Table 2.7: Mean deviations between AL and RE absolute forecast percentage errors for the UK and the Euro area

Horizon	UK			Euro area		
	Output	Inflation	All	Output	Inflation	All
	Smets and Wouters without nominal rigidities					
1	-1.30 (0.32)	0.02 (0.97)	26.85 (0.86)	-0.38 (0.63)	0.05 (0.90)	-0.05 (0.86)
4	-0.90 (0.45)	0.02 (0.97)	2.67 (0.90)	-0.91 (0.30)	0.05 (0.91)	-0.69 (0.12)
8	-0.34 (0.74)	0.01 (0.98)	-16.54 (0.65)	-1.09 (0.19)	0.05 (0.92)	-0.87 (0.06)
	Smets and Wouters without price rigidities					
1	-1.87 (0.24)	0.04 (0.93)	-32.40 (0.67)	-0.28 (0.57)	0.05 (0.89)	-0.04 (0.88)
4	-1.82 (0.10)	0.03 (0.94)	-28.06 (0.57)	-0.93 (0.27)	0.05 (0.89)	-0.63 (0.14)
8	-1.07 (0.28)	0.03 (0.95)	-50.76 (0.39)	-1.26 (0.14)	0.06 (0.90)	-0.86 (0.07)
	Smets and Wouters without wage rigidities					
1	-1.25 (0.37)	-1.55 (0.33)	37.18 (0.80)	0.52 (0.37)	-0.61 (0.47)	-0.28 (0.49)
4	-1.07 (0.48)	-1.01 (0.15)	6.87 (0.63)	0.47 (0.36)	-0.76 (0.18)	0.13 (0.79)
8	0.41 (0.71)	-0.97 (0.27)	0.79 (0.97)	0.05 (0.93)	-0.54 (0.38)	0.09 (0.79)
	Smets and Wouters with nominal rigidities					
1	-1.95 (0.22)	0.28 (0.81)	-37.21 (0.56)	-0.21 (0.68)	-0.59 (0.63)	-0.42 (0.20)
4	-2.01 (0.14)	-1.65 (0.48)	5.73 (0.86)	-0.28 (0.59)	-0.83 (0.15)	-0.50 (0.15)
8	-0.22 (0.80)	-1.18 (0.35)	9.87 (0.56)	0.13 (0.82)	-1.20 (0.17)	-0.40 (0.21)
	Smets and Woutres with financial accelerator					
1	0.50 (0.64)	-0.67 (0.59)	-35.55 (0.39)	-0.07 (0.85)	-0.12 (0.93)	-0.02 (0.95)
4	-2.17 (0.14)	0.40 (0.70)	-4.68 (0.74)	0.60 (0.39)	0.29 (0.59)	0.08 (0.78)
8	0.17 (0.81)	1.80 (0.10)	-0.45 (0.97)	0.33 (0.51)	0.09 (0.93)	-0.21 (0.61)

Notes: Values in parentheses denote p-values of two samples t-tests for zero mean of absolute forecast differentials.

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2.6 Appendix

2.6.1 Tables

Table 2.8: Prior distributions

	Prior distribution	Prior mean	Prior s.d.
ρ^a	Beta Distribution	0.500	0.2000
ρ^b	Beta Distribution	0.500	0.2000
ρ^g	Beta Distribution	0.500	0.2000
ρ^{q^s}	Beta Distribution	0.500	0.2000
ρ^{m^s}	Beta Distribution	0.500	0.2000
ρ	Normal Distribution	4.000	1.5000
σ	Normal Distribution	1.500	0.3750
η	Beta Distribution	0.700	0.1000
ξ^w	Beta Distribution	0.500	0.1000
ξ^p	Beta Distribution	0.500	0.1000
ι^w	Beta Distribution	0.500	0.1500
ι^p	Beta Distribution	0.500	0.1500
ψ	Beta Distribution	0.500	0.1500
Θ^p	Normal Distribution	1.250	0.1250
κ^π	Normal Distribution	1.500	0.2500
κ^r	Beta Distribution	0.750	0.1000
cry	Normal Distribution	0.125	0.0500
$\kappa^{\Delta y}$	Normal Distribution	0.125	0.0500
$\bar{\pi}100$	Gamma Distribution	$\frac{1}{T}\sum_t dl\bar{P}_t$	0.1000
$100(\dot{\frac{1}{\beta}} - 1)$	Gamma Distribution	0.250	0.1000
$\bar{l}$	Normal Distribution	0.000	2.0000
γ	Normal Distribution	$\frac{1}{T}\sum_t dlGDP_t$	0.1000
κ^{ϵ^a}	Normal Distribution	0.500	0.2500
α	Normal Distribution	0.300	0.0500
σ^l	Normal Distribution	2.000	0.5000
ρ^π	Beta Distribution	0.500	0.2000
ρ^w	Beta Distribution	0.500	0.2000
ρ^{ϵ^w}	Beta Distribution	0.500	0.2000
ρ^{ϵ^p}	Beta Distribution	0.500	0.2000
ρ	Beta Distribution	0.500	0.289
	Standard Errors of Shocks		
ϵ^a	Inverse Gamma Distribution	0.100	2.0000

(Continued on next page)

(continued)

	Prior distribution	Prior mean	Prior s.d.
ϵ^b	Inverse Gamma Distribution	0.100	2.0000
ϵ^g	Inverse Gamma Distribution	0.100	2.0000
ϵ^q	Inverse Gamma Distribution	0.100	2.0000
ϵ^m	Inverse Gamma Distribution	0.100	2.0000
ϵ^π	Inverse Gamma Distribution	0.100	2.0000
ϵ^w	Inverse Gamma Distribution	0.100	2.0000
	Financial Accelerator		
lev	Normal Distribution	1.7000, 0.2000	
ω	Normal Distribution	0.0100	0.5000
	Adaptive Learning		
ρ	Beta Distribution	0.500	0.2887

Table 2.9: Symbols and notation for the model

Variable	**Description**
	Endogenous
L^{obs}	hours worked observational variable
r^{obs}	policy rate observational variable
π^{obs}	inflation observational variable
Δy	output growth observational variable
Δc	consumption growth observational variable
Δi	investment growth observational variable
Δw	wage growth observational variable
ϵ^w	residual for wage equation
ϵ^π	residual for inflation equation
$\hat{mc}$	marginal cost
$\hat{u}$	capital utilization rate
$\hat{r}^k$	rental rate on capital services
$\hat{k}$	capital
$\hat{Q}^k$	price of capital
$\hat{c}$	consumption
$\hat{i}$	investment
$\hat{y}$	output
$\hat{l}$	hours worked
$\hat{pi}$	inflation
$\hat{w}$	wages

Symbol	Description
$\hat{r}$	risk free interest rate
$\hat{f}$	external finance cost
$\hat{n}$	net worth
$pr\hat{e}m$	external finance premium
a	shock on productivity
b	shock on demand
g	shock in government expenditure
q^s	shock to capital productivity
m^s	monetary policy shock
ϵ^p	cost price push shock
ϵ^w	cost wage push shock
$\hat{k}^p$	capital services
$l\hat{e}v$	leverage ratio
Parameters	
ω	elasticity of external risk premium
ζ^w	Kimball parameter in final goods aggregation function
ζ^p	Kimball parameter in labour market aggregation function
α	capital intensity in production function
ψ	reaction of capital utilization rate to rental rate r^k
β	discount parameter
ρ	adjustment cost parameter
δ	depreciation rate
σ	inverse elasticity of intertemporal substitution
h	habit parameter
Θ^p	fixed costs parameter
ι^w	wage indexation
ξ^w	Calvo parameter for wages
ι^p	price indexation
ξ^p	Calvo parameter prices
σ^l	elasticity of labor supply to real wage
Λ^w	wage mark up
κ^π	monetary policy parameter for inflation
$\kappa^{\Delta y}i$	monetary policy parameter for output growth
κ^r	persistency in policy rate
ρ^a	AR 1 coefficient for technology shock
ρ^b	AR 1 coefficient for demand shock
ρ^g	AR 1 coefficient for government expenditure
ρ^{q^s}	AR 1 coefficient for investment shocks
ρ^{m^s}	AR 1 coefficient for money suplply
γ	trend in the economy
Λ^p	Kimball parameter in final goods aggregation function
$\frac{r^k k}{y}$	steady state share spend for capital services to output

Symbol	Description
$\frac{1}{\lambda^w}\frac{1-\alpha}{\alpha r^k \frac{k}{c}}$	Kimball parameter in final goods aggregation function
$\frac{wl}{y}$	steady state share of wage bill to output
ρ	gain parameter for adaptive learning
ρ^{π}	AR 1 for inflation
ρ^{w}	AR 1for wages
ρ^{ϵ^w}	MA 1 term for price shock
ρ^{ϵ^p}	MA 1 for wage shock

2.6.2 Baseline model

The model includes 14 endogenous variables. As can be seen in equations (2.18), (2.12), (2.15), (2.24) and (2.27), expectations with respect to future realisations of variables have to be formed. For example, the consumption Euler equation (2.18) requires agents to form expectations about future price levels $\hat{\pi}_{[t+1]}$. All in all, the setup requires agents to form beliefs about seven forward-looking variables $\mathbf{y}_t^f$. The standard approach is to assume RE.

In the following the log-linearized model equations are presented as stated in Slobodyan & Wouters (2012). A complete list of symbols is reported in Table 2.9.

Optimal marginal cost:

$$\hat{mc}_t = \alpha\, \hat{r}_t^k + (1-\alpha)\ \hat{w}_t - a_t. \tag{2.8}$$

Optimal capital utilization rate:

$$\hat{u}_t = \hat{r}_t^k \, \frac{1}{\frac{\psi}{1-\psi}}. \tag{2.9}$$

Return on capital:

$$\hat{r}_t^k = \hat{w}_t + \hat{l}_t - \hat{k}_t. \tag{2.10}$$

Capital utilization:

$$\hat{k}_t = \hat{u}_t + \hat{k}_{t-1}^p. \tag{2.11}$$

Investment Euler equation:

$$\hat{i}_t = i_1\, \hat{i}_{t-1} + (1-i_1)\, \hat{i}_{t+1}^e + i_2\, \hat{p}_t^k + q_t^s, \tag{2.12}$$

$$i_1 = \frac{1}{1+\beta\gamma^{1-\sigma}\,\gamma}, \tag{2.13}$$

$$i_2 = \frac{i_1}{\gamma^2\rho}. \tag{2.14}$$

Optimal price of capital:

$$\hat{p}^k_t = (-\hat{r}_t) + \hat{\pi}^e_{t+1} + b_t\, q_1 + q_2\, \hat{r}^k_{t+1} + (1-q_2)\, \hat{p}^{k,e}_{t+1}, \tag{2.15}$$

$$q_1 = \frac{\sigma\left(1+\frac{\eta}{\gamma}\right)}{1-\frac{\eta}{\gamma}}, \tag{2.16}$$

$$q_2 = \frac{\bar{r}^k}{\bar{r}^k + (1-\delta)}. \tag{2.17}$$

Consumption Euler equation:

$$\hat{c}_t = b_t + (1-c_1)\,\hat{c}_{t-1} + c_1\,\hat{c}^e_{t+1} + c_2\,\left(\hat{l}_t - \hat{l}^e_{t+1}\right) - c_3\,\left(\hat{r}_t - \hat{\pi}^e_{t+1}\right), \tag{2.18}$$

$$c_1 = \frac{1}{1+\frac{\eta}{\gamma}}, \tag{2.19}$$

$$c_2 = \frac{(\sigma-1)\,\frac{\bar{w}\,\bar{L}}{\bar{c}}}{\sigma}\, c_1, \tag{2.20}$$

$$c_3 = \frac{c_1\,(1-\frac{\eta}{\gamma})}{\sigma}. \tag{2.21}$$

Resource constraint:

$$\hat{y}_t = \hat{c}_t\,\frac{\bar{c}}{\bar{y}} + \hat{i}_t\,\frac{\bar{i}}{\bar{y}} + g_t + \hat{u}_t\,\frac{r^k k}{y}. \tag{2.22}$$

Production function:

$$\hat{y}_t = \Theta^p\,\left(a_t + \alpha\,\hat{k}_t + (1-\alpha)\,\hat{l}_t\right). \tag{2.23}$$

New Keynesian Phillips curve for prices:

$$\hat{\pi}_t - \iota^p\,\hat{\pi}_{t-1} = \pi_1(\,\hat{\pi}^e_{t+1} - \iota^p\hat{\pi}_t) - \pi_2\,\hat{mc}^{-1}_t + \epsilon^p_t, \tag{2.24}$$

$$\pi_1 = \beta\gamma^{1-\sigma}\gamma, \tag{2.25}$$

$$\pi_2 = \frac{(1-\xi^p\pi_1)\,(1-\xi^p)}{\xi^p\,(1+(\Theta^p-1))\,\zeta^p}. \tag{2.26}$$

New Keynesian Phillips curve for wages:

$$\hat{\pi}^w_t - \iota^w \hat{\pi}^w_{t-1} = \pi_1(\hat{\pi}^{w,e}_{t+1} - \iota^w \hat{\pi}^w_t) - \pi_3 \hat{\mu}^w_t + \epsilon^w_t, \tag{2.27}$$

$$\pi_3 = \frac{(1-\xi^w \pi_1)(1-\xi^w)}{\xi^w (1+(\phi^w - 1))\zeta^w}, \tag{2.28}$$

$$\hat{\mu}^w_t = \hat{w}_t - w_1 \hat{c}_t + (1-w_1)\hat{c}_{t-1} - \sigma^l \hat{L}_t, \tag{2.29}$$

$$w_1 = \frac{1}{1-\frac{\eta}{\sigma}}. \tag{2.30}$$

Monetary policy rule:

$$\hat{r}_t = \hat{\pi}_t \kappa^\pi (1-\kappa^r) + (1-\kappa^r) \kappa^{\Delta y} (\hat{y}_t - a_t \Theta^p) + \kappa^{\Delta y} i (\Delta\hat{y}_t - \Theta^p \Delta a_t) + \kappa^r \hat{r}_{t-1} + m^s_t. \tag{2.31}$$

Law of motion for capital:

$$\hat{k}^p_t = k_1 \hat{k}^p_{t-1} + (1-k_1)\hat{i}_t + k_2 q^s_t, \tag{2.32}$$

$$k_1 = 1 - \frac{\bar{i}}{\bar{k}}, \tag{2.33}$$

$$k_2 = \frac{\bar{i}}{\bar{k}} \gamma^2 (1 + \beta \gamma^{1-\sigma} \gamma). \tag{2.34}$$

2.6.2.1 Shocks

All shocks are assumed to follow an AR(1) process.

Temporary technology shock:

$$a_t = a_{t-1} \rho^a + \epsilon^a_t. \tag{2.35}$$

Risk premium shock:

$$b_t = \rho^b b_{t-1} + \epsilon^b_t. \tag{2.36}$$

Government expenditure shock:

$$g_t = \rho^g g_{t-1} + \epsilon^g_t + \epsilon^a_t \kappa^{\epsilon^a}. \tag{2.37}$$

Capital utilization shock:

$$q^s_t = \rho^{q^s} q^s_{t-1} + \epsilon^q_t. \tag{2.38}$$

Monetary policy shock:

$$m^s_t = \rho^{m^s} m^s_{t-1} + \epsilon^m_t. \tag{2.39}$$

2.6.3 Model without nominal rigidities

The exclusion of nominal rigidities will alternate the og-linearized version of the model. The exclusion of price rigidities leads to a Calvo parameter for prices ξ^p to zero. In the log-linearized version this would lead to indeterminacy of the steady-state because eq. (2.24) is not defined. The nonlinear version of the model shows that prices are set with a markup over marginal costs. We replace (2.24) by

$$\hat{\pi}_t = \epsilon_t^p. \tag{2.40}$$

This illustrates that a model without price stickiness is not informative about deviations from trend inflation.

The exclusion of wage rigidities leads to a Calvo parameter for wages ξ^w equal to zero. In the log-linearized version this would lead to indeterminacy of the steady-state because eq. (2.27) is not defined. The nonlinear version of the model shows that wages are set with a markup over the marginal rate of substitution. We replace (2.27) by

$$\hat{w}_t = w_1\, \hat{c}_t + (1 - w_1)\hat{c}_{t-1} - \sigma^l \hat{L}_t + \epsilon_t^w, \tag{2.41}$$

$$w_1 = \frac{1}{1 - \frac{\eta}{\sigma}}. \tag{2.42}$$

Here the shock ϵ_t^w captures movements in the markup of wages.

2.6.4 Model with financial accelerator

Here only the equations which are modified by inclusion of the financial accelerator or added are reported. The notation and equations follows the work by Merola (2015).

Resource constraint:

$$\hat{y}_t = \hat{c}_t\, \frac{\bar{c}}{\bar{y}} + \hat{i}_t\, \frac{\bar{i}}{\bar{y}} + g_t + \hat{u}_t\, \frac{r^k k}{y} + \frac{k}{y} f(1 - \frac{r}{f})(1 - \frac{1}{lev})(f_t + \hat{p}_{t-1}^k + \hat{k}_t). \tag{2.43}$$

Optimal price of capital:

$$\hat{p}_t^k = -\left(\hat{f}_t + b_t\right) + \frac{r^k}{r^k + (1-\delta)} r_{t+1}^{k,e} + \frac{1-\delta}{r^k + (1-\delta)} \hat{p}_{t+1}^{k,e} \tag{2.44}$$

External finance costs:

$$\hat{f}_{t+1}^e = \hat{r}_t - \hat{\pi}_{t+1}^e + \omega\left(\hat{p}_{t+1}^k + \hat{k}_{t+1} - \hat{n}_t\right) \tag{2.45}$$

External finance premium:

$$pr\hat{e}m_t = \hat{f}_{t+1}^e - \hat{r}_t - \hat{\pi}_{t+1}^e = \omega\left(\hat{p}_{t+1}^k + \hat{k}_{t+1} - \hat{n}_t\right) \tag{2.46}$$

Law of motion for net worth:

$$\frac{1}{v\,f}\hat{n}_t = \bar{lev}\hat{f}_t - \omega\left(\bar{lev}-1\right)\left(\hat{p}^k_{t-1}+\hat{k}_t\right) - \left(\bar{lev}-1\right)\left(\hat{r}_{t-1}-\hat{\pi}_t\right) + \left[\omega(lev-1)+1\right]\,\hat{n}_{t-1} \tag{2.47}$$

Leverage ratio:

$$\hat{lev}_t = \hat{p}^k_t + \hat{k}_t - \hat{n}_{t-1} \tag{2.48}$$

2.6.5 Adaptive learning

Slobodyan & Wouters (2012) assume that agents forming beliefs about future outcomes by applying forecasting models, relying on autoregressive models of order two. They refer to this approach by adaptive learning. The forecasting model is expressed by

$$\underbrace{\begin{pmatrix} y^f_{1t} \\ y^f_{2t} \\ \vdots \\ y^f_{mt} \end{pmatrix}}_{\mathbf{y}^f_t} = \underbrace{\begin{pmatrix} \mathbf{X}_{1,t-1} & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{X}_{2,t-1} & \cdots & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{X}_{m,t-1} \end{pmatrix}}_{\mathbf{X}_{t-1}} \underbrace{\begin{pmatrix} \beta_{1t} \\ \beta_{2t} \\ \vdots \\ \beta_{mt} \end{pmatrix}}_{\beta_t} + \underbrace{\begin{pmatrix} u_{1t} \\ u_{2t} \\ \vdots \\ u_{mt} \end{pmatrix}}_{\mathbf{u}_t} . \tag{2.49}$$

In the notation of state-space models, equation (2.49) is the space equation. The coefficient vector of the forecasting model β is described by a VAR process. It constitutes the state equation. To estimate the forecasting parameters in β, agents rely on the Kalman filter. The coefficient vector β is updated at each point in time. Thus, agents do not use a static setup to form beliefs, but follow a flexible forecasting approach that adapts to the most recent data available. This is also the main difference between adaptive learning and rational expectations. Like this, Slobodyan & Wouters (2012) consider agents that apply Bayesian learning, i.e. AL when forming beliefs about relevant variables, rather than assuming rational expectations. The idea of AL seems to be intuitively more convincing, as less knowledge is demanded by agents. Specifically, agents are not required to rely on or even know the model structure when forming expectations.

In total, the Kalman filter is used by Slobodyan & Wouters (2012) at two points. First, it is used on the level of agents for learning (inner state-space system, as described above). Second, it is also applied for the estimation of the model (outer state-space system) using real data and a maximum likelihood approach. The state equation of the outer system is the perceived law of motion and the space is represented by the observation equation that will later be introduced in equation (2.55).

For the understanding of this approach, this distinction is of crucial importance. The parameter vector β is not estimated via maximum likelihood at any stage – neither within the inner system, nor within the outer system. It is determined by state estimation – which is the original purpose of the Kalman filter framework – within the inner state-space system. Increasing the number of lags that are represented in the state equation of the state estimation of the inner system must therefore not be confused with an increase of the lags of the equation which is the basis for the likelihood evaluation.

2.6.5.1 Inner state-space model

For the inner state space system the beliefs β_t are the state and evolve as follows[14]:

$$\begin{aligned}
\beta_{t|t} =& \beta_{t|t} + \mathbf{P}_{t|t-1}\mathbf{X}_{t-1}[\mathbf{\Sigma} + \mathbf{X}_{t-1}^{\top}\mathbf{P}_{t|t-1}\mathbf{X}_{t-1}]^{-1}(\mathbf{y}_t^f - \mathbf{X}_{t-1}^{\top}\,\beta_{t|t-1}), && (2.50)\\
\beta_{t+1|t} - \bar{\beta} =& \mathbf{F}(\beta_{t|t} - \bar{\beta}), && (2.51)\\
\mathbf{P}_{t|t} =& \mathbf{P}_{t|t-1} + \mathbf{P}_{t|t-1}\mathbf{X}_{t-1}[\Sigma + \mathbf{X}_{t-1}^{\top}\mathbf{P}_{t|t-1}\mathbf{X}_{t-1}]^{-1}\,\mathbf{X}_{t-1}^{\top}\mathbf{P}_{t|t-1}, && (2.52)\\
\mathbf{P}_{t+1|t} =& \mathbf{F}\,\mathbf{P}_{t|t}\mathbf{F}^{\top} + \mathbf{V}, && (2.53)
\end{aligned}$$

where $\mathbf{V}$ is the covariance matrix of the shocks $\mathbf{v}_t$, $\mathbf{\Sigma}$ the covariance matrix of prediction errors and $\mathbf{P}_{t|t-1}$ is the mean square prediction error. Note that the speed of learning is determined by the diagonal matrix F with main diagonal element ρ. This approach is the baseline scenario in Slobodyan & Wouters (2012) and is the inner Kalman filter for the forecast model. The beliefs β_t are updated according to (2.50). We just replace $\mathbf{x}$ by β, $\mathbf{z}$ by $\mathbf{y}^f$, $\mathbf{H}$ by $\mathbf{X}$, $\mathbf{R}$ by $\mathbf{\Sigma}$ and use that the Kalman-gain is $\mathbf{K}_t = \mathbf{P}_{t|t-1}\mathbf{X}_{t-1}[\mathbf{\Sigma}+\mathbf{X}_{t-1}^{\top}\mathbf{P}_{t|t-1}\mathbf{X}_{t-1}]^{-1}$.

2.6.5.2 Estimation

Under AL the state-space representation of the model is slightly different as under RE. Our outer state-space system used to estimate the structural parameters of the model takes the form

$$\xi_t = \mu_t + \mathbf{T}_t\xi_{t-1} + \mathbf{R}_t\varepsilon_t\ , \tag{2.54}$$

$$\begin{pmatrix} \mathbf{d}lGDP_t \\ \mathbf{d}lCons_t \\ \mathbf{d}lINV_t \\ \mathbf{d}lWag_t \\ \mathbf{l}HOURS_t \\ \mathbf{d}lP_t \\ \mathbf{R}_t^{MM} \end{pmatrix} = \begin{pmatrix} \gamma \\ \gamma \\ \gamma \\ \gamma \\ \bar{L} \\ \bar{\pi} \\ \bar{R} \end{pmatrix} + \begin{pmatrix} d\hat{y}_t \\ d\hat{c}_t \\ d\hat{i}_t \\ d\hat{w}_t \\ \hat{L}_t \\ \hat{\pi}_t \\ \hat{r}_t \end{pmatrix}. \tag{2.55}$$

The state vector ξ_t contains all endogenous variables as well as lagged exogenous shocks and exogenous variables. In contrast to RE, not only elements of μ_t associated with observational equations are possibly nonzero but also all other endogenous variables might have nonzero coefficients. Furthermore, the transition matrix $\mathbf{T}_t$ is now time varying, because it is a nonlinear function of the time varying state of the inner state space system for the

[14] Different from the previous section, time and iteration step are both indexed by t.

believes β_t. In the literature, equation (2.54) is also called the actual law of motion derived from the perceived law of motion and the model equations.

We estimate the model for the United States of America, the Euro area and the United Kingdom . The same priors for the structural parameters of the model (see Table 2.8) are applied as in Slobodyan & Wouters (2012), except for the trend parameters γ and $\bar{\pi}$ denoting the constants in the observational equations (2.55) which are sample specific. The estimation approach for the deep parameters of the model follows the work by Chang et al. (2002).

Therefore, for each country the prior mean for the trend coefficients is the estimated sample average. Other parameters remain unchanged and are identical to Slobodyan & Wouters (2012).

To run the estimation in Dynare, we use the code provided by Slobodyan & Wouters (2012) in Dynare 3.6.5. For $\beta_{t|t-1}$, $\mathbf{P}_{t|t-1}$, $\mathbf{\Sigma}$ and $\mathbf{V}$ we use RE equilibrium consistent believes. Those are derived by using the implied theoretical moments of the explanatory variables used to predict the forward-looking variables. One can simply compute the initial believes as follows:

$$\beta_{1|0} = E\{(\mathbf{X}^\top \mathbf{X})^{-1}\}^{-1} E(\mathbf{X}^\top \mathbf{Y}), \tag{2.56}$$

$$\mathbf{\Sigma} = E(\mathbf{y}_t^f - \mathbf{X}_{t-1}^\top \beta_{1|0})(\mathbf{y}_t^f - \mathbf{X}_{t-1}^\top \beta_{1|0})^\top, \tag{2.57}$$

$$\mathbf{P}_{1|0} = \sigma_0(\mathbf{X}^\top \mathbf{\Sigma} \mathbf{X})^{-1}, \tag{2.58}$$

$$\mathbf{V} = \sigma_v(\mathbf{X}_{\mathbf{\Sigma}}^\top \mathbf{X})^{-1}, \tag{2.59}$$

Chapter 3

Power Generation and Structural Change: Quantifying Economic Effects of the Coal Phase-Out in Germany* †

Abstract

In the fight against global warming, the reduction of greenhouse gas emissions is a major objective. In particular, a decrease in electricity generation by coal could contribute to reducing CO_2 emissions. We study potential economic consequences of a coal phase-out in Germany, using a multi-region dynamic general equilibrium model. Four regional phase-out scenarios before the end of 2040 are simulated. We find that the worst case phase-out scenario would lead to an increase in the aggregate unemployment rate by about 0.13 [0.09 minimum; 0.18 maximum] percentage points from 2020 to 2040. The effect on regional unemployment rates varies between 0.18 [0.13; 0.22] and 1.07 [1.00; 1.13] percentage points in the lignite regions. A faster coal phase-out can lead to a faster recovery. The coal phase-out leads to migration from German lignite regions to German non-lignite regions and reduces the labour force in the lignite regions by 10,100 [6,300; 12,300] people by 2040. A coal phase-out until 2035 is not worse in terms of welfare, consumption and employment compared to a coal-exit until 2040.

Keywords: Dynamic General Equilibrium Model, Labour Market Friction, Energy, Structural Change
JEL: E17, O11, O21, O44, Q28

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3.1 Introduction

Economic growth and development are accompanied by structural change. Technological progress, international competition and shifting preferences, for example, affect the industry structure and the regional distribution of economic activities. An important source of structural change is the decarbonization of the economy that many countries are trying to achieve. According to the Paris Agreement 2015, greenhouse gas emissions, which are a major driver of global warming, need to be reduced strongly in order to prevent the global average temperature to increase further. Germany aims to reduce its greenhouse gas emissions by 40% by 2020 and by 55% by 2030 compared to 1990 (Figure 3.5 in the Appendix). A possible way to achieve these targets is to stop producing electricity from lignite. Among the electricity-producing technologies, burning lignite is the one that generates the most CO_2 emissions per unit of electricity. In 2014, the lignite industry accounted for 25% of the total electricity generation but for 50% of CO_2 emissions in the electricity sector in Germany (Icha 2013) and employed about 21,000 persons (0.05% of all employees).

The German government plans to shut down lignite coal power plants by 2038. The coal phase-out will trigger two types of structural change: First, other energy sources will replace lignite to produce electricity. New technologies and industries will develop while the lignite coal power plants will disappear. Second, since lignite coal industries are regionally concentrated, regional effects of the coal phase-out vary across the country. In the lignite regions, employment may decline, unemployment may be elevated during the process of structural change and average labour income may decrease because salaries in the lignite industry are above average. Other regions will be affected through production linkages as well as income and price effects. Overall, the lignite coal phase-out will trigger or amplify structural change in terms of both sectoral composition and regional distribution. In order to achieve a broad consensus about the coal-phase out throughout Germany, the federal government inaugurated a commission on growth, structural change and employment to develop a plan for the stepwise reduction of electricity generation by lignite.[1] The political decision process needs to be informed about the sectoral and regional consequences of various phase-out pathways. Existing studies investigating the potential economic consequences of a coal phase-out in Germany have assessed the current economic situation of the lignite regions using descriptive statistics (e.g. Markwardt & Zundel 2017). Others have focused on the consequences for energy markets (see, e.g. Heinrichs & Markewitz 2015). Studies quantifying the potential employment effects have used static input-output models (see Frondel et al. 2018). Welsch (1998) investigates the potential economic effects of a hard-coal and nuclear phase-out for Germany on the national level with a dynamic general equilibrium model. However, many important aspects as labour market frictions, migration and regional distribution have been neglected in these studies. We contribute to the literature by incorporating these aspects into a dynamic general equilibrium model with multiple sectors and multiple regions. Im-

[1] See Federal Ministry for Economic Affairs and Energy:

`https://www.bmwi.de/Redaktion/DE/Downloads/E/einsetzung-der-kommission-wachstum-strukturwandel-beschaeftigung.pdf?__blob=publicationFile` portant features of the model are an imperfect labour market (hiring costs like in Blanchard & Galí (2010)), market power, trade and migration between lignite and non-lignite regions and fiscal transfers among regions. We use the model to assess the economic effects of various coal phase-out pathways that differ with respect to regional timing and speed of power plant shutdowns. First, we specify a Null-Scenario in which the share of lignite in electricity production is constant but the population is decreasing due to demographic change. Second, we define a baseline scenario, in which the political measures implemented before 2015 reduce the share of lignite in electricity production to about 48%. This already contributes to structural change but is by far not sufficient to achieve the emission targets. We draw model parameters from specific probability distributions in order to account for the uncertainty about the exact structure of the economy. Due to the already implemented political measures to reduce electricity generation by lignite in Germany, employment will drop by 4,500 to 18,000 persons until 2035 and the unemployment rate will increase by 0.01 to 0.04 percentage points. Then, we model coal phase-out scenarios in which the emission targets are actually met. The decline in employment may amount to 74,800 persons and the unemployment rate might increase by up to 0.18 percentage points, depending on the specific decommissioning plan. Regional employment effects differ depending on the regional importance of the lignite industry. Absolute effects will be largest in Rhineland and relative effects will be largest in Lusatia. We show in detail how the effects depend on the persistence in unemployment benefits and wages, preferences for local production, and the magnitude and persistence of market power. We also assess the welfare effects of the decommissioning plans currently under consideration. It turns out that none of these plans clearly dominates the others. The paper is structured as follows: Section 3.2 reports the current economic profiles of the lignite regions and describes the phase-out scenarios. Section 3.3 explains the multi-sector multi-region dynamic general equilibrium model. The calibration of the model is described in Section 3.4. The results and a sensitivity analysis are presented in Section 4.4. Section 4.6 summarizes the main results of the paper.

3.2 The Lignite Industry in Germany

3.2.1 Status quo

Lignite industries are located in four German regions: Central Germany, Lusatia, the Rhineland, and Helmstedt. In the smallest of these territories, Helmstedt, lignite has no longer been extracted since 2016. However, renaturation activities in Helmstedt have been employing some people since 2016. The lignite regions can be defined in various ways. For the economic analysis of regional structural change, labour market regions are a reasonable regional unit. labour market regions consist of several counties with intensive commuting flows (Kosfeld & Werner 2012) implying that the majority of workers in a region are living in the same region. First, we cluster counties into lignite and non-lignite territories by sorting all counties with an active lignite mine or power plant with an installed capacity of at least 50 MW into one of the three active lignite territories Central Germany, Lusatia, or Rhineland. Table 3.5 in the Appendix tabulates the identified territories. We then define lignite regions as labour market regions which include at least one county belonging to a lignite territory. Overall, we consider four regions: three lignite labour market regions and the rest of Germany. Table 3.1 reports the employment shares in 2014 for each region and sector. In 2014, roughly 0.05% of the workforce worked in the lignite industry. The Lusatia region, located in East Germany, has the highest employment share in the lignite sector, and Central Germany the lowest. In the rest of Germany, only about 500 people are employed in the lignite sector. In all lignite regions, unemployment rates are above the average national level. Gross

Table 3.1: Employment shares

Region	Energy		Non-Energy	Unemployment Rate	Total
	Lignite Coal	Non-Lignite Coal			
Rest of Germany	0.001%	0.64%	94.03%	5.32%	100%
Central Germany	0.15%	0.67%	90.01%	9.17%	100%
Lusatia	1.54%	0.56%	86.92%	10.97%	100%
Rhineland	0.31%	0.74%	91.61%	7.34%	100%
Germany	0.05%	0.65%	93.64%	5.67%	100%

Note: Employment shares by region and sector in 2014.
Sources: German Federal Statistical Office, German Federal Agency for Employment and own calculations.

value-added shares are similar to employment shares, see Table 3.6 in the Appendix. More important is the role of the lignite industry as a high wage paying regional employer. Wages are retrieved from balance sheet data of the three major companies operating the lignite mines and power plants.[2] Wages are high compared to other sectors. The average annual compensation (including social security contributions) in Lusatia for a worker in 2014 was about 26,500 euro. In contrast, the average annual compensation in Lusatia for a worker in the lignite sector in 2014 was about 66,000 euro. Labour shares are reported in Table 3.7 in the Appendix.

3.2.2 Phase-out paths

We start with a Null-Scenario in which the share of lignite in total electricity production stays constant but the expected demographic change is taken into account. According to official projections, the labour force will shrink by 3.5 million people by 2040 (Figure 3.6). Employment will decrease because more old employees will be retired in the years to come than young employees enter the labour market. Given the large regional variation in demographic dynamics it is important to isolate the employment effects triggered by the coal phase-out

[2] Balance sheets for RWE Power AG, Vattenfall Mining and Generation, and MIBRAG are provided at `https://www.unternehmensregister.de/ureg/`

to the ongoing regional demographics without coal phase-out. Some specific measures to reduce the share of lignite in electricity production have already been decided. These measures constitute our baseline scenario, see Table 3.3. They are described in Bundesregierung (2017).[3] In this scenario, electricity generation by lignite coal is reduced by 28% until 2030 and by 52% until 2040 in relation to the level of 2014, see Table 3.2. This is not sufficient to meet the greenhouse gas emission targets.

Additional actions to achieve the greenhouse gas emission reduction goals are implemented in scenarios named Phase-Out-2035-Weak, Phase-Out-2040-Age, Phase-Out-2040-Balanced, and Phase-Out-2035-Strong. The scenarios differ with respect to speed and regional distribution of emission reduction. Phase-Out-2035-Weak and Phase-Out-2035-Strong only consider reductions in lignite and exclude additional reductions in hard coal. Without further capacity management for hard coal power plants the installed capacity in 2030 is 18 GW and this requires a capacity reduction to 10 GW for lignite power plants. Therefore, a total phase-out by 2035 is necessary to be consistent with the German greenhouse gas emission targets. Phase-Out-2035-Strong only deviates from Phase-Out-2035-Weak by assuming a strong initial decline in 2020. Phase-Out-2040-Age and Phase-Out-2040-Balanced consider an additional reduction in hard coal electricity generation in Germany. Lignite power plant capacity needs to be reduced until 2030 to about 9 GW for a path where hard coal power plant capacity is 10 GW in 2030. These paths lead to a total coal phase-out in Germany by 2040. In order to meet the GHG emission targets in the scenarios Phase-Out-2040-Age and Phase-Out-2040-Balanced it requires an additional hard coal power plant capacity reduction of roughly 30% compared to the installed capacity in 2014.

3.3 Model

We use a dynamic general equilibrium model for Germany with four regions and three sectors. An overview of the model structure is depicted in Figure 3.1.[4] In order to include trade flows between regions, we need to differentiate between destination regions $r \in \{1, \ldots, R\}$ and regions of origin $o \in \{1, \ldots, R\}$ for traded goods. Regions are populated by a continuum of representative households h. Household members i^r live and work in the same region. Each household supplies labour to a representative firm f in its own region. Firms operate in the energy and non-energy sector $k \in \{E, NE\}$. The energy sector can allocate labour to the lignite sector or to other energy sources $s \in \{LC, NLC\}$ to produce energy. Further,

[3] EU regulation No. 525/2013 of the EU Parliament makes it mandatory for every member state to report historic and projected future developments of anthropogenic GHG emissions on the national level. The German government assumes an annual reduction rate of EU emission allowances of 1.74% until 2020 and after 2021 by 2.2% as well as the introduction of a Market Stability Reserve (MSR). Second, the federal government estimates that by 2035 the share of renewable energy sources in electricity consumption will be roughly 60%. Third, subsidies to increase the capacity of combined heat and power plants using natural gas will disincentive investments to increase the lifetime of current coal fired power plants. We consider the net electricity generation of lignite reported for the scenario „Mit-Weiteren-Maßnahmen“ as our baseline scenario and assume uniform percentage reductions in the regions.

[4] The notation is summarized in Tables 3.8–3.9.

Table 3.2: Net electricity generation by lignite coal

Year	Germany	Central Germany	Lusatia	Rhineland
Null-Scenario				
2014	100%	100%	100%	100%
2020	107%	107%	107%	107%
2025	108%	108%	108%	108%
2030	108%	108%	108%	108%
2035	109%	109%	109%	109%
2040	111%	111%	111%	111%
Baseline				
2014	100%	100%	100%	100%
2020	81%	81%	81%	81%
2025	82%	82%	82%	82%
2030	72%	72%	72%	72%
2035	48%	48%	48%	48%
2040	48%	48%	48%	48%
Phase-Out-2035-Weak				
2014	100%	100%	100%	100%
2020	81%	81%	81%	79%
2025	44%	56%	56%	31%
2030	24%	15%	15%	31%
2035	0%	0%	0%	0%
2040	0%	0%	0%	0%
Phase-Out-2040-Age				
2014	100%	100%	100%	100%
2020	81%	81%	81%	79%
2025	63%	70%	70%	54%
2030	42%	54%	54%	28%
2035	21%	13%	13%	28%
2040	0%	0%	0%	0%
Phase-Out-2040-Balanced				
2014	100%	100%	100%	100%
2020	80%	83%	80%	79%
2025	61%	80%	44%	67%
2030	43%	54%	42%	41%
2035	22%	1%	22%	29%
2040	0%	0%	0%	0%
Phase-Out-2035-Strong				
2014	100%	100%	100%	100%
2020	60%	60%	60%	60%
2025	44%	56%	56%	31%
2030	24%	15%	15%	31%
2035	0%	0%	0%	0%
2040	0%	0%	0%	0%

Note: Net electricity generation reduction compared to the base year 2014 in percent.
Sources: The Baseline path is based on Bundesregierung (2017). Phase-Out-2035-Weak, Phase-Out-2040-Age and Phase-Out-2040-Balanced are based on Öko-Institut, Büro für Energiewirtschaft und technische Planung (BET) & Klinski (2017). Phase-Out-2035-Strong investigates the potential impact for the case that Germany will meet its 2020 target.

Table 3.3: Definition of scenarios

Path	Description
Null-Scenario	No change in the share of electricity generation by lignite in total electricity generation.
Baseline	Reduction of lignite electricity generation due to already implemented political actions.
Phase-Out-2035-Weak	Complete shutdown of lignite power plants by 2035 without further actions to reduce hard coal electricity generation.
Phase-Out-2040-Age	Complete shutdown by 2040 according to age criteria and further reduction in hard coal electricity generation.
Phase-Out-2040-Balanced	Complete shutdown by 2040 with balanced regional capacity contributions and further reduction in hard coal electricity generation.
Phase-Out-2035-Strong	Complete shutdown by 2035 as in Phase-Out-2035-Weak and a reduction of lignite electricity generation by 40% in 2020.

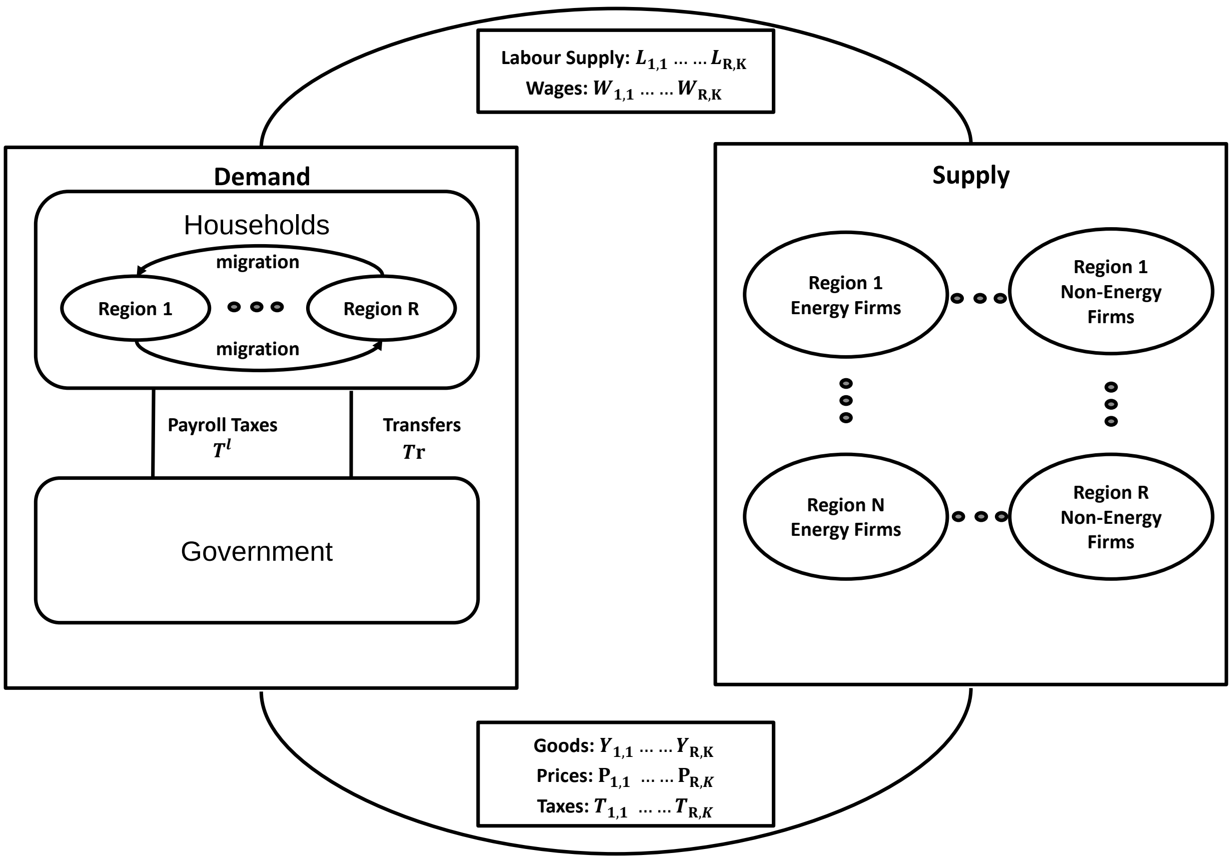
Sources: The Baseline path is based on Bundesregierung (2017). In Phase-Out-2035-Weak and Phase-Out-2035-Strong, net electricity generation falls according to the path "Kapa. nur BK" in Öko-Institut, Büro für Energiewirtschaft und technische Planung (BET) & Klinski (2017), except for the year 2020 in Phase-Out-2035-Strong. In Phase-Out-2040-Age and Phase-Out-2040-Balanced, net electricity generation falls according to the path "Kapa. BK&SK" in Öko-Institut, Büro für Energiewirtschaft und technische Planung (BET) & Klinski (2017).

households differentiate between three different employment opportunities indexed by $l \in \{E^{LC}, E^{NLC}, NE\}$. The central government collects taxes from labour income τ^l and charges a value-added tax on production τ.

The government consumes G_t, pays lump-sum transfers Tr_t, and unemployment benefits U_tB_t financed by tax revenues Tax_t. We assume a balanced government budget. Unemployment benefits are adjusted according to the development of national wages and with a backward-looking component to reflect rigidity in the adjustment of long-term unemployment benefits and wages. This specification allows for a sluggish adjustment of benefits, reflecting empirical evidence. All products produced in a given period are consumed and firms have no access to an inventory technology, i.e. we assume market clearing.[5]

[5] For further details see the Section D.2 in the Online Appendix.

Figure 3.1: Model overview



Source: own exhibition.

3.3.1 Households

3.3.1.1 Consumption and labour

Representative households maximize utility $u(C_{r,1,1,t}(h), \ldots, C_{r,R,K,t}(h), N_{r,1,t}, \ldots, N_{r,L,t}(h))$ with respect to (henceforth w.r.t.) consumption $C_{r,o,k,t}(h)$ and sectoral labour $N_{r,l,t}(h)$, given the utility function

$$
\begin{aligned}
u(C_{r,1,1,t}(h), \dots, C_{r,R,K,t}(h), N_{r,1,t}, \dots, N_{r,L,t}(h)) = \\
\Big[\sum_{k=1}^{K} \omega_k^{c\,\frac{1}{\eta^c}} C_{r,k,t}(h)^{\frac{\eta^c-1}{\eta^c}}\Big]^{\frac{\eta^c}{\eta^c-1}} - \sum_{l}^{L} \frac{z_t\, A_{r,l}^L N_{r,l,t}(h)^{1+\sigma_{r,l}^L}}{1+\sigma_{r,l}^L}, \\
C_{r,k,t}(h) = \Big[\sum_{o=1}^{R} w_{r,o,k}^{d\;\frac{1}{\eta_k^M}} C_{r,o,k,t}(h)^{\frac{\eta_k^M-1}{\eta_k^M}}\Big]^{\frac{\eta_k^M}{\eta_k^M-1}}.
\end{aligned}
\tag{3.1}
$$

The utility from consumption depends on sectoral consumption k from different regions o and is transformed into utility by a standard CES function. The elasticity of substitution between sectors η^c defines whether sectoral consumption goods are complements or substitutes. Preference shares ω_k^c define for equal prices of both consumption goods the share of consumption expenditure. The disutility of labour is sector-specific l and region-specific r through a disutility parameter $A_{r,l}^L$. The inverse Frisch elasticity is given by $\sigma_{r,l}^L$ and defines the elasticity between wages and employment. The budget constraint of the representative household is

$$
\begin{aligned}
P_{r,t}^c\, C_{r,t}(h) = P_{r,t}^c\, Tr_t(h) + P_{r,t}^c\, B_t\,(pop_{r,t}(h) - N_{r,t}(h)) \dots \\
+ \sum_{k}^{K} \Pi_{r,k,t}(h)\, P_{r,k,t} \; + (1-\tau^L)\, P_{r,t}^c W_{r,t} N_{r,t}(h).
\end{aligned}
\tag{3.2}
$$

$$
P_{r,t}^c C_{r,t}(h) = \sum_{k}^{K} \sum_{o}^{R} P_{o,k,t} C_{r,o,k,t}(h).
$$

$$
P_{r,t}^c W_{r,t} N_{r,t}(h) = \sum_{l}^{L} P_{r,l,t} W_{r,l,t} N_{r,l,t}(h).
$$

Households have no access to bonds or other assets to save money. Their income sources are the net profits of firms $\sum_k^K \Pi_{r,k,t}(h)\, P_{r,k,t}$, net labour income $\sum_l^L (1-\tau^l)\, P_{r,l,t}\, W_{r,l,t}\, N_{r,l,t}(h)$, lump-sum transfers from the state $Tr_t(h)$, and unemployment benefits $P_{r,t}^c B_t U_{r,t}(h)$. Households maximize utility (3.1) subject to the budget constraint (3.2) with respect to sectoral consumption and employment in each sector. The first order condition (henceforth first-order condition) for sectoral and regional consumption is

$$
C_{r,k,t}(h) = w_k^c \left(\frac{P_{r,k,t}^c}{P_{r,t}^c}\right)^{-\eta^c} C_{r,t}(h). \tag{3.3}
$$

$$
C_{r,o,k,t}(h) = w_{r,o,k}^d \left(\frac{P_{o,k,t}}{P_{r,k,t}^c}\right)^{-\eta_k^M} C_{r,k,t}(h). \tag{3.4}
$$

We derive this expression by assuming that the Lagrange multiplier of the optimization problem reflecting the marginal utility of relaxing the budget constraint is the inverse regional price level $P^c_{r,t}$. We can express the regional aggregate price index for consumption as

$$P^c_{r,t} = \Big(\sum_k w^c_k {P^c_{r,k,t}}^{1-\eta^c} \Big)^{\frac{1}{1-\eta^c}}. \tag{3.5}$$

Further, regional sector specific aggregate price indexes for consumption are given by

$$P^c_{r,k,t} = \Big(\sum_o w^d_{r,o,k} {P_{o,k,t}}^{1-\eta^M_k} \Big)^{\frac{1}{1-\eta^M_k}}. \tag{3.6}$$

Households derive income from labour and are compensated by the government for unemployed household members. Firms that produce intermediate goods hire household members. Households only send their members to work if the nominal wage compensates for the disutility of working and the unemployment benefits. The first-order condition for labour is

$$P_{r,l,t} W_{r,l,t} = z_t A^L_{r,l} N_{r,l,t}(h)^{\sigma^L_{r,l}} P^c_{r,t} + B_t. \tag{3.7}$$

The left-hand side of (3.7) defines the nominal regional sectoral wage. The associated marginal disutility by increasing labour supply in this sector and region is represented by the first term on the right-hand side. Furthermore, the outside option of being unemployed is also considered.

All households in a region are identical by assumption. Therefore, per capita variables $x_{r,t} = \frac{X_{r,t}}{pop_{r,t}} = \frac{\int_0^1 X_{r,t}(h)\mathrm{d}h}{\int_0^1 pop_{r,t}(h)\mathrm{d}h}$ are identical to individual variables $x_{r,t} = X_{r,t}(h)$ and we can drop the index h.

3.3.1.2 Migration

Migration is an important mechanism for regional economic adjustments after a regional sector-specific shock. Smets & Beyer (2015) show that migration flows in the U.S. can explain up to 50% of the long-run adjustment to region-specific economic shocks. After the reunification of Germany, East Germany lost up to 15% of its inhabitants since 1990, also in response to higher unemployment rates in East Germany. Accordingly, household members in our model can migrate to different regions in Germany. Most of the migrants have been between 20 and 30 years old belonging to cohorts entering the labour force (see Kühntopf & Stedtfeld 2012). Our approach to model intra-national migration as response to the coal phase-out reflects this finding. Migration is therefore more rigid than implied by standard classical economic models, because of hidden migration costs due to the potential loss of social networks, cultural preferences or real estate investments. Every German citizen can freely choose where to live and work.

In each period t the labour force population pop_t consists of individuals $i_t = \{1, \dots, pop_t\}$. A fraction $1 - \rho^{pop}$ of individuals enters the labour force in period t and the other fraction ρ^{pop} has been part of the labour force in the previous period. Individuals entering the labour force in the current period actively decide in what region n they want to live and work. The decision problem of an individual for one specific region is modelled by a random utility maximization problem, standard in the empirical migration literature (see Beine et al. 2016).

Individuals participate in the labour market for T periods. At the beginning of their working life, they decide where to work and live, taking into account the utility at the end of their working life

$$U^L_{i,r,T|t} = \log \left\{ (C_{r,T|t}(h) - \sum_l^L (1+\sigma^L_{r,l})^{-1} A^L_{r,l} N_{r,l,T|t}(h)^{1+\sigma^L_{r,l}}) \right\} + \eta^{pop}_{n,T|t} + \eta_{i,r} \tag{3.8}$$

given the information in period t. The first part of (3.8) is the utility function of the representative household at the end of the working life and $\eta^{pop}_{n,T|t}$ denotes that part of utility which depends on the economic fundamentals and common unobservable characteristics of the region among individuals given the information available at time t. The law of motion of exponential average regional attractiveness is given by

$$\epsilon^{pop}_{r,t} = \rho^{pop}\, \epsilon^{pop}_{r,t-1} + (1-\rho^{pop})\, \exp(U^L_{i,r,T|t}).$$

It is the weighted average of the utility derived from living in region r. The individual-specific stochastic component $\eta_{i,r}$ follows the Gumbell distribution (see McFadden et al. 1973). The probability of an individual choosing region r in period t is

$$\Pr(i=r|t) = \frac{\exp(U^L_{i,r,T|t})}{\sum_o \exp(U^L_{i,o,T|t})}. \tag{3.9}$$

The fraction $(1-\rho^{pop})$ choose to live in region r at time t with probability $\Pr(i=r|t)$ the remaining individuals ρ^{pop} stay at their current living and working place. Therefore, the regional shares in the labour force are also given by $w^{pop}_{r,t} = \rho^{pop} w^{pop}_{r,t-1} + (1-\rho^{pop})\Pr(i=r|t)$.

3.3.2 Firms

3.3.2.1 Producers of final goods

In each region, there is a continuum of firms f in the energy and non-energy sectors, producing differentiated goods. These goods are combined into a final good $Y_{r,k,t} = \left(\int_0^1 Y_{r,k,t}(f)^{\frac{1}{\lambda_{r,k,t}}} \mathrm{d}f \right)^{\lambda_{r,k,t}}$ in each sector, which is sold to the households. Firms operating in the final goods sector are perfectly competitive and have no market power. This set-up allows including price-setting power by firms (see Petrella & Santoro 2011). The profit maximization problem of the final goods firm in each sector looks as follows

$$\max_{Y_{r,k,t}(f)} P_{r,k,t}\, Y_{r,k,t} - \int_0^1 P_{r,k,t}(f)\, Y_{r,k,t}(f) \mathrm{d}f, \tag{3.10}$$
$$\text{s.t.} Y_{r,k,t} = \Big(\int_0^1 Y_{r,k,t}(f)^{\frac{1}{\lambda_{r,k,t}}} \mathrm{d}f \Big)^{\lambda_{r,k,t}}.$$

The first-order condition of the final goods producer w.r.t. an intermediate good is also the demand curve for each intermediate good given by

$$Y_{r,k,t}(f) = \Big(\frac{P_{r,k,t}(f)}{P_{r,k,t}} \Big)^{\frac{\lambda_{r,k,t}}{\lambda_{r,k,t}-1}} Y_{r,k,t}. \tag{3.11}$$

Living costs depend on migration and regional attractiveness. We assume that the market power of a firm depends on the attractiveness of the region it operates in. A higher attractiveness leads to a higher share of the population and therefore to more demand for housing services. Further, a greater number of people in one region will increase the demand for local services and increase the bargaining position of domestic firms. The market power of a firm $\lambda_{r,k,t}$ follows an auto-regressive process of order one. Firms have a higher market power in regions with a higher attractiveness $\epsilon^{pop}_{r,t}$ and therefore with a higher share of the population. Other unobserved determinants of the market power in a region and sector are summarized by the parameter $\sigma^{\lambda}_{r,k}$.

$$\lambda_{r,k,t} = \rho^{\lambda} \lambda_{r,k,t-1} + (1-\rho^{\lambda})\, \epsilon^{pop}_{r,t}\, \sigma^{\lambda}_{r,k}. \tag{3.12}$$

3.3.2.2 Non-energy sector intermediate goods producers

Producers of intermediate goods in the non-energy sector use labour $N_{r,k,t}$. They face adjustment costs $MC^{h}_{r,k,t+h}$, so that a fraction of their production is used by adjusting their employment stock. We introduce hiring costs as in Blanchard & Galí (2010) with a non-cyclical and a cyclical component. Non-cyclical components include, e.g. training costs. Cyclical hiring costs depend on the tightness in the current labour market $\frac{H_{r,t+h}}{U^{s}_{r,t+h}}$. A structural change in the lignite regions is likely to increase unemployment rates for a longer time period. A region's losing a key industry leads to higher unemployment rates in that regions for decades, compared to the national average – e.g. the Ruhrgebiet in Germany or the Rust Belt in the US. A higher labour supply and a smaller labour demand will shift wage bargaining power from employees to employers. Including a cyclical component in hiring costs captures this bargaining shift. The optimization problem of the firm is

$$\max_{N_{r,k,t}(f)} \sum_{h=0}^{\infty} \beta^h \Big\{ (1-\tau_{r,k,t+h})\, P_{r,k,t+h}(f)\, Y_{r,k,t+h}(f) - W_{r,k,t+h}\, N_{r,k,t+h}(f) \Big\} \tag{3.13}$$

$$\text{s.t. } Y_{r,k,t+h}(f) = \epsilon_{r,k,t+h} \Big(A_{r,k,t+h}(f) N_{r,k,t+h}(f)^{\alpha_{r,k}} - \frac{1}{\Psi} MC^h_{r,k,t+h}(f) H_{r,k,t+h}(f)^{\Psi} \Big), \tag{3.14}$$

$$MC^h_{r,k,t+h} = B^h_{r,k} \left\{ \psi + (1-\psi) \left(\frac{H_{r,t+h}}{U^s_{r,t+h}} \right)^{v} \right\} pop^{1-\psi}_{r,t+h}, \tag{3.15}$$

$$H_{r,k,t+h}(f) = N_{r,k,t+h}(f) - \Big(\mu^{pop}_{r,t} - \delta \Big) N_{r,k,t+h-1}(f), \tag{3.16}$$

$$P_{r,k,t+h}(f) = \left(\frac{Y_{r,k,t+h}(f)}{Y_{r,k,t+h}} \right)^{\frac{1-\lambda_{r,k,t+h}}{\lambda_{r,k,t+h}}} P_{r,k,t+h}. \tag{3.17}$$

The index f can be omitted when prices are flexible. All firms behave identically, and therefore aggregated variables are the same as individual variables, see Christiano et al. (2010). We can derive the first-order condition with respect to labour by plugging the constraints into the objective function and taking the first derivative with respect to labour. Labour market friction is the only source of intertemporal optimization. The first-order condition for firms is

$$\begin{aligned} &\frac{P_{r,k,t}}{\lambda_{r,k,t}} \alpha_{r,k}\, \epsilon_{r,k,t}\, A_{r,k,t}\, (1-\tau_{r,k,t})\, N^{\alpha_{r,k}-1}_{r,k,t} - \frac{P_{r,k,t}}{\lambda_{r,k,t}}\, MC^h_{r,k,t}\, \epsilon_{r,k,t}\, (1-\tau_{r,k,t}) H^{\Psi-1}_{r,k,t} \dots \\ &\qquad + \Big(\mu^{pop}_{r,t+1} - \delta \Big)\, \beta\, \frac{P_{r,k,t+1}}{\lambda_{r,k,t+1}}\, MC^h_{r,k,t+1}\, \epsilon_{r,k,t+1}\, (1-\tau_{r,k,t+1}) H^{\Psi-1}_{r,k,t+1} = W_{r,k,t}. \end{aligned} \tag{3.18}$$

3.3.2.3 Producers of intermediate goods in the energy sector

The intertemporal optimization problem of producers of intermediate goods in the energy sector is very similar to the problem of those in the non-energy sector. Energy firms can produce energy by allocating labour between the lignite and non-lignite sectors $s \in \{LC, NLC\}$. They face hiring costs in each input sector. The intertemporal optimization problem of the energy firm is the following:

$$\max_{N_{r,k,s,t}(f)} \sum_{h=0}^{\infty} \beta^h \Big[P_{r,k,t+h}(f)\, Y_{r,k,t+h}(f) - \sum_s \Big\{ W_{r,k,s,t+h}\, N_{r,k,s,t+h}(f) + Tax_{r,k,s,t+h}(f) \Big\} \Big] \tag{3.19}$$

$$\text{s.t. } Y_{r,k,t+h}(f) = \Big(\sum_s \phi_{r,k,s}^{\frac{1}{\eta^b}} Y_{r,k,s,t}(f)^{\frac{\eta^b-1}{\eta^b}} \Big)^{\frac{\eta^b}{\eta^b-1}}, \tag{3.20}$$

$$Y_{r,k,s,t+h}(f) = \epsilon_{r,k,s,t+h} \Big(A_{r,k,s,t+h}(f) N_{r,k,s,t+h}(f)^{\alpha_{r,k,s}} - \frac{1}{\Psi} MC^h_{r,k,s,t+h}(f) H_{r,k,s,t+h}(f)^{\Psi} \Big), \tag{3.21}$$

$$MC^h_{r,k,s,t+h} = B^h_{r,k,s} \left\{ \psi + (1-\psi) \left(\frac{H_{r,t+h}}{U^s_{r,t+h}} \right)^{v} \right\} pop^{1-\Psi}_{r,t+h}, \tag{3.22}$$

$$H_{r,k,s,t+h}(f) = N_{r,k,t+h}(f) - \left(\mu^{pop}_{r,t} - \delta \right) N_{r,k,t+h-1}(f), \tag{3.23}$$

$$P_{r,k,t+h}(f) = \left(\frac{Y_{r,k,t+h}(f)}{Y_{r,k,t+h}} \right)^{\frac{1-\lambda_{r,k,t+h}}{\lambda_{r,k,t+h}}} P_{r,k,t+h}. \tag{3.24}$$

Energy firms allocate labour to the input sectors according to their relative marginal productivity of labour and the respective wage paid to the workers, and the respective taxes. The first-order condition for firms is

$$\begin{aligned} &\underbrace{\frac{P_{r,k,s,t}(f)}{\lambda_{r,k,t}}\, \alpha_{r,k,s}\, A_{r,k,s,t}\, (1-\tau_{r,k,s,t})\, N^{\alpha_{r,k,s}-1}_{r,k,s,t} - \frac{P_{r,k,s,t}(f)}{\lambda_{r,k,t}}\, MC^h_{r,k,s,t}\, \epsilon_{r,k,s,t}\, (1-\tau_{r,k,s,t}) H^{\Psi-1}_{r,k,s,t}}_{\text{1) contemporaneous increase in production}} \dots \\ &\quad + \underbrace{(\mu^{pop}_{r,t+1} - \delta)\, \beta\, \frac{P_{r,k,s,t+1}(f)}{\lambda_{r,k,t+1}}\, MC^h_{r,k,s,t+1}\, \epsilon_{r,k,s,t+1}\, (1-\tau_{r,k,s,t+1}) H^{\Psi-1}_{r,k,s,t+1}}_{\text{2) avoided future hiring costs}} = W_{r,k,s,t}. \end{aligned} \tag{3.25}$$

As for the non-energy firm, the marginal product of labour for the respective input sector equals the marginal cost. The marginal cost is the respective wage.

$$P_{r,k,s,t}(f) = \frac{\partial Y_{r,k,t}(f)}{\partial Y_{r,k,s,t}(f)} = \phi_{r,k,s}^{\frac{1}{\eta^b}} \Big(\frac{Y_{r,k,s,t}(f)}{Y_{r,k,t}(f)} \Big)^{\frac{1}{\eta^b}} P_{r,k,t}(f), \tag{3.26}$$

$$\frac{\partial Tax_{r,k,s,t}(f)}{\partial Y_{r,k,s,t}(f)} = \tau_{r,k,s,t}\, P_{r,k,s,t}(f). \tag{3.27}$$

The marginal product of labour in one energy input sector depends on the marginal product of energy (3.26) and the marginal tax burden (3.27). We could also assume that energy input firms are independent companies selling to a competitive energy wholesaler. The energy wholesaler would need to pay a price according to (3.26) for the inputs. The first term on the left-hand side of (3.25) states the contemporaneous increase in energy production by increasing labour in one input sector less hiring costs. Hiring one more person today will reduce hiring costs in the next period, as captured by the second term in (3.25).

3.4 Calibration and Simulation

We simulate a deterministic transition of the economy from an initial steady-state to a terminal steady-state.[6] 3.7.1 provides a detailed description of the calibration of the initial and terminal steady-states.

The initial steady-state reflects the state of the German economy in 2014. The regional gross value-added shares of the initial steady-state are identical to the shares reported in Table 3.6. To match the reported shares we set regional and sector specific productivity $a_{r,l}$ accordingly. Therefore, the model will consider the contribution of each sector to overall gross value-added in the region. Our initial calibration also matches initial labour cost shares as reported in Table 3.7 by setting the labour productivity exponent $\alpha_{r,l}$ in the production function accordingly. Due to hiring costs and value-added taxes the parameter is not identical to the labour cost shares. We need to calibrate the slope of the labour supply curve $A_{r,l}$ to match the employment shares reported in Table 3.1.

The terminal steady-state is computed by alternating regional sector-specific productivity shocks to the lignite sector $\epsilon_{r,E,LC,t}$ to match the relative net electricity generation reported in Table 3.2. All structural parameters of the model are not changed as response to the coal phase-out. We are not explicitly modeling the political actions described in Section 3.2.2. We assume that the government has access to instruments to reduce the regional net electricity generation by lignite as reported by the electricity market model. One instrument is the decommissioning of power plants. Our model has no capital as input to the production of the intermediate goods producers. Nevertheless, decommissioning power plants affects capital utilization and capital stocks in the industry. The computation of productivity shocks is a simplified way to implement the decommissioning plan. Another instrument are regional lignite coal specific value-added tax rates (see Golosov et al. 2014, p.57). Because of legal constraints they are hard to implement and are not considered here.

We explicitly model the evolution of regional and sectoral gross value-added of the lignite coal industry $P_{r,E,LC,t}\,Y_{r,E,LC,t}$. We set $\epsilon_{r,E,LC,t}$ such that net electricity generation $Y_{r,E,LC,t}$ compared to 2014 corresponds to the reported net electricity generation by the electricity market model with a tolerance level of $\pm 2\%$. This approach requires that all potential fluctuations of the ratio between intermediate inputs and net electricity are included in the region and sector specific price $P_{r,E,LC,t}$.

Unemployment rates of the labour market regions are converging since the beginning of the 2000's. One of the main reasons of a convergence of unemployment rates is migration from lignite coal regions to other regions in Germany. In the long-run the lignite coal phase-out will decrease the number of people staying in or moving to the region. We assume that migration as a response to the coal phase-out will reduce the unemployment rate to the values for the year 2014 in the long-run.

[6] The model is implemented in Dynare (see Adjemian et al. 2006).

Table 3.4: Parameter space

Parameter	Interval	Description	Source for the mean
η_E^m	$\mathcal{U}(760, 840)$	elasticity of substitution between regions for energy	estimated from regional national accounts data
I_E^{Home}	$\mathcal{U}(0.475, 0.525)$	home bias energy	calibrated
I_{NE}^{Home}	$\mathcal{U}(0.8075, 0.8925)$	home bias non-energy	Hristov (2016)
σ^L	$\mathcal{U}(0.2375, 0.2625)$	inverse Frisch elasticity / excluding lignite Rest of Germany	King & Rebelo (1999)
$\bar{\lambda}_r$	$\mathcal{U}(1.1875, 1.3125)$	market power in region n at start	calibrated
η_r^b	$\mathcal{U}(19.57, 21.63)$	elasticity of substitution between lignite coal and non lignite coal in region n	calibrated
x	$\mathcal{U}(0.2131, 0.2355)$	steady-state job finding rate	according to long-term unemployed share
β	$\mathcal{U}(0.9975, 0.9984)$	discount factor	Hristov (2016)
v	$\mathcal{U}(0.95, 1.05)$	hiring cost elasticity to labour market tightness	Blanchard & Galí (2010)
ρ_ϵ^{pop}	$\mathcal{U}(0.9921, 0.9929)$	persistence in living preferences	calibrated
ρ^λ	$\mathcal{U}(0.855, 0.945)$	posterior mode of mark-up shocks	estimated by Smets & Wouters (2007)
ρ^b	$\mathcal{U}(0.8075, 0.8925)$	AR(1) coefficient for adjustment replacement rate	estimated from OECD data
η^c	$\mathcal{U}(0.7125, 0.7875)$	elasticity of substitution between energy and non-energy sector	estimated from national accounts data
$\frac{\kappa}{w\,n}$	$\mathcal{U}(0.0617, 0.0683)$	relation of hiring costs to wage bill	estimated by Christiano et al. (2016)

In order to evaluate the sensitivity of the results to our calibrated parameter values we define a parameter space. The parameter space is summarized in Table 3.4. We define marginal uniform distributions $\mathcal{U}(a, b)$ for the reported parameters. We will report the simulation results for all parameters set to the mean of their respective distributions. For our sensitivity analysis we will draw 1200 parameter combinations and simulate all paths. We conduct a univariate sensitivity analysis by changing only one parameter at the time and all other parameters are set to their respective mean. We report the sensitivity of our results for the minimum, the first quartile, the mean/median, the third quartile and the maximum of the respective univariate distributions.

3.5 Results

A reduction of net electricity generation by lignite according to Table 3.2 described in Section 3.2 will directly affect the demand for workers in the lignite industry, temporarily increase unemployment rates, reduce labour income and lead to migration. To explain the main results of the simulations we need to refer to simulation results of other variables. Therefore, we report results for other variables in Table 3.11 to Table 3.17 and Figure 3.9 to

Figure 3.15 in the Appendix.[7]

The reduction in lignite employees is depicted in Figure 3.2. The implemented climate policy measures captured by the Baseline scenario will reduce the number of employees in the lignite industry by 9,200 [9,200; 9,300] by 2040 compared to the Null-Scenario. The number of employees in the lignite industry compared to the Null-Scenario in the Rhineland, Lusatia and Central Germany will decrease by approximately 45% in each region compared to 2014. Additional political measures will reduce the number of lignite employees in the Rhineland, Lusatia, and Central Germany by 4,800 [4,700; 4,800], 3,800 [3,800; 3,800] and 1,100 [1,100; 1,100] people by 2040, respectively. Number in brackets indicate the smallest and largest difference to the Null-Scenario simulated for the time period 2014 to 2040 and based on the results of the multivariate sensitivity analysis.

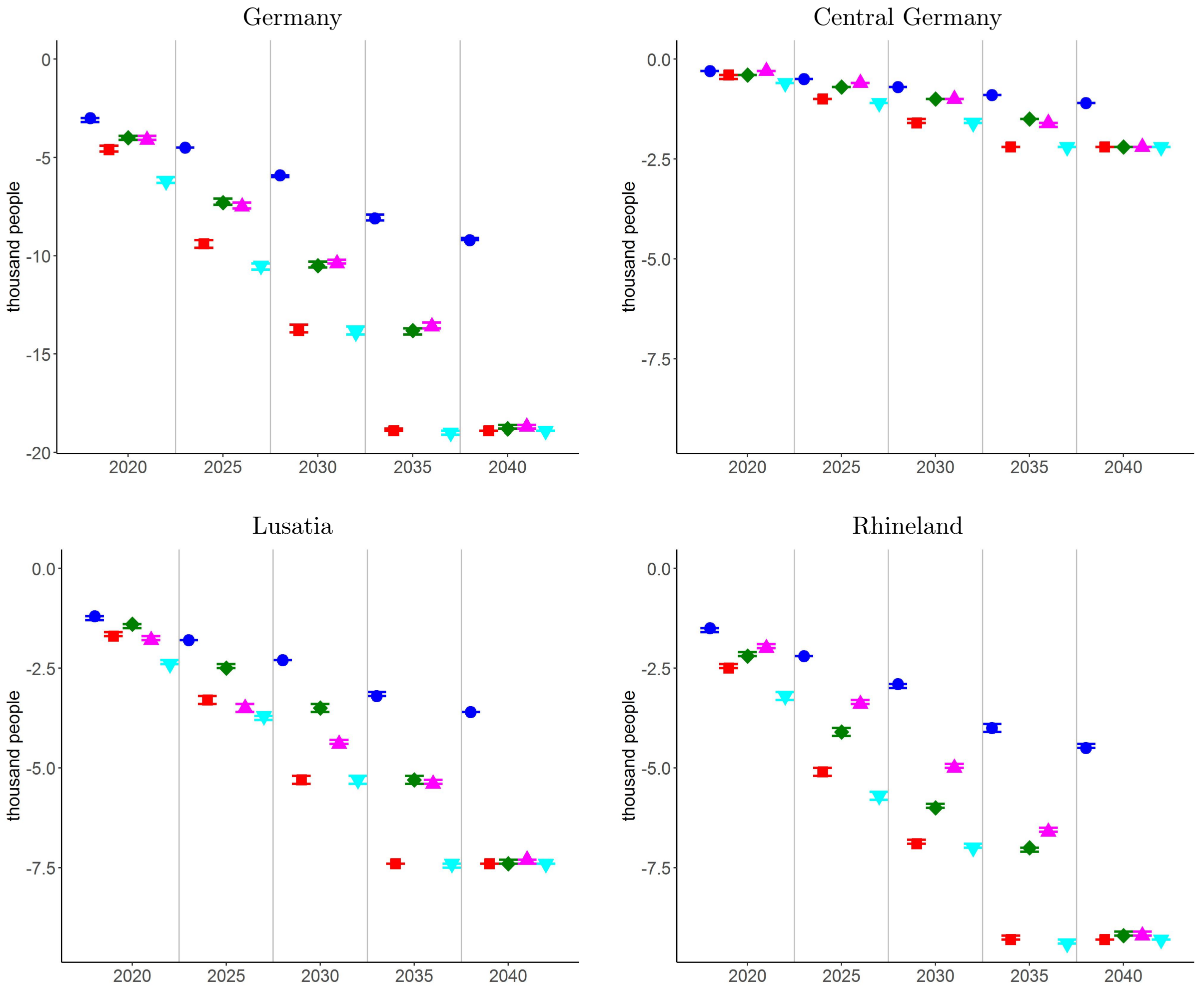
The direct employment effects will trigger negative indirect and induced employment effects, but also positive employment effects in other sectors by reducing labour costs and expanding other energy sources. Negative effects exceed the positive employment effects reflected by an increase in unemployment rates as depicted in Figure 3.3. Unemployment rates increase in the Baseline scenario by 0.02 [0.01; 0.03], 0.06 [0.04; 0.07], 0.48 [0.44; 0.51], and 0.10 [0.09; 0.11] percentage points between 2014 and 2040 compared to the Null-Scenario in the rest of Germany, Central Germany, Lusatia, and the Rhineland, respectively. There are up to 3,400 [-1,800; 9,700], 1,200 [1,000; 1,400], 5,000 [3,700; 5,700] and 5,500 [4,800; 6,100] fewer people employed between 2020 and 2040 compared to the Null-Scenario in the rest of Germany, Central Germany, Lusatia, and the Rhineland, respectively.

A total phase-out of coal increases the unemployment rates by 0.11 [0.06; 0.16] (Phase-Out-2040-Age, Phase-Out-2040-Balanced), 0.18 [0.13; 0.22] (Phase-Out-2035-Weak, Phase-Out-2040-Age, Phase-Out-2040-Balanced), 1.07 [1.00; 1.13] (Phase-Out-2035-Weak, Phase-Out-2035-Strong) and 0.25 [0.20; 0.28] [(Phase-Out-2035-Weak, Phase-Out-2035-Strong) percentage points in the rest of Germany, Central Germany, Lusatia, and the Rhineland. Therefore, up to 36,300 [20,400; 55,000] (Phase-Out-2040-Balanced), 2,800 [2,100; 3,300] (Phase-Out-2035-Weak), 9,500 [7,500; 10,600] (Phase-Out-2035-Weak) and 11,400 [9,900; 12,600] (Phase-Out-2040-Balanced) more people are unemployed compared to the Baseline in the rest of Germany, Central Germany, Lusatia, and the Rhineland. A total phase-out can lead to a maximum reduction in employment in Germany by up to 55,100 [36,300; 74,800] people in 2035 (Phase-Out-2040-Balanced). Only in the scenarios Phase-Out-2040-Age and Phase-Out-2040-Balanced the national unemployment rate will be above the value for the Null-Scenario in 2040. For the scenarios Phase-Out-2035-Weak and Phase-Out-2035-Strong the national unemployment rate is close to the value for the Null-Scenario in 2040. The recovery process is mainly driven by the rest of Germany and not the lignite coal regions itself.

The recovery process is mainly caused by lower real wages in the regions. This will also decrease labour income in the lignite regions permanently as depicted in Figure 3.11 in the Appendix. The fall in labour income is the greatest in Lusatia compared to all other regions. This even triggers in addition to the previous reasons a non-negative response in

[7] More results are reported in Table D.1 to Table D.24 and Figure D.1 to Figure D.15 in the Online Appendix.

Figure 3.2: Simulation results for employment in lignite sector

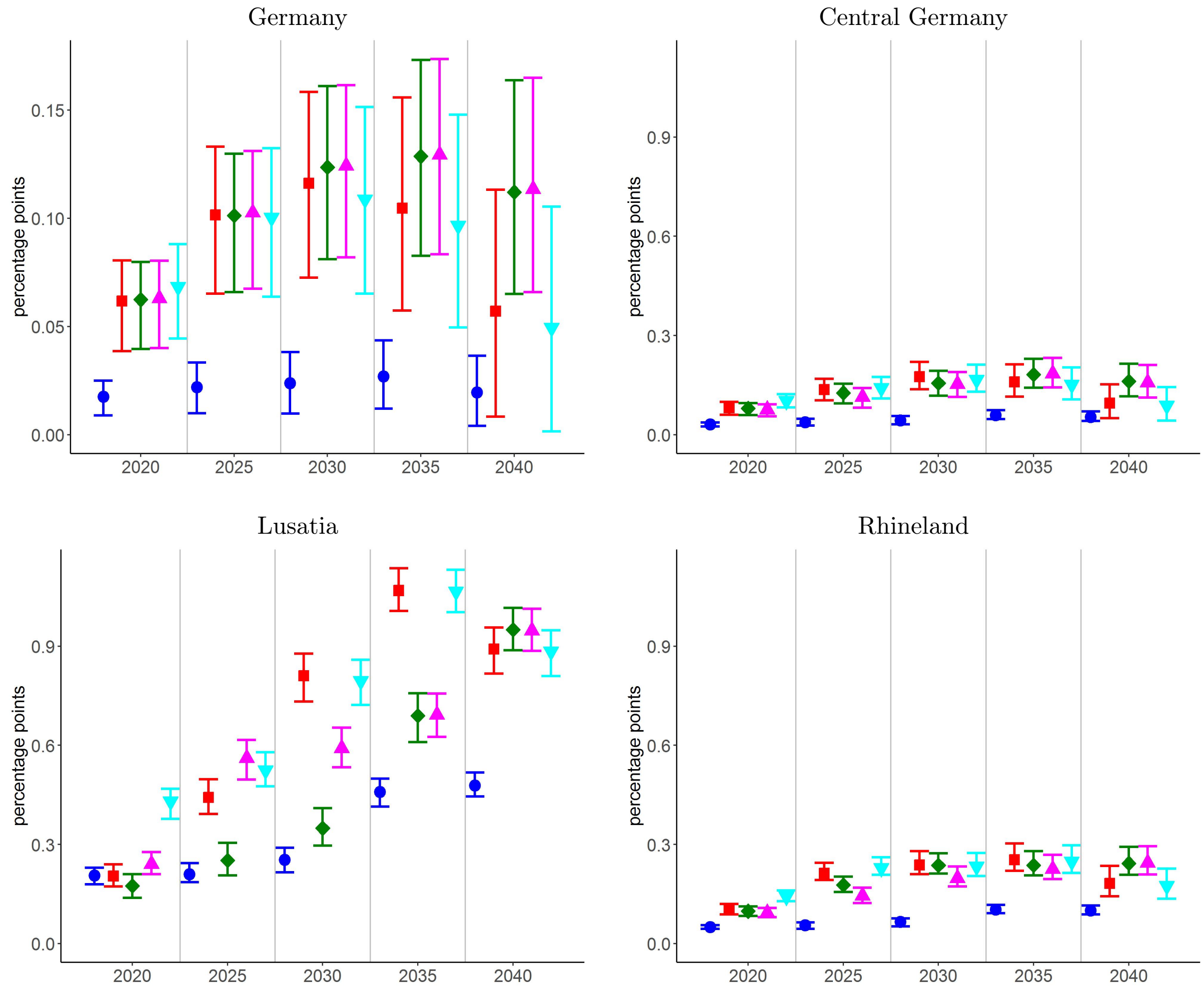


Note: Difference compared to the Null-Scenario in thousand people, Baseline (blue circle), Phase-Out-2035-Weak (red square), Phase-Out-2040-Age (green diamond), Phase-Out-2040-Balanced (magenta triangle point-up) and Phase-Out-2035-Strong (cyan triangle point-down). Horizontal lines indicate the maximum and minimum value observed for 1200 simulations.

non-energy employment rates. Nevertheless, in absolute terms, migration leads to a smaller labour force and to a smaller number of employees in the non-energy sector in Lusatia. A reduction in wages will reduce unemployment benefits in the long-run and trigger an increase in employment rates in Germany. The outside option of not working becomes less attractive. The lignite industry pays relatively high wages and overall wages will fall after the industry is no longer a potential employer. Due to this fall, unemployment rates also fall, because lower overall wages will increase demand for employees and reduce unemployment benefit rates.

Welfare depends on consumption and labour disutility as formulated in (3.1). There is no

Figure 3.3: Simulation results for unemployment rates



Note: Difference compared to the Null-Scenario in percentage points, Baseline (blue circle), Phase-Out-2035-Weak (red square), Phase-Out-2040-Age (green diamond), Phase-Out-2040-Balanced (magenta triangle point-up) and Phase-Out-2035-Strong (cyan triangle point-down). Horizontal lines indicate the maximum and minimum value observed for 1200 simulations.

phase-out path clearly dominating the other phase-out paths in terms of aggregate discounted future welfare (see Table 3.15 and Table 3.16). Only differences in discounted cumulative welfare for Lusatia differ notably and indicate that Phase-Out-2035-Weak is welfare efficient for Lusatia. Migration responds to new long-run differentials in utility. A total phase-out will reduce the attractiveness of the lignite regions Lusatia and the Rhineland and increase the attractiveness of the rest of Germany and Central Germany, which will result in lower and higher labour force shares, respectively. Nevertheless, the attractiveness of Central Germany does not increase sufficiently to attract more people, and leads to a more or less unaffected labour force share (see Figure 3.10 in the Appendix). Migration decreases the labour force by

5,600 [3,000; 7,000] and 4,300 [3,000; 5,100] in Lusatia and the Rhineland in 2040 compared to the Null-Scenario, respectively. In Central Germany, the labour force only decreases by 200 [200; 300] people by 2040. Compared to the Baseline path, migration between Lusatia, the Rhineland and the rest of Germany increases by roughly 4,000 people or 80%.

The previous results depend on the calibrated parameter values. We specify subjective probability distributions for a systematic sensitivity analysis.[8] Further, we investigate what parameters drive the simulated maximum increase in the unemployment rate between 2014 and 2040. Figure 3.4 depicts how the simulated maximum increase depend on the four most important parameters.

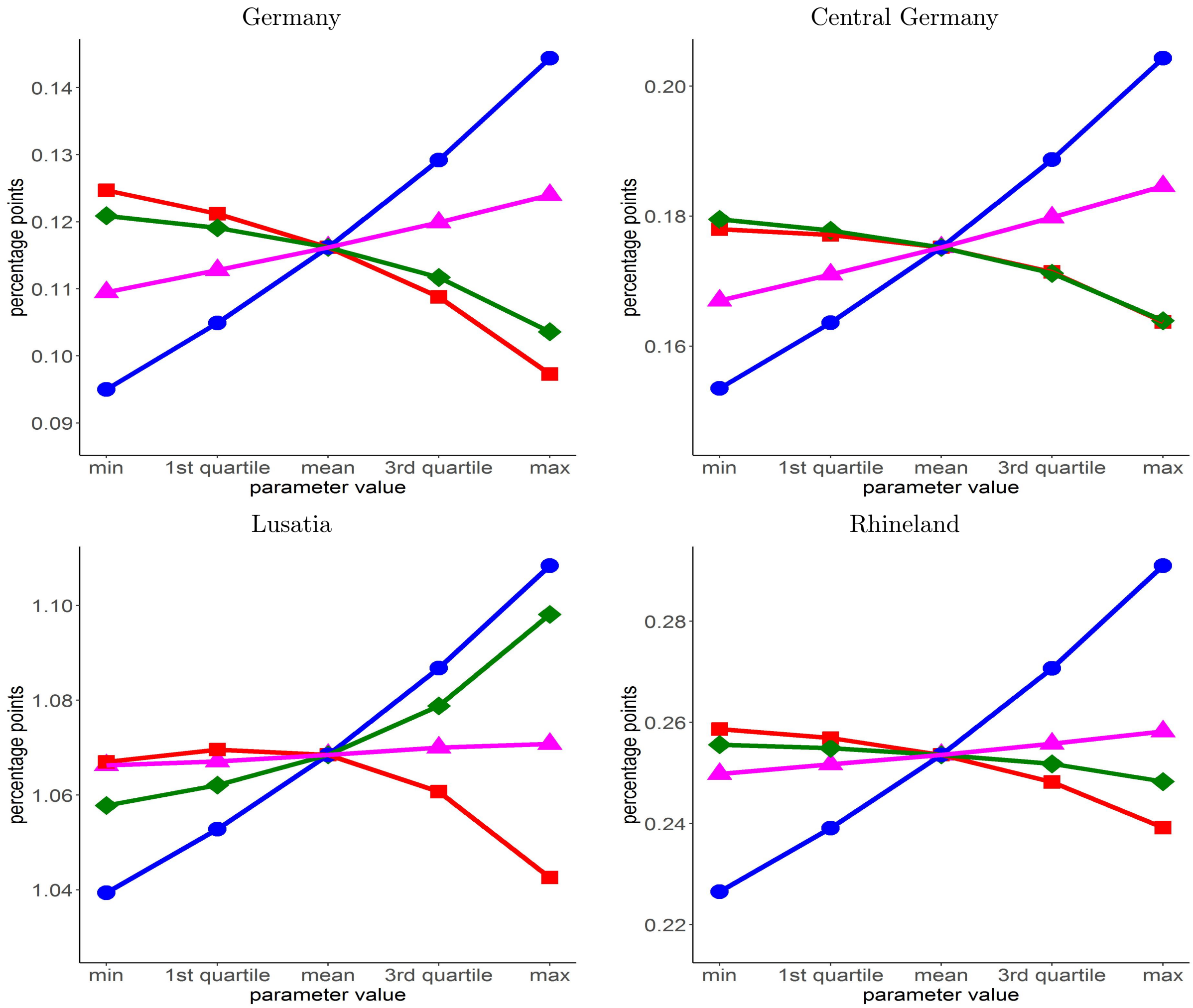
Figure 3.4 reports the sensitivity analysis for Phase-Out-2035-Weak. The reported parameters are the same for all phase-out paths. The maximum increase in the unemployment rate depends on the persistence in unemployment benefits. Higher persistence in unemployment benefits will reduce wage flexibility. At the mean the maximum increase in unemployment is about 0.12 percentage points. At the minimum value the maximum increase decreases by roughly 0.02 percentage points and at the maximum parameter value the maximum increase will be 0.02 percentage points higher. Future adjustments of unemployment benefits and wages have to be considered as potential policy tool to reduce the employment effects. Further, our introduction of rigid unemployment benefits also captures other rigidities in the adjustment of wages to changing economic fundamentals such as collective wage agreements. But reducing the rigidity in unemployment benefits will increase the maximum drop in wages. Nevertheless, the quantity effect dominates the price effect for labour compensation and more flexible unemployment benefits will reduce the maximum drop in labour compensation. The relationship between the maximum increase in the unemployment rate and the persistence in unemployment benefits is the same across all regions.

A home bias parameter for non-energy products set to the maximum value can increase the unemployment rate by less than 0.01 percentage points. Reducing the home bias parameter to its minimum value will decrease the maximum increase by roughly 0.02 percentage points. The simulated maximum increase in the lignite regions increases with a higher parameter value for the non-energy home bias. A higher home bias in the non-energy sector will reduce the demand in rest of Germany for non-energy products produced in the lignite regions. New jobs in the non-energy sector to replace the old jobs in the lignite industry require demand. A potential policy might be to stimulate demand for non-energy products from lignite regions.

The persistence in market power and the initial steady-state of market power have the weakest effect on the maximum increase in the unemployment rate. Unemployment rates depend positively on the persistence. Market power increases in regions experiencing a greater inflow of migration. More persistent market power will reduce the speed of adjustment. Therefore, the simulated maximum increase in the unemployment rate declines with a higher persistence in market power. A higher initial value for market power increases the maximum increase in the unemployment rate.

[8] The detailed results for all parameters are discussed in the Online Appendix D.3.

Figure 3.4: Sensitivity analysis for maximum increase in the unemployment rate



Note: Change in maximum employment drop between 2014 to 2040 for Phase-Out-2035-Weak compared to the Null-Scenario in percentage points changing the value of only one parameter. The most important parameters for the maximum increase in the German unemployment rate in descending order are: persistence in unemployment benefits ρ^b (blue circle), home bias non-energy I_{NE}^H (red square), persistence of market power ρ^λ (green diamond), steady-state value of market power $\bar{\lambda}$ (magenta triangle point-up). We report the change in the maximum drop for the minimum, first quartile, median/mean, third quartile and maximum parameter value.

3.6 Conclusions

In January 2019, the Commission on Growth, Structural Change and Employment in Germany proposed a plan for the stepwise reduction of electricity generation by lignite. The proposal suggests a total phase-out until 2038 and includes an option to phase-out until 2035, if this does not threaten the security of electricity supply. Our analysis shows, that a phase-out until 2035 is not worse than a phase-out until 2040 in terms of discounted cumulative welfare and might even be preferable in terms of the national unemployment rate. A phase-out until 2035 leads to a faster increase of other energy sources by increasing energy prices. This causes more employment in the rest of Germany. Albeit our simulation results do not explicitly model other energy sources we very likely underestimate the required employees in the non-lignite energy sector to replace the lignite industry. An earlier exit date is therefore very unlikely to increase negative employment effects. Nevertheless, this finding depends not only on the technical feasibility of the phase-out paths, but also on the assumption that migration is only determined by long-run variables and does not vary with the timing of the decommissioning plan. Therefore, migration takes place in all total phase-out scenarios at the same speed. Our sensitivity analysis identifies that the persistence in unemployment benefits, the demand for domestic non-energy products and the persistence in market power are important for the maximum drop in employment, labour income and consumption. Policy measures to reduce the impact of a coal phase-out should focus on the flexibility of wages and unemployment benefits, but should also lower formal and informal costs of starting a business to reduce market power.

The potential employment effects in absolute terms seem to be large, but with regard to the labour force of Germany rather small. Moreover, the labour force in Germany will decrease by 3.5 million by 2040, i.e. 8% of the labour force in 2014, due to demographic change. Compared to the effects of demographic developments in Germany, the lignite phase-out has relatively small effects. Furthermore, our analysis excluded any potential technical progress in other energy sectors, such as the renewable energy sector. Potential technological improvements in these sectors might crowd out lignite as an energy source. Neither have increasing extraction costs been considered. These developments would reduce the potential economic effects of a politically induced lignite phase-out in Germany. Our results show that postponing the phase-out will only move negative effects more into the future.

Our analysis did not consider potential effects of higher energy prices on the current account of Germany. In recent years, Germany has been a net electricity exporter and, hence, a coal phase-out might turn Germany into a net electricity importer, i.e. importing electricity that might be generated by lignite in neighbouring countries, such as the Czech Republic and Poland. However, this seems to be unlikely due to capacity constraints in these countries (Matthes et al. 2018).

An unsettled issue is whether a coal-phase out is the abatement-cost minimizing policy to achieve the national greenhouse gas emissions targets. The decommissioning of coal fired power plants is an additional national measure parallel to the European Union Emissions Trading Scheme. Hybrid regulations to reduce greenhouse gas emissions are inefficient compared to purely market based mechanisms (Böhringer et al. 2006). Most studies investigating the abatement costs of different policies use static estimates (see Gillingham & Stock 2018) and ignore intertemporal dependencies. For instance, Lin & Chen (2019) show that higher electricity prices lead to more innovations in the renewable energy sector in the long run. Our analysis ignores the costs of stranded assets implied by the transition from lignite electricity generation to non-lignite electricity generation (see Rozenberg et al. 2020). The main source of stranded assets induced by a lignite phase-out in Germany is a shorter life time of lignite power plants and mining fields. The book value of old lignite power plants is already close to zero. If more efficient and younger power plants operate longer, then the effect of stranded assets is reduced. Further, coal fired power plants can be modified to run based on other energy sources, reducing also the opportunity costs to continue the operation of coal fired power plants based on lignite. Future research should evaluate the impact of different policies and stranded assets on dynamic abatement costs. Nevertheless, the major share of abatement costs associated with a lignite phase-out in Germany is very likely a lower labour market income in the lignite regions.

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3.7 Appendix

3.7.1 Calibration

3.7.1.1 Initial Steady State

Annual trend inflation is assumed to be 2% and the corresponding gross trend inflation π^c is set to 1.02. The annual real per capita technological trend growth rate is set to 0.75% corresponding to a real per capita gross growth rate μ^{z^*} of 1.0075. The discount factor β is set to 0.998.[9] Employment shares are set in such a way that the values in Table 3.1 are obtained. Furthermore, to reflect the relative importance of each sector for regional labour income, we set the labour productivity exponent in the production function $\alpha_{r,l}$ to be

$$\alpha_{r,l} = \frac{1 + (1 - \tau_{r,l})\, \epsilon_{r,l}\, ((1 - (\mu^{pop} - \delta)\, \beta\, \mu^z\, \pi^c)\, \psi\, \kappa^w_{r,l}\, \frac{n_{r,l}}{h_{r,l}}}{(1 - \tau_{r,k})\, \epsilon_{r,l}\, (\frac{1}{\phi^w_{r,l}} + \kappa^w_{r,l})}. \tag{3.28}$$

We take into account the labour cost shares $\phi^w_{r,l} = \frac{w_{r,l} n_{r,l}}{y_{r,l}}$ reported in Table 3.6. The share of hiring costs relative to the wage bill $\kappa^w_{r,l} = \frac{\kappa_{r,l}}{w_{r,l}\, n_{r,l}}$ is 6.5%, in line with Christiano et al. (2016). The same formula holds for the energy input sectors. The exogenous separation rate δ is set such that the job finding rate x_t is 22%. A short-term unemployed person in Germany (less than one year) receives 60% of the last year's average net wage, depending on their family status, and a long-term unemployed person in Germany receives a compensation of 42% of the current net wage in Germany, including housing and other assistance.[10] The share of unemployed who are long-term unemployed in steady-state is $(1-x)^4$ and is around 37%. The effective labour tax rate τ^l is set to 35% (see Hristov 2016, p. 22). Unemployment benefits for a representative unemployed person in steady-state is a weighted average over unemployment benefits for short-term and long-term benefits. Therefore, unemployment benefits are set to 35% relative to the real gross wage $b = \psi^w\, w$ in steady-state. A regression of annual real unemployment benefits on past realizations shows that the persistence parameter ρ^b is estimated to be 0.85. The inverse Frisch elasticity of substitution $\sigma^L_{r,l}$ is set to 0.25 (see King & Rebelo 1999, p. 975).

In order to match the share of gross value-added to total production in Germany $\phi^y_{r,k}$

[9] The calibration of the parameters is summarized in Table 3.10.

[10] Replacement rates for long-term and short-term unemployed are reported by the OECD: `http://www.oecd.org/els/soc/benefits-and-wages.htm`

reported in Table 3.6, the productivity parameters $a_{r,k}$ are calculated by

$$\Omega^w_{r,k} = 1 - \frac{\kappa^w_{r,k}}{\frac{1}{\phi^w_{r,k}} + \epsilon_{r,k}\kappa^w_{r,k}}, \tag{3.29}$$

$$\Delta_{r,k,s} = \epsilon_{r,k,s}\, n_{r,k,s}^{\alpha_{r,k,s}}\, \Omega^w_{r,k,s} \left(\frac{\phi_{r,k,s}\, {\theta^y_{r,k,s}}^{-\eta^b}}{1-\tau_{r,k,s}}\right)^{\frac{1}{\eta^b-1}}, \tag{3.30}$$

$$\Delta_{r,k} = \begin{cases} \sum_{d=1}^{R} \frac{\phi_{d,k,s}^{\frac{1}{\eta^b}}\, \theta^y_{r,k}\, (1-\tau_{r,k,s})\, \epsilon_{r,k,s}\, \left(\frac{\sum_{r=1}^{S} \phi_{r,k,r}\, \Delta_{r,k,s}}{\Delta_{r,k,r}}\right)}{(1-\tau_{r,k})\, w_r^{pop}} & \text{,if } k = \text{E}, \\ \frac{\theta^y_{r,k}}{(1-\tau_{r,k})\, w_r^{pop}\, \gamma_{r,k}\, \epsilon_{r,k}\, n_{r,k}^{\alpha_{r,k}}\, \Omega^w_{r,k}} & \text{,if } k = \text{NE}, \end{cases} \tag{3.31}$$

$$a_{r,k} = \bar{a} \sum_{r=1}^{R} \sum_{h=1}^{K} \frac{1}{K\, R}\, \frac{\theta^y_{r,h}\, \Delta_{r,k}}{\theta^y_{r,k}\, \Delta_{d,h}}, \tag{3.32}$$

$$a_{r,k,s} = \left(\Delta_{r,k,s} \sum_{q=1}^{S} \frac{\phi_{r,k,q}}{\Delta_{r,k,q}}\right)^{-1} a_{r,k}. \tag{3.33}$$

The productivity parameters are rescaled such that on average $\bar{a} = 1$. In our special case, taxation is the same for each sector and region. The tax rate $\tau_{r,l}$ on sales is 19%, which corresponds to the value-added tax in Germany. Net value-added shares are identical to gross value-added shares, because tax rates are the same for each sector.

The CES demand weights $\omega^d_{r,o,k}$ are calibrated to reflect a home bias and transaction costs for trade between regions. Furthermore, the relative productivity profile and the size of the population are taken into consideration.

$$\omega^{pop,d}_{r,o,k} = \begin{cases} I^{HomeBias}_{r,k} & \text{if } r = d \\ (1 - I^{HomeBias}_{r,k})\, \frac{w_r^{pop}}{\sum_{o\notin r}^{R} w_o^{pop}} & \text{else} \end{cases}. \tag{3.34}$$

$$\omega^d_{r,o,k} = \frac{\omega^{pop,d}_{r,o,k}\, a_{r,k}}{\sum_{o=1}^{R} \omega^{pop,d}_{r,o,k}\, a_{o,k}}. \tag{3.35}$$

In the non-energy sector, non-tradable and tradable goods are combined. According to Hristov (2016), the non-tradable share in Germany is 0.56 and the tradable home bias is 0.6, therefore we have a home bias in the non-energy sector of $0.85 \approx 0.56 + 0.6 * (1 - 0.56)$. The home bias share for the energy sector is set to 0.5. The scaling coefficients for marginal hiring costs $B_{r,l}$ are

$$B^h_{r,l} = \frac{mc^h_{r,l}}{\Omega + (1-\Omega)\left(\frac{h_r}{u^s_r}\right)^v} \text{ with } mc^h_{r,l} = \psi\, \kappa^w_{r,l}\, w_{r,l}\, n_{r,l} h_{r,l}^{-\psi}. \tag{3.36}$$

Adjustment costs to the employment stock of a firm are quadratic $\psi = 2$. To fulfill the first-order conditions for wages, the disutility parameters $A^L_{r,l}$ are set such that

$$A^L_{r,l} = \left(1 - \tau^l\right) \frac{\gamma_{r,l}\, w^*_{r,l} - b_r}{\gamma^c_r\, n_{r,l}^{\sigma^L_{r,l}}}. \tag{3.37}$$

The elasticity of substitution between the energy and non-energy sectors η^c is estimated from gross-value-added data. The point estimate is 0.75, implying that the energy and non-energy sector are complements. Therefore, a price increase in one sector causes a reduction in demand for the other sector. To estimate the elasticity of substitution, we use (3.3).

The regional elasticity of substitution for non-energy products η^m_{NE} is estimated by pooled OLS with national accounts data for the German states. A point estimate of 1.15 is estimated in line with an estimate from Hristov (2016) for tradable regional products between European countries and Germany.

The regional elasticity of substitution for energy products η^m_E is estimated by pooled ordinary least squares with national accounts data for the German states. The point estimate is 800, in line with the fact that electricity and other products of the energy sector from different regions are perfect substitutes.

Unfortunately, there is no data source with which to estimate the elasticity of substitution between lignite and non-lignite $\eta^b \in (1, \ldots, \infty]$. Therefore, we calibrate this parameter to the smallest value such that a permanent sector productivity shock to lignite in one region triggers a non-negative employment reaction in the non-lignite energy sector of the region. This reaction depends on the relative elasticities of substitution between and within regions. The smallest value fulfilling this condition is $\eta^b = 20.6$. This value indicates also that other inputs to the energy sector are almost perfect substitutes.

3.7.1.2 Terminal Steady State

We simulate permanent shocks to sector productivity $\epsilon_{r,E,LC,t}$ of lignite in Germany. A decommissioning plan implies a stepwise reduction of sector specific productivity. We assume that the decommissioning plan is certain and irreversible.

The shutdown of lignite power plants implies new long-run differentials in sector productivity of German lignite regions and the rest of Germany. Our simulation is the transition from one deterministic steady-state to another. The terminal steady-state is calculated by solving the static equations of the model given the new sector productivity profile. More precisely, it is necessary to find new employment shares such that the first-order conditions of the households with respect to labour are satisfied. It is also necessary to find the relative prices given arbitrary employment shares such that the market clearing conditions hold.

Unobserved characteristics of regional attractiveness $\eta^{pop}_{r,t}$ adjust such that in the terminal steady-state regional total employment shares are the same as before. Migration leads to different population shares $w^{pop}_{r,t}$ and to different demands for products from each region.

A higher attractiveness of a region increases its population density, triggers higher housing prices and, therefore, alters the desired mark-ups in all sectors because the population density increases. This idea originates from Grossmann et al. (2017), who postulate that migration flows increase prices in regions with higher population densities through higher housing prices. We further assume that firms operating in regions with a higher attractiveness leading to higher population density can charge higher mark-ups than those in regions with lower attractiveness and population density.

The auto-correlation coefficient ρ^{pop} is set such that a population shock has a half-life of 22.5 years, corresponding to one-half of the time an individual participates in the labour force. Our implied annual share of individuals actively deciding to migrate is 3%. New individuals are assumed to have different preferences for where to live. In our set-up, the long-run attractiveness $\eta_r^{pop} = f_{r,T|t}(u(\{C_{r,k,T|t}(h)\}_{k=1}^{K}, \{N_{r,l,T|t}(h)\}_{l=1}^{L}))$ is a function of the terminal steady-state values of the endogenous variables of the model $\bar{Z}$ and exogenous variables X such that regional employment rates return to their original steady-state. Write $\tilde{Z}$ for the steady-state vector of endogenous variables without regional employment rates n_r and regional preferences η_r^{pop}. We can express regional employment shares as a function of living preferences

$$\bar{n}_r = f\{\eta_r^{pop}, \tilde{Z}(\bar{n}_r, \eta_r^{pop}, X), X\}. \tag{3.38}$$

Therefore, the steady-state is given such that $\bar{n}_r$ corresponds to the initial value given the new vector of exogenous and endogenous variables. The steady-state values of endogenous variables without regional employment rates depend on the regional employment rates, living preferences, and steady-state values of the exogenous variables. We are only able to find a numerical solution and not an analytical solution.

3.7.2 Tables

Table 3.5: Lignite labour market regions

Central Germany	Lusatia	Rhineland
	Territory	
Landkreis Leipzig	Landkreis Elbe-Elster	Rhein-Kreis Neuss
Stadt Leipzig	Landkreis Oberspreewald-Lausitz	Kreis Düren
Burgenlandkreis	Landkreis Spree-Neiße	Rhein-Erft-Kreis
Nordsachsen	Stadt Cottbus	Städteregion Aachen
Saalekreis	Landkreis Bautzen	Kreis Heinsberg
Stadt Halle	Landkreis Görlitz	Kreis Euskirchen
Landkreis Mansfeld-Südharz		Stadt Mönchengladbach
	Labour Market Region	
Erzgebirgskreis		Düsseldorf
Mittelsachsen		Krefeld
Zwickau		Leverkusen
		Mettmann
		Kreis Heinsberg
		Mettmann
		Rheinisch-Bergischer Kreis
		Viersen

Note: The counties belonging to territories using lignite and counties building a labour market region with the former ones are tabulated.
Sources: German Federal Ministry for Economic Affairs and Kosfeld & Werner (2012).

Table 3.6: Gross value-added shares

Region	Energy		Non-Energy	Total
	Lignite Coal	Non-Lignite Coal		
Rest of Germany	0.002	1.81	98.18	88.66
Central Germany	0.42	2.94	96.64	2.70
Lusatia	3.86	4.92	91.22	0.86
Rhineland	0.60	2.08	97.32	7.78
Germany	0.09	1.89	98.02	100.00

Note: Gross value-added shares in 2014 in percent. Total states the share of gross value-added of the region in national gross value-added.
Sources: German Federal Statistical Office and own calculations.

Table 3.7: Labour shares

Region	Energy		Non-Energy	Total
	Lignite Coal	Non-Lignite Coal		
Rest of Germany	50.87	35.65	56.85	56.28
Central Germany	54.48	31.55	57.52	56.75
Lusatia	60.09	24.75	55.61	56.75
Rhineland	58.42	46.32	57.53	54.27
Germany	58.37	36.15	56.91	57.30

Note: Labour shares for 2014 in percent. The ratio is the wage sum of the respective sector divided by gross value added in the sector.
Sources: German Federal Statistical Office and own calculations.

Table 3.8: Symbols of variables

Symbol	Description
z	exogenous common trend
tax	tax
n	employment
y	output
b	unemployment benefit
u	unemployment
U	utility
h	hiring rate
w	real wage
c	consumption
g	government spending
tr	government transfers
γ	relative producer prices
γ^c	relative consumption prices
u^s	unemployment before hiring
u	unemployment rate
w^{pop}	population weight
τ	effective tax rate for the firm
h	hiring rate
x	job finding rate
mc^h	marginal hiring cost
κ	hiring cost
w^*	optimal real wage
λ	mark-up
$\pi^{profits}$	profits
ϵ	technology shocks
ϵ^l	labour preference shock
ϵ^h	hiring cost shock
ϵ^{pop}	preference shock for living

Table 3.9: Symbols of parameters

Symbol	Description
R	regions
K	sectors
S	input sectors
ω_k^c	CES weight for sector k
η^c	elasticity of substitution between energy and non-energy
η^m	regional elasticity of substitution
a	productivity constants
ϕ	CES shares for energy production with lignite coal
A^L	disutility to labour
B	marginal hiring cost constants
Ω	share of business cycle invariant hiring costs
I_{NE}^{Home}	home bias
ω^d	regional demand preferences
σ^L	inverse Frisch elasticity of labour
σ^λ	constant in law of motion of mark-up equation
$\sigma^{\epsilon^{pop}}$	constant in law of motion of regional attractiveness
α	labour share
δ	separation rate
β	discount factor
v	hiring cost elasticity
ψ	exponent for hiring costs
η^c	elasticity of substitution between sectors
η^b	elasticity of substitution between coal and non coal
τ^l	tax rate on labour
π^c	steady-state inflation
μ^z	growth rate of exogenous trend z_t
μ^{pop}	population growth rate
f	AR(1) coefficient for real wage rigidity
ρ^b	AR(1) coefficient for adjustment replacement rate
ρ_ϵ	persistence productivity shock
ρ_ϵ^{pop}	AR(1) coefficient for living preferences
ρ^λ	persistence in mark-up

Table 3.10: Parameter values

Parameter	Value	Description	Source
η_E^m	800	elasticity of substitution between regions for energy	estimated from regional national account data
$\bar{\tau}$	0.190	VAT tax rate	Hristov (2016)
I_E^{Home}	0.500	home bias	calibrated
I_{NE}^{Home}	0.8500	home bias	calibrated according to Hristov (2016)
σ^L	0.25 / 0.001	inverse Frisch elasticity / inverse Frisch elasticity lignite Rest of Germany	King & Rebelo (1999) and calibration
$\bar{\lambda}_n$	1.250	market power in region n at start	calibrated
η_n^b	20.600	elasticity of substitution between lignite coal and non lignite coal in region n	calibrated
δ	$\frac{x}{1-x}\frac{1-n}{n}\mu^{pop}$	separation rate	computed
x	0.22	steady-state job finding rate	according to long-term unemployed share
β	0.998	discount factor	Hristov (2016)
υ	1.000	hiring cost elasticity to labour market tightness	Blanchard & Galí (2010)
ρ_ϵ^{pop}	0.9925	persistence in living preferences	calibrated
ρ^λ	0.900	posterior mode of mark-up shocks	estimated by Smets & Wouters (2007)
ρ^b	0.85	AR(1) coefficient for adjustment replacement rate	estimated from OECD data
ψ	2.000	exponent for hiring costs	calibrated
η^c	0.750	elasticity of substitution between energy and non-energy sector	estimated from national accounts data
τ^l	0.350	tax rate on labour	Hristov (2016)
Ω	0.950	share of invariant business cycle varying hiring costs	Christiano et al. (2016)
ζ^w	0.350	unemployment benefits replacement ratio	estimated
π^c	1.005	steady-state inflation	long-run inflation target of the ECB
μ^z	1.002	steady-state growth rate	potential growth rate according to IWH
μ^{pop}	0.999	steady-state population growth rate	average of projected labour force growth (see Figure 3.6).

Table 3.11: Employees in lignite sector

Year	Germany	Rest of Germany	Central Germany	Lusatia	Rhineland
			Null-Scenario		
2014	21.2 [0.0;0.0]	0.5 [0.0;0.0]	2.4 [0.0;0.0]	8.1 [0.0;0.0]	10.2 [0.0;0.0]
2020	-0.1 [0.0;0.0]	-0.4 [0.0;0.0]	0.0 [0.0;0.0]	0.1 [0.0;0.0]	0.2 [0.0;0.0]
2025	-0.7 [0.0;0.0]	-0.5 [0.0;0.0]	0.0 [0.0;0.0]	-0.1 [0.0;0.0]	-0.1 [0.0;0.0]
2030	-1.4 [0.0;0.0]	-0.5 [0.0;0.0]	-0.1 [0.0;0.0]	-0.4 [0.0;0.0]	-0.4 [0.0;0.0]
2035	-2.0 [0.0;0.0]	-0.5 [0.0;0.0]	-0.2 [0.0;0.0]	-0.6 [0.0;0.0]	-0.7 [0.0;0.0]
2040	-2.3 [0.0;0.0]	-0.5 [0.0;0.0]	-0.2 [0.0;0.0]	-0.7 [0.0;0.0]	-0.9 [0.0;0.0]
			Baseline		
2020	-3.0 [0.0;0.2]	0.0 [0.0;0.0]	-0.3 [0.0;0.0]	-1.2 [0.0;0.1]	-1.5 [0.0;0.1]
2025	-4.5 [0.0;0.0]	0.0 [0.0;0.0]	-0.5 [0.0;0.0]	-1.8 [0.0;0.0]	-2.2 [0.0;0.0]
2030	-5.9 [0.0;0.1]	0.0 [0.0;0.0]	-0.7 [0.0;0.0]	-2.3 [0.0;0.0]	-2.9 [0.0;0.1]
2035	-8.1 [-0.2;0.1]	0.0 [0.0;0.0]	-0.9 [0.0;0.0]	-3.2 [-0.1;0.0]	-4.0 [-0.1;0.1]
2040	-9.2 [-0.1;0.0]	0.0 [0.0;0.0]	-1.1 [0.0;0.0]	-3.6 [0.0;0.0]	-4.5 [-0.1;0.0]
			Phase-Out-2035-Weak		
2020	-4.6 [-0.2;0.1]	0.0 [0.0;0.0]	-0.4 [0.0;0.1]	-1.7 [-0.1;0.0]	-2.5 [-0.1;0.0]
2025	-9.4 [-0.2;0.2]	0.0 [0.0;0.0]	-1.0 [0.0;0.0]	-3.3 [-0.1;0.1]	-5.1 [-0.1;0.1]
2030	-13.8 [-0.3;0.1]	0.0 [0.0;0.0]	-1.6 [-0.1;0.0]	-5.3 [-0.1;0.1]	-6.9 [-0.1;0.0]
2035	-18.9 [-0.1;0.0]	0.0 [0.0;0.0]	-2.2 [0.0;0.0]	-7.4 [0.0;0.0]	-9.3 [-0.1;0.0]
2040	-18.9 [0.0;0.0]	0.0 [0.0;0.0]	-2.2 [0.0;0.0]	-7.4 [0.0;0.0]	-9.3 [0.0;0.0]
			Phase-Out-2040-Age		
2020	-4.0 [-0.1;0.1]	0.0 [0.0;0.0]	-0.4 [0.0;0.0]	-1.4 [0.0;0.1]	-2.2 [-0.1;0.0]
2025	-7.3 [-0.2;0.1]	0.0 [0.0;0.0]	-0.7 [0.0;0.0]	-2.5 [-0.1;0.0]	-4.1 [-0.1;0.1]
2030	-10.5 [-0.2;0.1]	0.0 [0.0;0.0]	-1.0 [0.0;0.0]	-3.5 [-0.1;0.1]	-6.0 [-0.1;0.0]
2035	-13.8 [-0.1;0.2]	0.0 [0.0;0.0]	-1.5 [0.0;0.0]	-5.3 [-0.1;0.1]	-7.0 [0.0;0.1]
2040	-18.8 [-0.2;0.0]	0.0 [0.0;0.0]	-2.2 [0.0;0.0]	-7.4 [-0.1;0.0]	-9.2 [-0.1;0.0]
			Phase-Out-2040-Balanced		
2020	-4.1 [-0.2;0.0]	0.0 [0.0;0.0]	-0.3 [0.0;0.0]	-1.8 [-0.1;0.0]	-2.0 [-0.1;0.0]
2025	-7.5 [-0.2;0.1]	0.0 [0.0;0.0]	-0.6 [0.0;0.0]	-3.5 [-0.1;0.1]	-3.4 [-0.1;0.0]
2030	-10.4 [-0.2;0.0]	0.0 [0.0;0.0]	-1.0 [0.0;0.0]	-4.4 [-0.1;0.0]	-5.0 [-0.1;0.0]
2035	-13.6 [-0.2;0.1]	0.0 [0.0;0.0]	-1.6 [0.0;0.1]	-5.4 [-0.1;0.0]	-6.6 [-0.1;0.0]
2040	-18.7 [-0.1;0.1]	0.0 [0.0;0.0]	-2.2 [0.0;0.0]	-7.3 [0.0;0.1]	-9.2 [-0.1;0.0]
			Phase-Out-2035-Strong		
2020	-6.2 [-0.2;0.1]	0.0 [0.0;0.0]	-0.6 [0.0;0.0]	-2.4 [-0.1;0.0]	-3.2 [-0.1;0.1]
2025	-10.5 [-0.1;0.2]	0.0 [0.0;0.0]	-1.1 [0.0;0.0]	-3.7 [0.0;0.1]	-5.7 [-0.1;0.1]
2030	-13.9 [-0.3;0.1]	0.0 [0.0;0.0]	-1.6 [-0.1;0.0]	-5.3 [-0.1;0.1]	-7.0 [-0.1;0.0]
2035	-19.0 [-0.1;0.1]	0.0 [0.0;0.0]	-2.2 [0.0;0.0]	-7.4 [0.0;0.1]	-9.4 [-0.1;0.0]
2040	-18.9 [0.0;0.0]	0.0 [0.0;0.0]	-2.2 [0.0;0.0]	-7.4 [0.0;0.0]	-9.3 [0.0;0.0]

Note: Simulation results for employees in the lignite industry in thousand people. Values for the Null-Scenario are reported as change to the base year 2014 and for the year 2014 actual values are reported and for the year 2014 actual values are reported. Values for other scenarios are differences to the Null-Scenario in the respective year. Values in brackets denote the minimum and maximum difference from the reported value obtained from 1200 simulations.

Table 3.12: Unemployment rates

Year	Germany	Rest of Germany	Central Germany	Lusatia	Rhineland
			Null-Scenario		
2014	5.67 [-0.00;0.00]	5.32 [0.00;0.00]	9.17 [0.00;0.00]	10.97 [-0.00;0.00]	7.34 [-0.00;0.00]
2020	0.00 [-0.00;0.00]	0.00 [-0.00;0.00]	0.00 [-0.00;0.00]	-0.00 [-0.00;0.00]	0.00 [-0.00;0.00]
2025	0.00 [-0.00;0.00]	0.00 [-0.00;0.00]	0.00 [-0.00;0.00]	-0.00 [-0.00;0.00]	0.00 [-0.00;0.00]
2030	0.00 [-0.00;0.00]	0.00 [-0.00;0.00]	-0.00 [-0.00;0.00]	-0.00 [-0.00;0.00]	-0.00 [-0.00;0.00]
2035	-0.00 [-0.00;0.00]	-0.00 [-0.00;0.00]	-0.00 [-0.00;0.00]	-0.00 [-0.00;0.00]	-0.00 [-0.00;0.00]
2040	-0.00 [-0.00;0.00]	-0.00 [-0.00;0.00]	-0.00 [-0.00;0.00]	-0.00 [-0.00;0.00]	-0.00 [-0.00;0.00]
			Baseline		
2020	0.02 [-0.01;0.01]	0.01 [-0.01;0.01]	0.03 [-0.01;0.01]	0.21 [-0.02;0.03]	0.05 [-0.01;0.01]
2025	0.02 [-0.01;0.01]	0.02 [-0.01;0.01]	0.04 [-0.01;0.01]	0.21 [-0.03;0.02]	0.06 [-0.01;0.01]
2030	0.02 [-0.01;0.01]	0.02 [-0.01;0.01]	0.04 [-0.01;0.01]	0.25 [-0.04;0.04]	0.07 [-0.01;0.01]
2035	0.03 [-0.02;0.01]	0.01 [-0.02;0.02]	0.06 [-0.02;0.01]	0.46 [-0.04;0.04]	0.10 [-0.01;0.01]
2040	0.02 [-0.02;0.02]	0.01 [-0.02;0.02]	0.05 [-0.02;0.01]	0.48 [-0.04;0.03]	0.10 [-0.01;0.01]
			Phase-Out-2035-Weak		
2020	0.06 [-0.02;0.02]	0.06 [-0.02;0.02]	0.08 [-0.02;0.02]	0.20 [-0.04;0.03]	0.10 [-0.02;0.02]
2025	0.10 [-0.03;0.04]	0.09 [-0.03;0.04]	0.14 [-0.03;0.03]	0.44 [-0.05;0.05]	0.21 [-0.03;0.02]
2030	0.12 [-0.04;0.04]	0.10 [-0.04;0.05]	0.18 [-0.05;0.04]	0.81 [-0.07;0.08]	0.24 [-0.04;0.03]
2035	0.10 [-0.05;0.05]	0.08 [-0.05;0.05]	0.16 [-0.05;0.05]	1.07 [-0.07;0.06]	0.25 [-0.05;0.03]
2040	0.06 [-0.06;0.05]	0.04 [-0.06;0.05]	0.10 [-0.06;0.05]	0.89 [-0.06;0.07]	0.18 [-0.05;0.04]
			Phase-Out-2040-Age		
2020	0.06 [-0.02;0.02]	0.06 [-0.02;0.02]	0.08 [-0.02;0.02]	0.17 [-0.04;0.04]	0.10 [-0.01;0.01]
2025	0.10 [-0.03;0.04]	0.09 [-0.03;0.04]	0.13 [-0.03;0.03]	0.25 [-0.05;0.05]	0.18 [-0.03;0.02]
2030	0.12 [-0.04;0.04]	0.11 [-0.04;0.04]	0.16 [-0.04;0.04]	0.35 [-0.06;0.05]	0.24 [-0.04;0.03]
2035	0.13 [-0.04;0.05]	0.11 [-0.05;0.05]	0.18 [-0.05;0.04]	0.69 [-0.07;0.08]	0.24 [-0.04;0.03]
2040	0.11 [-0.05;0.05]	0.09 [-0.05;0.05]	0.16 [-0.05;0.05]	0.95 [-0.07;0.06]	0.24 [-0.05;0.04]
			Phase-Out-2040-Balanced		
2020	0.06 [-0.02;0.02]	0.06 [-0.02;0.02]	0.08 [-0.02;0.02]	0.24 [-0.04;0.03]	0.09 [-0.01;0.01]
2025	0.10 [-0.03;0.04]	0.09 [-0.03;0.04]	0.11 [-0.03;0.03]	0.56 [-0.06;0.06]	0.14 [-0.02;0.02]
2030	0.12 [-0.04;0.04]	0.11 [-0.04;0.04]	0.15 [-0.04;0.04]	0.59 [-0.06;0.06]	0.20 [-0.03;0.03]
2035	0.13 [-0.04;0.05]	0.11 [-0.05;0.05]	0.18 [-0.05;0.04]	0.69 [-0.06;0.07]	0.23 [-0.04;0.03]
2040	0.11 [-0.05;0.05]	0.09 [-0.05;0.05]	0.16 [-0.05;0.05]	0.95 [-0.07;0.06]	0.24 [-0.05;0.04]
			Phase-Out-2035-Strong		
2020	0.07 [-0.02;0.02]	0.06 [-0.02;0.03]	0.10 [-0.02;0.02]	0.43 [-0.04;0.05]	0.14 [-0.02;0.01]
2025	0.10 [-0.03;0.04]	0.08 [-0.03;0.04]	0.14 [-0.03;0.03]	0.52 [-0.06;0.05]	0.23 [-0.03;0.02]
2030	0.11 [-0.04;0.04]	0.09 [-0.04;0.05]	0.17 [-0.05;0.04]	0.79 [-0.06;0.07]	0.23 [-0.04;0.03]
2035	0.10 [-0.05;0.05]	0.07 [-0.05;0.05]	0.15 [-0.05;0.04]	1.07 [-0.07;0.06]	0.25 [-0.05;0.03]
2040	0.05 [-0.06;0.05]	0.03 [-0.06;0.05]	0.09 [-0.06;0.05]	0.88 [-0.06;0.07]	0.17 [-0.05;0.04]

Note: Simulation results for unemployment rates. Values for the Null-Scenario are reported as change to the base year 2014 and for the year 2014 actual values are reported. Values for other scenarios are differences to the Null-Scenario in the respective year. Values in brackets denote the minimum and maximum difference from the reported value obtained from 1200 simulations.

Table 3.13: Labour force

Year	Germany	Rest of Germany	Central Germany	Lusatia	Rhineland
			Null-Scenario		
2014	45782.6 [0.0;0.0]	40375.7 [0.0;0.0]	1611.5 [0.0;0.0]	521.9 [0.0;0.0]	3273.5 [0.0;0.0]
2020	910.7 [0.0;0.0]	803.1 [-0.0;0.0]	32.1 [-0.0;0.0]	10.4 [-0.0;0.0]	65.1 [-0.0;0.0]
2025	-263.8 [-0.0;0.0]	-232.7 [-0.0;0.0]	-9.3 [-0.0;0.0]	-3.0 [-0.0;0.0]	-18.8 [-0.0;0.0]
2030	-1915.2 [-0.0;0.0]	-1689.1 [-0.0;0.0]	-67.4 [-0.0;0.0]	-21.8 [-0.0;0.0]	-136.9 [-0.0;0.0]
2035	-3194.1 [-0.0;0.0]	-2817.0 [-0.0;0.0]	-112.4 [-0.0;0.0]	-36.4 [-0.0;0.0]	-228.4 [-0.0;0.0]
2040	-3872.6 [-0.0;0.0]	-3415.3 [-0.0;0.0]	-136.3 [-0.0;0.0]	-44.1 [-0.0;0.0]	-276.9 [-0.0;0.0]
			Baseline		
2020	-0.0 [-0.0;0.0]	2.1 [-0.8;0.5]	-0.1 [-0.0;0.0]	-1.0 [-0.3;0.5]	-0.9 [-0.2;0.3]
2025	0.0 [-0.0;0.0]	3.5 [-1.4;0.8]	-0.2 [-0.0;0.0]	-1.7 [-0.4;0.8]	-1.5 [-0.3;0.5]
2030	0.0 [-0.0;0.0]	4.5 [-1.8;1.1]	-0.3 [-0.1;0.1]	-2.3 [-0.6;1.1]	-2.0 [-0.4;0.7]
2035	0.0 [-0.0;0.0]	5.4 [-2.1;1.3]	-0.3 [-0.1;0.1]	-2.7 [-0.7;1.3]	-2.3 [-0.5;0.8]
2040	0.0 [-0.0;0.0]	6.1 [-2.4;1.4]	-0.4 [-0.1;0.1]	-3.1 [-0.8;1.4]	-2.7 [-0.6;0.9]
			Phase-Out-2035-Weak		
2020	-0.0 [-0.0;0.0]	3.4 [-1.3;0.8]	-0.1 [-0.0;0.0]	-1.9 [-0.5;0.9]	-1.5 [-0.3;0.5]
2025	-0.0 [-0.0;0.0]	5.7 [-2.2;1.2]	-0.1 [-0.0;0.0]	-3.2 [-0.8;1.5]	-2.5 [-0.5;0.8]
2030	0.0 [-0.0;0.0]	7.4 [-2.8;1.6]	-0.1 [-0.1;0.0]	-4.1 [-1.0;1.9]	-3.2 [-0.6;1.0]
2035	0.0 [-0.0;0.0]	8.9 [-3.4;1.9]	-0.1 [-0.1;0.0]	-4.9 [-1.2;2.2]	-3.8 [-0.7;1.2]
2040	0.0 [-0.0;0.0]	10.1 [-3.8;2.2]	-0.2 [-0.1;0.0]	-5.6 [-1.4;2.6]	-4.3 [-0.8;1.3]
			Phase-Out-2040-Age		
2020	-0.0 [-0.0;0.0]	3.4 [-1.3;0.8]	-0.1 [-0.0;0.0]	-1.9 [-0.5;0.9]	-1.5 [-0.3;0.5]
2025	-0.0 [-0.0;0.0]	5.7 [-2.2;1.2]	-0.1 [-0.0;0.0]	-3.2 [-0.8;1.5]	-2.5 [-0.5;0.8]
2030	0.0 [-0.0;0.0]	7.4 [-2.8;1.6]	-0.1 [-0.1;0.0]	-4.1 [-1.0;1.9]	-3.2 [-0.6;1.0]
2035	0.0 [-0.0;0.0]	8.9 [-3.4;1.9]	-0.1 [-0.1;0.0]	-4.9 [-1.2;2.2]	-3.8 [-0.7;1.2]
2040	0.0 [-0.0;0.0]	10.1 [-3.8;2.2]	-0.2 [-0.1;0.0]	-5.6 [-1.4;2.6]	-4.3 [-0.8;1.3]
			Phase-Out-2040-Balanced		
2020	-0.0 [-0.0;0.0]	3.4 [-1.3;0.8]	-0.1 [-0.0;0.0]	-1.9 [-0.5;0.9]	-1.5 [-0.3;0.5]
2025	-0.0 [-0.0;0.0]	5.7 [-2.2;1.2]	-0.1 [-0.0;0.0]	-3.2 [-0.8;1.5]	-2.5 [-0.5;0.8]
2030	0.0 [-0.0;0.0]	7.4 [-2.8;1.6]	-0.1 [-0.1;0.0]	-4.1 [-1.0;1.9]	-3.2 [-0.6;1.0]
2035	0.0 [-0.0;0.0]	8.9 [-3.4;1.9]	-0.1 [-0.1;0.0]	-4.9 [-1.2;2.2]	-3.8 [-0.7;1.2]
2040	0.0 [-0.0;0.0]	10.1 [-3.8;2.2]	-0.2 [-0.1;0.0]	-5.6 [-1.4;2.6]	-4.3 [-0.8;1.3]
			Phase-Out-2035-Strong		
2020	-0.0 [-0.0;0.0]	3.4 [-1.3;0.8]	-0.1 [-0.0;0.0]	-1.9 [-0.5;0.9]	-1.5 [-0.3;0.5]
2025	-0.0 [-0.0;0.0]	5.7 [-2.2;1.2]	-0.1 [-0.0;0.0]	-3.2 [-0.8;1.5]	-2.5 [-0.5;0.8]
2030	0.0 [-0.0;0.0]	7.4 [-2.8;1.6]	-0.1 [-0.1;0.0]	-4.1 [-1.0;1.9]	-3.2 [-0.6;1.0]
2035	0.0 [-0.0;0.0]	8.9 [-3.4;1.9]	-0.1 [-0.1;0.0]	-4.9 [-1.2;2.2]	-3.8 [-0.7;1.2]
2040	0.0 [-0.0;0.0]	10.1 [-3.8;2.2]	-0.2 [-0.1;0.0]	-5.6 [-1.4;2.6]	-4.3 [-0.8;1.3]

Note: Simulation results for the labour force by region in thousand people. Values for the Null-Scenario are reported as change to the base year 2014 and for the year 2014 actual values are reported. Values for other scenarios are differences to the Null-Scenario in the respective year. Values in brackets denote the minimum and maximum difference from the reported value obtained from 1200 simulations.

Table 3.14: Employees

Year	Germany	Rest of Germany	Central Germany	Lusatia	Rhineland
Null-Scenario					
2014	43188.7 [0.0;0.0]	38227.0 [0.0;0.0]	1463.8 [0.0;0.0]	464.7 [0.0;0.0]	3033.2 [0.0;0.0]
2020	858.1 [-0.3;0.3]	759.5 [-0.2;0.3]	29.1 [-0.1;0.0]	9.2 [0.0;0.0]	60.3 [0.0;0.0]
2025	-249.6 [-0.3;0.4]	-221.0 [-0.3;0.4]	-8.4 [0.0;0.0]	-2.7 [0.0;0.0]	-17.5 [0.0;0.0]
2030	-1807.1 [-0.4;0.5]	-1599.5 [-0.4;0.5]	-61.2 [0.0;0.0]	-19.5 [0.0;0.0]	-126.9 [0.0;0.0]
2035	-3013.2 [-0.4;0.5]	-2667.1 [-0.4;0.4]	-102.1 [0.0;0.0]	-32.4 [0.0;0.1]	-211.6 [0.0;0.0]
2040	-3653.0 [-0.5;0.5]	-3233.4 [-0.4;0.4]	-123.8 [-0.1;0.0]	-39.3 [0.0;0.0]	-256.5 [0.0;0.1]
Baseline					
2020	-8.1 [-4.0;3.6]	-3.0 [-4.6;3.6]	-0.6 [-0.1;0.1]	-2.0 [-0.3;0.5]	-2.5 [-0.2;0.3]
2025	-10.0 [-5.4;5.2]	-3.4 [-6.3;5.2]	-0.8 [-0.1;0.2]	-2.6 [-0.3;0.8]	-3.2 [-0.3;0.3]
2030	-10.5 [-6.2;6.3]	-2.4 [-7.2;6.4]	-0.9 [-0.1;0.2]	-3.3 [-0.5;0.8]	-3.9 [-0.4;0.4]
2035	-11.5 [-6.5;7.0]	-0.3 [-7.6;7.4]	-1.2 [-0.2;0.2]	-4.7 [-0.7;1.1]	-5.3 [-0.6;0.7]
2040	-8.1 [-6.3;7.0]	3.5 [-7.3;7.4]	-1.1 [-0.1;0.2]	-5.0 [-0.7;1.3]	-5.5 [-0.6;0.7]
Phase-Out-2035-Weak					
2020	-28.9 [-10.8;8.7]	-19.9 [-10.6;8.4]	-1.3 [-0.3;0.3]	-2.8 [-0.4;0.8]	-4.9 [-0.5;0.5]
2025	-46.2 [-16.5;14.4]	-29.6 [-16.4;13.9]	-2.3 [-0.6;0.5]	-5.1 [-0.7;1.3]	-9.2 [-0.9;1.1]
2030	-50.9 [-19.2;18.5]	-30.0 [-19.1;17.6]	-2.8 [-0.6;0.7]	-7.7 [-1.0;1.7]	-10.4 [-1.0;1.4]
2035	-44.5 [-20.0;21.9]	-21.2 [-20.0;19.8]	-2.5 [-0.7;0.8]	-9.5 [-1.1;2.0]	-11.3 [-1.2;1.5]
2040	-23.8 [-20.0;23.7]	-3.6 [-20.3;21.3]	-1.5 [-0.6;0.9]	-9.2 [-1.2;2.2]	-9.5 [-1.3;1.5]
Phase-Out-2040-Age					
2020	-29.0 [-10.4;8.2]	-20.5 [-10.5;7.9]	-1.3 [-0.3;0.3]	-2.6 [-0.4;0.9]	-4.6 [-0.4;0.6]
2025	-46.0 [-16.0;13.2]	-31.8 [-16.0;12.7]	-2.1 [-0.5;0.5]	-4.1 [-0.6;1.4]	-8.0 [-0.7;0.9]
2030	-54.2 [-18.7;16.6]	-35.9 [-18.7;16.1]	-2.5 [-0.6;0.6]	-5.4 [-0.9;1.6]	-10.4 [-1.1;1.3]
2035	-54.6 [-19.4;19.1]	-33.4 [-19.7;18.4]	-2.8 [-0.6;0.7]	-7.7 [-1.0;2.1]	-10.7 [-1.1;1.5]
2040	-47.0 [-19.6;21.6]	-23.7 [-19.9;19.9]	-2.5 [-0.6;0.8]	-9.5 [-1.2;2.2]	-11.3 [-1.1;1.6]
Phase-Out-2040-Balanced					
2020	-29.5 [-10.8;8.0]	-20.7 [-10.6;7.9]	-1.3 [-0.4;0.2]	-3.0 [-0.4;0.7]	-4.5 [-0.5;0.5]
2025	-46.7 [-16.0;13.1]	-32.1 [-15.9;12.7]	-1.9 [-0.6;0.5]	-5.7 [-0.8;1.4]	-7.0 [-0.7;0.8]
2030	-54.6 [-18.8;16.3]	-36.3 [-18.7;15.9]	-2.5 [-0.7;0.5]	-6.6 [-0.9;1.6]	-9.2 [-1.0;1.2]
2035	-55.1 [-19.7;18.8]	-34.1 [-19.8;18.3]	-2.9 [-0.7;0.7]	-7.7 [-1.0;2.0]	-10.4 [-1.1;1.5]
2040	-47.5 [-19.6;21.5]	-24.2 [-19.9;20.0]	-2.4 [-0.6;0.8]	-9.5 [-1.3;2.2]	-11.4 [-1.2;1.5]
Phase-Out-2035-Strong					
2020	-31.9 [-11.1;9.3]	-20.1 [-11.1;9.0]	-1.7 [-0.3;0.3]	-4.0 [-0.6;0.9]	-6.1 [-0.6;0.6]
2025	-45.6 [-16.6;14.7]	-28.1 [-16.6;14.1]	-2.3 [-0.5;0.6]	-5.5 [-0.6;1.3]	-9.7 [-0.9;1.2]
2030	-47.8 [-19.3;18.7]	-27.2 [-19.2;17.6]	-2.7 [-0.6;0.7]	-7.6 [-0.9;1.7]	-10.3 [-1.1;1.3]
2035	-41.1 [-20.0;22.0]	-18.1 [-19.9;19.8]	-2.4 [-0.8;0.8]	-9.5 [-1.1;2.0]	-11.1 [-1.2;1.5]
2040	-20.7 [-19.9;23.3]	-0.8 [-20.1;21.2]	-1.4 [-0.6;0.8]	-9.2 [-1.3;2.2]	-9.3 [-1.3;1.5]

Note: Simulation results for the total number of employees by region in thousand people. Values for the Null-Scenario are reported as change to the base year 2014 and for the year 2014 actual values are reported. Values for other scenarios are differences to the Null-Scenario in the respective year. Values in brackets denote the minimum and maximum difference from the reported value obtained from 1200 simulations.

Table 3.15: Discounted welfare

Year	Germany	Rest of Germany	Central Germany	Lusatia	Rhineland
			Null-Scenario		
2014	100.00 [0.00;0.00]	100.00 [0.00;0.00]	100.00 [0.00;0.00]	100.00 [0.00;0.00]	100.00 [0.00;0.00]
2020	95.31 [-0.91;1.14]	95.31 [-0.91;1.14]	95.31 [-0.91;1.14]	95.31 [-0.91;1.14]	95.31 [-0.91;1.14]
2025	91.57 [-1.61;2.00]	91.57 [-1.61;2.00]	91.57 [-1.61;2.00]	91.57 [-1.61;2.00]	91.56 [-1.62;1.99]
2030	87.97 [-2.26;2.77]	87.97 [-2.26;2.77]	87.97 [-2.26;2.77]	87.97 [-2.26;2.77]	87.97 [-2.26;2.77]
2035	84.52 [-2.86;3.48]	84.52 [-2.86;3.48]	84.52 [-2.86;3.48]	84.52 [-2.86;3.48]	84.52 [-2.86;3.48]
2040	81.20 [-3.42;4.11]	81.20 [-3.42;4.11]	81.20 [-3.42;4.11]	81.20 [-3.42;4.11]	81.20 [-3.42;4.11]
			Baseline		
2020	-0.02 [-0.01;0.01]	-0.02 [-0.01;0.00]	-0.04 [-0.01;0.01]	-0.24 [-0.03;0.02]	-0.07 [-0.01;0.01]
2025	-0.02 [-0.01;0.01]	-0.03 [-0.02;0.00]	-0.03 [-0.02;0.01]	0.00 [-0.04;0.03]	-0.02 [-0.01;0.02]
2030	-0.02 [-0.02;0.01]	-0.02 [-0.01;0.01]	-0.03 [-0.02;0.02]	-0.04 [-0.04;0.07]	-0.04 [-0.02;0.01]
2035	-0.03 [-0.02;0.01]	-0.03 [-0.02;0.01]	-0.07 [-0.03;0.01]	-0.33 [-0.04;0.05]	-0.10 [-0.02;0.01]
2040	-0.02 [-0.01;0.02]	-0.02 [-0.01;0.02]	-0.04 [-0.01;0.02]	-0.14 [-0.04;0.05]	-0.06 [-0.01;0.02]
			Phase-Out-2035-Weak		
2020	-0.05 [-0.03;0.01]	-0.05 [-0.03;0.01]	-0.06 [-0.02;0.02]	-0.09 [-0.06;0.05]	-0.05 [-0.02;0.01]
2025	-0.09 [-0.04;0.02]	-0.08 [-0.03;0.03]	-0.10 [-0.03;0.03]	-0.16 [-0.09;0.07]	-0.21 [-0.03;0.03]
2030	-0.10 [-0.04;0.03]	-0.10 [-0.03;0.03]	-0.17 [-0.03;0.03]	-0.57 [-0.09;0.11]	-0.08 [-0.04;0.04]
2035	-0.10 [-0.03;0.04]	-0.12 [-0.04;0.03]	-0.02 [-0.03;0.04]	-0.08 [-0.07;0.10]	-0.10 [-0.04;0.04]
2040	-0.07 [-0.03;0.04]	-0.08 [-0.02;0.04]	0.02 [-0.03;0.04]	-0.06 [-0.09;0.13]	0.01 [-0.04;0.04]
			Phase-Out-2040-Age		
2020	-0.05 [-0.03;0.01]	-0.05 [-0.03;0.01]	-0.07 [-0.02;0.01]	-0.22 [-0.05;0.05]	-0.07 [-0.02;0.01]
2025	-0.08 [-0.03;0.02]	-0.08 [-0.03;0.02]	-0.10 [-0.03;0.02]	-0.18 [-0.08;0.09]	-0.13 [-0.03;0.03]
2030	-0.09 [-0.03;0.03]	-0.09 [-0.03;0.03]	-0.11 [-0.03;0.03]	-0.16 [-0.08;0.12]	-0.18 [-0.04;0.03]
2035	-0.10 [-0.04;0.04]	-0.11 [-0.04;0.03]	-0.18 [-0.04;0.03]	-0.66 [-0.10;0.14]	-0.10 [-0.03;0.03]
2040	-0.10 [-0.03;0.04]	-0.11 [-0.02;0.04]	-0.02 [-0.04;0.04]	-0.12 [-0.08;0.13]	-0.07 [-0.03;0.05]
			Phase-Out-2040-Balanced		
2020	-0.05 [-0.03;0.01]	-0.05 [-0.03;0.01]	-0.08 [-0.03;0.01]	-0.07 [-0.06;0.05]	-0.09 [-0.02;0.02]
2025	-0.08 [-0.03;0.02]	-0.08 [-0.03;0.02]	-0.08 [-0.03;0.02]	-0.53 [-0.08;0.06]	-0.10 [-0.03;0.03]
2030	-0.09 [-0.03;0.03]	-0.09 [-0.03;0.03]	-0.13 [-0.04;0.02]	-0.11 [-0.07;0.09]	-0.17 [-0.03;0.03]
2035	-0.10 [-0.04;0.04]	-0.11 [-0.04;0.03]	-0.22 [-0.03;0.02]	-0.32 [-0.08;0.12]	-0.13 [-0.04;0.03]
2040	-0.10 [-0.03;0.04]	-0.11 [-0.02;0.04]	0.00 [-0.03;0.04]	-0.14 [-0.07;0.14]	-0.08 [-0.04;0.05]
			Phase-Out-2035-Strong		
2020	-0.06 [-0.03;0.01]	-0.05 [-0.02;0.02]	-0.11 [-0.03;0.01]	-0.48 [-0.06;0.04]	-0.13 [-0.02;0.01]
2025	-0.09 [-0.03;0.02]	-0.09 [-0.03;0.02]	-0.09 [-0.03;0.02]	0.07 [-0.08;0.06]	-0.16 [-0.03;0.03]
2030	-0.09 [-0.03;0.03]	-0.10 [-0.04;0.03]	-0.17 [-0.03;0.03]	-0.58 [-0.08;0.12]	-0.07 [-0.04;0.03]
2035	-0.10 [-0.04;0.04]	-0.11 [-0.03;0.04]	-0.01 [-0.04;0.03]	-0.05 [-0.06;0.11]	-0.07 [-0.04;0.04]
2040	-0.06 [-0.03;0.04]	-0.08 [-0.03;0.04]	0.02 [-0.03;0.03]	-0.06 [-0.09;0.13]	0.02 [-0.03;0.04]

Note: Simulation results for discounted welfare per capita as index. Values for the Null-Scenario are reported as change to the base year 2014. Values for other scenarios are differences to the Null-Scenario in the respective year. Values in brackets denote the minimum and maximum difference from the reported value obtained from 1200 simulations.

Table 3.16: Discounted cumulative welfare

Year	Germany	Rest of Germany	Central Germany	Lusatia	Rhineland
			Null-Scenario		
2014	0.61 [-0.01;0.01]	0.64 [-0.01;0.01]	0.38 [-0.02;0.04]	0.38 [-0.03;0.04]	0.40 [-0.03;0.04]
2020	14.33 [-0.16;0.22]	15.01 [-0.26;0.30]	8.89 [-0.54;0.84]	8.95 [-0.56;0.85]	9.46 [-0.61;0.96]
2025	25.75 [-0.37;0.51]	26.97 [-0.55;0.62]	15.98 [-0.97;1.55]	16.09 [-1.02;1.58]	17.01 [-1.09;1.75]
2030	36.73 [-0.64;0.90]	38.47 [-0.90;1.06]	22.80 [-1.40;2.31]	22.95 [-1.47;2.35]	24.26 [-1.57;2.60]
2035	47.27 [-0.97;1.36]	49.52 [-1.32;1.58]	29.34 [-1.85;3.09]	29.55 [-1.92;3.14]	31.22 [-2.04;3.47]
2040	57.40 [-1.35;1.90]	60.13 [-1.78;2.18]	35.63 [-2.35;3.89]	35.88 [-2.45;3.96]	37.92 [-2.58;4.36]
			Baseline		
2020	-0.00 [-0.00;0.00]	-0.00 [-0.00;0.00]	0.00 [-0.00;0.00]	0.01 [-0.00;0.00]	0.00 [-0.00;0.00]
2025	-0.00 [-0.00;0.00]	-0.00 [-0.00;0.00]	-0.00 [-0.00;0.00]	-0.00 [-0.00;0.00]	-0.00 [-0.00;0.00]
2030	-0.00 [-0.00;0.00]	-0.01 [-0.00;0.00]	-0.00 [-0.00;0.00]	-0.00 [-0.00;0.00]	-0.01 [-0.00;0.00]
2035	-0.01 [-0.01;0.00]	-0.01 [-0.00;0.00]	-0.01 [-0.00;0.00]	-0.00 [-0.01;0.01]	-0.01 [-0.00;0.00]
2040	-0.01 [-0.01;0.01]	-0.01 [-0.01;0.01]	-0.01 [-0.01;0.00]	-0.02 [-0.01;0.01]	-0.02 [-0.01;0.00]
			Phase-Out-2035-Weak		
2020	-0.00 [-0.00;0.00]	-0.00 [-0.00;0.00]	-0.00 [-0.00;0.00]	0.01 [-0.00;0.00]	0.00 [-0.00;0.00]
2025	-0.01 [-0.00;0.00]	-0.01 [-0.00;0.00]	-0.01 [-0.00;0.00]	0.01 [-0.01;0.01]	-0.00 [-0.00;0.00]
2030	-0.02 [-0.01;0.01]	-0.02 [-0.01;0.01]	-0.01 [-0.01;0.01]	0.00 [-0.01;0.01]	-0.02 [-0.01;0.01]
2035	-0.03 [-0.01;0.01]	-0.04 [-0.01;0.01]	-0.02 [-0.01;0.01]	-0.02 [-0.02;0.02]	-0.03 [-0.01;0.01]
2040	-0.04 [-0.02;0.02]	-0.05 [-0.02;0.02]	-0.02 [-0.01;0.01]	-0.02 [-0.02;0.02]	-0.03 [-0.01;0.01]
			Phase-Out-2040-Age		
2020	-0.00 [-0.00;0.00]	-0.00 [-0.00;0.00]	-0.00 [-0.00;0.00]	0.01 [-0.00;0.00]	0.00 [-0.00;0.00]
2025	-0.01 [-0.00;0.00]	-0.01 [-0.00;0.00]	-0.01 [-0.00;0.00]	-0.00 [-0.01;0.01]	-0.00 [-0.00;0.00]
2030	-0.02 [-0.01;0.01]	-0.02 [-0.01;0.01]	-0.02 [-0.01;0.00]	-0.02 [-0.01;0.01]	-0.01 [-0.01;0.01]
2035	-0.03 [-0.01;0.01]	-0.04 [-0.01;0.01]	-0.02 [-0.01;0.01]	-0.03 [-0.02;0.02]	-0.03 [-0.01;0.01]
2040	-0.04 [-0.02;0.01]	-0.05 [-0.02;0.01]	-0.03 [-0.01;0.01]	-0.05 [-0.02;0.03]	-0.04 [-0.01;0.01]
			Phase-Out-2040-Balanced		
2020	-0.00 [-0.00;0.00]	-0.00 [-0.00;0.00]	-0.00 [-0.00;0.00]	0.01 [-0.00;0.00]	0.00 [-0.00;0.00]
2025	-0.01 [-0.00;0.00]	-0.01 [-0.00;0.00]	-0.01 [-0.00;0.00]	0.01 [-0.01;0.01]	-0.01 [-0.00;0.00]
2030	-0.02 [-0.01;0.01]	-0.02 [-0.01;0.01]	-0.01 [-0.01;0.00]	-0.02 [-0.01;0.01]	-0.02 [-0.01;0.01]
2035	-0.03 [-0.01;0.01]	-0.04 [-0.01;0.01]	-0.02 [-0.01;0.01]	-0.03 [-0.01;0.02]	-0.03 [-0.01;0.01]
2040	-0.04 [-0.02;0.01]	-0.05 [-0.02;0.01]	-0.03 [-0.01;0.01]	-0.05 [-0.02;0.02]	-0.04 [-0.01;0.01]
			Phase-Out-2035-Strong		
2020	-0.00 [-0.00;0.00]	-0.00 [-0.00;0.00]	-0.00 [-0.00;0.00]	0.02 [-0.00;0.00]	0.00 [-0.00;0.00]
2025	-0.01 [-0.00;0.00]	-0.01 [-0.00;0.00]	-0.01 [-0.00;0.00]	-0.01 [-0.01;0.00]	-0.01 [-0.00;0.00]
2030	-0.02 [-0.01;0.01]	-0.03 [-0.01;0.01]	-0.02 [-0.01;0.01]	-0.00 [-0.01;0.01]	-0.02 [-0.01;0.01]
2035	-0.03 [-0.01;0.01]	-0.04 [-0.01;0.01]	-0.02 [-0.01;0.01]	-0.02 [-0.02;0.01]	-0.03 [-0.01;0.01]
2040	-0.04 [-0.02;0.02]	-0.05 [-0.02;0.02]	-0.02 [-0.01;0.01]	-0.03 [-0.02;0.02]	-0.03 [-0.01;0.01]

Note: Simulation results for stationary discounted cumulative welfare per capita in utils. Values for the Null-Scenario are reported as change to the base year 2014 and for the year 2014 initial values are reported. Values for other scenarios are differences to the Null-Scenario in the respective year. Values in brackets denote the minimum and maximum difference from the reported value obtained from 1200 simulations.

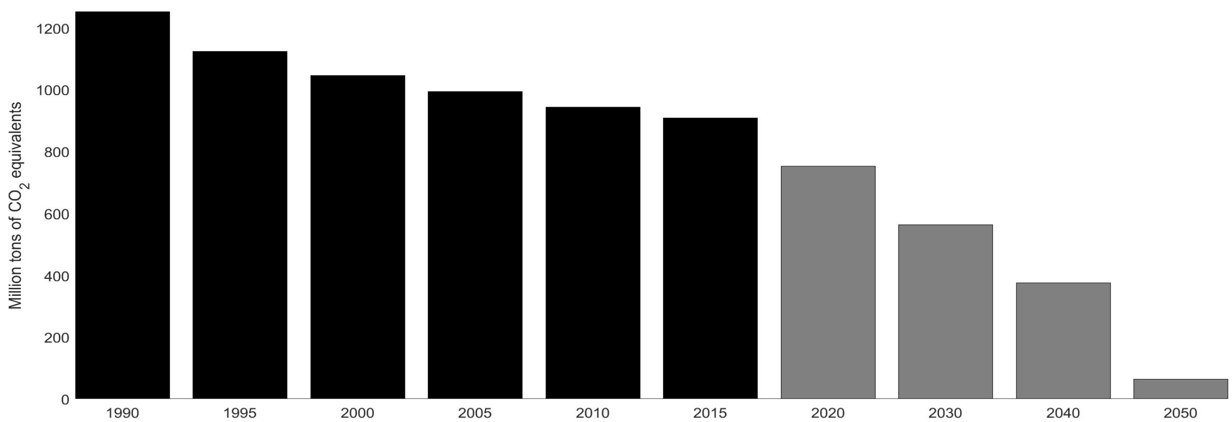
Table 3.17: Real consumption per capita

Year	Germany	Rest of Germany	Central Germany	Lusatia	Rhineland
			Null-Scenario		
2014	100.00 [0.00;0.00]	100.00 [0.00;0.00]	100.00 [0.00;0.00]	100.00 [0.00;0.00]	100.00 [0.00;0.00]
2020	104.58 [0.00;0.00]	104.58 [0.00;0.00]	104.58 [0.00;0.00]	104.59 [0.00;0.00]	104.58 [0.00;0.00]
2025	108.56 [0.00;0.00]	108.56 [0.00;0.00]	108.56 [0.00;0.00]	108.57 [0.00;0.01]	108.56 [0.00;0.00]
2030	112.70 [0.00;0.00]	112.70 [0.00;0.00]	112.70 [0.00;0.00]	112.70 [0.00;0.00]	112.70 [0.00;0.00]
2035	116.99 [0.00;0.00]	116.99 [0.00;0.00]	116.99 [0.00;0.00]	116.99 [0.00;0.00]	116.99 [0.00;0.00]
2040	121.44 [0.00;0.00]	121.44 [0.00;0.00]	121.44 [0.00;0.00]	121.44 [0.00;0.00]	121.44 [0.00;0.00]
			Baseline		
2020	-0.02 [-0.01;0.01]	-0.01 [-0.01;0.01]	-0.05 [-0.01;0.01]	-0.39 [-0.02;0.02]	-0.08 [-0.01;0.01]
2025	-0.02 [-0.02;0.01]	-0.02 [-0.01;0.01]	-0.05 [-0.02;0.01]	-0.21 [-0.03;0.03]	-0.06 [-0.02;0.01]
2030	-0.03 [-0.02;0.02]	-0.03 [-0.02;0.01]	-0.07 [-0.02;0.02]	-0.35 [-0.04;0.04]	-0.09 [-0.02;0.01]
2035	-0.04 [-0.03;0.02]	-0.03 [-0.02;0.02]	-0.12 [-0.02;0.02]	-0.85 [-0.04;0.03]	-0.18 [-0.02;0.02]
2040	-0.03 [-0.03;0.02]	-0.02 [-0.02;0.03]	-0.10 [-0.02;0.02]	-0.69 [-0.04;0.04]	-0.15 [-0.02;0.02]
			Phase-Out-2035-Weak		
2020	-0.05 [-0.02;0.02]	-0.05 [-0.02;0.02]	-0.08 [-0.02;0.02]	-0.28 [-0.05;0.04]	-0.08 [-0.02;0.02]
2025	-0.10 [-0.04;0.03]	-0.09 [-0.04;0.03]	-0.15 [-0.04;0.03]	-0.56 [-0.07;0.06]	-0.30 [-0.04;0.03]
2030	-0.13 [-0.05;0.05]	-0.13 [-0.05;0.04]	-0.27 [-0.05;0.04]	-1.33 [-0.09;0.06]	-0.20 [-0.04;0.05]
2035	-0.14 [-0.05;0.06]	-0.14 [-0.05;0.06]	-0.11 [-0.06;0.06]	-0.96 [-0.11;0.09]	-0.25 [-0.06;0.06]
2040	-0.09 [-0.06;0.07]	-0.10 [-0.05;0.07]	-0.05 [-0.04;0.06]	-0.96 [-0.09;0.09]	-0.11 [-0.05;0.05]
			Phase-Out-2040-Age		
2020	-0.05 [-0.02;0.02]	-0.05 [-0.02;0.01]	-0.09 [-0.03;0.01]	-0.39 [-0.04;0.03]	-0.10 [-0.02;0.02]
2025	-0.09 [-0.04;0.03]	-0.09 [-0.04;0.03]	-0.14 [-0.04;0.03]	-0.49 [-0.07;0.06]	-0.21 [-0.04;0.02]
2030	-0.13 [-0.05;0.05]	-0.13 [-0.05;0.04]	-0.19 [-0.05;0.04]	-0.62 [-0.08;0.09]	-0.30 [-0.05;0.04]
2035	-0.15 [-0.06;0.05]	-0.14 [-0.05;0.06]	-0.30 [-0.05;0.05]	-1.51 [-0.11;0.10]	-0.24 [-0.06;0.05]
2040	-0.15 [-0.06;0.06]	-0.16 [-0.06;0.06]	-0.11 [-0.06;0.06]	-1.04 [-0.08;0.10]	-0.23 [-0.06;0.06]
			Phase-Out-2040-Balanced		
2020	-0.05 [-0.02;0.02]	-0.05 [-0.02;0.01]	-0.09 [-0.02;0.01]	-0.28 [-0.06;0.04]	-0.12 [-0.02;0.01]
2025	-0.09 [-0.04;0.04]	-0.09 [-0.04;0.03]	-0.11 [-0.04;0.03]	-1.01 [-0.07;0.04]	-0.16 [-0.04;0.03]
2030	-0.13 [-0.05;0.04]	-0.13 [-0.05;0.04]	-0.20 [-0.04;0.04]	-0.68 [-0.07;0.05]	-0.28 [-0.04;0.04]
2035	-0.15 [-0.06;0.05]	-0.14 [-0.05;0.06]	-0.35 [-0.04;0.04]	-1.08 [-0.07;0.09]	-0.26 [-0.05;0.05]
2040	-0.15 [-0.06;0.07]	-0.16 [-0.06;0.06]	-0.09 [-0.05;0.06]	-1.08 [-0.09;0.10]	-0.24 [-0.06;0.06]
			Phase-Out-2035-Strong		
2020	-0.06 [-0.02;0.02]	-0.05 [-0.02;0.02]	-0.13 [-0.02;0.02]	-0.77 [-0.05;0.03]	-0.17 [-0.02;0.02]
2025	-0.10 [-0.04;0.03]	-0.09 [-0.03;0.04]	-0.14 [-0.04;0.03]	-0.36 [-0.08;0.06]	-0.26 [-0.04;0.03]
2030	-0.12 [-0.05;0.05]	-0.12 [-0.05;0.05]	-0.27 [-0.05;0.04]	-1.34 [-0.08;0.07]	-0.19 [-0.05;0.05]
2035	-0.14 [-0.06;0.05]	-0.14 [-0.05;0.05]	-0.10 [-0.06;0.05]	-0.93 [-0.11;0.08]	-0.21 [-0.06;0.06]
2040	-0.09 [-0.06;0.06]	-0.09 [-0.05;0.07]	-0.05 [-0.05;0.05]	-0.95 [-0.09;0.09]	-0.10 [-0.05;0.05]

Note: Simulation results for real consumption per capita as index. Values for the Null-Scenario are reported as change to the base year 2014. Values for other scenarios are differences to the Null-Scenario in the respective year. Values in brackets denote the minimum and maximum difference from the reported value obtained from 1200 simulations.

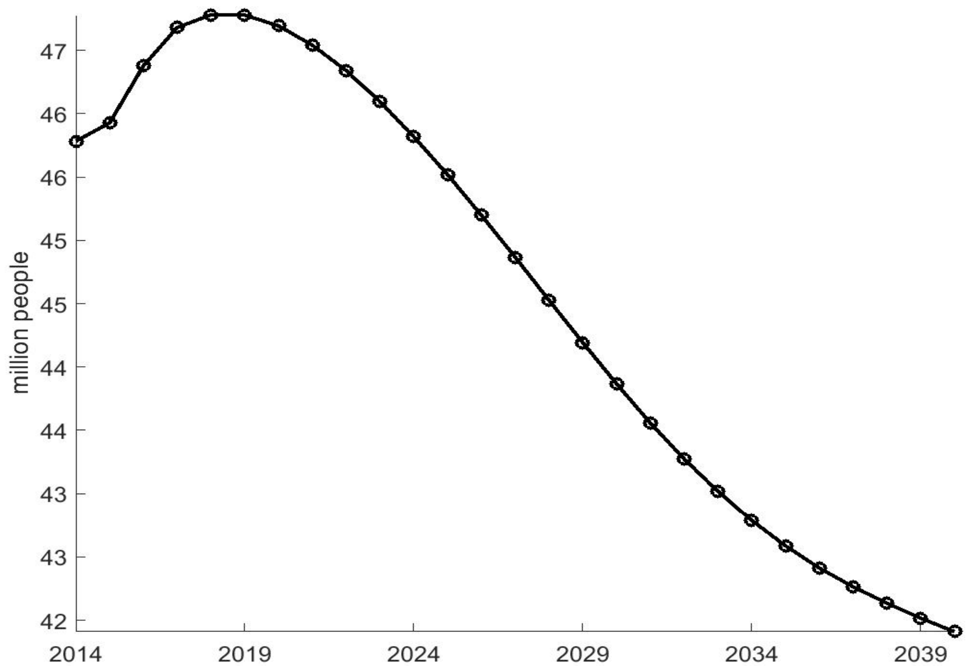
3.7.3 Figures

Figure 3.5: Greenhouse gas emissions of Germany



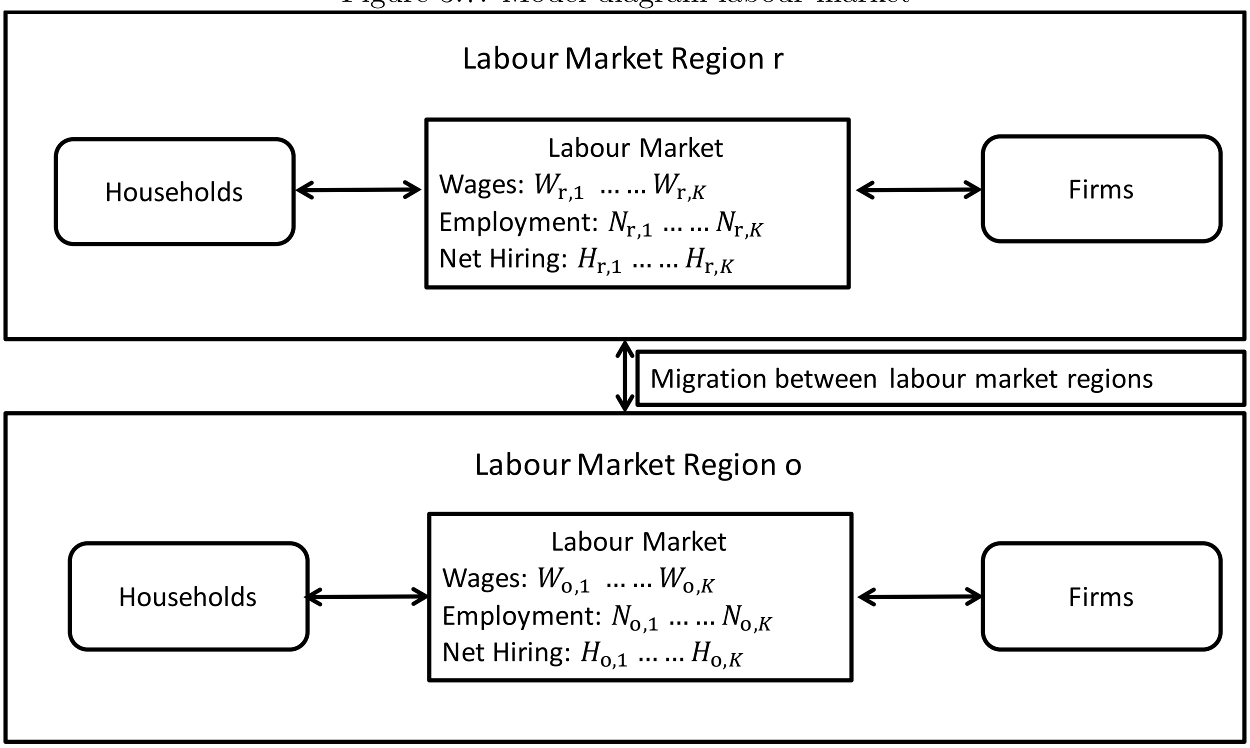
Note: Greenhouse gas emissions in Germany. Black bars are historical values and grey bars are national targets.
Sources: German Environment Agency, National Inventory Reports for the German Greenhouse Gas Inventory 1990 to 2016 (as of 01/2018) and initial forecast for 2017 (UBA press release 08/2018).

Figure 3.6: Labour force projection



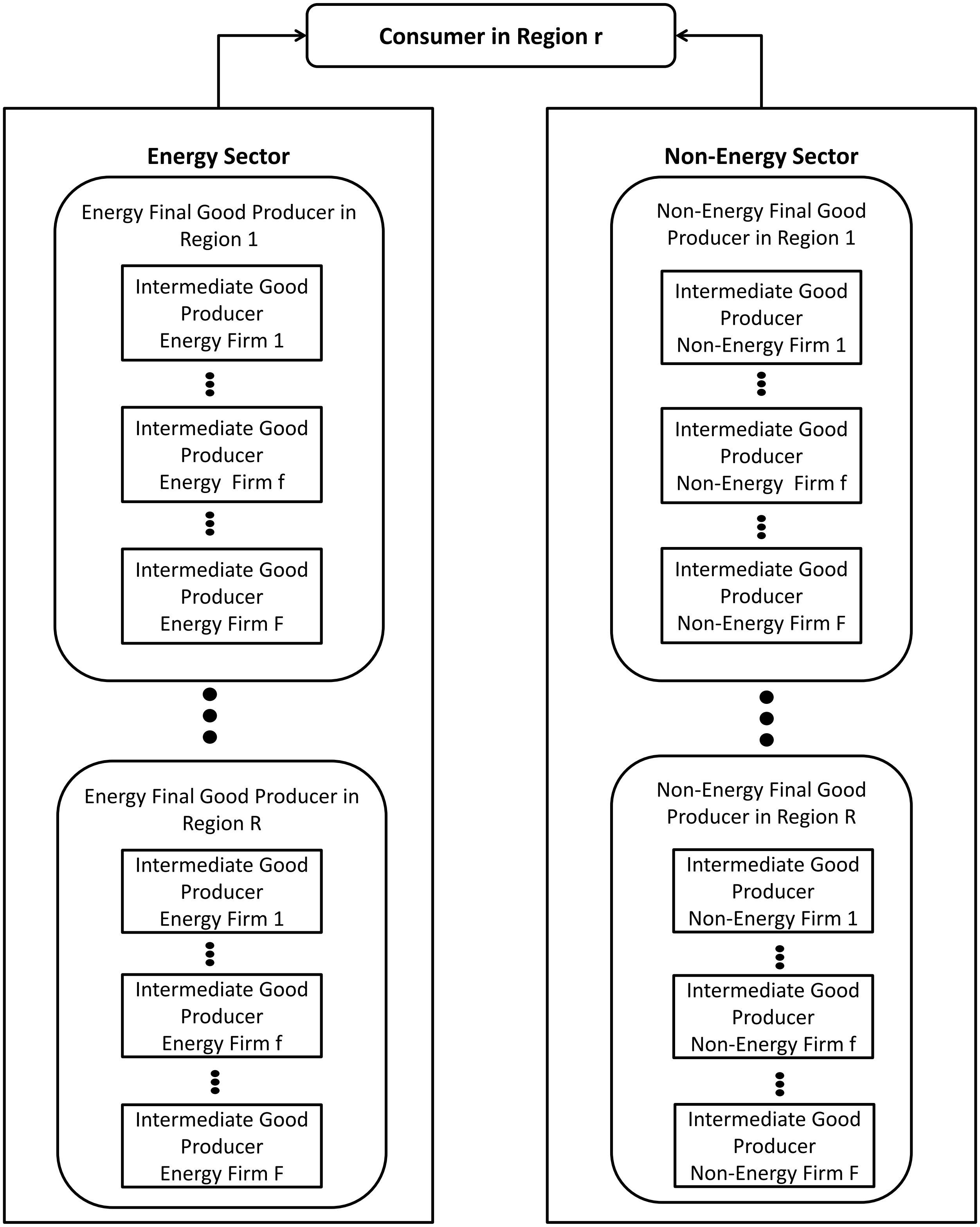
Sources: Eurostat, OECD.

Figure 3.7: Model diagram labour market



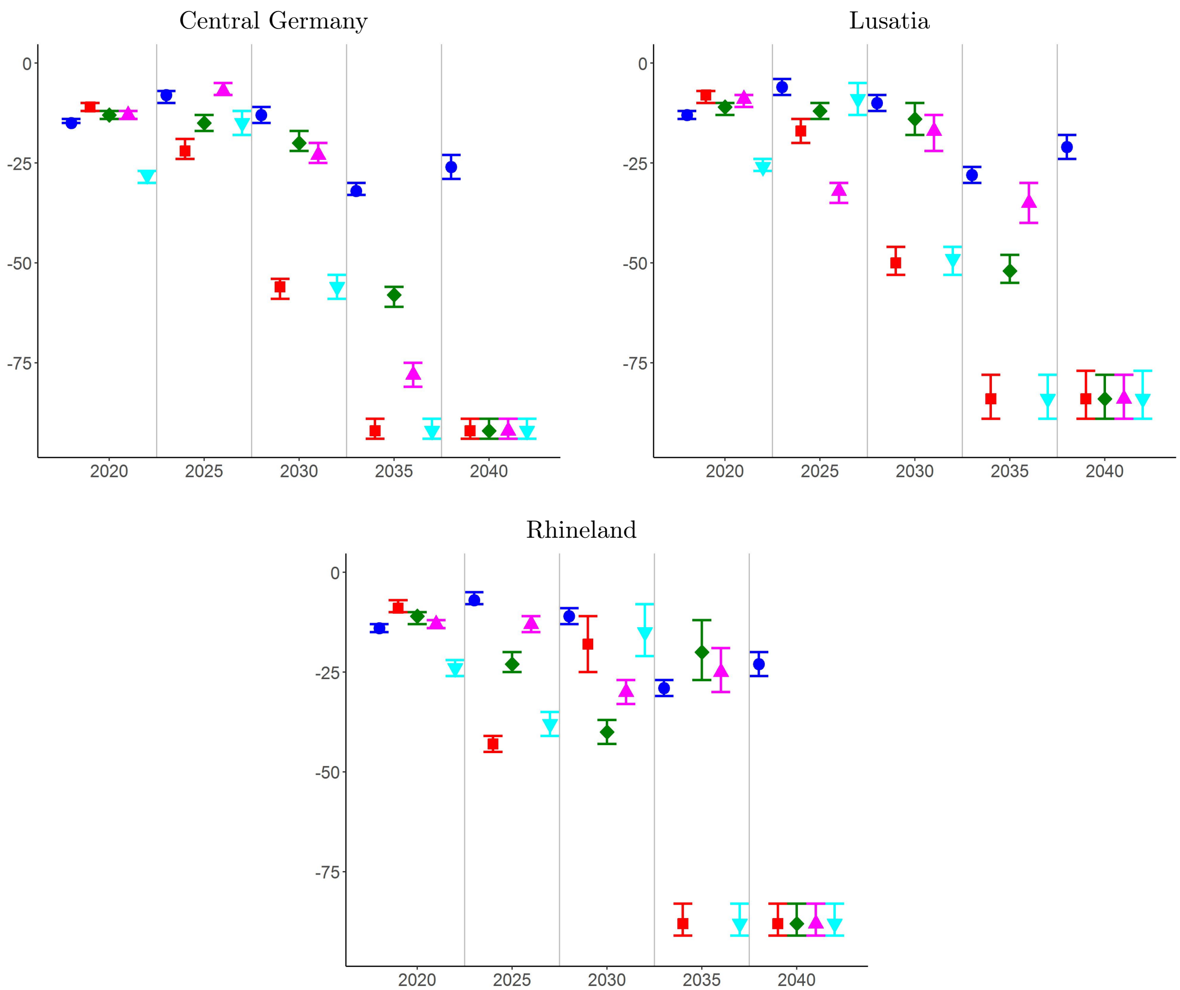
Source: own exhibition.

Figure 3.8: Model diagram production



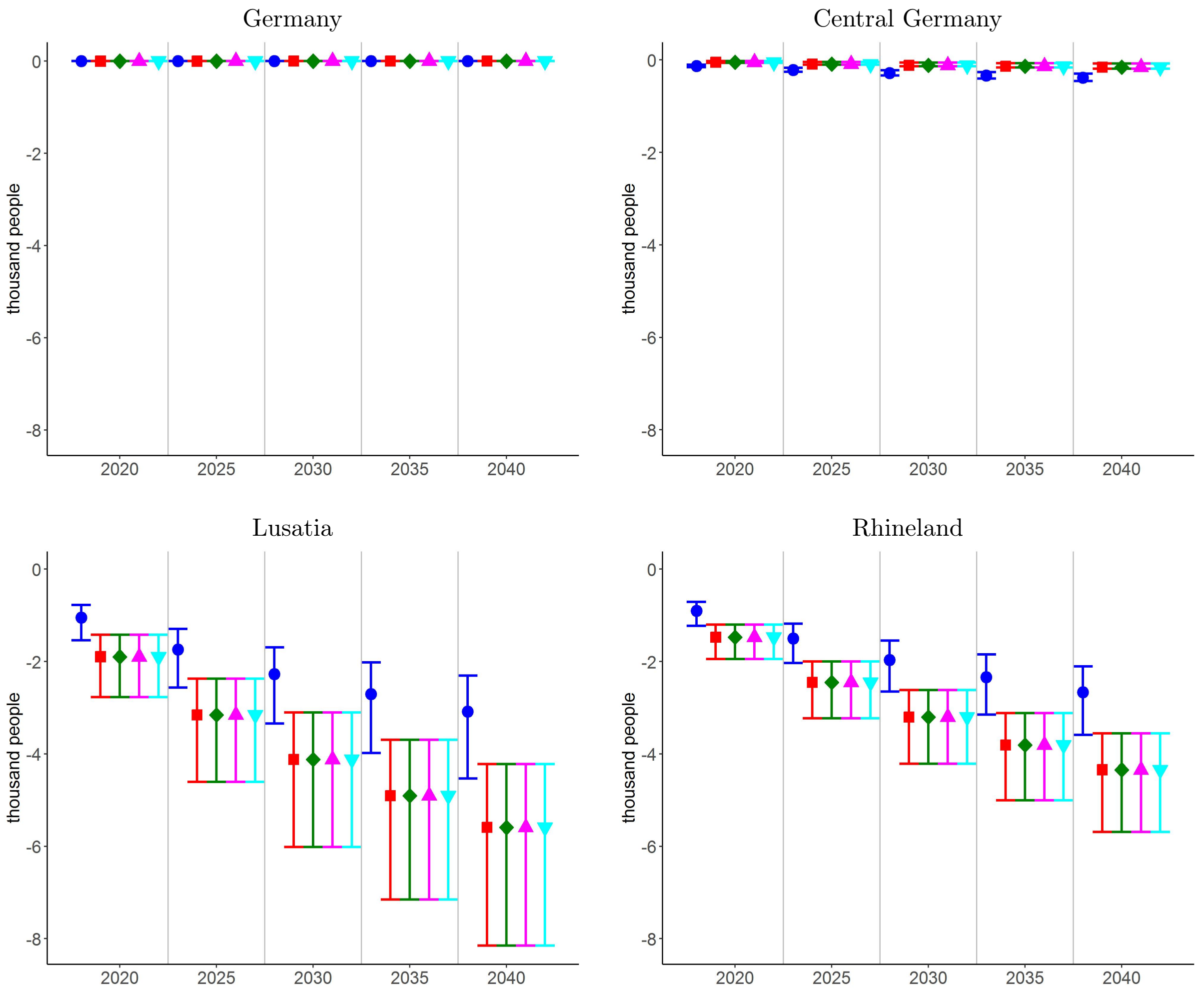
Source: own exhibition.

Figure 3.9: Simulation results for regional lignite coal specific productivity



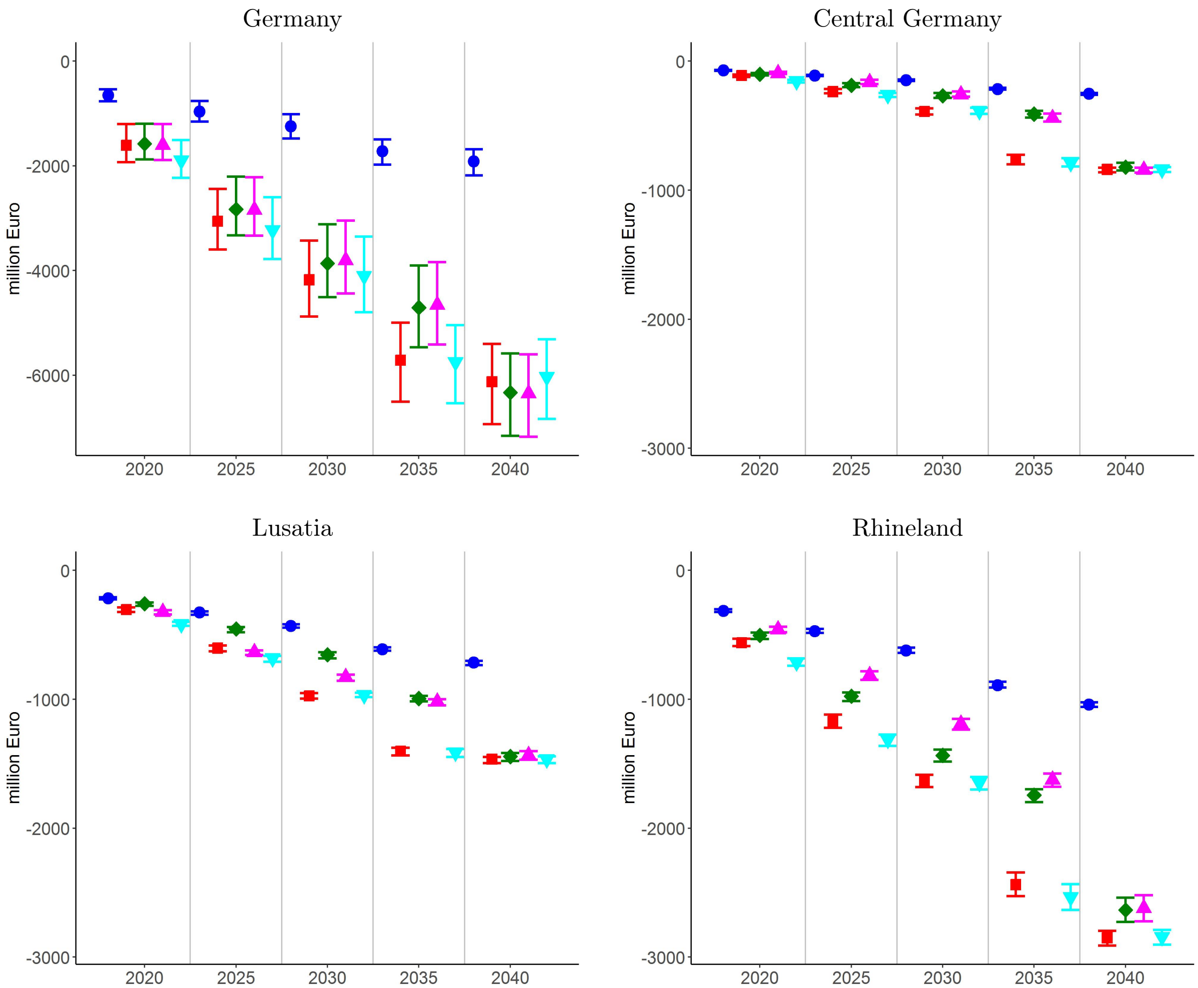
Note: Difference compared to the Null-Scenario in percentage points, Baseline (blue circle), Phase-Out-2035-Weak (red square), Phase-Out-2040-Age (green diamond), Phase-Out-2040-Balanced (magenta triangle point-up) and Phase-Out-2035-Strong (cyan triangle point-down). Horizontal lines indicate the maximum and minimum value observed for 1200 simulations.

Figure 3.10: Simulation results for labour force



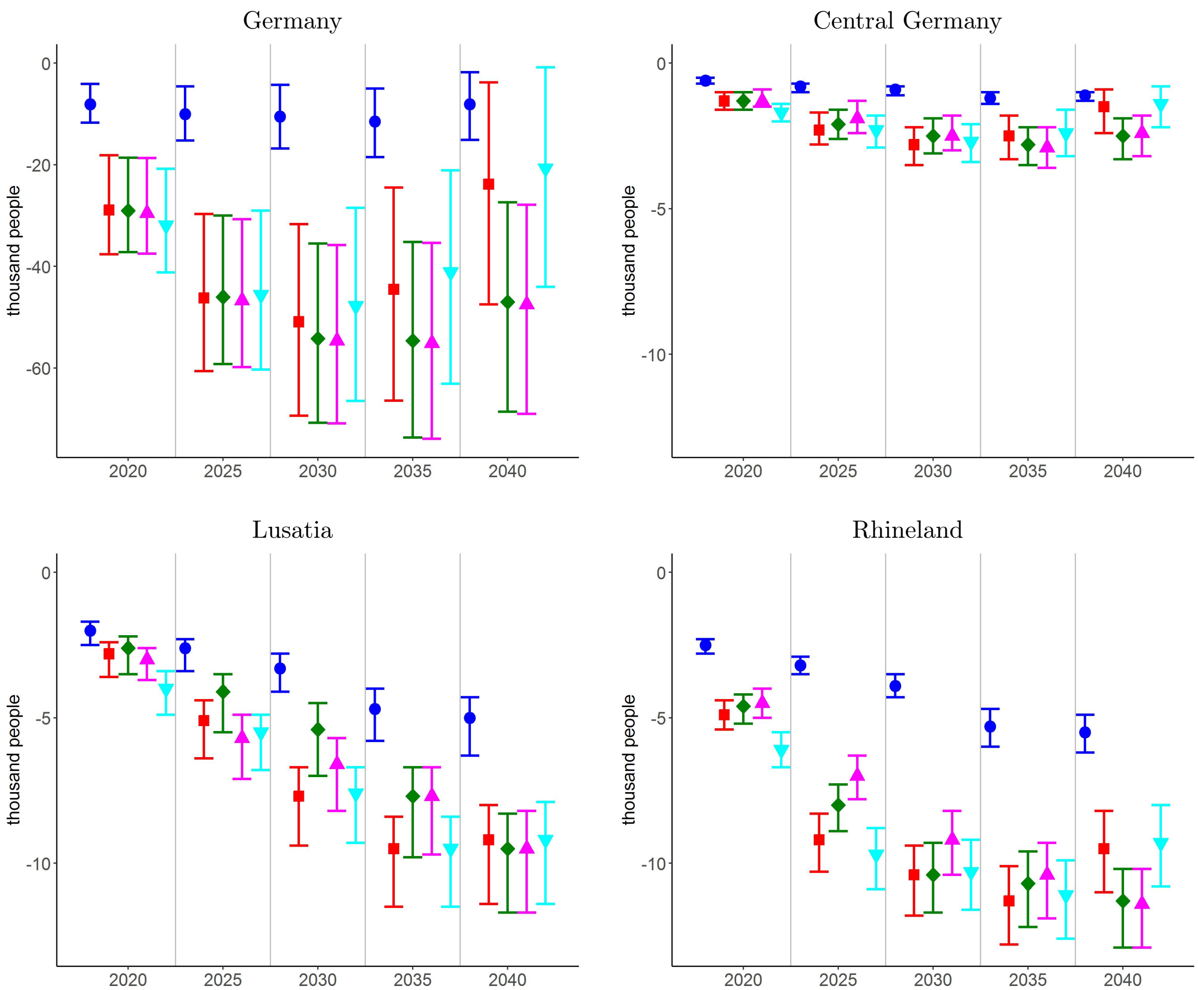
Note: Difference compared to the Null-Scenario in thousand people, Baseline (blue circle), Phase-Out-2035-Weak (red square), Phase-Out-2040-Age (green diamond), Phase-Out-2040-Balanced (magenta triangle point-up) and Phase-Out-2035-Strong (cyan triangle point-down). Horizontal lines indicate the maximum and minimum value observed for 1200 simulations.

Figure 3.11: Simulation results for labour income



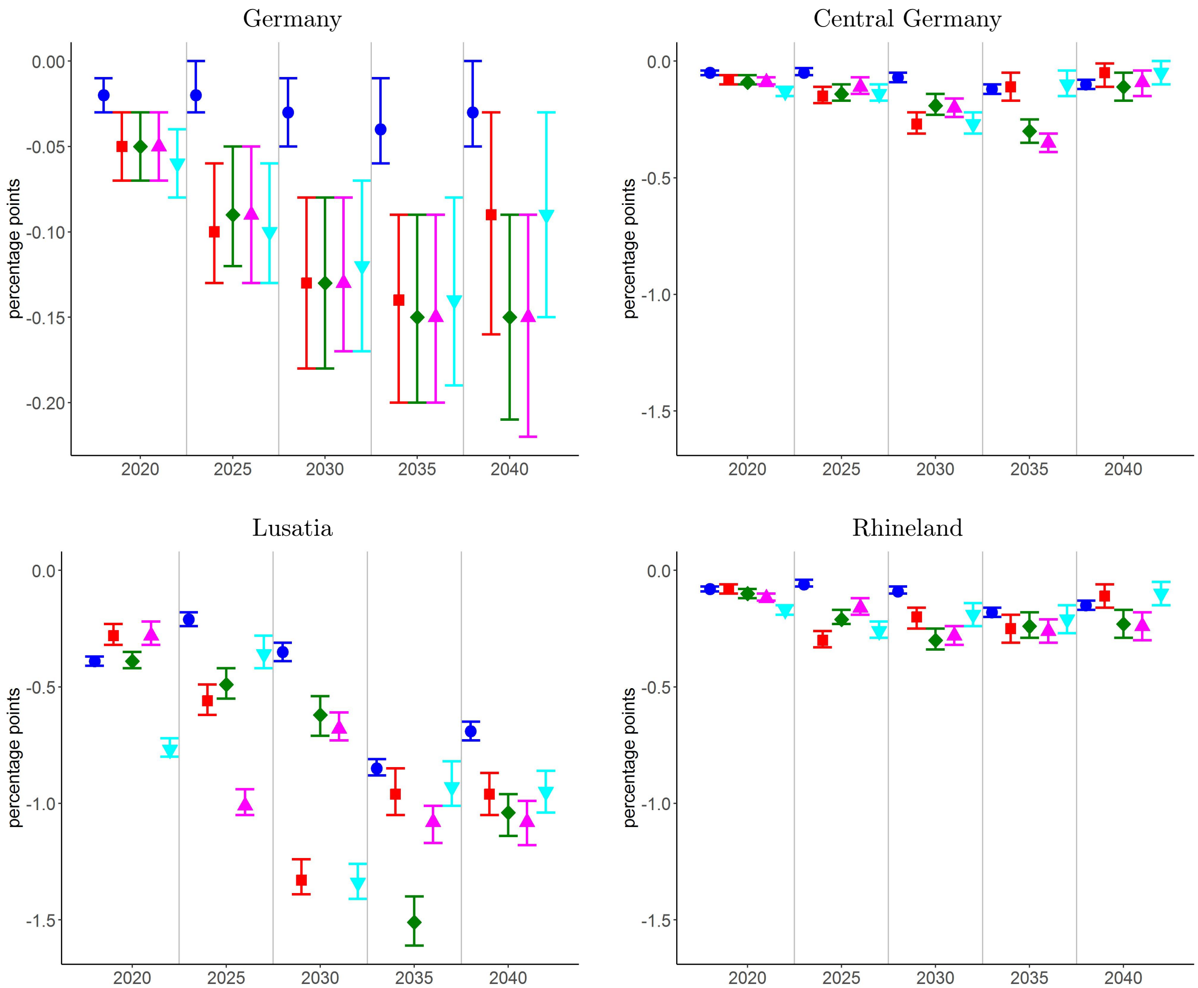
Note: Difference compared to the Null-Scenario in million euro, Baseline (blue circle), Phase-Out-2035-Weak (red square), Phase-Out-2040-Age (green diamond), Phase-Out-2040-Balanced (magenta triangle point-up) and Phase-Out-2035-Strong (cyan triangle point-down). Horizontal lines indicate the maximum and minimum value observed for 1200 simulations.

Figure 3.12: Simulation results for total employment



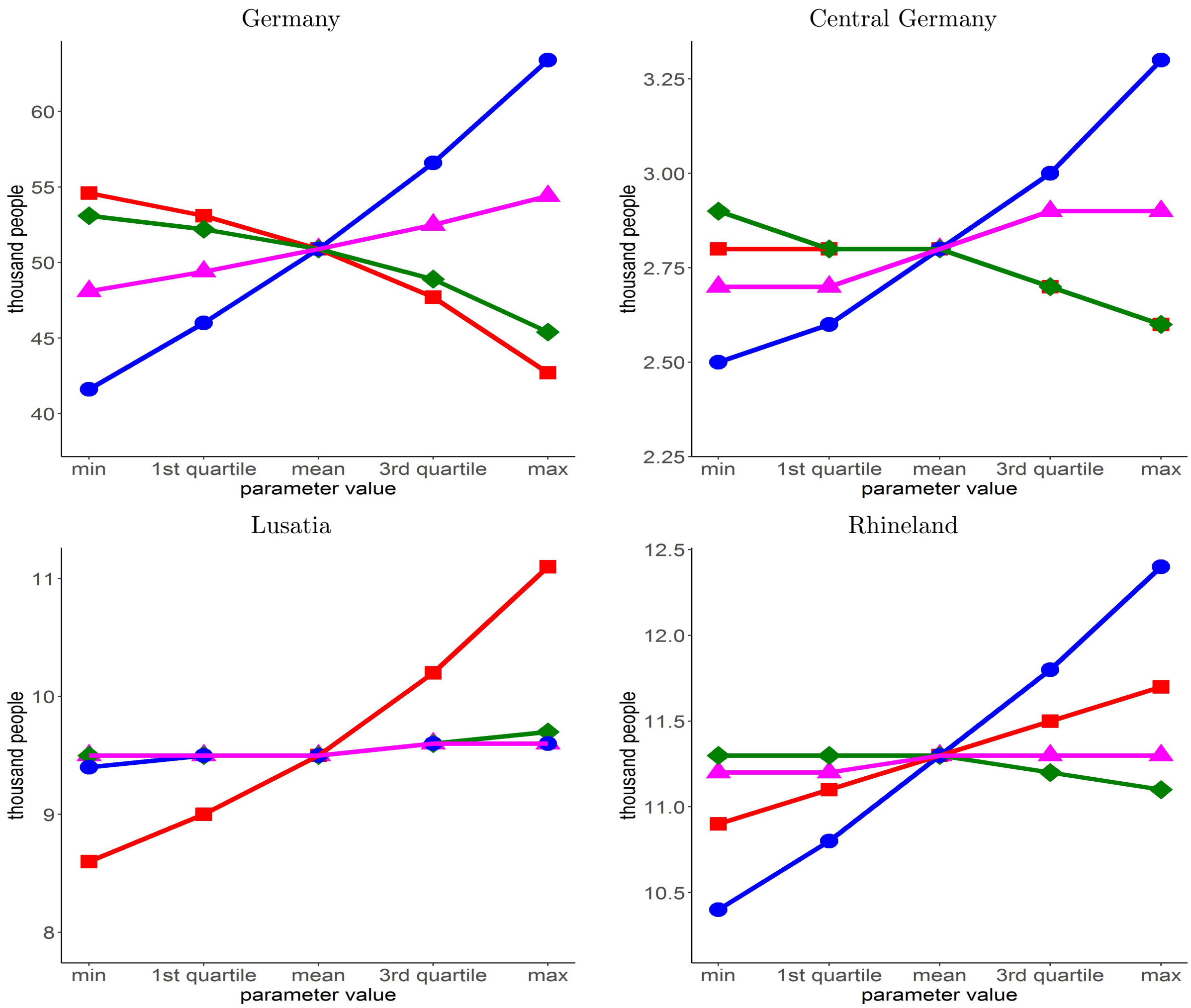
Note: Difference compared to the Null-Scenario in thousand people, Baseline (blue circle), Phase-Out-2035-Weak (red square), Phase-Out-2040-Age (green diamond), Phase-Out-2040-Balanced (magenta triangle point-up) and Phase-Out-2035-Strong (cyan triangle point-down). Horizontal lines indicate the maximum and minimum value observed for 1200 simulations.

Figure 3.13: Simulation results for consumption



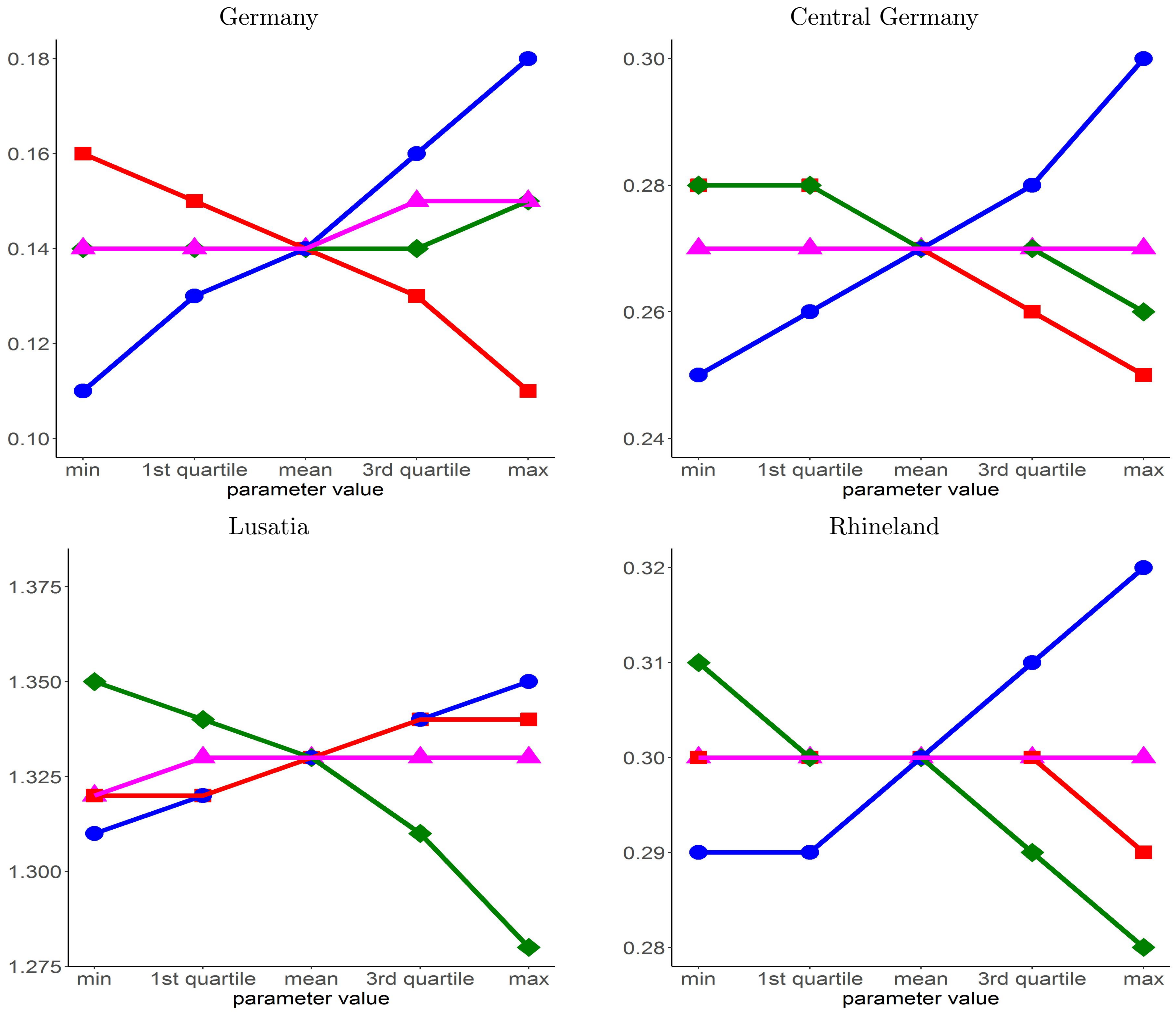
Note: Difference compared to the Null-Scenario in percentage points, Baseline (blue circle), Phase-Out-2035-Weak (red square), Phase-Out-2040-Age (green diamond), Phase-Out-2040-Balanced (magenta triangle point-up) and Phase-Out-2035-Strong (cyan triangle point-down). Horizontal lines indicate the maximum and minimum value observed for 1200 simulations.

Figure 3.14: Sensitivity analysis for maximum employment drop



Note: Difference compared to the Null-Scenario in thousand people, Baseline (blue circle), Phase-Out-2035-Weak (red square), Phase-Out-2040-Age (green diamond), Phase-Out-2040-Balanced (magenta triangle point-up) and Phase-Out-2035-Strong (cyan triangle point-down). Horizontal lines indicate the maximum and minimum value observed for 1200 simulations.

Figure 3.15: Sensitivity analysis for maximum consumption drop



Note: Change in maximum real consumption per capita drop between 2014 to 2040 for Phase-Out-2035-Weak compared to the Null-Scenario in percentage points changing the value of only one parameter. The most important parameters for the maximum German cumulative consumption drop in descending order (compare with legend) are: persistence in unemployment benefits ρ^b (blue circle), home bias non-energy I^H_{NE} (red square), persistence of market power ρ^λ (green diamond), inverse Frisch elasticity σ^L (magenta triangle point-up). We report the change in the maximum drop for the minimum, first quartile, median/mean, third quartile and maximum parameter value.

3.8 Online Appendix

3.8.1 Model equations

3.8.1.1 National aggregates

This section collects all the equations of the model associated with national aggregates.

national resource constraint

$$y_t = c_t + g_t \tag{3.39}$$

national consumption

$$c_t = \sum_{r=1}^{R} \gamma^c_{r\,t} \, {w^{pop}_r}_t \, c_{r\,t} \tag{3.40}$$

national gross value-added

$$y_t = \sum_{r=1}^{R} \sum_{k=1}^{K} {w^{pop}_r}_t \, \gamma_{r,k_t} \, y_{r,k_t} \tag{3.41}$$

national government budget constraint

$$g_t + b_t \, u_t + tr_t = tax_t \tag{3.42}$$

$$tax_t = \sum_{r=1}^{R} \sum_{k=1}^{K} (\gamma_{r,k,t} \, \tau^l \, w_{r,k,t} \, n_{r,k,t} + y_{r,k,t} \, w^{pop}_{r,t} \, \gamma_{r,k,t} \, \tau_{r,k,t}) \tag{3.43}$$

national real unemployment benefits

$$b_t = (\rho^b)^{\frac{1}{4}} \, b_{t-1} + (1 - (\rho^b)^{\frac{1}{4}}) \, \zeta^w \, w_{t-1} \tag{3.44}$$

national employment rate

$$n_t = \sum_{r=1}^{R} \sum_{k=1}^{K} w^{pop}_{r,t} \, n_{r,k,t} \tag{3.45}$$

national hiring rate

$$h_t = \sum_{r=1}^{R} \sum_{k=1}^{K} w^{pop}_{r,t} \, h_{r,k,t} \tag{3.46}$$

national unemployment rate

$$u_t = 1 - \sum_{r=1}^{R} w_{r,t}^{pop}\, n_{r,t} \tag{3.47}$$

national wage bill

$$n_t\, w_t = \sum_{r=1}^{R} \gamma_{r,t}^{c}\, n_{r,t}\, w_{r,t} \tag{3.48}$$

3.8.1.2 Regional aggregates

This section collects all equations of the model associated with regional aggregates.

regional price index for consumption

$$\gamma_{r,t}^{c} = \sum_{k=1}^{K} \left(\omega_k^c\, {\gamma_{r,k}}_t^{1-\eta^c} \right)^{\frac{1}{1-\eta^c}} \tag{3.49}$$

regional budget constraint

$$\gamma_{r,t}^{c}\, c_{r,t} = \sum_{k=1}^{K} (\gamma_{r,k,t}\, y_{r,k,t}\, \left(1 - \tau_{r,k,t}\right) - \tau^{l}\, \gamma_{r,k,t}\, n_{r,k,t}\, w_{r,k,t}) + \gamma_{r,t}^{c}\, \left(b_{r,t}\, u_{r,t} + tr_t\right) \tag{3.50}$$

regional wage bill

$$\gamma_{r,t}^{c}\, n_{r,t}\, w_{r,t} = \sum_{k=1}^{K} \gamma_{r,k,t}\, w_{r,k,t}\, n_{r,k,t} \tag{3.51}$$

regional aggregate production

$$y_{r,t} = \sum_{k=1}^{K} \gamma_{r,k,t}\, y_{r,k,t} \tag{3.52}$$

regional employment rate

$$n_{r,t} = \sum_{k=1}^{K} n_{r,k,t} \tag{3.53}$$

regional unemployment rate

$$u_{r,t} = 1 - n_{r,t} \tag{3.54}$$

regional unemployed looking for a job

$$u^s_{r,t} = 1 - (1 - \frac{\delta}{\mu^{pop}_{r,t}})\, n_{r,t-1} \tag{3.55}$$

regional hiring rate

$$h_{r,t} = \sum_{k=1}^{K} h_{r,k,t} \tag{3.56}$$

law of motion for population weight

$$w^{pop}_{r,t} = \rho^{pop} w^{pop}_{r,t-1} + (1 - \rho^{pop})\, \frac{\bar{\epsilon}^{pop}_r}{\sum_{d=1}^{R} \bar{\epsilon}^{pop}_d} \tag{3.57}$$

law of motion for living preferences

$$\epsilon^{pop}_{r,t} = \rho^{pop} \epsilon^{pop}_{r,t-1} + (1 - \rho^{pop})\, \sigma^{\epsilon^{pop}}_r \bar{U}_r \, \exp(\eta^{\epsilon^{pop}}_r) \tag{3.58}$$

regional population growth

$$\mu^{pop}_{r,t} = \frac{w^{pop}_{r,t}}{w^{pop}_{r,t-1}} \mu^{pop}_t \tag{3.59}$$

3.8.2 Regional energy sector

This section collects all the equations of the model associated with the regional energy input sector.

regional energy production function

$$y_{r,k,t} = \Big(\sum_{s=1}^{S} \phi_{r,k,s}^{\frac{1}{\eta^b}} y_{r,k,st}^{\frac{\eta^b-1}{\eta^b}} \Big)^{\frac{\eta^b}{\eta^b-1}} \tag{3.60}$$

input production function

$$y_{r,k,s,t} = \epsilon_{r,k,s,t} \left(a_{r,k,s}\, n_{r,k,s,t}^{\alpha_{r,k,s}} - \frac{1}{\psi}\, mc^h_{r,k,s,t}\, h^{\psi}_{r,k,s,t} \right) \tag{3.61}$$

wage from first-order condition of households

$$w_{r,k,s,t} = \frac{b_t + \gamma^c_{r,t}\, A^L_{r,k,s}\, n_{r,k,s,t}^{\sigma^L_{r,k,s}}}{(1 - \tau^l)\; \gamma_{r,k,s,t}} \tag{3.62}$$

law of motion for labour

$$n_{r,k,s,t} = h_{r,k,s,t} + \left(1 - \frac{\delta}{\mu^{pop}_{r,t}}\right) n_{r,k,s,t-1} \tag{3.63}$$

marginal hiring cost

$$mc^h_{r,k,s,t} = B^h_{r,k,s} \left(\Psi + (1-\Psi)\left(\frac{h_{r,t}}{u^s_{r,t-1}}\right)^v\right) \tag{3.64}$$

first-order condition for employment

$$\begin{aligned} &\frac{1}{\lambda_{r,k,t}} \left(\alpha_{r,k,s}\, a_{r,k,s}\, \epsilon_{r,k,s,t}\, (1-\tau_{r,k,s,t})\, n^{\alpha_{r,k,s}-1}_{r,k,s,t} - mc^h_{r,k,s,t}\, \epsilon_{r,k,s,t}\, (1-\tau_{r,k,s,t})\, h^{\psi-1}_{r,k,s,t}\right) \\ &+ \left(\mu^{pop}_{r,t+1} - \delta\right) \beta\, \mu^z\, \pi^c \frac{1}{\lambda_{r,k,t+1}} \frac{\gamma_{r,k,s,t+1}}{\gamma_{r,k,s,t}} (1-\tau_{r,k,s,t+1})\, \epsilon_{r,k,s,t+1}\, mc^h_{r,k,s,t+1}\, h^{\psi-1}_{r,k,s,t+1} = w_{r,k,s,t} \end{aligned} \tag{3.65}$$

marginal product for input of energy production

$$\gamma_{r,k,s,t} = \phi^{\frac{1}{\eta^b}}_{r,k,s} \left(\frac{y_{r,k,s,t}}{y_{r,k,t}}\right)^{\frac{1}{\eta^b}} \gamma_{r,k,t} \tag{3.66}$$

taxes on regional production

$$\tau_{r,k,s,t} = \rho^\tau\, \tau_{r,k,s_{t-1}} + (1-\rho^\tau)\, \bar{\tau}_{r,k,s}\, exp\left(\eta^\tau_{r,k,s_t}\right) \tag{3.67}$$

tax revenues from regional energy production

$$\tau_{r,k,t}\, \gamma_{r,k,t}\, y_{r,k,t} = \sum_{s=1}^{S} \tau_{r,k,s,t}\, \gamma_{r,k,s,t}\, y_{r,k,s,t} \tag{3.68}$$

law of motion for markups

$$\lambda_{r,k,s,t} = \rho^\lambda\, \lambda_{r,k,s,t-1} + (1-\rho^\lambda)\, \sigma^\lambda_{r,s,k} \phi^\lambda\, \epsilon^{pop}_{r,t} \tag{3.69}$$

regional energy wages

$$\gamma_{r,k,t}\, w^*_{r,k,t} = \sum_{s=1}^{S} w^*_{r,k,s,t}\, \gamma_{r,k,s,t} \tag{3.70}$$

regional energy employment rate

$$n_{r,k,t} = \sum_{s=1}^{S} n_{r,k,s,t} \tag{3.71}$$

regional energy hiring rate

$$h_{r,k_t} = \sum_{s=1}^{S} h_{r,k,s,t} \tag{3.72}$$

regional marginal energy hiring costs

$$\gamma_{r,k,t}\, mc^h_{r,k,t} = \sum_{s=1}^{S} mc^h_{r,k,s,t}\, \gamma_{r,k,s,t} \tag{3.73}$$

3.8.2.1 Regional non-energy sector

This section collects all the equations of the model associated with the regional energy input sector.

production function

$$y_{r,k,t} = \epsilon_{r,k,t} \left(a_{r,k}\, n_{r,k,t}^{\alpha_{r,k}} - \frac{1}{\psi}\, mc^h_{r,k,t}\, h^{\psi}_{r,k,t} \right) \tag{3.74}$$

wage from first-order condition of households

$$w_{r,k,t} = \frac{b_t + \gamma^c_{r,t}\, A^L_{r,k}\, n_{r,k,t}^{\sigma^L_{r,k}}}{(1-\tau^l)\ \gamma_{r,k,t}} \tag{3.75}$$

law of motion for labour

$$n_{r,k,t} = h_{r,k,t} + \left(1 - \frac{\delta}{\mu^{pop}_{r,t}}\right) n_{r,k,t-1} \tag{3.76}$$

marginal hiring cost

$$mc^h_{r,k,t} = B^h_{r,k} \left(\Psi + (1-\Psi) \left(\frac{h_{r,t}}{u_{r,t-1}} \right)^{v} \right) \tag{3.77}$$

first-order condition for employment

$$\frac{1}{\lambda_{r,k,t}} \left(\alpha_{r,k}\, a_{r,k}\, \epsilon_{r,k,t}\ (1-\tau_{r,k,t})\ n_{r,k,t}^{\alpha_{r,k}-1} - mc_{r,k,t}^{h}\, \epsilon_{r,k,t}\ (1-\tau_{r,k,t})\ h_{r,k,t}^{\psi-1} \right)$$
$$+\frac{1}{\lambda_{r,k,t+1}}\ \left(\mu_{r,t+1}^{pop} - \delta\right)\ \beta\, \mu^{z}\, \pi^{c}\, \frac{\gamma_{r,k,t+1}}{\gamma_{r,k,t}}\ (1-\tau_{r,k,t+1})\ \epsilon_{r,k,t+1}\, mc_{r,k,t+1}^{h}\, h_{r,k,t+1}^{\psi-1} = w_{r,k,t} \tag{3.78}$$

taxes on regional production

$$\tau_{r,k,t} = \rho^{\tau}\ \tau_{r,k_{t-1}} + (1-\rho^{\tau})\ \bar{\tau}_{r,k}\, exp\left(\eta_{r,k_{t}}^{\tau}\right) \tag{3.79}$$

law of motion for markups

$$\lambda_{r,k,t} = \rho^{\lambda}\ \lambda_{r,k,t-1} + (1-\rho^{\lambda})\, \sigma_{r,k}^{\lambda} \phi^{\lambda}\ \epsilon_{r,t}^{pop} \tag{3.80}$$

3.8.2.2 Household demand equations

This section collects all the equations of the model associated with household demand.

regional demand for sector consumption

$$c_{r,k,t} = \omega_{k}^{c}\ \left(\frac{\gamma_{r,k_{t}}^{c}}{\gamma_{r\,t}^{c}}\right)^{(-\eta^{c})}\ c_{r,t} \tag{3.81}$$

regional sector consumption price index

$$\gamma_{r,k_{t}}^{c} = \sum_{d=1}^{N} \left(\omega_{r,d,k}^{d}\ \gamma_{d,k_{t}}{}^{1-\eta_{k}^{m}}\right)^{\frac{1}{1-\eta_{k}^{m}}} \tag{3.82}$$

regional demand for consumption from other regions

$$c_{r,d,k,t} = \omega_{r,d,k}^{d}\ \left(\frac{\gamma_{d,k,t}}{\gamma_{r,k,t}^{c}}\right)^{\left(-\eta_{k}^{m}\right)}\ c_{r,k,t} \tag{3.83}$$

market clearing

$$\left(1-\tau_{r,k_{t}}\right)\ w_{r\ \ t}^{pop}\, y_{r,k_{t}} = \sum_{d=1}^{R} w_{d,t}^{pop}\ c_{r,d,k,t} \tag{3.84}$$

3.8.3 National aggregates, derivation and scaling

3.8.3.1 Government

The national government consumes G_t, pays lump-sum transfers Tr_t, and unemployment benefits $(U_t B_t)$ financed by tax revenues (Tax). We assume a balanced government budget

$$P_t^c G_t + P_t^c U_t B_t + P_t^c Tr_t = P_t^c Tax_t, \tag{3.85}$$

$$P_t^c Tax_t = \sum_{r=1}^{N} \sum_{k=1}^{K} (\tau^l \, P_{r,k,t} \, W_{r,k,t} \, N_{r,k,t} + \tau_{r,k,t} \, P_{r,k,t} \, Y_{r,k,t}). \tag{3.86}$$

Unemployment benefits are adjusted according to the development of national wages and with a backward-looking component to reflect rigidity in the adjustment of long-term unemployment benefits and wages. This specification allows for a sluggish adjustment of benefits, reflecting empirical evidence.

$$B_t = (\rho^b)^{\frac{1}{4}} B_{t-1} + (1 - (\rho^b)^{\frac{1}{4}}) \zeta^w W_{t-1}. \tag{3.87}$$

3.8.3.2 Market clearing

We assume market clearing. All products produced in a given period are consumed and firms have no access to an inventory technology. Therefore, sectoral production in one region is the sum of regional consumption from all regions.

$$Y_{r,k,t} \, (1 - \tau_{r,k,t}) = \sum_o C_{r,o,k,t}. \tag{3.88}$$

Consumption expenditures in one region and one sector is the sum of the products consumed from different regions purchased for the respective price.

$$P_{r,k,t}^C \, C_{r,k,t} = \sum_o P_{o,k,t} \, C_{r,o,k,t}, \tag{3.89}$$

$$P_{r,t}^C \, C_{r,t} = \sum_k P_{r,k,t}^C \, C_{r,k,t}. \tag{3.90}$$

Overall regional consumption expenditures have to be equal to the sectoral consumption expenditures. Note that the budget constraint of the representative household requires that a household's income from work, net profits and government transfers has to equal its consumption expenditure. If we sum all regional budget constraints, we get an expression for the gross value-added as from the expenditure approach

$$P_t^C Y_t = P_t^C \, C_t + P_t^C \, G_t. \tag{3.91}$$

Total gross value-added from the production approach in the economy $P_t^C \, Y_t$ is the sum of all goods evaluated at their market price

$$P_t^C Y_t = \sum_r \sum_k P_{r,k,t} Y_{r,k,t}. \tag{3.92}$$

3.8.4 National aggregates

The national consumption, gross value-added and government expenditures are given by the following identities:

$$P_t^c C_t = \sum_{r=1}^{R} P_{r,t}^c C_{r,t}, \tag{3.93}$$

$$P_t^c Y_t = \sum_{r=1}^{R} \sum_{k=1}^{K} P_{r,k,t} Y_{r,k,t}, \tag{3.94}$$

$$P_t^c\, Y_t = P_t^c\, G_t + P_t^c\, C_t. \tag{3.95}$$

3.8.5 Scaling of variables

In the following we refer to the vector of endogenous variables in the model by Z_t. To make the model trend stationary. we assume that all real non-stationary variables grow with a common trend z_t with the growth rate $\mu^z = \frac{z_t}{z_{t-1}}$. Furthermore, all nominal variables are scaled by the consumption price level P_t^c with trend inflation $\pi^c = \frac{P_t^c}{P_{t-1}^c}$ and are transformed into regional per capita variables.

$$C_t = z_t\, pop_t\, c_t, \tag{3.96}$$

$$Y_{r,k,t} = z_t\, pop_{r,t}\, y_{r,k,t}, \tag{3.97}$$

$$A_{r,k,t} = z_t\, pop_{r,t}^{1-\alpha_{r,k}}\, a_{r,k}, \tag{3.98}$$

$$MC_{r,k,t}^h = z_t\, pop_{r,t}^{1-\psi}\, mc_{r,k,t}^h, \tag{3.99}$$

$$W_{r,k,t} = P_{r,k,t}\, z_t\, w_{r,k,t}, \tag{3.100}$$

$$P_{r,k,t} = \gamma_{r,k,t}\, P_t^c, \tag{3.101}$$

$$P_{r,t}^c = \gamma_{r,t}^c\, P_t^c. \tag{3.102}$$

$$\tag{3.103}$$

3.8.5.1 Population growth, migration and labour market flows

The working age population pop_t in Germany at time t is the previous working age population pop_{t-1} minus exits EX_t plus entries EN_t in the respective period. The gross growth rate μ_t^{pop} of the working age population is defined by $\frac{pop_t}{pop_{t-1}}$. It is easy to see that the growth rate of the working age population is determined by the entry rate $en_t = \frac{EN_t}{pop_{t-1}}$ and exit rate

$ex_t = \frac{EX_t}{pop_{t-1}}$.

$$pop_t = pop_{t-1} + EN_t - EX_t, \tag{3.104}$$
$$\mu_t^{pop} - 1 = en_t - ex_t. \tag{3.105}$$

The stock of employed and unemployed people grows at the same speed as the working age population itself. A exogenous separation rate δ and endogenous net hiring rate h_t lead to transitions between the state of unemployment and employment. The stock of employed people evolves from newly hired people and already employed as follows

$$N_t = H_t + en_t N_{t-1} - ex_t N_{t-1} + N_{t-1} - \delta\, N_{t-1}, \tag{3.106}$$
$$n_t = h_t + (1 - \frac{\delta}{\mu_t^{pop}})\, n_{t-1}. \tag{3.107}$$

This law of motion holds for every sector and region. Given a sector invariant separation rate, we are able to express the unemployment rate at the beginning of the period in one region by

$$u_t^s = u_{t-1} - \left(1 - \frac{\delta}{\mu_t^{pop}}\right) n_{t-1}. \tag{3.108}$$

Now we are able to define the job finding probability $x_t = \frac{h_t}{u_t^s}$. Using the definition of the unemployment rate, we get

$$u_t = 1 - n_t, \tag{3.109}$$
$$u_t = 1 - \left\{h_t + \left(1 - \frac{\delta}{\mu_t^{pop}}\right) n_{t-1}\right\}, \tag{3.110}$$
$$u_t = (1 - x_t)\, u_{t-1} + (1 - x_t)\, \frac{\delta}{\mu_t^{pop}}\, n_{t-1}, \tag{3.111}$$
$$u_t = (1 - x_t) \sum_{i=1}^{\infty} \left\{\left(\prod_{j=0}^{i-1} (1 - x_{t-j})\right) \frac{\delta}{\mu_{t-i+1}^{pop}}\, n_{t-i}\right\}. \tag{3.112}$$

We can use the last expression to define the probability for an individual to be long-term or short-term unemployed. The probability of a person to be unemployed for up to one year in period t is $\prod_{i=1}^{4}(1 - x_{t-i})$. In steady-state this corresponds to $(1 - (1 - x)^4)$. The separation rate will be set such that this probability is 63%, to match German data.

The model only considers net migration. Regional population growth $\mu_{r,t}^{pop}$ is given by

$$\mu_{r,t}^{pop} = \frac{pop_{r,t}}{pop_{r,t-1}}, \tag{3.113}$$
$$\mu_{r,t}^{pop} = \mu_t^{pop}\, \frac{w_{r,t}^{pop}}{w_{r,t-1}^{pop}}. \tag{3.114}$$

3.8.5.2 Producers of intermediate goods in the energy sector

The problem of the producer of intermediate goods in the energy sector is slightly more complicated than the problem of the producer of intermediate goods in the non-energy sector. An producer of intermediate goods in the energy sector can either hire workers for the lignite sector or for the non-lignite sector. Intermediate firms with labour face monopolistic competition and, therefore, choose a production plan considering the demand for their products from the producers of final goods. The optimization problem of the firm is

$$\max_{N_{r,k,s,t}(f)} \sum_{h=0}^{\infty} \beta^h \Big\{ (1-\tau_{r,k,t+h})\, P_{r,k,t+h}(f)\, Y_{r,k,t+h}(f) - W_{r,k,s,t+h}\, N_{r,k,s,t+h}(f) \Big\} \tag{3.115}$$

$$\text{s.t. } Y_{r,k,t+h}(f) = \Big(\sum_s \phi_{r,k,s}^{\frac{1}{\eta^b}} Y_{r,k,s,t}(f)^{\frac{\eta^b-1}{\eta^b}} \Big)^{\frac{\eta^b}{\eta^b-1}}, \tag{3.116}$$

$$Y_{r,k,s,t+h}(f) = \epsilon_{r,k,s,t+h} \Big(A_{r,k,s,t+h}(f) N_{r,k,s,t+h}(f)^{\alpha_{r,k,s}} - \frac{1}{\Psi} MC^h_{r,k,s,t+h}(f) H_{r,k,s,t+h}(f)^{\Psi} \Big), \tag{3.117}$$

$$MC^h_{r,k,s,t+h} = B^h_{r,k,s} \Bigg\{ \psi + (1-\psi) \Bigg(\frac{H_{r,t+h}}{U^s_{r,t+h}} \Bigg)^{v} \Bigg\} pop^{1-\psi}_{r,t+h}, \tag{3.118}$$

$$H_{r,k,s,t+h}(f) = N_{r,k,t+h}(f) - \Big(\mu^{pop}_{r,t} - \delta \Big) N_{r,k,t+h-1}(f), \tag{3.119}$$

$$P_{r,k,t+h}(f) = \Bigg(\frac{Y_{r,k,t+h}(f)}{Y_{r,k,t+h}} \Bigg)^{\frac{1-\lambda_{r,k,t+h}}{\lambda_{r,k,t+h}}} P_{r,k,t+h}. \tag{3.120}$$

We can use the envelope theorem to obtain the following first-order condition with respect to $N_{r,k,s,t}(f)$

$$\begin{aligned} W_{r,k,s,t} = {} & (1-\tau_{r,k,t}) \frac{\mathrm{d}Y_{r,k,t}(f)}{\mathrm{d}Y_{r,k,s,t}(f)} \frac{\mathrm{d}Y_{r,k,s,t}(f)}{\mathrm{d}N_{r,k,s,t}(f)} \Big(\frac{\mathrm{d}P_{r,k,t}(f)}{\mathrm{d}Y_{r,k,t}(f)} Y_{r,k,t}(f) + P_{r,k,t}(f) \Big), \dots \\ & + \beta\,(1-\tau_{r,k,t+1}) \frac{\mathrm{d}Y_{r,k,s,t+1}(f)}{\mathrm{d}N_{r,k,s,t}(f)} \frac{\mathrm{d}Y_{r,k,s,t+1}(f)}{\mathrm{d}N_{r,k,s,t}(f)} \Big(\frac{\mathrm{d}P_{r,k,t+1}(f)}{\mathrm{d}Y_{r,k,t+1}(f)} Y_{r,k,t+1}(f) + P_{r,k,t+1}(f) \Big), \end{aligned} \tag{3.121}$$

$$\begin{aligned} \frac{\mathrm{d}Y_{r,k,t}(f)}{\mathrm{d}Y_{r,k,s,t}(f)} &= \phi_{r,k,s}^{\frac{1}{\eta^b}} \Big(\frac{Y_{r,k,t}(f)}{Y_{r,k,s,t}(f)} \Big)^{\frac{1}{\eta^b}}, \\ \frac{\mathrm{d}Y_{r,k,s,t}(f)}{\mathrm{d}N_{r,k,s,t}(f)} &= \epsilon_{r,k,s,t}\, z_t \Big(\alpha_{r,k,s}\, A_{r,k,s,t}\, N_{r,k,s,t}^{\alpha_{r,k,s}-1} - MC^h_{r,k,s,t}\, H^{\Psi-1}_{r,k,s,t} \Big), \\ \frac{\mathrm{d}Y_{r,k,s,t+1}(f)}{\mathrm{d}N_{r,k,s,t}(f)} &= (\mu^{pop}_{r,t+1} - \delta) MC^h_{r,k,s,t+1}\, \epsilon_{r,k,s,t+1}\, z_{t+1}\, H^{\Psi-1}_{r,k,s,t+1}, \\ \frac{\mathrm{d}P_{r,k,t}(f)}{\mathrm{d}Y_{r,k,t}(f)} &= \frac{1-\lambda_{r,k,t}}{\lambda_{r,k,t}} \Big(\frac{Y_{r,k,t}(f)}{Y_{r,k,t}} \Big)^{\frac{1-\lambda_{r,k,t}}{\lambda_{r,k,t}}} P_{r,k,t} \frac{1}{Y_{r,k,t}(f)}. \end{aligned}$$

Replacing all derivatives with their respective expressions we obtain the following first order condition:

$$\frac{P_{r,k,s,t}}{\lambda_{r,k,t}}\,\alpha_{r,k,s}\,A_{r,k,s,t}\,(1-\tau_{r,k,t})\,N_{r,k,s,t}^{\alpha_{r,k,s}-1} - \frac{P_{r,k,s,t}}{\lambda_{r,k,t}}\,MC_{r,k,t}^{h}\,\epsilon_{r,k,t}\,H_{r,k,s,t}^{\Psi-1}\dots \tag{3.122}$$

$$+\frac{P_{r,k,s,t+1}}{\lambda_{r,k,t+1}}\,(\mu_{r,t+1}^{pop}-\delta)\,\beta\frac{\lambda_{r,k,t}}{\lambda_{r,k,t+1}}\,MC_{r,k,s,t+1}^{h}\,\epsilon_{r,k,s,t+1}\,(1-\tau_{r,k,t+1})H_{r,k,s,t+1}^{\Psi-1} = W_{r,k,s,t},$$

$$P_{r,k,s,t} = \phi_{r,k,s}^{\frac{1}{\eta^b}}\left(\frac{Y_{r,k,t}(f)}{Y_{r,k,s,t}(f)}\right)^{\frac{1}{\eta^b}} P_{r,k,t}. \tag{3.123}$$

For the non-energy sector, $\frac{\mathrm{d}Y_{r,k,t}(f)}{\mathrm{d}Y_{r,k,s,t}(f)} = 1$ and the index s can be omitted.

3.8.6 Sensitivity analysis

Most of the structural model parameters are calibrated to match the German economy in 2014. The remaining ones, such as the inverse Frisch elasticity of substitution σ^L, are taken from the literature or are estimated, e.g. the persistence in unemployment benefits ρ^b. It is important to quantify how sensitive the reported results are with respect to these parameters. We construct an interval with the 95%, 97.5%, 102.5% and 105% values of the calibrated parameter value. For the persistence in regional attractiveness $\rho^{\epsilon^{pop}}$ we construct an interval around the implied average time an employee stays in the labour force. The interval around the discount factor is constructed around the implied interest rate $R = \frac{\mu^z\,\pi^c}{\beta}$ in a model with bonds. The sensitivity analysis is conducted for the following parameters: Discount factor β, elasticity of substitution between lignite coal and non-lignite coal η^b, regional elasticity of substitution for energy products η_E^m, regional elasticity of substitution for non-energy products η_{NE}^m, home bias energy products I_E^{Home}, home bias non-energy products I_{NE}^{Home}, share of hiring costs in wage sum $\frac{\kappa}{w\,n}$, long-run market power $\bar{\lambda}_l$, persistence in unemployment benefits ρ^b, persistence in regional attractiveness ρ_ϵ^{pop}, persistence in market power ρ^λ, inverse Frisch elasticity σ^L, labour market tightness hiring cost elasticity v and long-run job finding rate $x = \frac{h}{u}$.

In order to evaluate the sensitivity of the results with respect to each parameter, we report the maximum drop in the employment rate between the Null-Scenario and the respective scenario[11] for the time period 2014–2040 (see Tables 3.26–3.41). A one percent change in the inverse Frisch elasticity does not change the maximum drop in the employment rate by more than one percent (see Table 3.26). Labour supply reacts less to changes in wages if the Frisch elasticity is lower and vice versa. Wages react more to labour supply changes if the Frisch elasticity is lower. In Table 3.27 the maximum percentage drop in real regional wages are reported for different values of the inverse Frisch elasticity. A lower Frisch elasticity leads to more volatile wages and less volatile labour.

The persistence parameter for unemployment benefits determines how fast unemployment benefits react to changes in wages. We do not distinguish between long-term unemployment

[11] The maximum drop in employment is defined as follows: $\min\left(\left\{n_{r,t}^{Scenario} - n_{r,t}^{Null-Scenario}\right\}_{t=1}^{144}\right)$.

benefits and short-term unemployment benefits. A one percent increase in the persistence of unemployment benefits increases the maximum drop in the employment rate by up to 5 percent (see Table 3.28). A higher persistence in the adjustment of unemployment benefits leads to a lower adjustment of wages required by workers. Recovery in the model is achieved through migration and lower wages and a higher persistence in unemployment benefits leads to a slower adjustment process.

The sensitivity of employment to the elasticity of substitution between lignite coal and non-lignite coal η^b is very low, as reported in Table 3.29. A higher elasticity of substitution will increase the employment effects and indicates that less gross valued added from non-lignite coal is required to replace lignite coal to produce energy. Therefore, fewer people will find a job in the non-lignite coal energy sector. The results regarding employment rates are also very insensitive to variations in the regional elasticity of substitution between energy and non-energy products (see Table 3.30 and Table 3.31). An increase in the elasticity of substitution between energy and non-energy products will increase the ability of households to replace energy products by non-energy products while deriving the same utility. A one percent increase in the elasticity of substitution leads to a less than one percent reduction in the maximum drop in employment (see Table 3.32). Variations in the home bias for energy products also have no impact on the employment effects, as shown in Table 3.33. A one percent change in the home bias for non-energy products will trigger a more than one percent change in the maximum drop in the national employment rate (see Table 3.34). A higher home bias will reduce the maximum employment drop in the rest of Germany and Central Germany, but increase the maximum drop in Lusatia and the Rhineland. It is harder to generate new jobs in the non-energy sector for Lusatia and the Rhineland if demand from the rest of Germany for non-energy products is lower.

Market power will increase the maximum drop in employment, as shown in Table 3.35. Most of the increase is caused by a higher drop in the rest of Germany. In Central Germany, the Rhineland, and Lusatia, the impact on the maximum drop is negligible. A higher persistence in market power determines how quickly firms adjust their mark-ups in response to the change of attractiveness of the region they operate in. As stated before, attractiveness determines migration flows and affects the market power. A lower persistence leads to a faster adjustment of market power. Table 3.36 reports the results of the sensitivity analysis for the persistence parameter in market power. Firms will adjust their desired mark-up not as quickly according to their new market power if the persistence parameter is higher. As is known from standard micro theory, higher market power leads to lower output and lower demand for labour. Therefore, a slower adjustment to the new market power by firms will reduce the maximum drop in the employment rate.

The speed of migration in the model is determined by the persistence in the attractiveness of the region. As described before, we assume that entrants to the labour force decide where to live and work. We assume that after 22.5 years (roughly half the time an individual stays in the labour force) the labour force is populated to 50% by individuals who have chosen their working and living place after the coal phase-out path was announced. We construct the interval around the half-life an individual stays in the labour force (22.5 years) to compute the respective persistence parameters ρ_{ϵ}^{pop}. Table 3.37 shows that the maximum drop in employment changes are less than one percent if the half-time an individual stays in the labour force changes by one percent. A higher persistence in attractiveness leads to lower drops in the employment rate. A lower persistence implies that a higher share of people migrate each period. They only consider long-run developments in their decisions. This result reveals that it is not possible to change this parameter without altering the assumption about the process of migration. Reducing the persistence in the attractiveness of regions implies a higher share of population migrating each period. Altering the parameter requires altering the assumption about when individuals decide about their living and working place.

In Blanchard & Galí (2010) the elasticity of hiring costs with respect to the job finding rate is assumed to be unity. In Table 3.38 we document the sensitivity of the maximum drop in employment with respect to the elasticity of hiring costs. The results suggest that the drop in employment is only marginally affected by the elasticity of marginal hiring costs to the job finding rate. The quarterly job finding rate in Germany is assumed to be 22.43% and determines the exogenous separation rate in each period. This parameter implicitly determines steady-state hiring costs. A higher job finding rate will increase the exogenous separation rate. A higher long-run job finding rate will increase the maximum drop in the employment rate (see Table 3.39). It is easier to find new workers for firms and, therefore, incentives to do labour herding are reduced. The same argumentation holds for the share of hiring costs relative to the wage sum (see Table 3.40). An increase in the discount factor leads to a higher maximum drop in the employment rate (see Table 3.41). A lower decrease in the discount factor implies that future profits have a lower present value for firms. Their incentive to herd labour to increase future profits is lower.

3.8.7 Tables

Table 3.18: Regional and lignite specific productivity

Year	Germany	Rest of Germany	Central Germany	Lusatia	Rhineland
Null-Scenario					
2014	- [-;-]	100.00 [0.00;0.00]	100.00 [0.00;0.00]	100.00 [0.00;0.00]	100.00 [0.00;0.00]
2020	- [-;-]	-71.00 [-3.00;3.00]	0.00 [0.00;0.00]	0.00 [0.00;0.00]	0.00 [0.00;0.00]
2025	- [-;-]	-67.00 [-6.00;5.00]	0.00 [0.00;0.00]	0.00 [0.00;0.00]	0.00 [0.00;0.00]
2030	- [-;-]	-67.00 [-5.00;5.00]	0.00 [0.00;0.00]	0.00 [0.00;0.00]	0.00 [0.00;0.00]
2035	- [-;-]	-67.00 [-5.00;5.00]	0.00 [0.00;0.00]	0.00 [0.00;0.00]	0.00 [0.00;0.00]
2040	- [-;-]	-67.00 [-5.00;5.00]	0.00 [0.00;0.00]	0.00 [0.00;0.00]	0.00 [0.00;0.00]
Baseline					
2020	- [-;-]	-1.00 [-1.00;0.00]	-15.00 [-1.00;0.00]	-13.00 [-1.00;1.00]	-14.00 [-1.00;1.00]
2025	- [-;-]	0.00 [0.00;1.00]	-8.00 [-1.00;2.00]	-6.00 [-2.00;2.00]	-7.00 [-2.00;1.00]
2030	- [-;-]	-1.00 [-1.00;0.00]	-13.00 [-2.00;2.00]	-10.00 [-2.00;2.00]	-11.00 [-2.00;2.00]
2035	- [-;-]	-1.00 [-1.00;0.00]	-32.00 [-2.00;1.00]	-28.00 [-2.00;2.00]	-29.00 [-2.00;2.00]
2040	- [-;-]	-1.00 [-1.00;0.00]	-26.00 [-3.00;3.00]	-21.00 [-3.00;3.00]	-23.00 [-3.00;3.00]
Phase-Out-2035-Weak					
2020	- [-;-]	-1.00 [-1.00;0.00]	-11.00 [-1.00;1.00]	-8.00 [-1.00;2.00]	-9.00 [-2.00;1.00]
2025	- [-;-]	-1.00 [-1.00;0.00]	-22.00 [-3.00;2.00]	-17.00 [-3.00;3.00]	-43.00 [-2.00;2.00]
2030	- [-;-]	-1.00 [-1.00;0.00]	-56.00 [-2.00;3.00]	-50.00 [-4.00;3.00]	-18.00 [-7.00;7.00]
2035	- [-;-]	-1.00 [-1.00;1.00]	-92.00 [-3.00;2.00]	-84.00 [-6.00;5.00]	-88.00 [-5.00;3.00]
2040	- [-;-]	-1.00 [-1.00;1.00]	-92.00 [-3.00;2.00]	-84.00 [-7.00;5.00]	-88.00 [-5.00;3.00]
Phase-Out-2040-Age					
2020	- [-;-]	-1.00 [-1.00;0.00]	-13.00 [-1.00;1.00]	-11.00 [-1.00;2.00]	-11.00 [-1.00;2.00]
2025	- [-;-]	-1.00 [-1.00;0.00]	-15.00 [-2.00;2.00]	-12.00 [-2.00;2.00]	-23.00 [-3.00;2.00]
2030	- [-;-]	-1.00 [-1.00;0.00]	-20.00 [-3.00;2.00]	-14.00 [-4.00;4.00]	-40.00 [-3.00;3.00]
2035	- [-;-]	-1.00 [-1.00;0.00]	-58.00 [-2.00;3.00]	-52.00 [-4.00;3.00]	-20.00 [-8.00;7.00]
2040	- [-;-]	-1.00 [-1.00;1.00]	-92.00 [-3.00;2.00]	-84.00 [-6.00;5.00]	-88.00 [-5.00;3.00]
Phase-Out-2040-Balanced					
2020	- [-;-]	-1.00 [-1.00;1.00]	-13.00 [-1.00;1.00]	-9.00 [-1.00;2.00]	-13.00 [-1.00;1.00]
2025	- [-;-]	-1.00 [-1.00;0.00]	-7.00 [-2.00;1.00]	-32.00 [-2.00;3.00]	-13.00 [-2.00;2.00]
2030	- [-;-]	-1.00 [-1.00;0.00]	-23.00 [-3.00;2.00]	-17.00 [-4.00;5.00]	-30.00 [-3.00;3.00]
2035	- [-;-]	-1.00 [-1.00;0.00]	-78.00 [-3.00;3.00]	-35.00 [-5.00;5.00]	-25.00 [-6.00;5.00]
2040	- [-;-]	-1.00 [-1.00;1.00]	-92.00 [-3.00;2.00]	-84.00 [-6.00;5.00]	-88.00 [-5.00;3.00]
Phase-Out-2035-Strong					
2020	- [-;-]	-1.00 [-1.00;1.00]	-28.00 [-1.00;2.00]	-26.00 [-2.00;1.00]	-24.00 [-2.00;2.00]
2025	- [-;-]	-1.00 [-1.00;0.00]	-15.00 [-3.00;3.00]	-9.00 [-4.00;4.00]	-38.00 [-3.00;3.00]
2030	- [-;-]	-1.00 [-1.00;0.00]	-56.00 [-3.00;3.00]	-49.00 [-3.00;4.00]	-15.00 [-7.00;6.00]
2035	- [-;-]	-1.00 [-1.00;1.00]	-92.00 [-3.00;2.00]	-84.00 [-6.00;5.00]	-88.00 [-5.00;3.00]
2040	- [-;-]	-1.00 [-1.00;1.00]	-92.00 [-3.00;2.00]	-84.00 [-7.00;5.00]	-88.00 [-5.00;3.00]

Note: Simulation results for regional lignite productivity. Values in brackets denote the minimum and maximum difference from the reported value obtained from 1200 simulations.

Table 3.19: Net electricity generation in lignite sector

Year	Germany	Rest of Germany	Central Germany	Lusatia	Rhineland
		Null-Scenario			
2014	100.00 [0.00;0.00]	- [-;-]	100.00 [0.00;0.00]	100.00 [0.00;0.00]	100.00 [0.00;0.00]
2020	107.00 [0.00;0.00]	- [-;-]	107.00 [0.00;0.00]	107.00 [0.00;0.00]	107.00 [0.00;0.00]
2025	108.00 [0.00;0.00]	- [-;-]	108.00 [0.00;0.00]	108.00 [0.00;0.00]	108.00 [0.00;0.00]
2030	108.00 [0.00;0.00]	- [-;-]	108.00 [0.00;0.00]	108.00 [0.00;0.00]	108.00 [0.00;0.00]
2035	109.00 [0.00;0.00]	- [-;-]	109.00 [0.00;0.00]	109.00 [0.00;0.00]	109.00 [0.00;0.00]
2040	111.00 [0.00;0.00]	- [-;-]	111.00 [0.00;0.00]	111.00 [0.00;0.00]	111.00 [0.00;0.00]
		Baseline			
2020	81.00 [-1.00;0.00]	- [-;-]	81.00 [-1.00;0.00]	81.00 [-1.00;0.00]	81.00 [-1.00;0.00]
2025	81.00 [0.00;0.00]	- [-;-]	82.00 [0.00;1.00]	81.00 [0.00;0.00]	82.00 [0.00;1.00]
2030	72.00 [0.00;1.00]	- [-;-]	72.00 [0.00;0.00]	71.00 [0.00;0.00]	72.00 [0.00;1.00]
2035	48.00 [-1.00;0.00]	- [-;-]	48.00 [-1.00;0.00]	48.00 [-1.00;0.00]	48.00 [-1.00;0.00]
2040	48.00 [0.00;1.00]	- [-;-]	48.00 [0.00;1.00]	48.00 [0.00;1.00]	48.00 [0.00;1.00]
		Phase-Out-2035-Weak			
2020	80.00 [0.00;0.00]	- [-;-]	81.00 [0.00;0.00]	81.00 [0.00;0.00]	79.00 [0.00;0.00]
2025	44.00 [0.00;0.00]	- [-;-]	56.00 [0.00;0.00]	56.00 [0.00;0.00]	32.00 [-1.00;0.00]
2030	23.00 [0.00;0.00]	- [-;-]	16.00 [-1.00;0.00]	16.00 [-1.00;0.00]	30.00 [-1.00;0.00]
2035	0.00 [0.00;0.00]	- [-;-]	0.00 [0.00;0.00]	0.00 [0.00;0.00]	0.00 [0.00;0.00]
2040	0.00 [0.00;0.00]	- [-;-]	0.00 [0.00;0.00]	0.00 [0.00;0.00]	0.00 [0.00;0.00]
		Phase-Out-2040-Age			
2020	80.00 [0.00;0.00]	- [-;-]	81.00 [0.00;0.00]	81.00 [0.00;0.00]	79.00 [0.00;0.00]
2025	61.00 [0.00;0.00]	- [-;-]	70.00 [0.00;1.00]	69.00 [0.00;0.00]	54.00 [0.00;0.00]
2030	41.00 [0.00;0.00]	- [-;-]	55.00 [0.00;0.00]	54.00 [0.00;0.00]	28.00 [-1.00;0.00]
2035	21.00 [0.00;1.00]	- [-;-]	15.00 [0.00;1.00]	15.00 [0.00;1.00]	26.00 [-1.00;0.00]
2040	0.00 [0.00;0.00]	- [-;-]	0.00 [0.00;0.00]	0.00 [0.00;0.00]	0.00 [0.00;0.00]
		Phase-Out-2040-Balanced			
2020	80.00 [0.00;0.00]	- [-;-]	83.00 [0.00;0.00]	79.00 [-1.00;0.00]	79.00 [0.00;0.00]
2025	61.00 [0.00;0.00]	- [-;-]	80.00 [0.00;0.00]	44.00 [0.00;0.00]	67.00 [0.00;0.00]
2030	43.00 [0.00;0.00]	- [-;-]	54.00 [0.00;0.00]	41.00 [0.00;0.00]	41.00 [0.00;0.00]
2035	22.00 [0.00;0.00]	- [-;-]	2.00 [-1.00;0.00]	22.00 [0.00;0.00]	28.00 [-1.00;0.00]
2040	0.00 [0.00;0.00]	- [-;-]	0.00 [0.00;0.00]	0.00 [0.00;0.00]	0.00 [0.00;0.00]
		Phase-Out-2035-Strong			
2020	60.00 [0.00;0.00]	- [-;-]	60.00 [0.00;0.00]	60.00 [0.00;1.00]	60.00 [0.00;0.00]
2025	44.00 [0.00;0.00]	- [-;-]	56.00 [0.00;0.00]	56.00 [0.00;0.00]	32.00 [0.00;0.00]
2030	23.00 [0.00;0.00]	- [-;-]	17.00 [0.00;1.00]	16.00 [-1.00;0.00]	30.00 [-1.00;0.00]
2035	0.00 [0.00;0.00]	- [-;-]	0.00 [0.00;0.00]	0.00 [0.00;0.00]	0.00 [0.00;0.00]
2040	0.00 [0.00;0.00]	- [-;-]	0.00 [0.00;0.00]	0.00 [0.00;0.00]	0.00 [0.00;0.00]

Note: Simulation results for net electricity generation by lignite relative to 2014 values. Values in brackets denote the minimum and maximum difference from the reported value obtained from 1200 simulations.

Table 3.20: Employees in non-lignite sector

Year	Germany	Rest of Germany	Central Germany	Lusatia	Rhineland
Null-Scenario					
2014	297.4 [0.0;0.0]	259.5 [0.0;0.0]	10.8 [0.0;0.0]	2.9 [0.0;0.0]	24.2 [0.0;0.0]
2020	6.2 [0.0;0.0]	5.4 [0.0;0.0]	0.2 [0.0;0.0]	0.1 [0.0;0.0]	0.5 [0.0;0.0]
2025	-1.3 [0.0;0.0]	-1.2 [0.0;0.0]	0.0 [0.0;0.0]	0.0 [0.0;0.0]	-0.1 [0.0;0.0]
2030	-12.1 [0.0;0.0]	-10.6 [0.0;0.0]	-0.4 [0.0;0.0]	-0.1 [0.0;0.0]	-1.0 [0.0;0.0]
2035	-20.4 [0.0;0.0]	-17.8 [0.0;0.0]	-0.7 [0.0;0.0]	-0.2 [0.0;0.0]	-1.7 [0.0;0.0]
2040	-24.8 [0.0;0.0]	-21.7 [0.0;0.0]	-0.9 [0.0;0.0]	-0.2 [0.0;0.0]	-2.0 [0.0;0.0]
Baseline					
2020	2.6 [-0.1;0.2]	2.3 [-0.1;0.1]	0.1 [0.0;0.0]	0.0 [0.0;0.0]	0.2 [0.0;0.1]
2025	3.5 [-0.1;0.1]	3.2 [-0.1;0.1]	0.1 [0.0;0.0]	0.0 [0.0;0.0]	0.2 [0.0;0.0]
2030	4.7 [-0.2;0.2]	4.3 [-0.1;0.2]	0.1 [0.0;0.0]	0.0 [0.0;0.0]	0.3 [-0.1;0.0]
2035	6.3 [-0.2;0.4]	5.6 [-0.2;0.2]	0.2 [0.0;0.1]	0.0 [0.0;0.0]	0.5 [0.0;0.1]
2040	6.7 [-0.3;0.2]	6.1 [-0.2;0.2]	0.2 [0.0;0.0]	0.0 [0.0;0.0]	0.4 [-0.1;0.0]
Phase-Out-2035-Weak					
2020	4.0 [-0.3;0.3]	3.6 [-0.2;0.2]	0.2 [0.0;0.1]	0.0 [0.0;0.0]	0.2 [-0.1;0.0]
2025	7.5 [-0.3;0.4]	6.8 [-0.3;0.3]	0.2 [0.0;0.0]	0.0 [0.0;0.0]	0.5 [0.0;0.1]
2030	10.2 [-0.2;0.5]	9.2 [-0.2;0.4]	0.3 [0.0;0.1]	0.0 [0.0;0.0]	0.7 [0.0;0.1]
2035	11.7 [-0.5;0.3]	10.6 [-0.4;0.3]	0.3 [0.0;0.0]	0.0 [-0.1;0.0]	0.8 [-0.1;0.0]
2040	12.3 [-0.4;0.5]	11.1 [-0.4;0.4]	0.4 [0.0;0.1]	0.0 [0.0;0.0]	0.8 [-0.1;0.1]
Phase-Out-2040-Age					
2020	3.3 [-0.2;0.1]	3.0 [-0.2;0.1]	0.1 [0.0;0.0]	0.0 [0.0;0.0]	0.2 [0.0;0.0]
2025	5.6 [-0.3;0.2]	5.2 [-0.2;0.2]	0.1 [0.0;0.0]	0.0 [0.0;0.0]	0.3 [-0.1;0.0]
2030	7.9 [-0.4;0.3]	7.2 [-0.3;0.3]	0.2 [0.0;0.0]	0.0 [0.0;0.0]	0.5 [-0.1;0.0]
2035	10.1 [-0.4;0.4]	9.1 [-0.3;0.3]	0.3 [0.0;0.1]	0.0 [-0.1;0.0]	0.7 [-0.1;0.0]
2040	11.7 [-0.4;0.5]	10.6 [-0.4;0.4]	0.3 [-0.1;0.0]	0.0 [0.0;0.0]	0.8 [0.0;0.1]
Phase-Out-2040-Balanced					
2020	3.3 [-0.2;0.1]	3.0 [-0.2;0.1]	0.1 [0.0;0.0]	0.0 [0.0;0.0]	0.2 [0.0;0.0]
2025	5.7 [-0.2;0.3]	5.2 [-0.2;0.2]	0.1 [0.0;0.0]	0.0 [0.0;0.0]	0.4 [0.0;0.1]
2030	7.8 [-0.4;0.3]	7.1 [-0.3;0.3]	0.2 [0.0;0.0]	0.0 [0.0;0.0]	0.5 [-0.1;0.0]
2035	9.9 [-0.5;0.3]	9.0 [-0.3;0.3]	0.2 [-0.1;0.0]	0.0 [-0.1;0.0]	0.7 [-0.1;0.0]
2040	11.6 [-0.5;0.4]	10.5 [-0.4;0.3]	0.3 [-0.1;0.0]	0.0 [0.0;0.0]	0.8 [0.0;0.1]
Phase-Out-2035-Strong					
2020	5.2 [-0.3;0.1]	4.7 [-0.2;0.1]	0.2 [0.0;0.0]	0.0 [0.0;0.0]	0.3 [-0.1;0.0]
2025	7.9 [-0.3;0.2]	7.2 [-0.3;0.2]	0.2 [0.0;0.0]	0.0 [0.0;0.0]	0.5 [0.0;0.0]
2030	10.3 [-0.3;0.4]	9.3 [-0.3;0.3]	0.3 [0.0;0.1]	0.0 [0.0;0.0]	0.7 [-0.1;0.1]
2035	11.8 [-0.5;0.4]	10.7 [-0.4;0.4]	0.3 [0.0;0.0]	0.0 [-0.1;0.0]	0.8 [-0.1;0.0]
2040	12.3 [-0.4;0.5]	11.1 [-0.4;0.4]	0.4 [0.0;0.1]	0.0 [0.0;0.0]	0.8 [-0.1;0.1]

Note: Simulation results for employees in the non-lignite industry in thousand people. Values for the Null-Scenario are reported as change to the base year 2014 and for the year 2014 actual values are reported. Values for other scenarios are differences to the Null-Scenario in the respective year. Values in brackets denote the minimum and maximum difference from the reported value obtained from 1200 simulations.

Table 3.21: Employees in non-energy sector

Year	Germany	Rest of Germany	Central Germany	Lusatia	Rhineland
			Null-Scenario		
2014	42870.0 [0.0;0.0]	37966.9 [0.0;0.0]	1450.6 [0.0;0.0]	453.7 [0.0;0.0]	2998.8 [0.0;0.0]
2020	852.1 [-0.2;0.3]	754.6 [-0.2;0.3]	28.9 [0.0;0.0]	9.0 [0.0;0.0]	59.6 [0.0;0.0]
2025	-247.4 [-0.3;0.5]	-219.2 [-0.3;0.4]	-8.3 [0.0;0.1]	-2.6 [0.0;0.0]	-17.3 [0.0;0.0]
2030	-1793.5 [-0.6;0.5]	-1588.3 [-0.4;0.5]	-60.7 [-0.1;0.0]	-19.0 [0.0;0.0]	-125.5 [-0.1;0.0]
2035	-2990.7 [-0.4;0.5]	-2648.6 [-0.4;0.5]	-101.2 [0.0;0.0]	-31.7 [0.0;0.0]	-209.2 [0.0;0.0]
2040	-3625.8 [-0.5;0.5]	-3211.1 [-0.4;0.4]	-122.7 [-0.1;0.0]	-38.4 [0.0;0.0]	-253.6 [0.0;0.1]
			Baseline		
2020	-7.6 [-3.8;3.5]	-5.3 [-4.5;3.5]	-0.4 [-0.1;0.1]	-0.8 [-0.3;0.5]	-1.1 [-0.2;0.3]
2025	-9.1 [-5.3;5.0]	-6.6 [-6.2;5.1]	-0.4 [-0.2;0.1]	-0.9 [-0.4;0.7]	-1.2 [-0.3;0.3]
2030	-9.2 [-6.0;6.2]	-6.6 [-7.1;6.3]	-0.3 [-0.1;0.2]	-1.0 [-0.5;0.9]	-1.3 [-0.3;0.4]
2035	-9.7 [-6.5;6.9]	-6.1 [-7.8;7.2]	-0.4 [-0.2;0.2]	-1.4 [-0.7;1.1]	-1.8 [-0.6;0.6]
2040	-5.7 [-6.5;6.9]	-2.6 [-7.3;7.3]	-0.2 [-0.1;0.3]	-1.4 [-0.7;1.3]	-1.5 [-0.7;0.7]
			Phase-Out-2035-Weak		
2020	-28.2 [-10.7;8.5]	-23.4 [-10.6;8.3]	-1.1 [-0.4;0.3]	-1.1 [-0.4;0.8]	-2.6 [-0.4;0.5]
2025	-44.4 [-16.5;14.1]	-36.4 [-16.4;13.8]	-1.6 [-0.6;0.4]	-1.8 [-0.7;1.3]	-4.6 [-0.9;1.0]
2030	-47.3 [-18.8;18.5]	-39.2 [-19.1;17.2]	-1.5 [-0.5;0.7]	-2.4 [-1.0;1.7]	-4.2 [-0.9;1.3]
2035	-37.4 [-20.1;22.0]	-31.9 [-20.0;19.7]	-0.6 [-0.7;0.8]	-2.1 [-1.1;1.9]	-2.8 [-1.1;1.5]
2040	-17.2 [-20.3;23.7]	-14.7 [-20.4;21.3]	0.3 [-0.8;0.8]	-1.8 [-1.2;2.3]	-1.0 [-1.2;1.5]
			Phase-Out-2040-Age		
2020	-28.5 [-10.7;7.9]	-23.5 [-10.5;7.8]	-1.1 [-0.4;0.2]	-1.2 [-0.5;0.8]	-2.7 [-0.4;0.4]
2025	-44.6 [-15.9;12.7]	-37.0 [-15.9;12.5]	-1.6 [-0.5;0.4]	-1.7 [-0.7;1.3]	-4.3 [-0.7;0.8]
2030	-51.8 [-18.4;16.1]	-43.2 [-18.7;15.7]	-1.7 [-0.5;0.6]	-1.9 [-0.8;1.7]	-5.0 [-1.0;1.2]
2035	-51.1 [-19.6;19.1]	-42.7 [-19.8;18.2]	-1.6 [-0.7;0.7]	-2.4 [-1.1;2.0]	-4.4 [-1.0;1.5]
2040	-39.8 [-19.6;21.9]	-34.2 [-19.8;19.8]	-0.6 [-0.7;0.8]	-2.1 [-1.2;2.3]	-2.9 [-1.2;1.5]
			Phase-Out-2040-Balanced		
2020	-28.7 [-10.6;8.0]	-23.7 [-10.5;7.8]	-1.1 [-0.4;0.2]	-1.2 [-0.4;0.7]	-2.7 [-0.4;0.5]
2025	-45.0 [-15.7;12.8]	-37.3 [-15.8;12.5]	-1.5 [-0.5;0.4]	-2.2 [-0.7;1.4]	-4.0 [-0.7;0.8]
2030	-52.1 [-18.3;15.9]	-43.4 [-18.6;15.6]	-1.7 [-0.5;0.5]	-2.2 [-0.9;1.7]	-4.8 [-0.9;1.2]
2035	-51.5 [-19.4;18.8]	-43.2 [-19.7;18.0]	-1.5 [-0.7;0.7]	-2.3 [-1.1;2.0]	-4.5 [-1.1;1.5]
2040	-40.6 [-20.0;21.5]	-34.8 [-20.0;19.7]	-0.6 [-0.8;0.7]	-2.2 [-1.3;2.2]	-3.0 [-1.2;1.5]
			Phase-Out-2035-Strong		
2020	-30.8 [-11.0;9.2]	-24.8 [-11.1;8.9]	-1.2 [-0.3;0.4]	-1.6 [-0.5;0.9]	-3.2 [-0.5;0.6]
2025	-43.1 [-16.4;14.6]	-35.3 [-16.5;13.9]	-1.5 [-0.5;0.5]	-1.8 [-0.6;1.3]	-4.5 [-0.9;1.1]
2030	-44.2 [-18.9;18.8]	-36.5 [-19.1;17.3]	-1.4 [-0.5;0.7]	-2.3 [-1.0;1.7]	-4.0 [-0.9;1.3]
2035	-33.9 [-20.0;22.1]	-28.9 [-19.9;19.7]	-0.5 [-0.7;0.8]	-2.0 [-1.0;2.0]	-2.5 [-1.1;1.5]
2040	-14.0 [-20.0;23.5]	-11.9 [-20.1;21.2]	0.5 [-0.7;0.9]	-1.8 [-1.3;2.2]	-0.8 [-1.3;1.5]

Note: Simulation results for employees in non-energy sector in thousand people. Values for the Null-Scenario are reported as change to the base year 2014 and for the year 2014 actual values are reported. Values for other scenarios are differences to the Null-Scenario in the respective year. Values in brackets denote the minimum and maximum difference from the reported value obtained from 1200 simulations.

Table 3.22: Real gross value-added total

Year	Germany	Rest of Germany	Central Germany	Lusatia	Rhineland
			Null-Scenario		
2014	2631268 [0;0]	2332970 [0;0]	71090 [0;0]	22606 [0;0]	204602 [0;0]
2020	175310 [-10;15]	155419 [-9;13]	4740 [-1;0]	1509 [0;0]	13642 [-1;2]
2025	208876 [-20;24]	185183 [-17;22]	5646 [-1;1]	1797 [0;0]	16250 [-2;2]
2030	210052 [-27;30]	186225 [-24;27]	5678 [-1;1]	1807 [0;0]	16342 [-2;3]
2035	232242 [-31;30]	205899 [-28;27]	6277 [-1;1]	1998 [0;1]	18068 [-2;3]
2040	293908 [-30;31]	260573 [-28;27]	7944 [0;2]	2528 [0;1]	22863 [-2;3]
			Baseline		
2020	-552 [-367;236]	276 [-444;275]	-80 [-4;4]	-310 [-31;50]	-438 [-38;48]
2025	-551 [-513;368]	272 [-572;384]	-84 [-10;7]	-314 [-32;61]	-425 [-33;40]
2030	-593 [-694;491]	550 [-807;529]	-112 [-14;8]	-444 [-50;94]	-587 [-52;67]
2035	-914 [-884;621]	1031 [-1083;714]	-185 [-13;10]	-750 [-77;132]	-1010 [-96;121]
2040	-676 [-881;650]	1269 [-1091;734]	-186 [-14;10]	-762 [-77;133]	-997 [-96;112]
			Phase-Out-2035-Weak		
2020	-1335 [-644;507]	-416 [-685;506]	-98 [-15;12]	-323 [-38;72]	-498 [-38;48]
2025	-2713 [-1200;977]	-456 [-1365;1030]	-172 [-35;25]	-631 [-66;124]	-1454 [-124;187]
2030	-3216 [-1433;1294]	-437 [-1582;1317]	-349 [-25;28]	-1165 [-112;205]	-1265 [-91;99]
2035	-3237 [-1571;1533]	112 [-1742;1551]	-292 [-58;44]	-1291 [-107;200]	-1766 [-144;208]
2040	-1973 [-1675;1716]	1234 [-1866;1719]	-267 [-49;44]	-1350 [-132;240]	-1590 [-150;191]
			Phase-Out-2040-Age		
2020	-1366 [-667;493]	-398 [-725;506]	-99 [-15;11]	-341 [-42;78]	-528 [-44;63]
2025	-2472 [-1132;887]	-765 [-1241;909]	-146 [-32;22]	-511 [-63;124]	-1050 [-85;121]
2030	-3179 [-1442;1200]	-775 [-1620;1242]	-192 [-40;29]	-697 [-87;167]	-1515 [-125;175]
2035	-3469 [-1624;1460]	-458 [-1853;1507]	-363 [-27;31]	-1250 [-137;252]	-1398 [-104;133]
2040	-3401 [-1677;1636]	55 [-1892;1662]	-296 [-65;47]	-1361 [-128;239]	-1799 [-144;217]
			Phase-Out-2040-Balanced		
2020	-1383 [-672;491]	-396 [-734;508]	-95 [-15;11]	-333 [-35;70]	-559 [-51;75]
2025	-2506 [-1134;887]	-786 [-1228;905]	-97 [-38;25]	-834 [-84;149]	-789 [-66;88]
2030	-3162 [-1435;1189]	-812 [-1617;1229]	-208 [-33;25]	-824 [-83;154]	-1318 [-121;179]
2035	-3460 [-1629;1450]	-493 [-1867;1504]	-439 [-31;35]	-1107 [-116;218]	-1421 [-116;149]
2040	-3441 [-1691;1635]	24 [-1907;1665]	-288 [-62;49]	-1367 [-130;241]	-1810 [-145;216]
			Phase-Out-2035-Strong		
2020	-1782 [-742;569]	-182 [-851;615]	-170 [-12;12]	-589 [-58;93]	-841 [-66;90]
2025	-2695 [-1120;963]	-541 [-1234;984]	-170 [-34;25]	-595 [-54;102]	-1389 [-110;148]
2030	-3017 [-1426;1298]	-270 [-1565;1319]	-347 [-24;28]	-1166 [-114;212]	-1234 [-91;95]
2035	-3036 [-1540;1520]	237 [-1684;1526]	-282 [-58;45]	-1287 [-107;199]	-1704 [-136;185]
2040	-1760 [-1654;1705]	1425 [-1859;1709]	-262 [-48;44]	-1348 [-132;241]	-1575 [-154;191]

Note: Simulation results for real gross value-added in million euros. Values for the Null-Scenario are reported as change to the base year 2014 and for the year 2014 actual values are reported. Values for other scenarios are differences to the Null-Scenario in the respective year. Values in brackets denote the minimum and maximum difference from the reported value obtained from 1200 simulations.

Table 3.23: Real gross value-added in lignite sector

Year	Germany	Rest of Germany	Central Germany	Lusatia	Rhineland
			Null-Scenario		
2014	2448 [0;0]	56 [0;0]	295 [0;0]	873 [0;0]	1224 [0;0]
2020	105 [0;0]	-54 [0;0]	20 [0;0]	58 [0;0]	81 [0;0]
2025	135 [0;1]	-55 [0;1]	24 [0;0]	69 [0;0]	97 [0;0]
2030	136 [0;0]	-55 [0;0]	24 [0;0]	69 [0;0]	98 [0;0]
2035	157 [0;0]	-55 [0;0]	27 [0;0]	77 [0;0]	108 [0;0]
2040	212 [0;0]	-55 [0;0]	33 [0;0]	97 [0;0]	137 [0;0]
			Baseline		
2020	-605 [-6;1]	0 [0;0]	-74 [0;0]	-222 [-2;1]	-309 [-4;0]
2025	-634 [-1;10]	0 [-1;0]	-78 [0;1]	-233 [-1;4]	-323 [0;5]
2030	-873 [-2;3]	0 [0;0]	-107 [0;1]	-320 [-1;1]	-446 [-1;1]
2035	-1457 [-25;3]	0 [0;0]	-180 [-3;0]	-533 [-9;2]	-744 [-13;1]
2040	-1518 [-3;12]	0 [0;0]	-187 [-1;1]	-555 [-1;5]	-776 [-1;6]
			Phase-Out-2035-Weak		
2020	-636 [-1;2]	0 [0;0]	-74 [0;0]	-223 [-1;1]	-339 [0;1]
2025	-1534 [-4;4]	0 [-1;0]	-153 [0;0]	-454 [-1;2]	-927 [-4;3]
2030	-2024 [-1;3]	0 [0;0]	-270 [-1;1]	-800 [-4;3]	-954 [-5;7]
2035	-2603 [-2;1]	0 [0;0]	-322 [0;0]	-949 [-1;1]	-1332 [-1;0]
2040	-2659 [0;0]	0 [0;0]	-328 [0;0]	-970 [0;0]	-1361 [0;0]
			Phase-Out-2040-Age		
2020	-635 [-1;3]	0 [0;1]	-74 [0;0]	-223 [-1;1]	-338 [0;1]
2025	-1119 [-1;2]	0 [-1;0]	-114 [0;0]	-339 [-1;2]	-666 [0;0]
2030	-1601 [-2;4]	0 [0;0]	-158 [0;0]	-469 [-1;2]	-974 [-2;2]
2035	-2111 [-1;2]	0 [0;0]	-279 [-1;1]	-823 [-4;4]	-1009 [-5;6]
2040	-2658 [-1;1]	0 [0;0]	-328 [0;0]	-969 [0;1]	-1361 [-1;0]
			Phase-Out-2040-Balanced		
2020	-645 [0;2]	0 [0;1]	-70 [0;0]	-237 [0;1]	-338 [0;0]
2025	-1147 [-1;2]	0 [-1;0]	-83 [0;0]	-559 [0;2]	-505 [-1;0]
2030	-1564 [-2;2]	0 [0;0]	-160 [0;1]	-584 [-2;1]	-820 [-1;0]
2035	-2059 [-3;2]	0 [0;0]	-315 [-1;2]	-760 [-1;1]	-984 [-3;0]
2040	-2658 [-2;1]	0 [0;0]	-328 [0;0]	-969 [-1;1]	-1361 [-1;0]
			Phase-Out-2035-Strong		
2020	-1124 [-2;2]	0 [0;1]	-138 [0;0]	-411 [-1;1]	-575 [-1;0]
2025	-1540 [-2;2]	0 [0;0]	-153 [0;0]	-455 [-1;1]	-932 [-2;1]
2030	-2022 [-3;1]	0 [0;0]	-270 [-1;1]	-799 [-4;3]	-953 [-6;5]
2035	-2603 [-1;1]	0 [0;0]	-322 [0;0]	-949 [0;1]	-1332 [-1;0]
2040	-2659 [0;0]	0 [0;0]	-328 [0;0]	-970 [0;0]	-1361 [0;0]

Note: Simulation results for gross value-added at constant prices in the lignite industry in million euros. Values for the Null-Scenario are reported as change to the base year 2014 and for the year 2014 actual values are reported. Values for other scenarios are differences to the Null-Scenario in the respective year. Values in brackets denote the minimum and maximum difference from the reported value obtained from 1200 simulations.

Table 3.24: Real gross labour income

Year	Germany	Rest of Germany	Central Germany	Lusatia	Rhineland
			Null-Scenario		
2014	1482827 [0;0]	1312983 [0;0]	40342 [0;0]	12267 [0;0]	117235 [0;0]
2020	98766 [-12;11]	87442 [-9;11]	2689 [-3;0]	818 [0;0]	7817 [0;0]
2025	117634 [-10;11]	104142 [-10;12]	3205 [0;0]	975 [0;0]	9312 [-1;0]
2030	118288 [-16;12]	104724 [-15;14]	3222 [-1;0]	979 [0;0]	9363 [-1;0]
2035	130789 [-15;13]	115790 [-15;12]	3562 [0;0]	1085 [0;3]	10352 [-1;0]
2040	165535 [-21;18]	146555 [-17;14]	4507 [-3;0]	1372 [0;0]	13101 [-1;4]
			Baseline		
2020	-647 [-108;120]	-47 [-114;106]	-71 [-4;4]	-216 [-7;9]	-313 [-11;11]
2025	-962 [-200;190]	-55 [-215;183]	-111 [-5;5]	-326 [-8;18]	-470 [-16;14]
2030	-1244 [-232;232]	-47 [-237;225]	-147 [-6;6]	-430 [-12;15]	-620 [-20;19]
2035	-1716 [-223;263]	0 [-221;245]	-215 [-8;5]	-611 [-13;13]	-890 [-25;19]
2040	-1912 [-229;271]	94 [-256;262]	-252 [-6;9]	-713 [-10;23]	-1041 [-18;18]
			Phase-Out-2035-Weak		
2020	-1607 [-407;322]	-630 [-379;304]	-112 [-9;11]	-304 [-16;20]	-561 [-30;27]
2025	-3059 [-618;537]	-1049 [-572;486]	-236 [-19;13]	-603 [-19;25]	-1171 [-51;51]
2030	-4177 [-749;704]	-1184 [-705;641]	-389 [-24;24]	-973 [-22;21]	-1631 [-45;51]
2035	-5712 [-717;791]	-1107 [-754;727]	-764 [-38;35]	-1403 [-27;34]	-2438 [-94;89]
2040	-6124 [-724;809]	-971 [-715;750]	-839 [-14;23]	-1464 [-17;32]	-2850 [-53;60]
			Phase-Out-2040-Age		
2020	-1579 [-386;300]	-714 [-370;277]	-102 [-9;9]	-259 [-10;17]	-504 [-22;29]
2025	-2830 [-621;499]	-1212 [-590;459]	-187 [-16;14]	-455 [-14;26]	-976 [-29;38]
2030	-3863 [-747;644]	-1505 [-704;594]	-268 [-20;18]	-655 [-20;27]	-1435 [-45;48]
2035	-4706 [-801;760]	-1559 [-758;694]	-410 [-26;27]	-994 [-21;23]	-1743 [-45;56]
2040	-6330 [-749;825]	-1430 [-789;765]	-821 [-33;26]	-1444 [-27;35]	-2635 [-96;93]
			Phase-Out-2040-Balanced		
2020	-1599 [-398;291]	-722 [-372;276]	-94 [-12;6]	-324 [-14;19]	-459 [-21;22]
2025	-2838 [-621;495]	-1225 [-587;459]	-164 [-19;13]	-636 [-16;20]	-813 [-30;37]
2030	-3801 [-755;635]	-1523 [-708;590]	-259 [-25;16]	-828 [-19;28]	-1191 [-39;45]
2035	-4655 [-817;756]	-1572 [-766;695]	-441 [-34;28]	-1018 [-18;29]	-1624 [-47;56]
2040	-6349 [-746;824]	-1452 [-783;766]	-843 [-16;22]	-1433 [-31;36]	-2621 [-100;101]
			Phase-Out-2035-Strong		
2020	-1897 [-387;335]	-616 [-358;297]	-154 [-10;11]	-416 [-16;14]	-711 [-28;28]
2025	-3230 [-628;550]	-974 [-597;504]	-262 [-16;15]	-681 [-18;28]	-1313 [-40;50]
2030	-4104 [-750;694]	-1105 [-707;637]	-386 [-24;22]	-966 [-17;18]	-1647 [-45;54]
2035	-5749 [-707;784]	-1013 [-751;727]	-788 [-37;28]	-1413 [-26;35]	-2535 [-100;99]
2040	-6030 [-717;806]	-886 [-705;746]	-837 [-15;23]	-1464 [-20;32]	-2843 [-53;60]

Note: Simulation results for real gross labour income in Germany in million euros. Values for the Null-Scenario are reported as change to the base year 2014 and for the year 2014 actual values are reported. Values for other scenarios are differences to the Null-Scenario in the respective year. Values in brackets denote the minimum and maximum difference from the reported value obtained from 1200 simulations.

Table 3.25: Real gross labour income in lignite sector

Year	Germany	Rest of Germany	Central Germany	Lusatia	Rhineland
			Null-Scenario		
2014	1428 [0;0]	28 [0;0]	161 [0;0]	524 [0;0]	715 [0;0]
2020	62 [0;0]	-23 [0;0]	7 [0;0]	31 [0;0]	47 [0;0]
2025	77 [0;0]	-28 [0;0]	14 [0;0]	38 [0;0]	53 [0;0]
2030	81 [0;0]	-28 [0;0]	13 [0;0]	37 [0;0]	59 [0;0]
2035	92 [0;0]	-28 [0;0]	12 [0;0]	44 [0;0]	64 [0;0]
2040	123 [0;0]	-28 [0;0]	18 [0;0]	57 [0;0]	76 [0;0]
			Baseline		
2020	-272 [-2;15]	0 [0;0]	-28 [0;0]	-103 [-1;7]	-141 [-1;8]
2025	-401 [-2;2]	0 [0;0]	-45 [0;0]	-152 [-1;1]	-204 [-1;1]
2030	-539 [-4;6]	0 [0;0]	-63 [0;1]	-200 [-2;1]	-276 [-2;7]
2035	-763 [-11;7]	0 [0;0]	-85 [0;1]	-286 [-6;2]	-392 [-8;7]
2040	-878 [-7;4]	0 [0;0]	-104 [-1;0]	-327 [-1;2]	-447 [-7;2]
			Phase-Out-2035-Weak		
2020	-398 [-18;5]	0 [0;0]	-36 [-1;6]	-140 [-8;1]	-222 [-9;1]
2025	-816 [-14;11]	0 [0;0]	-86 [0;1]	-274 [-7;7]	-456 [-9;9]
2030	-1169 [-14;9]	0 [0;0]	-135 [-6;1]	-431 [-6;7]	-603 [-6;2]
2035	-1511 [-4;2]	0 [0;0]	-173 [0;0]	-565 [0;1]	-773 [-4;1]
2040	-1551 [0;0]	0 [0;0]	-179 [0;0]	-581 [0;0]	-791 [0;0]
			Phase-Out-2040-Age		
2020	-351 [-9;8]	0 [0;0]	-35 [0;0]	-118 [-1;7]	-198 [-8;1]
2025	-643 [-14;3]	0 [0;0]	-63 [-1;0]	-210 [-8;1]	-370 [-7;5]
2030	-925 [-16;6]	0 [0;0]	-90 [-1;0]	-297 [-8;6]	-538 [-8;2]
2035	-1209 [-9;10]	0 [0;0]	-133 [0;1]	-446 [-8;6]	-630 [-2;5]
2040	-1548 [-6;1]	0 [0;0]	-179 [0;0]	-581 [-3;0]	-788 [-3;1]
			Phase-Out-2040-Balanced		
2020	-358 [-16;2]	0 [0;0]	-28 [0;0]	-148 [-8;1]	-182 [-8;1]
2025	-653 [-11;6]	0 [0;0]	-54 [0;0]	-289 [-6;6]	-310 [-8;2]
2030	-911 [-9;5]	0 [0;0]	-89 [0;1]	-363 [-4;2]	-459 [-8;2]
2035	-1194 [-15;6]	0 [0;0]	-143 [-1;5]	-450 [-7;1]	-601 [-8;1]
2040	-1545 [-4;3]	0 [0;0]	-179 [0;0]	-578 [0;3]	-788 [-4;0]
			Phase-Out-2035-Strong		
2020	-541 [-14;9]	0 [0;0]	-54 [-1;0]	-199 [-8;1]	-288 [-7;8]
2025	-895 [-6;11]	0 [0;0]	-94 [-1;0]	-302 [-2;6]	-499 [-6;8]
2030	-1176 [-19;4]	0 [0;0]	-135 [-6;0]	-431 [-6;5]	-610 [-8;1]
2035	-1514 [-3;4]	0 [0;0]	-173 [0;0]	-565 [0;3]	-776 [-3;1]
2040	-1551 [0;0]	0 [0;0]	-179 [0;0]	-581 [0;0]	-791 [0;0]

Note: Simulation results for real gross labour income in the lignite industry in million euros. Values for the Null-Scenario are reported as change to the base year 2014 and for the year 2014 actual values are reported. Values for other scenarios are differences to the Null-Scenario in the respective year. Values in brackets denote the minimum and maximum difference from the reported value obtained from 1200 simulations.

Table 3.26: Sensitivity analysis for inverse Frisch elasticity

σ^L	Baseline	Phase-Out-2035-Weak	Phase-Out-2040-Age	Phase-Out-2040-Balanced	Phase-Out-2035-Strong
			Germany		
0.2375	0.0272	0.1175	0.1303	0.1310	0.1100
0.2437	0.0271	0.1168	0.1295	0.1302	0.1094
0.2500	0.0270	0.1162	0.1286	0.1294	0.1087
0.2562	0.0270	0.1155	0.1277	0.1285	0.1081
0.2625	0.0269	0.1148	0.1268	0.1277	0.1074
			Rest of Germany		
0.2375	0.0173	0.0970	0.1130	0.1146	0.0895
0.2437	0.0172	0.0964	0.1123	0.1139	0.0889
0.2500	0.0172	0.0958	0.1115	0.1131	0.0884
0.2562	0.0171	0.0953	0.1107	0.1124	0.0878
0.2625	0.0171	0.0947	0.1099	0.1116	0.0872
			Central Germany		
0.2375	0.0599	0.1774	0.1845	0.1870	0.1687
0.2437	0.0596	0.1763	0.1833	0.1857	0.1676
0.2500	0.0593	0.1752	0.1820	0.1844	0.1665
0.2562	0.0590	0.1741	0.1808	0.1831	0.1654
0.2625	0.0587	0.1730	0.1795	0.1818	0.1643
			Lusatia		
0.2375	0.4801	1.0709	0.9523	0.9491	1.0676
0.2437	0.4793	1.0697	0.9511	0.9479	1.0665
0.2500	0.4787	1.0685	0.9500	0.9468	1.0654
0.2562	0.4779	1.0673	0.9488	0.9456	1.0642
0.2625	0.4772	1.0660	0.9476	0.9443	1.0631
			Rhineland		
0.2375	0.1036	0.2559	0.2452	0.2467	0.2497
0.2437	0.1032	0.2548	0.2440	0.2455	0.2486
0.2500	0.1028	0.2536	0.2428	0.2443	0.2476
0.2562	0.1025	0.2525	0.2416	0.2431	0.2465
0.2625	0.1021	0.2514	0.2404	0.2420	0.2454

Note: Sensitivity analysis for the maximum increase in the unemployment rate in percentage points between 2014 and 2040 compared to the Null-Scenario for the respective region.

Table 3.27: Sensitivity analysis for wages depending on inverse Frisch elasticity

σ^L	Baseline	Phase-Out-2035-Weak	Phase-Out-2040-Age	Phase-Out-2040-Balanced	Phase-Out-2035-Strong
			Germany		
0.2375	-0.119	-0.387	-0.332	-0.332	-0.390
0.2437	-0.122	-0.390	-0.332	-0.335	-0.393
0.2500	-0.122	-0.393	-0.338	-0.335	-0.396
0.2562	-0.122	-0.393	-0.338	-0.338	-0.396
0.2625	-0.125	-0.396	-0.341	-0.341	-0.399
			Rest of Germany		
0.2375	-0.003	-0.067	-0.035	-0.035	-0.070
0.2437	-0.006	-0.067	-0.035	-0.038	-0.070
0.2500	-0.006	-0.070	-0.038	-0.038	-0.073
0.2562	-0.006	-0.070	-0.038	-0.038	-0.073
0.2625	-0.006	-0.070	-0.041	-0.041	-0.073
			Central Germany		
0.2375	-0.573	-2.112	-1.974	-2.043	-2.115
0.2437	-0.577	-2.126	-1.985	-2.057	-2.130
0.2500	-0.581	-2.137	-1.999	-2.068	-2.141
0.2562	-0.584	-2.148	-2.010	-2.079	-2.152
0.2625	-0.591	-2.159	-2.021	-2.090	-2.163
			Lusatia		
0.2375	-4.985	-10.621	-10.356	-10.257	-10.625
0.2437	-4.981	-10.628	-10.363	-10.265	-10.632
0.2500	-4.977	-10.640	-10.375	-10.272	-10.644
0.2562	-4.973	-10.647	-10.382	-10.280	-10.651
0.2625	-4.970	-10.655	-10.390	-10.287	-10.659
			Rhineland		
0.2375	-0.724	-2.230	-1.948	-1.930	-2.230
0.2437	-0.727	-2.238	-1.959	-1.943	-2.241
0.2500	-0.732	-2.248	-1.969	-1.953	-2.251
0.2562	-0.735	-2.256	-1.979	-1.964	-2.259
0.2625	-0.737	-2.266	-1.990	-1.971	-2.266

Note: Sensitivity analysis for the maximum drop in wages in percentage points between 2014 and 2040 compared to the Null-Scenario for the respective region.

Table 3.28: Sensitivity analysis for AR(1) for unemployment benefits

ρ^b	Baseline	Phase-Out-2035-Weak	Phase-Out-2040-Age	Phase-Out-2040-Balanced	Phase-Out-2035-Strong
			Germany		
0.8075	0.0191	0.0950	0.1072	0.1082	0.0868
0.8287	0.0228	0.1049	0.1166	0.1175	0.0970
0.8500	0.0270	0.1162	0.1286	0.1294	0.1087
0.8712	0.0320	0.1292	0.1426	0.1431	0.1223
0.8925	0.0382	0.1444	0.1593	0.1596	0.1383
			Rest of Germany		
0.8075	0.0119	0.0746	0.0946	0.0958	0.0684
0.8287	0.0141	0.0846	0.1023	0.1034	0.0767
0.8500	0.0172	0.0958	0.1115	0.1131	0.0884
0.8712	0.0212	0.1089	0.1256	0.1270	0.1020
0.8925	0.0260	0.1241	0.1422	0.1434	0.1179
			Central Germany		
0.8075	0.0513	0.1535	0.1591	0.1617	0.1440
0.8287	0.0549	0.1636	0.1697	0.1722	0.1545
0.8500	0.0593	0.1752	0.1820	0.1844	0.1665
0.8712	0.0645	0.1887	0.1966	0.1987	0.1806
0.8925	0.0707	0.2043	0.2137	0.2157	0.1970
			Lusatia		
0.8075	0.4699	1.0394	0.9209	0.9177	1.0360
0.8287	0.4738	1.0528	0.9343	0.9311	1.0495
0.8500	0.4787	1.0685	0.9500	0.9468	1.0654
0.8712	0.4845	1.0868	0.9686	0.9654	1.0840
0.8925	0.4916	1.1084	0.9909	0.9876	1.1061
			Rhineland		
0.8075	0.0955	0.2265	0.2209	0.2172	0.2201
0.8287	0.0989	0.2391	0.2283	0.2297	0.2328
0.8500	0.1028	0.2536	0.2428	0.2443	0.2476
0.8712	0.1076	0.2707	0.2601	0.2616	0.2650
0.8925	0.1134	0.2910	0.2810	0.2824	0.2857

Note: Sensitivity analysis for the maximum increase in the unemployment rate in percentage points between 2014 and 2040 compared to the Null-Scenario for the respective region.

Table 3.29: Sensitivity analysis for elasticity of substitution between lignite and non-lignite

η^b	Baseline	Phase-Out-2035-Weak	Phase-Out-2040-Age	Phase-Out-2040-Balanced	Phase-Out-2035-Strong
			Germany		
19.5700	0.0270	0.1163	0.1285	0.1293	0.1088
20.0850	0.0270	0.1163	0.1285	0.1293	0.1088
20.6000	0.0270	0.1162	0.1286	0.1294	0.1087
21.1150	0.0270	0.1162	0.1286	0.1294	0.1087
21.6300	0.0270	0.1161	0.1286	0.1294	0.1087
			Rest of Germany		
19.5700	0.0172	0.0959	0.1115	0.1131	0.0884
20.0850	0.0172	0.0959	0.1115	0.1131	0.0884
20.6000	0.0172	0.0958	0.1115	0.1131	0.0884
21.1150	0.0172	0.0958	0.1115	0.1131	0.0884
21.6300	0.0172	0.0958	0.1115	0.1132	0.0883
			Central Germany		
19.5700	0.0594	0.1753	0.1822	0.1846	0.1668
20.0850	0.0593	0.1752	0.1821	0.1845	0.1667
20.6000	0.0593	0.1752	0.1820	0.1844	0.1665
21.1150	0.0592	0.1751	0.1820	0.1843	0.1664
21.6300	0.0592	0.1751	0.1820	0.1842	0.1664
			Lusatia		
19.5700	0.4792	1.0703	0.9522	0.9489	1.0679
20.0850	0.4789	1.0690	0.9511	0.9478	1.0666
20.6000	0.4787	1.0685	0.9500	0.9468	1.0654
21.1150	0.4784	1.0673	0.9490	0.9458	1.0642
21.6300	0.4782	1.0663	0.9481	0.9449	1.0630
			Rhineland		
19.5700	0.1031	0.2548	0.2437	0.2451	0.2486
20.0850	0.1030	0.2542	0.2432	0.2447	0.2481
20.6000	0.1028	0.2536	0.2428	0.2443	0.2476
21.1150	0.1027	0.2532	0.2424	0.2439	0.2471
21.6300	0.1026	0.2527	0.2420	0.2436	0.2466

Note: Sensitivity analysis for the maximum increase in the unemployment rate in percentage points between 2014 and 2040 compared to the Null-Scenario for the respective region.

Table 3.30: Sensitivity analysis for regional elasticity of substitution energy

η_E^m	Baseline	Phase-Out-2035-Weak	Phase-Out-2040-Age	Phase-Out-2040-Balanced	Phase-Out-2035-Strong
			Germany		
760.0000	0.0270	0.1162	0.1286	0.1294	0.1087
780.0000	0.0270	0.1162	0.1286	0.1294	0.1087
800.0000	0.0270	0.1162	0.1286	0.1294	0.1087
820.0000	0.0270	0.1162	0.1286	0.1294	0.1087
840.0000	0.0270	0.1162	0.1286	0.1294	0.1087
			Rest of Germany		
760.0000	0.0172	0.0959	0.1115	0.1131	0.0884
780.0000	0.0172	0.0958	0.1115	0.1131	0.0884
800.0000	0.0172	0.0958	0.1115	0.1131	0.0884
820.0000	0.0172	0.0958	0.1115	0.1131	0.0884
840.0000	0.0172	0.0958	0.1115	0.1131	0.0884
			Central Germany		
760.0000	0.0593	0.1752	0.1820	0.1844	0.1665
780.0000	0.0593	0.1752	0.1820	0.1844	0.1665
800.0000	0.0593	0.1752	0.1820	0.1844	0.1665
820.0000	0.0593	0.1752	0.1820	0.1844	0.1665
840.0000	0.0593	0.1752	0.1820	0.1844	0.1665
			Lusatia		
760.0000	0.4786	1.0684	0.9500	0.9468	1.0653
780.0000	0.4787	1.0685	0.9500	0.9468	1.0653
800.0000	0.4787	1.0685	0.9500	0.9468	1.0654
820.0000	0.4787	1.0685	0.9500	0.9468	1.0654
840.0000	0.4787	1.0685	0.9501	0.9469	1.0654
			Rhineland		
760.0000	0.1028	0.2536	0.2428	0.2443	0.2475
780.0000	0.1028	0.2536	0.2428	0.2443	0.2476
800.0000	0.1028	0.2536	0.2428	0.2443	0.2476
820.0000	0.1029	0.2536	0.2428	0.2443	0.2476
840.0000	0.1029	0.2537	0.2428	0.2443	0.2476

Note: Sensitivity analysis for the maximum increase in the unemployment rate in percentage points between 2014 and 2040 compared to the Null-Scenario for the respective region.

Table 3.31: Sensitivity analysis for regional elasticity of substitution non-energy

η_{NE}^{m}	Baseline	Phase-Out-2035-Weak	Phase-Out-2040-Age	Phase-Out-2040-Balanced	Phase-Out-2035-Strong
			Germany		
1.0735	0.0257	0.1141	0.1264	0.1272	0.1066
1.1017	0.0264	0.1152	0.1276	0.1283	0.1077
1.1300	0.0270	0.1162	0.1286	0.1294	0.1087
1.1582	0.0277	0.1172	0.1296	0.1304	0.1097
1.1865	0.0282	0.1181	0.1305	0.1313	0.1106
			Rest of Germany		
1.0735	0.0160	0.0937	0.1093	0.1110	0.0863
1.1017	0.0166	0.0948	0.1104	0.1121	0.0874
1.1300	0.0172	0.0958	0.1115	0.1131	0.0884
1.1582	0.0177	0.0968	0.1124	0.1141	0.0893
1.1865	0.0182	0.0977	0.1134	0.1150	0.0902
			Central Germany		
1.0735	0.0582	0.1736	0.1802	0.1824	0.1650
1.1017	0.0588	0.1744	0.1811	0.1834	0.1658
1.1300	0.0593	0.1752	0.1820	0.1844	0.1665
1.1582	0.0598	0.1761	0.1829	0.1853	0.1673
1.1865	0.0602	0.1768	0.1837	0.1862	0.1680
			Lusatia		
1.0735	0.4760	1.0621	0.9441	0.9408	1.0590
1.1017	0.4773	1.0654	0.9471	0.9438	1.0622
1.1300	0.4787	1.0685	0.9500	0.9468	1.0654
1.1582	0.4799	1.0714	0.9527	0.9496	1.0683
1.1865	0.4810	1.0743	0.9553	0.9522	1.0711
			Rhineland		
1.0735	0.1017	0.2514	0.2406	0.2421	0.2453
1.1017	0.1023	0.2526	0.2417	0.2433	0.2465
1.1300	0.1028	0.2536	0.2428	0.2443	0.2476
1.1582	0.1034	0.2546	0.2438	0.2453	0.2486
1.1865	0.1038	0.2556	0.2447	0.2462	0.2496

Note: Sensitivity analysis for the maximum increase in the unemployment rate in percentage points between 2014 and 2040 compared to the Null-Scenario for the respective region.

Table 3.32: Sensitivity analysis for elasticity of substitution between energy and non-energy products

η^c	Baseline	Phase-Out-2035-Weak	Phase-Out-2040-Age	Phase-Out-2040-Balanced	Phase-Out-2035-Strong
			Germany		
0.7125	0.0280	0.1178	0.1299	0.1306	0.1106
0.7312	0.0275	0.1171	0.1292	0.1299	0.1097
0.7500	0.0270	0.1162	0.1286	0.1294	0.1087
0.7687	0.0265	0.1155	0.1279	0.1287	0.1079
0.7875	0.0262	0.1146	0.1274	0.1282	0.1070
			Rest of Germany		
0.7125	0.0177	0.0976	0.1129	0.1145	0.0903
0.7312	0.0175	0.0968	0.1122	0.1138	0.0894
0.7500	0.0172	0.0958	0.1115	0.1131	0.0884
0.7687	0.0170	0.0951	0.1108	0.1124	0.0875
0.7875	0.0166	0.0942	0.1103	0.1119	0.0865
			Central Germany		
0.7125	0.0599	0.1765	0.1830	0.1853	0.1680
0.7312	0.0596	0.1758	0.1825	0.1849	0.1673
0.7500	0.0593	0.1752	0.1820	0.1844	0.1665
0.7687	0.0589	0.1747	0.1816	0.1839	0.1659
0.7875	0.0587	0.1741	0.1812	0.1836	0.1652
			Lusatia		
0.7125	0.4791	1.0696	0.9509	0.9477	1.0667
0.7312	0.4788	1.0691	0.9504	0.9472	1.0661
0.7500	0.4787	1.0685	0.9500	0.9468	1.0654
0.7687	0.4784	1.0679	0.9495	0.9463	1.0647
0.7875	0.4781	1.0674	0.9490	0.9459	1.0641
			Rhineland		
0.7125	0.1034	0.2550	0.2438	0.2453	0.2491
0.7312	0.1031	0.2543	0.2434	0.2449	0.2483
0.7500	0.1028	0.2536	0.2428	0.2443	0.2476
0.7687	0.1027	0.2531	0.2422	0.2438	0.2469
0.7875	0.1024	0.2524	0.2418	0.2433	0.2462

Note: Sensitivity analysis for the maximum increase in the unemployment rate in percentage points between 2014 and 2040 compared to the Null-Scenario for the respective region.

Table 3.33: Sensitivity analysis for home bias energy

I_E^{Home}	Baseline	Phase-Out-2035-Weak	Phase-Out-2040-Age	Phase-Out-2040-Balanced	Phase-Out-2035-Strong
			Germany		
0.4750	0.0270	0.1162	0.1286	0.1294	0.1087
0.4875	0.0270	0.1162	0.1286	0.1294	0.1087
0.5000	0.0270	0.1162	0.1286	0.1294	0.1087
0.5125	0.0270	0.1162	0.1286	0.1294	0.1087
0.5250	0.0270	0.1162	0.1286	0.1294	0.1087
			Rest of Germany		
0.4750	0.0172	0.0958	0.1115	0.1131	0.0884
0.4875	0.0172	0.0958	0.1115	0.1131	0.0884
0.5000	0.0172	0.0958	0.1115	0.1131	0.0884
0.5125	0.0172	0.0958	0.1115	0.1131	0.0884
0.5250	0.0172	0.0958	0.1115	0.1131	0.0884
			Central Germany		
0.4750	0.0593	0.1752	0.1820	0.1844	0.1665
0.4875	0.0593	0.1752	0.1820	0.1844	0.1665
0.5000	0.0593	0.1752	0.1820	0.1844	0.1665
0.5125	0.0593	0.1752	0.1820	0.1844	0.1665
0.5250	0.0593	0.1752	0.1820	0.1844	0.1665
			Lusatia		
0.4750	0.4787	1.0685	0.9500	0.9468	1.0654
0.4875	0.4787	1.0685	0.9500	0.9468	1.0654
0.5000	0.4787	1.0685	0.9500	0.9468	1.0654
0.5125	0.4787	1.0685	0.9500	0.9468	1.0654
0.5250	0.4787	1.0685	0.9500	0.9468	1.0654
			Rhineland		
0.4750	0.1028	0.2536	0.2428	0.2443	0.2476
0.4875	0.1028	0.2536	0.2428	0.2443	0.2476
0.5000	0.1028	0.2536	0.2428	0.2443	0.2476
0.5125	0.1028	0.2536	0.2428	0.2443	0.2476
0.5250	0.1028	0.2536	0.2428	0.2443	0.2476

Note: Sensitivity analysis for the maximum increase in the unemployment rate in percentage points between 2014 and 2040 compared to the Null-Scenario for the respective region.

Table 3.34: Sensitivity analysis for home bias non-energy

I_{NE}^{Home}	Baseline	Phase-Out-2035-Weak	Phase-Out-2040-Age	Phase-Out-2040-Balanced	Phase-Out-2035-Strong
Germany					
0.8075	0.0328	0.1247	0.1372	0.1380	0.1170
0.8287	0.0304	0.1212	0.1337	0.1344	0.1136
0.8500	0.0270	0.1162	0.1286	0.1294	0.1087
0.8712	0.0220	0.1088	0.1211	0.1218	0.1014
0.8925	0.0143	0.0973	0.1093	0.1100	0.0901
Rest of Germany					
0.8075	0.0228	0.1053	0.1210	0.1226	0.0977
0.8287	0.0204	0.1013	0.1170	0.1186	0.0938
0.8500	0.0172	0.0958	0.1115	0.1131	0.0884
0.8712	0.0126	0.0879	0.1035	0.1052	0.0806
0.8925	0.0064	0.0761	0.0915	0.0932	0.0689
Central Germany					
0.8075	0.0619	0.1780	0.1852	0.1890	0.1690
0.8287	0.0610	0.1771	0.1843	0.1874	0.1683
0.8500	0.0593	0.1752	0.1820	0.1844	0.1665
0.8712	0.0562	0.1714	0.1777	0.1793	0.1630
0.8925	0.0507	0.1637	0.1695	0.1703	0.1556
Lusatia					
0.8075	0.4718	1.0670	0.9497	0.9465	1.0641
0.8287	0.4762	1.0696	0.9515	0.9484	1.0666
0.8500	0.4787	1.0685	0.9500	0.9468	1.0654
0.8712	0.4774	1.0607	0.9426	0.9393	1.0576
0.8925	0.4701	1.0426	0.9258	0.9224	1.0394
Rhineland					
0.8075	0.1036	0.2587	0.2481	0.2495	0.2530
0.8287	0.1036	0.2569	0.2460	0.2475	0.2510
0.8500	0.1028	0.2536	0.2428	0.2443	0.2476
0.8712	0.1007	0.2482	0.2373	0.2389	0.2419
0.8925	0.0963	0.2392	0.2287	0.2297	0.2326

Note: Sensitivity analysis for the maximum increase in the unemployment rate in percentage points between 2014 and 2040 compared to the Null-Scenario for the respective region.

Table 3.35: Sensitivity analysis for market power

$\bar{\lambda}$	Baseline	Phase-Out-2035-Weak	Phase-Out-2040-Age	Phase-Out-2040-Balanced	Phase-Out-2035-Strong
			Germany		
1.1875	0.0265	0.1095	0.1202	0.1207	0.1029
1.2188	0.0268	0.1128	0.1243	0.1249	0.1058
1.2500	0.0270	0.1162	0.1286	0.1294	0.1087
1.2813	0.0272	0.1199	0.1333	0.1342	0.1119
1.3125	0.0273	0.1240	0.1385	0.1395	0.1153
			Rest of Germany		
1.1875	0.0164	0.0895	0.1034	0.1048	0.0829
1.2188	0.0167	0.0925	0.1073	0.1088	0.0855
1.2500	0.0172	0.0958	0.1115	0.1131	0.0884
1.2813	0.0176	0.0994	0.1163	0.1178	0.0914
1.3125	0.0181	0.1034	0.1222	0.1233	0.0948
			Central Germany		
1.1875	0.0578	0.1670	0.1723	0.1741	0.1595
1.2188	0.0586	0.1710	0.1770	0.1791	0.1629
1.2500	0.0593	0.1752	0.1820	0.1844	0.1665
1.2813	0.0598	0.1798	0.1874	0.1900	0.1705
1.3125	0.0602	0.1846	0.1932	0.1960	0.1746
			Lusatia		
1.1875	0.4788	1.0663	0.9456	0.9441	1.0633
1.2188	0.4796	1.0671	0.9477	0.9455	1.0643
1.2500	0.4787	1.0685	0.9500	0.9468	1.0654
1.2813	0.4790	1.0700	0.9525	0.9488	1.0669
1.3125	0.4799	1.0708	0.9548	0.9502	1.0677
			Rhineland		
1.1875	0.1007	0.2498	0.2375	0.2387	0.2444
1.2188	0.1018	0.2517	0.2401	0.2414	0.2459
1.2500	0.1028	0.2536	0.2428	0.2443	0.2476
1.2813	0.1035	0.2558	0.2457	0.2474	0.2493
1.3125	0.1040	0.2582	0.2498	0.2508	0.2512

Note: Sensitivity analysis for the maximum increase in the unemployment rate in percentage points between 2014 and 2040 compared to the Null-Scenario for the respective region.

Table 3.36: Sensitivity analysis for AR(1) for market power

ρ^{λ}	Baseline	Phase-Out-2035-Weak	Phase-Out-2040-Age	Phase-Out-2040-Balanced	Phase-Out-2035-Strong
			Germany		
0.8550	0.0277	0.1209	0.1329	0.1336	0.1135
0.8775	0.0274	0.1191	0.1311	0.1319	0.1116
0.9000	0.0270	0.1162	0.1286	0.1294	0.1087
0.9225	0.0263	0.1117	0.1244	0.1252	0.1043
0.9450	0.0252	0.1036	0.1168	0.1175	0.0962
			Rest of Germany		
0.8550	0.0182	0.1010	0.1162	0.1178	0.0935
0.8775	0.0178	0.0990	0.1143	0.1160	0.0915
0.9000	0.0172	0.0958	0.1115	0.1131	0.0884
0.9225	0.0163	0.0910	0.1070	0.1086	0.0835
0.9450	0.0145	0.0821	0.0986	0.1002	0.0747
			Central Germany		
0.8550	0.0594	0.1795	0.1859	0.1883	0.1708
0.8775	0.0593	0.1778	0.1843	0.1867	0.1691
0.9000	0.0593	0.1752	0.1820	0.1844	0.1665
0.9225	0.0592	0.1712	0.1783	0.1807	0.1625
0.9450	0.0589	0.1639	0.1713	0.1737	0.1553
			Lusatia		
0.8550	0.4721	1.0578	0.9407	0.9374	1.0547
0.8775	0.4747	1.0621	0.9444	0.9412	1.0590
0.9000	0.4787	1.0685	0.9500	0.9468	1.0654
0.9225	0.4850	1.0788	0.9591	0.9559	1.0757
0.9450	0.4972	1.0981	0.9764	0.9732	1.0949
			Rhineland		
0.8550	0.1021	0.2556	0.2444	0.2460	0.2495
0.8775	0.1024	0.2549	0.2438	0.2454	0.2488
0.9000	0.1028	0.2536	0.2428	0.2443	0.2476
0.9225	0.1036	0.2518	0.2412	0.2427	0.2457
0.9450	0.1051	0.2483	0.2380	0.2395	0.2423

Note: Sensitivity analysis for the maximum increase in the unemployment rate in percentage points between 2014 and 2040 compared to the Null-Scenario for the respective region.

Table 3.37: Sensitivity analysis for AR(1) attractiveness

ρ_{ϵ}^{pop}	Baseline	Phase-Out-2035-Weak	Phase-Out-2040-Age	Phase-Out-2040-Balanced	Phase-Out-2035-Strong
			Germany		
0.9921	0.0277	0.1207	0.1338	0.1346	0.1132
0.9923	0.0273	0.1184	0.1311	0.1319	0.1110
0.9925	0.0270	0.1162	0.1286	0.1294	0.1087
0.9927	0.0267	0.1141	0.1261	0.1268	0.1067
0.9929	0.0265	0.1120	0.1237	0.1245	0.1046
			Rest of Germany		
0.9921	0.0180	0.1010	0.1175	0.1191	0.0935
0.9923	0.0176	0.0984	0.1145	0.1161	0.0909
0.9925	0.0172	0.0958	0.1115	0.1131	0.0884
0.9927	0.0168	0.0935	0.1086	0.1102	0.0860
0.9929	0.0164	0.0911	0.1062	0.1075	0.0836
			Central Germany		
0.9921	0.0588	0.1789	0.1863	0.1887	0.1702
0.9923	0.0590	0.1771	0.1841	0.1865	0.1684
0.9925	0.0593	0.1752	0.1820	0.1844	0.1665
0.9927	0.0595	0.1735	0.1800	0.1823	0.1648
0.9929	0.0597	0.1718	0.1780	0.1804	0.1631
			Lusatia		
0.9921	0.4627	1.0459	0.9253	0.9221	1.0427
0.9923	0.4709	1.0574	0.9379	0.9347	1.0543
0.9925	0.4787	1.0685	0.9500	0.9468	1.0654
0.9927	0.4862	1.0792	0.9617	0.9585	1.0761
0.9929	0.4936	1.0896	0.9731	0.9699	1.0864
			Rhineland		
0.9921	0.1007	0.2545	0.2437	0.2452	0.2484
0.9923	0.1019	0.2541	0.2432	0.2447	0.2480
0.9925	0.1028	0.2536	0.2428	0.2443	0.2476
0.9927	0.1039	0.2533	0.2424	0.2439	0.2472
0.9929	0.1048	0.2529	0.2419	0.2435	0.2468

Note: Sensitivity analysis for the maximum increase in the unemployment rate in percentage points between 2014 and 2040 compared to the Null-Scenario for the respective region.

Table 3.38: Sensitivity analysis for elasticity of marginal hiring costs to labour market tightness

v	Baseline	Phase-Out-2035-Weak	Phase-Out-2040-Age	Phase-Out-2040-Balanced	Phase-Out-2035-Strong
			Germany		
0.9500	0.0270	0.1161	0.1285	0.1293	0.1087
0.9750	0.0270	0.1162	0.1286	0.1293	0.1087
1.0000	0.0270	0.1162	0.1286	0.1294	0.1087
1.0250	0.0271	0.1162	0.1287	0.1294	0.1088
1.0500	0.0271	0.1163	0.1287	0.1295	0.1088
			Rest of Germany		
0.9500	0.0172	0.0958	0.1114	0.1130	0.0883
0.9750	0.0172	0.0958	0.1114	0.1131	0.0883
1.0000	0.0172	0.0958	0.1115	0.1131	0.0884
1.0250	0.0172	0.0959	0.1115	0.1132	0.0884
1.0500	0.0172	0.0959	0.1116	0.1132	0.0884
			Central Germany		
0.9500	0.0593	0.1751	0.1820	0.1843	0.1665
0.9750	0.0593	0.1752	0.1820	0.1844	0.1665
1.0000	0.0593	0.1752	0.1820	0.1844	0.1665
1.0250	0.0593	0.1753	0.1821	0.1844	0.1666
1.0500	0.0593	0.1753	0.1821	0.1845	0.1666
			Lusatia		
0.9500	0.4786	1.0684	0.9499	0.9467	1.0653
0.9750	0.4786	1.0684	0.9500	0.9468	1.0653
1.0000	0.4787	1.0685	0.9500	0.9468	1.0654
1.0250	0.4787	1.0685	0.9501	0.9469	1.0654
1.0500	0.4787	1.0686	0.9501	0.9469	1.0655
			Rhineland		
0.9500	0.1028	0.2535	0.2427	0.2442	0.2475
0.9750	0.1028	0.2536	0.2427	0.2443	0.2475
1.0000	0.1028	0.2536	0.2428	0.2443	0.2476
1.0250	0.1029	0.2537	0.2428	0.2444	0.2476
1.0500	0.1029	0.2538	0.2429	0.2444	0.2477

Note: Sensitivity analysis for the maximum increase in the unemployment rate in percentage points between 2014 and 2040 compared to the Null-Scenario for the respective region.

Table 3.39: Sensitivity analysis for job finding rate

$\frac{h}{u}$	Baseline	Phase-Out-2035-Weak	Phase-Out-2040-Age	Phase-Out-2040-Balanced	Phase-Out-2035-Strong
			Germany		
0.2131	0.0263	0.1131	0.1258	0.1266	0.1061
0.2187	0.0266	0.1147	0.1272	0.1279	0.1075
0.2243	0.0270	0.1162	0.1286	0.1294	0.1087
0.2299	0.0273	0.1177	0.1300	0.1306	0.1100
0.2355	0.0277	0.1192	0.1312	0.1318	0.1112
			Rest of Germany		
0.2131	0.0167	0.0929	0.1087	0.1104	0.0858
0.2187	0.0169	0.0944	0.1102	0.1118	0.0871
0.2243	0.0172	0.0958	0.1115	0.1131	0.0884
0.2299	0.0174	0.0973	0.1128	0.1144	0.0896
0.2355	0.0176	0.0987	0.1140	0.1156	0.0908
			Central Germany		
0.2131	0.0583	0.1708	0.1781	0.1803	0.1629
0.2187	0.0589	0.1731	0.1802	0.1824	0.1648
0.2243	0.0593	0.1752	0.1820	0.1844	0.1665
0.2299	0.0597	0.1772	0.1838	0.1863	0.1682
0.2355	0.0601	0.1793	0.1856	0.1881	0.1700
			Lusatia		
0.2131	0.4771	1.0632	0.9466	0.9422	1.0617
0.2187	0.4779	1.0660	0.9484	0.9446	1.0637
0.2243	0.4787	1.0685	0.9500	0.9468	1.0654
0.2299	0.4803	1.0703	0.9511	0.9488	1.0670
0.2355	0.4809	1.0724	0.9523	0.9505	1.0683
			Rhineland		
0.2131	0.1018	0.2512	0.2411	0.2425	0.2458
0.2187	0.1023	0.2524	0.2420	0.2435	0.2467
0.2243	0.1028	0.2536	0.2428	0.2443	0.2476
0.2299	0.1032	0.2549	0.2436	0.2451	0.2483
0.2355	0.1038	0.2560	0.2443	0.2458	0.2491

Note: Sensitivity analysis for the maximum increase in the unemployment rate in percentage points between 2014 and 2040 compared to the Null-Scenario for the respective region.

Table 3.40: Sensitivity analysis for share of hiring costs to wage bill

$\frac{\kappa}{wn}$	Baseline	Phase-Out-2035-Weak	Phase-Out-2040-Age	Phase-Out-2040-Balanced	Phase-Out-2035-Strong
			Germany		
0.0617	0.0273	0.1177	0.1301	0.1307	0.1099
0.0634	0.0272	0.1170	0.1293	0.1300	0.1093
0.0650	0.0270	0.1162	0.1286	0.1294	0.1087
0.0666	0.0269	0.1155	0.1279	0.1287	0.1082
0.0683	0.0267	0.1149	0.1272	0.1280	0.1076
			Rest of Germany		
0.0617	0.0174	0.0973	0.1129	0.1145	0.0895
0.0634	0.0173	0.0966	0.1121	0.1138	0.0889
0.0650	0.0172	0.0958	0.1115	0.1131	0.0884
0.0666	0.0171	0.0952	0.1108	0.1125	0.0878
0.0683	0.0170	0.0946	0.1102	0.1118	0.0873
			Central Germany		
0.0617	0.0595	0.1770	0.1838	0.1863	0.1681
0.0634	0.0594	0.1760	0.1830	0.1854	0.1673
0.0650	0.0593	0.1752	0.1820	0.1844	0.1665
0.0666	0.0591	0.1743	0.1812	0.1835	0.1658
0.0683	0.0590	0.1734	0.1803	0.1825	0.1651
			Lusatia		
0.0617	0.4807	1.0707	0.9516	0.9492	1.0674
0.0634	0.4802	1.0693	0.9510	0.9480	1.0664
0.0650	0.4787	1.0685	0.9500	0.9468	1.0654
0.0666	0.4781	1.0672	0.9490	0.9455	1.0644
0.0683	0.4776	1.0658	0.9480	0.9443	1.0634
			Rhineland		
0.0617	0.1030	0.2551	0.2440	0.2455	0.2486
0.0634	0.1028	0.2544	0.2433	0.2449	0.2481
0.0650	0.1028	0.2536	0.2428	0.2443	0.2476
0.0666	0.1027	0.2530	0.2422	0.2437	0.2471
0.0683	0.1026	0.2524	0.2417	0.2432	0.2466

Note: Sensitivity analysis for the maximum increase in the unemployment rate in percentage points between 2014 and 2040 compared to the Null-Scenario for the respective region.

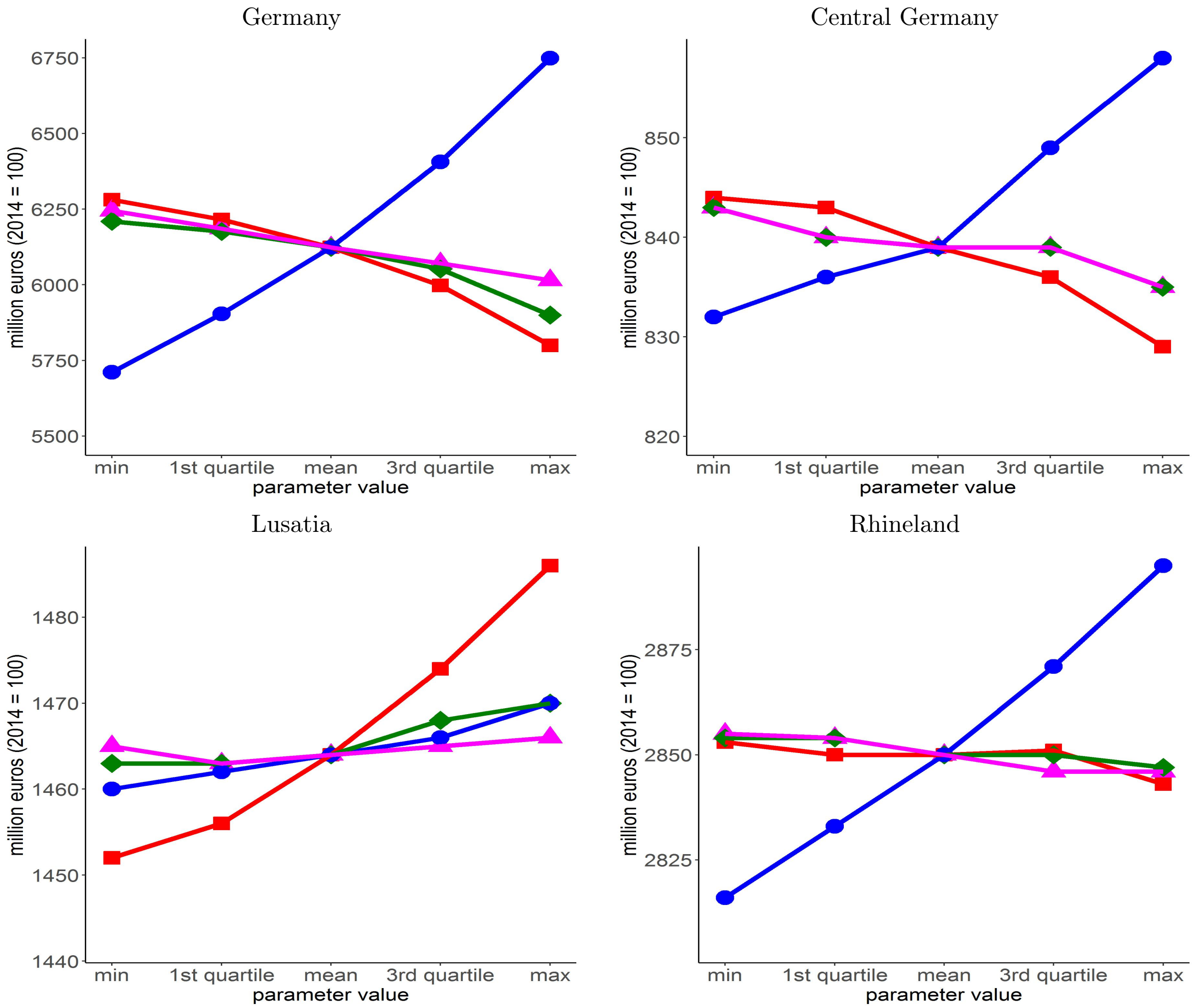
Table 3.41: Sensitivity analysis for discount factor

β	Baseline	Phase-Out-2035-Weak	Phase-Out-2040-Age	Phase-Out-2040-Balanced	Phase-Out-2035-Strong
			Germany		
0.9984	0.0269	0.1160	0.1283	0.1291	0.1085
0.9982	0.0270	0.1161	0.1285	0.1292	0.1086
0.9980	0.0270	0.1162	0.1286	0.1294	0.1087
0.9977	0.0272	0.1164	0.1288	0.1296	0.1089
0.9975	0.0272	0.1165	0.1289	0.1297	0.1090
			Rest of Germany		
0.9984	0.0171	0.0957	0.1112	0.1129	0.0881
0.9982	0.0171	0.0958	0.1113	0.1130	0.0883
0.9980	0.0172	0.0958	0.1115	0.1131	0.0884
0.9977	0.0172	0.0960	0.1117	0.1133	0.0885
0.9975	0.0173	0.0961	0.1118	0.1135	0.0887
			Central Germany		
0.9984	0.0591	0.1748	0.1817	0.1841	0.1662
0.9982	0.0592	0.1750	0.1819	0.1842	0.1664
0.9980	0.0593	0.1752	0.1820	0.1844	0.1665
0.9977	0.0594	0.1754	0.1823	0.1847	0.1668
0.9975	0.0594	0.1756	0.1824	0.1848	0.1669
			Lusatia		
0.9984	0.4793	1.0673	0.9493	0.9462	1.0648
0.9982	0.4785	1.0675	0.9497	0.9465	1.0650
0.9980	0.4787	1.0685	0.9500	0.9468	1.0654
0.9977	0.4788	1.0690	0.9505	0.9472	1.0659
0.9975	0.4790	1.0693	0.9508	0.9475	1.0662
			Rhineland		
0.9984	0.1026	0.2532	0.2422	0.2437	0.2470
0.9982	0.1028	0.2535	0.2425	0.2440	0.2473
0.9980	0.1028	0.2536	0.2428	0.2443	0.2476
0.9977	0.1029	0.2541	0.2432	0.2448	0.2480
0.9975	0.1030	0.2544	0.2435	0.2451	0.2483

Note: Sensitivity analysis for the maximum increase in the unemployment rate in percentage points between 2014 and 2040 compared to the Null-Scenario for the respective region.

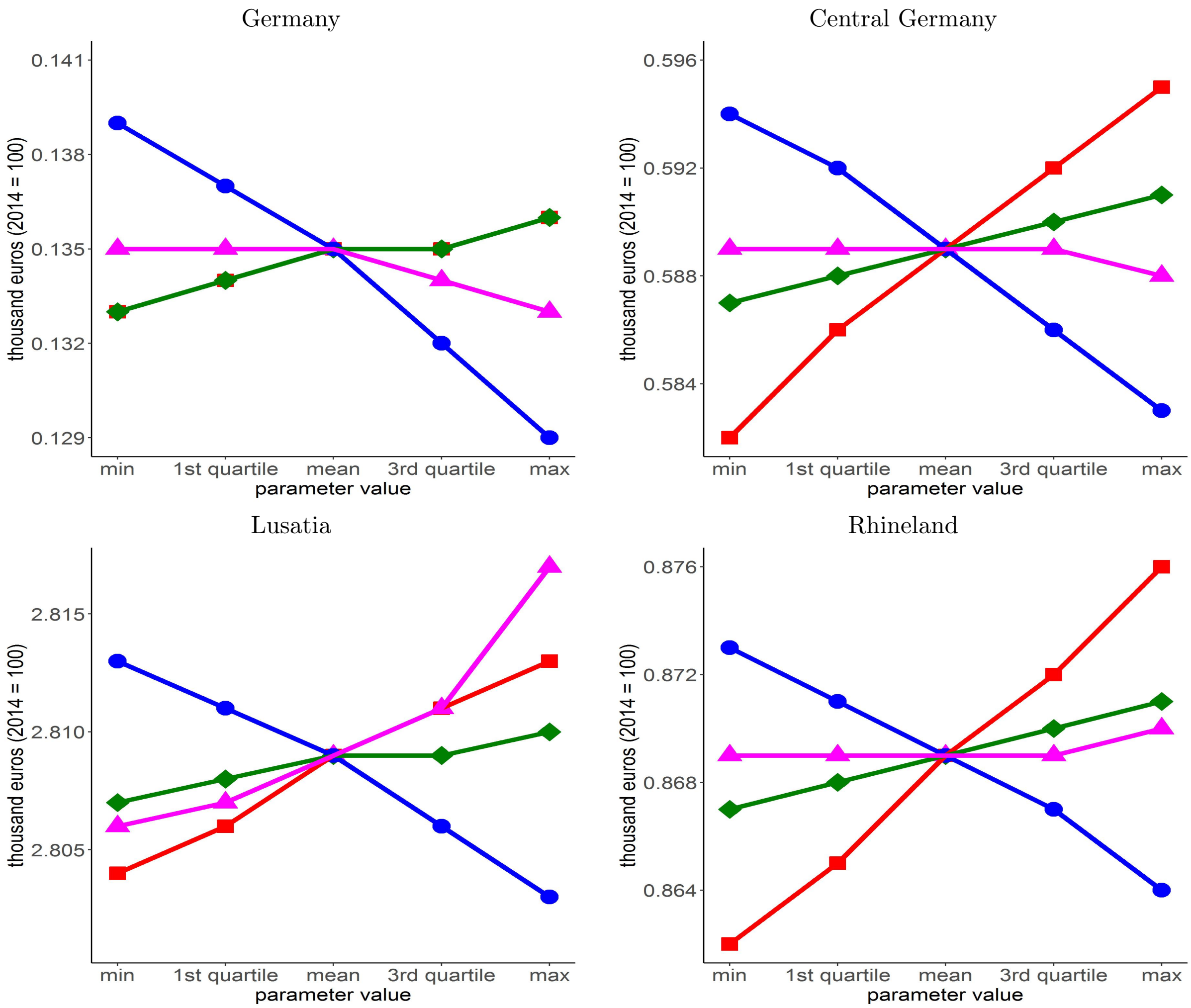
3.8.8 Figures

Figure 3.16: Sensitivity analysis for maximum drop in labour compensation



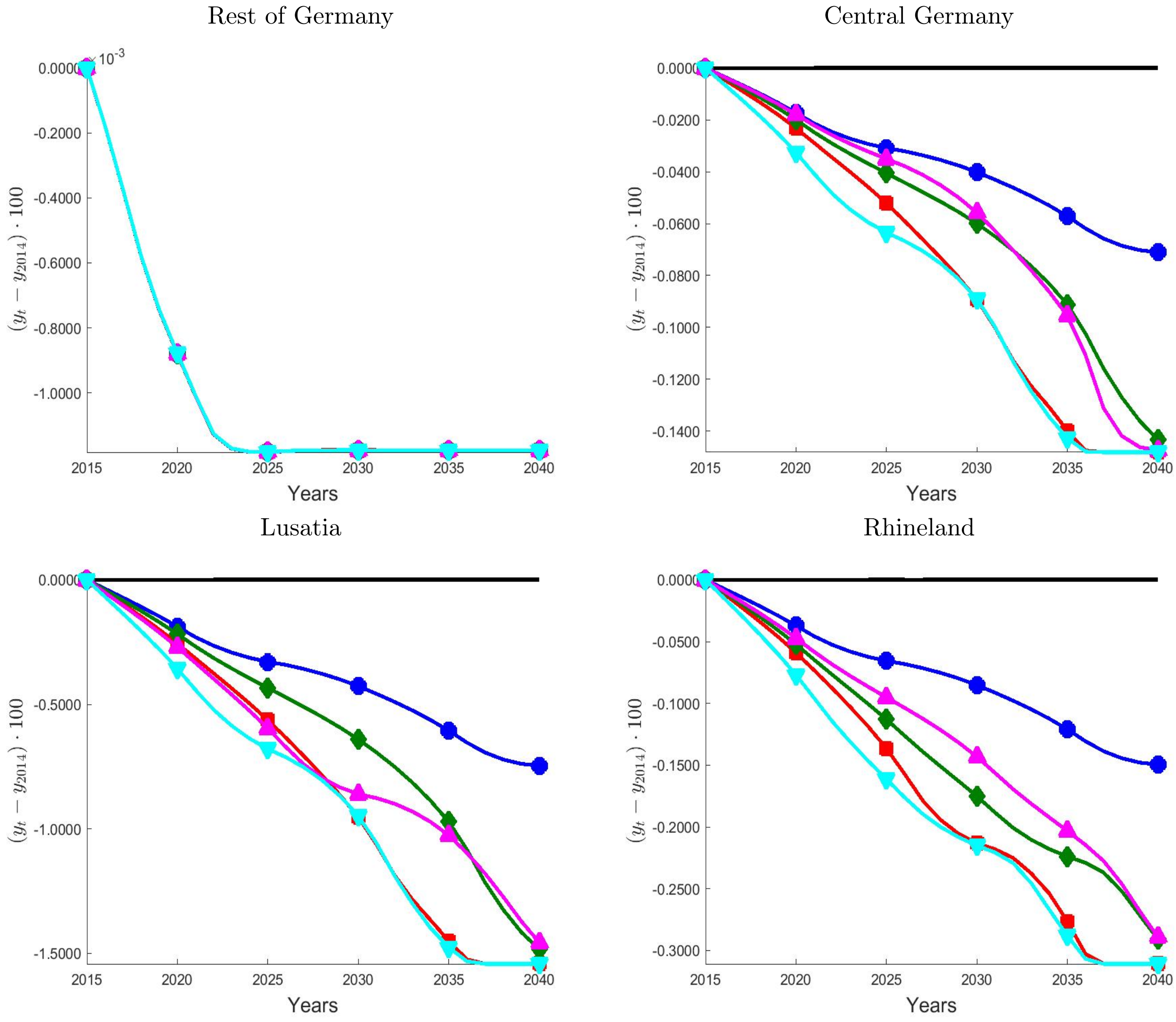
Note: Difference compared to the Null-Scenario in million euro , Baseline (blue circle), Scenario cenario 1 (red square), Phase-Out-2040-Age (green diamond), Phase-Out-2040-Balanced (magenta triangle point-up) and Phase-Out-2035-Strong (cyan triangle point-down). Horizontal lines indicate the maximum and minimum value observed for 1200 simulations.

Figure 3.17: Sensitivity analysis for maximum drop in wages



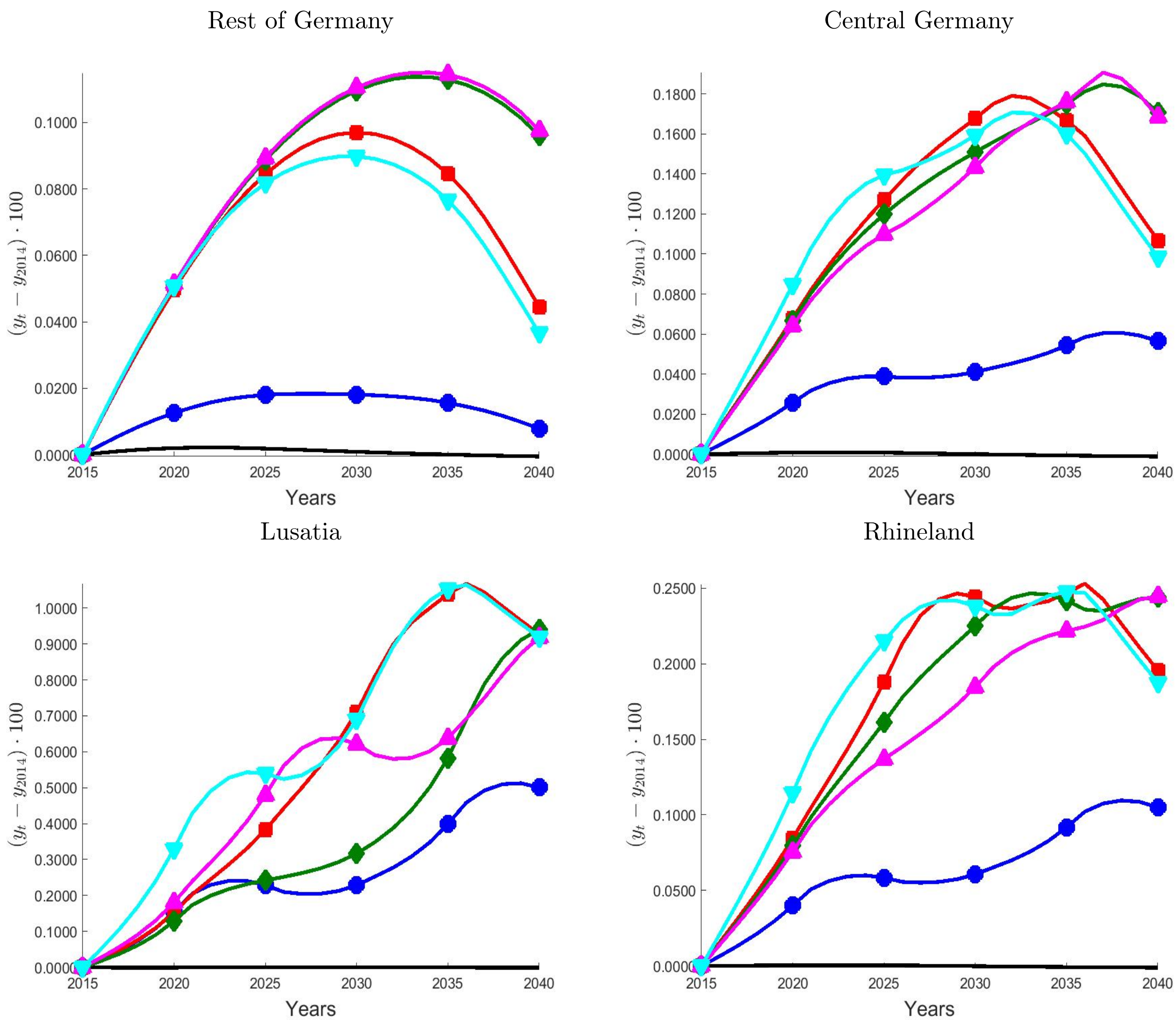
Note: Difference compared to the Null-Scenario in thousand euro, Baseline (blue circle), Phase-Out-2035-Weak (red square), Phase-Out-2040-Age (green diamond), Phase-Out-2040-Balanced (magenta triangle point-up) and Phase-Out-2035-Strong (cyan triangle point-down). Horizontal lines indicate the maximum and minimum value observed for 1200 simulations.

Figure 3.18: Simulation results for employment rates in lignite sector



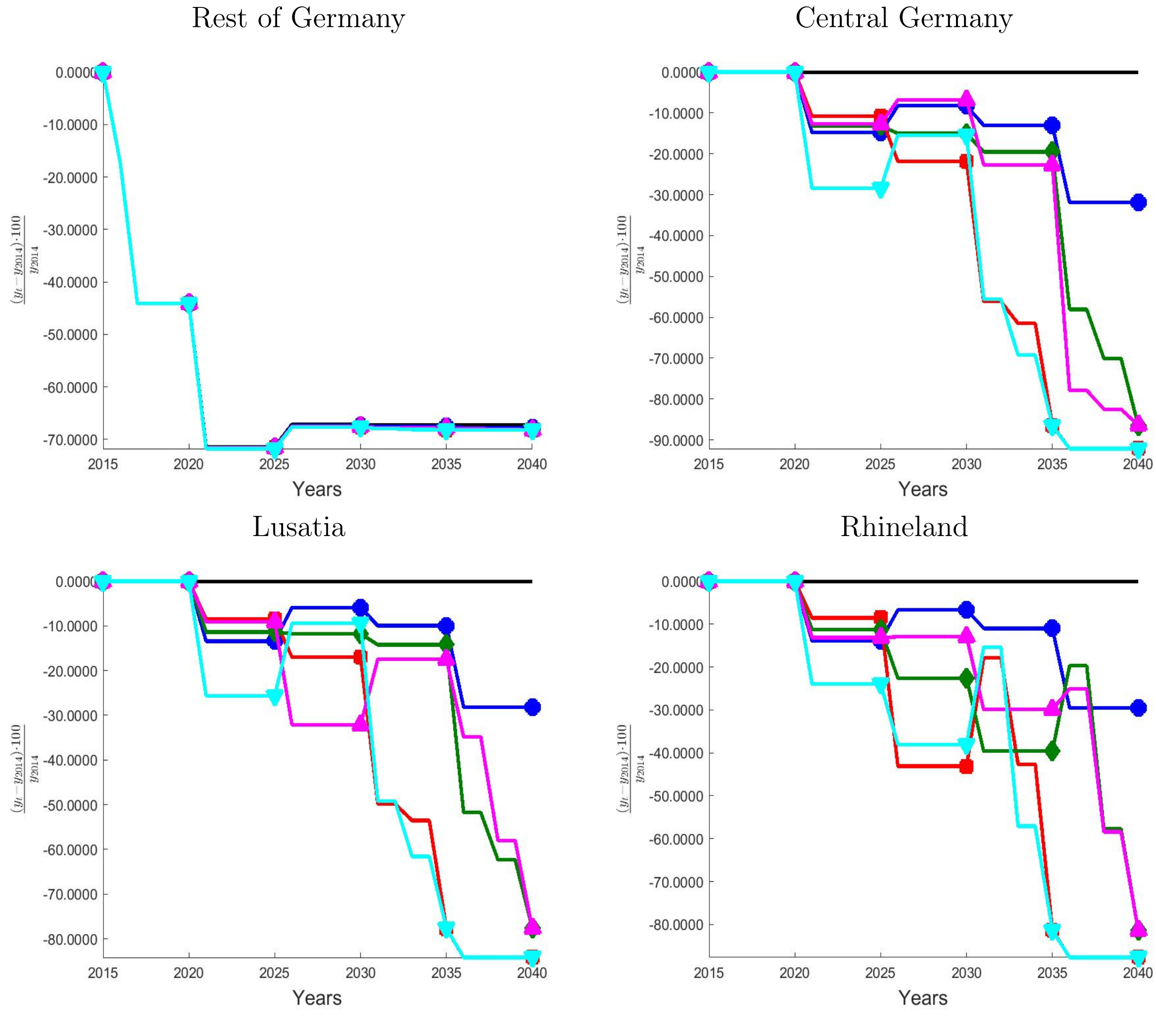
Note: The plots depict the simulation trajectories for the Null-Scenario (black solid line), Baseline (blue line with circle), Phase-Out-2035-Weak (red line with square), Phase-Out-2040-Age (green line with diamond), Phase-Out-2040-Balanced (magenta line with triangle pointing upward) and Phase-Out-2035-Strong (cyan line with triangle pointing downward).

Figure 3.19: Simulation results for unemployment rates



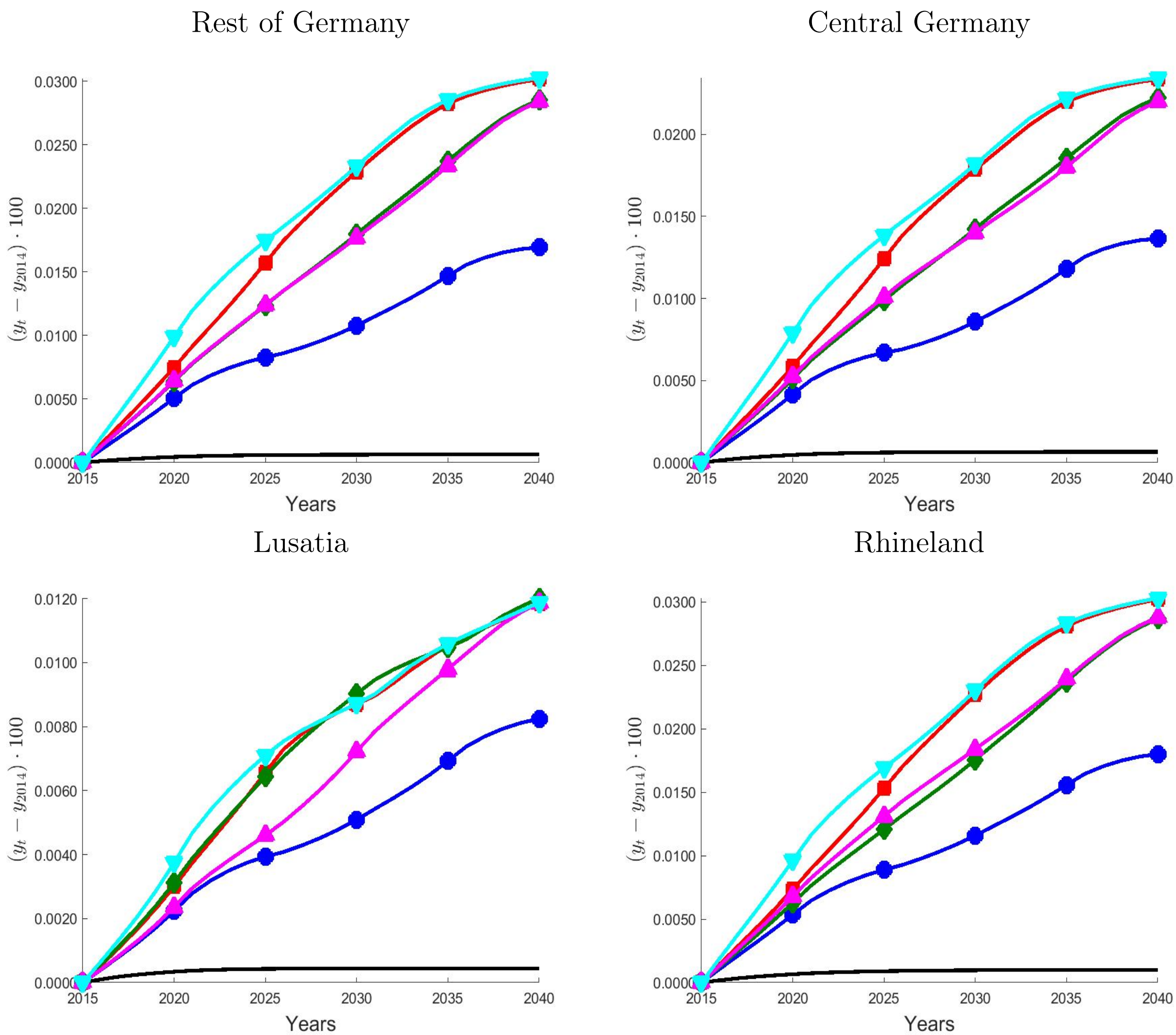
Note: The plots depict the simulation trajectories for the Null-Scenario (black solid line), Baseline (blue line with circle), Phase-Out-2035-Weak (red line with square), Phase-Out-2040-Age (green line with diamond), Phase-Out-2040-Balanced (magenta line with triangle pointing upward) and Phase-Out-2035-Strong (cyan line with triangle pointing downward).

Figure 3.20: Simulation trajectory for productivity shocks on lignite sectors



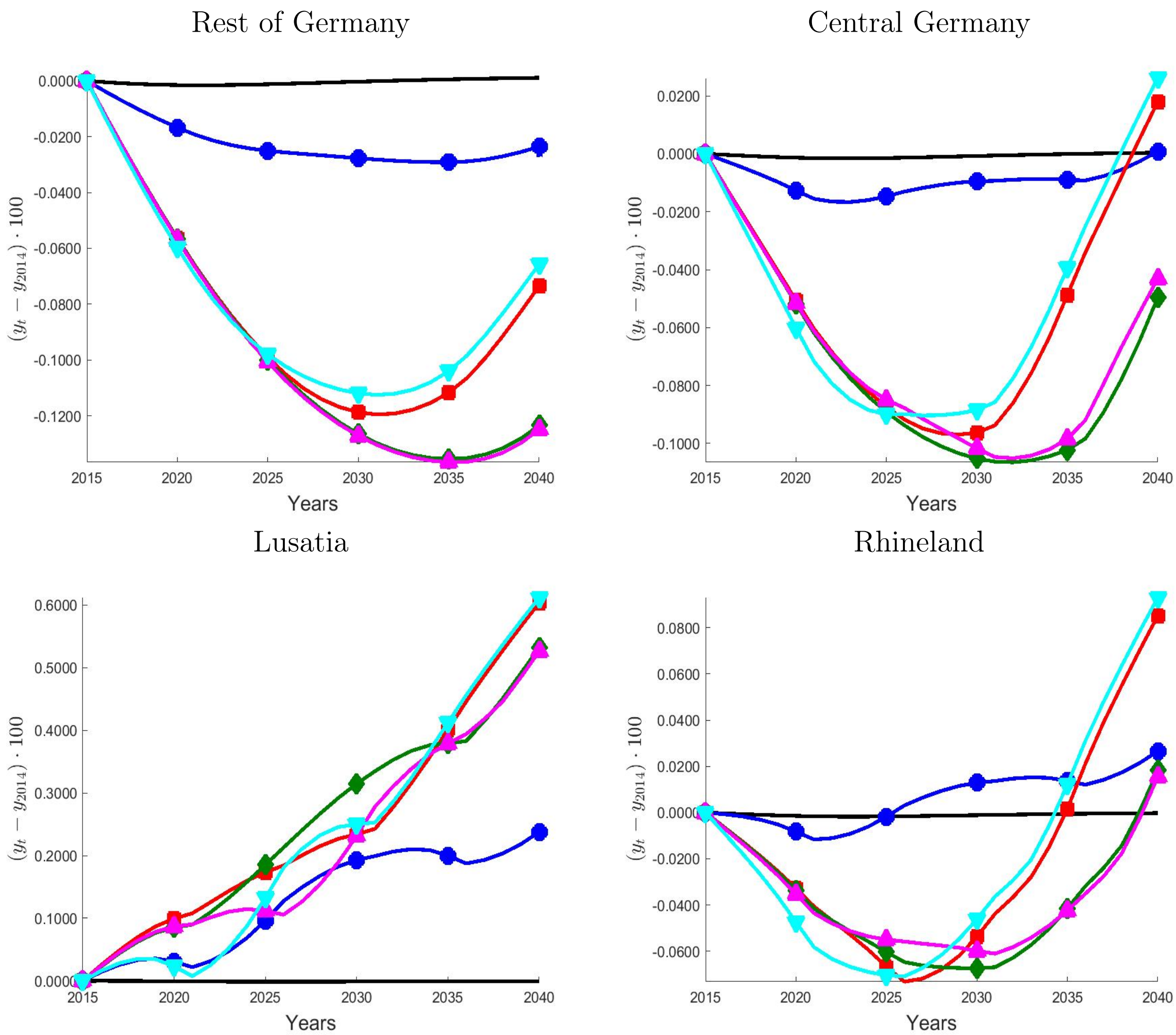
Note: The plots depict the simulation trajectories for the Null-Scenario (black solid line), Baseline (blue line with circle), Phase-Out-2035-Weak (red line with square), Phase-Out-2040-Age (green line with diamond), Phase-Out-2040-Balanced (magenta line with triangle pointing upward) and Phase-Out-2035-Strong (cyan line with triangle pointing downward).

Figure 3.21: Simulation trajectory for non-lignite employment rates



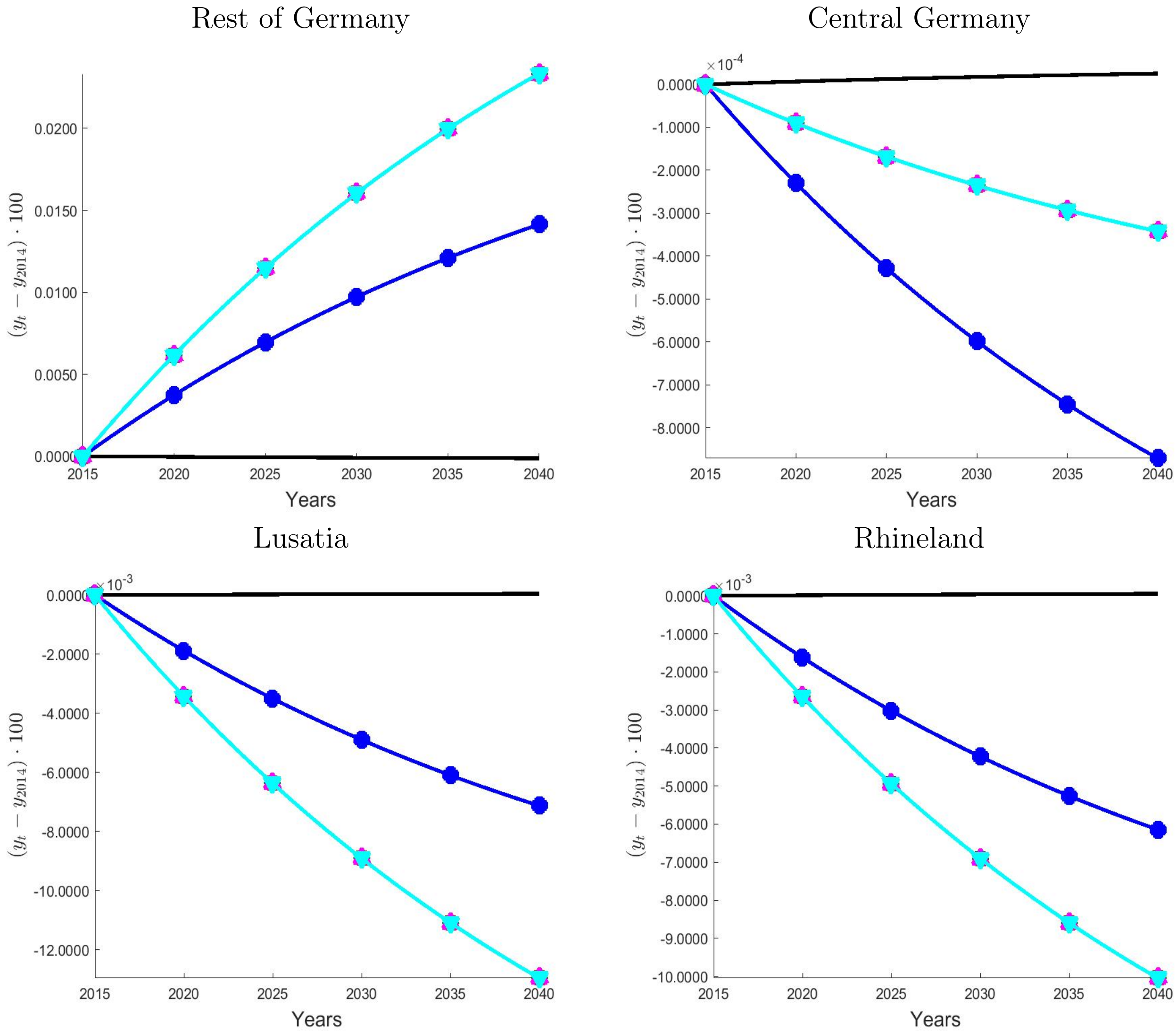
Note: The plots depict the simulation trajectories for the Null-Scenario (black solid line), Baseline (blue line with circle), Phase-Out-2035-Weak (red line with square), Phase-Out-2040-Age (green line with diamond), Phase-Out-2040-Balanced (magenta line with triangle pointing upward) and Phase-Out-2035-Strong (cyan line with triangle pointing downward).

Figure 3.22: Simulation trajectory for non-energy employment rates



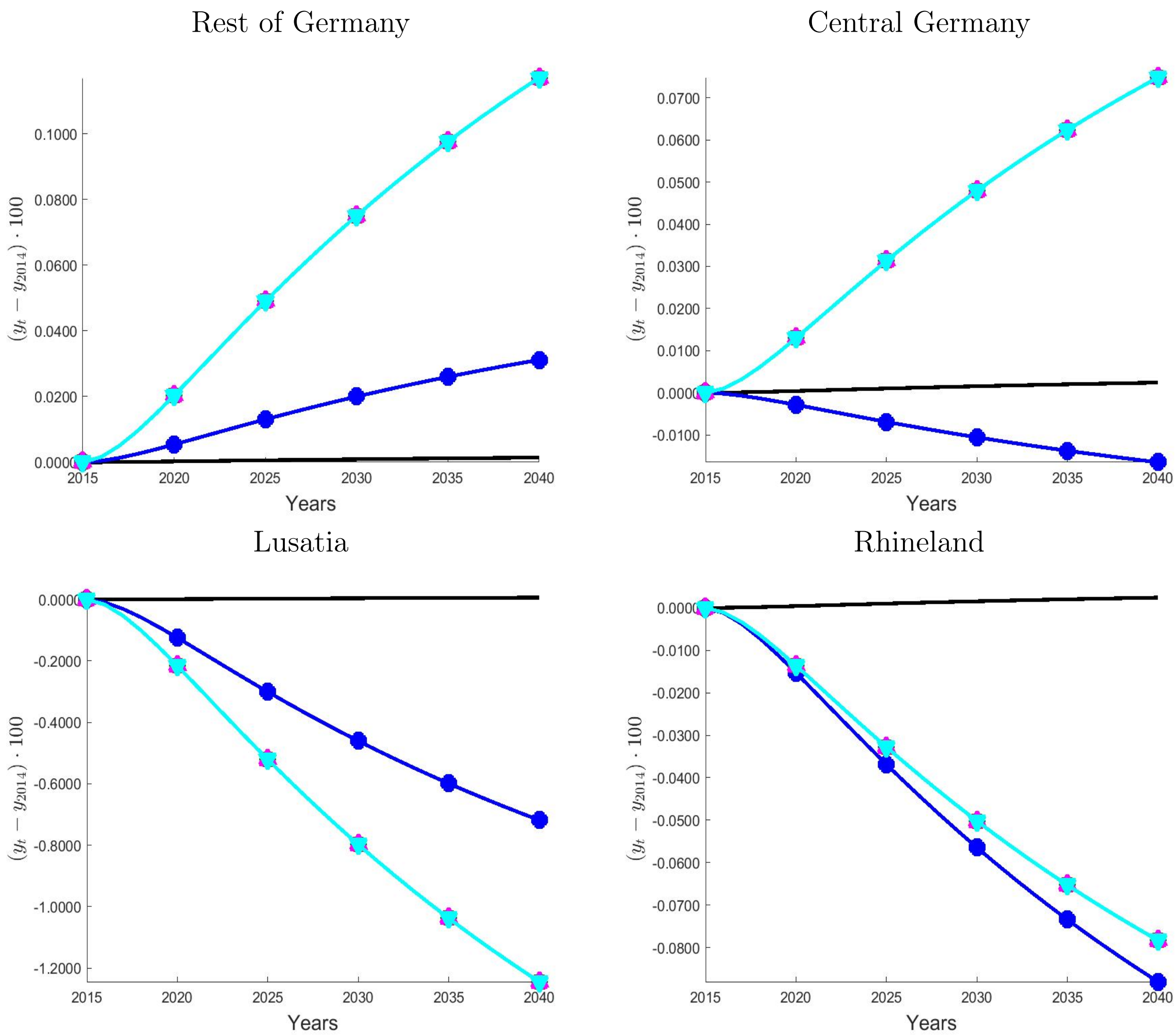
Note: The plots depict the simulation trajectories for the Null-Scenario (black solid line), Baseline (blue line with circle), Phase-Out-2035-Weak (red line with square), Phase-Out-2040-Age (green line with diamond), Phase-Out-2040-Balanced (magenta line with triangle pointing upward) and Phase-Out-2035-Strong (cyan line with triangle pointing downward).

Figure 3.23: Simulation trajectory for population shares



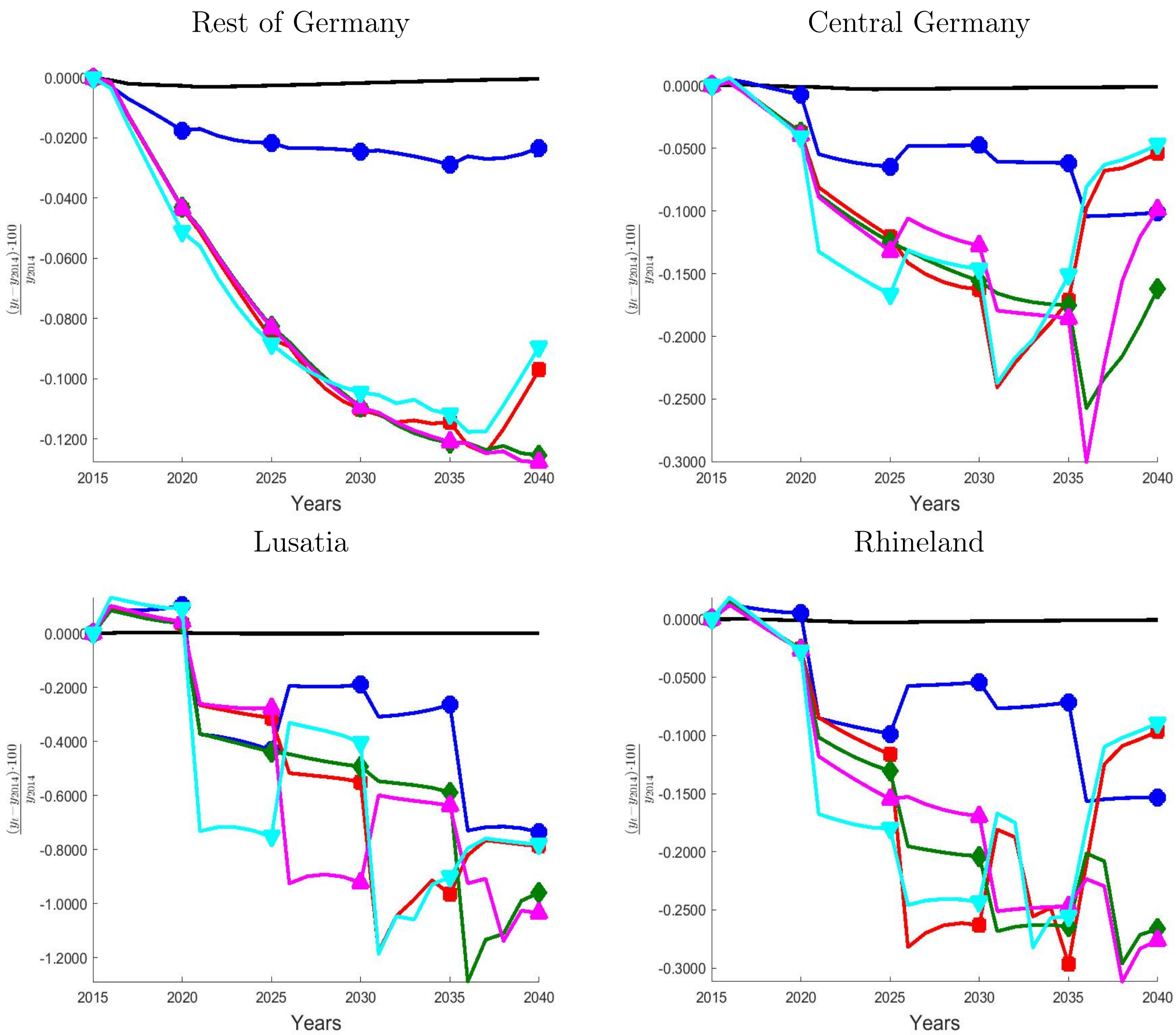
Note: The plots depict the simulation trajectories for the Null-Scenario (black solid line), Baseline (blue line with circle), Phase-Out-2035-Weak (red line with square), Phase-Out-2040-Age (green line with diamond), Phase-Out-2040-Balanced (magenta line with triangle pointing upward) and Phase-Out-2035-Strong (cyan line with triangle pointing downward).

Figure 3.24: Simulation trajectory for mark-ups



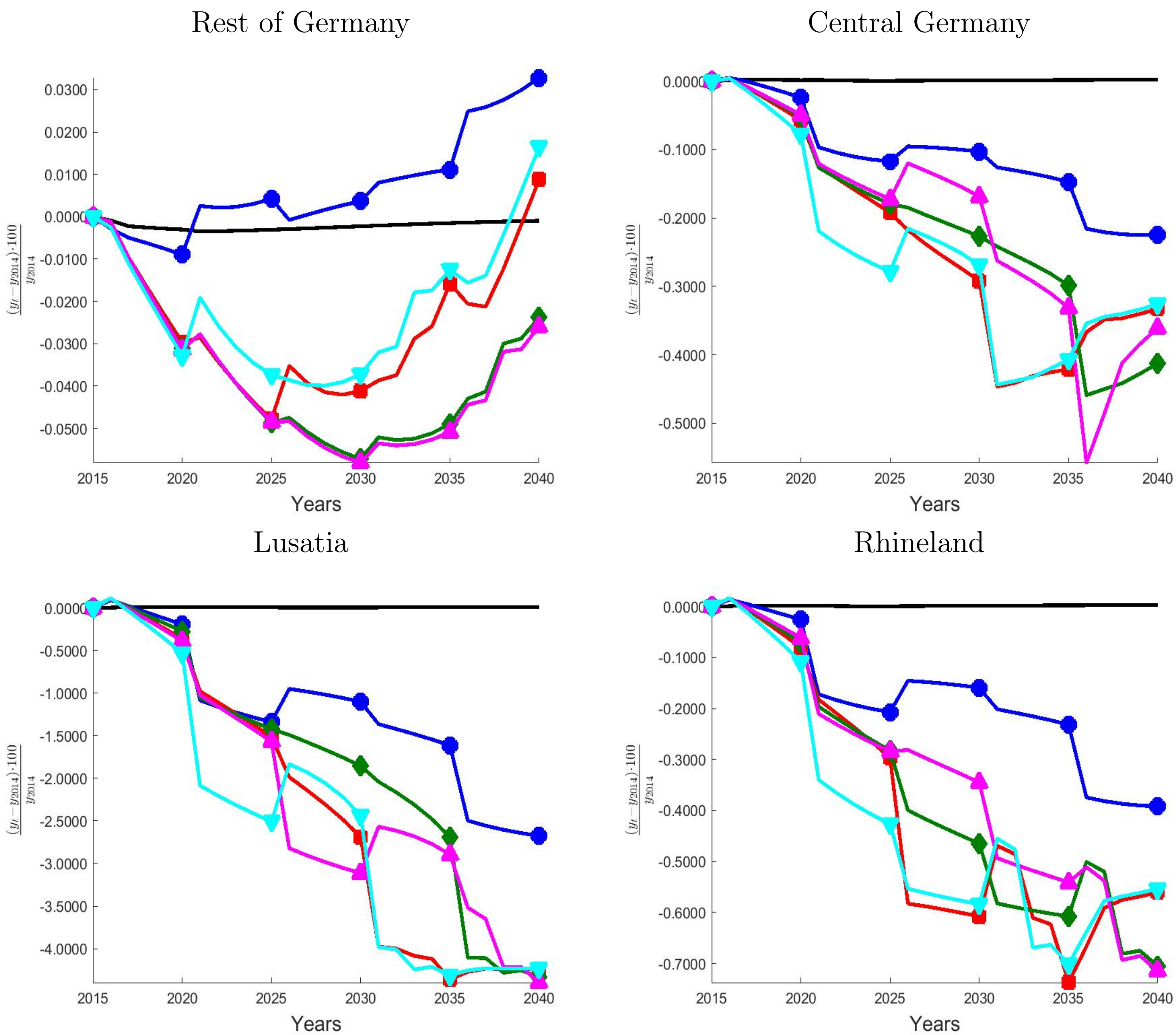
Note: The plots depict the simulation trajectories for the Null-Scenario (black solid line), Baseline (blue line with circle), Phase-Out-2035-Weak (red line with square), Phase-Out-2040-Age (green line with diamond), Phase-Out-2040-Balanced (magenta line with triangle pointing upward) and Phase-Out-2035-Strong (cyan line with triangle pointing downward).

Figure 3.25: Simulation trajectory for regional consumption



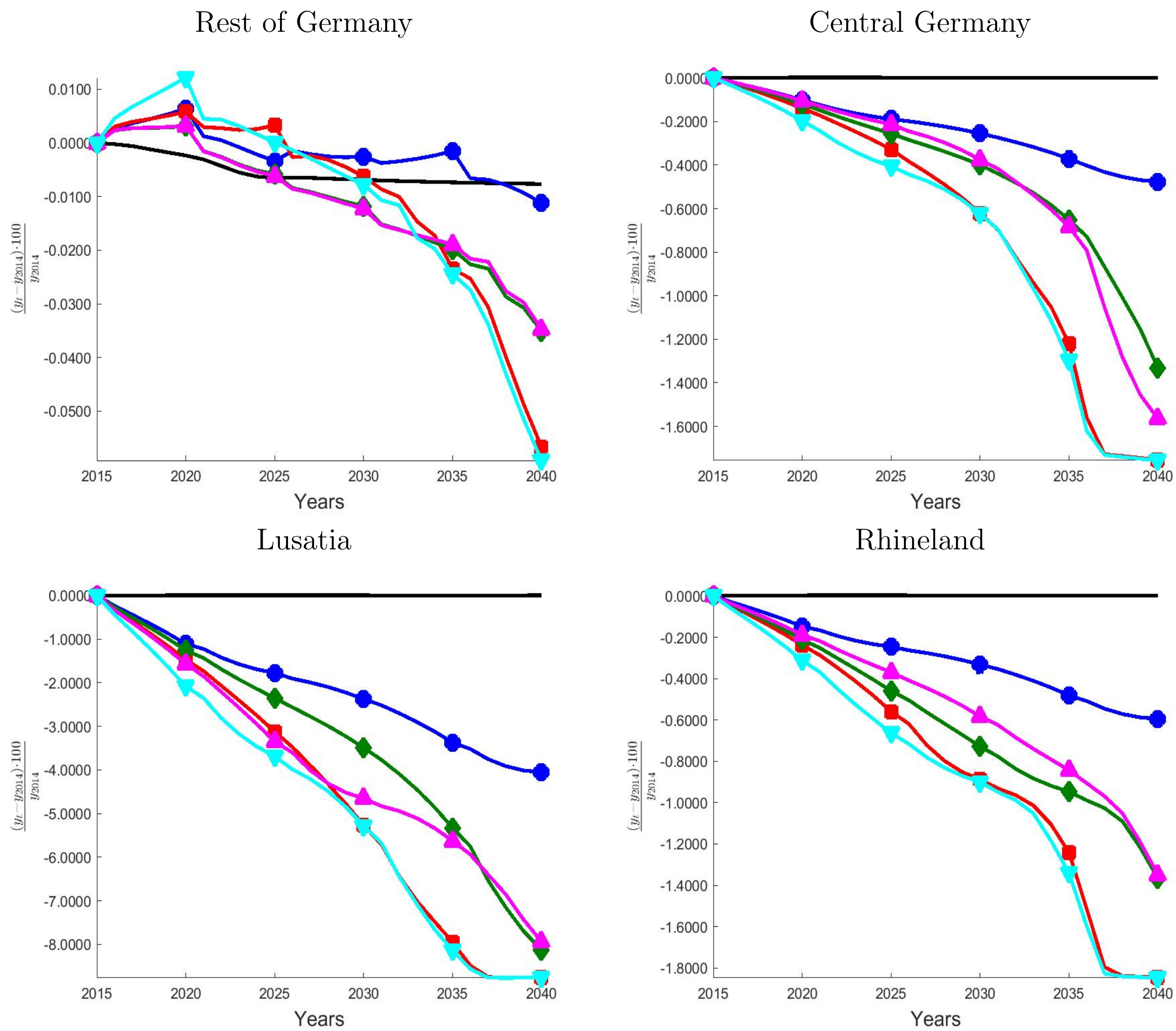
Note: The plots depict the simulation trajectories for the Null-Scenario (black solid line), Baseline (blue line with circle), Phase-Out-2035-Weak (red line with square), Phase-Out-2040-Age (green line with diamond), Phase-Out-2040-Balanced (magenta line with triangle pointing upward) and Phase-Out-2035-Strong (cyan line with triangle pointing downward).

Figure 3.26: Simulation trajectory for regional gross value-added



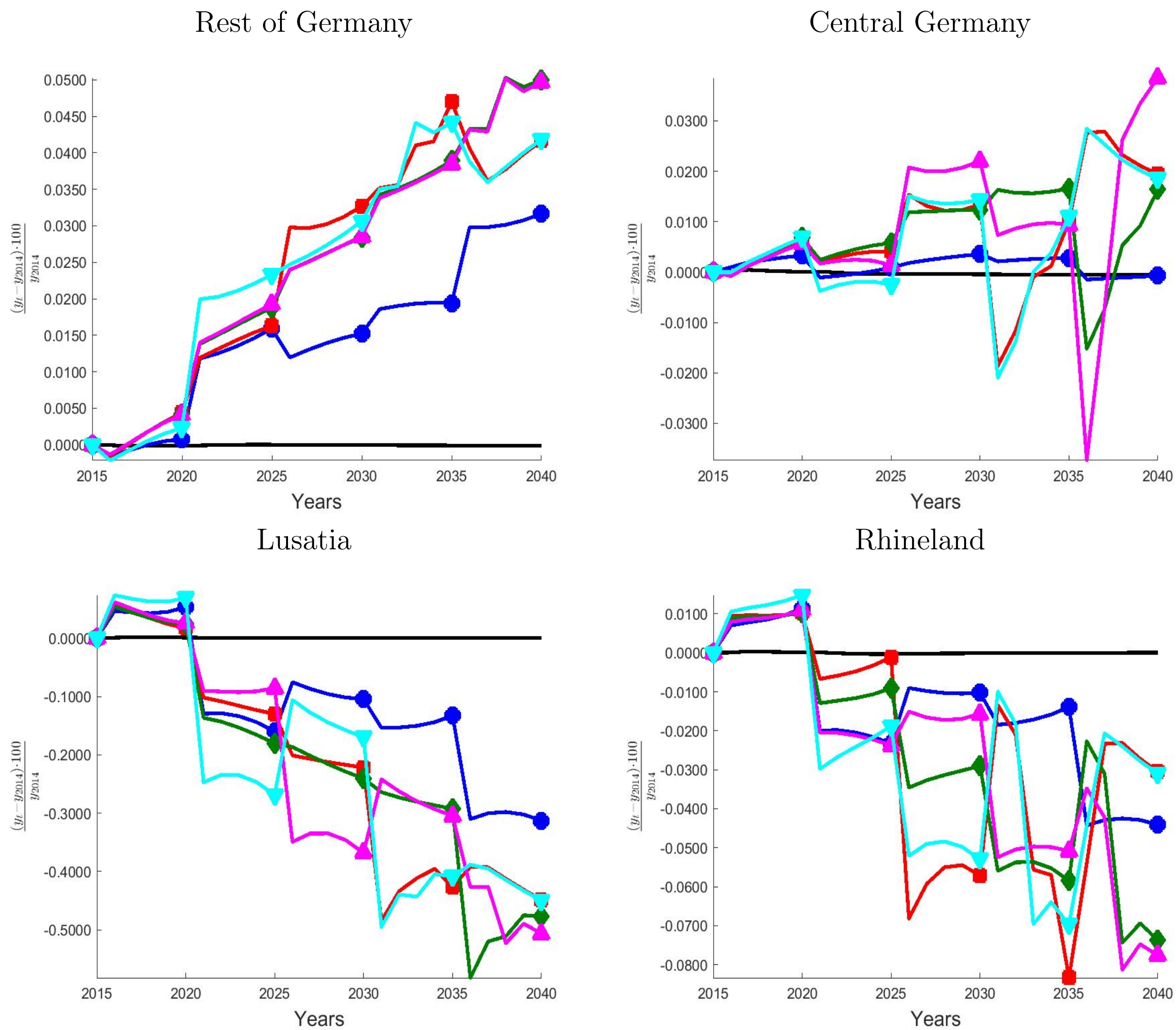
Note: The plots depict the simulation trajectories for the Null-Scenario (black solid line), Baseline (blue line with circle), Phase-Out-2035-Weak (red line with square), Phase-Out-2040-Age (green line with diamond), Phase-Out-2040-Balanced (magenta line with triangle pointing upward) and Phase-Out-2035-Strong (cyan line with triangle pointing downward).

Figure 3.27: Simulation trajectory for regional real wages



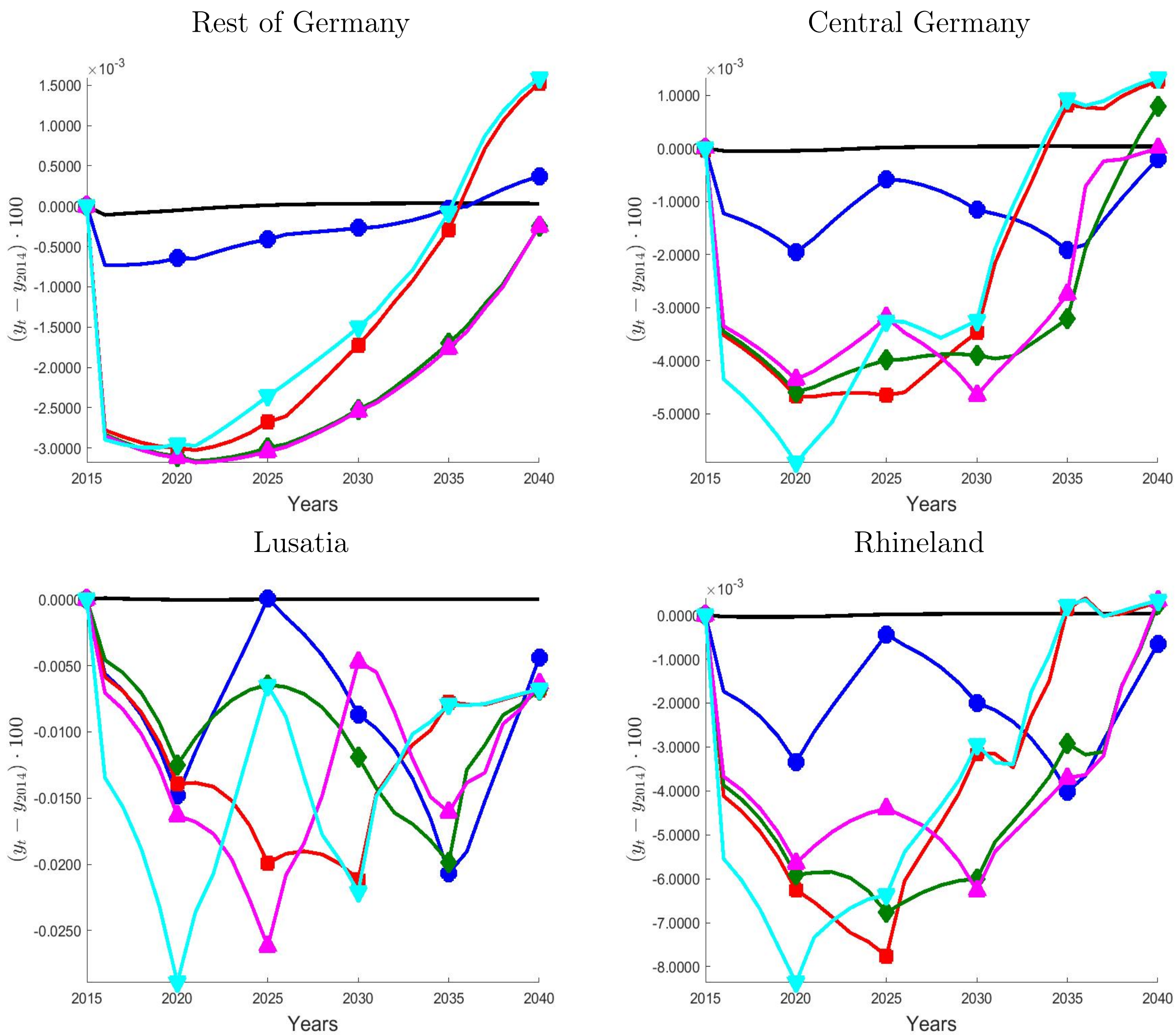
Note: The plots depict the simulation trajectories for the Null-Scenario (black solid line), Baseline (blue line with circle), Phase-Out-2035-Weak (red line with square), Phase-Out-2040-Age (green line with diamond), Phase-Out-2040-Balanced (magenta line with triangle pointing upward) and Phase-Out-2035-Strong (cyan line with triangle pointing downward).

Figure 3.28: Simulation trajectory for regional consumption price levels



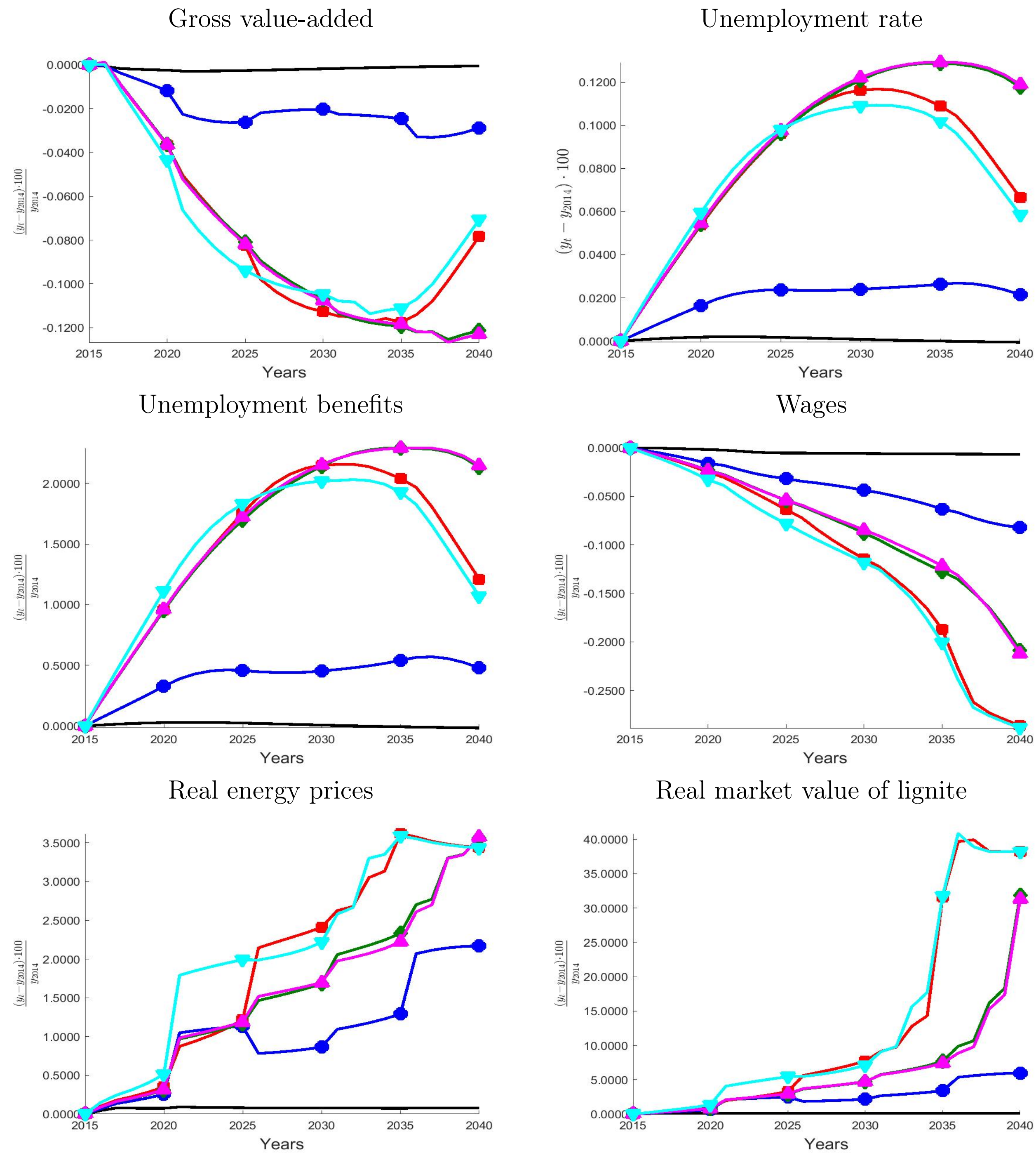
Note: The plots depict the simulation trajectories for the Null-Scenario (black solid line), Baseline (blue line with circle), Phase-Out-2035-Weak (red line with square), Phase-Out-2040-Age (green line with diamond), Phase-Out-2040-Balanced (magenta line with triangle pointing upward) and Phase-Out-2035-Strong (cyan line with triangle pointing downward).

Figure 3.29: Simulation trajectory for regional hiring rates



Note: The plots depict the simulation trajectories for the Null-Scenario (black solid line), Baseline (blue line with circle), Phase-Out-2035-Weak (red line with square), Phase-Out-2040-Age (green line with diamond), Phase-Out-2040-Balanced (magenta line with triangle pointing upward) and Phase-Out-2035-Strong (cyan line with triangle pointing downward).

Figure 3.30: Simulation trajectory for national aggregates



Note: The plots depict the simulation trajectories for the Null-Scenario (black solid line), Baseline (blue line with circle), Phase-Out-2035-Weak (red line with square), Phase-Out-2040-Age (green line with diamond), Phase-Out-2040-Balanced (magenta line with triangle pointing upward) and Phase-Out-2035-Strong (cyan line with triangle pointing downward).

Chapter 4

Is Risk the Fuel of the Business Cycle?[1]

Abstract

This paper develops a dynamic stochastic general equilibrium (DSGE) model with risky capital and oil as production factors. The production function of the representative firm is a nested constant elasticity of substitution function. The model is estimated using Bayesian techniques with economic data and on oil prices, production and consumption for the United States. The interaction between risk, investment decisions of firms, and the oil market are analysed, taking the short-run elasticity of substitution between oil and capital and the propagation mechanisms between risk in capital production and oil price movements into account. The model is used to reassess the contribution of the different potential drivers to the business cycle controlling for fluctuations in oil markets. Significant findings are that the contributions of financial market frictions and oil market disturbances to the US business cycle are low and that financial market disturbances mainly drove the Great Recession. The model can quantify the impact of climate change mitigation policies on the economy. Climate change mitigation policies, e.g. increasing oil taxes, to reduce crude oil consumption by 10% can cause a contraction of GDP by 1 to 2% and increases inflation. Monetary policy can stabilize inflation increasing the federal funds rate dependent on the degree of financial market imperfections by 0.15 to 0.40 percentage points annually.

JEL Codes: C32, C53, E37
Keywords: Business cycles, risk, energy, investment, DSGE, variance decomposition, structural models

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4.1 Introduction

Oil prices have been more volatile since the Yom Kippur war in 1973, and since then, macroeconomic research has been studying the relationship between oil prices and real economic activity. The Great Recession from 2007 to 2009 initiated a macroeconomic research agenda on the role of financial markets for the business cycle (see Christiano et al. 2014, Jermann & Quadrini 2012, Khan & Thomas 2013, Mian & Sufi 2014). We also know that oil and financial markets are interdependent (see Elder & Serletis 2009, 2010, Kilian 2008).

Suitable tools for investigating the macroeconomic role of oil (see Balke & Brown 2018, Bergholt et al. 2017, Dhawan & Jeske 2008, Milani 2009) and financial markets are general equilibrium models. A frequently used approach to model financial frictions is the so-called financial accelerator mechanism. This mechanism was introduced into a standard New-Keynesian DSGE (henceforth **NK-DSGE**) model by Bernanke et al. (1999). They showed that the accelerator could amplify small shocks, that might come from monetary policy or the oil market.

Christiano et al. (2014) (henceforth **CMR**) estimate a workhorse NK-DSGE model (see Christiano et al. 2005, Smets & Wouters 2003, 2007) (henceforth **CEE**) augmented by the financial accelerator mechanism described in Bernanke et al. (1999). Shocks to the credit market (risk shocks) can explain a majority of the US GDP growth variance, according to CMR. Quantitative financial variables (credit growth, networth) are necessary observables to achieve this result. Further, the estimated persistence in prices, wages and consumption are also important to obtain a dominant role of risk shocks for GDP growth.

Thus, CMR appear to have shown that risk is the fuel of the business cycle. However, they did not control for fluctuations in crude oil markets. Including crude oil market observables might change the estimated structural parameters. Persistence in wages and prices might be lower or higher, including oil. Estimated standard deviations of shocks are interdependent. Controlling for oil can change the contribution of other shocks to GDP growth and the business cycle.

The main objective of this paper is to study the interaction between oil and financial markets through the lens of an estimated DSGE model. This paper extends the model by CMR to include oil as production factor (henceforth **CMR–Oil**). It is essential to select a suitable benchmark model to isolate the effect of the interaction between oil markets and financial markets. This paper extends the CEE model (henceforth **CEE–Oil**). To capture the specific role of oil, one can switch from a Cobb-Douglas to a nested constant elasticity of substitution (henceforth **CES**) production function. There are two layers with the top layer combining labour and a composite production factor. The next layer combines oil and capital services to the composite production factor. Oil is used together with capital to produce output. In each layer, the production factors might be complements or substitutes, with the Cobb-Douglas production function as a particular case. It is standard to use Bayesian techniques to estimate the structural parameters of the model.

The results reveal that risk is not the main driver of the business cycle, but technology shocks are the main driver. However, risk shocks are an essential source for fluctuation. This result is not directly related to the inclusion of oil. The reason for a lower contribution of

Table 4.1: Overview of models

Abbreviation	Description
CEE	The workhorse model introduced by Christiano et al. (2005). It is a balanced growth model with price and wage rigidities.
CMR	The model introduced by Christiano et al. (2014) is based on Christiano et al. (2005) and includes financial frictions as described in Bernanke et al. (1999).
CEE–Oil	The CEE model with oil as production factor.
CMR–Oil	The CMR model with oil as production factor.

risk shocks to the business cycle is less persistent shocks to inflation, wages, demand and the monetary policy rule parameters.

The financial accelerator does not amplify oil market shocks in the CMR–Oil model, in contrast to the statement by Bernanke et al. (1999). Oil market shocks are essential to explain investment behaviour and less so to explain consumption. They drive changes in the permanent levels of consumption and investment, but not their growth rates. The theoretical variance decomposition for the CMR–Oil model reveals that oil explains less of the variation in investment compared to the CEE–Oil model. Oil market shocks explain about 11% of the variance without financial accelerator. With financial frictions, the contribution of oil market shocks to the variance in investment declines to almost 3%.

While a variance decomposition explains the theoretical second moments of the model variables, it does not describe specific historical episodes. Risk and oil market shocks might have been extraordinary drivers in particular episodes of the US business cycle since 1984. A historical decomposition reveals that risk shocks mainly contributed to the decline in GDP during the Great Recession. Otherwise, the contribution of risk shocks to the business cycle is low. Oil market variables have not been the leading cause of movements in GDP, investment or consumption growth. Oil market shocks moderately drive inflation. There is no remarkable difference between the historical decomposition of the variables using the CMR–Oil and the CEE–Oil model.

A striking result of the variance decomposition is that oil market variables explain less of the variance in GDP, consumption and investment with a financial accelerator. It contradicts the idea that the financial accelerator amplifies oil supply shocks. The opposite is true for monetary policy shocks. Impulse response functions to unexpected changes in the federal funds rate and unanticipated oil supply shocks support this picture. The financial accelerator mechanism amplifies the effect of monetary policy and reduces the impact of oil supply shocks.

Risk shocks, according to the historical decomposition, have been significant during the Great Recession. In contrast, oil supply shocks have not been significant during any historical episode in the last four decades. However, the US might recommit to the Paris Agreement enforced on November 4$^{\text{th}}$ 2016. A very likely consequence is the reduction of US oil consumption. Policymakers need to apply appropriate measures to reduce oil consumption to comply with the Paris Agreement. It is necessary to have adequate tools to assess the potential impact of mitigation measures on the economy. Golosov et al. (2014) use a calibrated dynamic general equilibrium model to evaluate mitigation measures and their effects on the economy. The estimated CEE–Oil and CMR–Oil model can assess the economic impact of mitigation policy. More precisely, the paper studies a reduction in oil consumption by an increase in oil taxes.

Impulse response functions derived from the structural CMR–Oil model show that a reduction in oil consumption by 10% causes a weak recession by -1 to -2%. An increase in the tax rate on oil will lead to inflation that is about 0.1 annual percentage points higher. Monetary policy may react to the rise in inflation. The federal funds rate needs to increase by 0.15 to 0.30 annual percentage points to stabilize price changes, according to the CMR–Oil model. In the CEE–Oil model an increase between 0.25 to 0.40 annual percentage points is required. Thus, more frictions in financing lead to lower changes needed in the federal funds rate to stabilize inflation.

In Section 4.2 I describe the CEE, CMR and the oil extended models. Section 4.3 describes the data and estimation procedure. Results are presented in Section 4.4 and discussed in Section 4.5. Section 4.6 concludes the paper.

4.2 The Model

This section describes the different models. Figure 4.1 is a graphical summary of all model versions. First, the section will non-technically discuss the CEE model. Second, the section will explain the modifications by CMR to include the financial accelerator into the CEE model. Third, the section will report the changes to fit oil as production factor into the CEE and CMR model.

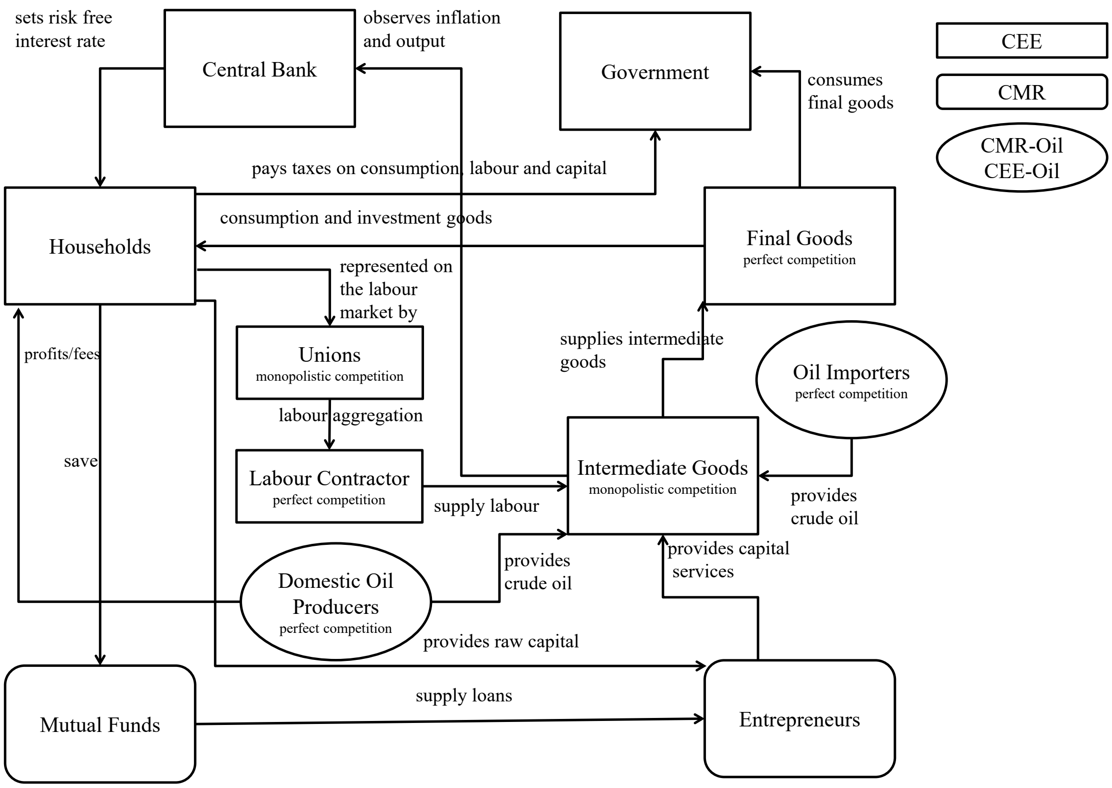
4.2.1 CEE

The baseline NK-DSGE model is depicted in Figure 4.1 and the equations are reported in Appendix 4.7.3.1.[2] I generally follow the description of Christiano et al. (2014) to describe the baseline DSGE model. All households j_h provide capital services K^s and hours worked h in each period t. Households either consume C or invest I final goods into their raw capital stock $\bar{K}_{t-1}$. The raw capital stock depreciates at a constant fraction δ. Capital services $K_t^s = u_t \bar{K}_{t-1}$ are rented to intermediate goods producing firms. Households face utilization costs $a(u_t)$ and investment adjustment cost $\mathcal{S}(\frac{I_t}{I_{t-1}})$. Investment adjustment costs depend on the growth rate in investment. The stock of raw capital evolves according to the standard law of motion.

The government charges a tax rate on consumption τ^c, labour τ^l and capital income τ^k. The government also collects taxes $Tax_{t+\kappa}$ and provides lump-sum transfers $Tr_{t+\kappa}$. Government expenditures G are financed by tax revenues. Households can purchase bonds B_t

[2] All symbols are explained in Table 4.5, Table 4.6 and Table 4.7 in the Appendix.

Figure 4.1: Model overview



Source: own exhibition.
Note: The diagram illustrates relationships between the different agents in the model. Rectangles represent agents present in the CEE model, rounded rectangles represent agents present in the CMR model and ellipses represent agents present in the CEE–Oil and CMR–Oil model.

and get an interest rate R_t. Households live infinitely and maximize intertemporal discounted utility (4.1) subject to their budget constraint (4.2).

$$\max_{\substack{\bar{K}_{j_h,t+\kappa+1}, I_{j_h,t+\kappa} \\ C_{j_h,t+\kappa}, B_{j_h,t+\kappa+1}}} \mathrm{E}_0 \sum_{\kappa=0}^{\infty} \beta^{\kappa} \left[\zeta_{c,t+\kappa} \Big\{ \ln(C_{j_h,t+\kappa} - bC_{j_h,t+\kappa-1}) \Big\} - \psi_L \int_0^1 \frac{h_{j_h,j_l,t+\kappa}^{1+\sigma_L}}{1+\sigma_L} dj_l \right], \tag{4.1}$$

$$\text{s.t.}(1+\tau^c)P_{t+\kappa}C_{j_h,t+\kappa} + B_{j_h,t+\kappa+1} + \Big(\frac{P_{t+\kappa}}{\Upsilon^{t+\kappa}\mu_{\Upsilon,t+\kappa}}\Big) I_{j_h,t+\kappa} + Tax_{t+\kappa} + Q_{\bar{K},t+\kappa}(1-\delta)\bar{K}_{t+\kappa}$$
$$= (1-\tau^l) \int_0^1 W_{j_h,j_l,t+\kappa} h_{j_h,j_l,t+\kappa} dj_l + R_{t+\kappa}B_{t+\kappa} + Q_{\bar{K},t+\kappa}\bar{K}_{j_h,t+\kappa+1} + \Delta_{j_h,t+\kappa} + Tr_{j_h,t+\kappa}. \tag{4.2}$$

Households discount the future with the discount factor β. In each period households utility depends positively on a weighted average of the current consumption level and the change to the previous period. Habit persistence b measures how important the current change in consumption is for utility. Working is associated with disutility, where the inverse Frisch elasticity σ^L measures how sensitive labour supply is to changes in wages. Each period the budget constraint (4.2) is binding.

Firms j_f use capital services K^s and homogenous working hours l to produce intermediate goods $Y_{j_f,t}$. A Cobb-Douglas function combines the two primary production factors. Firms have to pay wages W_t and a rental price for capital services $\tilde{r}_t^k P_t$. One can derive the demand for production factors from cost minimization subject to a given amount of output. Therefore marginal costs S_t depend directly on the market prices for the primary production factors. Fixed costs ensure zero profits in steady-state and reduce the incentives for new firms to enter the market (see Christiano et al. 2010).

$$\begin{aligned} &\min_{l_{j_f,t}, K^s_{j_f,t}} W_t l_{j_f,t} + P_t \tilde{r}_t^k K^s_{j_f,t}, \\ &\text{s.t.} Y_{j_f,t} = \epsilon_t \left(\frac{K^s_{j_f,t}}{\Upsilon^{t-1}} \right)^{\alpha_K} ({\epsilon^h}_t \, z_t l_{j_f,t})^{\alpha_N} - \phi_t z_t, \\ &\qquad l_{j_f,t} > 0, K^s_{j_f,t} > 0. \end{aligned} \tag{4.3}$$

These intermediate goods are imperfect substitutes to produce a final good Y_t using a constant elasticity of substitution production function. Parameter λ^f determines the degree of substitutability between the different products. Profit maximization of the final goods producer (4.4) implies that the overall price index P_t is a weighted average over all prices set by intermediate goods producers.

$$\begin{aligned} &\max_{Y_{j_f,t}} P_t Y_t - \int_0^1 P_{j_f,t} Y_{j_f,t} dj_f, \\ &\text{s.t. } Y_t = \left(\int_0^1 Y_{j_f,t}^{\frac{1}{\lambda^f}} dj_f \right)^{\lambda^f}. \end{aligned} \tag{4.4}$$

Intermediate goods-producing firms have price-setting power. They set their price $P_{j_f,t}$ to maximize expected discounted profits. Only a random fraction $1-\xi^p$ is allowed in each period to reset their price. All other intermediate firms update their prices according to an indexation rule $\tilde{\pi}_t P_{j_f,t-1}$. This two-stage production process, in combination with random price-setting, allows to model price rigidity. Further, it ensures that price inflation π_t can influence real economic variables in the model. The intertemporal expected discounted profit (4.5) is maximized choosing a optimal price $\tilde{P}_t$, subject to the demand for intermediate products (4.6).

$$\max_{\tilde{P}_t} \mathrm{E}_t \sum_{\kappa=0}^{\infty} (\beta\xi^p)^\kappa \lambda_{t+\kappa} (P_{j_f,t+\kappa} Y_{j_f,t+\kappa} - S_{t+\kappa} Y_{j_f,t+\kappa}), \tag{4.5}$$

$$\text{s.t.} Y_{j_f,t+\kappa} = Y_{t+\kappa} \left(\frac{\tilde{\Pi}_{t,t+\kappa} \tilde{P}_t}{P_{t+\kappa}} \right)^{-\frac{\lambda^f}{\lambda^f - 1}}. \tag{4.6}$$

Unions represent different types of labour, j_l and sell them to a labour contractor. Labour contractors sell homogenous labour l_t to the intermediate goods producing firm. A CES aggregation function bundles different types of labour. The parameter λ^w determines the degree of substitutability between the different types of labour. Total hours worked in each year in the economy is denoted by h_t. Similar to the problem of the intermediate goods producing firm only a fraction of unions $1-\xi^w$ is allowed to reset the wage. All other unions will reset their wage according to an indexation rule $W_{j_l,t} = \tilde{\pi}_t^w W_{j_l,t-1}$. Unions reset the wage to maximize the expected discounted wage bill less the foregone utility of the household working (4.7), subject to the demand for the specific type of labour by labour contractors (4.8). Unions take into account the disutility imposed on households by supplying labour to the intermediate goods-producing firms.

$$\max_{\tilde{W}_t} \mathrm{E}_t \sum_{\kappa=0}^{\infty} (\beta\xi^w)^\kappa \left[\lambda_{t+\kappa} \tilde{W}_t \tilde{\Pi}^w_{t,t+\kappa} h_{j_l,t+\kappa} (1 - \tau^l_{t+\kappa}) - \psi_L \frac{h^{1+\sigma_L}_{j_l,t+\kappa}}{1+\sigma_L} \right], \tag{4.7}$$

$$\text{s.t.} h_{j_l,t+\kappa} = l_{t+\kappa} \left(\frac{\tilde{\Pi}^w_{t,t+\kappa} \tilde{W}_t}{W_{t+\kappa}} \right)^{\frac{\lambda^w}{1-\lambda^w}}. \tag{4.8}$$

Monetary policy sets the risk free interest rate for bonds according to a Taylor rule (4.9). Christiano et al. (2014) state in their paper the monetary policy rule as stated in (4.9), with expected inflation and current GDP growth instead of past values. The risk free interest rate R_t responds to deviations in previous inflation π_{t-1} from its target and in GDP growth $\frac{C_{t-1}+I_{t-1}+G_{t-1}}{C_{t-2}+I_{t-2}+G_{t-2}}$ from its potential (see Bernanke et al. 1999). Government expenditures G_t are modelled as exogenous process.

$$\frac{1+R_t}{1+\bar{R}} = \left(\frac{1+R_{t-1}}{1+\bar{R}} \right)^{\tilde{\rho}} \left\{ \left(\frac{\pi_{t-1}}{\bar{\pi}} \right)^{1+\tilde{a}_\pi} \left(\frac{\mu^z_{t-1}}{\bar{\mu^z}} \frac{c_{t-1} + \frac{i_{t-1}}{\mu^\Upsilon_{t-1}} + g_{t-1}}{c_{t-2} + \frac{i_{t-2}}{\mu^\Upsilon_{t-2}} + g_{t-2}} \right)^{\tilde{a}_{\Delta y}} \right\}^{1-\tilde{\rho}} + \frac{\sigma^{x^p}}{4} x^p_t. \tag{4.9}$$

The economy follows a balanced growth path. All real variables have a common stochastic trend $z_t = \mu^z_t z_{t-1}$. This trend reflects long-run technological change leading to economic growth. Nominal variables are scaled by the nominal price level $P_t = \pi_t P_{t-1}$. Capital follows the common stochastic trend and has a specific deterministic trend of Υ^t. Temporary deviations from the balanced growth path are the result of shocks hitting the economy. The standard model comprises a shock to government expenditure g_t, total factor productivity ϵ_t, labour productivity ϵ_t^h, price mark-up shocks ϵ_t^p, wage mark-up shock ϵ^w, technological growth rate μ_t^z, shocks to the relative price of investment μ^Υ, consumption preference shock ζ_t^c, and investment adjustment cost shocks ζ_t^i. All shocks follow an autoregressive moving average (henceforth **ARMA**) process. Each shock is driven by a white noise process $\eta^{j_s}, j_s \in \{g, \epsilon, \epsilon^h, \epsilon^p, \epsilon^w, \mu^z, \mu^\Upsilon, \zeta^c, \zeta^i\}$.

4.2.2 CMR

CMR introduces entrepreneurs j_E and mutual funds j_{MF} to the CEE model. Appendix 4.7.3.3 reports different equations and modifications of the CMR model compared to the CEE model. In principle, the financial accelerator mechanism is caused by a conflict of interest between two agents (see Bernanke et al. 1999). Mutual funds use deposits (raw capital) from households to provide loans $B_{j_E,t+1}$ at the gross nominal interest rate Z_{t+1} to entrepreneurs. Mutual funds pay an interest rate R_t for households deposits. Entrepreneurs are owned by households and can either borrow or use their networth $N_{j_E,t}$ to produce effective capital $K_{j_E,t+1} = \omega_t \bar{K}_{j_E,t+1}$. Each household j_h owns a continuum of entrepreneurs j_E. All entrepreneurs experience in each period an idiosyncratic shock ω_t. This shock follows a log-normal distribution with an expectation equal to one and variance varying over time σ_t. This shock decides how much of the raw capital transforms into effective capital. Households still own raw capital, but they sell it to entrepreneurs in each period at a price $Q_{\bar{K},t-1}$. Mutual funds are operating under perfect competition to supply loans to entrepreneurs j_E using raw capital. These entrepreneurs are able to repay their loans with probability $1 - F_t(\bar{\omega}_{t+1})$, if their idiosyncratic productivity shock ω is bigger than a critical threshold $\bar{\omega}$. Entrepreneurs with an idiosyncratic productivity shock below this threshold file bankruptcy. Mutual funds need to verify whether entrepreneurs are bankrupt or not. This monitoring process is associated with costs $dcost(\bar{\omega})_t$, which are proportional by a factor μ to the earnings of the bankrupt entrepreneurs. The expected value of the assets of bankrupt entrepreneurs is given by $G_t(\bar{\omega}_{t+1})(1 + R_{t+1}^k)Q_{\bar{K},t}\bar{K}_{j_E,t+1}$. The term $G_t(\bar{\omega}_{t+1})$ represents the expected value of ω for bankrupt entrepreneurs. Costly state verification is an agency problem. Further, it introduces a wedge between the risk-free interest rate and the total return on raw capital R_t^k. This wedge is the credit spread and is a consequence of debt financing by entrepreneurs. Entrepreneurs choose the leverage ratio $L_t = \frac{N_{j_E,t}+B_{j_E,t+1}}{N_{j_E,t}}$ to maximize their expected profits subject to the cash constraint imposed by mutual funds.

Entrepreneurs solve the following optimization problem

$$\max_{L_t} \mathrm{E}_t \Big[\int_{\bar{\omega}_{t+1}}^{\infty} \{(1+R^k_{t+1})\omega Q_{\bar{K},t} K_{j_E,t+1} - B_{j_E,t+1}(1+Z_{t+1})\} f(\omega) d\omega \Big] \tag{4.10}$$
$$\text{s.t.}\{1-F_t(\bar{\omega}_{t+1})\}(1+Z_{t+1})B_{j_E,t+1} + (1-\mu)G_t(\bar{\omega}_{t+1})(1+R^k_{t+1})Q_{\bar{K},t}\bar{K}_{j_E,t+1}\ldots$$
$$\geq B_{j_E,t+1}(1+R_t).$$

Entrepreneurs do not accumulate infinite wealth because of an exogenous survival rate of γ_t. Entrepreneurs receive transfers from their households W^e each period. Entrepreneurs leaving the market $1-\gamma_t$ can consume a share Θ of their assets. Entrepreneurs transfer the remaining share of assets to households. The inclusion of entrepreneurs alternates resource constraint. The resource constraint derived from the budget constraint of households includes monitoring costs and transfers of entrepreneurs to households (see (4.11)). CMR include shocks to the survival rate of entrepreneurs η^{γ}_t and shocks to risk σ_t. These shocks are either anticipated η^s_t for $s \in [1,8]$ or unanticipated η^{σ}_t.

CMR also include long-term bonds $B^L_{j_h,t}$ to control for variations in the term structure between short-term and long-term bonds. The Spread between interest rates $\frac{1+R^L_t}{1+R_t}$ is determined by a term structure shock η^{term}_t. One can use long-run government bonds that have a one-year maturity and not a ten-year maturity. The one-year maturity requires less auxiliary variables for the leads included in the model. Solving the model is less time consuming, and therefore the estimation time is faster. Further, it allows running parameter identification tests discussed in Section 4.3.

4.2.3 CEE and CMR with oil

This section describes the inclusion of oil markets into the CEE and CMR model. Oil production, consumption and prices have a deterministic trend of Υ^{O^t}, which follows the approach for raw capital in CEE and CMR. A nested CES production function is introduced rather than the particular case of a Cobb-Douglas production function. First, the subsection explains the modifications to the budget constraint of the representative household, then the behavioural equations of oil producers. Third, the subsection describes the behaviour of the intermediate representative firm.

4.2.3.1 The representative household

The households optimization problem is the same as in CMR except that the budget constraint features now revenues from selling allowances to extract oil to local producers O^d_t. Households provide labour $h_{j_h,j_l,t}$ of type $j_l \in [0,1]$, raw capital $\bar{K}_{j_h,t}$ at price $Q_{\bar{K},t}$, consume final goods $C_{j_h,t}$ and invest into raw capital $I_{j_h,t}$. Further, they can purchase government bonds of one-quarter maturity $B_{j_h,t+1}$ and 4-quarter maturity $B^L_{j_h,t+4}$. The budget constraint is

$$(1+\tau^c)P_tC_{j_h,t} + B_{j_h,t+1} + B^L_{j_h,t+4} + \left(\frac{P_{t+k}}{\Upsilon^t \mu_{\Upsilon,t}}\right) I_{j_h,t} + Q_{\bar{K},t}\bar{K}_{j_h,t+1} + Tax_{t+\kappa} \quad (4.11)$$
$$= (1-\tau^l)\int_0^1 W_{j_h,j_l,t}h_{j_h,j_l,t}dj_l + R_tB_{j_h,t} + (R^L_t)^4 B^L_{j_h,t} + Q_{\bar{K},t}(1-\delta)\bar{K}_{j_h,t} + \Delta_{j_h,t}$$
$$+(1-\Theta)(1-\gamma_t)\{1-\Gamma_{t-1}(\bar{\omega}_t)\}R^k_t Q_{\bar{K},t-1}\bar{K}_{j_h,t} + \Gamma^d(O^d_{j_h,t}) + Tr_{t+\kappa}.$$

The modification of the budget constraint implies a modification of the resource constraint as well. One can drop the index j_h for households under the assumption of representative households. Total profits of domestic firms Δ_t include expenditures for oil $P^O_t\, O_t$ used in the production process. Oil is the only tradable production factor. One could also assume that domestic households do not possess all active oil suppliers in the US. Further, households receive transfers from entrepreneurs $(1-\Theta)\,(1-\gamma_t)\,\{1-\Gamma_{t-1}(\bar{\omega}_t)\}R^k_t Q_{\bar{K},t-1}\bar{K}_{j_h,t}$ leaving the market, after they consumed a fraction of their assets Θ.

4.2.3.2 Oil producers

There exists a continuum $j_p \in [0,1]$ of domestic oil producers d and oil importers im with access to infinite oil reserves. All domestic oil producers are identical, and the same is true for all oil importers. Homogeneity of suppliers rules out market power in the crude oil market. Oil reserves are infinite in the model, which contradicts reality. Domestic intermediate goods-producing firms buy oil $O^{d,im}_{j_p,t}$ for the same price P^O_t. Oil producers need to acquire the allowance and rig services to extract a barrel of oil from their respective households. It is also possible that the government sells the allowances and rig services to the household and transfers the revenues through tax cuts or subsidies back. The price of an allowance per barrel $\Gamma^{O,d,im}\left(O^{d,im}_t\right)$ is a function of the current extraction level $O^{d,im}_t$. Firms maximize profits choosing the amount of oil to extract

$$\max_{O^{d,im}_{j_p,t}} P^O_t\,(1-\tau^O_t)\,O^{d,im}_{j_p,t} - \Gamma^{O,d,im}_t(O^{d,im}_{j_p,t}). \quad (4.12)$$

The model simplifies the more complex tax system for oil production in the United States by a tax rate as a share on revenues τ^o_t. The log tax rate follows an auto-regressive process of order one as the other shocks.

The solution to the optimization problem is straight forward and represents the supply curve of the respective oil producers

$$P^O_t\,(1-\tau^O_t) = \frac{\partial\Gamma^{O,d,im}_t(O^{d,im}_t)}{\partial O^{d,im}_t} = \frac{\partial\left(\frac{\zeta^{O,d,im}_t}{\Upsilon^{O^t}\,\gamma^{O,d,im}}\,O^{d,im}_t\right)^{1+\sigma^O}}{\partial O^{d,im}_t} \quad (4.13)$$
$$= \left(\frac{\zeta^{O,d,im}_t}{\Upsilon^{O^t}\,\gamma^{O,d,im}}\right)^{1+\sigma^O}\left(O^{d,im}_t\right)^{\sigma^O}.$$

Oil producers reaction to oil price fluctuations is determined by $\sigma^O > 0$ the inverse price elasticity of oil supply to an increase in oil prices. The inverse price elasticity needs to be non-negative to ensure the existence of a maximum to the profit maximization problem. It also provides an upward sloping supply curve. A lower elasticity implies a steeper supply curve resembling very inelastic oil supply. Domestic and foreign oil producers have the same price elasticities, but different cost functions. Differences in the extraction cost $\gamma^{O,d,im} > 0$ of the respective reserves drive long-run differences in the supply curve. Idiosyncratic temporary shocks $\zeta_t^{O,d,im} > 0$ allow for temporary changes in the costs to supply oil. The exploitation of oil reservoirs might entail temporary different extraction costs depending on the remaining reserves or the quality of oil extracted. Providing imported oil also requires transportation costs, which fluctuate over time.

Total oil consumption in one period is domestic production, fewer oil exports plus oil imports. Therefore, the following identity has to hold in each period.

$$O_t = O_t^d - O_t^{ex} + O_t^{im}. \tag{4.14}$$

How much domestic oil is exported is not the result of an optimization problem. Domestic oil exports need to be greater than zero and smaller than the total amount of domestic oil production. Therefore, the following relation is specified

$$O_t^{ex} = \zeta_t^{O,ex} \, O_t^d, \tag{4.15}$$

$$log\left(\frac{\zeta_t^{O,ex}}{\bar{\zeta}^{O,ex}}\right) = \rho^{\zeta^{O,ex}} log\left(\frac{\zeta_t^{O,ex}}{\bar{\zeta}^{O,ex}}\right) + \eta^{O,ex}, \text{ for } \zeta_t^{O,ex} \in (0,\, 1). \tag{4.16}$$

The exogenous process $\zeta^{O^{ex}}$ follows an autoregressive process of order one and defines the share of exported oil.

4.2.3.3 The representative firm

Firms (j_f) produce intermediate goods $Y_{j_f,t}$ using capital services $K^s_{j_f,t}$, hours of homogenous labour $l_{j_f,t}$ and oil $O_{j_f,t}$. The production function for gross output $X_{j_f,t} = X(M_{j_f,t}, l_{j_f,t})$ is a nested constant elasticity of substitution function. Each firm has access to the same technology and can substitute between labour and a composite production factor $M_{j_f,t} = M(O_{j_f,t}, K^s_{j_f,t})$ from capital services and oil. The production elasticity of substitution $\eta^M \in (0,\infty)$ determines how easy it is for firms to substitute labour for other production factors. The degree of substitution between oil and capital services is captured by the production elasticity of substitution $\eta^O \in (0,\infty)$ and the degree of substitutability is $\rho^O = \frac{\eta^O - 1}{\eta^O}$. I further restrict the distribution parameters $\alpha_M \in (0,1)$ and $\alpha_O \in (0,1)$ of the CES production function in each stage to sum up to one.

$$X(M_{j_f,t}, l_{j_f,t}) = \begin{cases} \epsilon_t \, M_{j_f,t}^{\alpha_M} \, (z_t l_{j_f,t})^{1-\alpha_M} & \text{if } \eta^M = 1, \\ \epsilon_t \left[(\alpha_M)^{\frac{1}{\eta^M}} \, M_{j_f,t}^{\rho^M} + (1-\alpha_M)^{\frac{1}{\eta^M}} (z_t l_{j_f,t})^{\rho^M} \right]^{\frac{1}{\rho^M}} & \text{otherwise,} \end{cases} \tag{4.17}$$

$$M(O_{j_f,t}, K_{j_f,t}^s) = \begin{cases} \left(\epsilon_t^O \, \frac{O_{j_f,t}}{\Upsilon^{O^t}} \right)^{\alpha_O} \left(\epsilon_t^K \, \frac{K_{j_f,t}^s}{\Upsilon^{t-1}} \right)^{1-\alpha_O} & \text{if } \eta^O = 1, \\ \left\{ (1-\alpha_O)^{\frac{1}{\eta^O}} \left(\epsilon_t^K \, \frac{K_{j_f,t}^s}{\Upsilon^{t-1}} \right)^{\rho^O} + (\alpha_O)^{\frac{1}{\eta^O}} \left(\epsilon_t^O \, \frac{O_{j_f,t}}{\Upsilon^{O^t}} \right)^{\rho^O} \right\}^{\frac{1}{\rho^O}} & \text{otherwise.} \end{cases} \tag{4.18}$$

It requires a suitable capital stock to use crude oil efficiently. The composition of the capital stock is crucial for the ability of firms and households to abandon oil consumption. The effectiveness of the workforce depends less on crude oil usage. However, it is also possible to model labour and capital in one nest and combine the composite production factor with crude oil in the final stage. Nevertheless, the model follows the approach by Balke & Brown (2018) to model oil and capital services in one CES nest.

Firms face fixed costs $\phi_t z_t$ to produce net output $Y_{j_f,t}$, where $\bar{\phi}$ is set such that there are no profits in steady-state. Fixed cost ensure that profits are zero so that no new firm enters the market in steady-state. The intermediate good producing firms minimize the costs for a given production level.

$$Y_{j_f,t} = \begin{cases} X_{j_f,t} - \phi_t z_t, & \text{if } X_{j_f,t} > \phi_t z_t, \\ 0, & \text{else.} \end{cases} \tag{4.19}$$

Temporary total factor productivity shocks ϵ_t, temporary capital specific factor productivity shocks ϵ_t^K, temporary oil factor productivity shocks ϵ_t^O can change production factor demand. The optimization problem is

$$\begin{aligned} \min_{l_{j_f,t}, K_{j_f,t}^s, O_{j_f,t}} \; & W_t l_{j_f,t} + P_t \tilde{r}_t^k K_{j_f,t}^s + P_t^O O_{j_f,t}, \\ s.t. Y_{j_f,t} &= X(M(O_{j_f,t}, K_{j_f,t}^s), l_{j_f,t}) - \phi_t z_t, \\ l_{j_f,t} > 0,\; K_{j_f,t}^s > 0,\; & O_{j_f,t} > 0,\; M_{j_f,t} > 0,\; Y_{j_f,t} > 0. \end{aligned} \tag{4.20}$$

The corresponding Lagrangian, ignoring the non-negativity constraints, of the problem is

$$\mathcal{L}_t^{\mathrm{F,min}} = W_t \, l_{j_f,t} + P_t \, \tilde{r}_t^k \, K_{j_f,t}^s + P_t^O O_{j_f,t} + S_t \{ Y_{j_f,t} - (X(M_{j_f,t}, l_{j_f,t}) - \phi z_t) \}. \tag{4.21}$$

The first order conditions to (4.21) describe the demand for production factors by the representative firms.

$$\frac{\partial \mathcal{L}_t^{\mathrm{F,min}}}{\partial l_{j_f,t}} :0 = W_t - S_t z_t^{\frac{\eta^M - 1}{\eta^M}} \epsilon_t (\alpha_N)^{\frac{1}{\eta^O}} \left(\frac{X_{j_f,t}}{l_{j_f,t}}\right)^{\frac{1}{\eta^M}}, \tag{4.22}$$

$$\frac{\partial \mathcal{L}_t^{\mathrm{F,min}}}{\partial K^s_{j_f,t}} :0 = P_t \tilde{r}_t^k - P_t^M (1-\alpha_O)^{\frac{1}{\eta^O}} (\Upsilon^{t-1})^{-\rho^O} (\epsilon^K{}_t)^{\rho^O} \left(\frac{M_{j_f,t}}{K^s_{j_f,t}}\right)^{\frac{1}{\eta^O}}, \tag{4.23}$$

$$\frac{\partial \mathcal{L}_t^{\mathrm{F,min}}}{\partial O_{j_f,t}} :0 = P_t^O - P_t^M (\alpha_O)^{\frac{1}{\eta^O}} (\Upsilon^{O^t})^{-\rho^O} (\epsilon^O{}_t)^{\rho^O} \left\{\frac{M_{j_f,t}}{O_{j_f,t}}\right\}^{\frac{1}{\eta^O}}, \tag{4.24}$$

$$\frac{\partial \mathcal{L}_t^{\mathrm{F,min}}}{\partial S_t} :0 = X_{j_f,t} - X(l_{j_f,t}, M_{j_f,t}), \tag{4.25}$$

$$P_t^M = S_t \, z_t^{\rho^M} \, \epsilon_t \, \alpha_M^{\frac{1}{\eta^M}} \left(\frac{X_{j_f,t}}{l_{j_f,t}}\right)^{\frac{1}{\eta^M}}.$$

The constraint of the cost minimization is the CES production function for output. Appendix 4.7.5 discusses the sufficient conditions for a minimum. The shadow price of oil-capital composite goods P_t^M is equal to the marginal product $\frac{\partial X_{j_f,t}}{\partial M_{j_f,t}}$ times marginal costs S_t.

4.3 Estimation

This section describes the estimation procedure. It explains in detail the data used to estimate the structural model. Standard Bayesian estimation techniques are applied. Further, the section reports how priors for the structural parameters are selected. Finally, the estimated model is analysed using conventional screening tools.

The main issue with the estimation of medium-sized DSGE models is parameter identification. It is vital to obtain convergence using the Random Walk Metropolis-Hastings (RWMH) algorithm. First, one can check local parameter identification as defined in Iskrev (2010) at the prior mean before one should apply the RWMH algorithm. Further, the pairwise correlation between parameters does not exceed the upper bound of 0.99 and decrease the required number of draws for the RWMH algorithm to converge. Afterwards, a quasi-Newton with BFGS optimization routine delivers a posterior mode candidate. Parameter identification of the model is necessary at the posterior mode candidate[3]. In the next step, the scale parameter for the proposal distribution ensures an acceptance ratio for the RWMH algorithm of 0.25. It is important to note that some commonly used parameters are not estimated. Indexation parameters for inflation and wages are not estimated (ι^{π,μ^z}), and habit formation b. Including these parameters lead either to unidentified parameters at the prior mean or the candidate for the posterior mode. Therefore, these parameters are set to zero and excluded from the estimation. Further, I calibrate the monetary response variable to inflation ($\tilde{a}_\pi = 0.5$). The correlation between the monetary response variable and the mon-

[3] Here, the potential point is the mode found using the CSMINWEL algorithm introduced by Sims.

etary policy rigidity parameter ($\tilde{\rho}$) is very high. These changes make an exact replication of CMR or CEE impossible. Nevertheless, it ensures local identification of parameters by the data and model equations. It also ensures convergence of the RWMH after a reasonable amount of draws.

4.3.1 Data

I declare observable variables as introduced by Smets & Wouters (2003) and Christiano et al. (2005) to estimate the model. Those are GDP growth, GDP deflator as a measure for inflation, consumption growth, investment growth, hours worked, wage growth, federal funds rate and the relative price of investment (see Figure 4.8). The model includes additional variables to control for fluctuations in the financial market, as discussed in Christiano et al. (2014). The measure for net worth is the quarterly change in the DOW Jones Wilshire 5000 index. Credit growth is the change in loans to non-financial firms. The difference between interest rates on BAA-rated corporate bond yields and the interest rate on government bonds with a 10-year maturity measures the interest-rate spread. The observables include 1-year instead of 10-year constant maturity US government bonds to compute the term structure. This modification allows to introduce less auxiliary variables into the model and also to run identification screenings as proposed by Iskrev (2010). Figure 4.9 depicts the observed financial variables used to estimate the model.

The CMR–Oil model extends the set of observable variables compared to CMR by domestic crude oil production, consumption, and imports fewer changes in oil stocks growth rates. The Energy Information Administration (EIA) provides monthly historical data for crude oil field production, exports, imports and changes in the stock.[4] Further, the refinery acquisition cost of imported oil (see Kilian & Vigfusson 2013) corrected for inflation is observable for the growth in the real oil price changes. Figure 4.10 depicts oil market variables used to estimate the model. Growth rates in domestic field crude oil production, imported crude oil fewer changes in oil stocks and crude oil exports contain the necessary information to control for oil consumption in the US indirectly.

In Table 4.8, the p-values for Augmented Dickey-Fuller and Phillips-Perron tests are reported. The tests can not reject the null hypothesis of a unit root at the five percent significance level for hours worked using the Augmented-Dickey-Fuller test or the Phillips-Perron test. Nevertheless, hours worked is a stationary series following a standard convention in the literature. For the other variables, the test results are either not conclusive or indicate that one can reject the null hypothesis of a unit root with an error probability of less than 5%.

[4] One can download the data from `https://www.eia.gov/` under data for petroleum and other liquids. One can retrieve data for field production, exports, imports and stock changes from US crude oil supply and disposition under the subcategory summary (release date March 29th 2019). The subcategory prices (release date April 1st 2019) lists refinery acquisition costs.

4.3.2 Steady-state

The model finds a steady-state using two different algorithms. First, one can use an algorithm to calibrate the model to estimate it. This algorithm will find the share of assets eaten up by monitoring μ using a numerical approach, the threshold productivity value separating solvent and insolvent entrepreneurs $\bar{\omega}$, and the cross-sectional dispersion of productivity σ in turning raw into effective capital. Otherwise, structural parameter values ensure to match given long-run relationships.

Second, an algorithm is applied to compute impulse response functions to permanent shocks. It requires a numerical procedure for a given set of structural parameters.

4.3.2.1 Calibration

Appendix 4.7.4 describes the procedure to calibrate the model and find the steady-state and Table 4.9 reports the calibrated parameters. First, the algorithm sets $r^k = 0.0525$ approximately the value reported by CMR at the posterior mode of their model. The steady-state ratio between net worth and raw capital depends on the steady-state rental rate. This value corresponds to long-run equity to debt ratio of 2 approximately the observed ratio for the period 1984-Q2 to 2018-Q4.[5]. Further, production of y equals one. Therefore, the steady-state values of consumption c, investment i and government expenditure g are easily interpretable as shares. The model without financial accelerator does not feature an external finance premium. Therefore, the risk-free interest rate of R is twice as large as in the model with a financial accelerator.

Transfers of households to entrepreneurs w^e is equal to 0.005 identical to CMR. It is necessary to find monitoring costs μ such that the first-order condition of entrepreneurs and its respective constraint is satisfied. The bankruptcy probability $F(\bar{\omega}) = 0.56\%$ corresponds to the estimated mode by Christiano et al. (2014).

The solution of first-order conditions for the entrepreneur and its corresponding constraint does not depend on the answer to other endogenous variables. Therefore, it is possible to solve the remaining static equations independent of the credit market equilibrium. The procedure requires to guess a net output value y^z and to iteratively solve for all other endogenous variables. The algorithm calibrates the capital ϕ^K, the oil ϕ^O and the labour $1 - \phi^M$ cost shares. Hours worked h are equal to unity in steady-state as done in Christiano et al. (2014). Different from Christiano et al. (2014), The value of the disutility to work parameter ψ^L ensures the unity of hours worked in a steady-state.

An essential modification of the routine to find the steady-state is the inclusion of a nested CES production function but also including the particular case of the Cobb-Douglas production function. Distributional parameters of the CES function α^O, α^M depend on the steady-state expenditure shares and the ratio of oil consumption and output. Elasticities of substitutions η^M, η^O determine the value of distributional parameters.

[5] Compare with the series *Non-financial Corporate Business; Credit Market Debt as a percentage of the Market Value of Corporate Equities,%, Quarterly, Not Seasonally Adjusted* published by the Federal Reserve of St. Louis.

Table 4.2: Steady-state properties, model at priors versus data

Ratio	CEE–Oil Model	CMR–Oil Model	Sample averages
$\frac{i}{y}$	0.25	0.25	0.26
$\frac{c}{y}$	0.55	0.55	0.58
$\frac{g}{y}$	0.19	0.19	0.19
$\frac{\bar{k}-n}{n}$	–	0.5	0.5
R	0.021	0.011	0.009
$\frac{o}{y}$	0.002	0.002	0.002
$\frac{p^o\,o}{y}$	0.016	0.017	0.017
$\frac{o^{im}}{o}$	0.51	0.52	0.52

Notes: The sample range is 1984-Q2 to 2018-Q4. The first three ratios are computed as described in CMR. Debt to equity ratio corresponds to the inverse of the non-financial corporate business debt to equity ratio. The oil output ratio is computed for 2012 constant prices of the refinery acquisition costs and the deflator for GDP. The share of oil is the ratio between domestic oil consumption expenditures and GDP.
Sources: Own computation, Federal Reserve Bank of St. Louis, US Energy Information Administration.

4.3.2.2 Permanent shocks

It is necessary for the computation of impulse response functions to permanent shocks to modify the previous routine. It allows us to consider permanent shocks. However, the routine needs the following assumptions to compute the permanent effects:

1. Long-run mark-ups are constant.
2. Long-run growth rates of prices and permanent technology shocks do not change.
3. Long-run utilization and price of raw capital are constant.

Therefore, the new long-run level of output and the associated magnitude and relative demand for production factors will change. The algorithm allows changing all included arbitrary shocks permanently. Nevertheless, the transition path for large innovations might not be computable.

The routine computes the impulse response functions for permanent and temporary shocks using a deterministic simulation framework with perfect foresight. Therefore, the impulse response functions can be non-linear. This approach, as discussed in Lindé & Trabandt (2018), is more suitable to retrieve information for policy advice compared to impulse response functions derived from log-linearised models.

4.3.3 Priors for structural parameters

Table 4.10 reports the prior distributions for all 41 parameters. It is important to note that some commonly used parameters are not estimated. Indexation parameters for inflation and wages are not estimated (ι, ι^{w,μ^z}), and habit formation b. Including these parameters lead either to not identified parameters at the prior mean or the candidate for the posterior mode[6] using the local identification analysis introduced by Iskrev (2010). Further, estimating the monetary policy parameter $\tilde{a}_\pi$ leads to a pairwise correlation with the persistence parameter $\tilde{\rho}$ above 0.99. Therefore, these parameters are excluded from the estimation and set to zero.

For the estimation of the CMR–Oil and CEE–Oil model I first obtain priors for the standard structural parameters using posterior means and standard deviations from the estimation of the baseline CEE model. For the first stage (where I estimate the CEE model), I define usual priors. The price and wage rigidity parameters follow a Beta distribution with prior mean equal to 0.5 and a prior standard deviation of 0.1. The monetary policy parameter $\tilde{a}_{\Delta y}$, which captures the response to output growth has the usual Gaussian prior distribution with a prior mean of 0.3 and a prior standard deviation of 0.05. Standard deviations of shocks follow an inverse Gamma distribution and have identical prior means and standard deviations. Persistence parameters of the exogenous disturbances with equal prior means and standard deviations follow the commonly used Beta distribution. Table 4.10 reports the obtained posterior mean and standard deviation for the CEE model and the prior mean and standard deviation.

In contrast to Christiano et al. (2014) I do not estimate the steady-state bankruptcy probability $F(\bar{\omega}_t)$, because it leads to non identified parameters at the prior mean. Further, I exclude the share of assets used to monitor bankrupt entrepreneurs μ from the set of estimated parameters, because it is calibrated to ensure that lump-sum transfers w^e of entrepreneurs to their household is equal to 0.005 as in Christiano et al. (2014). The prior distribution for signal correlation is modified to ensure that the estimated correlation is bounded between minus one and one. The signal correlation for anticipated risk shocks, is estimated indirectly through an auxiliary parameter $\sigma(\xi_s, \xi_{s+1})$. The prior distribution of the parameter follows a Beta distribution and ensures that signal correlation is zero at the prior mean. Signal correlation $Corr(\xi_t^s, \xi_t^{s+1}) = 2\,\sigma(\xi_s, \xi_{s+1}) - 1$ is zero if the auxiliary parameter is equal to its prior mean of 0.5.

The main objective of this paper is to study the interaction between oil and financial markets through the lens of a dynamic stochastic general equilibrium model. The extension compared to the model described in Christiano et al. (2014) is the inclusion of oil as a production factor. Further, the model allows for the short-run oil supply to be neither perfectly elastic (see Milani 2009) nor inelastic to the oil price. Cost functions of domestic and foreign oil producers are convex, and the inverse oil supply price elasticity is given by σ^O. The prior mean of the inverse oil supply price elasticity is 10, such as in Baumeister & Hamilton (2019). The inverse oil supply price elasticity follows a Gamma distribution with a standard deviation equal to two. The nested CES production function with oil allows defining the oil demand price elasticity η^O. The prior mean of the oil demand price elasticity from Baumeister & Hamilton (2019) equals 0.1 and also follows a Gamma distribution with a standard deviation of 0.05. The Gamma distribution and standard deviation ensure that values above and below the prior mean have similar probability, but also restricting the

[6] Here the posterior mode candidate is the mode found using the CSMINWEL algorithm introduced by Sims.

parameter space to positive values. Further, the set of estimated parameters contains the elasticity of substitution between hours worked and the capital oil composite production factor. The prior mean is set to one with a standard deviation of 0.2 and follows the Gamma distribution function.

4.3.4 Posterior mode analysis

After finding a posterior mode candidate, an optimization routine finds a scale parameter for the RWMH algorithm with an acceptance ratio of 25%.[7] A target ratio of 25% is slightly above the range Roberts et al. (1997) suggested, but well in the range of usually applied acceptance ratios. The screening Brooks & Gelman (1998) analysis assesses whether the simulations are sufficient to reach convergence, based on four RWMH chains with a total length of one million per chain. Figure 4.13 depicts for both models the multivariate convergence diagnosis. The Online Appendix reports single parameter diagnostics. A burn-in period of about 1,600,000 draws is sufficient. After 1,600,000 draws the 80% inter-quantile range based on the posterior likelihood interval and the respective second and third central moments are indistinguishable close to each other and stabilize horizontally.

4.4 Results

First, this section compares the estimated structural parameters for the model with and without financial accelerator. Second, the section reports the variance and historical decomposition of the business cycle for the US economy. Third, temporary and permanent impulse responses to an exogenous shock affecting the oil supply curve are depicted based on the non-linear model equations. Fourth, the section discusses the potential recessionary effect of mitigation measures to reduce oil consumption.

4.4.1 Structural parameters

The interaction between oil and financial markets in the model might change the estimation results for the structural parameters common to both models. Table 4.3 reports the posterior mean for the different model parameters. The elasticity of substitution between the capital-oil composite production factor and hours worked is above one. It indicates that labour and capital are imperfect substitutes and not complements, according to the estimation results. Further, the posterior mean for the model with financial accelerator and oil is lower than without financial accelerator. The posterior mean of the CEE–Oil model is still part of the 90% credibility interval for the CMR–Oil model. The posterior mean for the inverse supply elasticity and the credibility intervals of oil are in both models very similar. The same is true for the demand elasticity of oil. Note, that the demand elasticity is below the prior mean and the supply elasticity above the prior mean. Therefore, oil demand reacts less to price changes than the oil supply.

[7] The optimization routine is implemented in Dynare using the mode compute option 6.

Table 4.3: Estimation results for structural parameters

Model	CEE–Oil model	CMR–Oil model
elasticity of subsitition between energy-capital composite good and labour	1.56	1.38
η^M	[1.29, 1.87]	[1.12, 1.70]
curvature of investment adjustment cost	5.58	6.88
$\mathcal{S}''$	[3.89, 7.75]	[5.13, 8.79]
curvature of utilization cost	1.12	1.12
$\sigma^{a(u)}$	[0.97, 1.27]	[0.97, 1.27]
weight on output growth in Taylor rule	0.37	0.38
$\tilde{a}_{\Delta y}$	[0.31, 0.44]	[0.31, 0.45]
weight on inflation in Taylor rule	-	-
$\tilde{a}_{\pi}$	[-]	[-]
Calvo parameter wages	0.32	0.33
ξ^w	[0.28, 0.36]	[0.29, 0.37]
Calvo parameter prices	0.44	0.43
ξ^p	[0.40, 0.47]	[0.40, 0.47]
AR(1) coefficient for risk free interest rate	0.79	0.83
$\tilde{\rho}$	[0.77, 0.81]	[0.81, 0.85]
demand price elasticity for oil consumption	0.10	0.11
η^O	[0.08, 0.14]	[0.08, 0.15]
inverse supply price elasticity for oil production	7.46	7.60
σ^O	[5.96, 9.46]	[6.04, 9.69]

Notes: The posterior mean and the 90% highest posterior density (HPD) interval for the respective parameters in parentheses are reported.

For the CMR–Oil model the posterior mean of the curvature parameter of investment adjustment cost is higher than for the CEE–Oil model. Both posterior means are part of the credibility interval of the other model and intervals overlap. The result indicates that changing investment levels will be less effective in the CMR–Oil model compared to the CEE–Oil model. Results for the curvature of capital utilization costs are very close in both models.

The Calvo parameter for wage stickiness is very low but is close to the one reported by CEE for the model without indexation. Including financial markets to the model leads to a decrease in the Calvo parameter for wage stickiness. Calvo parameters for price rigidity are slightly below the prior mean and indicate an average one-year duration of prices. The monetary policy parameter for output is very similar between both models. However, the monetary policy instrument is more rigid in the CMR–Oil model compared to the CEE–Oil model.

Estimation results for persistence parameters are reported in Table 4.11 in the Appendix. In the CMR–Oil model the persistence parameter for investment adjustment costs is smaller compared to the CEE–Oil model. It implies that investment adjustment costs are less persistent in a model with a financial accelerator. All other persistence parameters present in both models are very similar. Table 4.12 in the Appendix reports Estimation results for standard deviations of shocks. Here the standard deviations for investment adjustment costs and price mark-ups are different across the two models. The estimated anticipated signal correlation is weak (0.08) at the posterior mean. In 90% of the draws at the posterior mode, it does not exceed a moderate (0.32) magnitude.

The comparison of structural parameters reveals no tremendous difference between both models. Therefore, results for the variance and historical decomposition are mainly driven by including the financial accelerator.

4.4.2 Historical and variance decomposition

Table 4.4 reports the theoretical variance decomposition for the national account variables for the CMR, CEE–Oil and CMR–Oil model. In contrast to the results by CMR, risk shocks only explain one fifth instead of more than half of the theoretical variance of GDP growth. The main reason for this reduction is the monetary policy rule. Christiano et al. (2014) use a log-linearised version of (4.9). However, the monetary policy rule in their replication code is not the log-linearised version of (4.9).[8] This misspecification is the main reason for the divergence between results in Table 4.4 and Table 5 in Christiano et al. (2014).

Results of the variance decomposition using the estimated parameters by CMR show that risk shocks contribute about 21% in total to GDP growth. The contribution of risk is between 1.7% and 5.8%. Therefore, risk shocks are only a minor driver of GDP growth rates. In addition to the Taylor rule persistence parameters for consumption, inflation and wages are responsible for the drop. A lower persistence of prices and wages affect the contribution of risk to GDP growth. Less persistent habits lead to a lower contribution of risk to consumption behaviour.

Results at the posterior distribution of both model variants state that technology shocks, especially to the long-run growth rate, explain 38% to 59% of the theoretical variance of GDP growth. The introduction of financial frictions to the baseline model with oil leads to an increase in the contribution of monetary policy shocks to the theoretical variance of GDP growth. The second most important category are demand shocks. They explain 14% to 22% of the variance in GDP growth. Risk shocks and the marginal efficiency of investment are the main drivers of the growth rate in capital formation. The external finance premium, credit and equity growth rates are mainly driven by risk shocks as reported by Table 4.15 in the Appendix.

A first result is that the inclusion of financial frictions slightly reduces the theoretical variance contribution of oil market variables to GDP growth at the posterior mean. However, the credibility interval of the CEE–Oil model includes the posterior mean for the CMR–Oil model. The inclusion of a financial accelerator to the model does not affect the contribution of oil to the variance of GDP growth. One main reason for the observed reduction is a lower contribution of oil market variables to investment growth. It is noteworthy that shocks from the oil market have a lower contribution to the variance of investment, consumption and wage growth rates compared to their respective levels. It is valid for both models. Oil market disturbances explain only a small fraction of the theoretical variance of the federal funds rate and inflation.

As stated in Bernanke et al. (1999), the financial accelerator mechanism can amplify small shocks such as discretionary monetary policy. The theoretical variance decomposition

[8] The files are available under `https://www.aeaweb.org/articles?id=10.1257/aer.104.1.27`.

shows that unexpected movements in the federal funds rate contribute between 11% and 16% to the theoretical variance of GDP growth for the model with a financial accelerator. The contribution ranges between 8.5% and 12% for the CEE–Oil model. Nevertheless, the results can not verify the statement that the financial accelerator mechanism amplifies oil market shocks. In contrast, for the reported aggregates oil market shocks contribute less to GDP growth, consumption and investment with a financial accelerator. Here the main reason is, that risk shocks explain more of the variance in investment and reduce the contribution previously attributed to the oil market shocks.

Table 4.16 in the Appendix tabulates theoretical variance decomposition for the oil market variables. Domestic and foreign oil supply shocks do not affect each other. The contribution of domestic oil demand shocks is higher for home oil supply than for foreign production. Domestic and foreign oil supply shocks equally drive crude oil prices. Further, technology innovations and unexpected changes in domestic oil demand contribute with similar shares to the theoretical variance of oil prices. Including the financial accelerator into the model, shows that risk and investment shocks become as crucial as other technology innovations explaining the variance of oil prices. Otherwise, including the financial accelerator does not qualitatively alter the variance decomposition and also only slightly in a quantitative way.

Risk and the marginal efficiency of investment shocks mainly drive investment according to the variance decomposition. Figure 4.2 depicts the historical contribution of the marginal efficiency of investment (m.e.i.) and risk shocks to GDP growth. The inclusion of financial frictions reduces the contribution of the marginal efficiency of investment, especially during the Great Recession (through investment growth). The historical decomposition also reveals that risk shocks are the main driver of the external finance premium and credit growth. Further, the external finance premium reached its maximum observed value during the financial crisis, and this coincides with the time risk contributed the most to GDP and investment growth. The marginal efficiency of investment on the other side has only a small impact on the external finance premium and credit growth.

Figure 4.11 in the Appendix depicts oil market disturbances and their contribution to the business cycle. One can see here that the contribution of oil market disturbances to the oil price is almost identical in both models. The same result holds for GDP, investment and consumption growth. Therefore, the financial accelerator framework does not amplify the role of oil market disturbances on the US business cycle for the period 1984-Q2 to 2018-Q3. During the financial crisis, a tremendous oil price drop occurred. According to the historical decomposition at the posterior mean, the results show that the change in oil price was mainly due to oil market disturbances. It is clear that at the time, mostly lower oil demand driven by lower global economic activity (see, e.g. Ratti & Vespignani 2013) caused the fall in oil prices. A more detailed historical decomposition reveals that oil domestic productivity shocks (oil demand shocks) contributed the same share to the decline in oil prices as supply shocks. One potential explanation for the contribution of oil supply shocks is the closure of less profitable drilling wells, which implies that the remaining drilling wells are less expensive to operate.[9]

[9] The EIA publishes the number of US Crude Oil and Natural Gas Rotary Rigs in Operation under `https://www.eia.gov/dnav/ng/hist/e_ertrr0_xr0_nus_cM.htm`.

Table 4.4: Variance decomposition for national account variables at the posterior distribution

Variable	risk	investment	demand	financial	M.P.	markup	technol.	oil
				GDP growth				
CMR	21.1	4.3	39.8	0.2	1.4	19.2	14.0	0.0
CEE–Oil	0.0	11.4	17.0	0.0	10.3	8.6	51.7	0.8
	[0.0, 0.0]	[8.9, 13.6]	[13.9, 19.9]	[0.0, 0.0]	[8.5, 12.1]	[6.9, 10.3]	[44.8, 58.6]	[0.5, 1.1]
CMR–Oil	3.9	10.1	18.8	2.1	13.4	7.0	43.8	0.7
	[1.7, 5.8]	[7.6, 12.3]	[15.5, 21.9]	[0.6, 3.5]	[11.1, 16.0]	[5.6, 8.3]	[37.8, 49.9]	[0.5, 1.0]
				inflation				
CMR	51.2	20.0	3.0	0.4	1.9	13.9	9.6	0.0
CEE–Oil	0.0	18.9	5.2	0.0	8.5	14.3	51.4	1.2
	[0.0, 0.0]	[14.5, 23.4]	[4.2, 6.2]	[0.0, 0.0]	[6.4, 10.8]	[10.3, 17.7]	[41.7, 59.8]	[0.8, 1.6]
CMR–Oil	11.1	10.4	5.5	5.5	12.5	11.3	42.6	1.0
	[4.7, 17.1]	[7.8, 13.3]	[4.4, 6.6]	[1.6, 9.7]	[9.8, 15.4]	[8.0, 14.6]	[34.3, 50.7]	[0.6, 1.4]
				federal funds rate				
CMR	66.9	23.0	3.5	0.5	1.7	2.4	2.0	0.0
CEE–Oil	0.0	37.0	16.1	0.0	15.8	6.5	22.7	1.1
	[0.0, 0.0]	[30.2, 43.9]	[12.8, 19.6]	[0.0, 0.0]	[12.9, 18.7]	[4.1, 8.5]	[17.4, 27.7]	[0.6, 1.7]
CMR–Oil	23.4	13.1	15.1	14.1	12.4	4.4	17.0	0.3
	[10.9, 35.2]	[8.8, 17.6]	[11.6, 18.8]	[4.3, 23.9]	[9.4, 15.4]	[2.8, 6.1]	[12.6, 21.6]	[0.2, 0.5]
				investment growth				
CMR	68.1	21.8	1.0	0.7	0.7	6.5	1.2	0.0
CEE–Oil	0.0	75.1	0.4	0.0	0.3	6.4	15.8	1.1
	[0.0, 0.0]	[66.7, 82.2]	[0.2, 0.6]	[0.0, 0.0]	[0.1, 0.5]	[3.8, 8.6]	[10.9, 20.6]	[0.5, 1.7]
CMR–Oil	26.6	48.8	0.1	15.0	1.4	3.1	4.3	0.5
	[12.9, 40.3]	[37.8, 60.6]	[0.0, 0.1]	[5.3, 24.5]	[0.8, 1.8]	[2.2, 4.0]	[3.0, 5.6]	[0.3, 0.7]
				investment				
CMR	62.6	24.5	1.2	0.9	0.5	8.5	1.9	0.0
CEE–Oil	0.0	40.2	2.8	0.0	0.1	11.1	29.7	11.0
	[0.0, 0.0]	[30.0, 51.8]	[1.3, 4.1]	[0.0, 0.0]	[0.0, 0.1]	[6.1, 15.8]	[21.8, 38.5]	[2.3, 19.8]
CMR–Oil	29.1	14.6	0.1	33.9	1.7	5.5	8.8	3.7
	[12.0, 44.1]	[7.4, 20.4]	[0.0, 0.2]	[15.1, 52.8]	[1.0, 2.4]	[2.9, 8.5]	[5.1, 13.1]	[0.6, 7.0]
				consumption growth				
CMR	44.5	21.4	24.2	0.4	0.3	5.9	3.3	0.0
CEE–Oil	0.0	5.6	28.5	0.0	17.7	6.3	41.4	0.4
	[0.0, 0.0]	[4.2, 7.0]	[23.9, 32.5]	[0.0, 0.0]	[15.0, 21.0]	[4.9, 7.7]	[34.9, 48.0]	[0.3, 0.6]
CMR–Oil	2.7	3.2	27.6	2.0	19.3	5.6	39.2	0.4
	[1.1, 4.2]	[2.3, 4.1]	[23.0, 31.4]	[0.5, 3.4]	[16.1, 22.8]	[4.3, 6.8]	[32.8, 45.3]	[0.3, 0.6]
				consumption				
CMR	50.5	19.7	7.7	0.6	0.3	11.4	9.8	0.0
CEE–Oil	0.0	8.3	34.7	0.0	1.6	7.7	40.1	5.8
	[0.0, 0.0]	[5.5, 11.2]	[26.2, 44.4]	[0.0, 0.0]	[1.1, 2.0]	[5.5, 9.8]	[31.4, 49.7]	[1.1, 10.9]
CMR–Oil	8.6	4.3	26.4	10.4	2.3	8.3	34.8	3.3
	[3.4, 13.7]	[2.7, 5.8]	[18.4, 33.9]	[2.7, 18.0]	[1.6, 2.9]	[5.9, 10.6]	[25.7, 42.4]	[0.7, 6.0]
				wage growth				
CMR	4.0	3.0	2.9	0.0	0.2	59.8	30.0	0.0
CEE–Oil	0.0	0.4	0.8	0.0	0.0	40.8	56.9	0.8
	[0.0, 0.0]	[0.2, 0.6]	[0.5, 1.0]	[0.0, 0.0]	[0.0, 0.0]	[36.1, 45.9]	[46.6, 66.4]	[0.5, 1.2]
CMR–Oil	0.2	0.4	1.0	0.2	0.0	41.3	55.5	1.1
	[0.1, 0.4]	[0.2, 0.5]	[0.6, 1.3]	[0.0, 0.3]	[0.0, 0.1]	[36.5, 46.6]	[45.2, 65.3]	[0.6, 1.5]
				wage				
CMR	34.7	25.0	3.0	0.9	0.4	24.9	11.0	0.0
CEE–Oil	0.0	2.0	4.3	0.0	0.0	28.7	55.0	7.2
	[0.0, 0.0]	[1.1, 2.9]	[2.9, 6.0]	[0.0, 0.0]	[0.0, 0.0]	[19.7, 37.3]	[42.8, 67.8]	[1.4, 13.5]
CMR–Oil	2.6	1.2	2.6	8.1	0.4	31.6	45.9	4.7
	[0.8, 4.4]	[0.6, 1.7]	[1.7, 3.7]	[2.1, 14.3]	[0.2, 0.5]	[22.3, 41.1]	[33.8, 58.0]	[0.8, 8.7]

Note: Theoretical contribution of each shock group in percent to the total variance of the respective variable is reported. Results for the CMR model are computed using the parameter values of Christiano et al. (2014) as tabulated in Table 4.14. The variance decomposition for the CEE–Oil and CMR–Oil model are reported for the estimated posterior distribution. Values in parentheses represent 90% HPD interval of the model parameters. The shock groups are reported in Table 4.13.

4.4.3 Impulse response functions

The variance and historical decomposition both mainly reveal a crowding-out of m.e.i. for risk shocks. Bernanke et al. (1999) state that the financial accelerator can amplify the impact of small shocks such as discretionary monetary policy. The variance decomposition reveals a little amplification effect for monetary policy shocks, but the opposite for oil market disturbances. Figure 4.3 presents impulse response functions for discretionary monetary policy shocks on different variables.[10] The monetary policy shock increases the risk-free interest rate by more than two annualized percentage points for both models. This increase leads to a rise in the external finance premium by about 0.25 annual and the bankruptcy rate by 0.5 percentage points. This increase in the probability of insolvency and the external finance premium triggers an additional reduction in investment. With the financial accelerator, investment drops about four times more compared to the model without financial accelerator. Further, the 90% HPD interval is not overlapping and suggests that this difference is unlikely a random observation. Additionally, one can see that oil consumption also declines more persistently as a response to monetary policy shocks. However, the drop in GDP growth is only slightly more significant with financial accelerator compared to the model without financial accelerator. Consumption responds similarly to a monetary policy shock in both models. The model with financial accelerator simulates a more substantial drop in inflation for the model without financial accelerator. This greater magnitude in the decline of inflation links to a more persistent plunge in oil prices as a response to the more persistent decline in oil demand.

The financial accelerator might amplify oil supply shocks. Here, domestic and foreign oil supply shocks increase the oil price simultaneously by 50%. Figure 4.4 depicts the response to oil supply shocks for a selected number of variables. Oil consumption drops only by roughly five percent, reflecting the low price elasticity of oil demand. Investment will fall by approximately two percent with the financial accelerator mechanism and by 2.5% without credit market frictions. The resulting drop in GDP is indistinguishable for the two model variants. Inflation will increase by the same amount with and without financial accelerator. According to the monetary policy rule, the risk-free interest rate increases to reduce observed inflation. Monetary policy amplifies the drop in GDP. The external finance premium and bankruptcy probability both increase, but the decline in investment are lower with compared to the model without financial accelerator. Raw capital prices fall less in the model with financial accelerator compared to the model without financial accelerator. Household investments react less in the CMR–Oil model compared to the CEE–Oil model. A lower drop in raw capital prices is at odds with the previous findings for the monetary policy shock. For the monetary policy shock, an increase in the risk-free interest rate triggered a rise in the external finance premium this lead to a further decrease in the raw capital price. However, for the oil price shock, the external finance premium also increases. Nevertheless, the increase in the external finance premium is not sufficient to reduce the raw capital price more compared to the model without financial accelerator. The financial accelerator

[10] I compute impulse response functions for the non-linear version of the model using deterministic simulations.

mechanism also introduces rigidity for the raw capital price through the law of motion for net worth, the zero-profit condition of the mutual funds and the first-order condition of the entrepreneurs. Financial frictions have not the expected amplification effect for the oil supply shocks. In contrast, the reported impulse response functions suggest that the financial accelerator stabilizes investment compared to a model without credit market frictions.

Impulse response functions for permanent oil supply shock leads to a permanent increase in the price of oil and will permanently decrease oil consumption. The rise in oil prices triggers temporary initial higher inflation, an increase in GDP growth and an increase in the risk-free interest rate. Long-run stationary investment and consumption will decline, but consumption will initially increase. Here, the initial increase in consumption reflects less incentive to invest in the future capital stock, which is less productive. Therefore, households consume more disposable income. The bankruptcy probability and the external finance premium initially increase and permanently fall. This initial increase does not lead to a sharper drop in raw capital prices. In contrast, the raw capital price is more rigid and will not decline as much as without financial accelerator. Therefore, investment declines with a lower pace compared to the model without financial accelerator.

4.4.4 Mitigation and monetary policy

The historical decomposition did not attribute recessions to oil market disturbances. However, a future reduction in oil consumption to comply with the Paris Agreement might change this. It is possible to increase the tax on oil paid by suppliers τ^o to reduce oil consumption. Here, the increase in the oil tax rate ensures that oil consumption permanently falls by 10%. Also, the impact of mitigation policy on inflation requires discretionary monetary policy to mute the effect on inflation. Therefore, computed monetary policy shocks ensure that inflation does not deviate from its target value by more than 0.01 annual percentage points. Figure 4.6 reports the trajectories for oil tax rate, risk free interest rate, and inflation.[11] Oil tax rate needs to increase by more than 50 percentage points to reduce oil consumption permanently by 10%. As a result, the oil price will almost double. This increase in the oil price will then trigger inflation without any intervention by monetary policy authorities. Inflation will increase by more than 0.1 annual percentage points five quarters after the oil tax rate increase. After ten quarters inflation will be at least 0.05 percentage points lower compared to its initial value. The risk-free interest rate slightly increases, but after five quarters, it falls. The initial reaction of consumption is positive for the model without financial accelerator and negative for the model with a financial accelerator. Investment drops, but for the model, without the financial accelerator, the initial investment drop exceeds the one for the model with a financial accelerator.

In case monetary policy wants to stabilize inflation, it needs to discretionary deviate from its monetary policy rule. The interaction panel of Figure 4.6 shows the required response to the risk-free interest rate to stabilize inflation. The risk-free interest rate needs to increase by at least 0.15 annual percentage points for the first seven quarters to mute the impact of

[11] The response of oil market variables are depicted in Figure 4.12 in the Appendix.

the oil price increase. The required growth is by 0.1 yearly percentage points greater for the model with financial accelerator compared to the model without financial accelerator. This increase in the risk-free interest rate can stabilize inflation, but also leads to a faster decline in investment and consumption (see Figure 4.7). For the CEE–Oil model investment declines more severely compared to the CMR–Oil model.

4.5 Discussion

The comparison of the CEE–Oil and CMR–Oil model shows that the estimated structural parameters are very similar and do not reveal any credible difference at the posterior mean. Therefore differences between the models in explaining the macroeconomic variables are mainly caused by the financial accelerator mechanism. The variance decomposition reveals that risk shocks are the primary driver of investment, but not of consumption. The historical decomposition shows that risk explained most of the drop in real variables during the Great Recession. However, risk shocks are not the main driver of the business cycle in the US. Oil market shocks also contribute only with roughly one percent to GDP or inflation. Technology and demand shocks contribute the most to GDP growth according to the variance decomposition. More specifically, shocks to the long-run growth rate mainly drive GDP. For the sample period, 1984-Q2 to 2018-Q4 oil market shocks only played a minor role in the business cycle in the US.

The impulse responses reveal that monetary policy has more severe implications for investment in a model with financial accelerator compared to a model without financial accelerator. Monetary policy needs to monitor financial market imperfections to ensure that the selected policy instruments are adequate for the respective purpose. However, with the financial accelerator mechanism investment reacts less to oil supply shocks. Further, inflation is less volatile for the model with a financial accelerator. The degree of imperfections in the credit market determines the response of inflation and investment to oil supply shocks. The monetary policy response to oil market disturbances depends on the financial frictions.

Oil market variables have not been a major driver of the business cycle in the US, but this might change with ambitious future mitigation policies. It is in line with previous studies (see Mercure et al. 2018). More precisely, a reduction of oil consumption by 10%, in the long run, can lead to a decline in consumption by 0.6 to 1.6%. Further, the models predict a permanent reduction in investment by 3 to 7%. Financial market imperfection reduces the immediate response of investment to an increase in oil taxes. Inflation will be above the target rate for about six quarters. Afterwards, inflation will be below the target rate for the same number of quarters. The monetary authority can stabilize inflation, but the risk-free interest rate needs to increase substantially above the rate determined by the monetary policy rule. This increase in the risk-free interest rate will reduce consumption. Nevertheless, the reduction in investment is almost identical to the path without interaction.

The costs of deviating from the monetary policy rule are not only captured by a further decline in consumption or investment. There are also costs not directly measurable with a DSGE model. As stated in Fischer (1990), a discretionary monetary policy might lead to a loss in confidence and further to an increase in political pressure. Therefore, it seems not recommendable to mute the rise in inflation by deviating from the monetary policy rule. The reduction in consumption will increase political pressure to stick with the monetary policy rule.

The present study considers the interaction of financial markets and oil markets in a model for the US economy. Future research should reconsider some of the underlying assumptions of the model. First, the model considers oil as production factor without a differentiation of the usage of oil in the economy. Balke & Brown (2018) differentiates between oil for transportation and consumption. Mitigation policies will target the oil used in the transportation sector. Therefore, a more elaborate model will explicitly include alternatives to oil as an input to the transportation sector. Oil as a raw material in the chemical industry is still not easy to replace by other raw materials. Mitigation policies will target the reduction of oil as an energy source mainly applied in the transportation sector.

Another issue is that oil supply is not finite in the model, and the discovery of new reserves is costless. Hansen & Gross (2018) includes limited natural resources and introduces exploration activities to increase the reserves of natural resources for a small open economy. It seems worthwhile to extend the model to include such features. Nevertheless, this extension requires additional data to estimate the model. Identical extraction costs for domestic and foreign oil producers is a testable assumption.

4.6 Conclusion

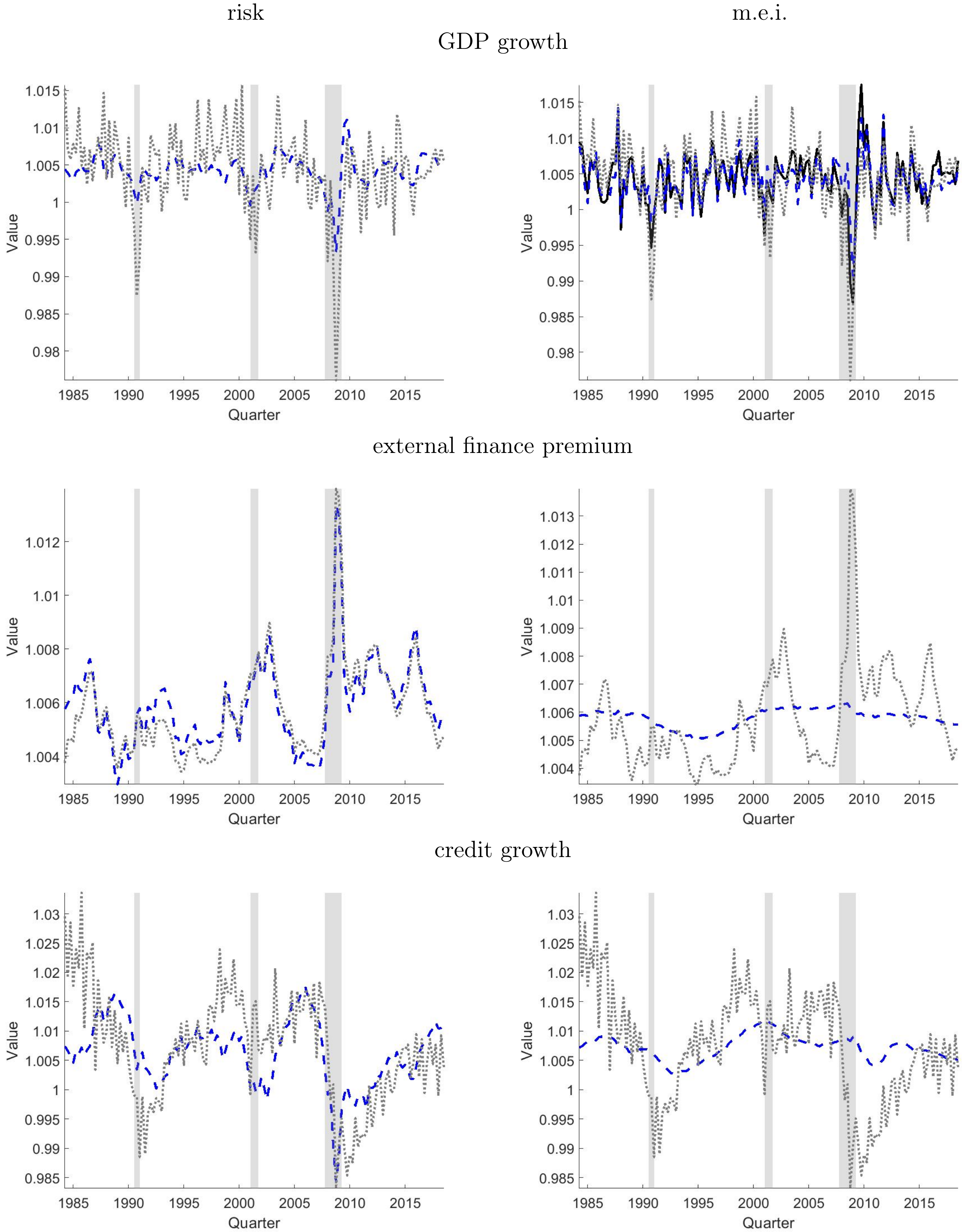
Is risk the fuel of the business cycle? The present study shows that disturbances from the credit market are not the main driver of the business cycle in the US. Nevertheless, they explain about one-fifth of the variance in GDP growth. Further, they are essential to explain investment behaviour. During the Great Recession, risk shocks have been the leading cause of a drop in investment and GDP.

Oil market shocks have not been a significant driver of the business cycle in the US between 1984-Q2 to 2018-Q4. These findings are based on the historical and theoretical variance decomposition of the US economy using a DSGE model with financial frictions and oil as a production factor. The impulse response functions at the posterior mean show that the financial accelerator amplifies the effect of monetary policy shocks on investment, but not for oil supply disturbances. In contrast to the statement in Bernanke et al. (1999), the response in investment to oil price shocks is not amplified compared to a model without financial accelerator.

In the future mitigation measures to reduce oil consumption can cause a recession. An increase in the oil tax rate by roughly 50 percentage points will decrease oil consumption permanently by 10%. This increase in the oil tax rate triggers higher oil prices by 50 to 90%. Inflation increases by 0.1 to 0.2 annual percentage points. Consumption permanently drops by 0.5 to 1.7% and investment by 3 to 7%. Monetary policy can stabilize inflation, but its reaction depends on financial market imperfections. The risk-free interest rate has to increase by 0.15 to 0.3 annual percentage points and without financial frictions by 0.25 to 0.45 annual percentage points, to mute the initial increase in inflation completely.

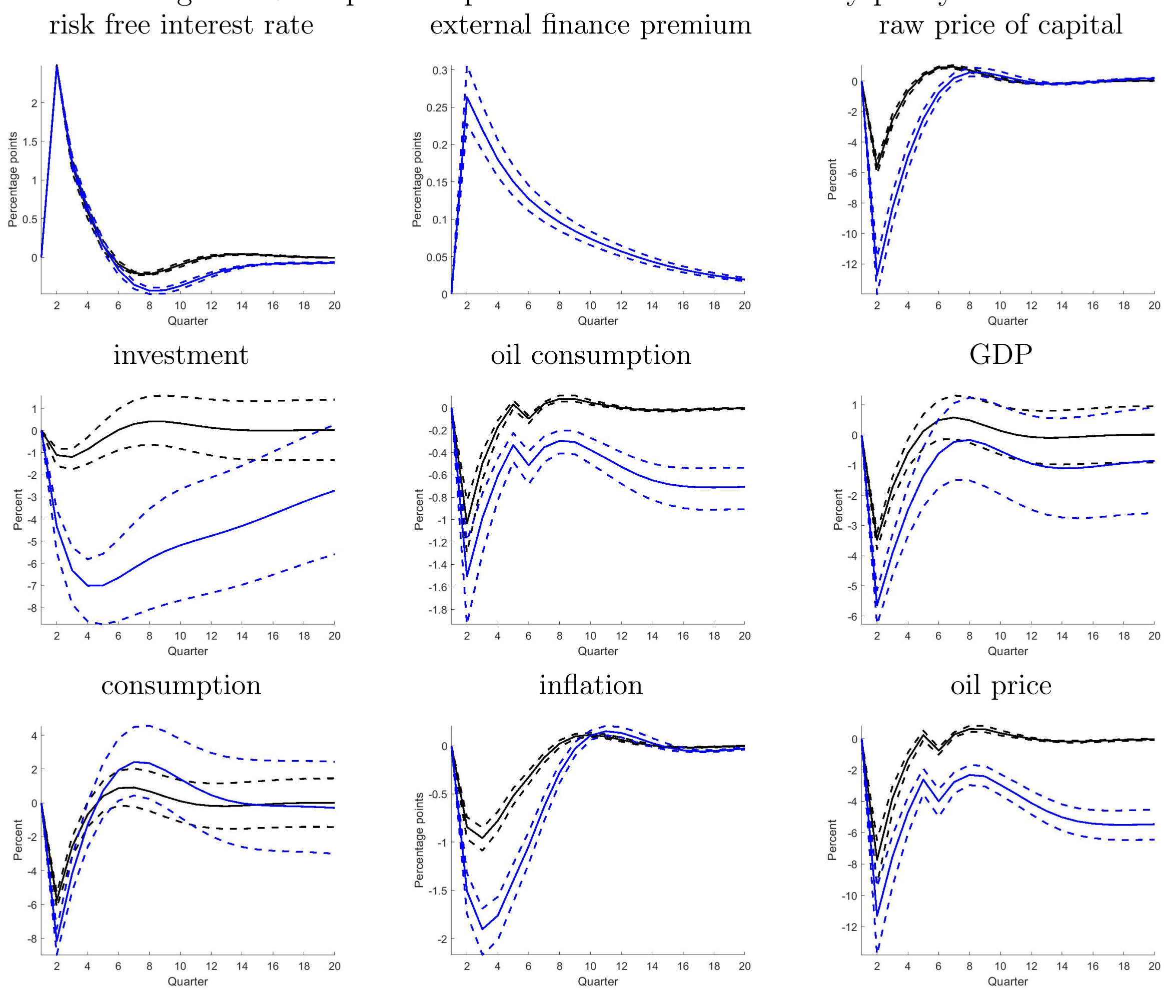
The developed model can study the interaction between financial and oil markets. Further, the model can analyse the impact of mitigation measures on the US economy. However, the discussion tackled some potential avenues for future modifications of the model. The model results are based on estimated parameters and the underlying estimation uncertainty and resemble the main contribution of the model to mitigation policy discussions.

Figure 4.2: Historical contribution of risk and m.e.i. shocks



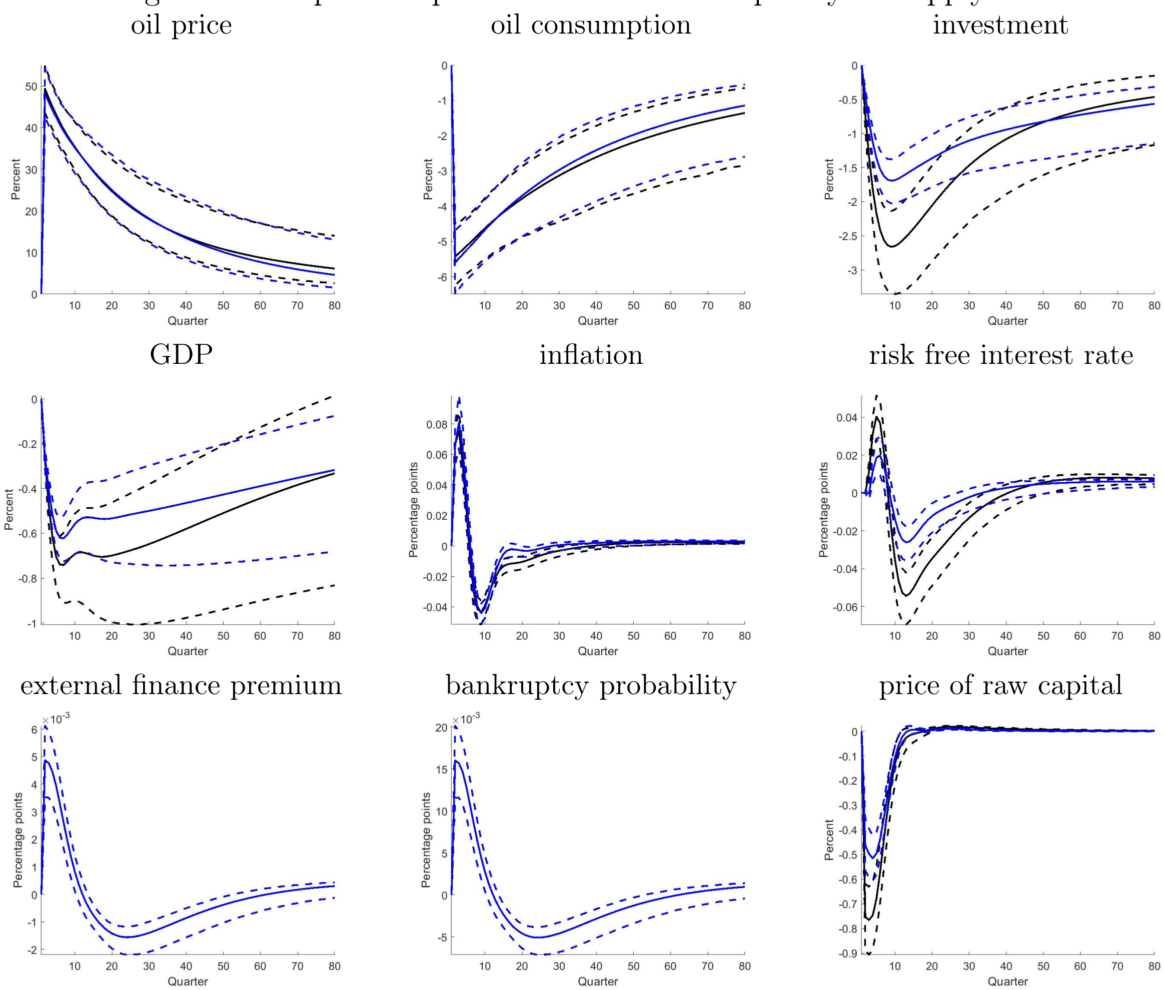
Note: The solid black line represents the historical decomposition for the CEE–Oil model, the shaded blue line for the CMR–Oil model, and the dotted gray line the observed data. Shaded areas represent National Bureau of Economic Research recessions as reported on `https://www.nber.org/cycles.html`.

Figure 4.3: Impulse response functions for monetary policy shocks



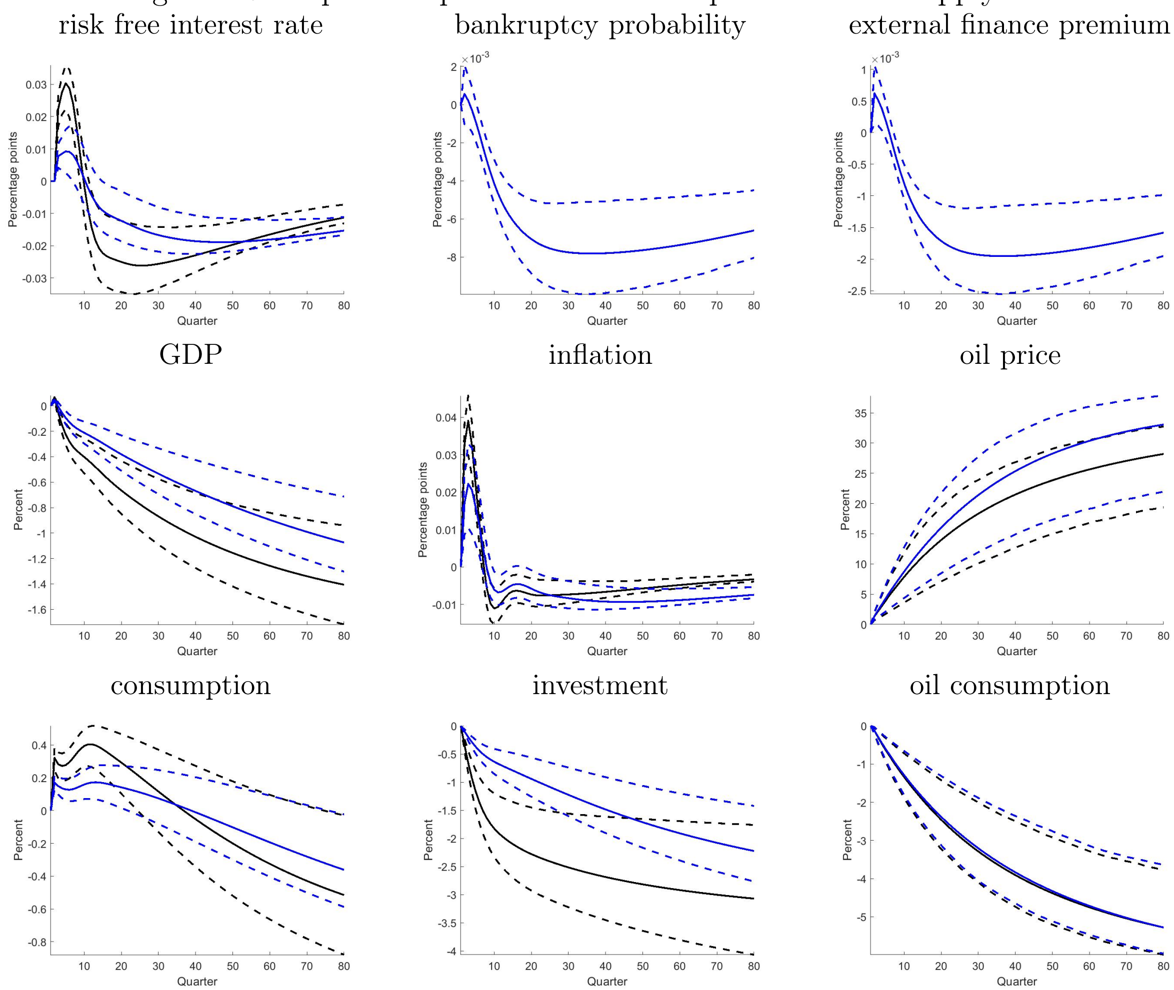
Note: Variables are expressed as percentage deviation from the sample mean/steady-state. The solid black line represents the impulse response function at the posterior mean for the CEE–Oil model and the solid blue line for the CMR–Oil model. Dashed lines represent the 90% HPD interval based on 1200 draws from the posterior distribution.

Figure 4.4: Impulse response functions for temporary oil supply shocks



Note: Variables are expressed as percentage deviation from the sample mean/steady-state. The solid black line represents the impulse response function at the posterior mean for the CEE–Oil model and the solid blue line for the CMR–Oil model. Dashed lines represent the 90% HPD interval based on 1200 draws from the posterior distribution.

Figure 4.5: Impulse response functions for permanent oil supply shocks

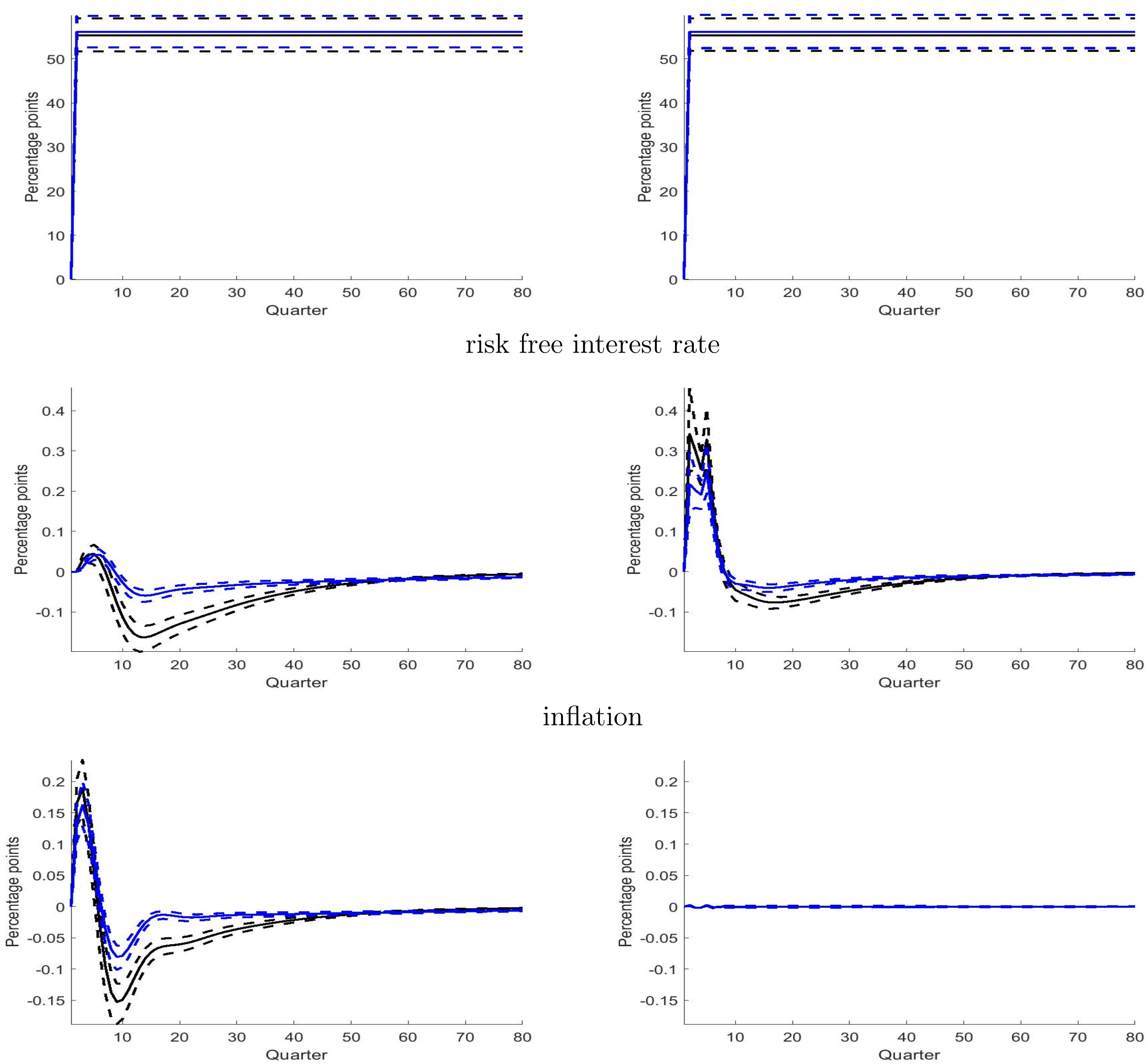


Note: Variables are expressed as percentage deviation from the sample mean/steady-state. The solid black line represents the impulse response function at the posterior mean for the CEE–Oil model and the solid blue line for the CMR–Oil model. Dashed lines represent the 90% HPD interval based on 1200 draws from the posterior distribution.

Figure 4.6: Trajectories for shocks to oil taxes: policy instruments and inflation

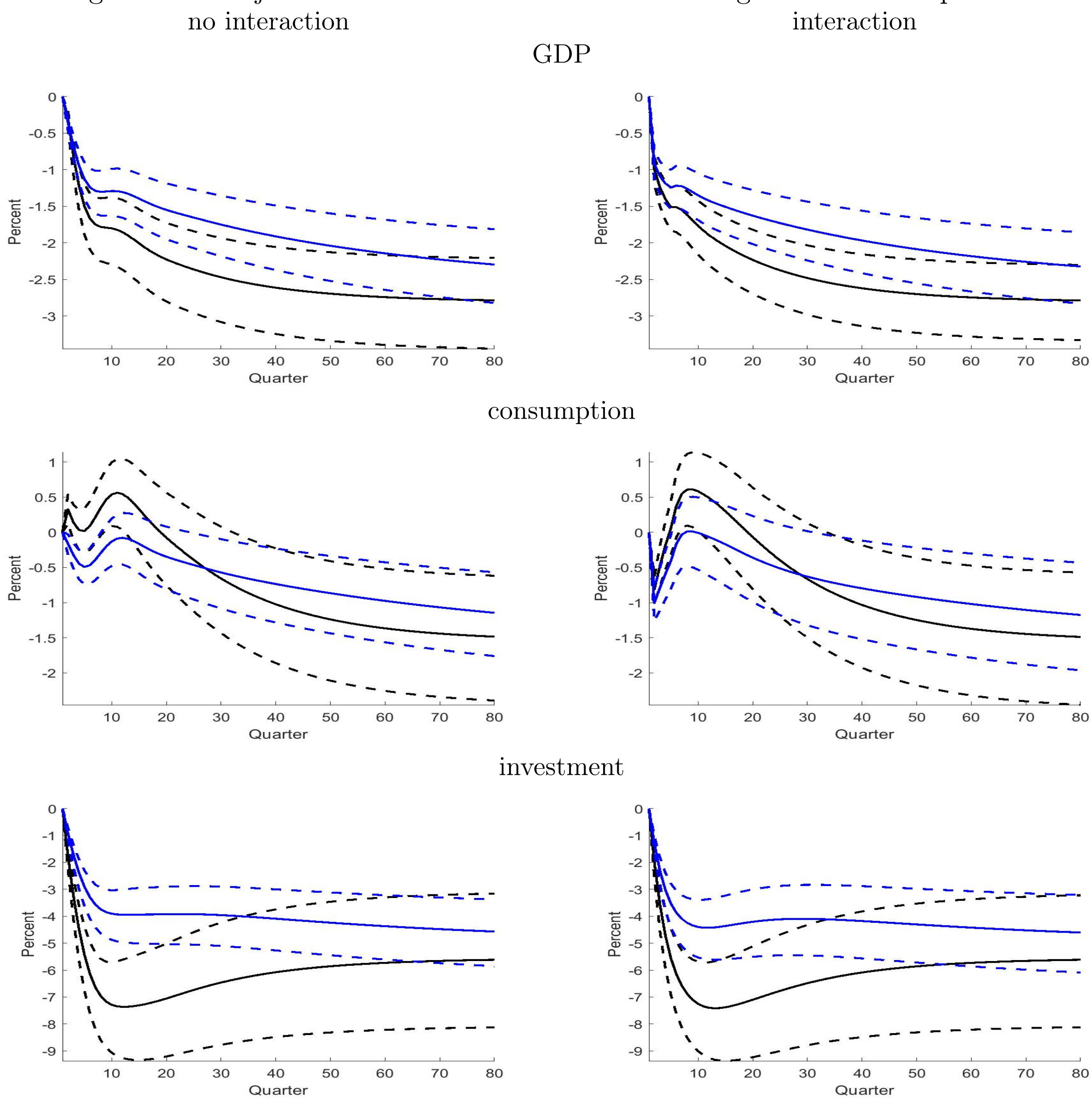
no interaction interaction

oil tax



Note: Variables are expressed as percentage deviation from the sample mean/steady-state. The solid black line represents the impulse response function at the posterior mean for the CEE–Oil model and the solid blue line for the CMR–Oil model. Dashed lines represent the 90% HPD interval based on 1200 draws from the posterior distribution.

Figure 4.7: Trajectories for shocks to oil taxes: GDP growth and components



Note: Variables are expressed as percentage deviation from the sample mean/steady-state. The solid black line represents the impulse response function at the posterior mean for the CEE–Oil model and the solid blue line for the CMR–Oil model. Dashed lines represent the 90% HPD interval based on 1200 draws from the posterior distribution.

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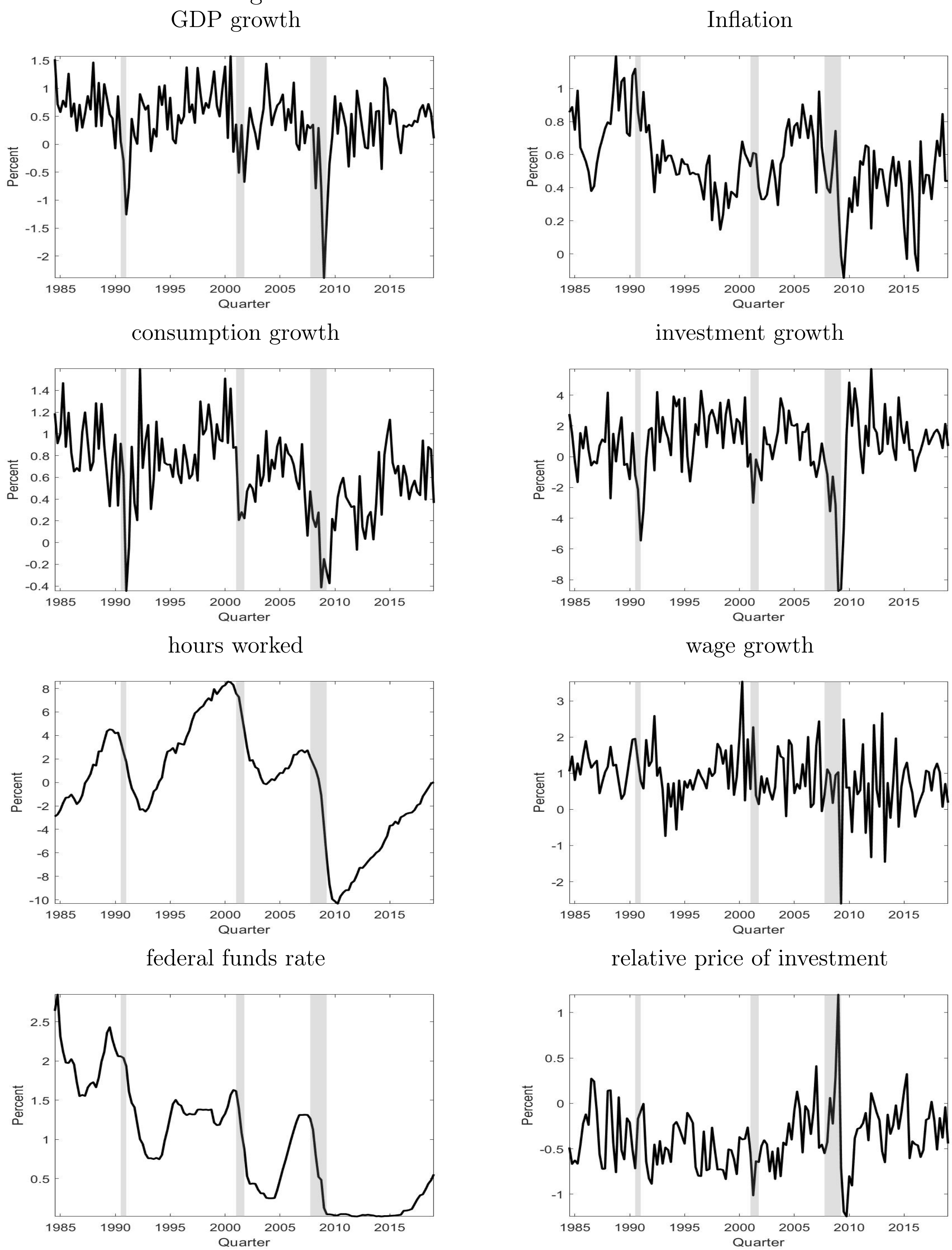
4.7 Appendix

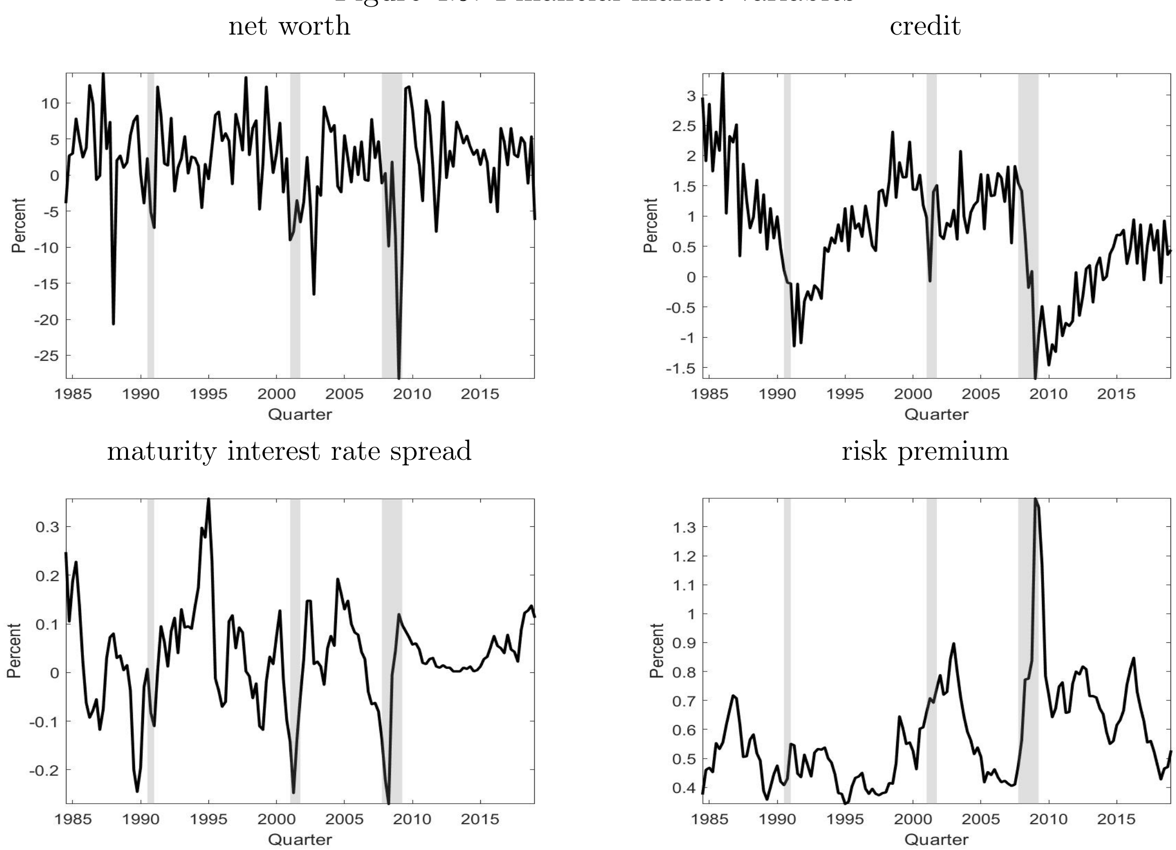
4.7.1 Figures

Figure 4.8: Standard macroeconomic variables



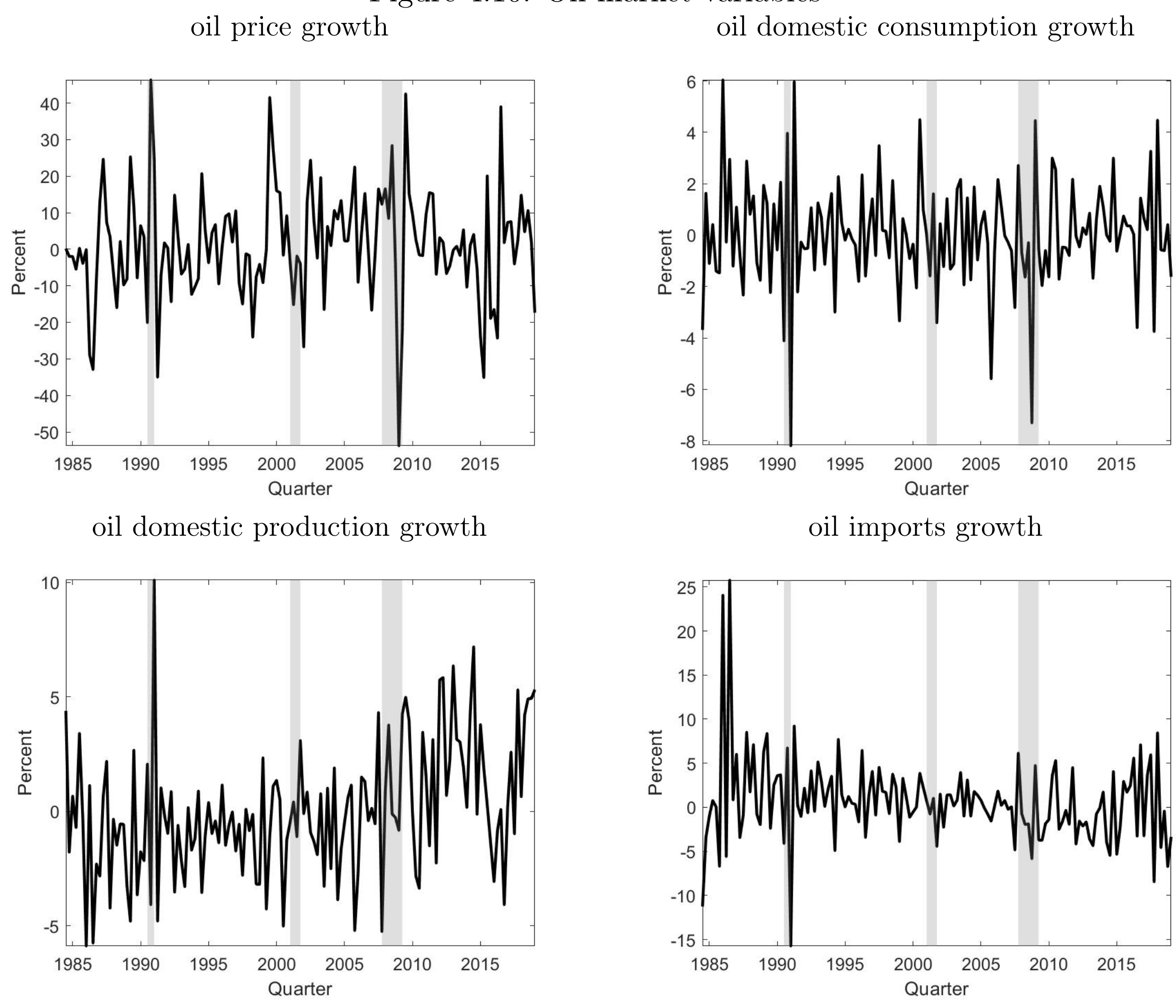
Notes: Shaded areas represent National Bureau of Economic Research recessions as reported on `https://www.nber.org/cycles.html`.
Sources: Own computation, Federal Reserve Bank of St. Louis.

Figure 4.9: Financial market variables



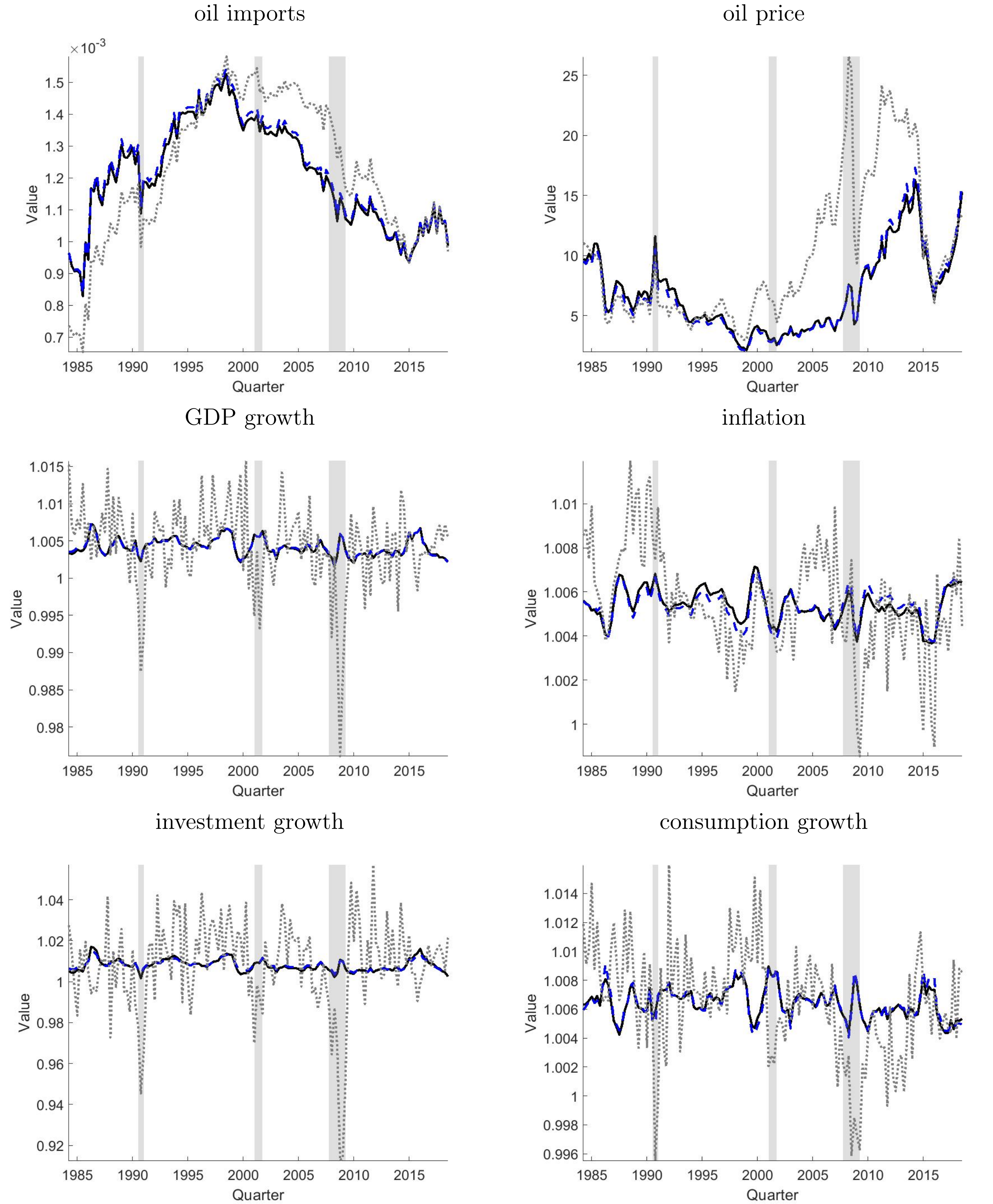
Notes: Shaded areas represent National Bureau of Economic Research recessions as reported on `https://www.nber.org/cycles.html`.
Sources: Own computation, Federal Reserve Bank of St. Louis.

Figure 4.10: Oil market variables



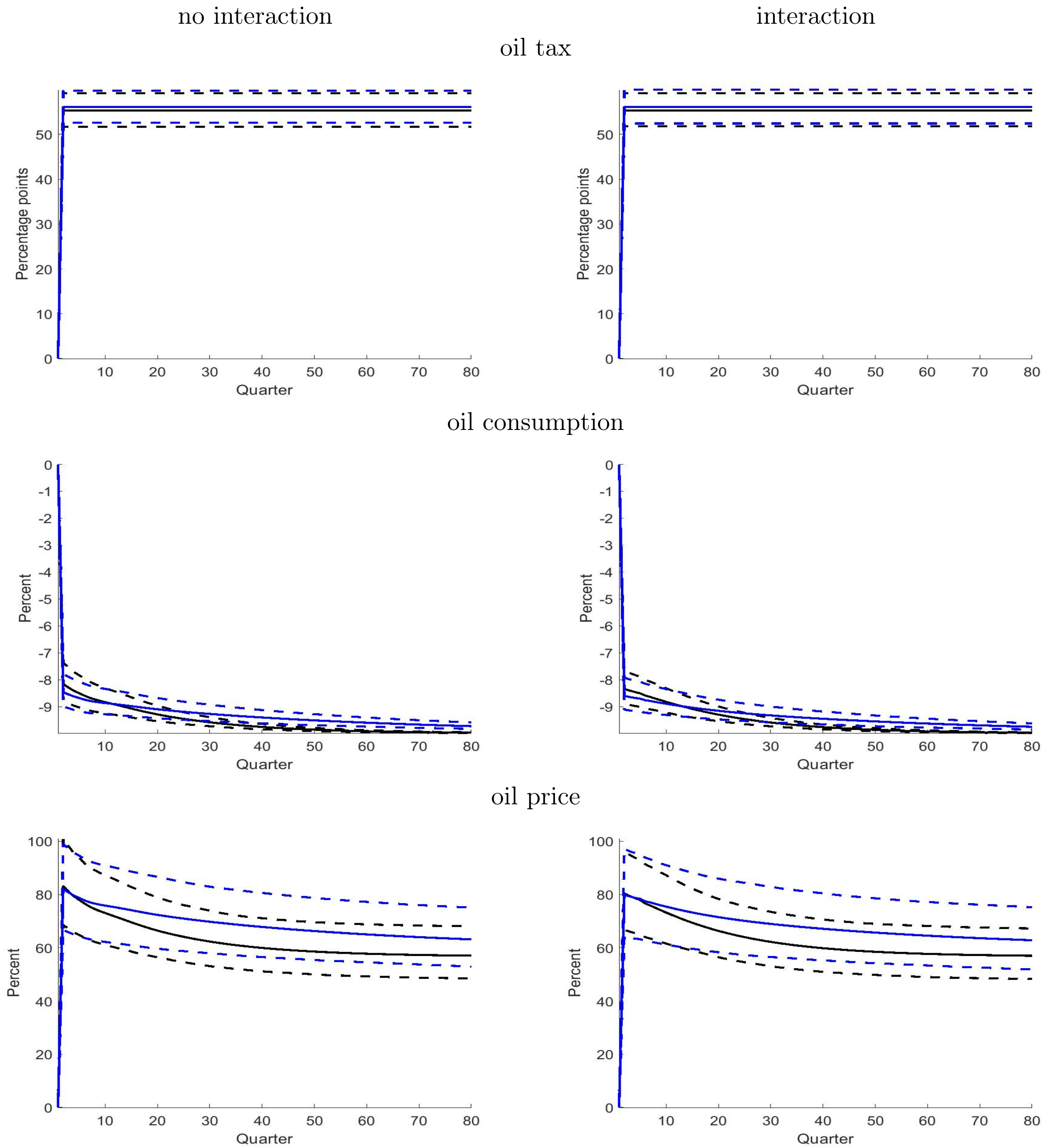
Notes: Shaded areas represent National Bureau of Economic Research recessions as reported on `https://www.nber.org/cycles.html`.
Sources: Own computation, US Energy Information Administration.

Figure 4.11: Historical contribution of oil market shocks



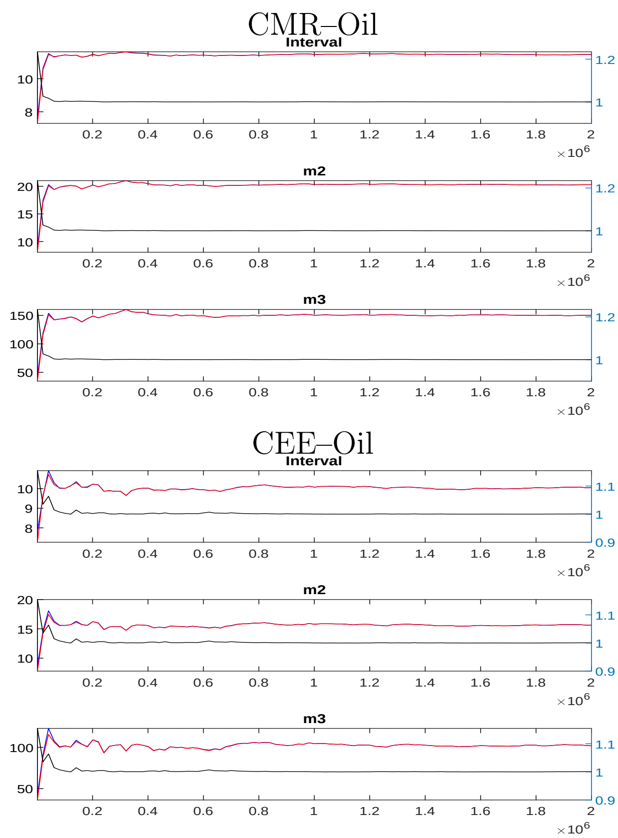
Notes: The solid black line represents the historical decomposition for the CEE–Oil model, the shaded blue line for the CMR–Oil model, and the dotted gray line the observed data. Shaded areas represent National Bureau of Economic Research recessions as reported on `https://www.nber.org/cycles.html`.
Sources: Own computation, Federal Reserve Bank of St. Louis, US Energy Information Administration.

Figure 4.12: Trajectories for shocks to oil taxes



Note: Variables are expressed as percentage deviation from the sample mean/steady-state. The solid black line represents the impulse response function at the posterior mean for the CEE-Oil model and the solid blue line for the CMR-Oil model. Dashed lines represent the 90% HPD interval based on 1200 draws from the posterior distribution. Sources: Own computation, Federal Reserve Bank of St. Louis, US Energy Information Administration.

Figure 4.13: Multivariate parameter convergence



Notes: The first row shows Brooks & Gelman (1998) convergence diagnostics for the 80% interval. The blue line depicts the pooled draws from all sequences, while the red line shows the mean interval range based on the draws of the individual sequences. Second and third central moments of the same statistic are depicted in row 2 and row 3. The grey line represents the ratio between the blue and red line depicted on the right y-axis. Here the statistics are based on the log-likelihood function.

4.7.2 Tables

Table 4.5: Endogenous variables

Variable		**Description**
stationary	**non-stationary**	
	z_t	long-run unit root technology shock
ϵ^k		temporary productivity shock composite good
ϵ^o		temporary productivity shock to oil usages
p^m	P^m	price of composite good
m	M	composite good
o	O	oil consumption
o^d	O^d	oil domestic production
o^{ex}	O^{ex}	oil exports
o^{im}	O^{im}	oil imports
p^o	P^O	oil price
τ^o		oil tax rate
ζ^o		domestic oil productivity shock
$\zeta^{o,im}$		domestic oil imports shock
$\zeta^{o,ex}$		domestic oil exports shock
τ^o		oil tax
o^{obs}		observational variable for oil consumption growth rate
$p^{o,obs}$		observational variable for relative price of oil growth rate
$o^{d,obs}$		observational variable for domestic oil production growth rate
$o^{im,obs}$		observational variable for oil imports growth rate
$o^{ex,obs}$		observational variable for oil exports growth rate
R^L		long-run interest rate
R^k		return on capital
n	N	net worth
$\bar{\omega}$		threshold for idiosyncratic risk
σ		risk
γ		fraction of entrepreneurs not leaving the market
$F(\bar{\omega})$		risk of bankruptcy
$F(\bar{\omega})$		expected value of $\bar{\omega}$
$dcost(\bar{\omega})$		monitoring cost
ξ^1		news to risk 1 periods ahead
ξ^2		news to risk 2 periods ahead
ξ^3		news to risk 3 periods ahead
ξ^4		news to risk 4 periods ahead
ξ^5		news to risk 5 periods ahead
ξ^6		news to risk 6 periods ahead
ξ^7		news to risk 7 periods ahead

Table 4.5 – Continued

Variable		**Description**
stationary	**non-stationary**	
ξ^8		news to risk 8 periods ahead
ζ_{term}		term structure
b^{obs}		observational variable for credit
$R^k - R^{L^{obs}}$		observational variable for relative price of risk premium
$S^{1,obs}$		observational variable for spread
n^{obs}		observational variable for net worth
c	C	consumption
g	G	government expenditure
i	I	investment
q	Q	price of raw capital
λ^z		marginal utility of consumption
y^z	Y	net output
ϕ		fix costs
h		hours worked
$\bar{k}$	$\bar{K}$	raw capital
u		utilization rate of raw capital
r^k	$\tilde{r}^k$	rental rate of capital
w	W	wage
s	S	real marginal cost
μ^z		long-run technology growth rate
μ^Υ		long-run investment growth rate
R		risk free interest rate
F^p		auxiliary variable for optimal price
K^p		auxiliary variable for optimal price
F^w		auxiliary variable for optimal wage
K^w		auxiliary variable for optimal wage
w^*		wage dispersion index
p^*		price distortion index
π		gross inflation
$\tilde{\pi}$		gross inflation of non-optimizing firms
$\tilde{\pi^w}$		gross wage inflation of non-optimizing unions
π^w		gross wage inflation
ϵ		temporary TFP shock
ϵ^h		temporary productivity shocks for hours worked
ζ^i		investment adjustment cost
ζ^c		consumption preference shock
ζ^h		labour supply preference shock
ϵ^w		wage mark up shock
ϵ^p		price mark up shock

Table 4.5 – Continued

Variable		**Description**
stationary	**non-stationary**	
y^{obs}		observational variable for GDP growth
h^{obs}		observational variable for hours worked
i^{obs}		investment observation
w^{obs}		observational variable for wages
c^{obs}		observational variable for consumption
$p^{i,obs}$		observational variable for relative price of investment
π^{obs}		inflation observation
R^{obs}		observational variable for risk free interest rate

Table 4.6: Exogenous variables

Shock	**Description**
η^{ϵ^k}	productivity shock for capital
η^{ϵ^o}	productivity shock for capitak
η^{ζ^o}	exogenous temporary oil cost shock
$\eta^{\zeta^{o,im}}$	exogenous temporary oil import shock
$\eta^{\zeta^{o,ex}}$	exogenous temporary oil export shock
η^{τ^o}	exogenous temporary oil tax shock
η^{γ}	survival rate of entrepreneurs
η^{σ}	unanticipated risk
η^{ξ^1}	news to risk 1 periods ahead
η^{ξ^2}	news to risk 2 periods ahead
η^{ξ^3}	news to risk 3 periods ahead
η^{ξ^4}	news to risk 4 periods ahead
η^{ξ^5}	news to risk 5 periods ahead
η^{ξ^6}	news to risk 6 periods ahead
η^{ξ^7}	news to risk 7 periods ahead
η^{ξ^8}	news to risk 8 periods ahead
η^{term}	term structure shock
η^{n}	measurement error net worth
η^{gamma}	survival rate of entrepreneurs
η^{x^p}	exogenous monetary policy shock
η^{ϵ^w}	exogenous temporary shock wage mark-up
η^{ϵ^p}	exogenous temporary shock price mark-up
η^{μ^Υ}	exogenous long-run investment shock
η^{μ^z}	exogenous long-run TFP shock

Table 4.6 – Continued

Shock	Description
η^{ϵ}	exogenous temporary TFP shock
η^{ϵ^h}	exogenous temporary productivity shock hours
η^{ζ^h}	labour supply preference shock
η^{ζ^c}	consumption preference shock
η^{ζ^i}	marginal efficiency of investment shock
η^{g}	exogenous shock to government expenditure

Table 4.7: Parameters

Parameter	Description
$\tilde{a}_{\Delta p^o}$	weight on oil inflation in Taylor rule
$\bar{\epsilon^k}$	steady-state capital technology shock
σ^{ϵ^k}	standard deviation capital technology shock
ρ^{ϵ^k}	AR(1) coefficient for capital technology shock
$\bar{\epsilon^o}$	steady-state oil productivity
σ^{ϵ^o}	standard deviation oil productivity
ρ^{ϵ^o}	AR(1) coefficient for oil productivity
α^O	distribution parameter for oill
α^M	distribution parameter for composite good
ρ^{μ^o}	AR(1) coefficient for oil productivity shocks
ρ^{ζ^o}	AR(1) coefficient for oil cost shocks
$\rho^{\zeta^{o,im}}$	AR(1) coefficient for oil imports shocks
$\rho^{\zeta^{o,ex}}$	AR(1) coefficient for oil exports shocks
ρ^{τ^o}	AR(1) coefficient for oil tax shocks
γ^o	oil extraction cost parameter
$\gamma^{o^e x}$	oil exports extraction cost parameter
$\gamma^{o^{im}}$	oil imports extraction cost parameter
η^O	inverse demand price elasticity for oil consumption
σ^O	inverse supply price elasticity for oil production
τ^o	tax on oil production
ζ^o	long-run value of cost push shock
$\zeta^{o,im}$	long-run value of oil imports
$\zeta^{o,ex}$	long-run value of oil exports
ϵ^o	long-run value of oil productivity shock
σ^{μ^o}	standard deviation productivity of oil
σ^{p^o}	standard deviation measurement error refinery acquisition price
σ^{ζ^o}	standard deviation oil supply shock
$\sigma^{\zeta^{o,im}}$	standard deviation oil imports shock
$\sigma^{\zeta^{o,ex}}$	standard deviation oil exports shock

Table 4.7 – Continued

Parameter	Description
σ^{τ^o}	standard deviation oil tax shock
$\frac{o}{y}$	long-run oil output ratio
$\frac{o^d}{o}$	long-run oil domestic output to oil consumption ratio
$\frac{o^{im}}{o}$	long-run oil imports to oil consumption ratio
$\frac{o^{ex}}{o^d}$	long-run oil exports to oil domestic ratio
$o^{ex,trend,obs}$	trend in net oil exports observation
$o^{im,trend,obs}$	trend in oil imports observation
$o^{d,trend,obs}$	trend in oil domestic production observation
$o^{trend,obs}$	trend in oil consumption observation
$p^{o,trend,obs}$	trend in oil price observation
Θ	share of consumed remaining assets of leaving entrepreneurs
$F(\bar{\omega})$	steady-state bankruptcy rate
$\bar{\gamma}$	steady-state survival rate of entrepreneurs
$\frac{\bar{n}}{\bar{k}}$	steady-state equity to asset ratio
ρ^{γ}	AR(1) coefficient for survival rate of entrepreneurs
μ	monitoring cost
ρ^{σ}	AR(1) coefficient for σ
ρ^{term}	AR(1) coefficient for term structure
$\overline{R^L - R}$	steady-state term structure
$\bar{\sigma}$	steady-state risk level
ω^e	transfers to entrepreneurs from households
σ^{σ}	standard deviation unanticipated risk shock
σ^{ξ}	standard deviation anticipated shock
$\sigma(\xi_t, \xi_{t-1})$	signal correlation
σ^{term}	standard deviation term structure shock
σ^{γ}	standard deviation survival rate entrepreneurs
σ^{n}	standard deviation measurement error net worth
$credit^{trend,obs}$	trend in consumption observation
$n^{trend,obs}$	trend in net worth observation
$premium^{trend,obs}$	trend in premium observation
$Spread1^{trend,1,obs}$	trend in spread 1 observation
α^K	distribution parameter capital
α^N	distribution parameter labour
ϕ^G	steady-state share of government expenditure on output
ϕ^O	steady-state share of oil on output
ϕ^K	steady-state share of capital on output
λ^f	elasticity of substitution for intermediate products
λ^w	elasticity of substitution for different labour types
η^M	elasticity of substitution between energy-capital composite good and labour
β	weight on risk in Taylor rule

Table 4.7 – Continued

Parameter	Description
δ	depreciation rate of capital
$\bar{\epsilon}$	steady-state technology shock
$\bar{\epsilon^h}$	steady-state labour productivity shock
$\bar{\epsilon^w}$	steady-state wage mark-up shock
$\bar{\mu^z}$	steady-state growth rate
$\bar{\mu^\Upsilon}$	steady-state investment growth rate
ψ^L	weight on disutlity on labour
$\bar{r^k}$	steady-state rental rate on capital services
$\sigma^{a(u)}$	curvature of utilization cost
ξ^p	Calvo parameter prices
ξ^w	Calvo parameter wages
$\tilde{\rho}$	AR(1) coefficient for risk fre interest rate
$\tilde{a}_\pi$	weight on inflation in Taylor rule
$\tilde{a}_{\Delta y}$	weight on output growth in Taylor rule
$\bar{\pi}$	steady-state inflation
ι	price indexing weight of inflation target
ι^{μ^z}	wage indexing weight on persistent technology growth
ι^w	wage indexing weight on inflation target
$\bar{R}$	steady-state interest rate
ρ^ϵ	AR(1) coefficient for tfp shocks
ρ^{ϵ^h}	AR(1) coefficient for hours shocks
ρ^{ϵ^p}	AR(1) coefficient for price mark-up shock
ρ^{ϵ^w}	AR(1) coefficient for wage mark-up shock
ρ^{μ^z}	AR(1) coefficient for μ^z
ρ^{μ^Υ}	AR(1) coefficient for μ^Υ
ρ^{ζ^c}	AR(1) coefficient for ζ^c
ρ^{ζ^i}	AR(1) coefficient for ζ^i
ρ^{ζ^h}	AR(1) coefficient for ζ^h
ρ^g	AR(1) coefficient for government expenditure
ρ^s	AR(1) coefficient for marginal cost
b	habit formation parameter
τ^c	consumption tax rate
τ^k	capital income tax rate
τ^l	labour income tax rate
$\mathcal{S}''$	curvature of investment adjustment cost
σ^L	curvature for the disutility to labour
υ	mean growth rate for capital
υ^o	mean growth rate for oil consumption
$\bar{\zeta^c}$	steady-state consumption preference
$\bar{\zeta^i}$	steady-state marginal efficiency of investment
$\bar{\zeta^h}$	steady-state marginal efficiency of labour

Table 4.7 – Continued

Parameter	Description
$\bar{g}$	steady-state government expenditure
$\bar{y}$	steady-state output
σ^{ϵ}	standard deviation technology
σ^{ϵ^h}	standard deviation technology hours worked
σ^{μ^z}	standard deviation growth rate shock
σ^{μ^Υ}	standard deviation investment specific growth rate
σ^{ζ^c}	standard deviation consumption preference shock
σ^{ζ^i}	standard deviation investment specific preference shock
σ^{ζ^h}	standard deviation labour preference shock
σ^{g}	standard deviation government expenditure shock
σ^{ϵ^p}	standard deviation price mark-up
σ^{ϵ^w}	standard deviation wage mark-up
σ^{x^p}	standard deviation monetary policy shock
$c^{trend,obs}$	trend in consumption observation
$gdp^{trend,obs}$	trend in GDP observation
$h^{trend,obs}$	trend in hours observation
$i^{trend,obs}$	trend in investment observation
$w^{trend,obs}$	trend in wage observation
$p^{i,trend,obs}$	trend in relative price of investment observation
$R^{trend,obs}$	trend in interest rate observation
$\pi^{trend,obs}$	trend in inflation observation

Table 4.9: Parameter Values

Parameter	Value	Description
ζ^o	1.000	long-run value of cost push shock
$\zeta^{o,f}$	1.000	long-run value of oil imports
ϵ^o	1.000	long-run value of oil productivity shock
$\frac{o}{y}$	0.002	long-run oil output ratio
$\frac{o^d}{o}$	0.474	long-run oil domestic output to oil consumption ratio
$\frac{o^f}{o}$	0.512	long-run oil imports to oil consumption ratio
$\frac{o^{ex}}{o^d}$	0.013	long-run oil exports to oil domestic ratio
Θ	0.005	share of consumed remaining assets of leaving entrepreneurs
$F(\bar{\omega})$	0.006	steady state bankruptcy rate
$\bar{\gamma}$	0.985	steady state survival rate of entrepreneurs
ω^e	0.005	transfers to entrpreneurs from households
ϕ^G	0.190	steady state share of government expenditure on output

Table 4.9 – Continued

Parameter	**Value**	**Description**
ϕ^O	0.017	steady state share of oil on output
ϕ^K	0.400	steady state share of capital on output
λ^f	1.200	elasticity of subsitition for intermediate products
λ^w	1.050	elasticity of subsitition for different labour types
β	0.999	weight on risk in Taylor rule
δ	0.025	depreciation rate of capital
$\bar{\epsilon}$	0.516	steady state technology shock
$\bar{\epsilon^h}$	1.000	steady state labour productivity shock
$\bar{\epsilon^w}$	1.000	steady state wage mark-up shock
$\bar{\epsilon^w}$	1.000	steady state wage mark-up shock
$\bar{\mu^z}$	1.004	steady state growth rate
$\bar{\mu^\Upsilon}$	1.000	steady state investment growth rate
$\bar{r^k}$	0.052	steady state rental rate on capital services
$\tilde{a}_\pi$	0.500	weight on inflation in Taylor rule
$\bar{\pi}$	1.006	steady state inflation
ι	0.000	price indexing weight of inflation target
ι^{μ^z}	0.000	wage indexing weight on persisitent technology growth
ι^w	0.000	wage indexing weight on inflation target
$\bar{R}$	0.011	steady state interest rate
b	0.000	habit formation parameter
τ^c	0.047	consumption tax rate
τ^k	0.320	capital income tax rate
τ^l	0.241	labour income tax rate
σ^L	1.000	curvature for the disutility to labour
υ	1.004	mean growth rate for capital
$\bar{\zeta^c}$	1.000	steady state consumption preference
$\bar{\zeta^i}$	1.000	steady state marginal efficiency of investment
$\bar{\zeta^h}$	1.000	steady state marginal efficiency of labour
$\bar{g}$	0.188	steady state government expenditure
$\bar{y}$	1.000	steady state output

Table 4.10: Prior information (parameters)

Parameter	Distribution	Mean	Std.dev.
	baseline parameters		

(Continued on next page)

Table 4.10: (continued)

Parameter	Distribution	Mean	Std.dev.
η^M	Gamma	1	0.2
$\mathcal{S}''$	Gaussian	4.9844	1.7662
$\sigma^{a(u)}$	Gaussian	1.0499	0.0965
$\tilde{a}_{\Delta y}$	Gaussian	0.3422	0.0461
ξ^w	Beta	0.2989	0.0342
ξ^p	Beta	0.4634	0.0364
$\tilde{\rho}$	Beta	0.7795	0.0196
σ^{ϵ}	Inv. Gamma	0.0091	0.0006
σ^{μ^z}	Inv. Gamma	0.011	0.0008
$\sigma^{\mu^{\Upsilon}}$	Inv. Gamma	0.0075	0.0005
σ^{ζ^i}	Inv. Gamma	0.035	0.0057
σ^{ζ^c}	Inv. Gamma	0.0161	0.0013
σ^g	Inv. Gamma	0.0203	0.0013
σ^{x^p}	Inv. Gamma	0.0089	0.0006
σ^{ϵ^p}	Inv. Gamma	0.0112	0.0008
ρ^{ϵ}	Beta	0.9033	0.0163
ρ^{μ^z}	Beta	0.0897	0.0444
$\rho^{\mu^{\Upsilon}}$	Beta	0.4813	0.1491
ρ^{ζ^i}	Beta	0.6529	0.0601
ρ^{ζ^c}	Beta	0.9711	0.0078
ρ^g	Beta	0.9276	0.0141
ρ^{ϵ^p}	Beta	0.8562	0.0348
oil market			
η^M	Gamma	1	0.2000
η^O	Gamma	0.1000	0.0500
σ^O	Gamma	10.0000	2.0000
σ^{ζ^o}	Inv. Gamma	0.1000	2.0000
$\sigma^{\zeta^{o,im}}$	Inv. Gamma	0.1000	2.0000
$\sigma^{\zeta^{o,ex}}$	Inv. Gamma	0.1000	2.0000
ρ^{ζ^o}	Beta	0.5000	0.2000
$\rho^{\zeta^{o,ex}}$	Beta	0.5000	0.2000
$\rho^{\zeta^{o,im}}$	Beta	0.5000	0.2000
ρ^{ϵ^o}	Beta	0.5000	0.2000
financial accelerator			
σ^{γ}	Inv. Gamma	0.1000	2.0000

(Continued on next page)

Table 4.10: (continued)

Parameter	Distribution	Mean	Std.dev.
σ^{ξ}	Inv. Gamma	0.1000	2.0000
σ^{σ}	Inv. Gamma	0.1000	2.0000
σ^{term}	Inv. Gamma	0.1000	2.0000
σ^{n}	Inv. Gamma	0.1000	2.0000
$\sigma(\xi_t, \xi_{t-1})$	Beta	0.5000	0.1000
ρ^{γ}	Beta	0.5000	0.2000
ρ^{σ}	Beta	0.5000	0.2000
ρ^{term}	Beta	0.5000	0.2000

Table 4.8: Tests for stationary observable variables

	Augmented Dickey-Fuller	Phillips-Perron
π^{obs}	0.22	0.01
y^{obs}	0.01	0.01
c^{obs}	0.08	0.01
i^{obs}	0.02	0.01
h^{obs}	0.21	0.73
b^{obs}	0.34	0.01
n^{obs}	0.01	0.01
$p^{i,obs}$	0.01	0.01
$premium^{obs}$	0.06	0.03
R^{obs}	0.01	0.21
$S^{1,obs}$	0.01	0.01
w^{obs}	0.01	0.01
$o^{im,obs}$	0.01	0.01
$o^{ex,obs}$	0.01	0.01
$o^{d,obs}$	0.01	0.01
$p^{o,obs}$	0.01	0.01

Note: p-values for the tests are reported.

Table 4.11: Estimation results for rigidity parameters

Model	CEE–Oil model	CMR–Oil model
AR(1) coefficient for TFP shocks	0.92	0.91
ρ^{ϵ}	[0.90, 0.93]	[0.89, 0.93]
AR(1) coefficient for μ^z	0.05	0.04
ρ^{μ^z}	[0.02, 0.10]	[0.01, 0.09]
AR(1) coefficient for μ^Υ	0.46	0.45
ρ^{μ^Υ}	[0.28, 0.64]	[0.28, 0.63]
AR(1) coefficient for ζ^i	0.63	0.57
ρ^{ζ^i}	[0.56, 0.69]	[0.49, 0.65]
AR(1) coefficient for government expenditure	0.93	0.94
ρ^g	[0.92, 0.95]	[0.92, 0.95]
AR(1) coefficient for price mark-up shock	0.87	0.90
ρ^{ϵ^p}	[0.83, 0.90]	[0.86, 0.92]
AR(1) coefficient for survival rate of entrepreneurs	-	0.56
ρ^{γ}	[-]	[0.31, 0.72]
AR(1) coefficient for σ	-	0.92
ρ^{σ}	[-]	[0.88, 0.94]
AR(1) coefficient for term structure	-	0.21
ρ^{term}	[-]	[0.16, 0.26]
AR(1) coefficient for oil cost shocks	0.99	0.99
ρ^{ζ^o}	[0.98, 1.00]	[0.97, 1.00]
AR(1) coefficient for oil exports shocks	-	-
$\rho^{\zeta^{o,ex}}$	[-]	[-]
AR(1) coefficient for oil imports shocks	0.96	0.96
$\rho^{\zeta^{o,f}}$	[0.93, 0.99]	[0.94, 0.99]
AR(1) coefficient for oil productivity	0.86	0.87
ρ^{ϵ^m}	[0.79, 0.92]	[0.81, 0.93]

Note: The posterior mean and the 90% highest posterior density (HPD) interval for the respective parameters are reported in parentheses.

Table 4.12: Estimation results for standard deviations

Model	CEE–Oil model	CMR–Oil model
standard deviation technology	0.01	0.01
σ^{ϵ}	[0.01, 0.01]	[0.01, 0.01]
standard deviation growth rate shock	0.01	0.01
σ^{μ^z}	[0.01, 0.01]	[0.01, 0.01]
standard deviation investment specific growth rate	0.01	0.01
$\sigma^{\mu^{\Upsilon}}$	[0.01, 0.01]	[0.01, 0.01]
standard deviation investment specific preference shock	0.03	0.03
σ^{ζ^i}	[0.03, 0.04]	[0.02, 0.03]
standard deviation consumption preference shock	0.01	0.02
σ^{ζ^c}	[0.01, 0.02]	[0.01, 0.02]
standard deviation labour preference shock	-	-
σ^{ζ^h}	[-]	[-]
standard deviation government expenditure shock	0.02	0.02
σ^{g}	[0.02, 0.02]	[0.02, 0.02]
standard deviation monetary policy shock	0.01	0.01
σ^{x^p}	[0.01, 0.01]	[0.01, 0.01]
standard deviation price mark-up	0.01	0.01
σ^{ϵ^p}	[0.01, 0.01]	[0.01, 0.01]
standard deviation oil productivity	0.03	0.03
σ^{ϵ^m}	[0.03, 0.04]	[0.03, 0.04]
standard deviation oil supply shock	0.03	0.03
σ^{ζ^o}	[0.03, 0.03]	[0.03, 0.03]
standard deviation oil imports shock	0.05	0.05
$\sigma^{\zeta^{o,f}}$	[0.04, 0.05]	[0.04, 0.05]
standard deviation oil exports shock	3.38	3.38
$\sigma^{\zeta^{o,ex}}$	[3.07, 3.74]	[3.07, 3.75]
standard deviation survival rate enetrepreneurs	-	0.01
σ^{γ}	[-]	[0.01, 0.01]
standard deviation anticipated shock	-	0.02
σ^{ξ}	[-]	[0.02, 0.02]
standard deviation unanticipated risk shock	-	0.04
σ^{σ}	[-]	[0.04, 0.05]
standard deviation term structure shock	-	0.02
σ^{term}	[-]	[0.01, 0.02]
standard deviation measurement error net worth	-	0.06
σ^{n}	[-]	[0.06, 0.07]
signal correlation	-	0.54
$\sigma(\xi_t, \xi_{t-1})$	[-]	[0.43, 0.67]

Note: The posterior mean and the 90% highest posterior density (HPD) interval for the respective parameters are reported in parentheses.

Table 4.13: Classification of shock groups

Group	Shocks
anticipated risk	η^{ξ^i} for $i \in \{1, \ldots, 8\}$
unanticipated risk	η^{σ}
risk	anticipated and unanticipated risk
financial	η^{γ}, η^{term}
investment	η^{ζ^i}, $\eta^{\mu^{\Upsilon}}$
monetary policy (M.P.)	η^{x^p}
fiscal policy	η^{g}
policy	fiscal policy and monetary policy
markup	η^{ϵ^p}
demand	η^{ζ^c}
domestic oil supply	$\eta^{\zeta^{o^d}}, \eta^{\zeta^{o^{ex}}}$
oil demand	η^{ϵ^o}
foreign oil supply	$\eta^{\zeta^{o^{im}}}$
oil supply	domestic and foreign oil supply
oil	oil supply and oil demand

Table 4.14: Parameter values for CMR replication

Description	Symbol	Value
Structural parameters		
share of consumed remaining assets of leaving entrepreneurs	Θ	0.005
steady state bankruptcy rate	$F(\bar{\omega})$	0.0056
steady state survival rate of entrepreneurs	$\bar{\gamma}$	0.985
monitoring cost	μ	0.3074
curvature of utilization cost	$\sigma^{a(u)}$	2.5356
Calvo parameter prices	ξ^p	0.7412
Calvo parameter wages	ξ^w	0.8128
AR(1) coefficient for risk free interest rate	$\tilde{\rho}$	0.8503
weight on inflation in Taylor rule	$\tilde{a}_\pi$	2.3965
weight on output growth in Taylor rule	$\tilde{a}_{\Delta y}$	0.3649
price indexing weight of inflation target	ι	0.8974
wage indexing weight on persisitent technology growth	ι^{μ^z}	0.9366
wage indexing weight on inflation target	ι^w	0.4891
habit formation parameter	b	0.7358
curvature of investment adjustment cost	S	10.78
Persistence parameters		
AR(1) coefficient for TFP shocks	ρ^ϵ	0.8089
AR(1) coefficient for hours shocks	ρ^{ϵ^h}	0.5
AR(1) coefficient for price mark-up shock	ρ^{ϵ^p}	0.9109
AR(1) coefficient for wage mark-up shock	ρ^{ϵ^p}	0.5
AR(1) coefficient for μ^z	ρ^{μ^z}	0.1459
AR(1) coefficient for μ^Υ	ρ^{μ^Υ}	0.987
AR(1) coefficient for ζ^c	ρ^{ζ^c}	0.8968
AR(1) coefficient for ζ^i	ρ^{ζ^i}	0.9087
AR(1) coefficient for ζ^h	ρ^{ζ^h}	0.5
AR(1) coefficient for government expenditure	ρ^g	0.9427
AR(1) coefficient for marginal cost	ρ^s	0.5
AR(1) coefficient for σ	ρ^σ	0.9706
AR(1) coefficient for term strucut	ρ^{term}	0.9744
Standard deviations of shocks		
standard deviation unanticipated risk shock	σ^σ	0.07
standard deviation anticipated shock	σ^ξ	0.0283
signal correlation	$\sigma(\xi_t, \xi_{t-1})$	0.6757
standard deviation term structure shock	σ^{term}	0.0016
standard deviation survival rate enetrepreneurs	σ^γ	0.0081
standard deviation measurement error net worth	σ^n	0
standard deviation technology	σ^ϵ	0.0046
standard deviation technology hours worked	σ^{ϵ^h}	0
standard deviation growth rate shock	σ^{μ^z}	0.0071
standard deviation investment specific growth rate	σ^{μ^Υ}	0.004
standard deviation consumption preference shock	σ^{ζ^c}	0.0233
standard deviation investment specific preference shock	σ^{ζ^i}	0.055
standard deviation labour preference shock	σ^{ζ^h}	0
standard deviation government expenditure shock	σ^g	0.0228
standard deviation wage mark-up	σ^{ϵ^w}	0
standard deviation price mark-up	σ^{ϵ^p}	0.011
standard deviation monetary policy shock	σ^{x^p}	0.0049

Notes: The parameter values are from Christiano et al. (2014) to compute the variance decomposition at the posterior mode as reported in Table 4.4.

Table 4.15: Variance decomposition for financial market variables at the posterior distribution

Variable	risk	investment	demand	financial	M.P.	markup	technol.	oil
				credit growth				
CEE–Oil model	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	[0.0, 0.0]	[0.0, 0.0]	[0.0, 0.0]	[0.0, 0.0]	[0.0, 0.0]	[0.0, 0.0]	[0.0, 0.0]	[0.0, 0.0]
CMR–Oil model	19.0	1.3	0.4	64.0	3.1	2.3	9.6	0.2
	[8.8, 28.6]	[0.8, 1.8]	[0.2, 0.5]	[56.9, 72.0]	[2.3, 3.9]	[1.5, 3.1]	[6.8, 12.6]	[0.1, 0.3]
				external finance premium				
CEE–Oil model	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	[0.0, 0.0]	[0.0, 0.0]	[0.0, 0.0]	[0.0, 0.0]	[0.0, 0.0]	[0.0, 0.0]	[0.0, 0.0]	[0.0, 0.0]
CMR–Oil model	75.8	1.5	0.4	19.9	1.5	0.1	0.9	0.0
	[45.7, 105.3]	[0.9, 2.1]	[0.3, 0.5]	[9.4, 30.7]	[1.0, 1.9]	[0.0, 0.1]	[0.5, 1.2]	[0.0, 0.0]
				equity growth				
CEE–Oil model	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	[0.0, 0.0]	[0.0, 0.0]	[0.0, 0.0]	[0.0, 0.0]	[0.0, 0.0]	[0.0, 0.0]	[0.0, 0.0]	[0.0, 0.0]
CMR–Oil model	15.7	1.6	0.1	11.6	4.3	0.2	0.4	0.1
	[7.7, 23.1]	[1.1, 2.2]	[0.0, 0.1]	[6.2, 16.8]	[3.2, 5.2]	[0.2, 0.3]	[0.3, 0.6]	[0.0, 0.1]
				term spread				
CEE–Oil model	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	[0.0, 0.0]	[0.0, 0.0]	[0.0, 0.0]	[0.0, 0.0]	[0.0, 0.0]	[0.0, 0.0]	[0.0, 0.0]	[0.0, 0.0]
CMR–Oil model	10.6	10.9	2.4	27.6	25.5	5.7	16.8	0.5
	[4.8, 16.5]	[8.1, 13.9]	[1.6, 3.0]	[20.7, 35.0]	[22.1, 29.3]	[3.6, 7.5]	[12.3, 21.0]	[0.2, 0.7]

Note: Contribution of each shock group in percent to the total theoretical variance of the respective variable is reported. Values in parentheses represent 90% HPD interval of the model parameters. The shock groups are reported in Table 4.13.

Table 4.16: Variance decomposition for oil market variables at the posterior distribution

Variable	risk, financial and inv.	policy, demand, markup	technol.	domestic oil supply	foreign oil supply	domestic oil demand
domestic oil supply growth						
CEE–Oil model	0.1	1.3	2.4	66.3	15.0	14.9
	[0.1, 0.1]	[0.9, 1.7]	[1.4, 3.5]	[51.9, 80.4]	[8.8, 20.3]	[11.7, 18.4]
CMR–Oil model	0.2	1.2	2.5	67.5	13.9	14.6
	[0.1, 0.3]	[0.8, 1.7]	[1.5, 3.6]	[52.4, 82.2]	[8.4, 19.8]	[11.2, 18.2]
domestic oil supply						
CEE–Oil model	0.2	0.5	1.1	89.6	6.7	1.9
	[0.0, 0.4]	[0.1, 1.0]	[0.2, 2.0]	[80.7, 98.9]	[0.7, 12.5]	[0.4, 3.3]
CMR–Oil model	1.4	0.8	1.3	85.5	8.4	2.6
	[0.2, 2.9]	[0.2, 1.4]	[0.3, 2.3]	[73.9, 97.6]	[1.1, 15.4]	[0.7, 4.6]
foreign oil supply growth						
CEE–Oil model	0.1	0.8	1.4	11.2	77.4	9.1
	[0.0, 0.1]	[0.4, 1.1]	[0.9, 2.0]	[6.3, 16.5]	[69.2, 84.0]	[6.4, 12.3]
CMR–Oil model	0.1	0.8	1.5	10.4	78.5	8.8
	[0.0, 0.2]	[0.4, 1.1]	[0.9, 2.0]	[5.3, 15.4]	[70.9, 85.6]	[6.0, 11.8]
foreign oil supply						
CEE–Oil model	0.3	0.7	1.5	11.1	83.8	2.6
	[0.1, 0.5]	[0.2, 1.3]	[0.3, 2.5]	[1.9, 21.4]	[71.7, 96.3]	[0.5, 4.2]
CMR–Oil model	1.5	0.8	1.3	8.1	85.5	2.7
	[0.1, 2.9]	[0.2, 1.5]	[0.3, 2.4]	[1.2, 15.6]	[75.3, 97.0]	[0.5, 4.5]
oil price growth						
CEE–Oil model	0.2	2.5	4.7	34.9	28.7	29.0
	[0.1, 0.2]	[1.8, 3.1]	[3.4, 5.9]	[27.8, 42.0]	[23.3, 34.9]	[22.1, 35.8]
CMR–Oil model	0.4	2.6	4.1	34.6	28.0	30.4
	[0.2, 0.6]	[1.9, 3.2]	[3.1, 5.4]	[27.3, 41.4]	[22.7, 33.5]	[23.0, 37.6]
oil price						
CEE–Oil model	1.2	3.1	6.2	42.7	35.7	11.1
	[0.5, 1.9]	[1.3, 5.0]	[3.0, 9.5]	[20.7, 62.4]	[17.3, 53.4]	[4.2, 17.5]
CMR–Oil model	6.4	3.6	6.0	34.4	37.0	12.6
	[1.2, 11.8]	[1.6, 5.7]	[3.1, 9.0]	[15.8, 53.2]	[18.1, 54.1]	[4.3, 19.9]

Note: Contribution of each shock group in % to the total theoretical variance of the respective variable is reported. Values in parentheses represent 90% HPD interval of the model parameters. The shock groups are reported in Table 4.13.

4.7.3 Model equations

4.7.3.1 CEE model equations

The CEE model consists of equations (4.26) to (4.49), which describe the behaviour of endogenous variables.Here the stationary version of the model is reported. The derivation of all model equations is provided in the Online Appendix. Shocks are described by (4.50) to (4.58).

4.7.3.1.1 Households

This block contains model equations describing the behaviour of representative households in the model.

Households face investment adjustment costs. These investment adjustment costs reduce the effectiveness of investments into the raw capital stock. Investment adjustment costs depend on the curvature parameter $\mathcal{S}''$, marginal efficiency of investment adjustment shocks ζ_t^i, the change in investment $\frac{i_t}{i_{t-1}}$, the growth rate of technological change μ_t^z and investment specific trend Υ.

$$\mathcal{S}\left(\frac{{\mu^z}_t\,\Upsilon\,{\zeta^i}_t\,i_t}{i_{t-1}}\right) = \left(exp\left(\sqrt{\frac{\mathcal{S}''}{2}}\left(\frac{{\mu^z}_t\,\Upsilon\,{\zeta^i}_t\,i_t}{i_{t-1}} - \Upsilon\,\bar{\mu^z}\right)\right)\right.$$
$$\left. +exp\left(-\sqrt{\frac{\mathcal{S}''}{2}}\left(\frac{{\mu^z}_t\,\Upsilon\,{\zeta^i}_t\,i_t}{i_{t-1}} - \Upsilon\,\bar{\mu^z}\right)\right) - 2\right). \tag{4.26}$$

Raw capital evolves according to a standard law of motion. Each period a constant fraction δ of the old capital stock depreciates. Investments into the capital stock are necessary to maintain and extend the raw capital stock.

$$\bar{k}_t = \frac{(1-\delta)}{{\mu^z}_t\,\Upsilon}\,\bar{k}_{t-1} + \left(1 - S\left(\frac{{\mu^z}_t\,\Upsilon\,{\zeta^i}_t\,i_t}{i_{t-1}}\right)\right)\,i_t. \tag{4.27}$$

From the intertemporal optimization problem of the household the first order condition with respect to consumption is the marginal utility of consumption. The marginal utility of consumption depends on preference shocks ζ_t^c, the discount factor β, habit formation b, the tax rate on consumption and the growth rate of technological change μ_t^z.

$${\lambda^z}_t\,(1+\tau^c) = \frac{{\mu^z}_t\,{\zeta^c}_t}{{\mu^z}_t\,c_t - b\,c_{t-1}} - \frac{\beta\,b\,{\zeta^c}_{t+1}}{c_{t+1}\,{\mu^z}_{t+1} - c_t\,b}. \tag{4.28}$$

Investments into the capital stock by households is a trade-off between foregone consumption today for future income. The first term in (4.29) represents foregone consumption today by increasing investment today. The second and third term represents the increase in potential consumption tomorrow by an increase in the capital stock.

$$0 = \frac{(-\lambda^z{}_t)}{\mu^\Upsilon{}_t} + \lambda^z{}_t \, q_t \left(1 - S\left(\frac{\mu^z{}_t \, \Upsilon \, \zeta^i{}_t \, i_t}{i_{t-1}} \right) - \frac{\partial S\left(\frac{\mu^z{}_t \, \Upsilon \, \zeta^i{}_t \, i_t}{i_{t-1}} \right)}{\partial \frac{i_t}{\Upsilon}} \right) + \frac{\beta \, \lambda^z{}_{t+1}}{\Upsilon \, \mu^z{}_{t+1}} \, q_{t+1} \, \frac{\partial S\left(\frac{\mu^z{}_{t+1} \, \Upsilon \, \zeta^i{}_{t+1} \, i_{t+1}}{i_t} \right)}{\partial \frac{i_t}{\Upsilon}} \left(\frac{\Upsilon \, \mu^z{}_{t+1} \, \zeta^i{}_{t+1} \, i_{t+1}}{i_t} \right)^2 . \tag{4.29}$$

Households provide capital services $k^s_t = u_t \, \bar{k}_{t-1}$ for a rental rate r^k_t. Utilization of raw capital u_t is associated with costs $a(u_t)$. The optimal utilization rate equates marginal costs and benefits.

$$r^k_t = \bar{r}^k \, exp\left(\sigma^{a(u)} \, (u_t - 1)\right). \tag{4.30}$$

In the CEE model raw capital is a control variable of households. The benefit of having one more unit of raw capital in the next period is additional discounted marginal consumption using revenues from renting capital services. This benefit equals the cost of foregone consumption today.

$$0 = \beta \, \frac{\lambda^z_{t+1}}{\mu^z_{t+1} \, \pi_{t+1}} \, r^k_{t+1} \, u_{t+1} \, (1 - \tau^k) - q_t \lambda^z_t + (1 - \delta)\beta \, q_{t+1} \, \lambda^z_{t+1}. \tag{4.31}$$

In addition to raw capital, households can also access short-term bonds b_t. Those bonds are purchased such that forgone consumption today λ^z_t equals potential additional consumption tomorrow. It is the Euler equation for bonds and is an implicit arbitrage condition between raw capital and bonds.

$$0 = (1 + R_t) \, \frac{\beta \, \lambda^z{}_{t+1}}{\pi_{t+1} \, \mu^z{}_{t+1}} - \lambda^z{}_t. \tag{4.32}$$

4.7.3.1.2 Production

The standard NK-DSGE model introduces a two layer production process of final goods. In the first stage the two primary production factors homogenous labour $l_t = h_t \, (w^*_t)^{\frac{\lambda^w}{\lambda^w - 1}}$ and capital services $\frac{u_t \, \bar{k}_{t-1}}{\mu^z{}_t \, \Upsilon}$ are used to produce intermediate goods. Homogenous labour depends on the wage dispersion index and total hours worked. Wage rigidity leads to a mismatch between the marginal product of the specific type of labour and its price. This mismatch determines the total level of homogenous labour supplied by labour contractors. Intermediate goods are transformed into a final good. The effectiveness of the transformation depends on the price dispersion index p^*_t. Therefore total final output y_t is given by

$$y_t = p^*{}_t{}^{\frac{\lambda^f}{\lambda^f - 1}} \, \epsilon_t \, \left(\frac{u_t \, \bar{k}_{t-1}}{\mu^z{}_t \, \Upsilon} \right)^{\alpha^K} \, \left(\epsilon^h{}_t \, h_t \, w^*{}_t{}^{\frac{\lambda^w}{\lambda^w - 1}} \right)^{\alpha^N} - \phi_t. \tag{4.33}$$

It is standard to include fixed costs to ensure that the no entry condition is fulfilled in steady-state. Fixed costs ϕ are set to ensure zero profits in steady-state. Further, I model fixed costs proportional to the previous year total final output.

$$\phi_t = \frac{1 - \frac{1}{\lambda^f}}{\frac{1}{\lambda^f}}\, y_{t-4}. \tag{4.34}$$

Intermediate goods producing firms demand capital services such that the associated relative marginal costs $\frac{{r^k}_t}{s_t}$ are equal to its marginal product.

$$\frac{{r^k}_t}{s_t} = \alpha^K \left(\frac{\phi_t + y_t\, {p^*_t}^{\frac{\lambda^f}{1-\lambda^f}}}{\frac{u_t\, \bar{k}_{t-1}}{{\mu^z}_t\, \Upsilon}} \right). \tag{4.35}$$

Firms producing intermediate goods demand hours worked such that the marginal product of an additional unit of homogenous labour equals its marginal cost.

$$\frac{w_t}{s_t} = \alpha^N \left(\frac{\phi_t + y_t\, {p^*_t}^{\frac{\lambda^f}{1-\lambda^f}}}{h_t\, {w^*}_t^{\frac{\lambda^w}{\lambda^w - 1}}} \right). \tag{4.36}$$

4.7.3.2 Price setting

Intermediate goods producing firms minimize costs associated with their primary production factors. However, they also maximize expected discounted profits. The expected discounted profits of intermediate goods producing firms depend on the optimal price $\tilde{p}_t$ they set today. Firms not able to reset their price use an indexation rule. The indexation rule is a weighted average between previous inflation π_{t-1} and the inflation target $\bar{\pi}$. The weight on past inflation is ι.

$$\tilde{\pi}_t = {\pi_{t-1}}^{1-\iota}\, \bar{\pi}^{\iota}. \tag{4.37}$$

The share of intermediate goods-producing firms $1 - \xi^p$ able to reset their price choose all the same price. The optimal price is given by $\tilde{p}_t = \frac{K^p_t}{F^p_t}$. Here we introduce two auxiliary variables to express infinite sums recursively. The denominator

$$F^p_t = y_t\, {\lambda^z}_t + \left(\frac{\tilde{\pi}_{t+1}}{\pi_{t+1}} \right)^{\frac{1}{1-\lambda^f}} \beta\, \xi^p\, F^p_{t+1}. \tag{4.38}$$

The numerator of the optimal price is the infinite sum of discounted expected marginal costs. Further, the shock ϵ^p_t are temporary deviations to the relationship between the optimal price and marginal costs.

$$K^p_t = s_t\, y_t\, {\lambda^z}_t\, \lambda^f\, {\epsilon^p}_t + \beta\, \xi^p \left(\frac{\tilde{\pi}_{t+1}}{\pi_{t+1}} \right)^{\frac{\lambda^f}{1-\lambda^f}} K^p_{t+1}. \tag{4.39}$$

A relationship between numerator K_t^p and denominator F_t^p can be derived from the price index.

$$K_t^p = F_t^p \left(\frac{1 - \xi^p \left(\frac{\tilde{\pi}_t}{\pi_t}\right)^{\frac{1}{1-\lambda^f}}}{1 - \xi^p} \right)^{1-\lambda^f} . \tag{4.40}$$

Price dispersion is a consequence of the random price setting mechanism. The price dispersion index depends on the optimal price and previous price dispersion.

$$p^*{}_t = \left((1 - \xi^p) \left(\frac{K_t^p}{F_t^p}\right)^{\frac{\lambda^f}{1-\lambda^f}} + \xi^p \left(\frac{\tilde{\pi}_t}{\pi_t}\, p^*_{t-1}\right)^{\frac{\lambda^f}{1-\lambda^f}} \right)^{\frac{1-\lambda^f}{\lambda^f}} . \tag{4.41}$$

4.7.3.2.1 Wage setting

Households provide different labour types $h_{j_h,j_l,t}$. Unions represent these labour types. Unions negotiate wages for each type of labour. Labour contractors use the different types of labour to provide homogenous labour l_t.

Unions can only renegotiate wages each period with a probability of $1-\xi^w$ and otherwise reset wages according to an wage inflation indexation rule $\tilde{\pi^w}_t$. This rule depends on previous price inflation, the inflation target, the long-run growth rate of technological change and the contemporaneous growth rate of technological change.

$$\tilde{\pi^w}_t = {\pi_{t-1}}^{1-\iota^w} \, \bar{\pi}^{\iota^w} \, \bar{\mu^z}^{1-\iota^{\mu^z}} \, {\mu^z{}_t}^{\iota^{\mu^z}} . \tag{4.42}$$

Nominal wages are $W_t = z_t\, P_t\, w_t$ a product of real wages, technological change and the current price level. Wage inflation π^w is a product of current price inflation and the growth rate of technological change.

$$\pi_t^w = \mu_t^z\, \pi_t. \tag{4.43}$$

The wage dispersion index like the price dispersion index depends on the previous level of wage dispersion and the current optimal wage set by negotiating unions. It measures the inefficiency in the labour market caused by rigid wage setting.

$$w_t^* = \left((1 - \xi^w) \left(\frac{1 - \xi^w \left(\frac{\frac{\tilde{\pi^w}_t}{\pi^w{}_t} w_{t-1}}{w_t} \right)^{\frac{1}{1-\lambda^w}}}{1 - \xi^w} \right)^{\lambda^w} + \xi^w \left(\frac{\frac{\tilde{\pi^w}_t}{\pi^w{}_t} w_{t-1}}{w_t} w^*{}_{t-1} \right)^{\frac{\lambda^w}{1-\lambda^w}} \right)^{\frac{1}{\frac{\lambda^w}{1-\lambda^w}}} . \tag{4.44}$$

Unions set wages $\tilde{w}_t = \frac{K_t^w\, \psi^L}{F_t^w\, w_t}$ to maximize expected discounted wage bills reduced by the implied disutility of households supplying labour. The denominator F_t^w is an auxiliary variable introduced to express an infinite sum.

$$F^w{}_t = \frac{h_t\, w^*{}_t{}^{\frac{\lambda^w}{\lambda^w-1}}\, \lambda^z{}_t\, \left(1-\tau^l\right)}{\lambda^w\, \epsilon^w{}_t} + \beta\, \xi^w\, \left(\frac{\tilde{\pi^w}_{t+1}}{\pi^w{}_{t+1}}\right)^{\frac{1}{1-\lambda^w}}\, \left(\frac{w_t}{w_{t+1}}\right)^{\frac{\lambda^w}{1-\lambda^w}}\, F^w{}_{t+1}. \tag{4.45}$$

The numerator K_t^w is like F_t^w also an auxiliary variable to express an infinite sum. This infinite sum captures the expected disutility of households to work for the optimal wage.

$$K_t^w = \left(h_t\, w^*{}_t{}^{\frac{\lambda^w}{\lambda^w-1}}\right)^{1+\sigma^L} + \beta\, \xi^w\, \left(\frac{w_t\, \frac{\tilde{\pi^w}_{t+1}}{\pi^w{}_{t+1}}}{w_{t+1}}\right)^{\frac{\lambda^w\left(1+\sigma^L\right)}{1-\lambda^w}}\, K_{t+1}^w. \tag{4.46}$$

It is possible to derive a relationship between the numerator and denominator for optimal wages using the wage index.

$$K_t^w = \frac{F^w{}_t\, w_t\, \left(\frac{1-\xi^w\left(\frac{\tilde{\pi^w}_t}{\pi^w{}_t}\right)^{\frac{1}{1-\lambda^w}}}{1-\xi^w}\right)^{1-\lambda^w\left(1+\sigma^L\right)}}{\zeta^h{}_t\, \psi^L}. \tag{4.47}$$

4.7.3.2.2 Monetary policy and resource constraint

A main objective of NK-DSGE models is the analysis of monetary policy. To model monetary policy the Taylor rule is included. The Taylor rule postulates that the risk-free interest rate is a function of deviations in previous inflation from its target value and deviations in GDP growth from it s potential. The parameters $\tilde{a}_\pi$ and $\tilde{a}_{\Delta y}$ govern the response of the monetary policy authority to the respective deviations. Further, monetary policy considers previous risk free interest rates and weights them with $\tilde{\rho}$. Potential discretionary deviations form the rule are captured by x_t^p measured in annualized terms.

$$\frac{1+R_t}{1+\bar{R}} = \left(\frac{1+R_{t-1}}{1+\bar{R}}\right)^{\tilde{\rho}} \left\{\left(\frac{\pi_{t-1}}{\bar{\pi}}\right)^{1+\tilde{a}_\pi} \left(\frac{\mu_{t-1}^z}{\bar{\mu^z}}\, \frac{c_{t-1}+\frac{i_{t-1}}{\mu_{t-1}^\Upsilon}+g_{t-1}}{c_{t-2}+\frac{i_{t-2}}{\mu_{t-2}^\Upsilon}+g_{t-2}}\right)^{\tilde{a}_{\Delta y}}\right\}^{1-\tilde{\rho}} + \frac{\sigma^{x^p}}{4} x_t^p. \tag{4.48}$$

The resource constraint can be derived from the budget constraint of the household. Total output used in the economy is either used for investment, consumption, government expenditure or eaten up by capital utilization costs.

$$y_t = c_t + \frac{i_t}{\mu_t^\Upsilon} + g_t + \frac{\bar{k}_{t-1}}{\mu^z{}_t\, \Upsilon}\, a(u_t). \tag{4.49}$$

4.7.3.2.3 Shocks

Shocks in the CEE model are responsible for fluctuations of the endogenous variables around the balanced growth path. These variables do not depend on the development of endogenous variables.

The standard NK-DSGE model does not explicitly model the behaviour of fiscal policy. It is therefore assumed that government expenditure follows an autoregressive process of order one.

$$log\left(\frac{g_t}{\bar{g}}\right) = \rho^g \, log\left(\frac{g_{t-1}}{\bar{g}}\right) + \sigma^g \, \eta^g_t. \tag{4.50}$$

In order to capture potential fluctuations on the supply side total factor productivity shocks are introduced. These shocks capture fluctuations in the efficiency of combining primary production factors to intermediate and final goods. These shocks have no direct impact on the relative productivity of the production factors.

$$log\left(\frac{\epsilon_t}{\bar{\epsilon}}\right) = \rho^\epsilon \, log\left(\frac{\epsilon_{t-1}}{\bar{\epsilon}}\right) + \sigma^\epsilon \, {\eta^\epsilon}_t. \tag{4.51}$$

Labour productivity shocks only affect the productivity of labour and have direct implications for the relative productivity of both production factors.

$$log\left(\frac{{\epsilon^h}_t}{\bar{\epsilon^h}}\right) = \rho^{\epsilon^h} \, log\left(\frac{{\epsilon^h}_{t-1}}{\bar{\epsilon^h}}\right) + \sigma^{\epsilon^h} \, {\eta^{\epsilon^h}}_t. \tag{4.52}$$

Cost-push shocks are shocks to the desired mark-up over marginal costs and are a standard shock included in NK-DSGE models. They mainly capture variations in the markup over the business cycle.

$$log\left(\frac{{\epsilon^p}_t}{\bar{\epsilon^p}}\right) = \rho^{\epsilon^p} \, log\left(\frac{{\epsilon^p}_{t-1}}{\bar{\epsilon^p}}\right) + \sigma^{\epsilon^p} \, {\eta^{\epsilon^p}}_t. \tag{4.53}$$

Wage markup shocks are similar to price mark-up shocks. They mainly capture variations in the wage markup over the business cycle.

$$log\left(\frac{{\epsilon^w}_t}{\bar{\epsilon^w}}\right) = \rho^{\epsilon^p} \, log\left(\frac{{\epsilon^w}_{t-1}}{\bar{\epsilon^w}}\right) + \sigma^{\epsilon^w} \, {\eta^{\epsilon^w}}_t. \tag{4.54}$$

Episodes of more and less rapid technological growth require a time varying growth rate. Nevertheless, this growth rate is independent of endogenous variables in the model.

$$log\left(\frac{{\mu^z}_t}{\bar{\mu^z}}\right) = \rho^{\mu^z} \, log\left(\frac{{\mu^z}_{t-1}}{\bar{\mu^z}}\right) + \sigma^{\mu^z} \, {\eta^{\mu^z}}_t. \tag{4.55}$$

The relative price for investment is driven by an exogenous shock. This shock is necessary to include the relative price of investment as an observable variable for the estimation of the model.

$$log\left(\frac{{\mu^\Upsilon}_t}{\bar{\mu^\Upsilon}}\right) = \rho^{\mu^\Upsilon} \, log\left(\frac{{\mu^\Upsilon}_{t-1}}{\bar{\mu^\Upsilon}}\right) + \sigma^{\mu^\Upsilon} \, {\eta^{\mu^\Upsilon}}_t. \tag{4.56}$$

Households preferences to consume might fluctuate over time. This is captured by temporary shocks to consumption preferences.

$$log\left(\frac{\zeta^c{}_t}{\bar{\zeta^c}}\right) = \rho^{\zeta^c}\, log\left(\frac{\zeta^c{}_{t-1}}{\bar{\zeta^c}}\right) + \sigma^{\zeta^c}\, \eta^{\zeta^c}{}_t. \tag{4.57}$$

Capital formation depends on the effectiveness of investment into the capital stock. This efficiency fluctuates over time due to an exogenous process.

$$log\left(\frac{\zeta^i{}_t}{\bar{\zeta^i}}\right) = \rho^{\zeta^i}\, log\left(\frac{\zeta^i{}_{t-1}}{\bar{\zeta^i}}\right) + \sigma^{\zeta^i}\, \eta^{\zeta^i}{}_t. \tag{4.58}$$

4.7.3.2.4 Observational Equations

Estimating the model requires to define observational variables. Standard observational variables are the main components of GDP. It is necessary to define suitable transformations of the observed variables and the model variables. Observational variables for the CEE model are consumption growth (4.59), GDP growth (4.60), hours worked (4.61), investment growth (4.62), wage growth (4.63), relative price of investment (4.64), inflation (4.65), and the risk free interest rate (4.66).

$$c_t^{obs} = \overline{c}^{obs}\, \frac{\mu^z{}_t\, c_t}{\bar{\mu^z}\, c_{t-1}}, \tag{4.59}$$

$$y_t^{obs} = \overline{y}^{obs}\, \frac{\mu^z{}_t \left(c_t + \frac{i_t}{\mu^\Upsilon{}_t} + g_t\right)}{\bar{\mu^z}\left(c_{t-1} + \frac{i_{t-1}}{\mu^\Upsilon{}_{t-1}} + g_{t-1}\right)}, \tag{4.60}$$

$$h_t^{obs} = \overline{h}^{obs}\, \frac{h_t}{(\bar{h})}, \tag{4.61}$$

$$i_t^{obs} = \overline{i}^{obs}\, \frac{\mu^z{}_t\, i_t}{\bar{\mu^z}\, i_{t-1}}, \tag{4.62}$$

$$w_t^{obs} = \overline{w}^{obs}\, \frac{\mu^z{}_t\, w_t}{\bar{\mu^z}\, w_{t-1}}, \tag{4.63}$$

$$p_t^{i,obs} = \overline{p}^{i,obs}\, \frac{\bar{\mu^\Upsilon}}{\mu^\Upsilon{}_t}, \tag{4.64}$$

$$\pi_t^{obs} = \overline{\pi}^{obs}\, \frac{\pi_t}{\bar{\pi}}, \tag{4.65}$$

$$R_t^{obs} = \overline{R}^{obs}\, exp\left(R_t - \bar{R}\right). \tag{4.66}$$

4.7.3.3 CMR model equations

The CMR model uses (4.26) to (4.48). Including the financial accelerator leads to modifications of the resource constraint. Further, (4.67), (4.69), (4.70), (4.71), (4.72), (4.73) are additional model equations. These equations describe the behaviour of entrepreneurs and mutual funds. The new resource constraint is now (4.74) and replaces (4.49). Further, the financial accelerator model will introduce new shocks to the model. These shocks drive the dispersion in the idiosyncratic productivity of entrepreneurs.

4.7.3.3.1 Entrepreneurs

The main modification of CMR compared to the CEE model is to introduce entrepreneurs and mutual funds as agents. Households do not supply capital services to the intermediate goods producing firms. Entrepreneurs provide now effective capital to intermediate goods producing firms. Mutual funds grant loans to entrepreneurs. Loans need to be repaid. The probability that an entrepreneur cannot repay the loans is given by $F(\bar{\omega}_t)$. Default probability increases with the threshold $\bar{\omega}_t$. Here Φ denotes the normal distribution and σ_t is the cross-sectional dispersion of ω.

$$F(\bar{\omega}_t) = \Phi\left(\frac{log\left(\bar{\omega}_t\right) + \frac{{\sigma_{t-1}}^2}{2}}{\sigma_{t-1}}\right). \tag{4.67}$$

The value of the assets of insolvent entrepreneurs depends on the expected value of ω below the threshold $\bar{\omega}$. This expected value is required to model monitoring costs and the credit spread.

$$G(\bar{\omega}_t) = \Phi\left(\frac{log\left(\bar{\omega}_t\right) + \frac{{\sigma_{t-1}}^2}{2}}{\sigma_{t-1}} - \sigma_{t-1}\right). \tag{4.68}$$

Entrepreneurs purchase raw capital from households. Profits of entrepreneurs depend on the return on raw capital purchases. The return on raw capital depends on inflation, current and past raw capital prices, the rental rate for effective capital services and the possibility to deduct taxes on depreciated capital.

$$1 + {R^k}_t = \frac{\pi_t\,\left(\left(1-\tau^k\right)\,\left(u_t\,r_t^k - a(u_t)\right) + (1-\delta)\,q_t\right)}{\Upsilon\,q_{t-1}} + \delta\,\tau^k. \tag{4.69}$$

Mutual funds operate under perfect competition and free entry. This rules out profits of mutual funds. The zero profit condition determines the leverage ratio for a given credit spread.

$$0 = 1 + \frac{\left(1+{R^k}_t\right)\,\frac{\bar{k}_{t-1}\,q_{t-1}}{n_{t-1}}\,\left(G(\bar{\omega}_t)\,(1-\mu) + \bar{\omega}_t\,\left(1-F(\bar{\omega}_t)\right)\right)}{1+R_{t-1}} - \frac{\bar{k}_{t-1}\,q_{t-1}}{n_{t-1}}. \tag{4.70}$$

Entrepreneurs optimal choice of leverage defines the threshold value $\bar{\omega}$ as a nonlinear function of the credit spread, given a dispersion value in the current period. An increase in the credit spread will reduce the threshold value separating insolvent and solvent entrepreneurs.

$$
\begin{aligned}
0 =& \frac{(1-(\bar{\omega}_{t+1}\;(1-F(\bar{\omega}_{t+1}))+G(\bar{\omega}_{t+1})))\;\left(1+{R^{k}}_{t+1}\right)}{1+R_t} \\
&+\frac{1-F(\bar{\omega}_{t+1})}{1-F(\bar{\omega}_{t+1})-\mu\,\Phi(\frac{log(\bar{\omega}_{t+1})+\frac{{\sigma_t}^2}{2}}{\sigma_t})\,\sigma_t}\left(\frac{1+{R^{k}}_{t+1}}{1+R_t}\left(\bar{\omega}_{t+1}\;(1-F(\bar{\omega}_{t+1}))\ldots\right.\right. \\
&\left.\left.+(1-\mu)\,G(\bar{\omega}_{t+1})\right)-1\right).
\end{aligned} \tag{4.71}
$$

Networth of the representative surviving entrepreneur is the sum of current profits (first term), transfers ω^e from households and previous net worth (last term) in (4.72).

$$
\begin{aligned}
n_t = q_{t-1}\,\bar{k}_{t-1}\,\frac{\gamma_t}{{\mu^{z}}_t\,\pi_t}&\left({R^k}_t - R_{t-1} - \left(1+{R^k}_t\right)\;\ldots\right. \\
(G(\bar{\omega}_t)+\bar{\omega}_t\;(1-F(\bar{\omega}_t)) - (G(\bar{\omega})_t\;(1-\mu)+\bar{\omega}_t\;(1-F(\bar{\omega})_t)))&\left.\vphantom{\frac{a}{b}}\right)+\omega^e+ \\
&\frac{n_{t-1}\;(1+R_{t-1})\;\gamma_t}{{\mu^z}_t\,\pi_t}.
\end{aligned} \tag{4.72}
$$

Mutual funds need to monitor default entrepreneurs. Monitoring costs will eat up some of the resources in the economy. Monitoring costs are by a factor μ proportional to their value of assets. Their value of assets depends on the expected value of ω below the threshold $\bar{\omega}$ given by $G(\bar{\omega}_t)$.

$$
dcost(\bar{\omega})_t = \frac{\bar{k}_{t-1}\,q_{t-1}\;\left(1+{R^k}_t\right)\;G(\bar{\omega})_t\,\mu}{{\mu^z}_t\,\Upsilon}. \tag{4.73}
$$

The resource constraint includes now additional terms. These additional terms are monitoring costs $dcost(\bar{\omega}_t)$ and assets used by exiting entrepreneurs $\frac{\Theta\,(1-\gamma_t)\,(n_t-\omega^e)}{\gamma_t}$.

$$
y_t = dcost(\bar{\omega}_t) + c_t + \frac{i_t}{{\mu^{\Upsilon}}_t} + g_t + \frac{\bar{k}_{t-1}}{{\mu^z}_t\,\Upsilon}\,a(u_t) + \frac{\Theta\;(1-\gamma_t)\;(n_t-\omega^e)}{\gamma_t}. \tag{4.74}
$$

4.7.3.3.2 Shocks

The CMR model features additional shocks. Shocks affect directly the financial variables. The most important shock is the so-called risk shock σ_t. This shock is the dispersion in the idiosyncratic productivity of entrepreneurs. This dispersion is driven by unanticipated shocks η^σ and anticipated shocks ξ^s.

$$log\left(\frac{\sigma_t}{\bar{\sigma}}\right) = \rho^{\pi^*}\,log\left(\frac{\sigma_{t-1}}{\bar{\sigma}}\right) + \sigma^\sigma\,{\eta^\sigma}_t + \sum_{s=1}^{S} log\left({\xi^s}_{t-s}\right). \tag{4.75}$$

Anticipated risk shocks are correlated $\sigma(\xi_t^s, \xi s + 1_t)$. The number of signals is a degree of freedom. CMR use 8 shocks in their baseline model.

$$log\left({\xi^s}_t\right) = \begin{cases} \sigma^\xi\,{\eta^{\xi^s}}_t + \left(2\,\sigma(\xi_t^s, \xi s + 1_t) - 1\right)\,log\left({\xi^{s+1}}_t\right) & \text{,if } s < S, \\ \sigma^\xi\,{\eta^{\xi^s}}_t & \text{,if } s = S. \end{cases} \tag{4.76}$$

The survival rate is time-varying and is also labelled equity shock. The survival rate defines how much networth from the previous period remains.

$$log\left(\frac{\gamma_t}{\bar{\gamma}}\right) = \rho^\gamma\,log\left(\frac{\gamma_{t-1}}{\bar{\gamma}}\right) + \sigma^\gamma\,{\eta^\gamma}_t. \tag{4.77}$$

Shocks to the term structure are also included. These shocks are responsible for wedges between the effective short-term risk free interest rates on short-term bonds R and long-term interest rates R^L.

$$log\left(\frac{\zeta_t^{term}}{\bar{\zeta}^{term}}\right) = \rho^{term}\,log\left(\frac{\zeta_{t-1}^{term}}{\bar{\zeta}_t^{term}}\right) + \sigma^{term}\,{\eta^{term}}_t. \tag{4.78}$$

4.7.3.3.3 Observational Equations

One of the main findings in CMR is that the contribution of risk shocks to the business cycle depends on the inclusion of quantitative variables describing the financial market. The CMR model is estimated in addition to the observables from the CEE model with data on extended credit growth (4.79), networth growth (4.80), credit spread (4.81) and the term structure (4.82).

$$\frac{b_t^{obs}}{\bar{b}^{obs}} = \frac{q_t\bar{k}_t - n_t}{q_{t-1}\bar{k}_{t-1} - n_{t-1}}\,\frac{\mu_t^z}{\mu^z}, \tag{4.79}$$

$$\frac{n_t^{obs}}{\bar{n}^{obs}} = \frac{n_t}{n_{t-1}}\,\frac{\mu_t^z}{\mu^z}, \tag{4.80}$$

$$\frac{premium_t^{obs}}{\overline{premium}^{obs}} = \exp\{\mu G_{t-1}(\bar{\omega}_t)\frac{q_{t-1}\bar{k}_t}{q_{t-1}\bar{k}_t - n_t} - \mu G(\bar{\omega})\frac{q\bar{k}}{q\bar{k} - n}\}, \tag{4.81}$$

$$\frac{S_t^{1,obs}}{\bar{S}^{1,obs}} = 1 + R_t^L - R_t. \tag{4.82}$$

4.7.3.4 CMR/CEE–Oil model equations

I will now outline the modifications of the CEE model and CMR model to include oil as production factor. To include oil as production factor I replace equations (4.33), (4.35), and (4.36) with equations (4.83), (4.84), (4.85), (4.86), (4.88) and (4.89). These equations describe the production process. It is also necessary to describe the behaviour of oil supplying firms. The behaviour of oil supplying firms is described by (4.90), (4.91), (4.92), and (4.93). It is necessary to modify the resource constraint to include oil as reported in (4.87). Oil market shocks are introduced with (4.94), (4.95), (4.96), (4.97), and (4.98). A shock for capital productivity is introduced as well (4.99).

4.7.3.4.1 Production

In contrast to the CEE and CMR model the production of final goods is described by a two layer CES production function. The upper layer of a nested CES production function in stationary firm and including price and wage dispersion combines capital-oil composite goods m_t and homogenous labour $l_t = h_t \, {w^*}_t^{\frac{\lambda^w}{\lambda^w-1}}$.

$$y_t = \begin{cases} {p^*}_t^{\frac{\lambda^f}{\lambda^f-1}} \epsilon_t \left({\alpha^M}^{\frac{1}{\eta^M}} \, m_t^{\frac{\eta^M-1}{\eta^M}} + {\alpha^N}^{\frac{1}{\eta^M}} \left({\epsilon^h}_t \, h_t \, {w^*}_t^{\frac{\lambda^w}{\lambda^w-1}} \right)^{\frac{\eta^M-1}{\eta^M}} \right)^{\frac{\eta^M}{\eta^M-1}} - \phi_t, & \text{if } \eta^M \neq 1, \\ {p^*}_t^{\frac{\lambda^f}{\lambda^f-1}} \epsilon_t \, m_t^{\alpha^M} \left({\epsilon^h}_t \, h_t \, {w^*}_t^{\frac{\lambda^w}{\lambda^w-1}} \right)^{\alpha^N} - \phi_t, & \text{if } \eta^M = 1. \end{cases} \tag{4.83}$$

The capital-oil composite production factor combines the primary production factors oil and effective capital. I include specific productivity shocks for both production factors.

$$m_t = \begin{cases} \left({\alpha^K}^{\frac{1}{\eta^O}} \left({\epsilon^m}_t \, \frac{u_t \, \bar{k}_{t-1}}{{\mu^z}_t \, \Upsilon} \right)^{\frac{\eta^O-1}{\eta^O}} + {\alpha^O}^{\frac{1}{\eta^O}} \left({\epsilon^o}_t \, o_t \right)^{\frac{\eta^O-1}{\eta^O}} \right)^{\frac{\eta^O}{\eta^O-1}}, & \text{if } \eta^O \neq 1, \\ \left({\epsilon^m}_t \, \frac{u_t \, \bar{k}_{t-1}}{{\mu^z}_t \, \Upsilon} \right)^{\alpha^K} \left({\epsilon^o}_t \, o_t \right)^{\alpha^O}, & \text{if } \eta^O \neq 1. \end{cases} \tag{4.84}$$

The demand for the capital composite production factor depends on the relative price $\frac{p_t^m}{s_t}$ and the marginal product represented by the right hand side of (4.85).

$$\frac{{p^m}_t}{s_t} = {\alpha^M}^{\frac{1}{\eta^M}} \, \epsilon_t^{\frac{\eta^M-1}{\eta^M}} \left(\frac{m_t}{\phi_t + y_t \, p_t^{*\frac{\lambda^f}{1-\lambda^f-1}}} \right)^{\frac{(-1)}{\eta^M}}. \tag{4.85}$$

The demand for hours worked depends on the relative price $\frac{w_t}{s_t}$ and its marginal product represented by the right hand side of (4.86).

$$\frac{w_t}{s_t} = {\alpha^N}^{\frac{1}{\eta^M}} \, \epsilon_t^{\frac{\eta^M-1}{\eta^M}} \, {\epsilon^h}_t^{\frac{\eta^M-1}{\eta^M}} \left(\frac{h_t \, {w^*}_t^{\frac{\lambda^w}{\lambda^w-1}}}{\phi_t + y_t \, p_t^{*\frac{\lambda^f}{1-\lambda^f}}} \right)^{\frac{(-1)}{\eta^M}}. \tag{4.86}$$

The resource constraint of the economy changes. It now features oil export revenues and oil import expenditures. Oil export revenues increase the funds disposable for different purposes. Oil import expenditures require goods to pay for them.

$$y_t = c_t + \frac{i_t}{\mu_t^{\Upsilon}} + g_t + \frac{\bar{k}_{t-1}\, a(u_t)}{{\mu^z}_t\, \Upsilon} - p_t^o\ \left({o^{ex}}_t - {o^f}_t\right) \qquad (4.87)$$
$$\begin{cases} +dcost(\bar{\omega})_t + \frac{\Theta\,(1-\gamma_t)\,(n_t-\omega^e)}{\gamma_t} & \text{, for CMR–Oil model,} \\ & \text{for CEE–Oil model.} \end{cases}$$

4.7.3.4.2 Oil market

Demand for oil in the economy is given by the first order condition of representative intermediate goods producers. Intermediate goods producers demand oil as long as its marginal cost does not exceed its marginal product (right hand side of (4.88)).

$$\frac{p_t^o}{{p^m}_t} = {\alpha^O}^{\frac{1}{\eta^O}}\ {\epsilon^o}_t^{\frac{\eta^O-1}{\eta^O}}\ \left(\frac{o_t}{m_t}\right)^{\frac{(-1)}{\eta^O}}. \qquad (4.88)$$

As for oil demand for capital is given by the first order condition of the intermediate goods producer. In case of $\eta^O = 1$ and an oil share equal to zero this equation is identical to (4.35).

$$\frac{{r^k}_t}{{p^m}_t} = {\alpha^K}^{\frac{1}{\eta^O}}\ {\epsilon^m}_t^{\frac{\eta^O-1}{\eta^O}}\ \left(\frac{\frac{u_t\,\bar{k}_{t-1}}{{\mu^z}_t\,\Upsilon}}{m_t}\right)^{\frac{(-1)}{\eta^O}}. \qquad (4.89)$$

The previous equations represent the behaviour of the demand side for oil in the economy. Now the supply side is considered. The first order condition derived from the profit maximization problem of domestic oil producers equates the marginal product $p_t^o(1-\tau_t^o)$ with the marginal cost of providing one more unit of oil (right hand side of(4.90)). This is the domestic oil supply curve.

$$p_t^o\ (1-{\tau^o}_t) = \left(\frac{{\zeta^o}_t}{\gamma^o}\right)^{1+\sigma^O}\ {o^d}_t^{\ \sigma^O}. \qquad (4.90)$$

Oil importers also supply oil according to a supply curve. This supply curve is derived from their profit maximization problem. Costs for supplying importing oil to domestic intermediate goods producers are not identical.

$$p_t^o\ (1-{\tau^o}_t) = \left(\frac{{\zeta^{o,im}}_t}{\gamma^{o^{im}}}\right)^{1+\sigma^O}\ {o_t^{im}}^{\sigma^O}. \qquad (4.91)$$

In contrast to domestic oil producers and oil importers the supply o foil exports is not the result of an optimization problem. It is modelled as the share of domestically produced oil, which is not consumed domestically.

$$o_t^{ex} = {o^d}_t {\zeta^{o,ex}}_t. \tag{4.92}$$

Domestically produced oil and imported oil represent the available oil in one period. This oil supply can either be consumed or exported as stated in (4.93).

$$o_t + o_t^{ex} = o_t^d + o_t^{im}. \tag{4.93}$$

4.7.3.4.3 Shocks

Costs for providing domestic crude oil can fluctuate over time. This motivates the inclusion of a domestic oil cost shock ζ^o.

$$log\left(\frac{{\zeta^o}_t}{\zeta^o}\right) = \rho^{\zeta^o}\, log\left(\frac{{\zeta^o}_{t-1}}{\zeta^o}\right) + \sigma^{\zeta^o}\, {\eta^{\zeta^o}}_t. \tag{4.94}$$

The same is true for import oil. Costs for providing imported crude oil might have different short-.term developments than costs for domestically produced oil. In order to capture such differences $\zeta^{o,im}$ is included.

$$log\left(\frac{{\zeta^{o,im}}_t}{\zeta^{o,im}}\right) = \rho^{\zeta^{o,im}}\, log\left(\frac{{\zeta^{o,im}}_{t-1}}{\zeta^{o,im}}\right) + \sigma^{\zeta^{o,im}}\, {\eta^{\zeta^{o,im}}}_t. \tag{4.95}$$

The share of oil exported relative to overall domestic oil production is not constant. Therefore, a shock to the share of exported oil $\zeta^{o,ex}$ is included.

$$log\left(\frac{{\zeta^{o,ex}}_t}{\zeta^{o,ex}}\right) = \rho^{\zeta^{o,ex}}\, log\left(\frac{{\zeta^{o,ex}}_{t-1}}{\zeta^{o,ex}}\right) + \sigma^{\zeta^{o,ex}}\, {\eta^{\zeta^{o,ex}}}_t. \tag{4.96}$$

In the US different tax rates on crude oil are applied in the federal states. I model only a simplified tax system. Taxes paid by oil suppliers τ^o are modelled as an autoregressive process of order one.

$$log\left(\frac{{\tau^o}_t}{\tau^o}\right) = \rho^{\tau^o}\, log\left(\frac{{\tau^o}_{t-1}}{\tau^o}\right) + \sigma^{\tau^o}\, {\eta^{\tau^o}}_t. \tag{4.97}$$

Demand for oil is also driven by the efficiency of oil. The quality of crude oil can vary over time. It is also possible that extending the quantity of used oil in refineries will impact the effect of oil on the productivity of the factor.

$$log\left(\frac{{\epsilon^o}_t}{\bar{\epsilon^o}}\right) = \rho^{\epsilon^o}\, log\left(\frac{{\epsilon^o}_{t-1}}{\bar{\epsilon^o}}\right) + \sigma^{\epsilon^o}\, {\eta^{\epsilon^o}}_t. \tag{4.98}$$

The same reasons to include productivity shocks for oil apply to effective capital. It is possible to use ϵ^k for permanent shocks. A combination of ϵ^o and ϵ^k is interesting to study potential mitigation scenarios.

$$log\left(\frac{\epsilon^k{}_t}{\bar{\epsilon^k}}\right) = \rho^{\epsilon^k}\, log\left(\frac{\epsilon^k{}_{t-1}}{\bar{\epsilon^k}}\right) + \sigma^{\epsilon^k}\, \eta^{\epsilon^k}{}_t. \tag{4.99}$$

4.7.3.4.4 Observational Equations

In addition to the observables in the MCR and CEE model I introduce observables for the oil market. The oil makret obseervagbles are oil consumption growth (4.100), domestic oil production growth (4.101), oil import growth (4.102), oil exports growth (4.103) and real price of oil changes (4.104).

$$o^{obs}{}_t = \frac{\mu^z{}_t}{\bar{\mu^z}}\frac{o_t}{o_{t-1}}\overline{o}^{obs}, \tag{4.100}$$

$$o^{d,obs}{}_t = \frac{\mu^z{}_t}{\bar{\mu^z}}\frac{o^d_t}{o^d_{t-1}}\overline{o}^{d,obs}, \tag{4.101}$$

$$o^{im,obs}{}_t = \frac{\mu^z{}_t}{\bar{\mu^z}}\frac{o^{im}_t}{o^{im}_{t-1}}\overline{o}^{im,obs}, \tag{4.102}$$

$$o^{ex,obs}{}_t = \frac{\mu^z{}_t}{\bar{\mu^z}}\frac{o^{ex}_t}{o^{ex}_{t-1}}\overline{o}^{ex,obs}, \tag{4.103}$$

$$p^{o,obs}_t = \overline{p}^{o,obs}\frac{p^o_t}{p^o_{t-1}}. \tag{4.104}$$

$$\tag{4.105}$$

4.7.4 Steady-state

4.7.4.1 Calibration

For the estimation of the model around a deterministic steady-state the following algorithm is used.

1. define the following steady-state shares:
   (a) rental rate on capital services r^k
   (b) capital expenditure share $\phi^K = \frac{r^k \frac{\bar{k}}{\mu^*_z \Upsilon}}{y}$
   (c) oil expenditure share $\phi^O = \frac{p^O o}{y}$
   (d) oil to output ratio $\frac{o}{y}$
   (e) oil and capital expenditure share $\phi^M = \phi^K + \phi^O$
   (f) domestic oil share $\theta^{\frac{o^d}{o}} = \frac{o^d}{o}$
   (g) oil exports share $\theta^{\frac{o^{ex}}{o^d}} = \frac{o^ex}{o^d}$

   (h) oil imports share $\theta^{\frac{o^{im}}{o}} = \frac{o^{im}}{o}$
   (i) oil tax rate τ^o
   (j) steady-state output $\bar{y}$

2. set the following variables to pre-defined values under a flexible price equilibrium:
   (a) mark-up λ^f
   (b) long-run growth rate μ_z^*
   (c) investment specific long-run growth rate μ^Υ
   (d) gross inflation and inflation target π, π^*
   (e) retained earnings of entrepreneurs γ
   (f) hours worked h, capital utilization rate u, price dispersion index p^* and wage dispersion index w^* are equal to one

3. compute the following variables:
   (a) marginal cost $s = \frac{1}{\lambda^f}$
   (b) fixed cost $\phi = \frac{1-s}{s}\, y$
   (c) price of raw capital $q = \frac{1}{\mu^\Upsilon}$
   (d) short-run and long-run interest rate $R = R^L = \frac{\pi \mu_z^*}{\beta} - 1$
   (e) return on capital $R^K = \frac{\{(1-\tau^k)\, r^k + 1 - \delta\}\pi}{\Upsilon} + \tau^k \delta - 1$
   (f) interest rate spread $s^p = \frac{1+R^K}{1+R}$

4. compute $\phi = y\, \frac{1-s}{s}$

5. compute $\bar{k} = \frac{\phi^K y \mu^z\, \Upsilon}{r^k (1+\psi^k\, R)}$

6. compute $w = \frac{\{1-(\phi^K+\phi^O)\} y\, s}{h}$

7. compute $o = \theta^{\frac{o}{y}}\, y$

8. compute $o^{im} = \theta^{\frac{o^{im}}{o}}\, o$

9. compute $o^d = \frac{o - o^{im}}{1-\phi^{o^{ex}}}$

10. compute $o^{ex} = \theta^{o^{ex}}\, o^d$

11. compute $p^o = \phi^O\, \frac{s\, y}{o}$

12. if $\eta^O = 1$ do
    (a) compute $\alpha^O = \frac{\phi^O}{\phi^O + \phi^K}$

(b) compute $\alpha^K = \frac{\phi^K}{\phi^O+\phi^K}$

(c) compute $p^M = \left(\frac{p^O}{\alpha^O}\right)^{\alpha^O} \left(\frac{r^k}{\alpha^K}\right)^{\alpha^K}$

(d) compute $m = \frac{y\,(\phi^O+\phi^K)}{p^M}$

13. if $\eta^O \neq 1$ do

(a) compute $p^M = \left(\frac{\phi^O}{\phi^O+\phi^K} p^{O\eta^O-1} + \frac{\phi^K}{\phi^O+\phi^K} r^{k\eta^O-1}\right)^{\frac{1}{\eta^O-1}}$

(b) compute $m = \frac{y\,(\phi^O+\phi^K)}{p^M}$

(c) compute $\alpha^O = \left(\frac{p^O}{p^M}\right)^{\eta^O} \frac{o}{m}$

(d) compute $\alpha^K = \left(\frac{r^k}{p^M}\right)^{\eta^O} \frac{\frac{u\,\bar{k}}{\Upsilon\mu^z}}{m}$

14. if $\eta^M = 1$ do

(a) compute $\alpha^M = \phi^O + \phi^K$

(b) compute $\alpha^N = 1 - \alpha^M$

(c) compute $\epsilon = s^{-1} \left(\frac{p^M}{\alpha^M}\right)^{\alpha^M} \left(\frac{w}{\alpha^N}\right)^{\alpha^N}$

15. if $\eta^M \neq 1$ do

(a) compute $\epsilon = s^{-1} \left(\phi^M p^{M\eta^M-1} + (1-\phi^M) w^{\eta^M-1}\right)^{\frac{1}{\eta^M-1}}$

(b) compute $\alpha^M = \left(\frac{p^M}{s}\right)^{\eta^M} \frac{m}{y+\phi}$

(c) compute $\alpha^N = \left(\frac{w}{s}\right)^{\eta^O} \frac{h}{y+\phi}$

16. compute $i = (1 - \frac{1-\delta}{\mu^z\,\Upsilon})\,\bar{k}$

17. compute $dcost = \frac{\mu\,G\,(1+r^k)\bar{k}}{\pi\mu^z}$

18. solve the contract problem of entrepreneurs for the monitoring cost parameter μ the bankruptcy threshold $\bar{\omega}$ and the idiosyncratic dispersion σ.
use numerical procedure to find μ such that $|\epsilon^\mu| < i^{Tol}$

(a) use numerical procedure to find $\bar{\omega}$ such that $|\epsilon^{\bar{\omega}}| < i^{Tol}$

i. use numerical procedure to find σ such that $|\epsilon^\sigma| < i^{Tol}$

A. guess σ

B. define $z = \frac{log(\bar{\omega})+0.5\,\sigma^2}{\sigma}$

C. calculate $\epsilon^{\sigma} = \bar{F} - \Phi(z)$

ii. define $\Gamma = \Phi(z-\sigma) + \bar{\omega}(1-\Phi(z))$

iii. define $G = \mu\,\Phi(z-\sigma)$

iv. define $n = \frac{(w^e + \frac{\gamma}{\pi\,\mu^z}(r^k - R - \mu G\,(1+r^k)\,\bar{k})}{1-\gamma\,\frac{1+R}{\pi\,\mu^z}}$

v. calculate $\epsilon^{\bar{\omega}} = (1-\Gamma)\,s^p - \frac{1-\bar{F}}{1-\bar{F}-\mu\omega\varphi(z)}\,(s^p(\Gamma - \mu\,G) - 1)$

(b) calculate $\epsilon^{\mu} = \frac{n}{k} - (1 - s^p\,(\Gamma - \mu\,G))$

19. compute $c = (1-\eta^g)\,y + (o^d - o)\,\frac{p^o}{\mu^o} - d - \Theta\,\frac{1-\gamma}{\gamma}\,(n - w^e) - \frac{i}{\mu^{\Upsilon}}$
20. compute $g = \frac{\eta^g}{1-\eta^g}\,(c + \frac{i}{\mu^{\Upsilon}})$
21. compute $\lambda^z = \frac{\zeta^c}{(1+\tau^c)\,c}\,\frac{\mu^z - b\beta}{\mu^z - b}$
22. compute $\Psi^L = \frac{(1-\tau^l)\,\lambda^z\,w\,h^{-\sigma^L}}{\zeta^h\,\lambda^w}$
23. compute $F^p = \frac{\lambda^z\,y}{1-\beta\,\xi^p}$
24. compute $K^p = \frac{\lambda^z\,y\,s\,\lambda^f}{1-\beta\,\xi^p}$
25. compute $F^w = \frac{h\,(1-\tau^l)\lambda^z}{\lambda^w\,(1-\beta\,\xi^w)}$
26. compute $K^w = \frac{h^{1+\sigma^L}}{1-\beta\,\xi^w}$

4.7.4.2 Permanent shock

For the computation of impulse response functions to permanent shocks I need to modify the steady-state routine.

1. solve the oil consumption identity, the first order condition defining labour supply, the first oder condition of entrepreneurs with respect to leverage ratio and the constraint of the optimality problem of entrepreneurs

2. guess the real price of oil p^o, the capital stock $\bar{k}$, the rental rate of capital r^k and the threshold value dividing entrepreneurs into solvent and insolvent firms $\bar{\omega}$ such that $\begin{pmatrix} |\epsilon^{p^o}| \\ |\epsilon^{\bar{k}}| \\ |\epsilon^{r^k}| \\ |\epsilon^{\bar{\omega}}| \end{pmatrix} < i^{Tol}$

3. set mark-up λ^f

4. set long-run growth rate μ_z^*

5. set investment specific long-run growth rate μ^{Υ}
6. set gross inflation and inflation target π, π^*
7. set retained earnings of entrepreneurs γ
8. set capital utilization rate u, price dispersion index p^* and wage dispersion index w^* to one
9. compute marginal cost $s = \frac{1}{\lambda^f}$
10. compute fixed cost $\phi = \frac{1-s}{s}\, y$
11. compute price of raw capital $q = \frac{1}{\mu^{\Upsilon}}$
12. compute short-run and long-run interest rate $R = R^L = \frac{\pi \mu_z^*}{\beta} - 1$
13. $o = \left(\frac{p^o}{r^k}\right)^{-\eta^O} \ \frac{\alpha^O}{\alpha^K}\, \frac{\bar{k}}{\mu^z\, \Upsilon}\, \epsilon^{o\eta^O - 1}$
14. $o^{im} = \left(\frac{p^o\,(1-\tau^o)}{\left(\frac{\zeta^{o^{im}}}{\gamma^{o^{im}}}\right)^{1+\sigma^o}}\right)^{\frac{1}{\sigma^o}}$
15. $o^d = \left(\frac{p^o\,(1-\tau^o)}{\left(\frac{\gamma^o}{\zeta^o}\right)^{1+\sigma^o}}\right)^{\frac{1}{\sigma^o}}$
16. $o^{ex} = \zeta^{o^{ex}}\, o^d$
17. if $\eta^O = 1$ do
    (a) $p^m = \left(\frac{r^k}{\alpha^K}\right)^{\alpha^K} \ \left(\frac{p^o}{\alpha^O}\right)^{\alpha^O}$
    (b) $m = \left(\epsilon^M\, \frac{\bar{k}}{\mu^z\, \Upsilon}\right)^{\alpha^K} \ (\epsilon^o\, o)^{\alpha^O}$
18. if $\eta^O \neq 1$ do
    (a) $p^m = \left(\alpha^K \ \left(\frac{r^k}{\epsilon^M}\right)^{1-\eta^O} + \alpha^O \ \left(\frac{p^o}{\epsilon^o}\right)^{1-\eta^O}\right)^{\frac{1}{1-\eta^O}}$
    (b) $m = \left(\alpha^{K\,\frac{1}{\eta^O}} \ \left(\epsilon^M\, \frac{\bar{k}}{\mu^z\, \Upsilon}\right)^{\frac{\eta^O-1}{\eta^O}} + \alpha^{O\,\frac{1}{\eta^O}}\, (\epsilon^o\, o)^{\frac{\eta^O-1}{\eta^O}}\right)^{\frac{\eta^O}{\eta^O-1}}$
19. $\rho^m = \frac{\eta^M - 1}{\eta^M}$
20. if $\eta^M = 1$

(a) $w = s\,\epsilon\left\{\left(\frac{p^m}{\alpha^M}\right)^{\alpha^M}\right\}^{\frac{-1}{\alpha^N}}\ \alpha^N$

(b) $h = m\,\left(\frac{w}{p^m}\right)^{-\eta^M}\ \frac{\alpha^N}{\alpha^M}\,\epsilon h^{\eta^M - 1}$

(c) $y = s\,\epsilon m^{\alpha^M}\,(\epsilon^h\,h)^{\alpha^N}$

21. if $\eta^M \neq 1$

(a) $w = \left((s*\epsilon)^(1-\eta^M) - \frac{\alpha^M}{\alpha^N}\,p^{m1-\eta^M}\right)^{\frac{1}{1-\eta^M}}\ \epsilon^h$

(b) $h = m\,\left(\frac{w}{p^m}\right)^{-\eta^M}\ \frac{\alpha^N}{\alpha^M}\,\epsilon h^{\eta^M - 1}$

(c) $y = s\,\epsilon\,\alpha^{M\frac{1}{\eta^M}}\ m^{\rho^m} + \alpha^{N\frac{1}{\eta^M}}\,(\epsilon^h\,h)^{\rho^m})^{\frac{1}{\rho^m}}$

22. $\phi = y\,\frac{1-s}{s}$

23. $n = \frac{\bar{k}\,\gamma}{\pi\,\mu^z}\,\frac{R^k - R - \mu\,G\,(1+R^k)}{1-\gamma\,\frac{1+R}{\pi\,\mu^z} - \frac{w^e}{(1-s^p\,(\Gamma-\mu\,G))\,\bar{k}}}$

24. compute $i = (1 - \frac{1-\delta}{\mu^z\,\Upsilon})\,\bar{k}$

25. compute $d = \frac{\mu\,G\,(1+r^k)\bar{k}}{\pi\mu^z}$

26. define $z = \frac{log(\bar{\omega})+0.5\,\sigma^2}{\sigma}$

27. compute $\bar{F} = \Phi(z)$

28. define $\Gamma = \Phi(z-\sigma) + \bar{\omega}(1-\Phi(z))$

29. compute $G = \mu\,\Phi(z-\sigma)$

30. define $n = \frac{(w^e + \frac{\gamma}{\pi\,\mu^z}(r^k - R - \mu G\,(1+r^k)\,\bar{k})}{1-\gamma\,\frac{1+R}{\pi\,\mu^z}}$

31. compute $c = (1-\eta^g)\,y + (o^d - o)\,\frac{p^o}{\mu^o} - d - \Theta\,\frac{1-\gamma}{\gamma}\,(n - w^e) - \frac{i}{\mu^\Upsilon}$

32. compute $g = \frac{\eta^g}{1-\eta^g}\,(c + \frac{i}{\mu^\Upsilon})$

33. compute $\lambda^z = \frac{\zeta^c}{(1+\tau^c)\,c}\,\frac{\mu^z - b\beta}{\mu^z - b}$

34. compute $F^p = \frac{\lambda^z\,y}{1-\beta\,\xi^p}$

35. compute $K^p = \frac{\lambda^z\,y\,s\,\lambda^f}{1-\beta\,\xi^p}$

36. compute $F^w = \frac{h\,(1-\tau^l)\lambda^z}{\lambda^w\,(1-\beta\,\xi^w)}$

37. compute $K^w = \frac{h^{1+\sigma^L}}{1-\beta\,\xi^w}$

38. compute residuals for the following model equations

    (a) compute $\epsilon^{\bar{k}} = \Psi^L - \frac{(1-\tau^l)\,\lambda^z\,w\,h^{-\sigma^L}}{\zeta^h\,\lambda^w}$

    (b) compute $\epsilon^{p^o} = o - (o^d - o^{ex} + o^{im})$

    (c) compute $\epsilon^{r^k} = w^e - \left(1 - \frac{\gamma}{\pi\,\mu^z}\right)\,(R^k - R - \mu\,G\,(1+R^k))\,\bar{k} - \frac{\gamma\,(1+R)}{\pi\,\mu^z}\,n$

    (d) compute $\epsilon^{\bar{\omega}} = (1-\Gamma)\,s^p - \frac{1-\bar{F}}{1-\bar{F}-\mu\omega\varphi(z)}\,(s^p(\Gamma - \mu\,G) - 1)$

4.7.5 Sufficient conditions for a minimum of the cost minimization problem

I will now discuss sufficient conditions for a minimum of the intermediate goods producing firm's cost minimization problem. Intermediate goods producing firms use homogenous labour $l_{j_f,t}$, capital services $K^s_{j_f,t}$ and crude oil $O_{j_f,t}$. These production factors are combined in a two layer nested CES function to produce intermediate goods $Y_{j_f,t}$. The cost minimization problem of the firm is

$$\begin{aligned} \min_{l_{j_f,t},K^s_{j_f,t},O_{j_f,t}} \; & W_t l_{j_f,t} + P_t \tilde{r}^k_t K^s_{j_f,t} + P^O_t O_{j_f,t}, \\ s.t. Y_{j_f,t} &= X(M(O_{j_f,t}, K^s_{j_f,t}), l_{j_f,t}) - \phi_t z_t, \\ l_{j_f,t} > 0,\; K^s_{j_f,t} > 0,\; & O_{j_f,t} > 0,\; M_{j_f,t} > 0,\; Y_{j_f,t} > 0. \end{aligned} \tag{4.106}$$

Intermediate goods producers pay wages $W_{j_f,t}$, rental rates on capital $P_t\tilde{r}^k_t$, and a price for crude oil P^O_t. In addition to variable costs firms also have fixed costs $z_t\phi_t$. The production functions for total output $X(M(O_{j_f,t}, K^s_{j_f,t}), h_{j_f,t})$ and the capital-oil composite production factor $M_{j_f,t} = M(O_{j_f,t}, K^s_{j_f,t})$ are given by

$$X(M_{j_f,t}, l_{j_f,t}) = \begin{cases} \epsilon_t\, M_{j_f,t}^{\alpha_M}\,(z_t l_{j_f,t})^{1-\alpha_M} \text{,if } \eta^M = 1, \\ \epsilon_t \left[(\alpha_M)^{\frac{1}{\eta^M}}\, M_{j_f,t}^{\rho^M} + (1-\alpha_M)^{\frac{1}{\eta^M}}\,(z_t l_{j_f,t})^{\rho^M}\right]^{\frac{1}{\rho^M}} \text{ otherwise.} \end{cases} \tag{4.107}$$

$$M(K^s_{j_f,t}, O_{j_f,t}) = \begin{cases} \left(\epsilon^O_t \frac{O_{j_f,t}}{\Upsilon^{O^t}}\right)^{\alpha_O} \left(\epsilon^K_t \frac{K^s_{j_f,t}}{\Upsilon^{t-1}}\right)^{1-\alpha_O} \text{,if } \eta^O = 1, \\ \left\{(1-\alpha_O)^{\frac{1}{\eta^O}} \left(\epsilon^K_t \frac{K^s_{j_f,t}}{\Upsilon^{t-1}}\right)^{\rho^O} + (\alpha_O)^{\frac{1}{\eta^O}} \left(\epsilon^O_t \frac{O_{j_f,t}}{\Upsilon^{O^t}}\right)^{\rho^O}\right\}^{\frac{1}{\rho^O}} \text{ otherwise.} \end{cases} \tag{4.108}$$

The corresponding Lagrangian, ignoring the non-negativity constraints, of the problem is

$$\mathcal{L}_t^{\mathrm{F,min}} = W_t\, l_{j_f,t} + P_t\, \tilde{r}_t^k\, K_{j_f,t}^s + P_t^O O_{j_f,t} + S_t\{Y_{j_f,t} - (X(M(O_{j_f,t}, K_{j_f,t}^s), l_{j_f,t}) - \phi z_t)\}. \tag{4.109}$$

The necessary conditions for a stationary point of (4.109) are

$$\frac{\partial \mathcal{L}_t^{\mathrm{F,min}}}{\partial l_{j_f,t}} : 0 = \quad W_t - X_{l,j_f,t} = W_t - S_t z_t^{\frac{\eta^M - 1}{\eta^M}} \epsilon_t (\alpha_N)^{\frac{1}{\eta^M}} \left(\frac{X_{j_f,t}}{l_{j_f,t}}\right)^{\frac{1}{\eta^M}}, \tag{4.110}$$

$$\frac{\partial \mathcal{L}_t^{\mathrm{F,min}}}{\partial K_{j_f,t}^s} : 0 = \quad P_t \tilde{r}_t^k - X_{K^s,j_f,t} = P_t \tilde{r}_t^k - P_t^M (1-\alpha_O)^{\frac{1}{\eta^O}} (\Upsilon^{t-1})^{-\rho^O} (\epsilon^K{}_t)^{\rho^O} \left(\frac{M_{j_f,t}}{K_{j_f,t}^s}\right)^{\frac{1}{\eta_O}}, \tag{4.111}$$

$$\frac{\partial \mathcal{L}_t^{\mathrm{F,min}}}{\partial O_{j_f,t}} : 0 = \quad P_t^O - X_{O,j_f,t} = P_t^O - P_t^M (\alpha_O)^{\frac{1}{\eta^O}} (\Upsilon^{O^t})^{-\rho^O} (\epsilon^O{}_t)^{\rho^O} \left\{\frac{M_{j_f,t}}{O_{j_f,t}}\right\}^{\frac{1}{\eta^O}}, \tag{4.112}$$

$$\frac{\partial \mathcal{L}_t^{\mathrm{F,min}}}{\partial S_t} : 0 = \quad X_{j_f,t} - X(l_{j_f,t}, M_{j_f,t}). \tag{4.113}$$

I define an auxiliary variable $P_t^M = S_t\, X_{M,j_f,t} = S_t\, z_t^{\rho^M}\, \epsilon_t\, \alpha_M^{\frac{1}{\eta^M}} \left(\frac{X_{j_f,t}}{l_{j_f,t}}\right)^{\frac{1}{\eta^M}}$ to define the partial derivative of total output $X_{j_f,t}$ with respect to $M_{j_f,t}$ times marginal costs. For the following analysis I will drop the time index t and the index for firms j_f.

I apply Theorem 1.14 in De la Fuente (2000) to check whether the solution to (4.110), (4.111), (4.112), (4.113). is indeed a minimizer of the cost function. The optimization problem consists of choice variables $\mathbf{x}_t = [O_{j_f,t}, K_{j_f,t}^s, h_{j_f,t}]^\top$, an objective function $\mathcal{F}(\mathrm{x}_t) = W_t\, h_{j_f,t} + P_t^O\, O_{j_f,t} + P_t\, r_t^k\, K_{j_f,t}^s$ and one constraint $\mathcal{G}(\mathbf{x}_t) = X(M_{j_f,t}, h_{j_f,t}) - X_{j_f,t}$. The optimization problem in compact notation is $\{\min_{\mathbf{x_t}} \mathcal{F}(\mathrm{x}_t); \mathcal{G}(\mathbf{x}_t) = 0\}$. According to Theorem 1.14 the objective function $\mathcal{F}$ needs to be pseudo-convex and all constraints quasi-concave $\mathcal{G}^j$ for a minimum. $\mathcal{F}$ is pseudo-convex for positive factor prices. All constraints need to be quasi-concave. If all constraints are concave they are also quasi-concave. I show that the CES production function is concave if $\rho^{M,O} \in (0,1)$ and $\alpha^{M,O} > 0$. The Hessian matrix of $X(M_{j_f,t}, h_{j_f,t})$ is denoted by $\mathcal{H}^X$.

$$\mathcal{H}^X = \begin{pmatrix} \frac{\partial^2 X}{\partial O^2} & \frac{\partial^2 X}{\partial O \partial K^s} & \frac{\partial^2 X}{\partial O \partial l} \\ \frac{\partial^2 X}{\partial O \partial K^s} & \frac{\partial^2 X}{\partial {K^s}^2} & \frac{\partial^2 X}{\partial K^s \partial l} \\ \frac{\partial^2 X}{\partial O \partial l} & \frac{\partial^2 X}{\partial K^s \partial l} & \frac{\partial^2 X}{\partial l^2} \end{pmatrix} = \begin{pmatrix} X_{OO} & X_{OK^s} & X_{Ol} \\ X_{K^s O} & X_{K^s K^s} & X_{K^s l} \\ X_{lO} & X_{lK^s} & X_{K^s K^s} \end{pmatrix}.$$

$$X_{OO} = -\frac{1}{\eta^M} X_M \left(\frac{1}{M} - \frac{X_M}{X} \right) M_O^2 - \frac{1}{\eta^O} \left(\frac{1}{O} - \frac{M_O}{M}) \, M_O \, X_M \right).$$

$$X_{K^s K^s} = -\frac{1}{\eta^M} X_M \left(\frac{1}{M} - \frac{X_M}{X} \right) M_{K^s}^2 - \frac{1}{\eta^O} \left(\frac{1}{O} - \frac{M_K^s}{M}) \, M_{K^s} \, X_M \right).$$

$$X_{ll} = -\frac{1}{\eta^M} X_l \left(\frac{1}{l} - \frac{X_l}{X} \right).$$

$$X_{OK^s} = -\frac{1}{\eta^M} X_M \left(\frac{1}{M} - \frac{X_M}{X} \right) M_{K^s} \, M_O + \frac{1}{\eta^O} M_O \, \frac{M_{K^s}}{M} X_M.$$

$$X_{Ol} = \frac{1}{\eta^M} X_l \, X_M \, M_O \, \frac{1}{X}.$$

In order to check that the matrices are negative semidefinite, the first principal minor needs to be harmful, and the sign of the principal minors are alternating. The leading principal minors $\kappa_{1,2,3}^{minor}$ are

$$\kappa_1^{minor} = X_{OO}, \tag{4.114}$$

$$\kappa_2^{minor} = X_{OO} \, X_{K^s K^s} - {X_{OK^s}}^2, \tag{4.115}$$

$$\begin{aligned} \kappa_3^{minor} &= X_{OO} \, X_{K^s K^s} \, X_{ll} + X_{OK^s} \, X_{Ol} \, X_{lO} + X_{Ol} \, X_{K^s O} \, X_{lK^s} \\ &\quad - X_{OK^s} \, X_{OK^s} \, X_{ll} - X_{OO} \, X_{Kl} \, X_{lK} - X_{Ol} \, X_{K^s K^s} \, X_{lO}. \end{aligned} \tag{4.116}$$

The first principal minor is the second derivative of output with respect to oil. This term is negative, if $\alpha^{M,O,N,K} > 0$ and $\eta^{M,O} > 0$. Further, note that $X(M,l)$ and $M(K^s,O)$ are both homogenous of degree one. This implies that the following identities hold

$$M = M_{K^s} \, K^s + M_O \, O, \tag{4.117}$$

$$X = X_M \, M + X_l \, l. \tag{4.118}$$

It is now necessary to show that the second principal minor is positive. Under the parameter restrictions this indeed is the case.

$$\kappa_2^{minor} = X_{OO}\, X_{K^s K^s} - X_{OK^s}{}^2,$$
$$\kappa_2^{minor} = \left(a\,\frac{K^s}{O} + b\,\frac{M_O}{M_{K^s}}\right)\left(a\,\frac{O}{K^s} + b\,\frac{M_{K^s}}{M_O}\right) - (a-b)^2,$$
$$\kappa_2^{minor} = \left(2 + \frac{M_O O}{M_{K^s} K^s} + \frac{M_{K^s} K^s}{M_O O}\right) a\, b > 0,$$
$$a = \frac{M_{K^s}\, M_O\, X_M}{M\,\eta^O} > 0,$$
$$b = \frac{M_{K^s}\, M_O\, X_M\, X_l\, l}{M\, X\, \eta^M} > 0.$$

The third principal minor is the determinant of the Hessian matrix $\mathcal{H}^X$. The production function is homogenous of degree one, and the determinant of the Hessian matrix is zero. One can derive the following expression for the determinant.

$$\kappa_3^{minor} = X_{OO}\, X_{K^s K^s}\, X_{ll} + X_{OK^s}\, X_{Ol}\, X_{lO} + X_{Ol}\, X_{K^s O}\, X_{lK^s}$$
$$- X_{OK^s}\, X_{OK^s}\, X_{ll} - X_{OO}\, X_{Kl}\, X_{lK} - X_{Ol}\, X_{K^s K^s}\, X_{lO},$$
$$\kappa_3^{minor} = -\frac{a\,b\,c\,d}{e} + \frac{a\,b\,c\,d}{e},$$
$$a = M_{K^s}^4\, M_O^4\, X_l^5\, X_M^6\,(X - X_l\, l),$$
$$b = M_{K^s}^2\, X_M \left(\frac{X_M}{X} - \frac{1}{M}\right)\frac{1}{\eta^M} + M_{K^s}\, X_M \left(\frac{M_{K^s}}{M} - \frac{1}{K^s}\right)\frac{1}{\eta^O},$$
$$c = M_O^2\, X_M \left(\frac{X_M}{X} - \frac{1}{M}\right)\frac{1}{\eta^M} + M_O\, X_M \left(\frac{M_O}{M} - \frac{1}{O}\right)\frac{1}{\eta^O},$$
$$d = (X\,\eta^M - X\,\eta^O + M\, X_M\,\eta^O)^2,$$
$$e = M^2\, X^7\, {\eta^M}^7\, {\eta^O}^2\, l.$$

The determinant is zero, and this implies that the Hessian matrix is negative semidefinite. The optimization problem satisfies the conditions for Theorem 1.14 to apply.

4.8 Online Appendix

4.8.1 Model derivation

4.8.1.1 Scaling and observational equations

I will now explicitly state the scaling of the different variables to transform the non-stationary model to a stationary model. The following scaling is applied:

$$q_t = \frac{Q_{\bar{K},t}\Upsilon^t}{P_t}, \quad y_{z,t} = \frac{Y_t}{z_t}, \quad i_t = \frac{I_t}{z_t \Upsilon^t}, \quad w_t = \frac{W_t}{z_t P_t}, \quad \lambda_{z,t} = P_t z_t \lambda_t,$$
$$k_t = \frac{\bar{K}_t}{z_{t-1}\Upsilon^{t-1}}, \quad \mu_t^z = \mu^z \frac{z_t}{z_{t-1}}, \quad c_t = \frac{C_t}{z_t}, \quad n_{t+1} = \frac{N_{t+1}}{P_t\, z_t},$$
$$r_t^k = \frac{\tilde{r}_t^k}{\Upsilon^t} \quad o_t^{d,im,ex} = \frac{O_t^{d,im,ex} z_t}{\Upsilon O^t}, \quad p_t^o = \frac{P_t^o}{P_t}\Upsilon O^t.$$

I have 17 observational equations, linking the model variables to the observed variables. The sample average of arbitrary variable x_t is denoted by $\bar{x}_t$.

$$
\begin{aligned}
\frac{y_t^{obs}}{\overline{y}^{obs}} &= \frac{c_t + \frac{i_t}{\mu_{\Upsilon,t}} + g_t}{c_{t-1} + \frac{i_{t-1}}{\mu_{\Upsilon,t-1}} + g_{t-1}} \frac{\mu_t^z}{\mu^z}. \\
\frac{c_t^{obs}}{\overline{c}^{obs}} &= \frac{c_t}{c_{t-1}} \frac{\mu_t^z}{\mu^z}. \\
\frac{i_t^{obs}}{\overline{i}^{obs}} &= \frac{i_t}{i_{t-1}} \frac{\mu_t^z}{\mu^z}. \\
\frac{b_t^{obs}}{\overline{b}^{obs}} &= \frac{q_t\bar{k}_t - n_t}{q_{t-1}\bar{k}_{t-1} - n_{t-1}} \frac{\mu_t^z}{\mu^z}. \\
\frac{n_t^{obs}}{\overline{n}^{obs}} &= \frac{n_t}{n_{t-1}} \frac{\mu_t^z}{\mu^z}. \\
\frac{premium_t^{obs}}{\overline{premium}^{obs}} &= \exp\{\mu G_{t-1}(\bar{\omega}_t)\frac{q_{t-1}\bar{k}_t}{q_{t-1}\bar{k}_t - n_t} - \mu G(\bar{\omega})\frac{q\bar{k}}{q\bar{k} - n}\}. \\
\frac{w_t^{obs}}{\overline{w}^{obs}} &= \frac{w_t}{w_{t-1}} \frac{\mu_t^z}{\mu^z}. \\
\frac{S_t^{1,obs}}{\overline{S}^{1,obs}} &= 1 + R_t^L - R_t. \\
\frac{h_t^{obs}}{\overline{h}^{obs}} &= \frac{h_t}{h}. \\
\frac{p_t^{i,obs}}{\overline{p}^{i,obs}} &= \frac{1}{\mu_{\Upsilon,t}}. \\
\frac{R_t^{obs}}{\overline{R}^{obs}} &= \exp(R_t - R). \\
\frac{\Pi_t^{obs}}{\overline{\Pi}^{obs}} &= \frac{\Pi_t}{\Pi}. \\
\frac{p_t^{O,obs}}{\overline{p}^{O,obs}} &= \frac{p_t^O}{p_{t-1}^O}. \\
\frac{o_t^{d,obs}}{\overline{o}^{d,obs}} &= \frac{o_t^d}{o_{t-1}^d} \frac{\mu_t^z}{\mu^z}.
\end{aligned}
$$

I demean the observed variables by their respective sample means. This approach allows to deal with different growth rates of oil market quantities in the sample. Sample means also include the deterministic trends Υ^O.

4.8.1.2 Final goods producers

The firms producing homogeneous output Y_t from $Y_{j_f,t}$ solve

$$\max_{Y_{j_f,t}} P_t Y_t - \int_0^1 P_{j_f,t} Y_{j_f,t} dj_f, \tag{4.119}$$
$$\text{s.t. } Y_t = \left(\int_0^1 Y_{j_f,t}^{\frac{1}{\lambda^f}} dj_f \right)^{\lambda^f}.$$

The firms are facing perfect competition and can not set their prices and have no influence on the input prices. Therefore the FOC w.r.t. $Y_{j_f,t}$ can be derived with the envelope theorem,

$$\begin{aligned} P_t \frac{dY_t}{dY_{j_f,t}} - P_{j_f,t} &= 0, \\ P_t \left(\frac{Y_t}{Y_{j_f,t}} \right)^{\frac{\lambda^f - 1}{\lambda^f}} - P_{j_f,t} &= 0. \end{aligned} \tag{4.120}$$

Solve for $Y_{j_f,t}$ and set back in definition for Y_t to get a relationship between P_t and $P_{j_f,t}$.

$$\begin{aligned} Y_t &= \left(\int_0^1 Y_{j_f,t}^{\frac{1}{\lambda^f}} dj_f \right)^{\lambda^f}, \\ Y_t &= \left[\int_0^1 \left\{ \left(\frac{P_{j_f,t}}{P_t} \right)^{-\frac{\lambda^f}{\lambda^f - 1}} Y_t \right\}^{\frac{1}{\lambda^f}} dj_f \right]^{\lambda^f}, \\ Y_t &= \left\{ \int_0^1 (P_{j_f,t}^{\frac{\lambda^f}{1-\lambda^f}})^{\frac{1}{\lambda^f}} dj_f \right\}^{\lambda^f} Y_t P_t^{\frac{\lambda^f}{\lambda^f - 1}}, \\ P_t &= \left(\int_0^1 P_{j_f,t}^{\frac{1}{1-\lambda^f}} \right)^{1-\lambda^f}. \end{aligned} \tag{4.121}$$

I need to express total output by firms $Y_t = \int_0^1 Y_{j_f,t} dj_f$ by total demand for output. Remember, that prices for production factors in the model are identical for all firms. Under the assumption of identical production functions, all firms use the same production factor ratios.

$$\int_0^1 Y_{j_f,t} dj_f = \epsilon_t \begin{cases} \epsilon_t \left(\frac{M_{j_f,t}}{z_t\, l_{j_f,t}}\right)^{\alpha_M} z_t \int_{j_f=0}^1 l_{j_f,t} dj_f - \phi_t z_t & \text{if } \eta^M = 1, \\ \epsilon_t \left[\alpha_M \left(\frac{M_{j_f,t}}{z_t\, l_{j_f,t}}\right)^{\frac{\eta^M-1}{\eta^M}} + (1-\alpha_M)(1)^{\frac{\eta^M-1}{\eta^M}}\right]^{\frac{\eta^M}{\eta^M-1}} z_t h_t - \phi_t z_t & \text{else.} \end{cases} \tag{4.122}$$

$$\int_0^1 Y_{j_f,t} dj_f = \begin{cases} \epsilon_t\, M_t^{\alpha_M} \left(z_t l_t\right)^{1-\alpha_M} - \phi_t z_t & \text{if } \eta^M = 1, \\ \epsilon_t \left[\alpha_M^{\frac{1}{\eta^M}} M_t^{\frac{\eta^M-1}{\eta^M}} + (1-\alpha_M)^{\frac{1}{\eta^M}} (z_t\, l_t)^{\frac{\eta^M-1}{\eta^M}}\right]^{\frac{\eta^M}{\eta^M-1}} - \phi_t z_t & \text{else.} \end{cases} \tag{4.123}$$

Using the demand for individual products of intermediate goods-producing firms I derive

$$\int_0^1 Y_{j_f,t} dj_f = Y_t \int_{j_f=0}^1 \left(\frac{P_{j_f,t}}{P_t}\right)^{\frac{\lambda^f}{1-\lambda^f}} dj_f. \tag{4.124}$$

$$Y_t = \left(\int_0^1 Y_{j_f,t} dj_f\right) \int_{j_f=0}^1 \left(\frac{P_{j_f,t}}{P_t}\right)^{\frac{\lambda^f}{\lambda^f-1}} dj_f. \tag{4.125}$$

I can write the current price dispersion level P_t^* as a function of prices set optimally in t and the ones which have to stick with the old prices. Due to the Calvo assumption only a share of $1-\xi^p$ can reset their prices, the others have to stick to the old prices.

$$P_t^* = \left(\int_0^1 P_{j_f,t}^{\frac{\lambda^f}{1-\lambda^f}}\right)^{\frac{1-\lambda^f}{\lambda^f}}, \tag{4.126}$$

$$P_t^* = \left\{\xi^p \left(\tilde{\Pi}_t P_{t-1}^*\right)^{\frac{\lambda^f}{1-\lambda^f}} + (1-\xi^p)\tilde{P}_t^{\frac{\lambda^f}{1-\lambda^f}}\right\}^{\frac{1-\lambda^f}{\lambda^f}},$$

$$p_t^* = \left\{\xi^p \left(\frac{\tilde{\Pi}_t}{\Pi_t} p_{t-1}^*\right)^{\frac{\lambda^f}{1-\lambda^f}} + (1-\xi^p)\tilde{p}_t^{\frac{\lambda^f}{1-\lambda^f}}\right\}^{\frac{1-\lambda^f}{\lambda^f}}. \tag{4.127}$$

Here I define $p_t^* = \frac{P_t^*}{P_t}$ and $\tilde{p}_t = \frac{\tilde{P}_t}{P_t}$ and use again the homogeneity of degree one. The current price level is a weighted average over optimally set prices and the price level of the past.

Note that $\frac{P_{t-1}^*}{P_t} = \frac{1}{\Pi_t} p_{t-1}^*$. The firms which can not optimize their prices, set them according to $P_t = \tilde{\Pi}_t P_{t-1}$, where $\tilde{\Pi}_t = (\Pi_t^*)^{\iota} \Pi_{t-1}^{1-\iota}$.

Now I can use the price dispersion index derived above to express total demand for the final good as a function of the price dispersion and the production factors.

$$Y_t = p_t^{*\frac{\lambda^f}{\lambda^f-1}} \begin{cases} \epsilon_t\, M_t^{\alpha_M} \left(z_t l_t\right)^{1-\alpha_M} - \phi_t z_t & \text{if } \eta^M = 1, \\ \epsilon_t \left[\alpha_M^{\frac{1}{\eta^M}} M_t^{\frac{\eta^M-1}{\eta^M}} + (1-\alpha_M)^{\frac{1}{\eta^M}} (z_t\, l_t)^{\frac{\eta^M-1}{\eta^M}}\right]^{\frac{\eta^M}{\eta^M-1}} - \phi_t z_t & \text{else.} \end{cases}$$

4.8.1.3 Intermediate goods producers

Let us turn to the optimization problem of the firms facing monopolistic competition. They seek to maximize

$$\max_{\tilde{P}_t} \mathrm{E}_t \sum_{\kappa=0}^{\infty} (\beta\xi^p)^\kappa \lambda_{t+\kappa}(P_{j_f,t+\kappa}Y_{j_f,t+\kappa} - S_{t+\kappa}Y_{j_f,t+\kappa}), \tag{4.128}$$

$$\text{s.t.} Y_{j_f,t+\kappa} = Y_{t+\kappa}\left(\frac{P_{j_f,t+\kappa}}{P_{t+\kappa}}\right)^{-\frac{\lambda^f}{\lambda^f-1}}, \tag{4.129}$$

$$P_{j_f,t+\kappa} = \tilde{\Pi}_{t,t+\kappa}\tilde{P}_t. \tag{4.130}$$

Firms optimizing their prices consider future states in which they are not able to reset their prices. Therefore they take into account that an optimal price set today $\tilde{P}_t$ might be effective forever.

Consider the fraction of prices $\frac{P_{j_f,t+\kappa}}{P_{t+\kappa}}$, I can plug in (4.130) in this expression to obtain

$$\frac{P_{j_f,t+\kappa}}{P_{t+\kappa}} = \frac{\tilde{\Pi}_{t,t+\kappa}\tilde{P}_t}{P_{t+\kappa}}.$$

Furthermore use $\tilde{p}_t = \frac{\tilde{P}_t}{P_t}$ and manipulate the price fraction such that

$$\left(\tilde{\Pi}_{t,t+\kappa}\right)\tilde{p}_t\frac{P_t}{P_{t+\kappa}} = \frac{\tilde{\Pi}_{t,t+\kappa}}{\Pi_{t,t+\kappa}}\tilde{p}_t. \tag{4.131}$$

Note that $\frac{P_{t+\kappa}}{P_t} = \Pi_{t,t+\kappa} = \prod_{h_\kappa=0}\Pi_{t+h_\kappa}$. For the following define $X_{t,t+\kappa} = \frac{\tilde{\Pi}_{t,t+\kappa}}{\Pi_{t,t+\kappa}}$. Now take the first derivative of (4.128) w.r.t $\tilde{P}_t$ set it to zero and make use of the envelope theorem.

$$0 = \mathrm{E}_t \sum_{\kappa=0}^{\infty} (\beta\xi^p)^\kappa \lambda_{t+\kappa}\left\{\left(\tilde{\Pi}_{t,t+\kappa}\tilde{P}_t - S_{t+\kappa}\right)\frac{dY_{j_f,t+\kappa}}{d\tilde{P}_t} + \tilde{\Pi}_{t,t+\kappa}Y_{j_f,t+\kappa}\right\}. \tag{4.132}$$

I know that only $Y_{j_f,t+\kappa}$ and $P_{j_f,t+\kappa}$ depend on $\tilde{P}_t$. It is therefore necessary to find the first derivative for these variables w.r.t. $\tilde{P}_t$. For $\frac{dP_{j_f,t+\kappa}}{d\tilde{P}_t}$ this is trivial and equals $\tilde{\Pi}_{t,t+\kappa}$. The first derivative is

$$\begin{aligned}\frac{dY_{j_f,t+\kappa}}{d\tilde{P}_t} &= Y_{t+\kappa}P_{t+\kappa}^{\frac{\lambda^f}{\lambda^f-1}}\frac{-\lambda^f}{\lambda^f-1}\tilde{P}_t^{\frac{-\lambda^f}{\lambda^f-1}-1}, \\ \frac{dY_{j_f,t+\kappa}}{d\tilde{P}_t} &= \frac{-\lambda^f}{\lambda^f-1}\frac{Y_{j_f,t+\kappa}}{\tilde{P}_t}.\end{aligned} \tag{4.133}$$

Now plug in (4.133) into (4.132) to obtain

$$0 = \mathrm{E}_t \sum_{\kappa=0}^{\infty} (\beta\xi^p)^\kappa \lambda_{t+\kappa} \left\{ \tilde{\Pi}_{t,t+\kappa} \frac{-1}{\lambda^f - 1} Y_{j_f,t+\kappa} + \frac{\lambda^f}{\lambda^f - 1} \frac{Y_{j_f,t+\kappa}}{\tilde{P}_t} S_{t+\kappa} \right\},$$

$$0 = \mathrm{E}_t \sum_{\kappa=0}^{\infty} (\beta\xi^p)^\kappa \lambda_{t+\kappa} \left\{ \tilde{\Pi}_{t,t+\kappa} Y_{j_f,t+\kappa} - \lambda^f \frac{Y_{j_f,t+\kappa}}{\tilde{P}_t} S_{t+\kappa} \right\}. \tag{4.134}$$

Use (4.129) to rearrange (4.134).

$$0 = \mathrm{E}_t \sum_{\kappa=0}^{\infty} (\beta\xi^p)^\kappa \lambda_{t+\kappa} \left\{ \tilde{\Pi}_{t,t+\kappa} \frac{-1}{\lambda^f - 1} Y_{j_f,t+\kappa} + \frac{\lambda^f}{\lambda^f - 1} \frac{Y_{j_f,t+\kappa}}{\tilde{P}_t} S_{t+\kappa} \right\}, \tag{4.135}$$

$$0 = \mathrm{E}_t \sum_{\kappa=0}^{\infty} (\beta\xi^p)^\kappa \lambda_{t+\kappa} \left\{ \tilde{\Pi}_{t,t+\kappa} Y_{j_f,t+\kappa} - \lambda^f \frac{Y_{j_f,t+\kappa}}{\tilde{P}_t} S_{t+\kappa} \right\}, \tag{4.136}$$

$$0 = \mathrm{E}_t \sum_{\kappa=0}^{\infty} (\beta\xi^p)^\kappa \lambda_{t+\kappa} Y_{t+\kappa} P_{t+\kappa} \left\{ (X_{t,t+\kappa}\tilde{p}_t)^{\frac{-1}{\lambda^f - 1}} - \lambda^f s_{t+\kappa} (X_{t,t+\kappa}\tilde{p}_t)^{\frac{-\lambda^f}{\lambda^f - 1}} \right\}. \tag{4.137}$$

In the above derivation I made use of several simplifications to obtain the last align. To get from (4.137) to (4.135) use the demand constraint and take $P_{t+\kappa}$ and $Y_{t+\kappa}$ out of the parentheses. Therefore you get the real marginal cost $s_{t+\kappa} = S_{t+\kappa}/P_{t+\kappa}$. For the first part of the sum I use (4.131). Now solve for $\tilde{p}_t$ to obtain the following fraction

$$\tilde{p}_t = \mathrm{E}_t \frac{\sum_{\kappa=0}^{\infty} (\beta\xi^p)^\kappa \lambda_{t+\kappa} Y_{t+\kappa} P_{t+\kappa} \lambda^f s_{t+\kappa} (X_{t,t+\kappa})^{\frac{-\lambda^f}{\lambda^f - 1}}}{\sum_{\kappa=0}^{\infty} (\beta\xi^p)^\kappa \lambda_{t+\kappa} Y_{t+\kappa} P_{t+\kappa} (X_{t,t+\kappa})^{\frac{-1}{\lambda^f - 1}}}.$$

Define auxiliary expressions for the numerator $K_{p,t}$ and denominator $F_{p,t}$ of (4.138). Derive the law of motions for these two. For the auxiliary expression $F_{p,t}$ the law of motion is derived by

$$F_{p,t} = \mathrm{E}_t \sum_{\kappa=0}^{\infty} (\beta\xi^p)^\kappa \lambda_{t+\kappa} Y_{t+\kappa} P_{t+\kappa} (X_{t,t+\kappa})^{\frac{-1}{\lambda^f - 1}}, \tag{4.138}$$

$$F_{p,t} = \lambda_t Y_t P_t + \mathrm{E}_t\, \beta\xi^p \, (X_{t,1})^{\frac{1}{1-\lambda^f}} \, F_{p,t+1}, \tag{4.139}$$

$$F_{p,t} = \lambda_t Y_t P_t + \mathrm{E}_t\, \beta\xi^p \left(\frac{\tilde{\Pi}_{t+1}}{\Pi_{t+1}} \right)^{\frac{1}{1-\lambda^f}} F_{p,t+1}. \tag{4.140}$$

Analogously the law of motion for $K_{p,t}$ is

$$K_{p,t} = \lambda_t Y_t P_t s_t \lambda^f + \mathrm{E}_t\, \beta\xi^p \left(\frac{\tilde{\Pi}_{t+1}}{\Pi_{t+1}} \right)^{\frac{\lambda^f}{1-\lambda^f}} K_{p,t+1}. \tag{4.141}$$

These two law of motions are used for the simulation and estimation of the model in Dynare. Therefore (4.127), (4.140) and (4.141) are entering the equilibrium conditions of the model.

Further, I know that the price index is a weighted average of optimal prices and not reset prices. I can derive the following relationship between numerator and denominator.

$$K_t^p = \left\{ 1 - \xi^p \left(\frac{\tilde{\Pi}_t}{\Pi_t} \right)^{1-\lambda^f} \right\}^{\frac{1}{1-\lambda^f}} F_t^p. \tag{4.142}$$

In contrast to Christiano et al. (2014), one can differentiate between the mark-up charged by a firm $\lambda_{f,t}$ over marginal cost and the elasticity of substitution between intermediate goods to produce final goods λ_f. This modification affects the law of motion of the price dispersion index. It is only possible to reformulate the price dispersion index recursively, assuming time-invariant elasticities of substitution.

The inflation adjustment rule (4.127), the law of motion for the denominator of the optimal price (4.140), the law of motion for the numerator of the optimal price (4.141), the relationship between numerator and denominator (4.142), and the price dispersion index (4.127) enter the model.

4.8.1.4 Labour contractor

After the unions negotiated for each type of labour $h_{j_l,t}$ the wages $W_{j_l,t}$, the labour contractor has to decide how much labour is supplied $l_t = (\int_0^1 h_{j_l,t}^{\frac{1}{\lambda^w}} dj_l)^{\lambda^w}$. Therefore a similar problem as for the final goods producer has to be solved. Here the optimization problem is the following

$$\max_{h_{j_l,t}} W_t l_t - \int_0^1 W_{t,j_l} h_{t,j_l} dj_l,$$

$$\max_{h_{j_l,t}} W_t \left(\int_0^1 h_{j_l,t}^{\frac{1}{\lambda^w}} dj_l \right)^{\lambda^w} - \int_0^1 W_{t,j_l} h_{j_l,t} dj_l. \tag{4.143}$$

(4.143) is a typical static profit optimization problem. The FOC condition is

$$0 = W_t l_t^{\frac{\lambda^w - 1}{\lambda^w}} h_{j_l,t}^{\frac{1-\lambda^w}{\lambda^w}} - W_{j_l,t}. \tag{4.144}$$

Now I can obtain an expression for the demanded labour $h_{j_l,t}$ of the different types relative to the total supplied labour h_t. Therefore solve (4.144) for $h_{j_l,t}$ to obtain

$$h_{j_l,t} = l_t \left(\frac{W_{j_l,t}}{W_t} \right)^{\frac{\lambda^w}{1-\lambda^w}}. \tag{4.145}$$

This labour demand function for each type can be used to express the current wage level W_t as a function of the different wages for the different labour types $W_{j_l,t}$. Plug (4.145) in

$l_t = (\int_0^1 h_{j_l,t}^{\lambda^w} dj_l)^{\frac{1}{\lambda^w}}$ to obtain

$$l_t = \left[\int_0^1 \left\{\left(\frac{W_{j_l,t}}{W_t}\right)^{\frac{\lambda^w}{1-\lambda^w}} l_t\right\}^{\frac{1}{\lambda^w}} dj_l\right]^{\lambda^w}, \tag{4.146}$$

$$W_t^{\frac{\lambda^w}{1-\lambda^w}} = \int_0^1 W_{j_l,t}^{\frac{\lambda^w}{1-\lambda^w}} dj_l, \tag{4.147}$$

$$W_t = \left(\int_0^1 W_{j_l,t}^{\frac{1}{1-\lambda^w}} dj_l\right)^{1-\lambda^w}. \tag{4.148}$$

Now one can derive an expression for the aggregate wage level depending on the different wages for the different labour types. Analogously to the price-setting problem only a fraction of unions ξ^w is allowed to reset their prices in period t. If they reset their prices in period t, all unions set their prices to the optimal wage $\tilde{W}_t$. The share $1-\xi^w$ of unions have to reset their wages according to the following rule

$$\begin{aligned} W_{j_l,t} &= \tilde{\Pi}_{w,t}(\mu_{z,t})^{\iota_\mu}(\mu_z)^{1-\iota_\mu}W_{t-1}, \\ \tilde{\Pi}_{w,t} &= (\Pi_t^{target})^{\iota_w}(\Pi_{t-1})^{1-\iota_w}{\mu_{t-1}^z}^{\iota_\mu}\,\mu^{z\iota_\mu}, &(4.149)\\ \Pi_t^w &= \Pi_t\,\mu_t^z. &(4.150)\end{aligned}$$

Define $\tilde{\Pi}_{t,t+\kappa}^w = \prod_{h=0}^{\kappa}\tilde{\Pi}_{t,t+h_\kappa}(\mu_{z,t+h_\kappa})^{\iota_\mu}(\mu_z)^{1-\iota_\mu}$ for further computations this will be useful. As for the intermediate firms wee need to derive a relationship between total homogenous hours supplied l_t and total hours worked $h_t = \int_0^1 h_{j_l,t}dj_l$. As for the intermediate firms unions can optimize their wages with probability ξ^w. Therefore the current wage dispersion level can be expressed as

$$W_t^* = \left[(1-\xi^w)\tilde{W}_t^{\frac{\lambda^w}{1-\lambda^w}} + \xi^w\left\{x_t^w W_{t-1}^*\right\}^{\frac{\lambda^w}{1-\lambda^w}}\right]^{\frac{1-\lambda^w}{\lambda^w}}. \tag{4.151}$$

Now divide the whole expression (4.151) by W_t and I get

$$w_t^* = \left[(1-\xi^w)\tilde{w}_t^{\frac{\lambda^w}{1-\lambda^w}} + \xi^w\left\{\frac{x_t^w}{\pi_t^w}w_{t-1}^*\right\}^{\frac{\lambda^w}{1-\lambda^w}}\right]^{\frac{1-\lambda^w}{\lambda^w}}. \tag{4.152}$$

Here I define $w_t^* = W_t^*/W_t$ and $\tilde{w}_t = \tilde{W}_t/W_t$.

4.8.1.5 Unions

Now one can turn to the optimization problem of the unions. They face similar to the intermediate goods producers monopolistic competition. Nevertheless, the unions are representing the households. Therefore they maximize the wage bill less the associated disutility to work. Their objective is

$$\max_{\tilde{W}_t} \mathrm{E}_t \sum_{\kappa=0}^{\infty} (\beta\xi^w)^\kappa \left[\lambda_{t+\kappa}\tilde{W}_t\tilde{\Pi}^w_{t,t+\kappa}h_{j_l,t+\kappa}(1-\tau^l_{t+\kappa}) - \psi_L \frac{h^{1+\sigma_L}_{j_l,t+\kappa}}{1+\sigma_L}\right], \tag{4.153}$$

$$\text{s.t.}h_{j_l,t+\kappa} = l_{t+\kappa}\left(\frac{\tilde{\Pi}^w_{t,t+\kappa}\tilde{W}_t}{W_{t+\kappa}}\right)^{\frac{\lambda^w}{1-\lambda^w}}. \tag{4.154}$$

The objective function (4.153) is the maximization of the wage bill and minimizing the disutility to work. Here the discounted net wage bill $\tilde{W}_t x^w_{t,t+\kappa}h_{j_l,t+\kappa}(1-\tau^l_{t+\kappa})$ expressed in utility terms $\lambda_{t+\kappa}$ is the revenue and the costs are the dis-utility to labour. Similar to the intermediate goods producer the unions have to consider the demand for their labour captured by the constraint (4.153).

Lets derive the FOC of the above optimization problem. This is done analogously as for the intermediate good producer. The FOC reads

$$\begin{aligned}0 = \mathrm{E}_t \sum_{\kappa=0}^{\infty}(\beta\xi^p)^\kappa \Bigg[&\lambda_{t+\kappa}x^w_{t,t+\kappa}\,h_{j_l,t+\kappa}\,(1-\tau^l_{t+\kappa}) + \frac{\lambda^w}{1-\lambda^w}\,\lambda_{t+\kappa}\,x^w_{t,t+\kappa}h_{j_l,t+\kappa}\,(1-\tau^l_{t+\kappa})\dots \\ &-\psi_L\frac{\lambda^w}{1-\lambda^w}\frac{h^{1+\sigma_L}_{j_l,t+\kappa}}{\tilde{W}_t}\Bigg],\end{aligned} \tag{4.155}$$

$$\begin{aligned}0 = \mathrm{E}_t \sum_{\kappa=0}^{\infty}(\beta\xi^p)^\kappa \Bigg[&\lambda_{t+\kappa}x^w_{t,t+\kappa}\tilde{W}_t\left(\frac{x^w_{t,t+\kappa}\tilde{W}_t}{W_{t+\kappa}}\right)^{\frac{\lambda^w}{1-\lambda^w}} l_{t+\kappa}(1-\tau^l_{t+\kappa})\dots \\ &-\lambda^w\psi_L\left(\frac{x^w_{t,t+\kappa}\tilde{W}_t}{W_{t+\kappa}}\right)^{\frac{\lambda^w}{1-\lambda^w}(1+\sigma_L)} l^{1+\sigma_L}_{t+\kappa}\Bigg],\end{aligned} \tag{4.156}$$

$$\begin{aligned}0 = \mathrm{E}_t \sum_{\kappa=0}^{\infty}(\beta\xi^p)^\kappa \Bigg[&\lambda_{t+\kappa}\Pi^w_{t,t+\kappa}W_t(X^w_{t,t+\kappa}\tilde{w}_t)^{\frac{1}{1-\lambda^w}}l_{t+\kappa}(1-\tau^l_{t+\kappa}) \\ &-\lambda^w\psi_L(X^w_{t,t+\kappa}\tilde{w}_t)^{\frac{\lambda^w}{1-\lambda^w}(1+\sigma_L)}l^{1+\sigma_L}_{t+\kappa}\Bigg].\end{aligned} \tag{4.157}$$

The FOC (4.157) is obtained by plugging in the demand constraint (4.154) in (4.155), rescale by W_t and define $X^w_{t,t+\kappa} = \frac{x^w_{t,t+\kappa}}{\prod_{h=0}^{\kappa}\Pi^w_{t+h}}$. I can now solve for $\tilde{w}_t$. Therefore divide (4.157) by $\tilde{w}_t^{\frac{\lambda^w}{1-\lambda^w}(1+\sigma_L)}$ and obtain

$$\tilde{w}_t^{\frac{1-\lambda^w(1+\sigma_L)}{1-\lambda^w}} = \mathrm{E}_t \frac{\sum_{\kappa=0}^{\infty}(\beta\xi^p)^\kappa \lambda^w \psi_L (X_{t,t+\kappa}^w)^{\frac{\lambda^w}{1-\lambda^w} l_{t+\kappa}(1+\sigma_L)}}{\sum_{\kappa=0}^{\infty}(\beta\xi^p)^\kappa \lambda_{t+\kappa}\Pi_{t,t+\kappa}^w W_t (X_{t,t+\kappa}^w)^{\frac{1}{1-\lambda^w}} l_{t+\kappa}(1-\tau_{t+\kappa}^l)}, \tag{4.158}$$

$$\tilde{w}_t = \mathrm{E}_t \left\{ \frac{\sum_{\kappa=0}^{\infty}(\beta\xi^p)^\kappa \lambda^w \psi_L \left(X_{t,t+\kappa}^w\right)^{\frac{\lambda^w}{1-\lambda^w}(1+\sigma_L)} l_{t+\kappa}^{1+\sigma_L}}{\sum_{\kappa=0}^{\infty}(\beta\xi^p)^\kappa \lambda_{t+\kappa}\Pi_{t,t+\kappa}^w W_t (X_{t,t+\kappa}^w)^{\frac{1}{1-\lambda^w}} l_{t+\kappa}(1-\tau_{t+\kappa}^l)} \right\}^{\frac{1-\lambda^w}{1-\lambda^w(1+\sigma_L)}}. \tag{4.159}$$

Now the fraction is split again in numerator K_t^w and denominator F_t^w. The law of motions is then derived analogously to the price equations. I can rewrite (4.159) as

$$\tilde{w}_t = \mathrm{E}_t \left(\frac{\psi_L K_{w,t}}{W_t F_{w,t}} \right)^{\frac{1-\lambda^w}{1-\lambda^w(1+\sigma_L)}}, \tag{4.160}$$

$$F_t^w = \frac{\lambda_t l_t (1-\tau_t^l)}{\lambda^w} + \mathrm{E}_t\, \beta\xi^w \Pi_{t+1}^w (X_{t,1}^w)^{\frac{1}{1-\lambda^w}}, \tag{4.161}$$

$$K_t^w = l_t^{1+\sigma_L} + \mathrm{E}_t\, \beta\xi^w (X_{t,1}^w)^{\frac{\lambda^w}{1-\lambda^w}(1+\sigma_L)} l_{t+1}^{1+\sigma_L}. \tag{4.162}$$

The wage index in each period states an implicit relationship between the numerator K_t^w and denominator F_t^w. One can use the wage index w_t to derive.

$$1 = \left[(1-\xi^w)\tilde{w}_t^{\frac{1}{1-\lambda^w}} + \xi^w \left\{ \frac{\tilde{\Pi}_t^w}{\Pi_t^w} \right\}^{\frac{1}{1-\lambda^w}} \right]^{1-\lambda^w},$$

$$\tilde{w}_t = \left[\frac{1-\xi^w \left\{ \frac{\tilde{\Pi}_t^w}{\Pi_t^w} \right\}^{\frac{1}{1-\lambda^w}}}{1-\xi^w} \right]^{1-\lambda^w},$$

$$K_t^w = \frac{w_t\, F_t^w}{\psi^L} \left[\frac{1-\xi^w \left\{ \frac{\tilde{\Pi}_t^w}{\Pi_t^w} \right\}^{\frac{1}{1-\lambda^w}}}{1-\xi^w} \right]^{1-\lambda^w}. \tag{4.163}$$

Consider again the aggregated labour input $h_t = \int_0^1 h_{j_l,t} dj_l$ can also be expressed as function of homogenous labour supply l_t. I know that

$$\begin{aligned} h_t &= \int_0^1 h_{j_l,t}, \\ &= \int_0^1 l_t \left(\frac{W_{j_l,t}}{W_t}\right)^{\frac{\lambda^w}{1-\lambda^w}} dj_l, \\ &= l_t (w_t^*)^{\frac{\lambda^w}{1-\lambda^w}}. \end{aligned} \tag{4.164}$$

One can solve (4.164) and solve for l_t and plug it back in (4.161) and (4.162).

For the model the wage block consists of (4.149), (4.150) (4.152), (4.161), (4.162) and (4.163).

4.8.1.6 Production

Firms produce intermediate goods $Y_{j_f,t}$ using capital services $K^s_{j_f,t}$, hours of labour $h_{j_f,t}$ and oil $O_{j_f,t}$ The production function is a nested constant elasticity of substitution function. Each firm has access to the same technology and can substitute between labour and a composite production factor $M_{j_f,t}$ from capital services and oil. The production elasticity of substitution η^M determines how easy it is for firms to substitute labour for other production factors. The degree of substitution between oil and capital services is captured by the production elasticity of substitution η^O. One can further restrict the distribution parameters α_M and α_O of the CES production function in each stage, to sum up to one in contrast to the paper by Cantore et al. (2015).

$$Y_{j_f,t} = \begin{cases} \epsilon_t \, M_{j_f,t}^{\alpha_M} \, (z_t l_{j_f,t})^{\alpha_N} - \phi_t z_t & \text{, if } \eta^M = 1, \\ \epsilon_t \left[\alpha_M{}^{\frac{1}{\eta^M}} \, M_{j_f,t}^{\frac{\eta^M-1}{\eta^M}} + \alpha_N^{\frac{1}{\eta^M}} (z_t \, l_{j_f,t})^{\frac{\eta^M-1}{\eta^M}} \right]^{\frac{\eta^M}{\eta^M-1}} - \phi_t z_t & \text{, else.} \end{cases} \tag{4.165}$$

$$M_{j_f,t} = \epsilon_t^M \begin{cases} \left(\epsilon_t^O \, \frac{O_{j_f,t}}{\Upsilon_t^O} \right)^{\alpha_O} \, \left(\epsilon_t^M \, \frac{K^s_{j_f,t}}{\Upsilon^{t-1}} \right)^{\alpha_K} & \text{, if } \eta^O = 1, \\ \left\{ \alpha_K^{\frac{1}{\eta^O}} \, (\epsilon_t^M \, \frac{K^s_{j_f,t}}{\Upsilon^{t-1}})^{\frac{\eta^O-1}{\eta^O}} + \alpha_O{}^{\frac{1}{\eta^M}} (\epsilon_t^O \, \frac{O_{j_f,t}}{\Upsilon_t^O})^{\frac{\eta^O-1}{\eta^O}} \right\}^{\frac{\eta^O}{\eta^O-1}} & \text{, else.} \end{cases} \tag{4.166}$$

$$\phi_t z_t = (\lambda^f - 1) \, Y_{j_f,t-4}. \tag{4.167}$$

$$\begin{aligned} \min_{l_{j_f,t}, K^s_{j_f,t}, O_{j_f,t}, M_{j_f,t}} \quad & W_t l_{j_f,t} + P_t \tilde{r}_t^k K^s_{j_f,t} + P_t^O O_{j_f,t}, \\ & s.t. (4.165), (4.166), \\ & \quad l_{j_f,t} > 0, K^s_{j_f,t} > 0, O_{j_f,t} > 0, M_{j_f,t} > 0. \end{aligned} \tag{4.168}$$

The corresponding Lagrangian of the problem is

$$\begin{aligned} \mathcal{L}_t^{\mathrm{F,min}} =& W_t \, l_{j_f,t} + P_t \tilde{r}_t^k K^s_{j_f,t} + P_t^O O_{j_f,t} + S_t \{ Y_{j_f,t} - (\mathcal{X}(M_{j_f,t}, l_{j_f,t}) - \phi z_t) \} \ldots \\ & + P_t^M \{ M_{j_f,t} - \mathcal{M}(O_{j_f,t}, K^s_{j_f,t}) \}. \end{aligned} \tag{4.169}$$

It is straightforward to solve (4.169). The FOCs are

$$\frac{\partial \mathcal{L}_t^{\mathrm{F,min}}}{l_{j_f,t}} :0 = W_t - S_t z_t^{\frac{\eta^M-1}{\eta^M}} \epsilon_t (\alpha_N)^{\frac{1}{\eta^O}} \left(\frac{X_{j_f,t}}{l_{j_f,t}}\right)^{\frac{1}{\eta^M}}, \tag{4.170}$$

$$\frac{\partial \mathcal{L}_t^{\mathrm{F,min}}}{M_{j_f,t}} :0 = P_t^M - (\alpha_M)^{\frac{1}{\eta^M}} (\Upsilon^{t-1})^{\rho^M} (\epsilon^K{}_t)^{\rho^M} \left(\frac{X_{j_f,t}}{M_{j_f,t}}\right)^{\frac{1}{\eta_M}}, \tag{4.171}$$

$$\frac{\partial \mathcal{L}_t^{\mathrm{F,min}}}{K_{j_f,t}^s} :0 = P_t \tilde{r}_t^k - P_t^M (1-\alpha_O)^{\frac{1}{\eta^O}} (\Upsilon^{t-1})^{\frac{1-\eta^O}{\eta^O}} (\epsilon^K{}_t)^{\frac{\eta^O-1}{\eta^O}} \left(\frac{M_{j_f,t}}{K_{j_f,t}^s}\right)^{\frac{1}{\eta_O}}, \tag{4.172}$$

$$\frac{\partial \mathcal{L}_t^{\mathrm{F,min}}}{O_{j_f,t}} :0 = P_t^O - P_t^M (\alpha_O)^{\frac{1}{\eta^O}} (\Upsilon^{O^t})^{\frac{1-\eta^O}{\eta^O}} (\epsilon^O{}_t)^{\frac{\eta^O-1}{\eta^O}} \left\{\frac{M_{j_f,t}}{O_{j_f,t}}\right\}^{\frac{1}{\eta^O}}, \tag{4.173}$$

$$\frac{\partial \mathcal{L}_t^{\mathrm{F,min}}}{P_t^M} :0 = M_{j_f,t} - \mathcal{M}(O_{j_f,t}, K_{j_f,t}^s), \tag{4.174}$$

$$\frac{\partial \mathcal{L}_t^{\mathrm{F,min}}}{S_t} :0 = X_{j_f,t} - \mathcal{X}(l_{j_f,t}, M_{j_f,t}). \tag{4.175}$$

I can transform the equations into stationary versions, I need to divide them with z_t and P_t.

$$w_t P_t z_t = s_t P_t z_t^{\frac{\eta^M-1}{\eta^M}} \epsilon_t^{\frac{\eta^M-1}{\eta^M}} \alpha_N^{\frac{1}{\eta^M}} \left(\frac{y_{j_f,t} z_t + \phi_t z_t}{l_{j_f,t}}\right)^{\frac{1}{\eta^M}}, \tag{4.176}$$

$$p_t^M P_t = s_t P_t \epsilon_t^{\frac{\eta^M-1}{\eta^M}} \alpha_M^{\frac{1}{\eta^M}} \left(\frac{y_{j_f,t} z_t + \phi_t z_t}{m_{j_f,t} z_t}\right)^{\frac{1}{\eta^M}}, \tag{4.177}$$

$$\frac{r_t^k}{\Upsilon^t} = p_t^M P_t \epsilon_t^{M \frac{\eta^O-1}{\eta^O}} \left(\frac{1}{\Upsilon^t}\right)^{\frac{\eta^O-1}{\eta^O}} \alpha_K^{\frac{1}{\eta^O}} \left(\frac{m_{j_f,t} z_t}{\frac{u_{j_f,t} \bar{k}_{j_f,t} z_{t-1} \Upsilon^{t-1}}{\Upsilon^t}}\right)^{\frac{1}{\eta_O}}, \tag{4.178}$$

$$P_t \frac{p_t^O}{\Upsilon^{O^t}} = p_t^M P_t \left(\frac{\epsilon_t^O}{\Upsilon^{O^t}}\right)^{\frac{\eta^O-1}{\eta^O}} \alpha_O^{\frac{1}{\eta^O}} \left\{\frac{m_{j_f,t} z_t}{o_{j_f,t} \Upsilon^{O^t} z_t}\right\}^{\frac{1}{\eta_O}}. \tag{4.179}$$

Now I need to consider the results from the previous subsections regarding the representation of $y_{j_f,t}$ and $l_{j_f,t}$ as a function of aggregate production y_t and total hours worked

h_t.

$$y_t = p_t^{*\frac{\lambda^f}{\lambda^f - 1}} \begin{cases} \epsilon_t \, m_t^{\alpha_M} \, (h_t \, w_t^{*\frac{\lambda^w}{\lambda^w - 1}})^{\alpha_N} - \phi_t & \text{, if } \eta^M = 1, \\ \epsilon_t \left[{\alpha_M}^{\frac{1}{\eta^M}} \, {m_t}^{\frac{\eta^M - 1}{\eta^M}} + \alpha_N^{\frac{1}{\eta^M}} \, (h_t \, w_t^{*\frac{\lambda^w}{\lambda^w - 1}})^{\frac{\eta^M - 1}{\eta^M}} \right]^{\frac{\eta^M}{\eta^M - 1}} - \phi_t z_t & \text{, else,} \end{cases} \tag{4.180}$$

$$m_t = \epsilon_t^M \begin{cases} \left((\epsilon_t^O \, o_t \right)^{\alpha_O} \, \frac{u_t \bar{k}_t}{\Upsilon \mu_t^z}^{\alpha_K} & \text{, if } \eta^O = 1, \\ \left\{ \alpha_K^{\frac{1}{\eta^O}} \, (\frac{K^s_{j_f,t}}{\Upsilon^{t-1}})^{\frac{\eta^O - 1}{\eta^O}} + {\alpha_O}^{\frac{1}{\eta^M}} \, (\epsilon_t^O \, \frac{O_{j_f,t}}{\Upsilon_t^O})^{\frac{\eta^O - 1}{\eta^O}} \right\}^{\frac{\eta^O}{\eta^O - 1}} & \text{, else,} \end{cases} \tag{4.181}$$

$$\phi_t = (\lambda^f - 1) \, y_{t-4}, \tag{4.182}$$

$$w_t = s_t {\epsilon_t}^{\frac{\eta^M - 1}{\eta^M}} {\alpha_N}^{\frac{1}{\eta^M}} \left(\frac{y_t \, p_t^{*\frac{\lambda^f}{1-\lambda^f}} + \phi_t}{h_t \, w_t^{*\frac{\lambda^w}{\lambda^w - 1}}} \right)^{\frac{1}{\eta^M}}, \tag{4.183}$$

$$p_t^M = s_t {\epsilon_t}^{\frac{\eta^M - 1}{\eta^M}} {\alpha_M}^{\frac{1}{\eta^M}} \left(\frac{y_t \, p_t^{*\frac{\lambda^f}{1-\lambda^f}} + \phi_t}{m_{j_f,t}} \right)^{\frac{1}{\eta^M}}, \tag{4.184}$$

$$r_t^k = p_t^M {\epsilon_t^M}^{\frac{\eta^O - 1}{\eta^O}} \, \alpha_K^{\frac{1}{\eta^O}} \left(\frac{m_t}{\frac{u_t \bar{k}_t}{\mu_t^z \, \Upsilon}} \right)^{\frac{1}{\eta^O}}, \tag{4.185}$$

$$p_t^O = p_t^M \, {\epsilon_t^M}^{\frac{\eta^O - 1}{\eta^O}} \, \left(\epsilon_t^O \right)^{\frac{\eta^O - 1}{\eta^O}} \, \alpha_O^{\frac{1}{\eta^O}} \left\{ \frac{m_t}{o_t} \right\}^{\frac{1}{\eta_O}}. \tag{4.186}$$

Equations (4.180), (4.181), (4.182), (4.183), (4.184), (4.185), (4.181) and (4.186) are part of the model.

4.8.1.7 Households

Households face a typical dynamic problem to maximize their discounted present utility. They have to find optimal level of consumption $C_{j_h,t+\kappa}$. Furthermore, they can either purchase short term risk-free bonds $B_{j_h,t+\kappa}$ used by mutual funds or long term risk-free bonds $B^L_{j_h,t+\kappa}$. Households are also able to invest in capital $I_{j_h,t+\kappa}$. The dynamic optimization problem for a representative household is

$$\max_{C_{j_h,t+\kappa},B_{j_h,t+\kappa+1},B^L_{j_h,t+\kappa+4},I_{j_h,t+\kappa},I_{j_h,t+\kappa+1}} \mathrm{E}_t \sum_{\kappa=0}^{\infty} \beta^{\kappa}\Big[\Big\{\zeta_{c,t+\kappa}\ln(C_{j_h,t+\kappa}-bC_{j_h,t+\kappa-1})\Big\}\dots \tag{4.187}$$

$$-\psi_L\int_0^1 \frac{h_{j_h,t+\kappa}(j_l)^{1+\sigma_L}}{1+\sigma_L}dj_l\Big],$$

$$\begin{aligned}
&\text{s.t.}(1+\tau^c)P_{t+\kappa}C_{j_h,t+\kappa}+B_{j_h,t+\kappa+1}+B^L_{j_h,t+\kappa+4}+\Big(\frac{P_{t+k}}{\Upsilon^{t+\kappa}\mu_{\Upsilon,t+\kappa}}\Big)I_{j_h,t+\kappa}+Tax_{t+\kappa}\\
&=\Delta^{O,j^d}_{t+\kappa}+\Gamma(O^{j^d}_{t+\kappa})+(1-\tau^l)\int_0^1 W_{j_h,t+\kappa}(j_l)h_{j_h,t+\kappa}(j_l)dj_l+R_{t+\kappa}B_{t+\kappa}+(R^L_{t+\kappa})^4B^L_{j_h,t+\kappa}\\
&\quad+Q_{\bar K,t+\kappa}\bar K_{j_h,t+\kappa+1}-Q_{\bar K,t+\kappa}(1-\delta)\bar K_{t+\kappa}+\Delta_{j_h,t+\kappa}\\
&\quad+(1-\Theta)(1-\gamma_{t+\kappa})\{1-\Gamma_{t+\kappa-1}(\bar\omega_{t+\kappa})\}R^k_{t+\kappa}Q_{\bar K,t+\kappa-1}\bar K_{j_h,t+\kappa}+Tr_{j_h,t+\kappa}.
\end{aligned}\tag{4.188}$$

The raw capital stock evolves according to a standard law of motion. This law of motion for capital features proportional depreciations and investment adjustment costs.

$$\bar K_{t+\kappa+1}=(1-\delta)\bar K_{t+\kappa}+\{1-\mathcal{S}(\zeta_{i,t+\kappa}I_{t+\kappa}/I_{t+\kappa-1})\}I_{t+\kappa}. \tag{4.189}$$

At every point in time, a household maximizes utility for each variable to optimize. It is necessary to set up a Lagrangian to solve this problem. The following Lagrangian has to be solved.

$$\begin{aligned}
L^H_t=&\,\mathrm{E}_t\sum_{k=0}^{\infty}\beta^{\kappa}\Big[\zeta_{c,t+\kappa}\ln(C_{j_h,t+\kappa}-bC_{j_h,t+\kappa-1})-\psi_L\int_0^1\frac{h_{j_h,t+\kappa}(j_l)^{1+\sigma_L}}{1+\sigma_L}dj_l\Big] \qquad (4.190)\\
&-\lambda_{j_h,t+\kappa}\Big\{(1+\tau^c)P_{t+\kappa}C_{j_h,t+\kappa}+B_{j_h,t+\kappa+1}+B^L_{j_h,t+\kappa+4}+\left(\frac{P_{t+k}}{\Upsilon^{t+\kappa}\mu_{\Upsilon,t+\kappa}}\right)I_{j_h,t+\kappa}\\
&+Tax_{j_h,t+\kappa}-(1-\tau^l)\int_0^1 W_{j_h,t}(j_l)h_{j_h,t+\kappa}(j_l)dj_l-R_{t+\kappa}B_{j_h,t+\kappa}-(R^L_{t+\kappa})^4B^L_{j_h,t+\kappa}\dots\\
&+Q_{\bar K,t+\kappa}\bar K_{j_h,t+\kappa+1}-Q_{\bar K,t+\kappa}(1-\delta)\bar K_{j_h,t+\kappa}-\Delta^{O,d}_{j_h,t+\kappa}+\Gamma(O^d_t)\dots\\
&-(1-\Theta)(1-\gamma_{t+\kappa})\{1-\Gamma_{t+\kappa-1}(\bar\omega_{t+\kappa})\}R^k_{t+\kappa}Q_{\bar K,t+\kappa-1}\bar K_{j_h,t+\kappa}-Tr_{j_h,t+\kappa}\Big\}\Big].
\end{aligned}$$

One can use the standard law of motion for raw capital $\bar K_{t+1}$ as a function of non depreciated previous capital and former investment I_t. Here δ is the standard depreciation rate of capital and $\mathcal{S}(\zeta_{i,t+\kappa}I_{t+\kappa}/I_{t+\kappa-1})$ is a convex adjustment cost function. This function punishes either to high investment today or to low investment in the past. Consider the Lagrangian and the law of motion for capital (4.189). To find the optimal level of investment in each period the effect of $I_{t+\kappa}$ on $\bar K_{t+\kappa+1}$ and $\bar K_{t+\kappa+2}$ has to be considered. The FOC w.r.t to $I_{t+\kappa}$

reads

$$\frac{dL_t^H}{dI_{t+\kappa}} = \mathrm{E}_t \Big\{ -\lambda_{t+\kappa} \frac{P_{t+\kappa}}{\Upsilon^{t+\kappa}\mu_{\Upsilon,t+\kappa}} + \lambda_{t+\kappa} Q_{\bar{K},t+\kappa} \frac{d\bar{K}_{t+\kappa+1}}{dI_{t+\kappa}} - \beta\lambda_{t+\kappa+1}(1-\delta)\, Q_{\bar{K},t+\kappa+1} \frac{d\bar{K}_{t+\kappa+1}}{dI_{t+\kappa}}$$
$$+ \beta\lambda_{t+\kappa+1} Q_{\bar{K},t+\kappa+1} \frac{d\bar{K}_{t+\kappa+2}}{dI_{t+\kappa}} \Big\}. \tag{4.191}$$

The first term reflects the marginal cost for investment expressed in expected utility terms today. The second term reflects the increase in raw capital revenue in the next period, while the third term mirrors the decrease in purchase costs for raw capital two periods ahead by decreasing adjustment costs. To see this more clearly it is necessary to look at the FOCs for $\bar{K}_{t+\kappa+1}$ and $\bar{K}_{t+\kappa+2}$,

$$\frac{d\bar{K}_{t+\kappa+1}}{dI_{t+\kappa}} = 1 - S(\zeta_{i,t+\kappa} I_{t+\kappa}/I_{t+\kappa-1}) - S^{'}(\zeta_{i,t+\kappa} I_{t+\kappa}/I_{t+\kappa-1}) \frac{\zeta_{i,t+\kappa} I_{t+\kappa}}{I_{t+\kappa-1}}, \tag{4.192}$$

$$\frac{d\bar{K}_{t+\kappa+2}}{dI_{t+\kappa}} = (1-\delta)\, \frac{d\bar{K}_{t+\kappa+1}}{dI_{t+\kappa}} + S^{'}(\zeta_{i,t+\kappa+1} I_{t+\kappa+1}/I_{t+\kappa})\zeta_{i,t+\kappa+1} \Big(\frac{I_{t+\kappa+1}}{I_{t+\kappa}} \Big)^2. \tag{4.193}$$

One can insert (4.192) and (4.193) in (4.191) to obtain the final FOC. For the final expression I use $x^I_{t+\kappa} = \frac{I_{t+\kappa}}{I_{t+\kappa-1}}$. In the following all FOCs are reported, which are then used in the model.

$$\frac{dL_t^H}{dC_{t+\kappa}} = \mathrm{E}_t \Big\{ \frac{\zeta_{c,t+\kappa}}{C_{t+\kappa} - bC_{t+\kappa-1}} - \frac{\zeta_{c,t+\kappa+1} b}{C_{t+\kappa+1} - bC_{t+\kappa}} - \lambda_{t+\kappa}(1+\tau^c) P_{t+\kappa} \Big\}, \tag{4.194}$$

$$\frac{dL_t^H}{dB_{t+\kappa+1}} = \mathrm{E}_t \Big\{ -\lambda_{t+\kappa} + \beta\lambda_{t+\kappa+1} R_{t+\kappa+1} \Big\}, \tag{4.195}$$

$$\frac{dL_t^H}{dB^L_{t+\kappa+4}} = \mathrm{E}_t \Big\{ -\lambda_{t+\kappa} + \beta^4 (\prod_{s=1}^{4} \zeta_{term,t+\kappa+s}) \lambda_{t+\kappa+4} (R^L_{t+\kappa+4})^4 \Big\}, \tag{4.196}$$

$$\frac{dL_t^H}{dI_{t+\kappa}} = \mathrm{E}_t \Big\{ -\lambda_{t+\kappa} \frac{P_{t+\kappa}}{\Upsilon^{t+\kappa}\mu_{\Upsilon,t+\kappa}} + \lambda_{t+\kappa} Q_{\bar{K},t+\kappa} (1 - S(\zeta_{i,t+\kappa} x^I_{t+\kappa}) \tag{4.197}$$
$$- \mathcal{S}^{'}(\zeta_{i,t+\kappa} x^I_{t+\kappa}))\zeta_{i,t+\kappa} x^I_{t+\kappa} + \beta\lambda_{t+\kappa+1} Q_{\bar{K},t+\kappa+1} \mathcal{S}^{'}(\zeta_{i,t+\kappa+1} x^I_{t+\kappa+1})\zeta_{i,t+\kappa+1} (x^I_{t+\kappa+1})^2,$$

$$\frac{dL_t^H}{d\bar{K}_{t+\kappa+1}} = \mathrm{E}_t \Big\{ \lambda_{t+\kappa} Q_{\bar{K},t+\kappa} - \beta\lambda_{t+\kappa+1} Q_{\bar{K},t+\kappa+1}(1-\delta) \Big\}. \tag{4.198}$$

Equations (4.194), (4.195), (4.196), and (4.197) are used in all model versions. The FOC for capital (4.198) is not used in the risk shock model with entrepreneurs.

4.8.1.8 Entrepreneurs

Christiano et al. (2014) is the main source for this section. The key modification of the risk shock model compared to the classic NK-DSGE model is the introduction of entrepreneurs. To each household belongs a large number of entrepreneurs of different types. Entrepreneurs

j_E purchase raw capital $\bar{K}_{t+1}$ from different households for the price $Q_{\bar{K},t-1}$. To finance these purchases, each entrepreneur has its net worth $N_{j_E,t}$ and access to loans $B_{j_E,t+1}$ from mutual funds. They purchase loans after production took place in the period t. $N_{j_E,t}$ introduces heterogeneity to entrepreneurs. One can assume that net worth $N_{j_E,t}$ in all periods satisfies the following conditions

- $N_{j_E,t} \geq 0 \quad \forall j_E,\, t,$
- $N_{j_E,t}$ has the density function $f_t(N_{j_E,t})$,
- $N_{j_E,t+1} = \int_0^\infty N_{j_E} f_t(N_{j_E}) dj_E$.

Let us consider the actions of an entrepreneur during one period. Each entrepreneur does the following actions during one period.

1. The entrepreneur purchases raw capital with the loans from mutual funds and its net worth. This leads to the following condition for each period t,

$$Q_{\bar{K},t}\bar{K}_{j_E,t+1} = N_{j_E,t} + B_{j_E,t+1}. \tag{4.199}$$

2. After raw capital is purchased an idiosyncratic shock hits each entrepreneur ω. This shock transforms raw capital to effective capital $K_{j_E,t+1} = \omega\bar{K}_{j_E,t+1}$. I assume that the idiosyncratic shock follows a log-normal distribution with an expectation equal to one and variance varying over time.

   - $\mathrm{E}(\omega) = 1$ and $\mathrm{Var}(\omega) = \sigma_t^2$,
   - $\log\omega \sim N(-\frac{\sigma^2}{2}, \sigma^2)$.

3. The entrepreneur has to decide how much capital services $u_{t+1}\omega\bar{K}_{j_E,t+1}$ she wants to provide at a competitive market rental rate r_{t+1}^k. Here the variable $u_{j_E,t+1}$ is the utilization rate for effective capital. The utilization of effective capital will produce costs $a(u_{t+1})$. Therefore the net revenues by capital services can be expressed as

$$\{u_{t+1}r_{t+1}^k - \tau^o a(u_{t+1})\}\omega\bar{K}_{j_E,t+1}\frac{P_{t+1}}{\Upsilon^{t+1}}(1-\tau^k). \tag{4.200}$$

   I find the optimal level of utilization by taking the first derivative with respect to u_{t+1}. Therefore the rental rate for capital is given by

$$r_{t+1}^k = a'(u_{t+1}). \tag{4.201}$$

   This FOC implies that optimal utilization rates are independent of the type of entrepreneur. All entrepreneurs face the same utilization costs and the same return on capital services. The utilization costs come later.

4. In $t+1$ the entrepreneurs will sell the non depreciated effective capital $(1-\delta)\omega\bar{K}_{j_E,t+1}$ to the households at price $Q_{\bar{K},t+1}$. Furthermore, it is assumed that entrepreneurs can deduct depreciated effective capital by $\delta\tau^k$ at historical costs $Q_{\bar{K}',t}$.

With the information from above, it is possible to determine the total return to effective capital in one period. Therefore one can set up the profit function for an N_{j_E} type entrepreneur. The costs for an entrepreneur purchasing raw capital from households are given by

$$C(\omega \bar{K}_{j_E,t+1}) = Q_{\bar{K},t}\omega \bar{K}_{j_E,t+1}.$$

The Revenues are given by

$$R(\omega \bar{K}_{j_E,t+1}) = (u_{t+1}r^k_{t+1} - \tau^o a(u_{t+1}))\omega \bar{K}_{j_E,t+1}\frac{P_{t+1}}{\Upsilon^{t+1}}(1-\tau^k)\omega \bar{K}_{j_E,t+1} + (1-\delta)Q_{\bar{K},t+1}\omega \bar{K}_{j_E,t+1}$$
$$+\delta\tau^k Q_{\bar{K}',t}\omega.$$

The total return of effective capital $1 + R^k_{t+1}$ can be derived by dividing the revenues by the costs.

$$1 + R^k_{t+1} = \frac{(u_{t+1}r^k_{t+1} - \tau^o a(u_{t+1}))\frac{P_{t+1}}{\Upsilon^{t+1}}(1-\tau^k) + (1-\delta)Q_{\bar{K},t+1} + \delta\tau^k Q_{\bar{K}',t}}{Q_{\bar{K},t}}. \tag{4.202}$$

In (4.202) there is no variable depending on the type of the entrepreneur. This is caused by the fact that all entrepreneurs will choose the same level of utilization. The return for raw capital is for each entrepreneur uncertain, because of the realization of ω and the return to raw capital is given by $\omega(1 + R^k_{t+1})$.

The most crucial decision of an entrepreneur is about its leverage $L_t = \frac{N_{j_E,t}+B_{j_E,t+1}}{N_{j_E,t}}$. This variable expresses the expenditures for raw capital relative to the net worth of an entrepreneur. Mutual funds lend loans $B_{j_E,t+1}$ to entrepreneurs at the gross nominal rate of interest Z_{t+1}. Therefore an entrepreneur has to repay $B_{j_E,t+1}(1 + Z_{t+1})$. Whether the entrepreneur is able to pay this amount depends on ω. Let $\bar{\omega}_{t+1}$ denote the threshold for the value of ω which separates entrepreneurs in insolvent and solvent ones. The total returns of effective capital are just enough to cover the loan costs, which translate into

$$(1 + R^k_{t+1})\bar{\omega}_{t+1}Q_{\bar{K},t}K_{j_E,t+1} = B_{j_E,t+1}(1 + Z_{t+1}). \tag{4.203}$$

It is assumed that entrepreneurs evaluate debt contracts according to their expected net worth in period $t+1$. They will maximize

$$\mathrm{E}_t\Big[\int_{\bar{\omega}_{t+1}}^{\infty}\{(1+R^k_{t+1})\omega Q_{\bar{K},t}K_{j_E,t+1}-B_{j_E,t+1}(1+Z_{t+1})\}f(\omega)d\omega\Big]=\dots \tag{4.204}$$

$$\mathrm{E}_t\{1-\Gamma_t(\bar{\omega}_{t+1})\}(1+R^k_{t+1})L_tN_{j_E,t},$$

$$\Gamma_t(\bar{\omega}_{t+1})=\{1-F_t(\bar{\omega}_{t+1})\}\bar{\omega}_{t+1}+G_t(\bar{\omega}_{t+1}), \tag{4.205}$$

$$F_t(\bar{\omega}_{t+1})=\int_0^{\bar{\omega}_{t+1}}f_t(\omega)d\omega, \tag{4.206}$$

$$G_t(\bar{\omega}_{t+1})=\int_0^{\bar{\omega}_{t+1}}\omega f_t(\omega)d\omega, \tag{4.207}$$

$$L_t=\frac{Q_{\bar{K},t}\bar{K}_{j_E,t+1}}{N_{j_E,t}}.$$

I define $\Gamma_t(\bar{\omega}_{t+1})$ and $G_t(\bar{\omega}_{t+1})$ for notational purposes. To obtain the right hand side of (4.204) insert (4.203) into the left hand side. I then obtain the following

$$\mathrm{E}_t\int_{\bar{\omega}_{t+1}}^{\infty}(\omega-\bar{\omega}_{t+1})f_t(\omega,\sigma_t)d\omega(1+R^k_{t+1})Q_{\bar{K},t}K_{j_E,t+1}\frac{N_{j_E,t}}{N_{j_E,t}}=\dots$$

$$\mathrm{E}_t\int_{\bar{\omega}_{t+1}}^{\infty}(\omega-\bar{\omega}_{t+1})f_t(\omega,\sigma_t)d\omega(1+R^k_{t+1})L_tN_{j_E,t},$$

$$\int_{\bar{\omega}_{t+1}}^{\infty}(\omega-\bar{\omega}_{t+1})f_t(\omega,\sigma_t)d\omega=1-G_t(\bar{\omega}_{t+1})-\{1-F(\bar{\omega}_{t+1})\}\bar{\omega}_{t+1}=\{1-\Gamma_t(\bar{\omega}_{t+1})\}.$$

Here I use the fact that $\lim_{\bar{\omega}_{t+1}\to\infty}G_t(\bar{\omega}_{t+1})=\int_0^{\infty}f(\omega)\omega d\omega=\mathrm{E}\,\omega=1$ and that I can arbitrary split the integral. This implies $\int_{\bar{\omega}_{t+1}}^{\infty}f(\omega)\omega d\omega=\int_0^{\infty}f(\omega)\omega d\omega-\int_0^{\bar{\omega}_{t+1}}f(\omega)\omega d\omega=1-G_t(\bar{\omega}_{t+1})$. Note that $1-\Gamma_t(\bar{\omega}_{t+1})$ is the share of average entrepreneurial earnings. Here $\int_{\bar{\omega}_{t+1}}^{\infty}f(\omega)\omega d\omega$ denotes the expected value of ω conditional that $\omega\geq\bar{\omega}_{t+1}$. On the other side $\bar{\omega}_{t+1}\int_{\bar{\omega}_{t+1}}^{\infty}f(\omega)d\omega$ weights $\bar{\omega}_{t+1}$ with the probability that $\Pr(\omega\geq\bar{\omega}_{t+1})$. Entrepreneurs with $\omega<\bar{\omega}_{t+1}$ are ignored, because their net worth in $t+1$ is zero. These entrepreneurs will go bankrupt, because they are not able to repay their obligations to the mutual funds.

This raises the question how mutual funds decide how much loan they grant to entrepreneurs. It is assumed that each mutual fund holds perfectly diversified loans of portfolios to entrepreneurs with different $N_{j_E,t}$. Mutual funds get deposits from households and they have to pay them back the principal times R_t. Therefore the opportunity costs of extending loans to entrepreneurs at rate Z_{t+1} is reflected by loans granted to households. If an entrepreneur goes bankrupt the mutual fund obtains $(1-\mu)\omega(1+R^k_{t+1})Q_{\bar{K},t}\bar{K}_{j_E,t+1}$. Here μ is the fraction of monitoring costs a mutual fund has to pay for knowing whether the entrepreneur is bankrupt or not. In this case the mutual fund gets all the effective capital of this entrepreneur. From solvent firms they get the promised $(1+Z_{t+1})B_{j_E,t+1}$. Due to the fact that they hold perfectly diversified portfolio and they are not allowed to discriminate a priori, they have to provide loans to every entrepreneur at the same rate of interest. A mutual fund extends loans to entrepreneurs according to

$$\{1 - F_t(\bar{\omega}_{t+1})\}(1 + Z_{t+1})B_{j_E,t+1} + (1-\mu)G_t(\bar{\omega}_{t+1})(1 + R^k_{t+1})Q_{\bar{K},t}\bar{K}_{j_E,t+1} \geq B_{j_E,t+1}(1 + R_t). \tag{4.208}$$

(4.208) is the cash constraint, stating that expected earnings from lending loans to entrepreneurs must be greater or equal to the amount, which mutual funds have to repay to the households. New mutual funds do not face entry costs. It is therefore not possible for a mutual fund to make expected nonzero profits. The inequality is equality under free entry. I can simplify this expression by inserting (4.203) and use $\frac{B_{j_E,t+1}}{Q_{\bar{K},t}\bar{K}_{j_E,t+1}} = \frac{L_t - 1}{L_t}$. The cash constraint is given by

$$\Gamma_t(\bar{\omega}_{t+1}) - \mu G_t(\bar{\omega}_{t+1}) = \frac{L_t - 1}{L_t}\frac{1 + R_t}{1 + R^k_{t+1}}. \tag{4.209}$$

An entrepreneur has to choose the optimal level of leverage given the realized ω according to the menu of contracts supplied by mutual funds (4.209). One can now set up the Lagrangian for the entrepreneur's optimization problems. The objective is given by (4.204) and the constraint is (4.209). One can obtain the following Lagrangian

$$\begin{aligned} L^E_t &= \mathrm{E}_t[\{1 - \Gamma_t(\bar{\omega}_{t+1})\}(1 + R^k_{t+1})L_t N_{j_E,t} \dots \\ &+ \mu^E_t\{\Gamma_t(\bar{\omega}_{t+1}) - \mu\, G_t(\bar{\omega}_{t+1}) - \frac{L_t - 1}{L_t}\frac{1 + R_t}{1 + R^k_{t+1}}\}]. \end{aligned} \tag{4.210}$$

Here it is important to know that the entrepreneur knows $\bar{\omega}_{t+1}$ before they optimize L_t. The FOC associated with $\bar{\omega}_{t+1}$ determines the value of the Lagrange multiplier. This multiplier is derived by

$$\frac{\partial L^E}{\partial \bar{\omega}_{t+1}} = \mathrm{E}_t[-\Gamma'_t(\bar{\omega}_{t+1})(1 + R^k_{t+1})L_t N_{j_E,t} + \mu^E_t\{\Gamma'_t(\bar{\omega}_{t+1}) - \mu G'_t(\bar{\omega}_{t+1})\}],$$

$$\mu^E_t = \mathrm{E}_t\,\frac{\Gamma'_t(\bar{\omega}_{t+1})(1 + R^k_{t+1})L_t N_{j_E,t}}{\Gamma'_t(\bar{\omega}_{t+1}) - \mu G'_t(\bar{\omega}_{t+1})}. \tag{4.211}$$

Now plug (4.211) into (4.210) and take the FOC w.r.t. L_t to get

$$\begin{aligned} \frac{dL^E}{dL_t} = \mathrm{E}_t\Bigg[&\{1 - \Gamma_t(\bar{\omega}_{t+1})\}\frac{1 + R^k_{t+1}}{1 + R_t} + \frac{\Gamma'_t(\bar{\omega}_{t+1})}{\Gamma'_t(\bar{\omega}_{t+1}) - \mu G'_t(\bar{\omega}_{t+1})}\Big[\frac{(1 + R^k_{t+1})}{(1 + R_t)}\{\Gamma_t(\bar{\omega}_{t+1}) \dots \\ &- \mu G_t(\bar{\omega}_{t+1})\} - 1\Big]\Bigg]. \end{aligned} \tag{4.212}$$

The standard debt contract is independent of the specific $N_{j_E,t}$ of an entrepreneur.

Note, that revenues of mutual funds $B_{t+1}\,Z_{t+1}$ are equal to $\Gamma_t(\bar{\omega}_{t+1})\,(1{+}R^k_{t+1})\,Q_{\bar{K},t}K_{j_E,t+1}$.

Further, the cash constraint can be solved for the risk free interest rate R_t. Therefore, it is possible to express the credit spread by

$$\begin{aligned} Z_t - R_t &= [\Gamma_{t-1}(\bar{\omega}_t) - \{\Gamma_{t-1}(\bar{\omega}_t) - \mu\, G_{t-1}(\bar{\omega}_t)\}]\; Q_{\bar{K},t-1} K_{j_E,t}, \\ Z_t - R_t &= \mu\, G_{t-1}(\bar{\omega}_t)\, Q_{\bar{K},t-1} K_{j_E,t}. \end{aligned} \tag{4.213}$$

This expression is used for the observational equation to estimate the model.

It is also necessary to derive the law of motion for N_{t+1}. One can define

$$V_t = \{1 - \Gamma_{t-1}(\bar{\omega}_t)\}(1 + R_t^k) Q_{\bar{K},t-1} \bar{K}_t, \tag{4.214}$$

$$V_t = [1 - \{1 - F_{t-1}(\bar{\omega}_t)\}\bar{\omega}_t - G_{t-1}(\bar{\omega}_t)](1 + R_t^k) Q_{\bar{K},t-1}, \tag{4.215}$$

$$\begin{aligned} V_t &= (1 + R_t^k) Q_{\bar{K},t-1} - [\{1 - F_{t-1}(\bar{\omega}_t)\}\bar{\omega}_t \\ &+ (1-\mu) G_{t-1}(\bar{\omega}_t)](1 + R_t^k) Q_{\bar{K},t-1} - \mu G_{t-1}(\bar{\omega}_t)(1 + R_t^k) Q_{\bar{K},t-1}, \end{aligned} \tag{4.216}$$

$$V_t = \{R_t^k - R_{t-1} - \mu G_{t-1}(\bar{\omega}_t)(1 + R_t^k) Q_{\bar{K},t-1}\} Q_{\bar{K},t-1} \bar{K}_t + (1 + R_t) N_t. \tag{4.217}$$

Here V_t in (4.214) represents the net worth of an entrepreneur minus lump sum transfers W_t^e from households and the transfers from entrepreneurs to households $1-\gamma_t$. The average share of entrepreneurial earnings received by entrepreneurs is $\{1 - \Gamma_{t-1}(\bar{\omega}_t)\}$, which is multiplied by the initial amount of investment $Q_{\bar{K},t-1}\bar{K}_t$ and the total return to capital $1+R_t^k$. I plug in (4.205) into (4.214) to obtain (4.215). Afterwards I use the fact that mutual funds earnings are equal to the second addend in (4.216) or $(1 + R_t)\,(Q_{\bar{K},t-1}\bar{K}_t - N_t)$, which follows from (4.203) and (4.208). Now you just rearrange terms in (4.216) to get to (4.217).

Now one can multiply this expression by the share of earnings not transferred to households γ_t and add lump-sum transfers W_t^e to get

$$N_{t+1} = \gamma_t\{R_t^k - R_{t-1} - \mu G_{t-1}(\bar{\omega}_t)(1 + R_t^k)\} Q_{\bar{K},t-1} \bar{K}_t + \gamma_t (1 + R_t) N_t + W^e. \tag{4.218}$$

Lets now take a look at the aggregates of the model. The aggregate, raw capital stock, capital services and loans extended are given by

$$\bar{K}_{t+1} = \int_0^\infty \bar{K}_{t+1}^N f_t(N) dN, \tag{4.219}$$

$$K_t^s = \int_0^\infty \int_0^\infty u_t \omega \bar{K}_t^N f_{t-1}(N) f(\omega, \sigma_t) d\omega dN = u_t \bar{K}_t, \tag{4.220}$$

$$B_{t+1} = \int_0^1 B_{t+1}^N f_t(N) dN = \int_0^1 (Q_{\bar{K},t} \bar{K}_{t+1}^N - N) f_t(N) dN = Q_{\bar{K},t} \bar{K}_{t+1} - N_{t+1}. \tag{4.221}$$

The following equations are used in the model: (4.202), (4.206), (4.207), (4.212), (4.209) and (4.218).

4.8.1.9 Monetary policy

Risk free interest rates for short-term bonds B_t are determined by the central bank. The central bank or the monetary authority is assumed to set R_t according to an interest rate rule,

$$\frac{1+R_t}{1+\bar{R}} = \left(\frac{1+R_{t-1}}{1+\bar{R}}\right)^{\tilde{\rho}} \left\{ \left(\frac{\pi_{t-1}}{\bar{\pi}}\right)^{1+\tilde{a}_\pi} \left(\frac{\mu_t^z}{\bar{\mu^z}} \frac{c_{t-1} + \frac{i_{t-1}}{\mu_{t-1}^{\Upsilon}} + g_{t-1}}{c_{t-2} + \frac{i_{t-2}}{\mu_{t-2}^{\Upsilon}} + g_{t-2}}\right)^{\tilde{a}_{\Delta y}} \right\}^{1-\tilde{\rho}} + \frac{\sigma^{x^p}}{4} x_t^p. \quad (4.222)$$

The specification is the same as the one used by Christiano et al. (2014), with an annual monetary policy shock x_t^p.

4.8.1.10 Resource constraint

The whole economy produces aggregate real output Y_t. This aggregate output consists of private real consumption C_t, government real consumption G_t, real monitoring costs by mutual funds D_t and real costs for providing capital services $a(u_t)\Upsilon^{-t}\bar{K}_t$. I can derive the following resource constraint from the budget constraint of the representative household

$$Y_t = D_t + G_t + C_t + \frac{I_t}{\Upsilon^t \mu_{\Upsilon,t}} + \tau_t^o a(u_t) \frac{\bar{K}_t}{\Upsilon^t} + \Theta \frac{1-\gamma_t}{\gamma_t P_t}(N_{t+1} - W^e) - P_t^O (O_t^{ex} - O_t^{im}). \quad (4.223)$$

Here one can use the fact that government expenditure is the sum of all lump-sum taxes Tax_t, taxes on capital, taxes on labour income, taxes on oil and less lump-sum transfers Tr_t to households and deductible taxes on capital depreciation. Profits of intermediate goods-producing firms are $\Delta_t = P_t Y_t - W_t h_t - \tilde{r}_t^k P_t u_t \bar{K}_{t-1} - P_t^O O_t$. Domestic oil-producing firms transfer profits $P_t^O O_t^d - \Gamma(O_t^d)$ to households. One can use the identity for oil consumption to replace domestic oil production O^d by domestic oil consumption O, oil exports O^{ex} and oil imports O^{im}. Domestic oil consumption expenditures will cancel out, but oil exports and imports remain in the resource constraint. In order to have monitoring costs by mutual funds and the share of net worth consumed by existing entrepreneurs, one can modify the following expressions from the budget constraint

$$B_{t+1} + Q_{\bar{K},t}\,(1-\delta)\bar{K}_t + W^e = (1+R_{t-1})\,B_t + \underbrace{Q_{\bar{K},t}\,\bar{K}_{t+1}}_{B_{t+1}+N_{t+1}} \dots \tag{4.224}$$
$$+ (1-\gamma_t)\,(1-\Theta)\,[1-\Gamma(\bar{\omega}_t)]\,(1+R^k_t)\,Q_{\bar{K},t-1}\,\bar{K}_t \dots$$
$$+ \{r^K_t u_t - a(u_t)\}\,P_t \Upsilon^{-t}\bar{K}_t + (1-\delta)\,Q_{\bar{K},t}\bar{K}_t - (1+R^k_t)\,Q_{\bar{K},t-1}\,\bar{K}_t,$$
$$a(u_t)\,P_t\Upsilon^{-t}\bar{K}_t = r^K_t u_t\,P_t\Upsilon^{-t}\bar{K}_t + N_{t+1} - W^e + (1-\gamma_t)\,(1-\Theta)\,\frac{N_{t+1}-W^e}{\gamma_t} \tag{4.225}$$
$$- \underbrace{\left((1+R^k_t)\,Q_{\bar{K},t-1}\,\bar{K}_t - (1+R_{t-1})\,B_t\right)}_{\frac{1}{\gamma_t}(N_{t+1}-W^e)},$$
$$a(u_t)\,P_t\,\Upsilon^{-t}\,\bar{K}_t = N_{t+1} - W^e + r^K_t\,u_t\,P_t\Upsilon^{-t}\bar{K}_t + (1-\gamma_t)\,(1-\Theta)\,\frac{N_{t+1}-W^e}{\gamma_t} \tag{4.226}$$
$$- \underbrace{\left((1+R^k_t)\,Q_{\bar{K},t-1}\,\bar{K}_t - (1+R_{t-1})\,B_t\right)}_{\frac{1}{\gamma_t}(N_{t+1}-W^e)+D_t} + \Theta\,(1-\gamma_t)\,\frac{N_{t+1}-W^e}{\gamma_t}.$$

The real monitoring costs D_t is the share of earnings of entrepreneurs spent for monitoring relative to the present price level,

$$D_t = \mu G_{t-1}(\bar{\omega}_t)(1+R^k_t)\frac{Q_{\bar{K},t-1}\bar{K}_t}{P_t}. \tag{4.227}$$

I assume that government expenditures is the product of z_t and g_t,

$$G_t = z_t g_t. \tag{4.228}$$

Further I know that oil imports and oil exports have a different trend and I assume that government expenditures is the product of z_t and g_t,

$$O_t = z_t\,\Upsilon^{O^t} o_t, \tag{4.229}$$
$$O^d_t = z_t\,\Upsilon^{O^t} o^d_t. \tag{4.230}$$

4.8.1.11 Utilization costs

I assume the following cost function for the utilization of effective capital into capital services $a(u_t)$. This function is given by

$$a(u_t) = r^k\{\exp(\sigma_a(u-1)) - 1\}\frac{1}{\sigma_a}, \tag{4.231}$$

where $\sigma_a > 0$ and r^k is the steady-state rental rate of capital. In steady-state $u = 1$ by the definition of $a(u)$. To see this just consider the first derivative of (4.232) set to zero. Here I get

$$a^{'}(u_t) = r^k\{\exp(\sigma_a(u-1))\} = 0 \iff u = 1. \tag{4.232}$$

The steady-state level of u is independent of r^k.

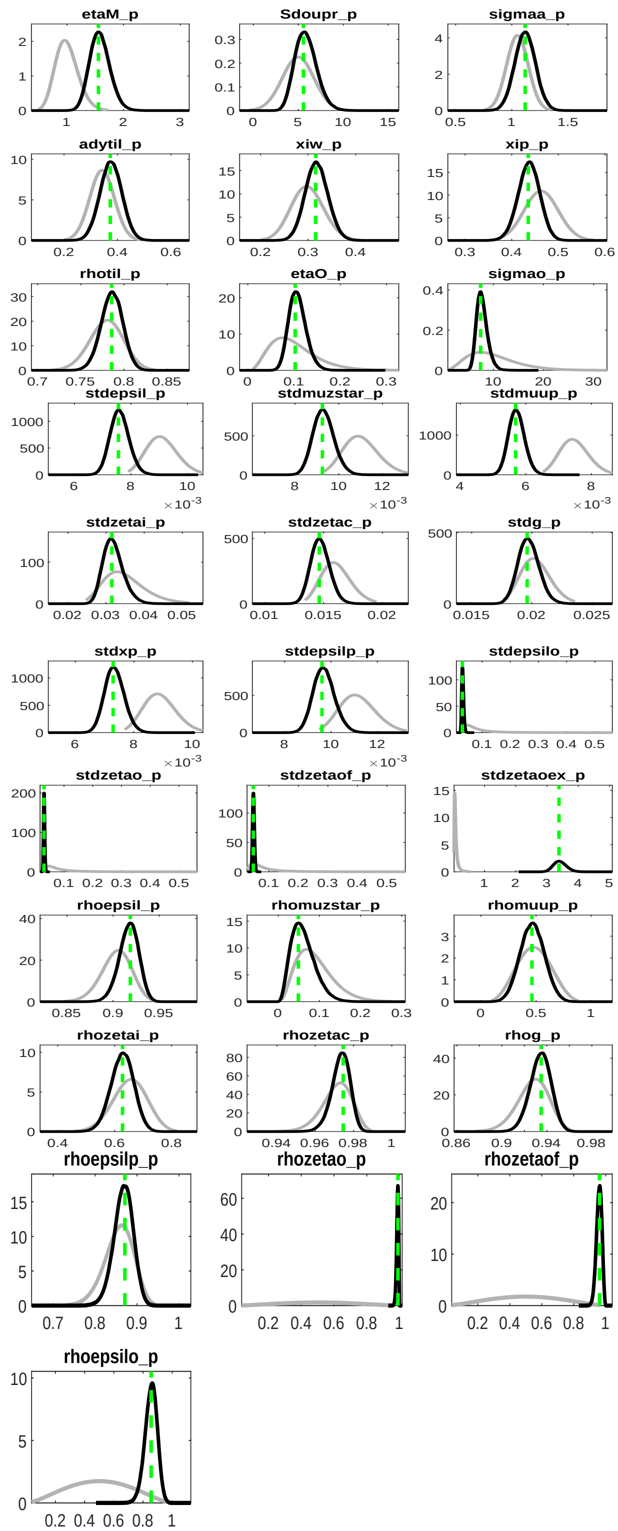
4.8.1.12 Investments adjustment costs

One can model the adjustment costs for investment such that the global minimum appears if investment today is equal to investment from yesterday. If this ratio is greater or smaller than in steady-state, the adjustment costs will increase. I therefore formulate the following adjustment cost function

$$S(\zeta_{I,t}x_t^I) = \frac{1}{2}[\exp\{\sqrt{S''}(\zeta_{I,t}x_t^I - \zeta_I x^I)\} + \exp\{-\sqrt{S''}(\zeta_{I,t}x_t^I - \zeta_I x^I)\} - 2]. \tag{4.233}$$

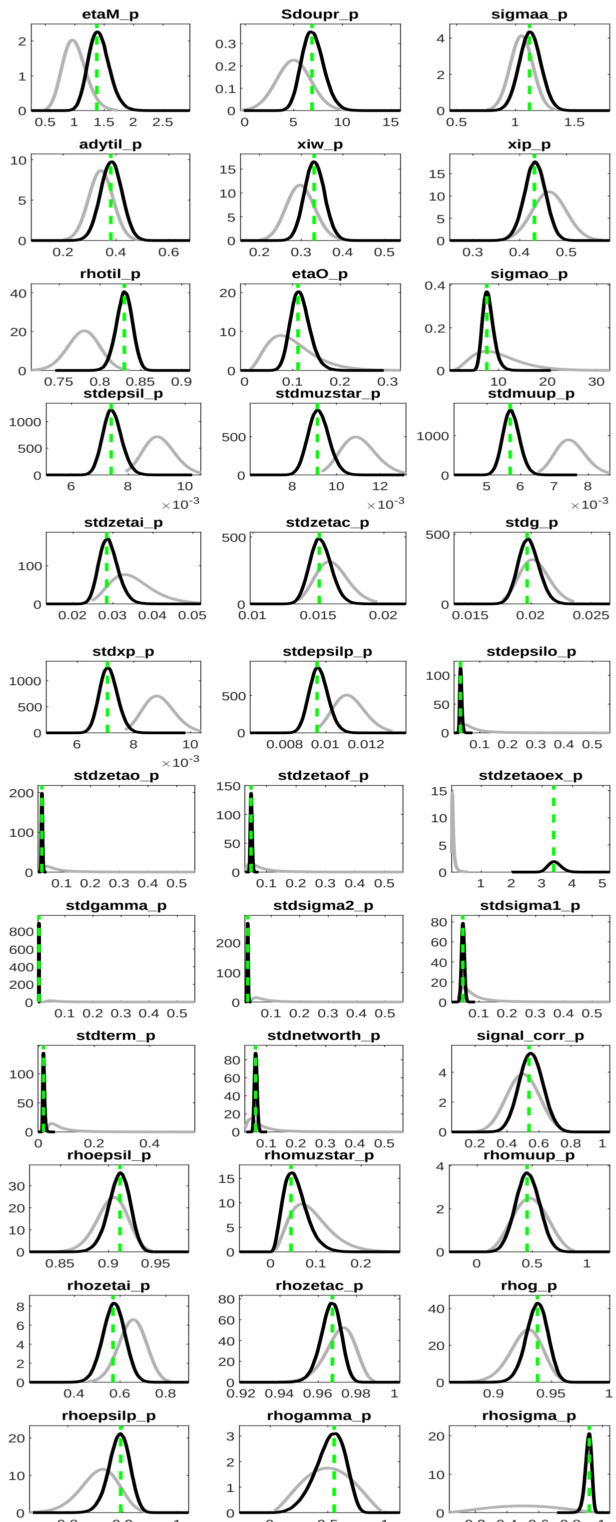
4.8.2 Figures

Figure 4.14: Priors and posteriors CEE-Oil



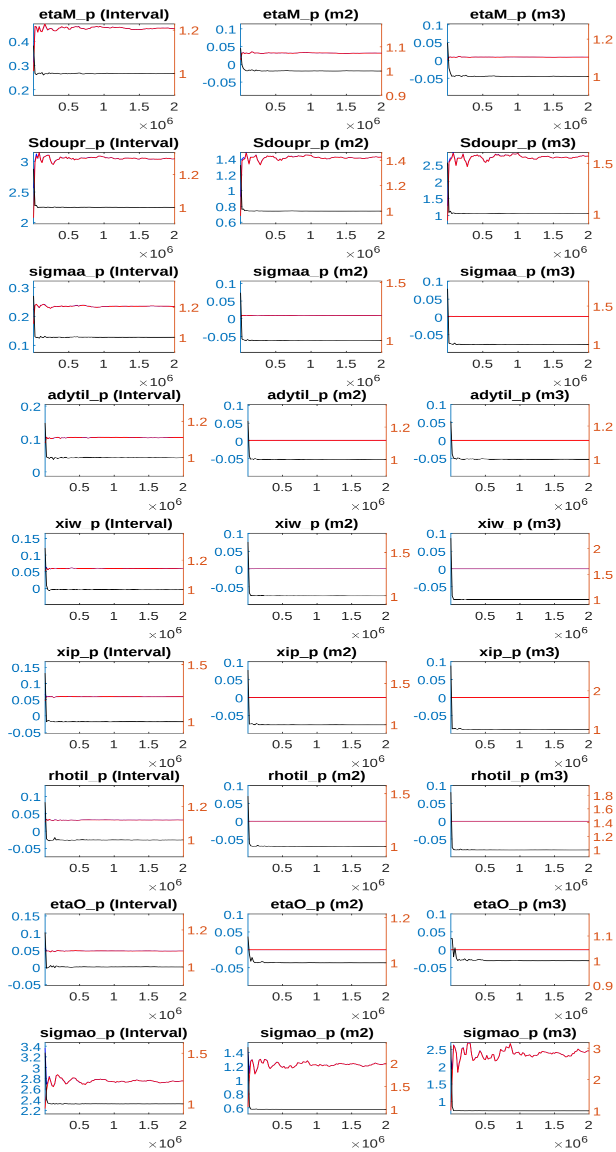
Notes: The grey line depicts the prior density and the black line the posterior density. The posterior mode is depicted by the green dashed line.

Figure 4.15: Priors and posteriors CMR-Oil



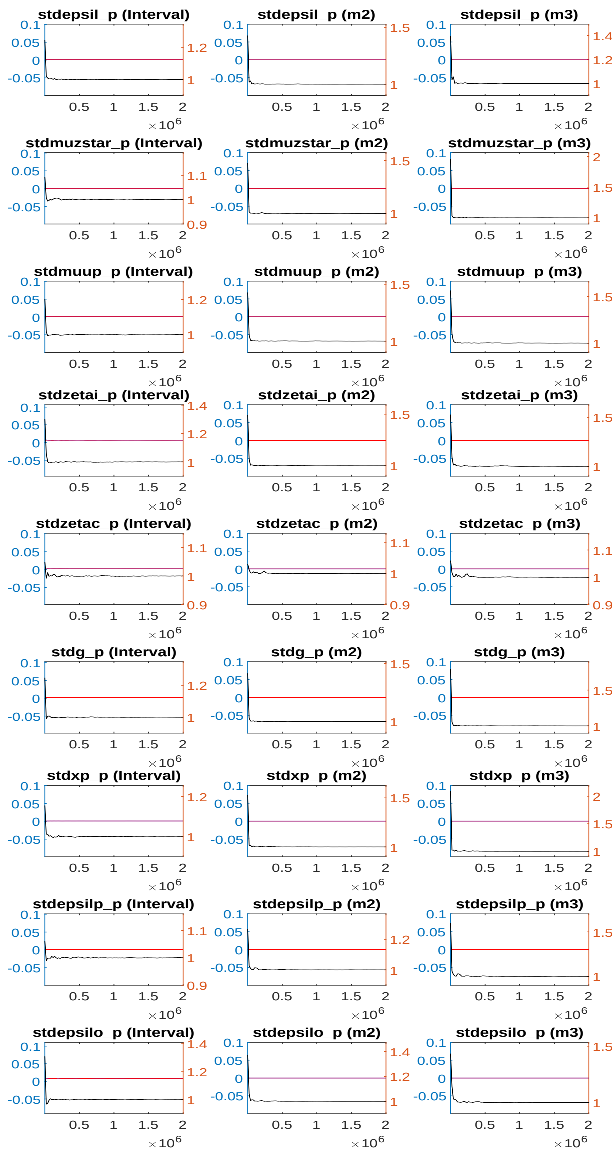
Notes: The grey line depicts the prior density and the black line the posterior density. The posterior mode is depicted by the green dashed line.

Figure 4.16: Parameter convergence CEE-Oil



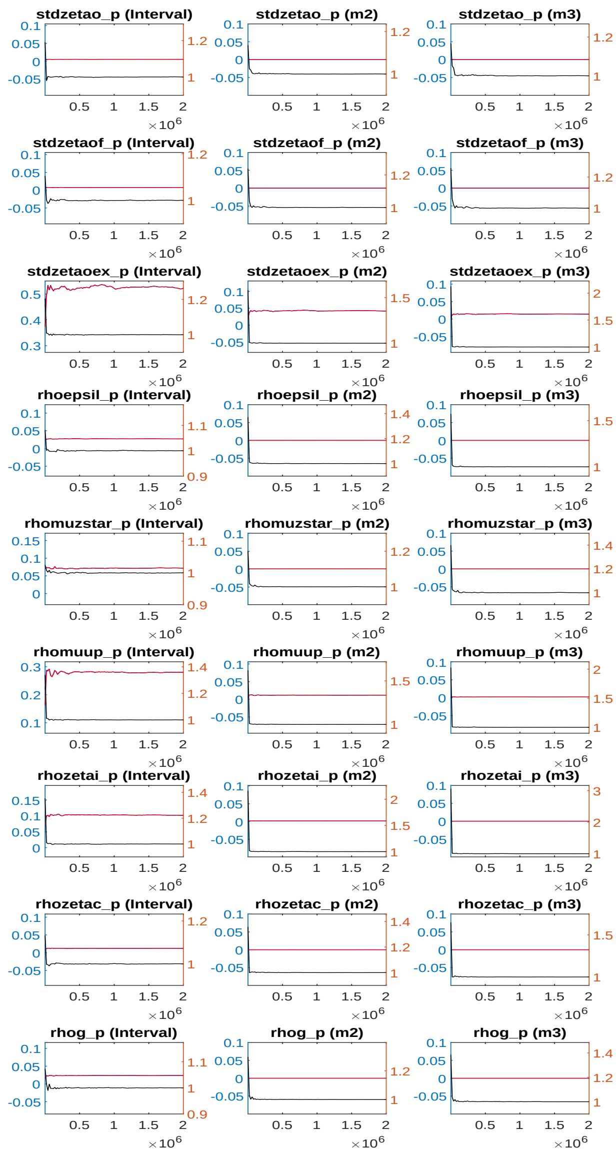
Notes: The first row shows Brooks & Gelman (1998) convergence diagnostics for the 80% interval. The blue line depicts the pooled draws from all sequences, while the red line shows the mean interval range based on the draws of the individual sequences. Second and third central moments of the same statistic are depicted in row 2 and row 3. The grey line represents the ratio between the blue and red line depicted on the right y-axis.

Figure 4.17: Parameter convergence CEE-Oil II



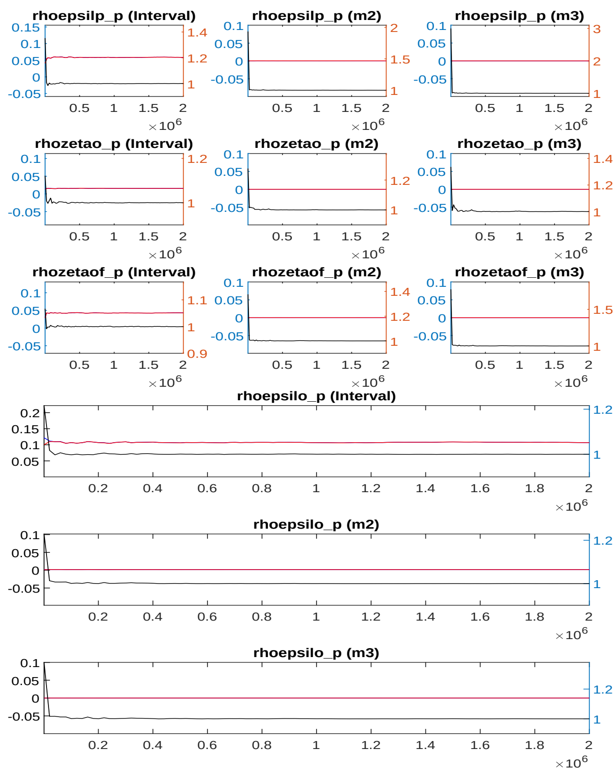
Notes: The first row shows Brooks & Gelman (1998) convergence diagnostics for the 80% interval. The blue line depicts the pooled draws from all sequences, while the red line shows the mean interval range based on the draws of the individual sequences. Second and third central moments of the same statistic are depicted in row 2 and row 3. The grey line represents the ratio between the blue and red line depicted on the right y-axis.

Figure 4.18: Parameter convergence CEE-Oil III



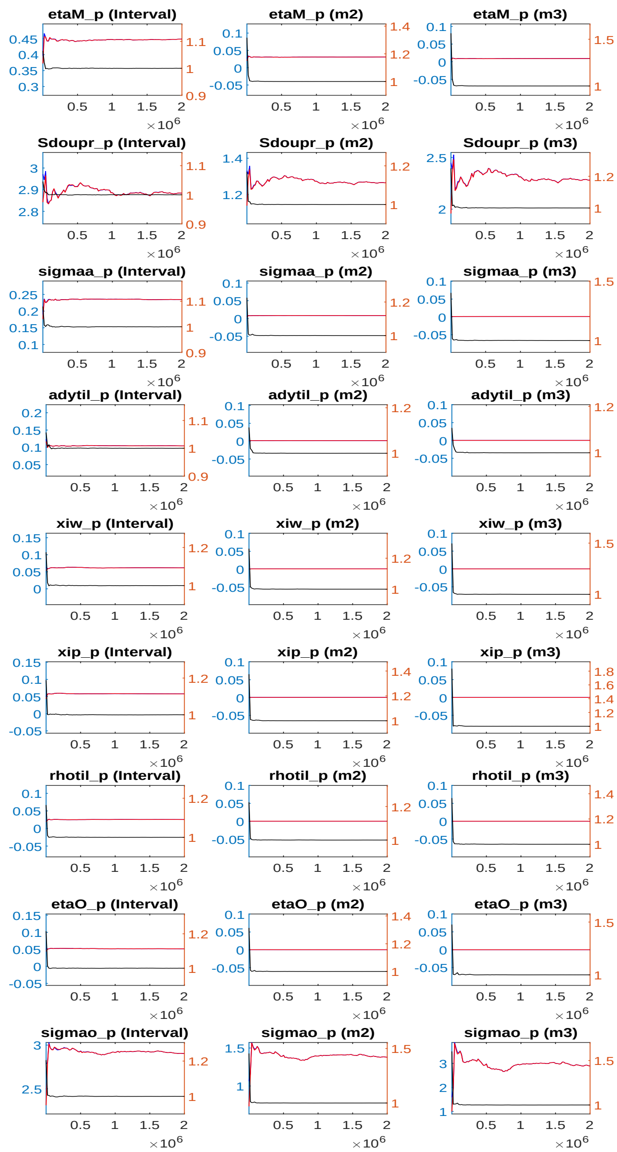
Notes: The first row shows Brooks & Gelman (1998) convergence diagnostics for the 80% interval. The blue line depicts the pooled draws from all sequences, while the red line shows the mean interval range based on the draws of the individual sequences. Second and third central moments of the same statistic are depicted in row 2 and row 3. The grey line represents the ratio between the blue and red line depicted on the right y-axis.

Figure 4.19: Parameter convergence CEE-Oil IV



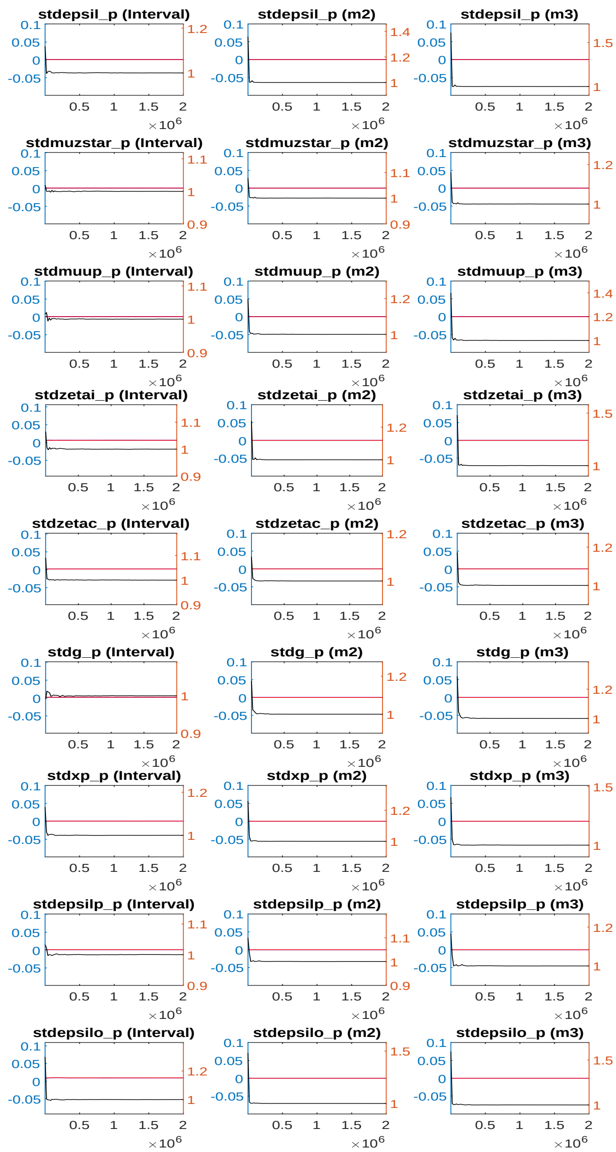
Notes: The first row shows Brooks & Gelman (1998) convergence diagnostics for the 80% interval. The blue line depicts the pooled draws from all sequences, while the red line shows the mean interval range based on the draws of the individual sequences. Second and third central moments of the same statistic are depicted in row 2 and row 3. The grey line represents the ratio between the blue and red line depicted on the right y-axis.

Figure 4.20: Parameter convergence CMR-Oil



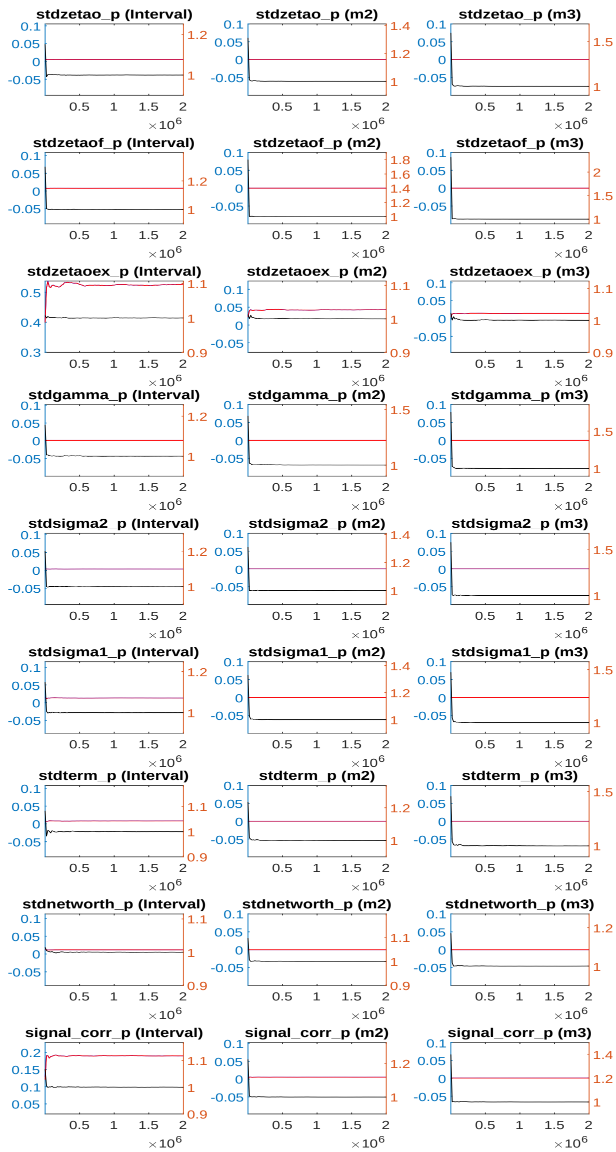
Notes: The first row shows Brooks & Gelman (1998) convergence diagnostics for the 80% interval. The blue line depicts the pooled draws from all sequences, while the red line shows the mean interval range based on the draws of the individual sequences. Second and third central moments of the same statistic are depicted in row 2 and row 3. The grey line represents the ratio between the blue and red line depicted on the right y-axis.

Figure 4.21: Parameter convergence CMR-Oil II



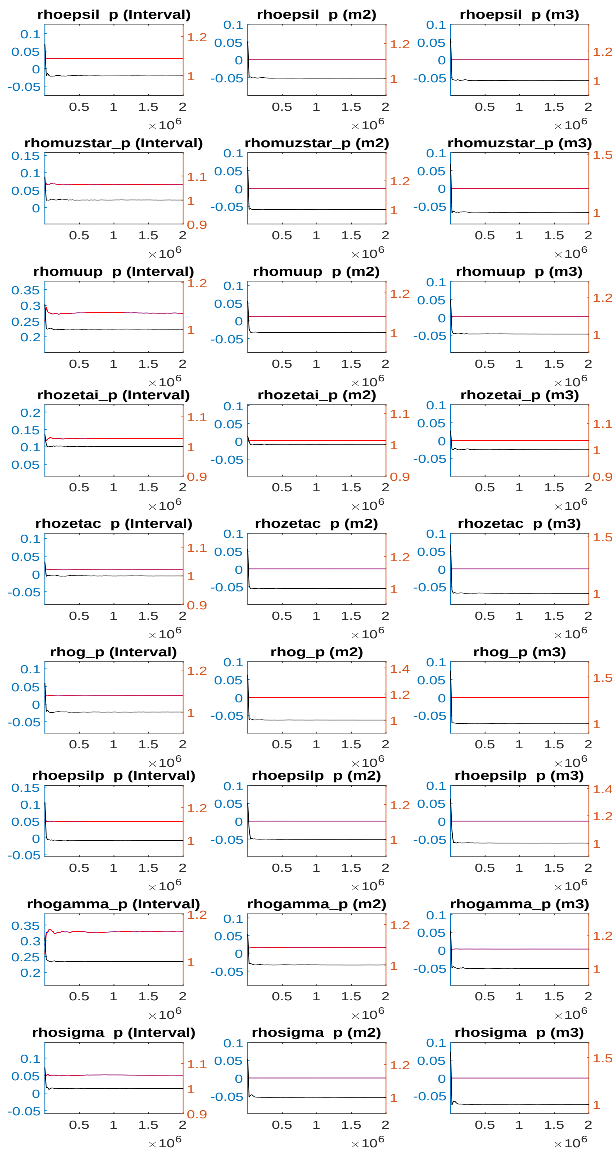
Notes: The first row shows Brooks & Gelman (1998) convergence diagnostics for the 80% interval. The blue line depicts the pooled draws from all sequences, while the red line shows the mean interval range based on the draws of the individual sequences. Second and third central moments of the same statistic are depicted in row 2 and row 3. The grey line represents the ratio between the blue and red line depicted on the right y-axis.

Figure 4.22: Parameter convergence CMR-Oil III



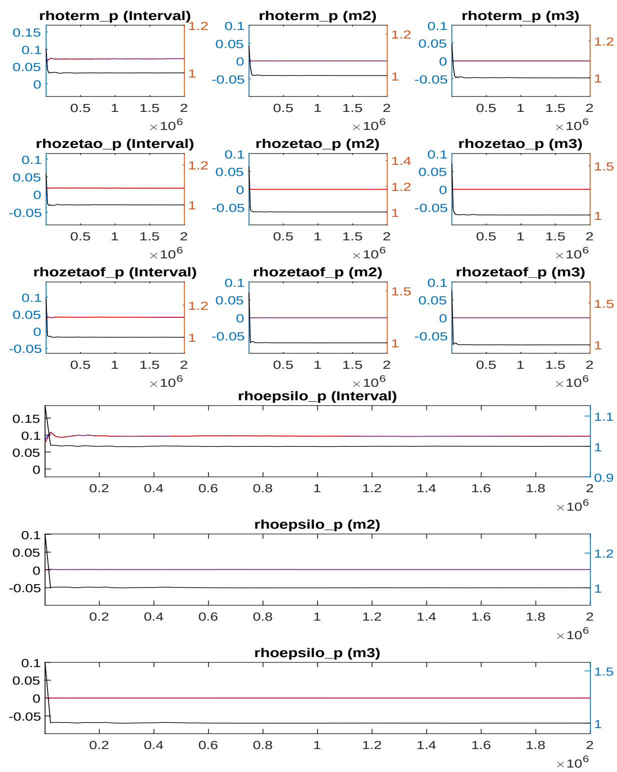
Notes: The first row shows Brooks & Gelman (1998) convergence diagnostics for the 80% interval. The blue line depicts the pooled draws from all sequences, while the red line shows the mean interval range based on the draws of the individual sequences. Second and third central moments of the same statistic are depicted in row 2 and row 3. The grey line represents the ratio between the blue and red line depicted on the right y-axis.

Figure 4.23: Parameter convergence CMR-Oil IV



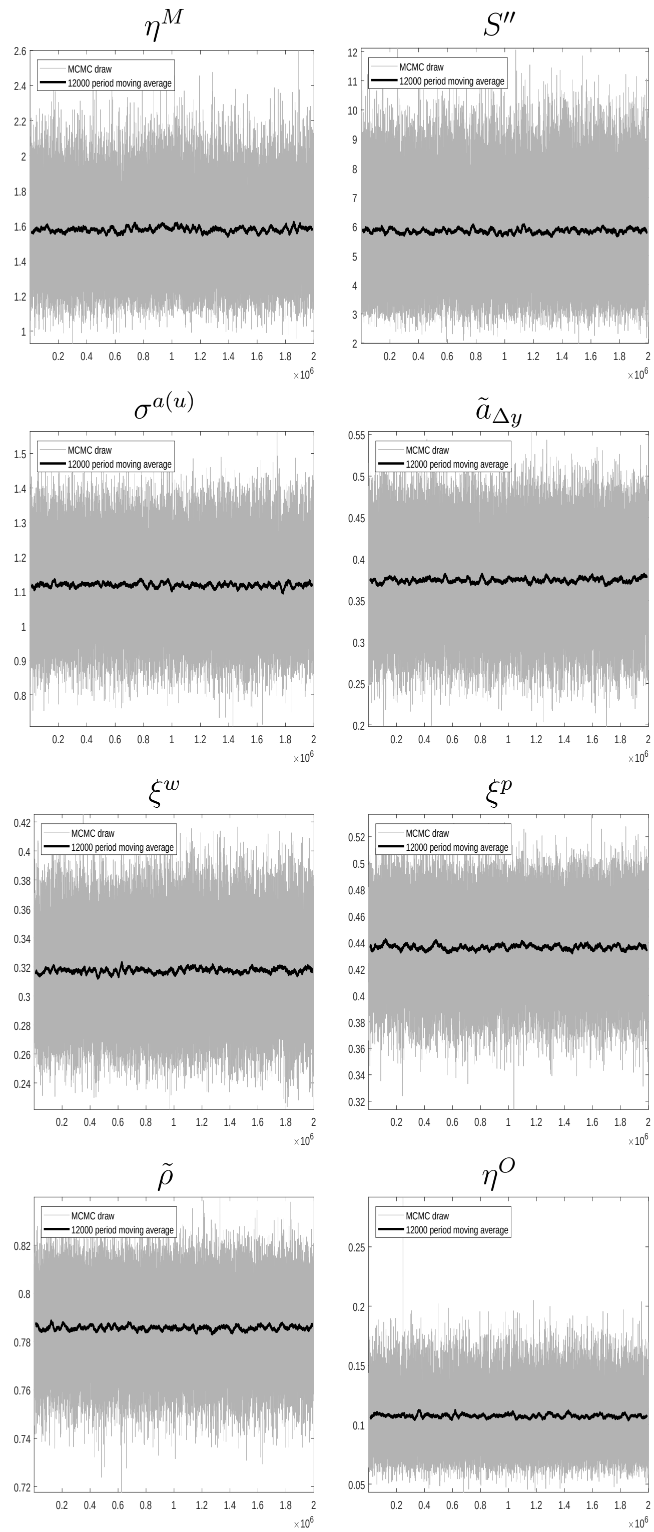
Notes: The first row shows Brooks & Gelman (1998) convergence diagnostics for the 80% interval. The blue line depicts the pooled draws from all sequences, while the red line shows the mean interval range based on the draws of the individual sequences. Second and third central moments of the same statistic are depicted in row 2 and row 3. The grey line represents the ratio between the blue and red line depicted on the right y-axis.

Figure 4.24: Parameter convergence CMR-Oil V



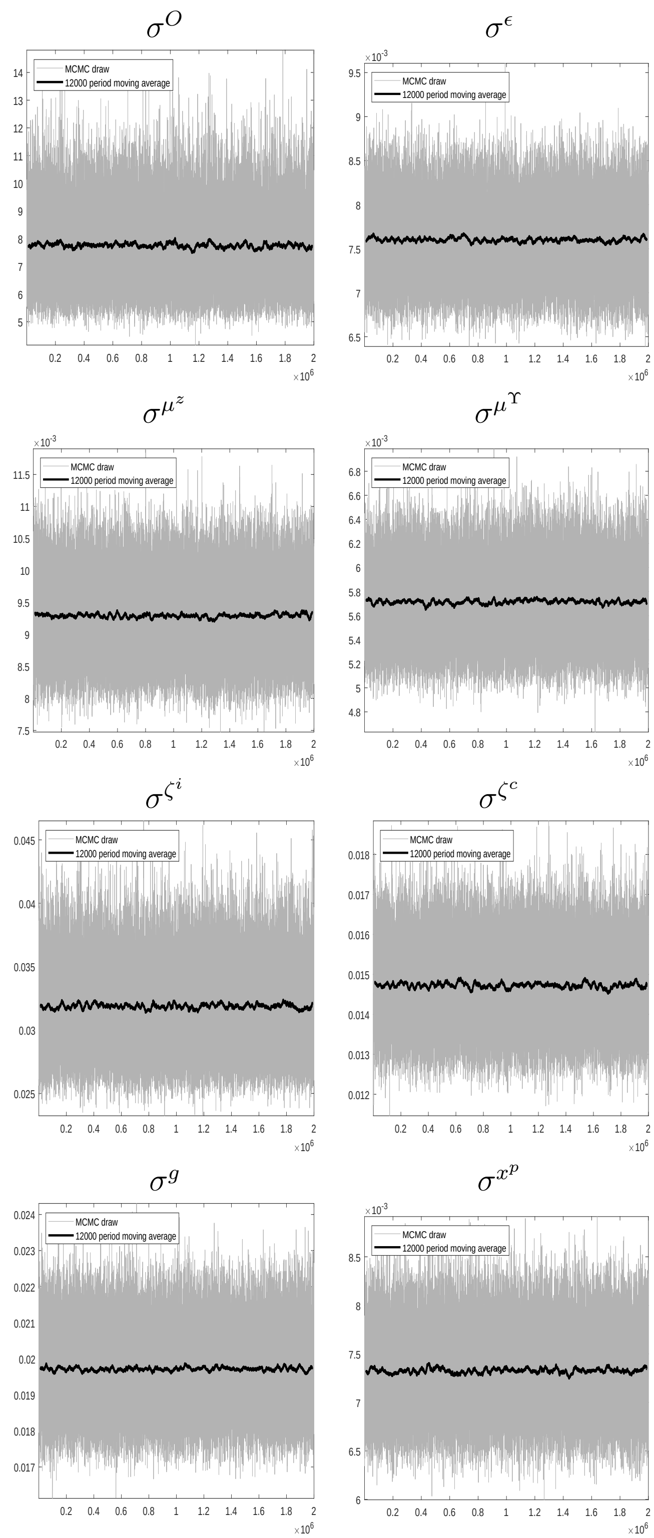
Notes: The first row shows Brooks & Gelman (1998) convergence diagnostics for the 80% interval. The blue line depicts the pooled draws from all sequences, while the red line shows the mean interval range based on the draws of the individual sequences. Second and third central moments of the same statistic are depicted in row 2 and row 3. The grey line represents the ratio between the blue and red line depicted on the right y-axis.

Figure 4.25: Trace plots for chain 1 CEE–Oil I



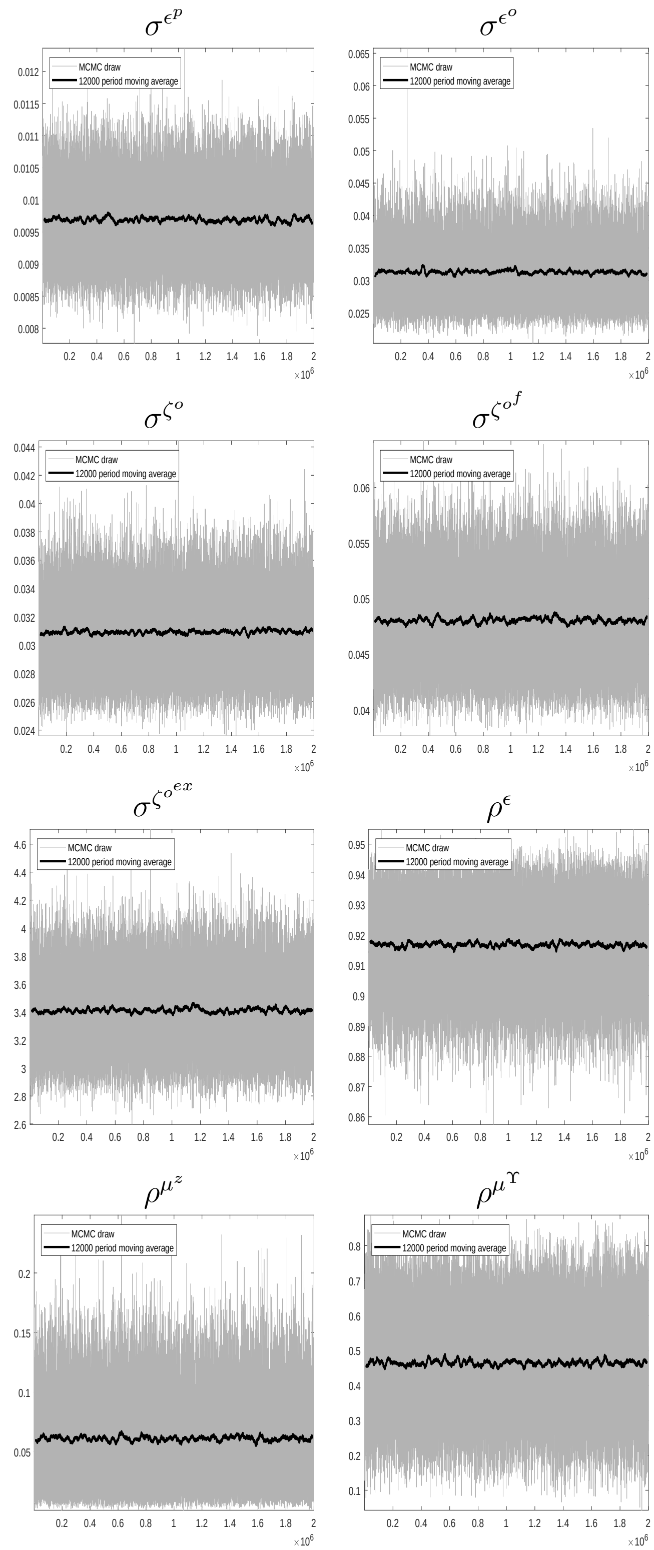
Notes: The grey line depicts parameter values and the back line the moving average.

Figure 4.26: Trace plots for chain 1 CEE–Oil II



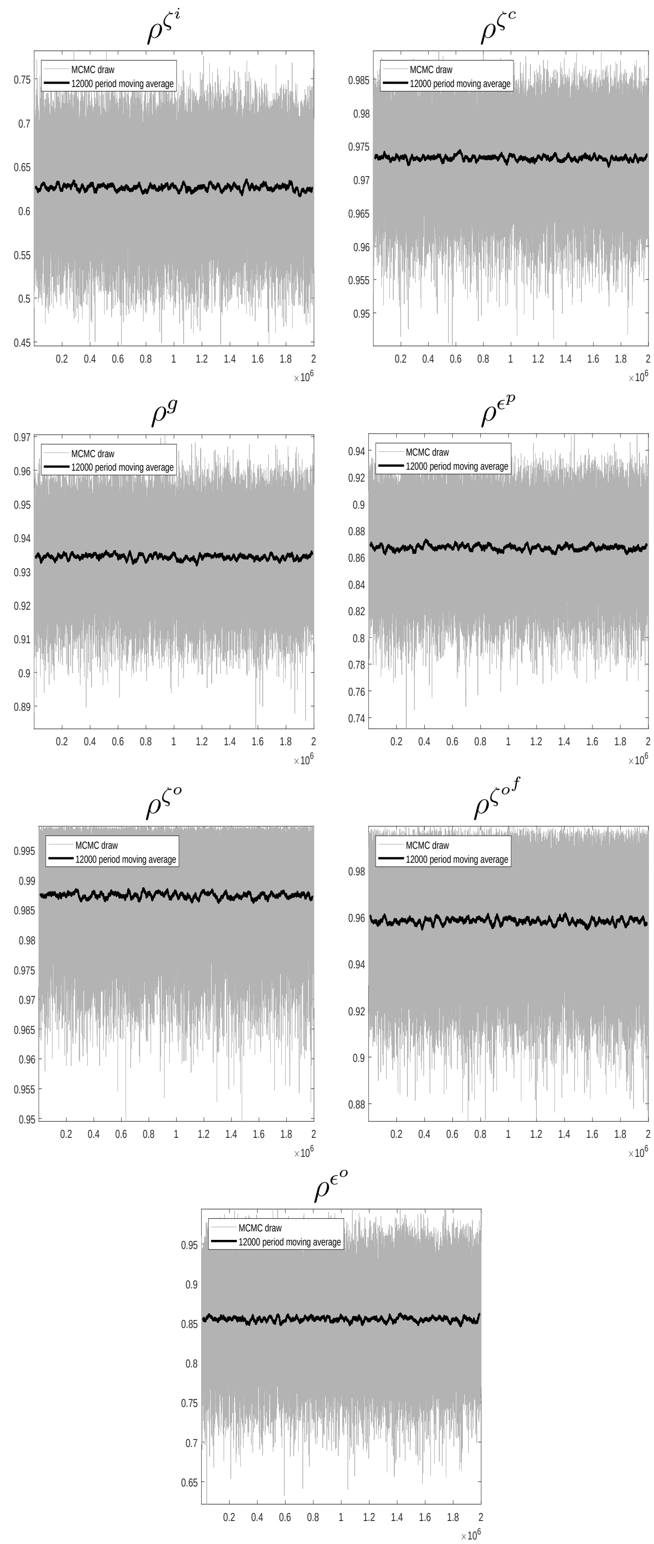
Notes: The grey line depicts parameter values and the back line the moving average.

Figure 4.27: Trace plots for chain 1 CEE–Oil III



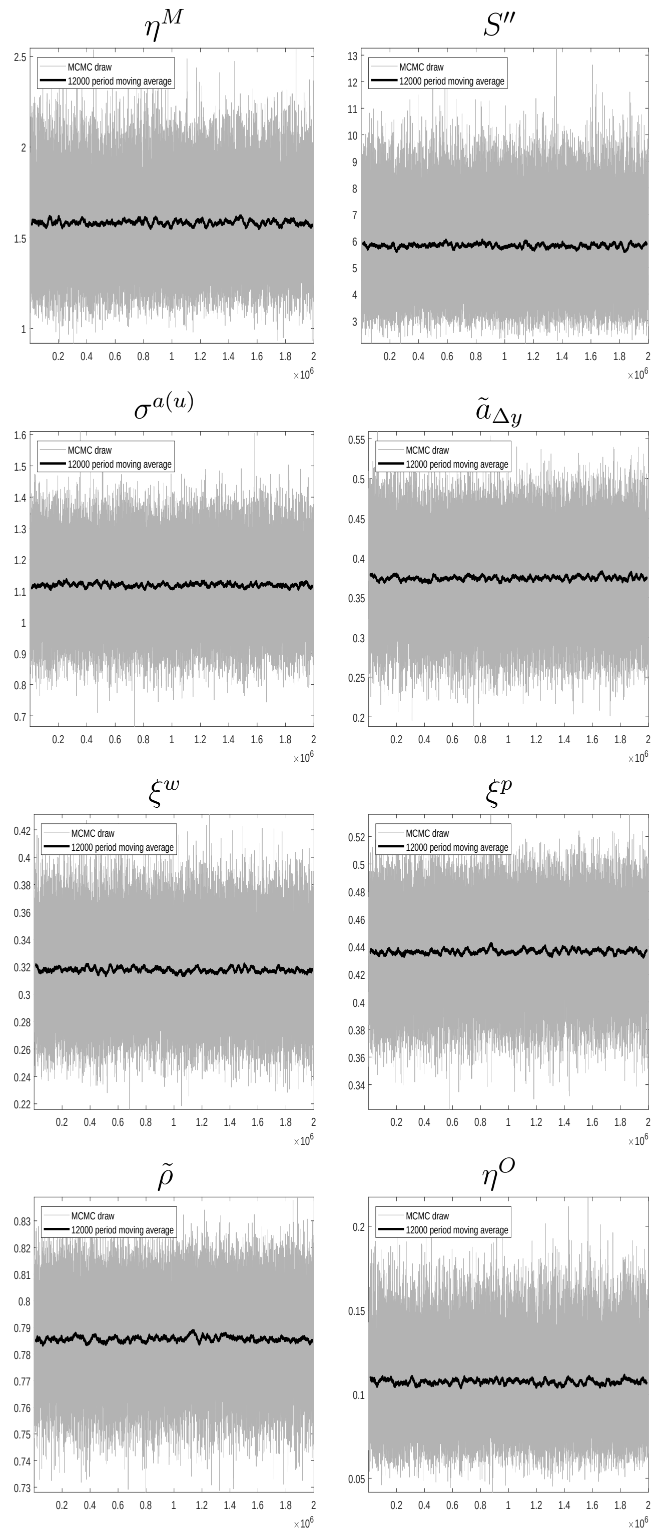
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 4.28: Trace plots for chain 1 CEE–Oil IV



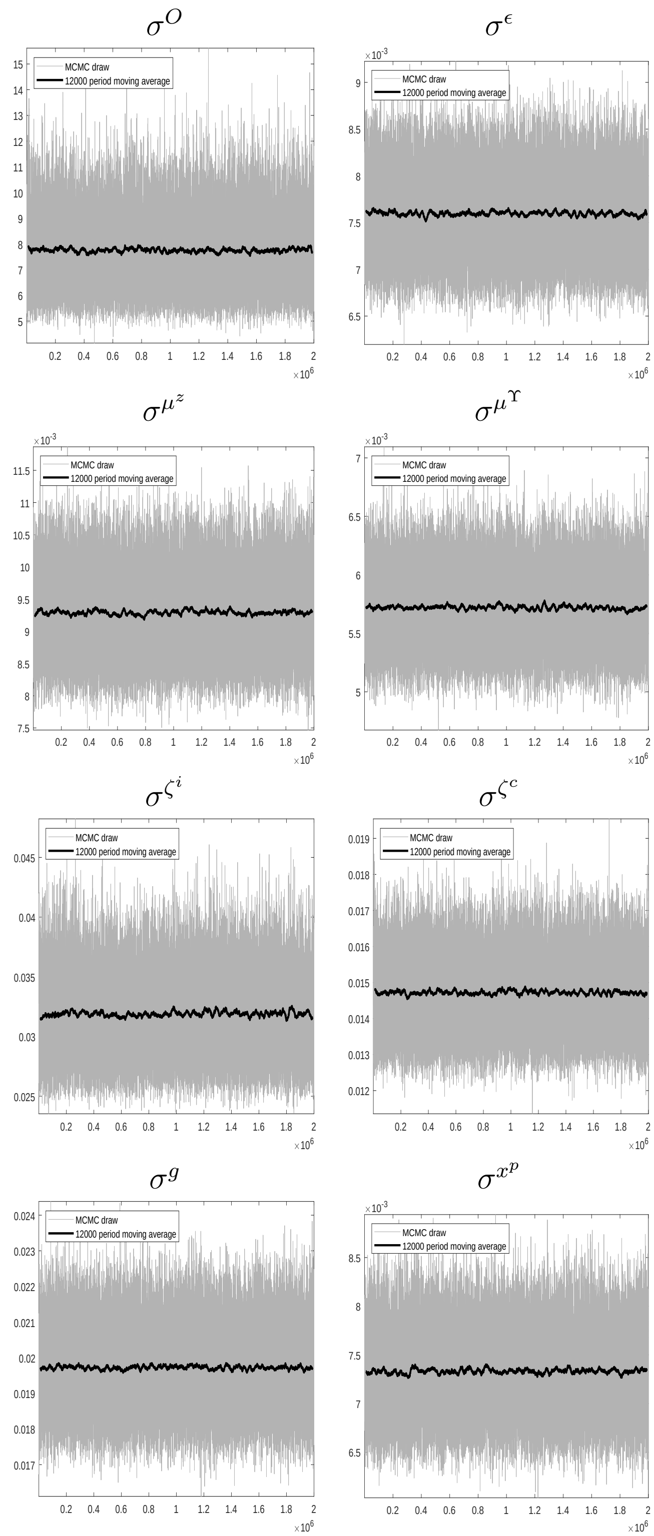
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 4.29: Trace plots for chain 2 CEE–Oil I



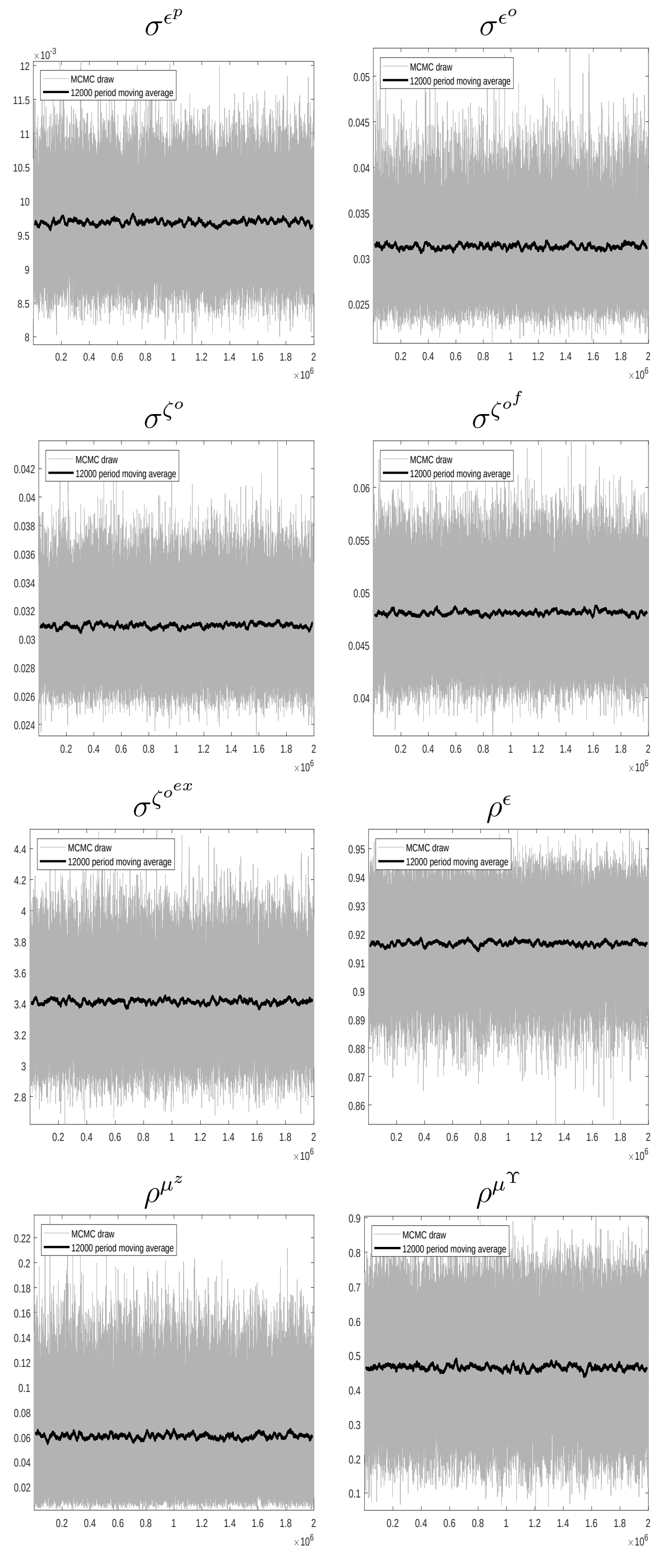
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 4.30: Trace plots for chain 2 CEE–Oil II



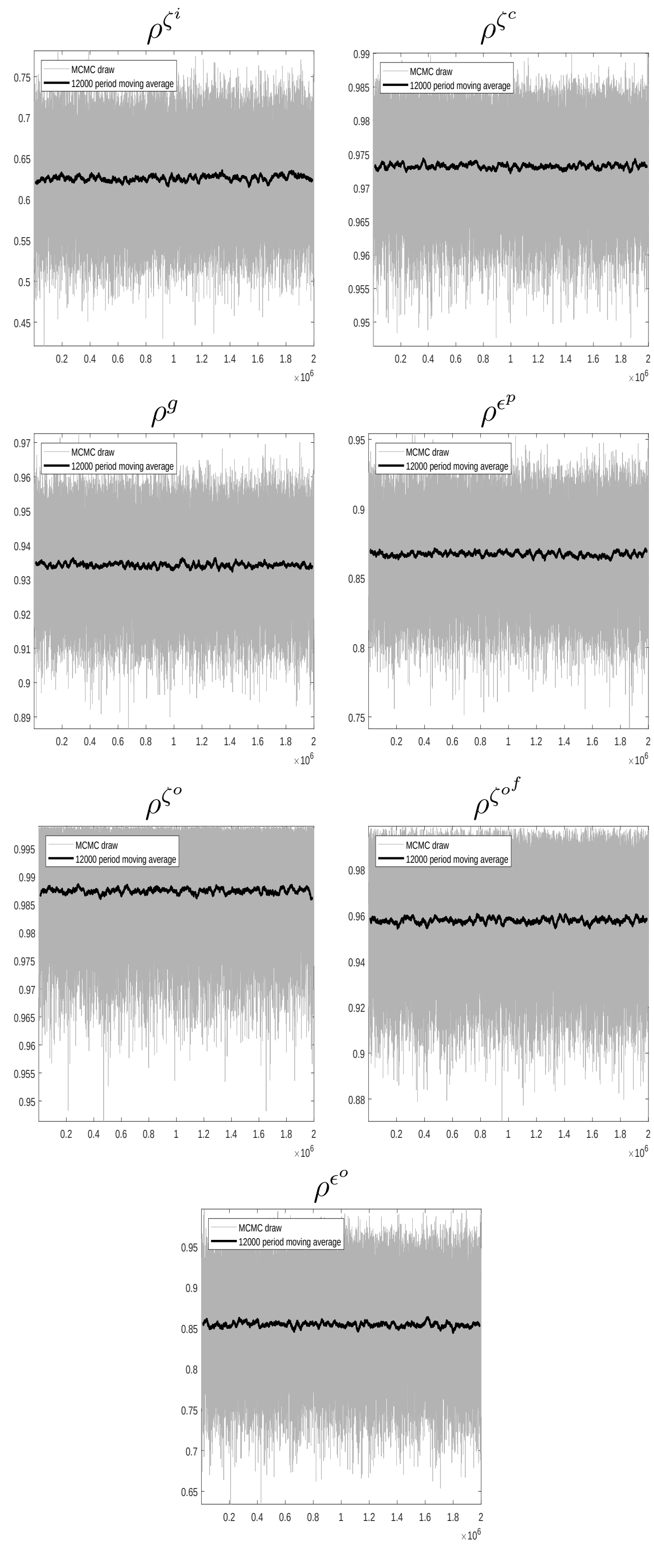
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 4.31: Trace plots for chain 2 CEE–Oil III



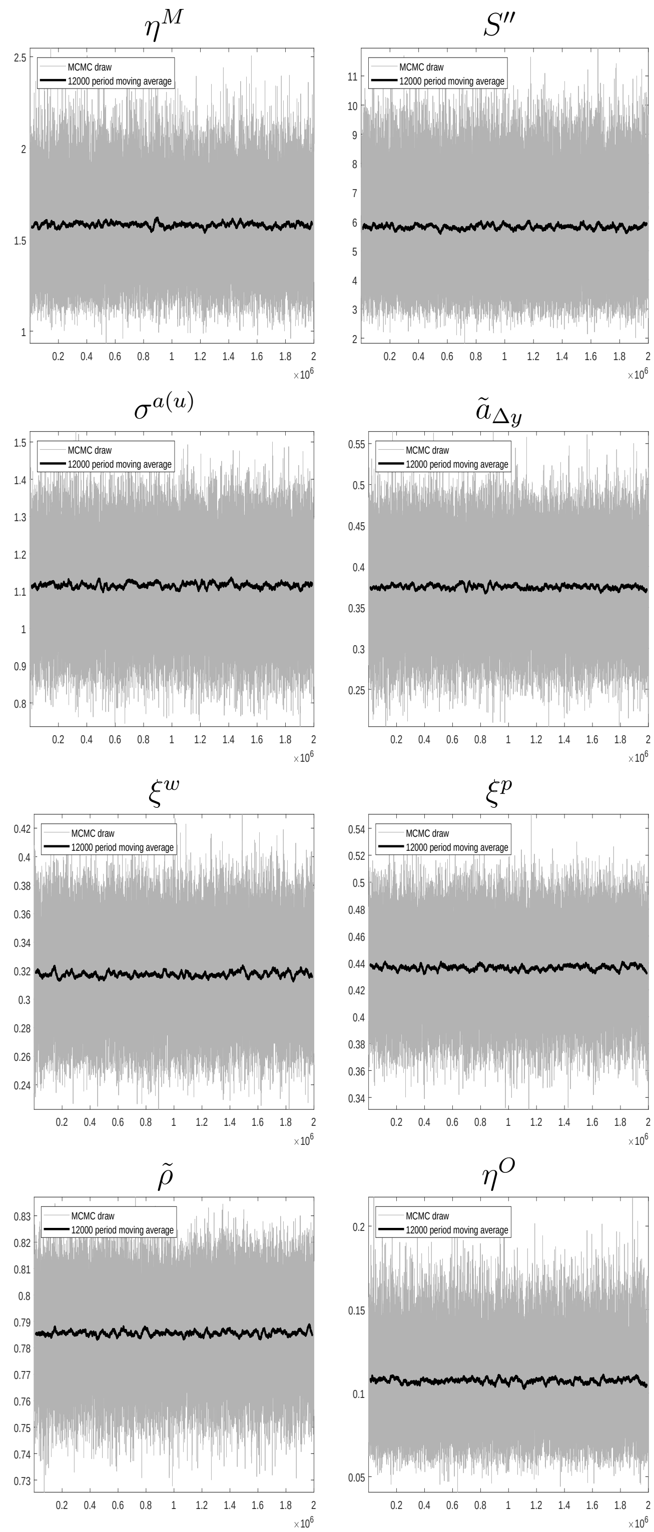
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 4.32: Trace plots for chain 2 CEE–Oil IV



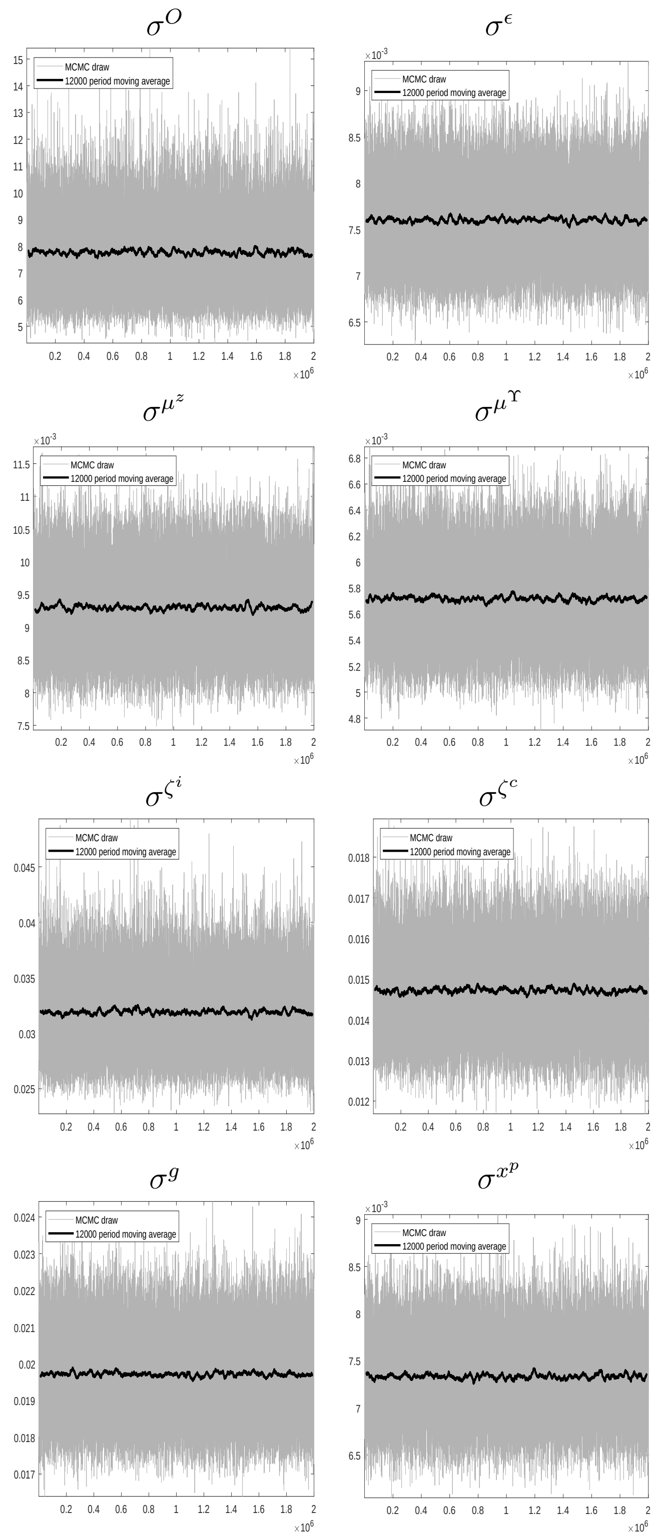
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 4.33: Trace plots for chain 3 CEE–Oil I



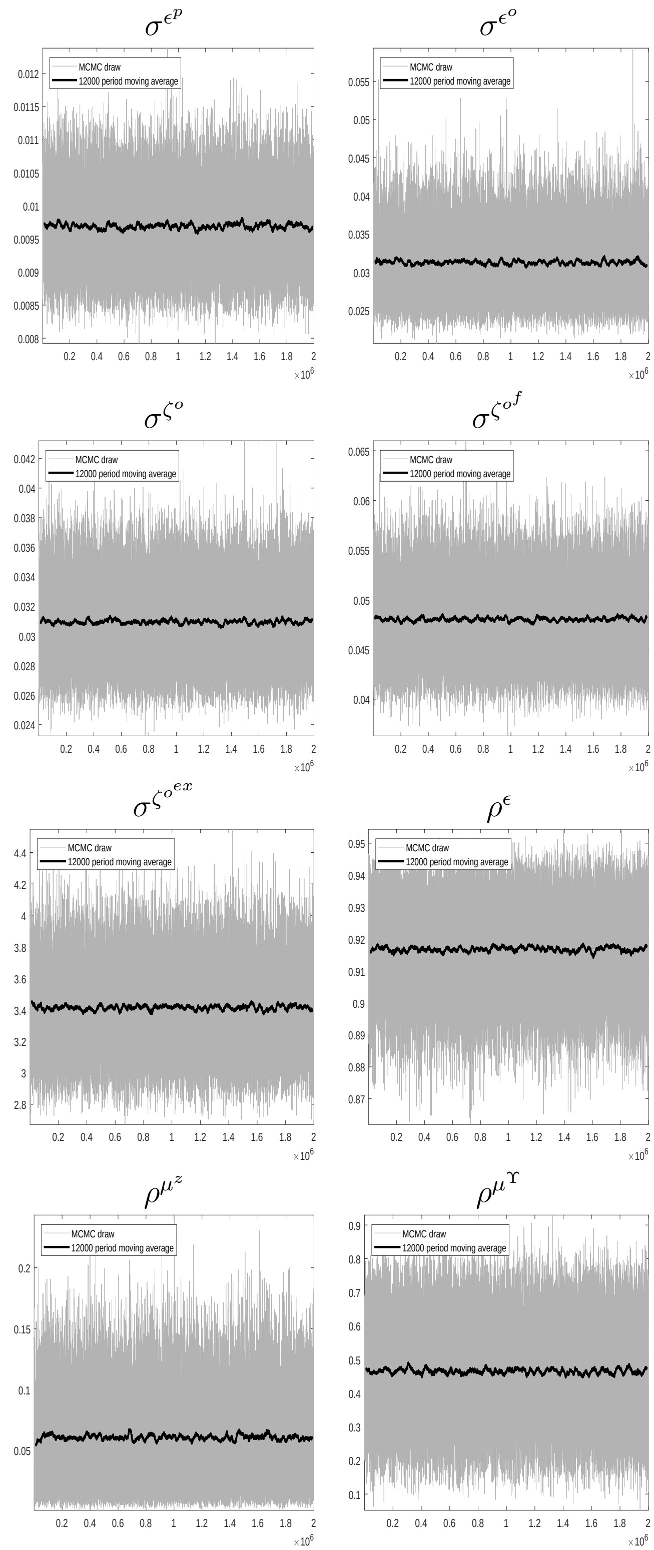
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 4.34: Trace plots for chain 3 CEE–Oil II



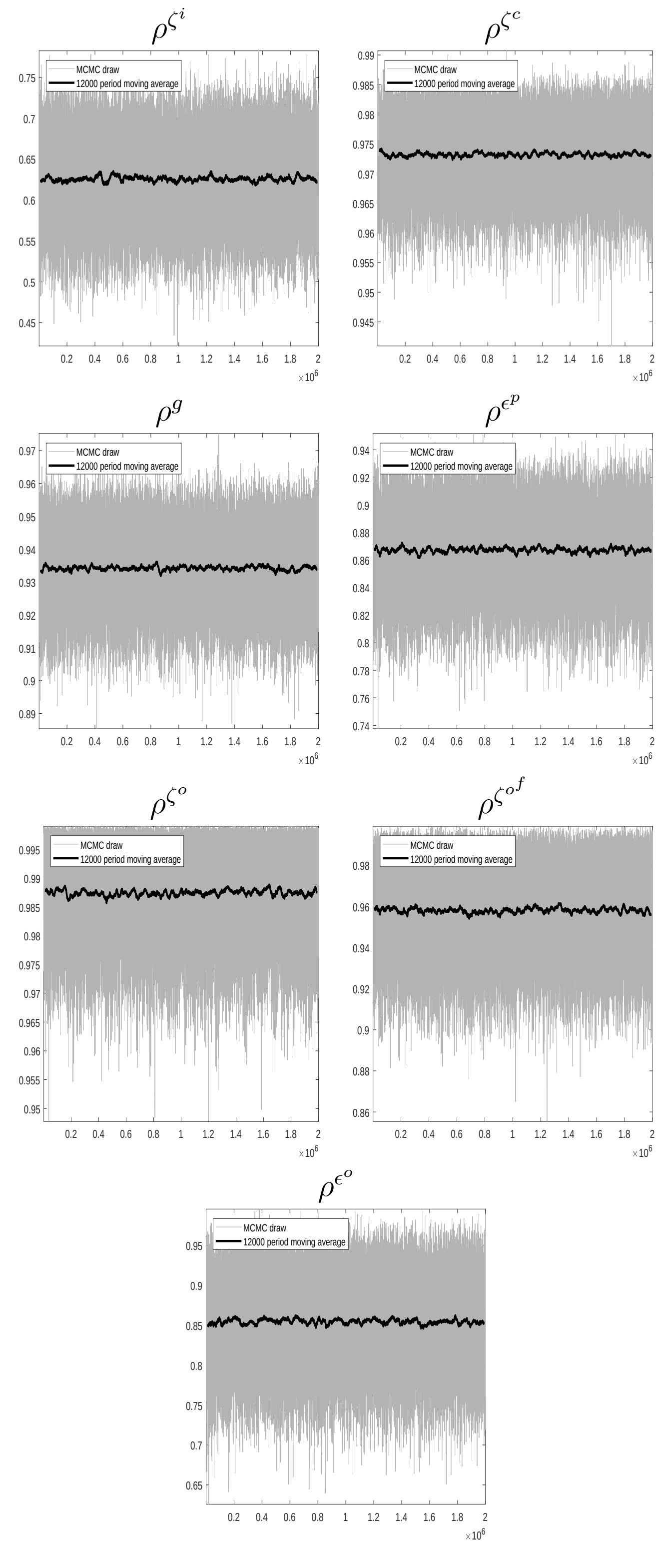
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 4.35: Trace plots for chain 3 CEE–Oil III



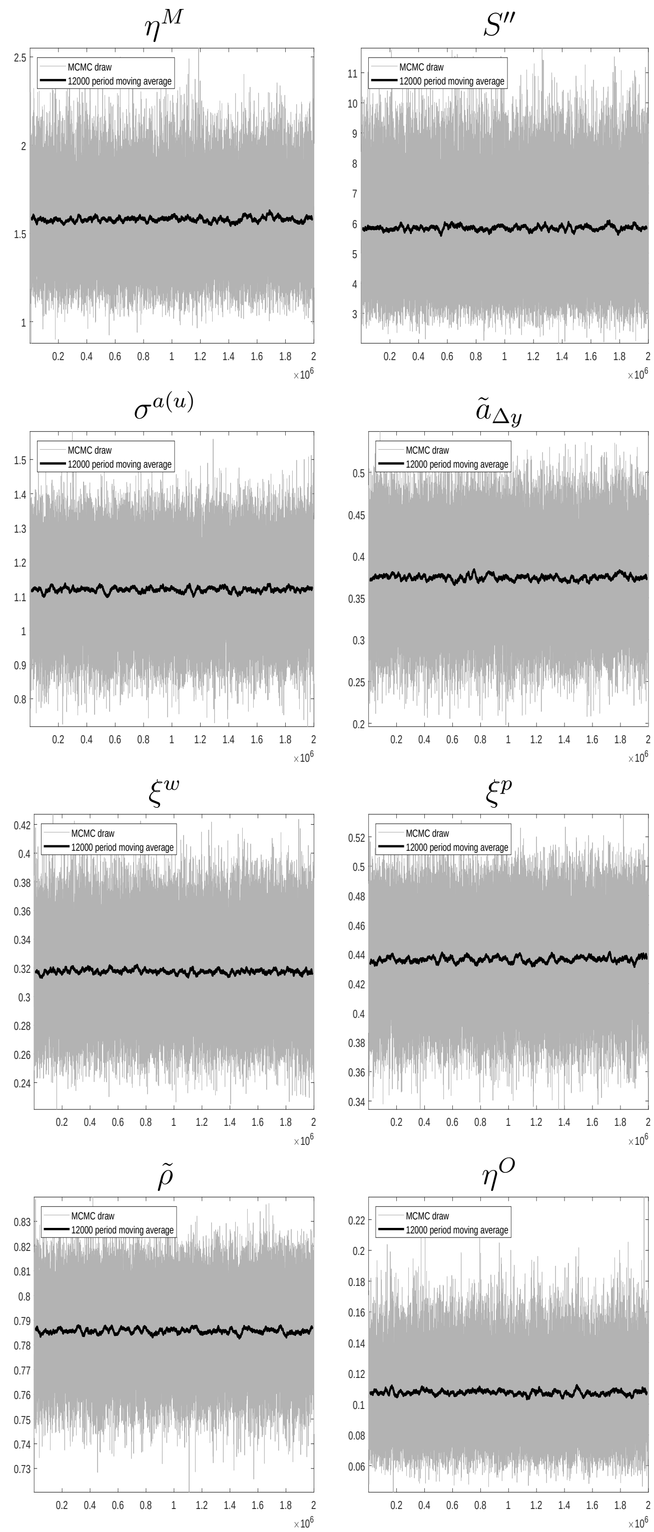
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 4.36: Trace plots for chain 3 CEE–Oil IV



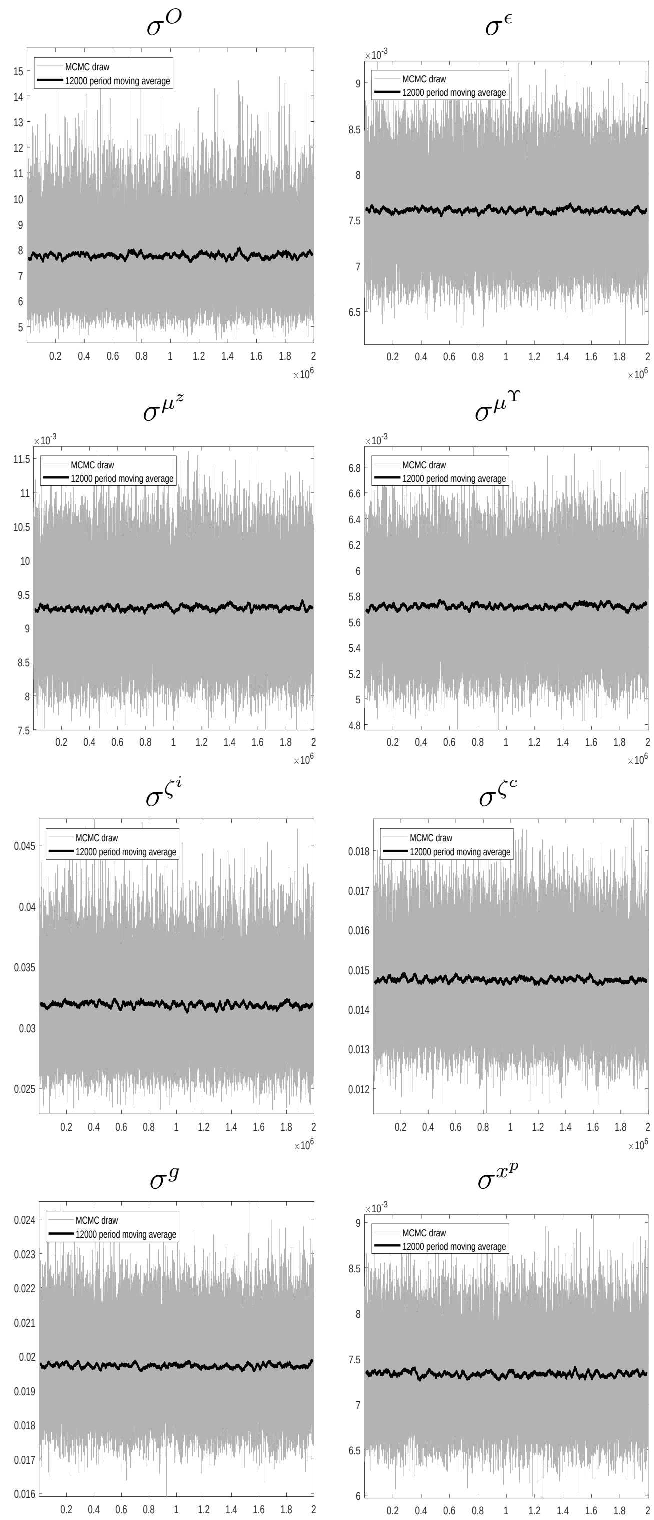
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 4.37: Trace plots for chain 4 CEE–Oil I



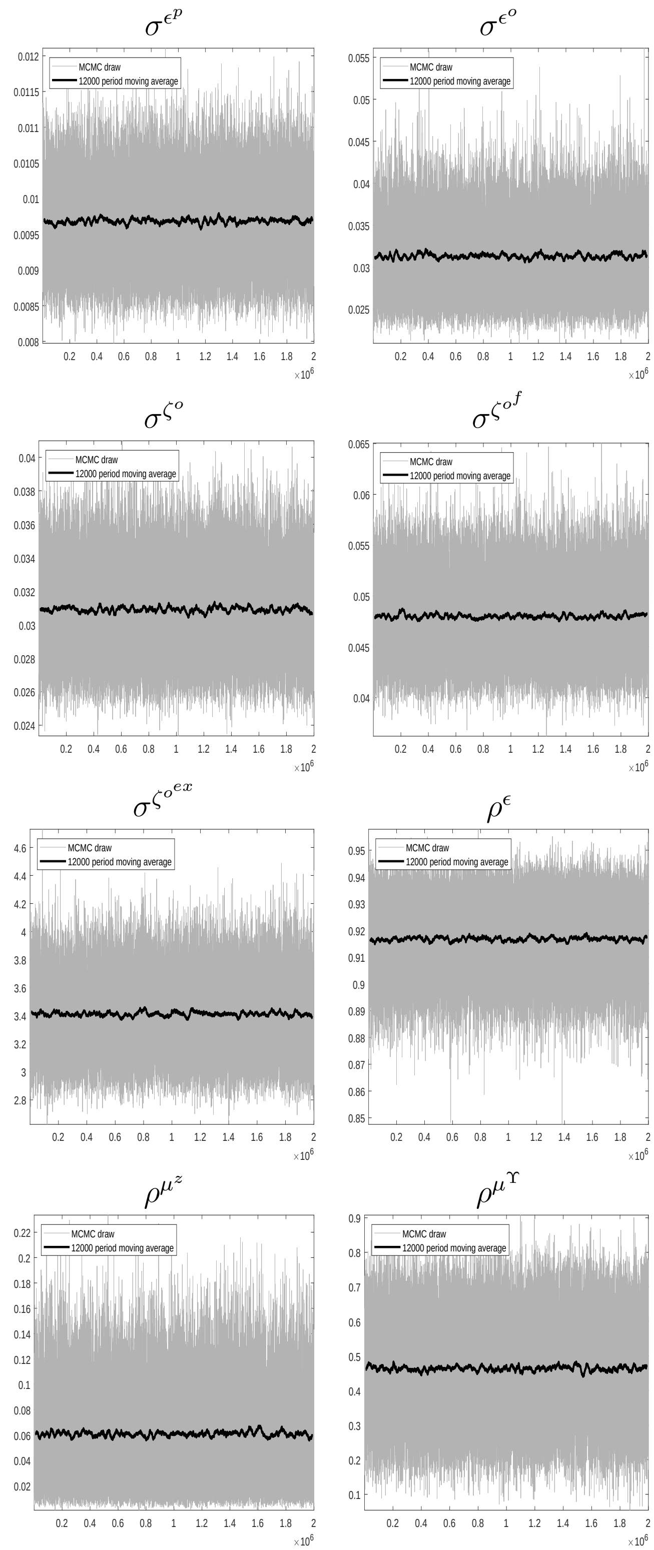
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 4.38: Trace plots for chain 4 CEE–Oil II



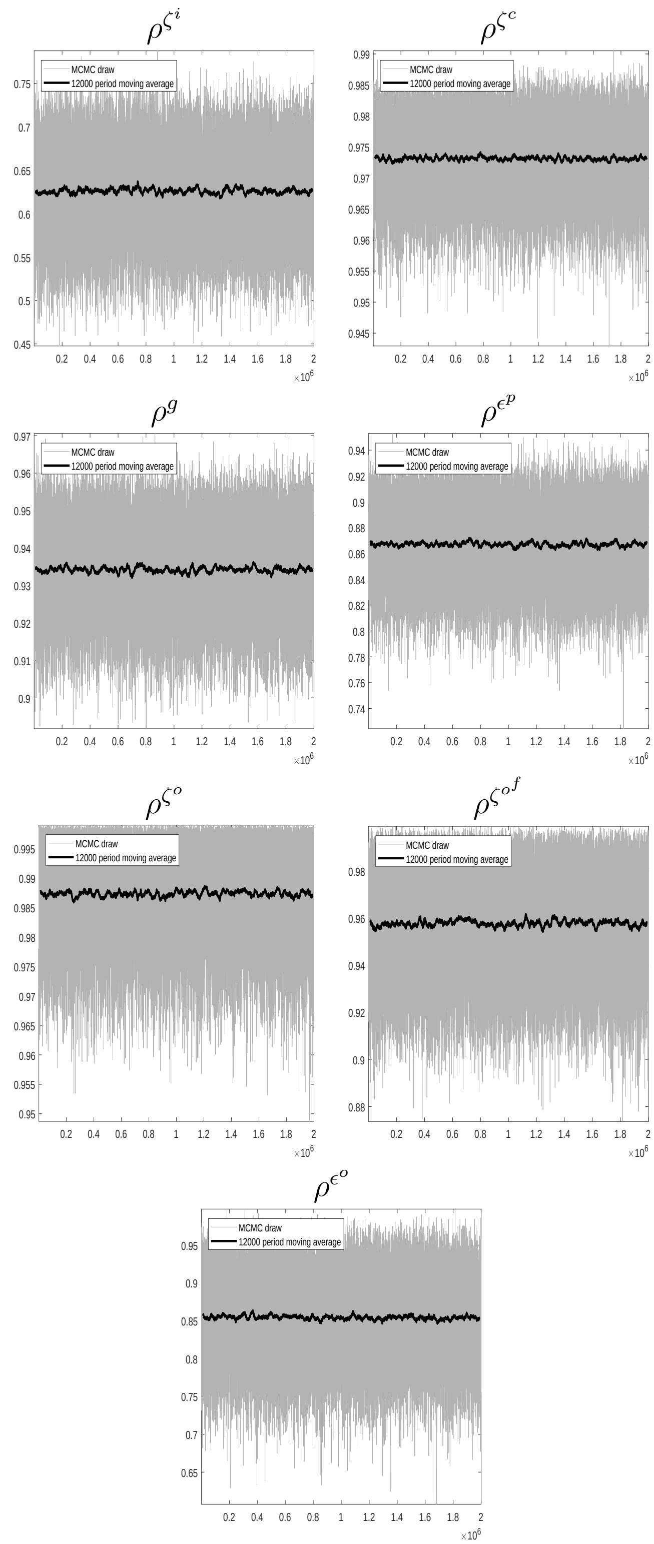
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 4.39: Trace plots for chain 4 CEE–Oil III



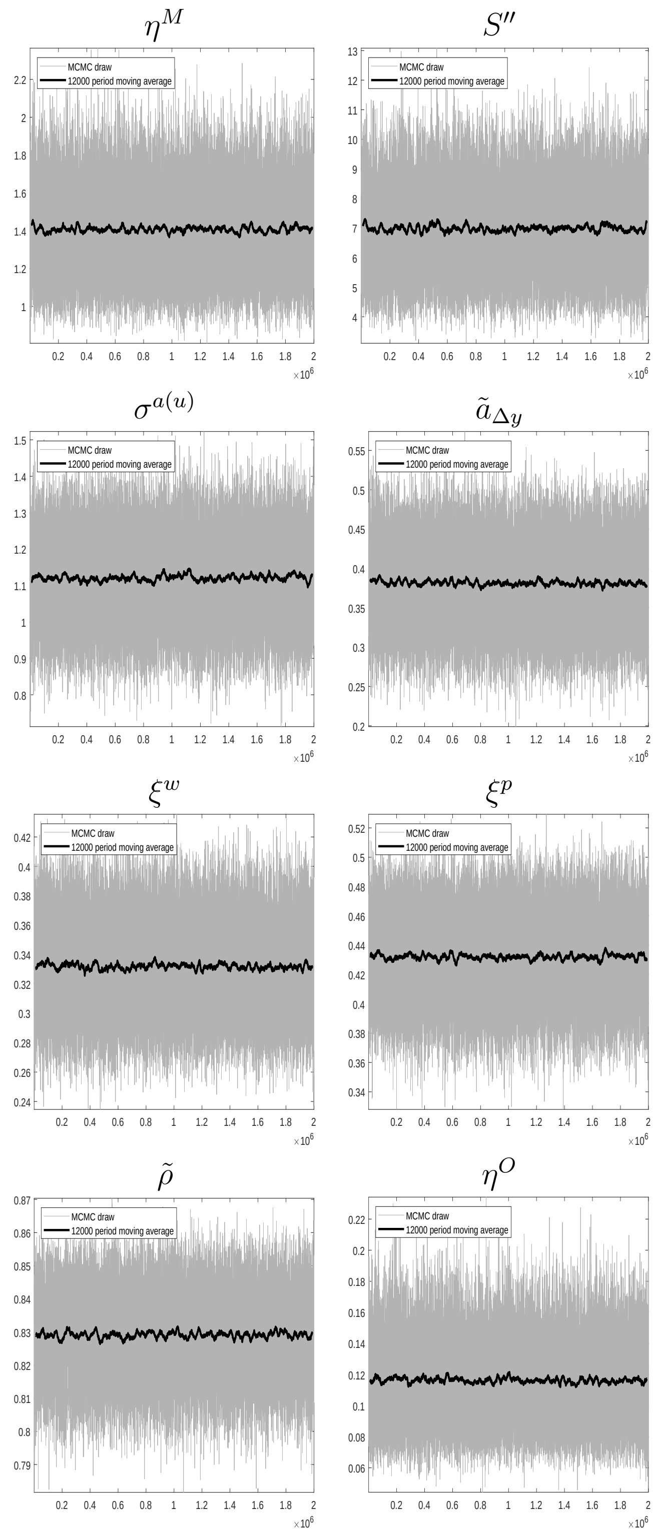
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 4.40: Trace plots for chain 4 CEE–Oil IV



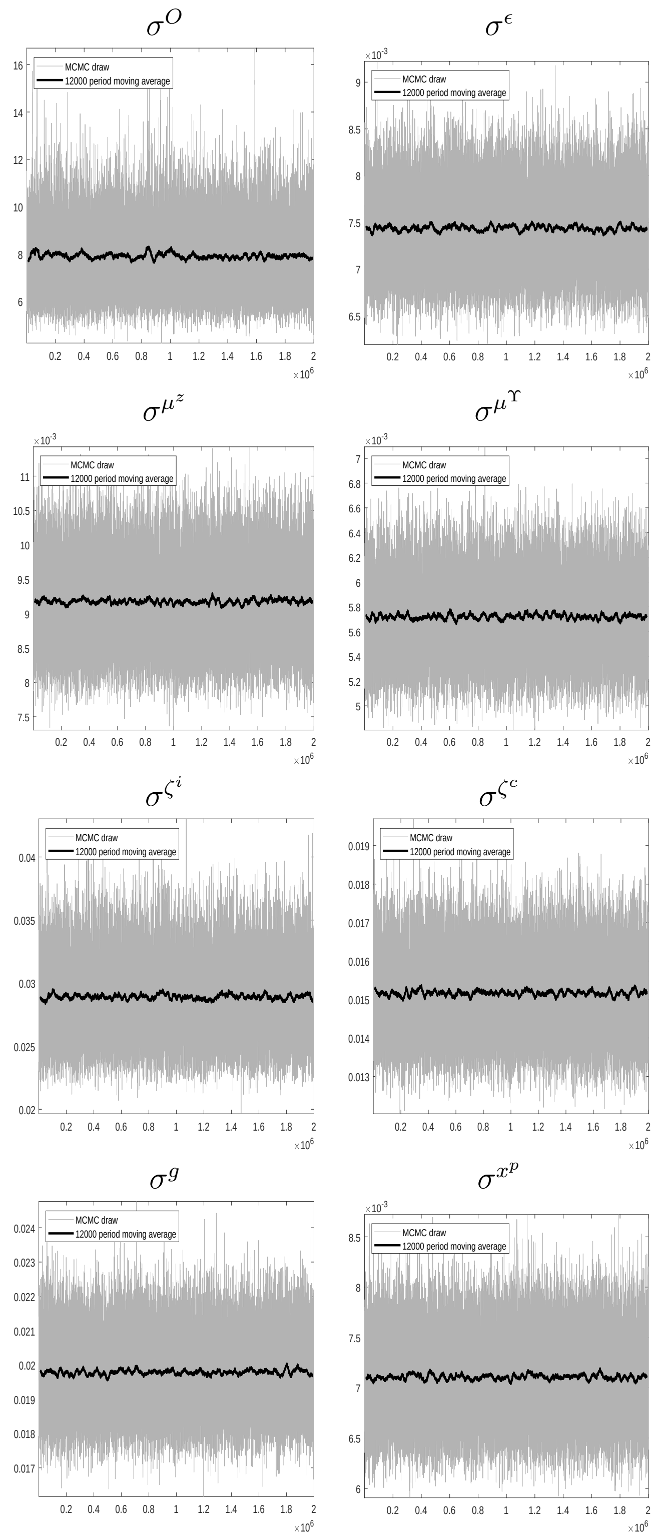
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 4.41: Trace plots for chain 1 CMR–Oil I



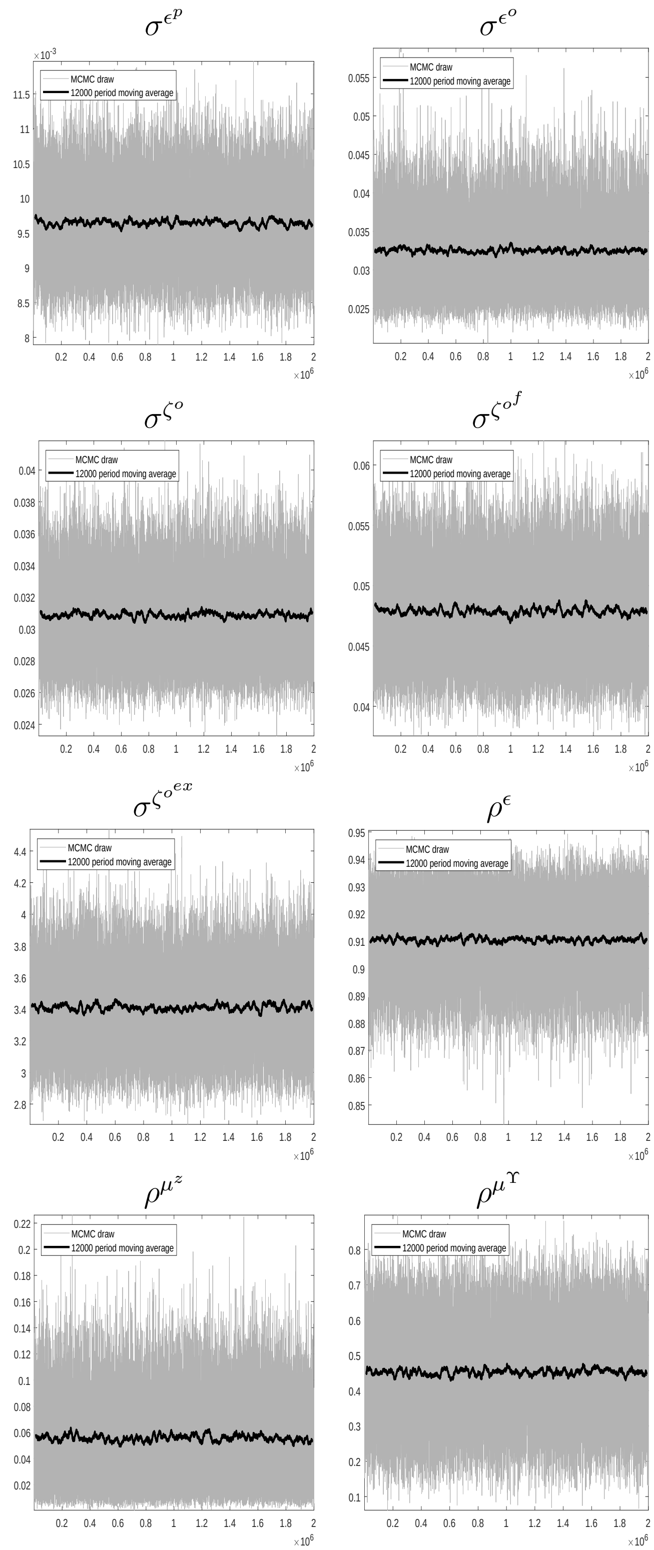
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 4.42: Trace plots for chain 1 CMR–Oil II



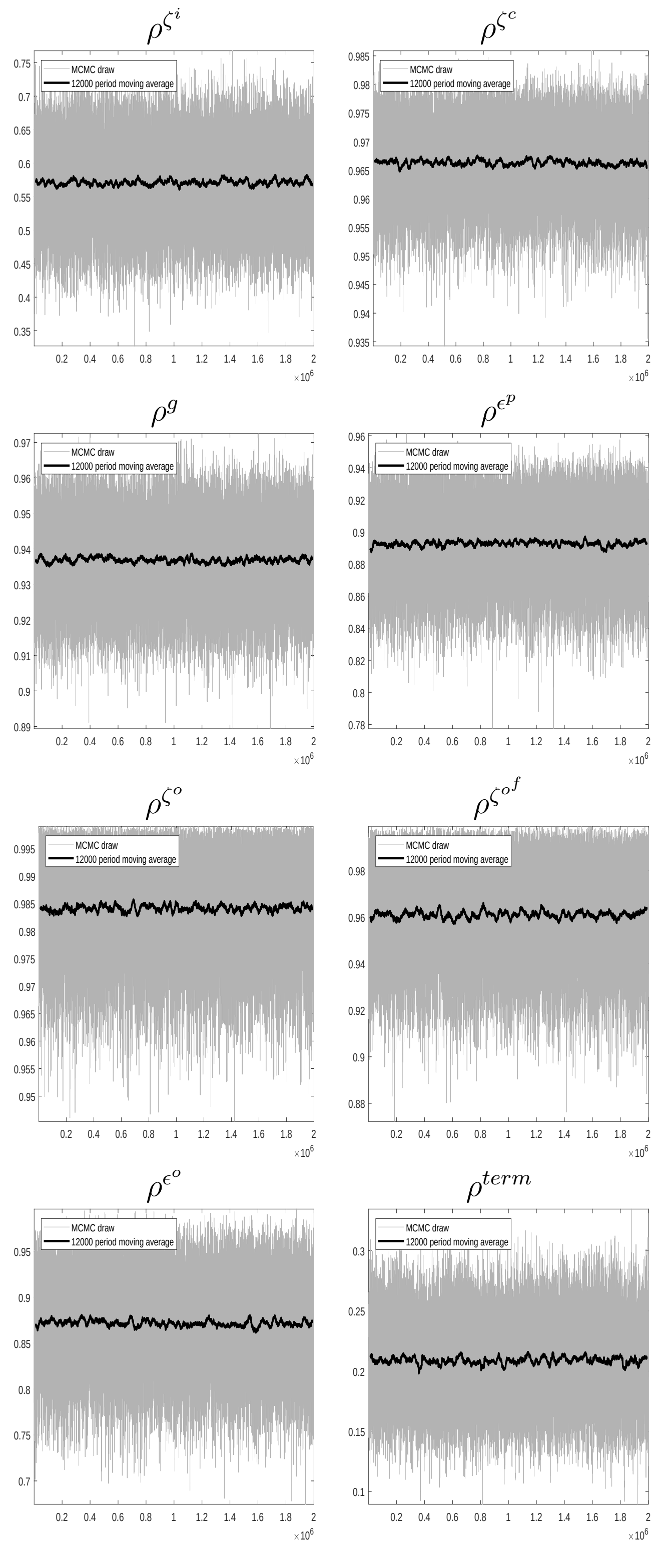
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 4.43: Trace plots for chain 1 CMR–Oil III



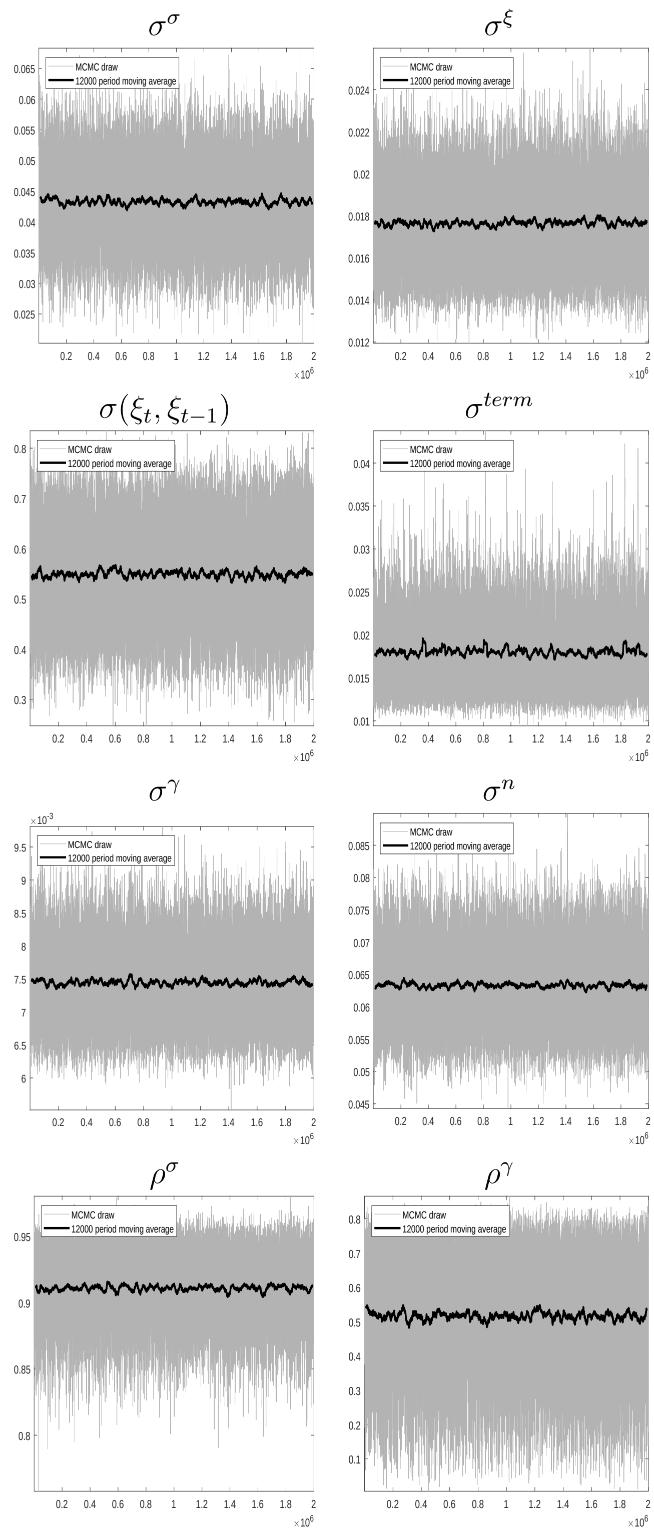
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 4.44: Trace plots for chain 1 CMR–Oil IV



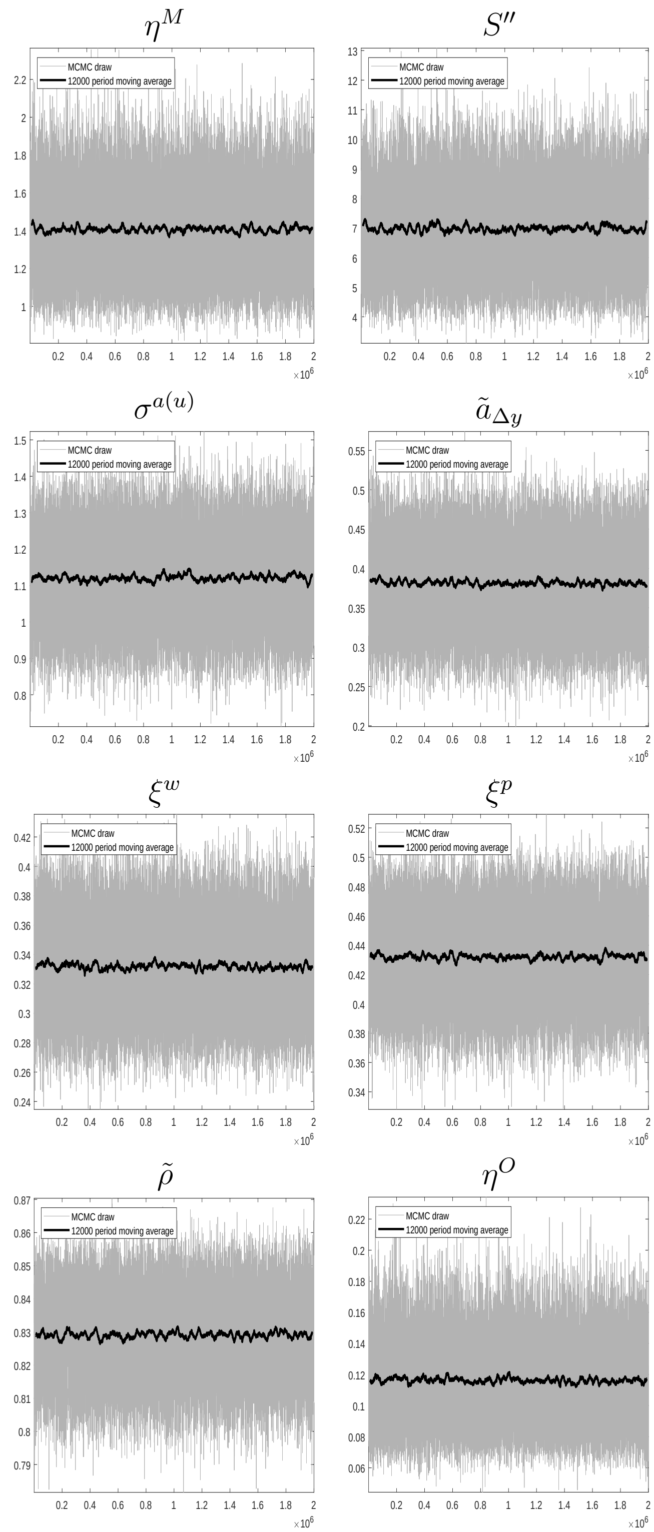
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 4.45: Trace plots for chain 1 CMR–Oil V



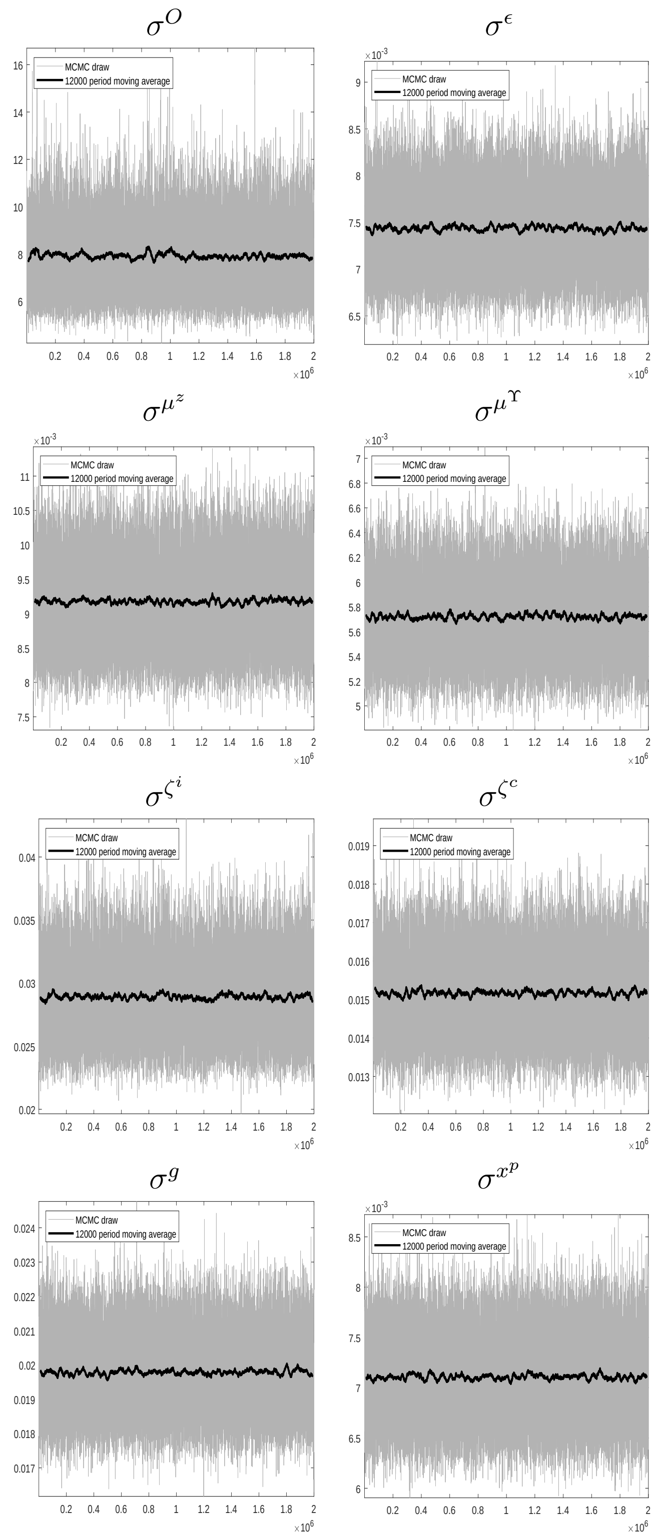
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 4.46: Trace plots for chain 2 CMR–Oil I



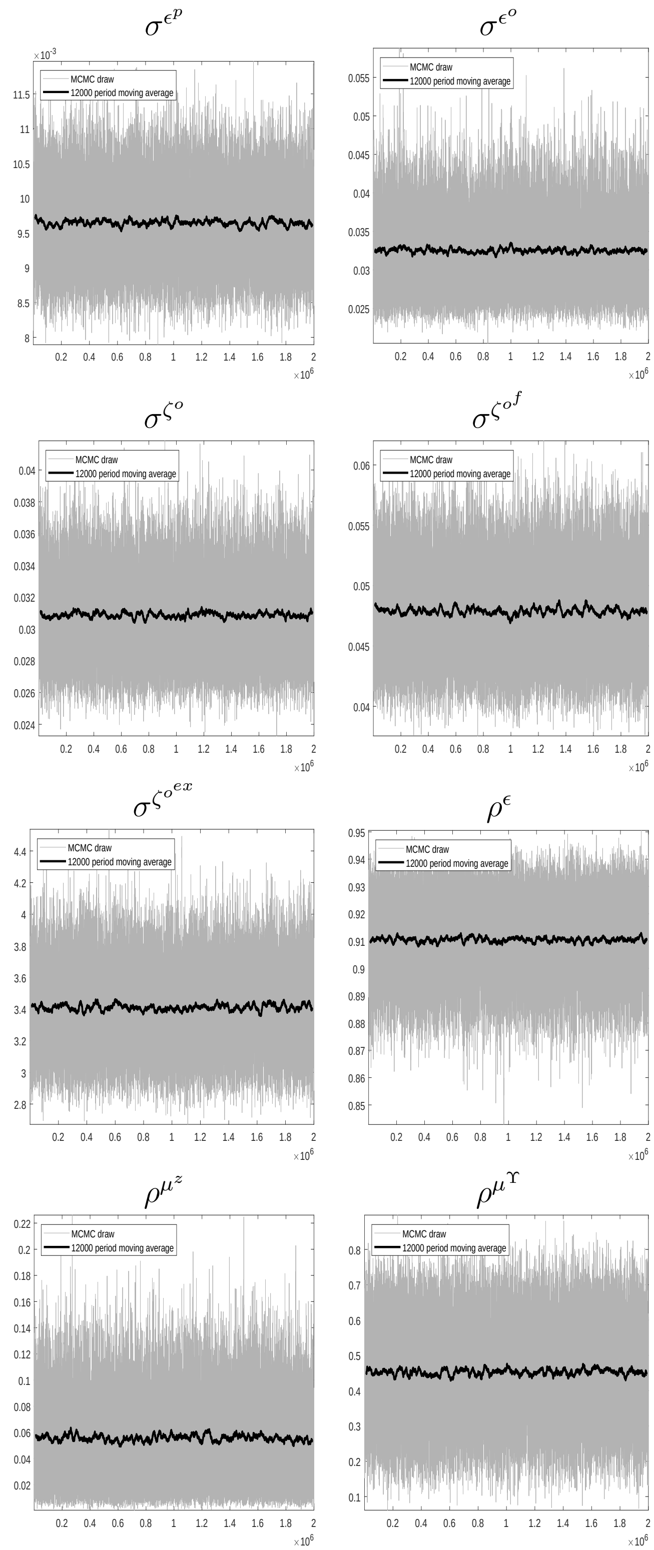
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 4.47: Trace plots for chain 2 CMR–Oil II



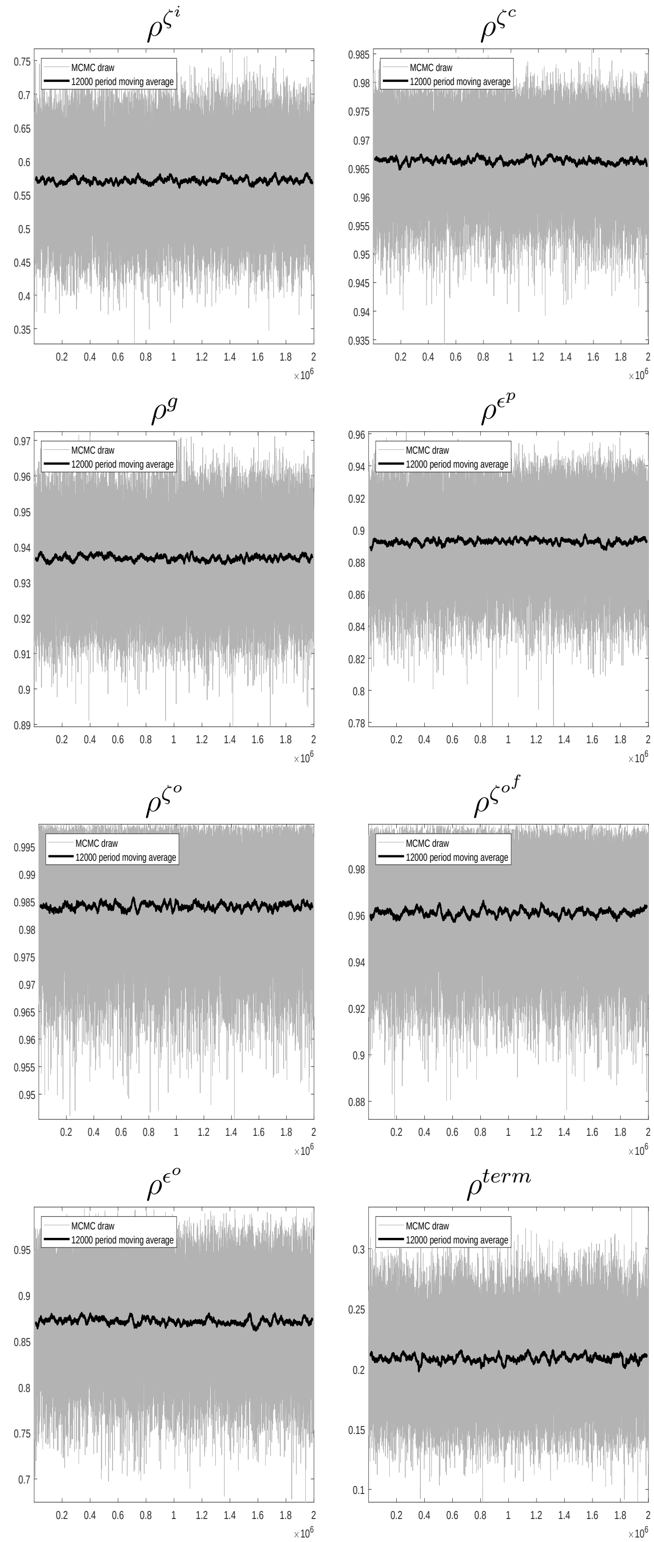
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 4.48: Trace plots for chain 2 CMR–Oil III



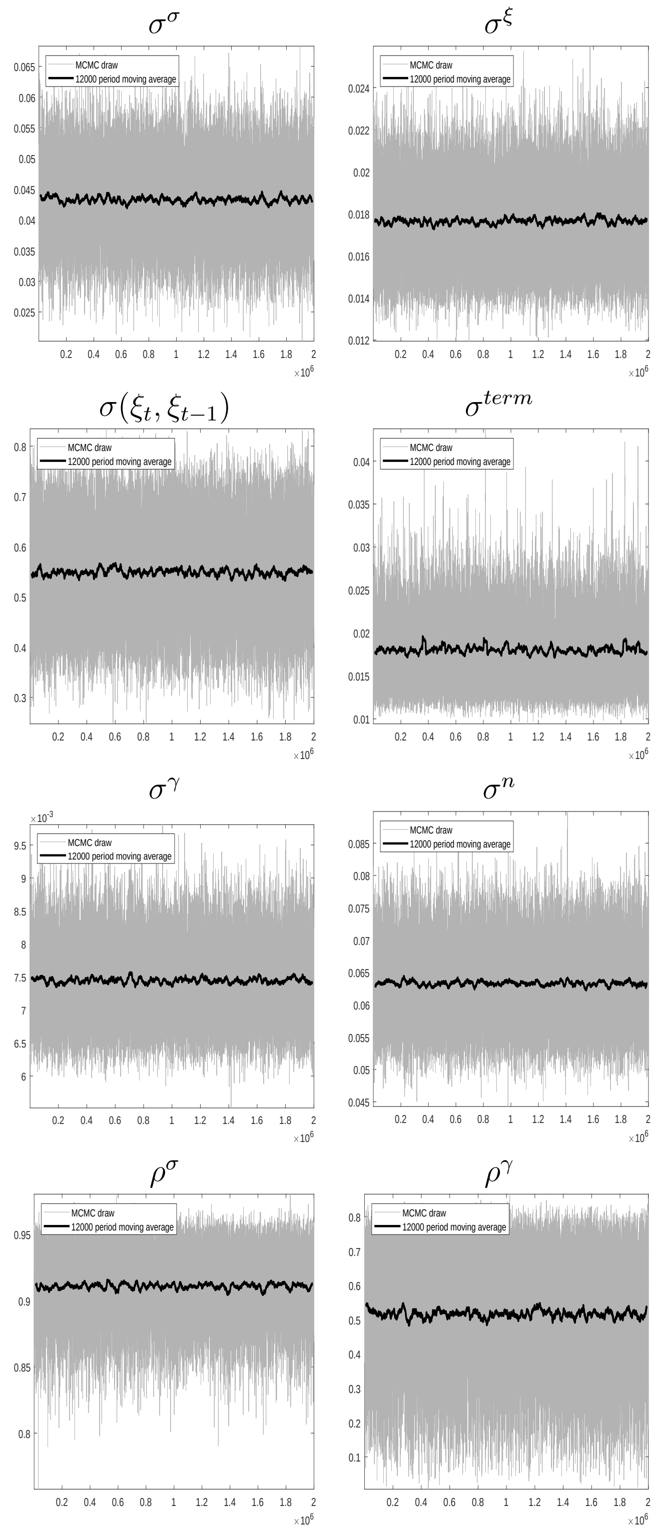
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 4.49: Trace plots for chain 2 CMR–Oil IV



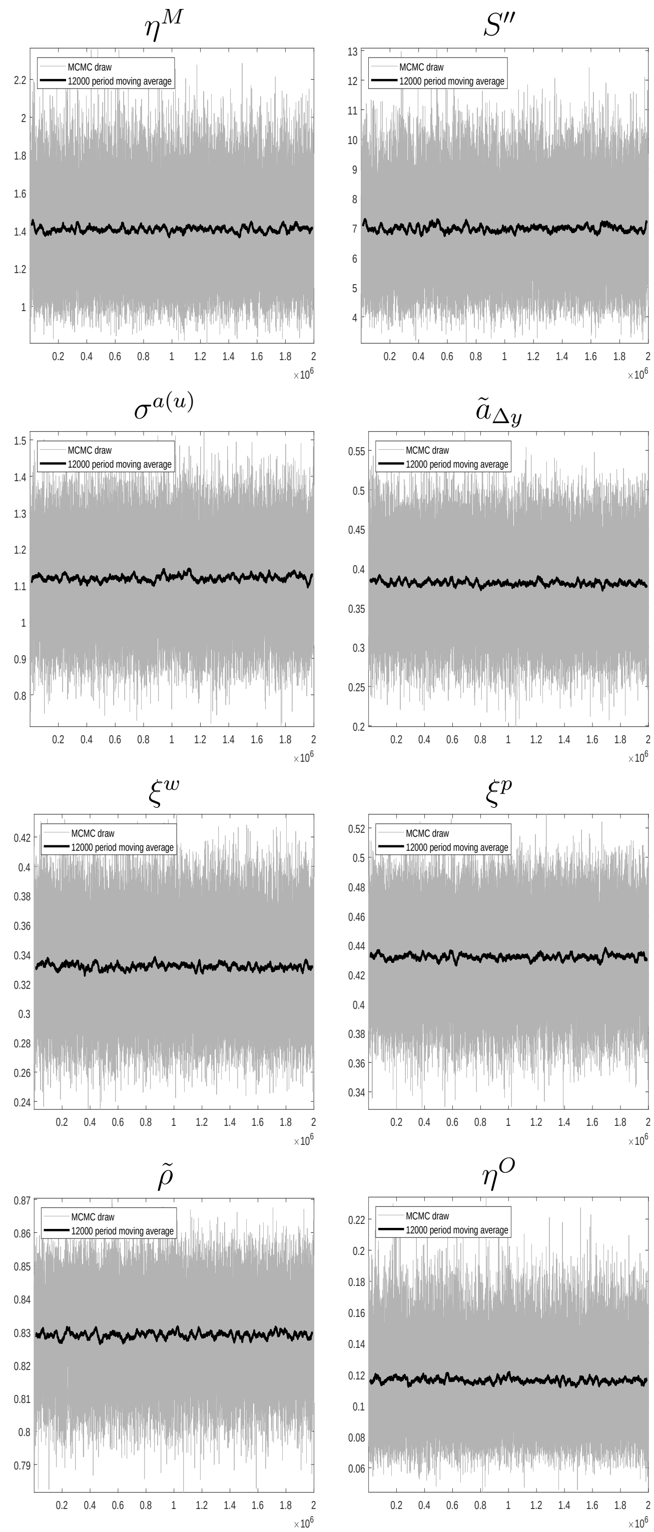
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 4.50: Trace plots for chain 2 CMR–Oil V



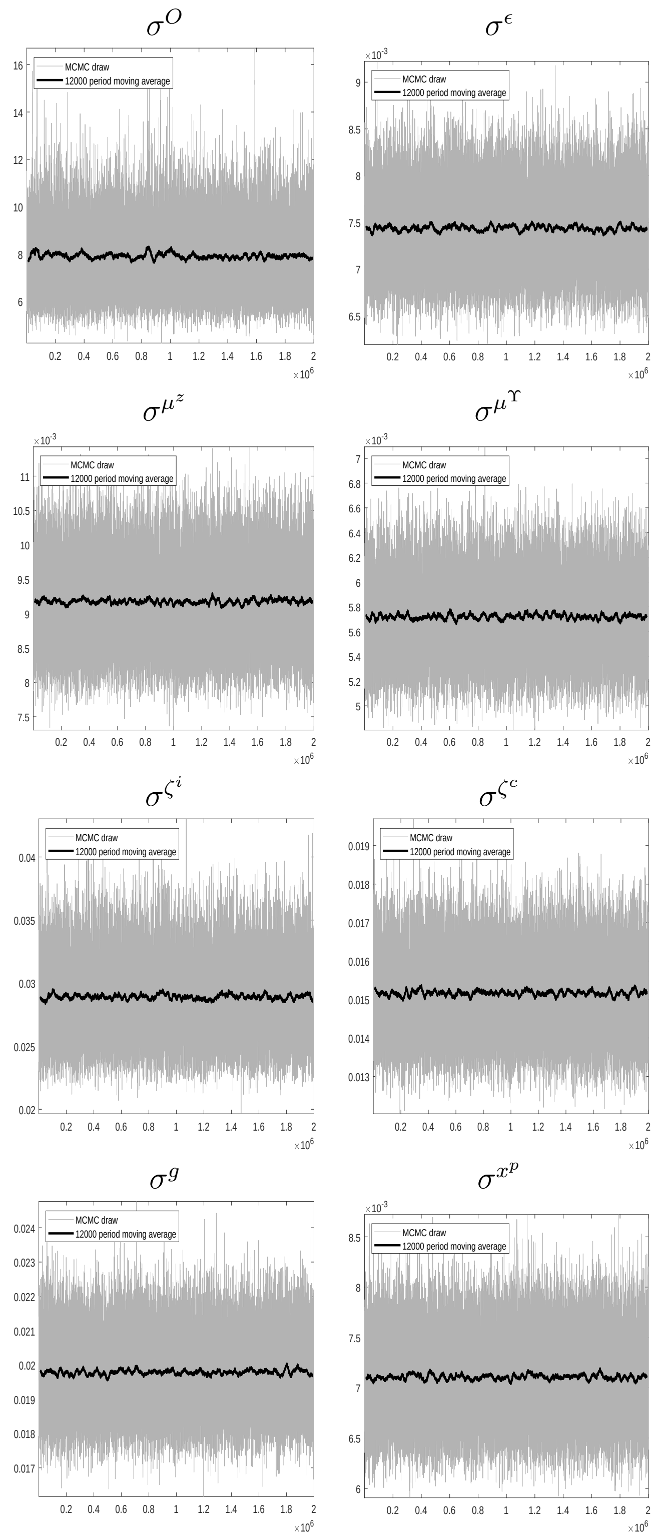
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 4.51: Trace plots for chain 3 CMR–Oil I



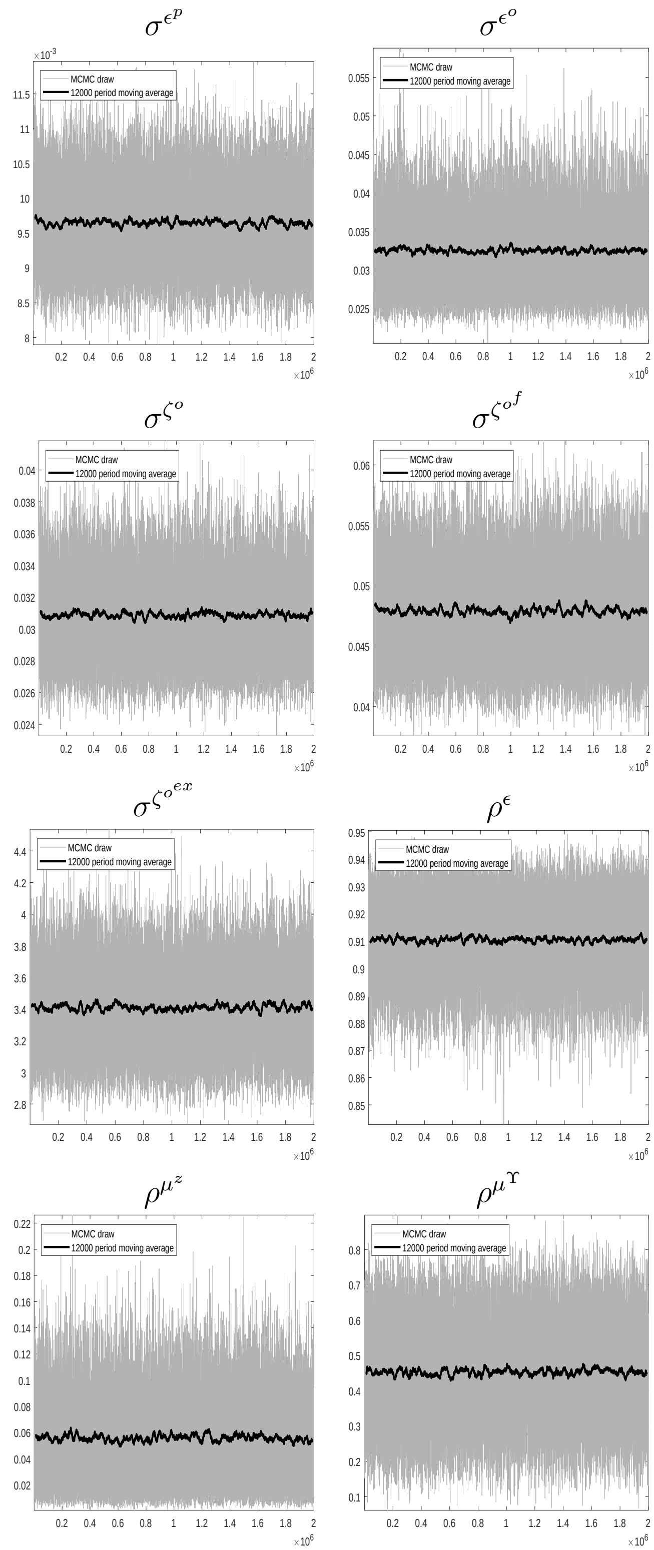
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 4.52: Trace plots for chain 3 CMR–Oil II



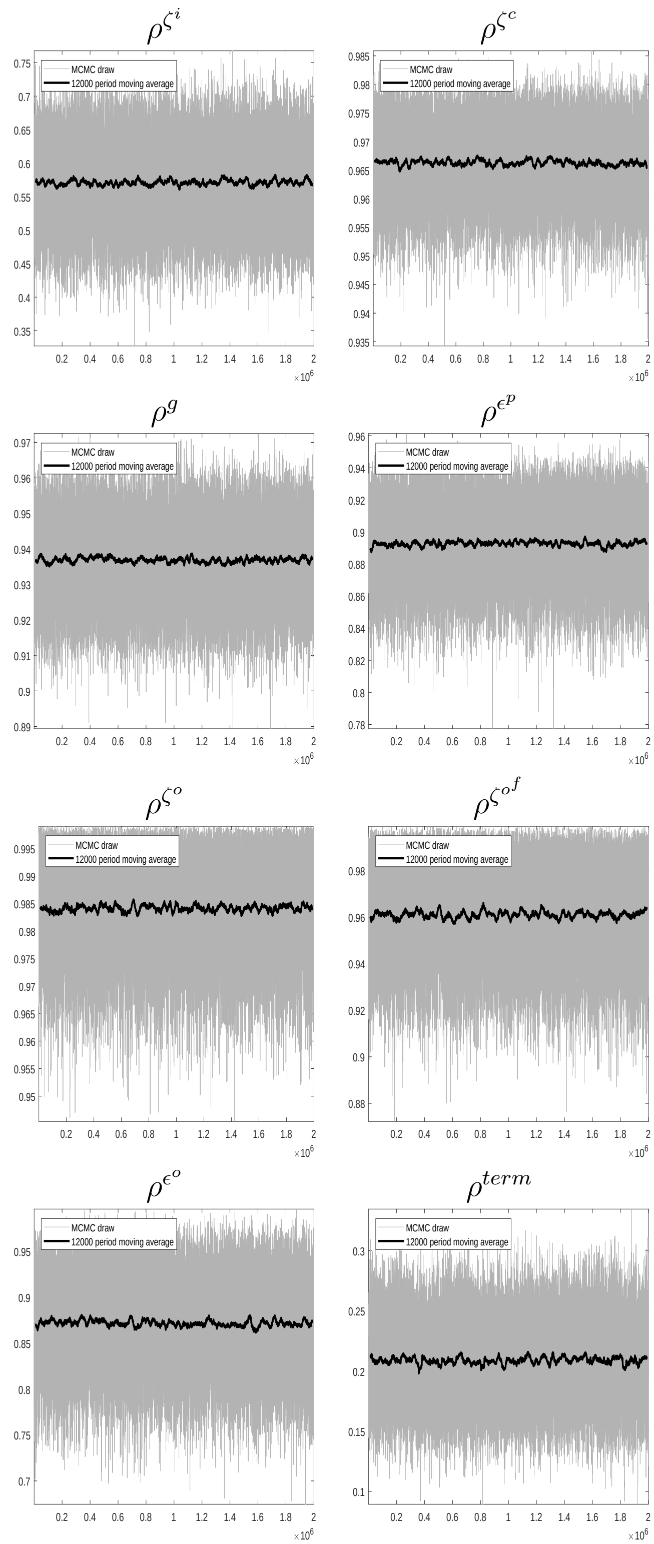
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 4.53: Trace plots for chain 3 CMR–Oil III



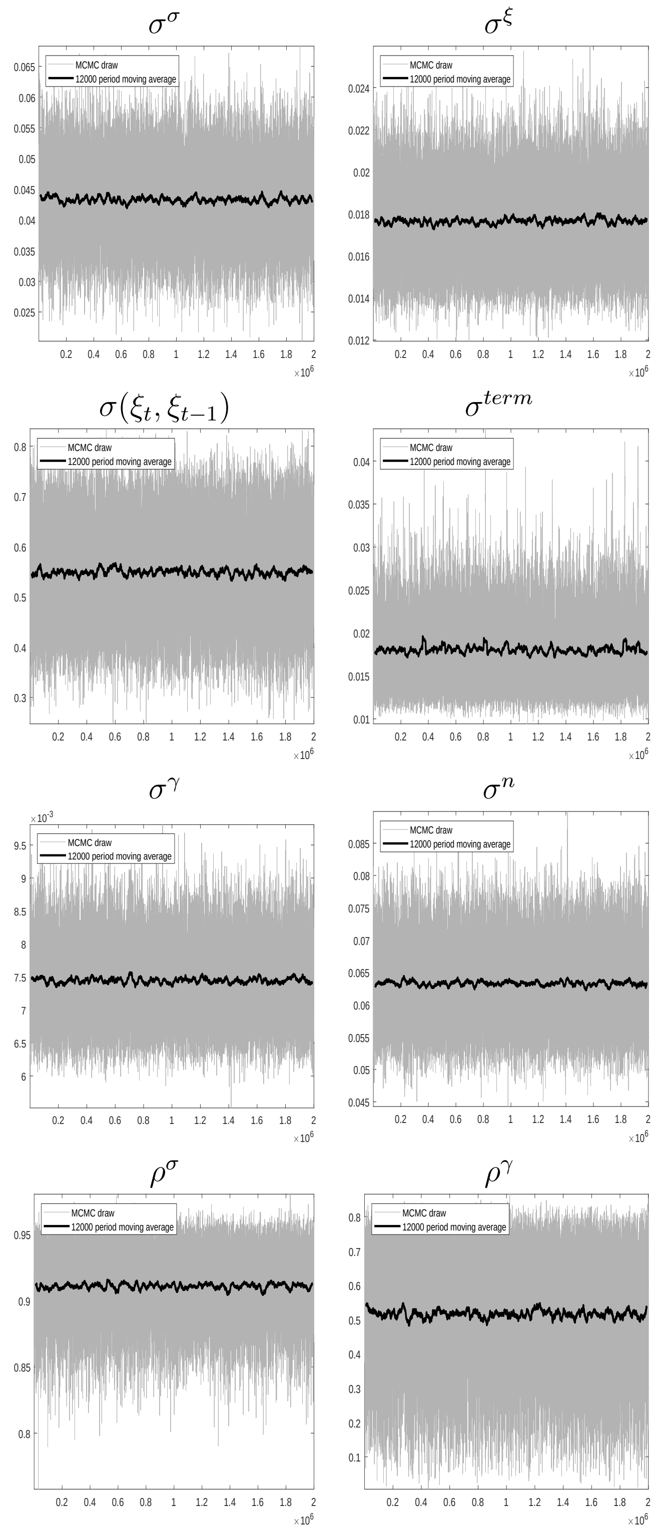
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 4.54: Trace plots for chain 3 CMR–Oil IV



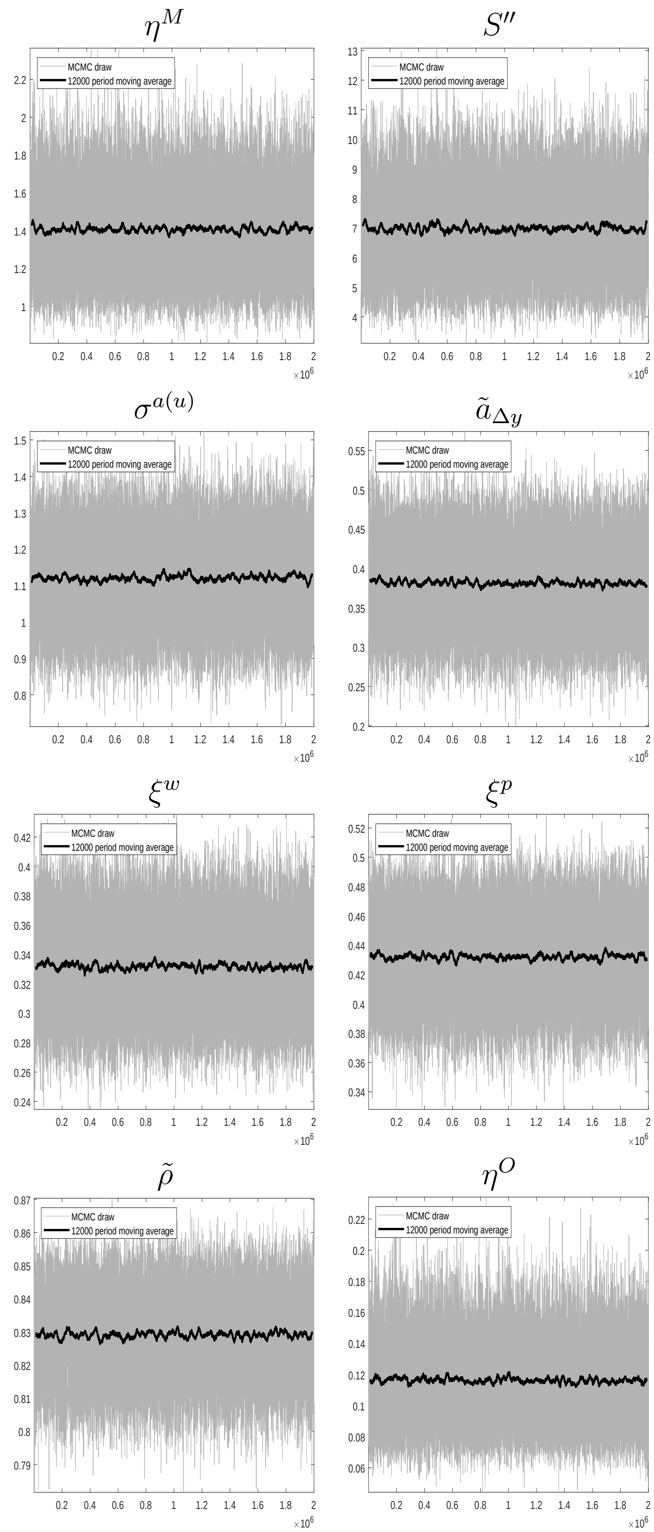
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 4.55: Trace plots for chain 3 CMR–Oil V



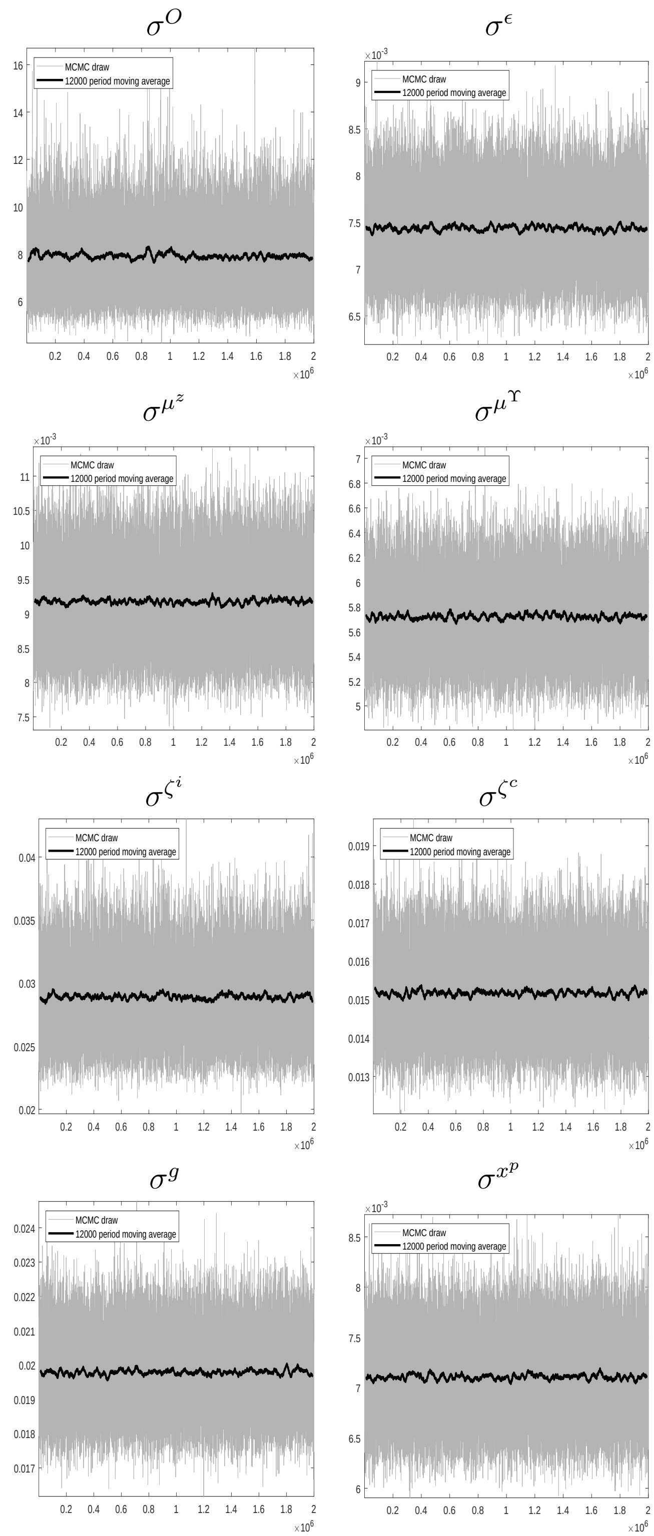
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 4.56: Trace plots for chain 4 CMR–Oil I



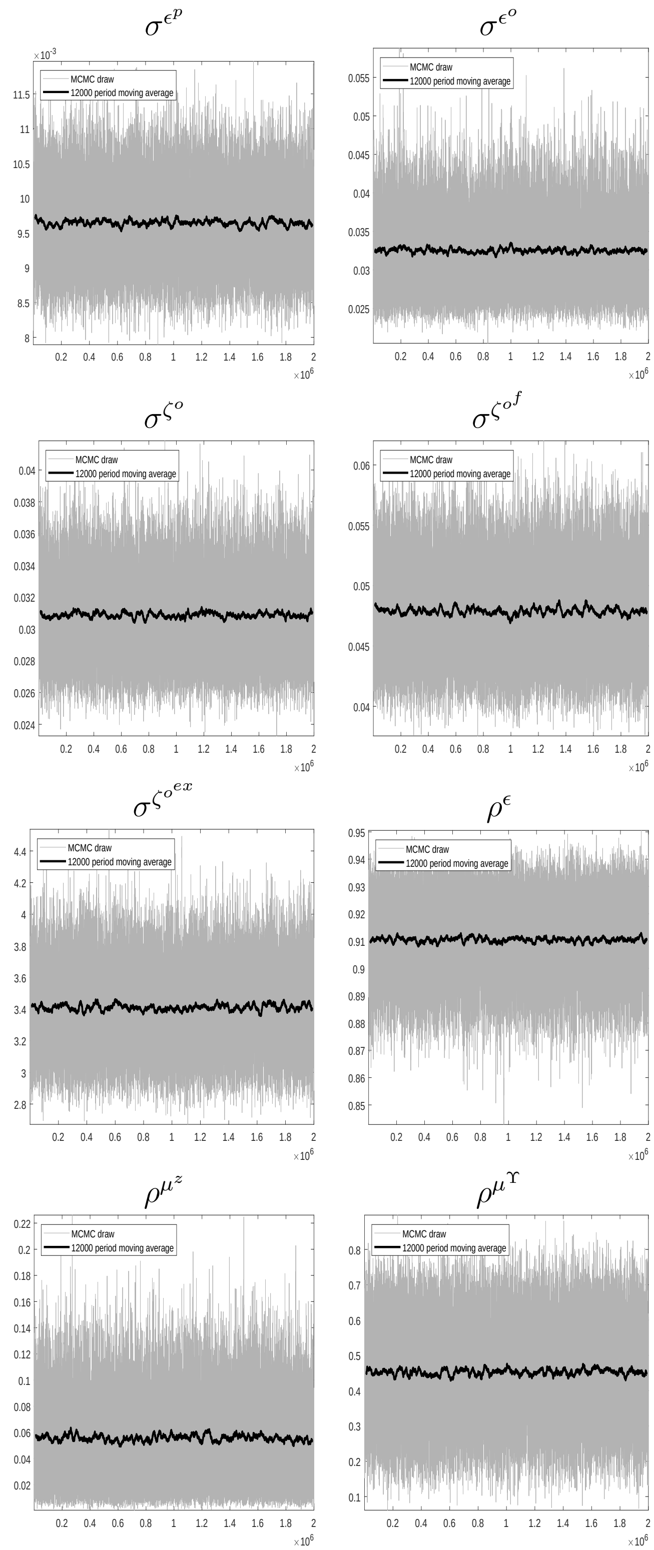
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 4.57: Trace plots for chain 4 CMR–Oil II



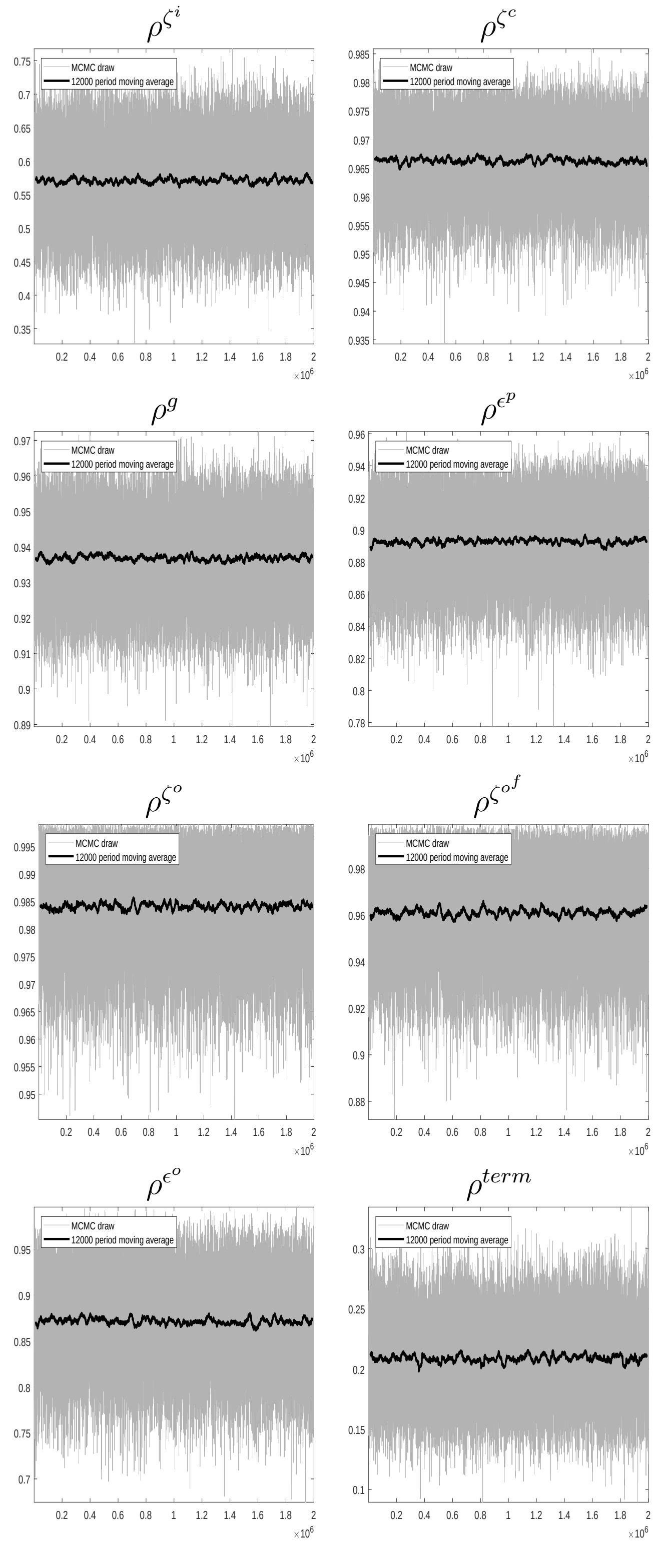
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 4.58: Trace plots for chain 4 CMR–Oil III



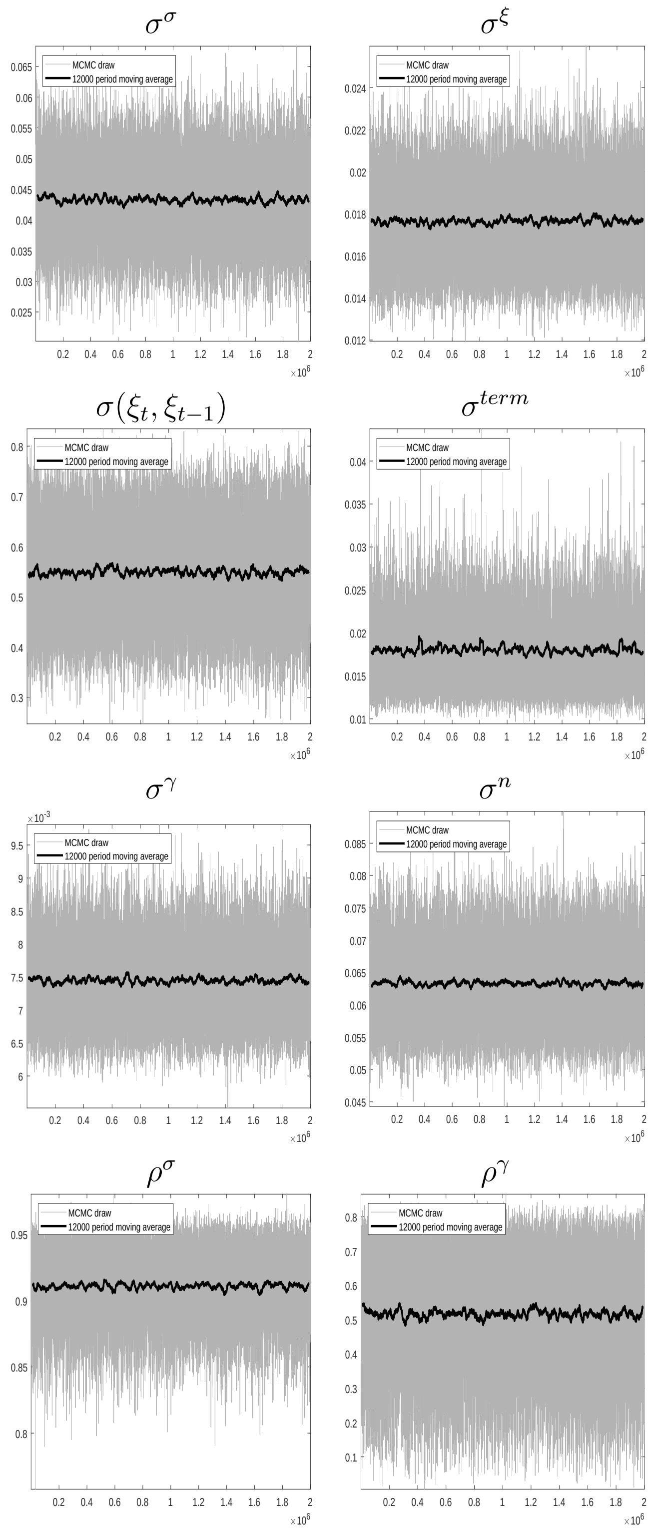
Notes: The grey line depicts parameter values and the black line the moving average.

Figure 4.59: Trace plots for chain 4 CMR–Oil IV



Notes: The grey line depicts parameter values and the black line the moving average.

Figure 4.60: Trace plots for chain 4 CMR–Oil IV



Notes: The grey line depicts parameter values and the black line the moving average.