EXPOSITIO
QVARVNDAM FORMVLARVM
DE
CENTRO GRAVITATIS

QVAM
AMPLISSIMI PHILOSOPHORVM ORDINIS
AVCTORITATE
PRO LOCO
IN EO RITE OBTINENDO
D. XXI. AVGVSTI MDCCXCIX
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PVBLICE DEFENDET

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§. 1.

Quot et quam diverſi argumenti quaeſtiones in eandem unius eiusdemque problematis formulam cogere Analyſis poſſit, non modo ea indagans, quae ipsi peculiaria habentur, ſed etiam tum, vires cum ſuas in Geometria et Mechanice exercet, claris docet exemplis. At harum formularum numerum ita reſtringere, ut eius nulla imminutio ſuperſit, ipſae vero ſtationem praebeant maxime opportunam, ex qua omnis matheseos campus conspiciatur apertus, conatuum mentis humanae longe audaciſſimus eſt, quippe qui efficiat, ut haec cognitionum humanarum pars ad ſummum deducatur culmen, quod ipſa rei natura petendum propoſuit.

In ſcripto autem noſtro, id tantum agetur, vt pateat problematum ſeriem formarum parabolicarum exemplis illuſtratam ad unum redire problema, atque ex hac parte, collectis atque inſtructis iis, quae disperſa deprehendebantur, ad editiſſimum illud culmen, quamlibet maxime e longinquo, via tendatur. Quo conſilio ſuscepto, ſi quid opuſculo noſtro commodi et utilitatis acceſſerit, operae tuliſſe pretium arbitrabimur.

§. 2.

Formulae nonnullae, quarum uſus infra frequentior erit.

I. Propoſita ſit aequatio:

$$g=\frac{\beta}{(\alpha+\beta+\gamma)\,a^{\frac{\gamma}{\alpha}}}\left(\frac{a^{\frac{\alpha+\beta+\gamma}{\alpha}}-x^{\frac{\alpha+\beta+\gamma}{\alpha}}}{a^{\frac{\beta}{\alpha}}-x^{\frac{\beta}{\alpha}}}\right)$$

Ex ea, diviſo ad dextram numeratore et denominatore per $a^{\frac{1}{\alpha}}-x^{\frac{1}{\alpha}}$, prodit altera:

$$g=\frac{\beta}{(\alpha+\beta+\gamma)\,a^{\frac{\gamma}{\alpha}}}\left(\frac{a^{\frac{\alpha+\beta+\gamma-1}{\alpha}}+a^{\frac{\alpha+\beta+\gamma-2}{\alpha}}x^{\frac{1}{\alpha}}+a^{\frac{\alpha+\beta+\gamma-3}{\alpha}}x^{\frac{2}{\alpha}}\ldots+x^{\frac{\alpha+\beta+\gamma-1}{\alpha}}}{a^{\frac{\beta-1}{\alpha}}+a^{\frac{\beta-2}{\alpha}}x^{\frac{1}{\alpha}}+a^{\frac{\beta-3}{\alpha}}x^{\frac{2}{\alpha}}\ldots+x^{\frac{\beta-1}{\alpha}}}\right)$$

ubi, poſito $x=a$, termini numeratoris omnes, $\alpha+\beta+\gamma$ numero, nec non termini denominatoris omnes, β numero, inter ſe aequantur, vertiturque aequatio in hanc:

$$g=\frac{\beta(\alpha+\beta+\gamma)}{(\alpha+\beta+\gamma)\beta}\,\frac{a^{\frac{\alpha+\beta+\gamma-1}{\alpha}}}{a^{\frac{\beta+\quad-1}{\alpha}}}=a$$

II. Eſt

$$\frac{1}{b^{\frac{\gamma}{\alpha}}-y^{\frac{\gamma}{\alpha}}}\left(\frac{(\alpha+\beta)\left(b^{\frac{\alpha+\beta+\gamma}{\alpha}}-y^{\frac{\alpha+\beta+\gamma}{\alpha}}\right)}{(\alpha+\beta+\gamma)\left(b^{\frac{\alpha+\beta}{\alpha}}-y^{\frac{\alpha+\beta}{\alpha}}\right)}-y^{\frac{\gamma}{\alpha}}\right)=\frac{1}{\alpha+\beta+\gamma}\left(\frac{(\alpha+\beta)\,b^{\frac{\alpha+\beta}{\alpha}}}{b^{\frac{\alpha+\beta}{\alpha}}-y^{\frac{\alpha+\beta}{\alpha}}}-\gamma\frac{y^{\frac{\gamma}{\alpha}}}{b^{\frac{\gamma}{\alpha}}-y^{\frac{\gamma}{\alpha}}}\right)$$

Namque terminis ad ſiniſtram poſitis eodem denominatore dato, invenitur:

$$\frac{(\alpha+\beta)b^{\frac{\alpha+\beta+\gamma}{\alpha}}-(\alpha+\beta)\,y^{\frac{\alpha+\beta+\gamma}{\alpha}}-(\alpha+\beta+\gamma)\,b^{\frac{\alpha+\beta}{\alpha}}\,y^{\frac{\gamma}{\alpha}}+(\alpha+\beta+\gamma)\,y^{\frac{\alpha+\beta+\gamma}{\alpha}}}{(\alpha+\beta+\gamma)\left(b^{\frac{\gamma}{\alpha}}-y^{\frac{\gamma}{\alpha}}\right)\left(b^{\frac{\alpha+\beta}{\alpha}}-y^{\frac{\alpha+\beta}{\alpha}}\right)}$$

at collecto termino ſecundo et quarto, haec terminorum diſpoſitio in promtu eſt:

$$\frac{(\alpha+\beta)\,b^{\frac{\alpha+\beta}{\alpha}}\left(b^{\frac{\gamma}{\alpha}}-y^{\frac{\gamma}{\alpha}}\right)-\gamma\,y^{\frac{\gamma}{\alpha}}\left(b^{\frac{\alpha+\beta}{\alpha}}-y^{\frac{\alpha+\beta}{\alpha}}\right)}{(\alpha+\beta+\gamma)\left(b^{\frac{\gamma}{\alpha}}-y^{\frac{\gamma}{\alpha}}\right)\left(b^{\frac{\alpha+\beta}{\alpha}}-y^{\frac{\alpha+\beta}{\alpha}}\right)}$$

Singulisque terminis, quibus numerator conſtat, diviſis, reperitur:

$$\frac{1}{(\alpha+\beta+\gamma)}\left(\frac{(\alpha+\beta)\,b^{\frac{\alpha+\beta}{\alpha}}}{b^{\frac{\alpha+\beta}{\alpha}}-y^{\frac{\alpha+\beta}{\alpha}}}-\frac{y^{\frac{\beta}{\alpha}}}{b^{\frac{\gamma}{\alpha}}-y^{\frac{\gamma}{\alpha}}}\right)$$

quod principio ad dextram poſuimus.

III. Quotientis

$$\frac{P}{Q}=\frac{1}{\alpha+\beta+\gamma}\left(\frac{(\alpha+\beta)\,b^{\frac{\alpha+\beta}{\alpha}}}{b^{\frac{\alpha+\beta}{\alpha}}-y^{\frac{\alpha+\beta}{\alpha}}}-\frac{\gamma\,y^{\frac{\beta}{\alpha}}}{b^{\frac{\gamma}{\alpha}}-y^{\frac{\gamma}{\alpha}}}\right)$$

in quo praeter y nihil eſt variabile, quaeritur valor, quando fiat, $y=b$. Terminis ad eundem denominatorem revocatis, proveniet,

$$\frac{P}{Q}=\frac{(\alpha+\beta)\,b^{\frac{\alpha+\beta+\gamma}{\alpha}}-(\alpha+\beta+\gamma)\,b^{\frac{\alpha+\beta}{\alpha}}\,y^{\frac{\gamma}{\alpha}}+\gamma\,y^{\frac{\alpha+\beta+\gamma}{\alpha}}}{(\alpha+\beta+\gamma)\left(b^{\frac{\alpha+\beta+\gamma}{\alpha}}-b^{\frac{\alpha+\beta}{\alpha}}\,y^{\frac{\gamma}{\alpha}}-b^{\frac{\gamma}{\alpha}}\,y^{\frac{\alpha+\beta}{\alpha}}+y^{\frac{\alpha+\beta+\gamma}{\alpha}}\right)}$$

Tum, poſito $y=b$, $\frac{P}{Q}$ ambiguum induit valorem $\frac{0}{0}$, cuius ambiguitas methodo Bernoulliana tollenda eſt, de qua videatur Kaestneri, Viri illuſtris, Analyſis infinitorum p. 428 Ed. 1799. Differentialibus ſcilicet ſingulis denominatoris et numeratoris evolutis, altero per alterum diviſo, factaque de more reductione invenitur,

$$\frac{dP}{dQ}=\frac{\gamma\left(b^{\frac{\alpha+\beta}{\alpha}}-y^{\frac{\alpha+\beta}{\alpha}}\right)}{\gamma\,b^{\frac{\alpha+\beta}{\alpha}}+(\alpha+\beta)\,b^{\frac{\gamma}{\alpha}}\,y^{\frac{\alpha+\beta-\gamma}{\alpha}}-(\alpha+\beta+\gamma)\,y^{\frac{\alpha+\beta}{\alpha}}}$$

Sed haec formula valorem $\frac{0}{0}$ retinet, poſito $y=b$, itaque iterata in eundem modum differentiatione, profertur:

$$\frac{d^2P}{d^2Q}=\frac{\gamma\,y^{\frac{\beta}{\alpha}}}{(\alpha+\beta+\gamma)\,y^{\frac{\beta}{\alpha}}-(\alpha+\beta+\gamma)\,b^{\frac{\gamma}{\alpha}}\,y^{\frac{\beta-\gamma}{\alpha}}}$$

quo exſeritur valor quotientis $\frac{P}{Q}$, eo caſu, quo ponitur $y=b$, reperitur ſcilicet:

$$\frac{P}{Q}=\frac{\gamma\,b^{\frac{\beta}{\alpha}}}{(\alpha+\beta+\gamma)\,b^{\frac{\beta}{\alpha}}-(\alpha+\beta-\gamma)\,b^{\frac{\beta}{\alpha}}}=\frac{1}{2}$$

§. 3.

PROBLEMA.

Data sit figura 1. *ABC, cuius ordinatae in axe AD perpendiculares hanc servant rationem:*

$$BC : MN = AD^{\frac{n}{m}} : AR^{\frac{n}{m}}$$

ubi $\frac{n}{m}$ *exponentem quemcunque constantem designat. Quaeritur in parte variabili MNBC centrum gravitatis.*

SOLVTIO.

Centrum gravitatis cum in axe situm sit, nihil, quod investigetur, superest, nisi ipsius a puncto A distantia. Referantur ad hoc punctum statica elementa et ponatur

$$AD = a$$
$$BC = b$$
$$AR = x$$
$$NM = y$$

et designent G et g centra gravitatis figurarum ABC, et $MNBC$ eritque per hypothesin

$$BC : MN = AD^{\frac{n}{m}} : AR^{\frac{n}{m}}$$

i. e. $$b : y = a^{\frac{n}{m}} : x^{\frac{n}{m}}$$

unde sequitur $$y = \frac{bx^{\frac{n}{m}}}{a^{\frac{n}{m}}}$$

Sed staticum momentum figurae AMN est:

$$\int xydx = \int \frac{bx^{\frac{n+m}{m}}}{a^{\frac{n}{m}}} dx = \frac{m}{n+2m} \frac{b}{a^{\frac{n}{m}}} x^{\frac{n+2m}{m}} + \text{Const.}$$

ipsaque area ANM, sive ponderum suorum summa,

$$\int ydx = \int \frac{bx^{\frac{n}{m}}}{a^{\frac{n}{m}}} dx = \frac{m}{n+m} \frac{b}{a^{\frac{n}{m}}} x^{\frac{n+m}{m}} + \text{Const.}$$

Conſtans quantitas adiicienda in utroque integralium nulla eſt, quod, poſito $x = o$, ANM evaneſcit, eamque ob cauſam ex eis, poſito $x = a$, ſequitur

$$\text{figurae } ABC \text{ momentum } = \frac{m}{n+2m} \cdot \frac{b\, a^{\frac{n+2m}{m}}}{a^{\frac{n}{m}}}$$

$$\text{eiusque area } = \frac{m}{n+m} \cdot \frac{b\, a^{\frac{n+m}{m}}}{a^{\frac{n}{m}}}$$

Quantum vero in variabili parte $MNBC$ centrum gravitatis G diſtet ab A, invenitur, differentia momentorum figurarum ABC et AMN per ipſarum arearum differentiam diviſa, itaque, facta reductione, eruitur

$$AG = \frac{(n+m)}{(n+2m)} \left(\frac{a^{\frac{n+2m}{m}} - x^{\frac{n+2m}{m}}}{a^{\frac{n+m}{m}} - x^{\frac{n+m}{m}}} \right)$$

Corollarivm I.

Ex formula pro AG inventa, ſequitur, poſito $x = o$, integrae figurae ABC centrum gravitatis g quantum diſtet ab A, ſcilicet

$$Ag = \frac{n+m}{n+2m}\, a.$$

Contra vero, poſito $x = a$, ex formula I. §. 2. ſi ſubſtituantur m; $n+m$; o pro α; β; γ, ſequitur $AG = a$. Hoc eſt, creſcente absciſſa x, creſcit pars auferenda AMN et propius accedit centrum G ad punctum D, donec, poſito $x = a$, evaneſcat $MNBC$ et concidant G et D.

Exemplum 1. Si ABC Fig. 1. Parabolam Apollonii exhibet ex eius aequatione $y^2 = px$ ſequitur $b : y = a^{\frac{1}{2}} : x^{\frac{1}{2}}$
ideoque hic est $m = 2$; $n = 1$ et $Ag = \frac{3}{5} a$

Exemplum 2. Si BAC Fig. 2. Neiliana eſt parabola, eiusque axis AD, ſequitur ex eius aequatione $x^3 = py^2$
$b : y = a^{\frac{3}{2}} : x^{\frac{3}{2}}$
hic ergo eſt $m = 2$; $n = 3$ et $Ag = \frac{5}{7} a$

Corollarium II.

Cum formula Corollarii $Ag = \frac{n+m}{n+2m} a$ nullam lineam praeter a involvat ab ea sola distantiam Ag pendere patet, minimeque a basi b. Itaque omnes areae parabolarum eiusdem generis, hoc est, quibus iidem exponentium m et n competunt valores, idem centrum gravitatis habent, dum vertex A et axis communes sint maneatque abscissae x valor immutatus. Sic Fig. 3. areae parabolarum eiusdem generis ABC et AEF, in quibus AD de axe est pars, centrum gravitatis g commune habent. De partibus $MNBC$, $PQEF$ earundem figurarum pariter monendum est, eas communi centro grauitatis G uti.

Corollarium III.

In problemate distantia centri gravitatis G Fig. 1. ab A expressa est per abscissas $AD = a$; $AR = x$, iam exprimenda proponitur distantia centri G ab R per ordinatas $MN = y$; $BC = b$, et earundem distantiam $RD = c$. Adsunt aequationes binae $c = a - x$ et $a^{\frac{n}{m}} y = b x^{\frac{n}{m}}$ eaeque hos proferunt abscissarum a et x valores:

$$a = \frac{c b^{\frac{m}{n}}}{b^{\frac{m}{n}} - y^{\frac{m}{n}}} \qquad x = \frac{c y^{\frac{m}{n}}}{b^{\frac{m}{n}} - y^{\frac{m}{n}}}$$

Quibus in formula pro AG inventa substitutis, detractoque AR, et superfluis factoribus deletis, exoritur:

$$RG = AG - AR = \frac{c}{b^{\frac{m}{n}} - y^{\frac{m}{n}}} \left(\frac{(n+m)}{(n+2m)} \frac{\left(b^{\frac{n+2m}{n}} - y^{\frac{n+2m}{n}}\right)}{\left(b^{\frac{n+m}{n}} - y^{\frac{n+m}{n}}\right)} - x^{\frac{m}{n}} \right)$$

sive secundum §. 2. II. substitutis n; m; m pro α; β; γ,

$$RG = \frac{c}{(n+2m)} \left(\frac{(n+m)\, b^{\frac{n+m}{m}}}{b^{\frac{n+m}{n}} - y^{\frac{n+m}{n}}} - \frac{m y^{\frac{m}{n}}}{b^{\frac{m}{n}} - y^{\frac{m}{n}}} \right).$$

Exemplum 1. Pro Apolloniana parabola ABC Fig. 1. est $m = 2$; $n = 1$ (vide Exemplum 1. Corollarii I.) itaque $RG = \frac{c}{5} \left(\frac{3b^3}{b^3 - y^3} - \frac{2y^2}{b^2 - y^2} \right)$.

Exemplum 2. Sit Fig. 4. *BAC* Triangulum iſosceles, *G* centrum gravitatis Trapezii *MNBC* erit. Namque

$$MN : BC = AR : RD$$

ſive $$y : b = x^1 : a^1$$

ergo $m = 1$; $n = 1$, et $RG = \frac{c}{3}\left(\frac{2b^2}{b^2 - y^2} - \frac{y}{b - y}\right) = \frac{c}{3}\left(1 + \frac{b}{b+y}\right)$.

Corollarium IV.

Exaequatis y et b, valor diſtantiae

$$GR = \frac{c}{n + 2m}\left(\frac{(n+m)\, b^{\frac{n+m}{n}}}{b^{\frac{n+n}{n}} - y^{\frac{n+m}{n}}} - \frac{m y^{\frac{m}{n}}}{b^{\frac{m}{n}} - y^{\frac{m}{n}}}\right)$$

eſt $= \frac{1}{2}c$ id quod ex §. 2. III. patet, poſitis, n; m; m pxo α; β; γ. Neque hic valor figurae repugnat, quo minor enim eſt differentia ordinatarum $MN = y$ et $BC = b$, eo magis ad medium rectae *RD* accedit *G* ipſumque medium conſequitur, quando, aequatis *MN* et *BC*, vertitur *MNBC* in rectangulum. Ex ipſius autem figurae propoſitae natura diiudicandum eſt, utrum *BC* et *RG* finitae ſint, nec ne, poſito $MN = BC$, h. e. $y = b$. Sic in exemplo 1. Corollarii praecedentis y nunquam poteſt aequare b, ſed in exemplo ſecundo poſſibile eſt, poſito enim $x = a$, vertitur triangulum in rectangulum infinitae altitudinis, ſed finitae manent b et c.

§. 4.

Problema.

Orta ſit fig. 5. $\alpha\beta\gamma$ ex fig. 1. ABC, §. 3. elementis quidem ita traductis, ut ordinatae parallelae maneant neque diſtantiam ſuam a vertice ſeu linea $A\alpha$ mutent mediasque ordinatas transeat curva $\alpha\varrho\delta$, quae curva gravitatis vocabitur, quia cuiusque elementi pondus in puncto interſectionis curvae huius et ordinatae collectum eſſe cogitari poteſt. Sit praeterea ex apice α demiſſa in $\beta\gamma$ perpendicularis $\alpha\varepsilon$, in qua capiantur, tum figurae $\alpha\beta\gamma$, tum curvae gravitatis $\alpha\varrho\delta$ abſciſſae, huiusque ordinatae rectangulae hanc rationem ſervent

$$\delta\varepsilon : \zeta\varrho = \alpha\varepsilon^{\frac{s}{r}} : \alpha\zeta^{\frac{s}{r}}$$

ubi $\frac{s}{r}$ exponentem quemlibet conſtantem deſignat: quaeritur centrum gravitatis in parte variabili $\mu\nu\beta\gamma$.

S o l v t i o.

Sit $\alpha\varepsilon = a = AD$ Fig. 1.

$\beta\gamma = b = BC$ - -

$\alpha\zeta = x = AR$ - -

$\mu\nu = y = MN$ - -

$\delta\varepsilon = k$

$\zeta\varrho = z$

Deſignet g' in figura $\alpha\beta\gamma$ centrum gravitatis et G' partis variabilis $\mu\nu\beta\gamma$, atque referantur ad lineas αL et $\alpha\varepsilon$ elementorum momenta.

Cum exortum ſit $\alpha\beta\gamma$ ex ABC, ita quidem traductis elementis, ut elementorum aequalium diſtantiae ab $A\alpha$ eadem manerent in $\alpha\beta\gamma$, quae fuerant in ABC; et eam quoque ob cauſſam utriusque momenta aequalia ſunt cumque eis centrorum G et G' diſtantiae ab $A\alpha$ i. e.

$$G'L = GA = \frac{n+m}{n+2m}\left(\frac{a^{\frac{n+2m}{m}} - x^{\frac{n+2m}{m}}}{a^{\frac{n+m}{m}} - x^{\frac{n+m}{m}}}\right) \qquad \text{(I.}$$

Itaque ſupereſt tantum, ut centri grauitatis G' diſtantia ab $\alpha\varepsilon$ reperiatur. Ex origine figurae $\alpha\beta\gamma$ ſequitur:

$$\beta\gamma : \mu\nu = \alpha\varepsilon^{\frac{n}{m}} : \alpha\zeta^{\frac{n}{m}}$$

i. e. $$b : y = a^{\frac{n}{m}} : x^{\frac{n}{m}}$$

ergo $$y = \frac{b\,x^{\frac{n}{m}}}{a^{\frac{n}{m}}}$$

Ordinatarum curvae gravitatis ratio erat:

$$\delta\varepsilon : \zeta\varrho = \alpha\varepsilon^{\frac{s}{r}} : \alpha\zeta^{\frac{s}{r}}$$

i. e. $$k : z = a^{\frac{s}{r}} : x^{\frac{s}{r}}$$

ergo $$z = \frac{k\,x^{\frac{s}{r}}}{a^{\frac{s}{r}}}$$

Hinc ſequitur figurae $\alpha\mu\nu$ momentum ad $\alpha\varepsilon$ relatum

$$\int zydx = \int \frac{kx^{\frac{s}{r}}}{a^{\frac{s}{r}}} \frac{bx^{\frac{n}{m}}}{a^{\frac{n}{m}}} dx = \int \frac{kbx^{\frac{sm+nr}{mr}}}{a^{\frac{sm+nr}{mr}}} dx = \frac{rm}{mr+ms+nr} \frac{kbx^{\frac{mr+ms+nr}{mr}}}{a^{\frac{ms+nr}{mr}}} + \text{Conſt.}$$

ubi quantitas conſtans adiicienda eſt nulla, evaneſcente $\alpha\mu\nu$ pro $x=o$. Figurae vero $\alpha\mu\nu$ et AMN, eiusdem ſunt areae ſupra repertae

$$\int ydx = \int \frac{bx^{\frac{n}{m}}}{a^{\frac{n}{m}}} dx = \frac{mr}{nr+mr} \frac{bx^{\frac{mr+nr}{mr}}}{a^{\frac{nr}{mr}}}$$

Quibus ex integralibus, poſito $x=a$, nanciscimur:

Momentum figurae $\alpha\beta\gamma$ ad $\alpha\varepsilon$ relatum $= \dfrac{mr}{mr+ms+nr} \dfrac{kba^{\frac{mr+ms+nr}{mr}}}{a^{\frac{ms+nr}{mr}}}$,

Aream figurae $\alpha\beta\gamma$ $= \dfrac{mr}{mr+nr} \dfrac{ba^{\frac{mr+nr}{mr}}}{a^{\frac{nr}{mr}}}$

Sed, quantum in parte variabili $\mu\nu\beta\gamma$ figurae $\alpha\beta\gamma$ centrum gravitatis G' diſtet ab αL, reperitur, differentia momentorum figurarum $\alpha\beta\gamma$ et $\alpha\mu\nu$ per ipſarum arearum differentiam diviſa, ſcilicet ſuperfluis rite deletis:

$$GK = \frac{mr+nr}{mr+ms+nr} \frac{k}{a^{\frac{ms}{mr}}} \left(\frac{a^{\frac{mr+ms+nr}{mr}} - x^{\frac{mr+ms+nr}{mr}}}{a^{\frac{mr+nr}{mr}} - x^{\frac{mr+nr}{mr}}} \right) \quad \text{(II.}$$

Corollarium I.

Reperiuntur ex formula I. et II. poſito $x=o$, centri gravitatis g' in figura $\alpha\beta\gamma$ diſtantiae ab αL et $\alpha\varepsilon$:

$$g'l = \frac{m+n}{n+2m} a$$

$$g'k = \frac{mr+nr}{mr+ms+nr} k$$

Sed, posito $x=a$, eaedem formulae I. et II. dant $GL=a$, et $GK=k$, uti ex §. 2. I. liquet, si pro α; β; γ, scribantur, m; $m+n$; o, cum quaeritur de formula I. et mr; ms; nr, cum de formula II. quaeritur. (His conferantur ea, quae supra monuimus §. 3. Cor. I.)

Exemplum. Sint $\alpha\mu\beta$ et $\alpha\nu\gamma$ fig. 5. parabolae Apollonianae ad diversas parametros p et q constructae, axem habeant communem $\alpha\varepsilon$, quaeritur centrum gravitatis g' in figura $\alpha\beta\gamma$.

$$\begin{array}{lrcl} \text{Est} & \beta\varepsilon : & \mu\zeta & = \alpha\varepsilon^{\frac{1}{2}} : \alpha\zeta^{\frac{1}{2}} \\ & \gamma\varepsilon : & \nu\zeta & = \alpha\varepsilon^{\frac{1}{2}} : \alpha\zeta^{\frac{1}{2}} \\ \text{itaque} & \beta\varepsilon-\gamma\varepsilon : & \mu\zeta-\nu\zeta & = \alpha\varepsilon^{\frac{1}{2}} : \alpha\zeta^{\frac{1}{2}} \\ \text{sive} & \beta\gamma : & \mu\nu & = \alpha\varepsilon^{\frac{1}{2}} : \alpha\zeta^{\frac{1}{2}} \\ \text{h. e.} & b : & y & = a^{\frac{1}{2}} : x^{\frac{1}{2}} \end{array}$$

est igitur $m=2$; $n=1$ et $lg'=\frac{3}{5}a$

Vt $g'k$ etiam eruatur, animadvertendum est, curvam gravitatis $\alpha\varrho\delta$ dividere ordinatas $\mu\nu$ et $\beta\gamma$ in partes aequales. Hinc sequitur:

$$\zeta\varrho=\tfrac{1}{2}(\mu\zeta+\nu\zeta)$$

h. e. $$z=\tfrac{1}{2}\,x^{\frac{1}{2}}(p^{\frac{1}{2}}+q^{\frac{1}{2}}).$$

Curva igitur gravitatis $\alpha\varrho\delta$ pariter est Parabola Apolloniana et

$$\varepsilon\delta:\zeta\varrho=\alpha\varepsilon^{\frac{1}{2}}:\alpha\zeta^{\frac{1}{2}}$$

i. e. $$k:z = a^{\frac{1}{2}} : x^{\frac{1}{2}}$$

unde sequitur $r=2$; $s=1$ et propterea

$$g'k=\tfrac{3}{4}k.$$

Scholion. Si ponatur hoc in exemplo $\beta\gamma:\gamma\varepsilon=d:e$, sequitur $\gamma\varepsilon=\frac{e}{d}\beta\gamma =\frac{e}{d}b$, cumque sit $k=\varepsilon\delta=\delta\gamma+\gamma\varepsilon=\frac{1}{2}b+\frac{e}{d}b=\frac{d+2e}{2d}b$, denique $g'k=\frac{3}{4}k =\frac{3}{8}\frac{d+2e}{d}b$. Evanescente v. c. $\varepsilon\gamma$, ipsum evanescit e fitque $\beta\alpha\gamma$ dimidium parabolae et $$g'k=\tfrac{3}{8}b$$ h. e. centrum gravitatis in dimidio parabola distat ab axe $\frac{3}{8}$ ordinatae imae.

Corollarium II.

In problemate §. 4. centri gravitatis G distantiae a rectis αL et $\alpha\varepsilon$ expressae erant per abscissas $\alpha\varepsilon=a$; $\alpha\zeta=x$; et ordinatam $\delta\varepsilon=k$, iam centri gravitatis G distantiae ab ordinata $\mu\nu$ et perpendiculari $\varrho\iota$ e media $\mu\nu$ in basin $\beta\gamma$ demissa, exprimendae ſunt per ordinatas $\mu\nu = y$; $\beta\gamma = b$; earundem ordinatarum diſtantiam $\varrho\iota = c$, et ordinatarum $\delta\varepsilon$; $\zeta\varrho$ differentiam $\delta\iota = f$.

Ex §. 3. Corollario III. huc transferendi ſunt valores:

$$1)\quad a = \frac{c b^{\frac{m}{n}}}{b^{\frac{m}{n}} - y^{\frac{m}{n}}} \qquad\qquad 2)\quad x = \frac{c y^{\frac{m}{n}}}{b^{\frac{m}{n}} - y^{\frac{m}{n}}}$$

cumque ſit $f = k - z$ et $z = \frac{k x^{\frac{s}{r}}}{a^{\frac{s}{r}}}$, ſive $z = \frac{k y^{\frac{ms}{nr}}}{b^{\frac{ms}{nr}}}$,

ſubſtitutis ſcilicet valoribus ex 1. et 2. per eliminationem eruitur:

$$3)\quad k = \frac{f b^{\frac{ms}{nr}}}{b^{\frac{ms}{nr}} - y^{\frac{ms}{nr}}} \qquad\qquad 4)\quad z = \frac{f y^{\frac{ms}{nr}}}{b^{\frac{ms}{nr}} - y^{\frac{ms}{nr}}}$$

Sed ſubſtitutis valoribus 1, 2, abſciſſarum a, x in formula I. diſtantiae LG deductoque valore abſciſſae $\alpha\zeta$, ſic deducitur hoc loco ex $GL - \alpha\zeta$:

$$G\sigma = \frac{c}{n+2m}\left(\frac{(n+m)\,b^{\frac{m+n}{n}}}{b^{\frac{m+n}{n}} - y^{\frac{m+n}{n}}} - \frac{m \cdot y^{\frac{m}{n}}}{b^{\frac{m}{n}} - y^{\frac{m}{n}}}\right) \quad \text{(III.}$$

uti ſupra §. 3. Coroll. III. ex $AG - AR$ inventum erat RG et $AG - AR$.

Ad explorandam quoque diſtantiam $G\vartheta$ valores literarum k; a; x; z ex aequationibus, 1, 2, 3, 4, in formula:

$$G\vartheta = Gk - \varrho\zeta = \frac{mr}{mr+ms+nr}\,\frac{k}{a^{\frac{ms}{mr}}}\left(\frac{a^{\frac{mr+ms+nr}{mr}} - x^{\frac{mr+ms+nr}{mr}}}{a^{\frac{ms+nr}{mr}} - x^{\frac{ms+nr}{mr}}}\right) - z$$

ſubſtituantur, et facta de more reductione, reperietur

$$G\vartheta = \frac{f}{b^{\frac{ms}{nr}} - y^{\frac{ms}{nr}}} \left(\frac{(mr+nr)}{(mr+ms+nr)} \frac{\left(b^{\frac{mr+ms+nr}{nr}} - y^{\frac{mr+ms+nr}{nr}}\right)}{\left(b^{\frac{mr+nr}{nr}} - y^{\frac{mr+nr}{nr}}\right)} - y^{\frac{ms}{nr}} \right)$$

unde ſequitur ſecundum §. 2. II. poſito $\alpha = nr$; $\beta = mr$; $\gamma = ms$

$$G\vartheta = \frac{f}{mr+ms+nr} \left(\frac{(mr+nr)\, b^{\frac{m+n}{n}}}{b^{\frac{m+n}{n}} - y^{\frac{m+n}{n}}} - \frac{ms\, y^{\frac{ms}{nr}}}{b^{\frac{ms}{nr}} - y^{\frac{ms}{nr}}} \right) \quad \text{(IV.}$$

Exemplum. Sint $\alpha\beta$; $\alpha\gamma$ iterum parabolae Apollonianae, uti in exemplo Coroll. I., quarum axis eſt $\alpha\varepsilon$, centri gravitatis G in $\mu\nu\beta\gamma$ diſtantiae ab $\mu\nu$, et $\varrho\iota$ ex formulis propoſitis III. et IV. poſito uti ſupra $m=2$; $n=1$; et $r=2$; $s=1$, reperiuntur

$$G\sigma = \frac{e}{5} \left(\frac{3b^3}{b^3 - y^3} - \frac{2y^2}{b^2 - y^2} \right)$$

$$G\vartheta = \frac{f}{4} \left(\frac{3b^3}{b^3 - y^3} - \frac{y}{b - y} \right)$$

§. 5.

P r o b l e m a.

Data ſint pyramidoidis APQS Fig. 6. baſis PQS, axis gravitatis AD ſit ſectio pqs baſi PQS parallela et homologae figurarum pqs et PQS ordinatae hanc ſervent rationem:

$$BC : MN = AD^{\frac{n}{m}} : AR^{\frac{n}{m}},$$

ubi $\frac{n}{m}$ exponentes quoscunque conſtantes deſignat: quaeritur centrum gravitatis in parte variabili pqsPQS pyramidoidis APQS.

S o l v t i o.

Sit $AD = a$
$BC = b$
$AR = x$
$NM = y$
$PQS = B$
$pqs = \Upsilon$

Deſignet h Centrum gravitatis in pyramidoide $APQS$ et H in parte eius variabili $pqsPQS$. Affertur ex hypotheſi

$$BC : MN = AD^{\frac{n}{m}} : AR^{\frac{n}{m}}$$

i. e. $$b : y = a^{\frac{n}{m}} : x^{\frac{n}{m}}$$

unde ſequitur $$y = \frac{bx^{\frac{n}{m}}}{a^{\frac{n}{m}}}$$

Cumque ſimilium figurarum areae ſint in ratione duplicata linearum ſuarum homologarum, alteram habemus proportionem

$$PQS : pqs = BC^2 : MN^2$$

i. e. $$B : Y = b^2 : y^2$$

itaque $$Y = \frac{By^2}{b^2} = \frac{Bx^{\frac{2n}{m}}}{a^{\frac{2n}{m}}}$$

Hinc ſequitur momentum ſtaticum ſolidi variabilis $Apqs$

$$\int xYdx = \int \frac{xBx^{\frac{2n}{m}}}{a^{\frac{2n}{m}}} dx = \frac{m}{(2n+2m)} \frac{B}{a^{\frac{2n}{m}}} x^{\frac{2n+2m}{m}} + \text{Conſt.}$$

nec non eiusdem ſolidi volumen, ſive ſumma ponderum ſuorum,

$$\int Ydx = \int \frac{Bx^{\frac{2n}{m}}}{a^{\frac{2n}{m}}} dx = \frac{m}{2n+m} \frac{B}{a^{\frac{2n}{m}}} x^{\frac{2n+m}{m}} + \text{Conſt.}$$

Vtroque in integrali quantitas conſtans adiicienda nulla eſt, cum ſolidum $Apqs$, poſito $x = o$, evanescat. Poſito vero, $x = a$, prodit integri pyramidoidis $APQS$

momentum ad A relatum $$= \frac{m}{2n+2n} \frac{B}{a^{\frac{2n}{m}}} a^{\frac{2n+2m}{m}}$$

eiusdemque volumen, sive ponderis summa $= \frac{m}{2n+m} \cdot \frac{B}{a^{\frac{2n}{m}}} \cdot a^{\frac{2n+m}{m}}$

At quantum in solido variabili *pqs PQS* gravitatis centrum *H* distet ab apice *A*, differentia momentorum solidorum *APQS* et *Apqs* per ponderum seu voluminum suorum differentiam divisa, reperitur, scilicet, superfluis rite deletis,

$$AH = \frac{(2n+m)}{(2n+2m)} \left(\frac{a^{\frac{2n+2m}{m}} - x^{\frac{2n+2m}{m}}}{a^{\frac{2n+m}{m}} - x^{\frac{2n+m}{m}}} \right)$$

Corollarium I.

Posito $x = o$ nanciscimur ex formula proposita centri gravitatis *h* in pyramidoide integro *APQS* distantiam ab *A*

$$Ah = \frac{2n+m}{2n+2m}\, a$$

Sed posito $x = a$, invenitur $AH = a$, secundum §. 2. I. cum substituantur m; $2n+m$; o; loco α; β; γ. Vt enim in figuris planis, quas supra protulimus sic etiam in solido *pqsPQS* gravitatis centrum *H*, eo propius accedit ad *D*, quo maior est abscissa *x* cumque ea solidum ablatum *Apqs*, et concidunt *H* et *D*, aequatis *x* et *a*, evanescente scilicet *pqsPQS*.

Exemplum 1. Gravitatis centrum *h* in paraboloide ex revolutione parabolae Apollonianae *ABC* fig. 1. circa axem suum generato, posito $m = 2$; $n = 1$ reperitur $Ah = \frac{2}{3}$.

Exemplum 2. Cum ponitur $m = 2$, $n = 3$ reperitur: $Ah = \frac{4}{5} a$ distantia centri gravitatis in paraboloide ex revolutione parabolae Neilianae *ABC* fig. 2. §. 3. Coroll. I. Exempl. 2. circa axem *AD* generato.

Corollarium II.

In figuris planis vidimus supra distantias centrorum gravitatis ab apicibus non ab ordinatarum magnitudine pendere, ita hic centrorum gravitatis *h* et *H* in pyramidoide *APQS* Fig. 6. et eius parte variabili *pqsPQS* distantiae *Ah*, *AH* ab apice *A*; eaedem manent dum quantitatum m; n; a; x; nulla mutetur. Neque enim magnitudo basium *B*, et *Y* neque ipsarum forma hoc loco spectatur, sed haec ad ratiocinia adhibenda sufficit, ut centra gravitatis figurarum *B* et *Y* sint in perpendiculari *AD* e baseos *B* gravitatis centro *D* erecta, earundemque figurarum ordinatae homologae, b et y servent rationem $a^{\frac{n}{m}} : x^{\frac{n}{m}}$; caetera vero, qualiacunque sint, distantiarum AH, Ah magnitudinem non afficiunt. Hoc adeo tum verum est, quum D et R, extra limites figurarum B, Y, posita sunt, quorum centra gravitatis designant, cuius rei exempla praebent solida pyramidoide excavata.

Corollarium III.

Cum quaeritur gravitatis centrum H in solido $PQSpqs$ Fig. 6. quantum distet ab R, eaque distantia RH exprimenda proponitur per ordinatas $MN = y$; $BC = b$ et basium B, Y distantiam $RD = c$, habemus tum, veluti §. 3. Coroll. III.

$$a = \frac{cb^{\frac{m}{n}}}{b^{\frac{m}{n}} - y^{\frac{m}{n}}} \quad \text{et} \quad x = \frac{cy^{\frac{m}{n}}}{b^{\frac{m}{n}} - y^{\frac{m}{n}}}$$

quibus valoribus abscissarum a et x substitutis in formula H, detractoque valore abscissae AR, superfluisque deletis, reperitur:

$$RH = \frac{c}{b^{\frac{m}{n}} - y^{\frac{m}{n}}} \left[\frac{2n+m}{2n+2m} \frac{\left(b^{\frac{2n+2m}{n}} - y^{\frac{2n+2m}{n}}\right)}{\left(b^{\frac{2n+m}{n}} - y^{\frac{2n+m}{n}}\right)} - y^{\frac{m}{n}} \right]$$

unde sequitur ex §. 2. II. cum ponitur $\alpha = n$; $\beta = n + m$; $\gamma = m$;

$$RH = \frac{c}{2n+2m} \left(\frac{(2n+m)\, b^{\frac{2n+m}{n}}}{b^{\frac{2n+m}{n}} - y^{\frac{2n+m}{n}}} - \frac{m\, y^{\frac{m}{n}}}{b^{\frac{m}{n}} - y^{\frac{m}{n}}} \right)$$

Exemplum 1. Posito $m = 2$; $n = 1$ in paraboloide Apolloniano truncato, cuius sectionem per axem exhibet $MNBC$ Fig. 1. reperitur, centri gravitatis H distantia ab R

$$RH = \frac{c}{3}\left(\frac{2b^4}{b^4 - y^4} - \frac{y^2}{b^2 - y^2}\right) = \frac{c}{3}\left(1 + \frac{b^2}{b^2 + y^2}\right)$$

Exemplum 2. Centrum gravitatis H in pyramide recta truncata, quantum distet ab R, posito $m = 1$; $n = 1$, (quippe est $b : y = \alpha : x$) reperitur

$$RH = \frac{c}{4}\left(\frac{3b^3}{b^3 - y^3} - \frac{y}{b - y}\right)$$

Pyramis autem hoc loco recta vocatur ea, cuius apicem transit perpendicularis e centro gravitatis *D* baseos erecta, et solida complectitur omnia, quorum axis gravitatis perpendicularis est in basi figuram arbitrariam habente, quaeque in apicem desinunt et in quibus ordinatarum sectionum homologarum haec est ratio: $b : y = a^{r} : x^{r}$.

Praeterea cum ex formula *RH* generali reditus ad distantiam *Ah* pateat, si fingatur pars ablata *Apqs* infinite parva, (evanescente enim y, c aequat a), deducitur hoc modo in exemplo pyramidum ex *RH*

$$ah = \tfrac{3}{4}a.$$

Corollarium IV.

Si quaeritur valor, quem, posito $y = a$, accipiat distantia

$$RH = \frac{c}{2n+2m}\left(\frac{(2n+m)\,b^{\frac{2n+m}{n}}}{b^{\frac{2n+m}{n}} - y^{\frac{2n+m}{n}}} - \frac{m\,y^{\frac{m}{n}}}{b^{\frac{m}{n}} - y^{\frac{m}{n}}}\right)$$

substituantur §. 2. III. n; $n+m$; m; pro α; β; γ et reperitur esse

$$RH = \tfrac{1}{2}c$$

Quo minor enim est differentia ordinatarum b et y, eo magis ad prisma accedit solidum *pqsPQS*, cuius centrum gravitatis in media altitudine c situm est. His conferantur, quae §. 3. Corollario IV. monuimus.

Scholion. Si quis formularum comparationem instituat, quas pro distantiis *AH*, *RH* et §. 3. pro distantiis *AG* et *RG* protulimus, non negligendum adesse animadvertet consensum inter distantias a punctis iisdem centrorum gravitatis in figuris et solidis. Huius formularum similitudinis usus infra docebitur.

§. 6.

Problema.

Solidum α PQS fig. 7. ex pyramidoide APQS §. 5. fig. 6. ortum sit, elementis quidem ita traductis versus perpendiculum ex α in planum baseos PQS demissum, ut latera sectionis cuiusque pqs basi parallelae parallela maneant homologis baseos lateribus v. c. ps et PS, atque eadem sit ipsius sectionis pqs in utraque figura 6. et 7.

distantia ab $A\alpha$, axis vero gravitatis AD transeat in curvam gravitatis $\alpha\varrho\delta$, cuius ordinatae hanc afferant proportionem

$$\delta\varepsilon : \varrho\zeta = \alpha\varepsilon^{\frac{s}{r}} : \alpha\zeta^{\frac{s}{r}}$$

ubi $\frac{s}{r}$ exponentem quemlibet constantem exprimit; quaeritur centrum gravitatis in variabili solido pqsPQS.

SOLVTIO.

Sit fig. 7. $\alpha\varepsilon = a = AD$ *Fig.* 6.
$\beta\gamma = b = BC$ - -
$\alpha\zeta = x = AR$ - -
$\mu\nu = y = MN$ - -
$\zeta\varepsilon = c = RD$ - -
$PQS = B = PQS$ - -
$pqs = \Upsilon = pqs$ - -
$\delta\varepsilon = k$
$\zeta\sigma = z$
$\delta\iota = f$

Designet h' gravitatis centrum in pyramidoide αPQS, H' in solido variabili *pqsPQS*. Quod vero propter originem solidi αPQS ex pyramidoide $APQS$ sectiones *pqs* fig. 6. et fig. 7. loco quidem, non figura differunt, ordinatarum $\beta\gamma$ et $\mu\nu$ eadem est ratio, quae ordinatarum erat homologarum *BC*, *MN* fig 6. §. 5. nempe:

$$\beta\gamma : \mu\nu = \alpha\varepsilon^{\frac{n}{m}} : \alpha\zeta^{\frac{n}{m}}$$

i. e. $$b : y = a^{\frac{n}{m}} : x^{\frac{n}{m}}$$

itaque $$y = \frac{bx^{\frac{n}{m}}}{a^{\frac{n}{m}}}$$

eandemque ob causam areae quoque sectionum ab apicibus A, α aequidistantium *pqs* fig. 6. et *pqs* fig. 7. aequales sunt, cum ipsis elementorum momentis ad αA relatis: hinc sequitur esse $H'L = HA$ et $H'\sigma = HR$ §. 5. nempe:

$$H'L = \frac{(2n+m)}{(2n+2m)} \left(\frac{a^{\frac{2n+2m}{m}} - x^{\frac{2n+2m}{m}}}{a^{\frac{2n+m}{m}} - x^{\frac{2n+m}{m}}} \right) \quad (\text{I.}$$

$$H'\sigma = \frac{c}{(2n+2m)}\left(\frac{(2n+m)\,b^{\frac{2n+m}{n}}}{b^{\frac{2n+m}{n}} - y^{\frac{2n+n}{m}}} - \frac{m\,y^{\frac{m}{n}}}{b^{\frac{m}{n}} - y^{\frac{m}{n}}}\right) \quad (11.$$

Supereſt, ut quaeratur, quantum centrum gravitatis H' in ſolido variabili $pqsPQS$ diſtet a perpendicularibus $\alpha\varepsilon$ et $\varrho\iota$, quarum altera ex apice α, altera e centro gravitatis ϱ ſectionis pqs demiſſa eſt in baſin. Huic inveſtigationi binae, quae ſequuntur, proportiones inſerviunt:

$$\delta\varepsilon : \varrho\zeta = \alpha\varepsilon^{\frac{s}{r}} : \alpha\zeta^{\frac{s}{r}} \quad \text{et} \quad PQS : pqs = \beta\gamma^2 : \mu\nu^2$$

$$k : z = a^{\frac{s}{r}} : x^{\frac{s}{r}} \qquad B : Y = b^2 : y^2 .$$

quibus ſequitur $\quad z = \dfrac{kx^{\frac{s}{r}}}{a^{\frac{s}{r}}} \quad$ et $\quad Y = \dfrac{By^2}{b^2} = \dfrac{Bx^{\frac{2n}{m}}}{a^{\frac{2n}{m}}}$

Hinc deducitur ſtaticum momentum ſolidi αpqs ad $\alpha\varepsilon$ relatum

$$\int zY\,dx = \int \frac{kx^{\frac{s}{r}}}{a^{\frac{s}{r}}}\,\frac{Bx^{\frac{2n}{m}}}{a^{\frac{2n}{m}}} = \int \frac{kBx^{\frac{2nr+ms}{mr}}}{a^{\frac{sm+2nr}{rm}}}\,dx = \frac{mr}{mr+ms+2nr}\,\frac{kBx^{\frac{mr+ms+2nr}{mr}}}{a^{\frac{ms+2nr}{mr}}} + \text{Conſt.}$$

Volumen eiusdem ſolidi $\alpha pqs = Apqs$ fig. 6. eſt vti ſupra §. 5.

$$\int Y\,dx = \int \frac{Bx^{\frac{2n}{m}}}{a^{\frac{2n}{m}}}\,dx = \frac{mr}{mr+2nr}\,\frac{Bx^{\frac{2nr+mr}{mr}}}{a^{\frac{2nr}{mr}}} + \text{Conſt.}$$

Sed in utroque integrali quantitas conſtans nulla eſt, quia pro $x=0$ in nihilum abit αpqs. Itaque poſito $x=0$, invenitur totius pyramidoidis αPQS

ſtaticum momentum $\quad = \dfrac{mr}{rm+ms+2nr}\,\dfrac{kBa^{\frac{mr+ms+2nr}{mr}}}{a^{\frac{sm+2nr}{mr}}}$, eiusdemque volumen $= \frac{mr}{mr + 2nr} \frac{B a^{\frac{mr + 2nr}{mr}}}{a^{\frac{2nr}{mr}}}$

Quantum vero centrum gravitatis H' in ſolido variabili diſtet a perpendiculari $\alpha\varepsilon$, eruitur, differentia momentorum pyramidoidis αPQS et ſolidi αpqs ad $\alpha\varepsilon$ relatorum diviſa per differentiam voluminorum eorundem ſolidorum, ſcilicet, ubi deleta ſunt ſuperflua,

$$H'K = \frac{mr + 2nr}{mr + ms + 2nr} \frac{k}{a^{\frac{ms}{mr}}} \left(\frac{a^{\frac{mr + ms + 2nr}{mr}} - x^{\frac{mr + ms + 2nr}{mr}}}{a^{\frac{mr + 2nr}{mr}} - x^{\frac{mr + 2nr}{mr}}} \right) \quad \text{(III.}$$

Quo denique centri gravitatis H' diſtantia ab $\varrho\iota$ per ordinatas $\beta\gamma = b$; $\mu\nu = y$; et ordinatarum $\varrho\zeta$; $\varepsilon\delta$, differentiam $\varrho\iota = f$ expreſſa reperiatur, valores adſunt literarum α; x; k; z; hoc loco, uti §. 4. Coroll. II. evolvendi, ubi repertum erat

$$a = \frac{c b^{\frac{m}{n}}}{b^{\frac{m}{n}} - y^{\frac{m}{n}}}; \quad x = \frac{c y^{\frac{m}{n}}}{b^{\frac{m}{n}} - y^{\frac{m}{n}}}; \quad k = \frac{f b^{\frac{ms}{nr}}}{b^{\frac{ms}{nr}} - y^{\frac{ms}{nr}}}; \quad z = \frac{f y^{\frac{ms}{nr}}}{b^{\frac{ms}{nr}} - y^{\frac{ms}{nr}}};$$

His valoribus in formula III. ſubſtitutis, deductoque de ea valore ordinatae z, tandem, formula rite contracta, haec provenit:

$$H'\vartheta = H'k - \zeta\varrho = \frac{f}{b^{\frac{ms}{nr}} - y^{\frac{ms}{nr}}} \left(\frac{(mr + 2nr)\left(b^{\frac{mr + ms + 2nr}{nr}} - y^{\frac{mr + ms + 2nr}{nr}} \right)}{(mr + ms + 2nr)\left(b^{\frac{mr + 2nr}{nr}} - y^{\frac{mr + 2nr}{nr}} \right)} - y^{\frac{ms}{nr}} \right)$$

quae congruit cum ſequente

$$H'\vartheta = \frac{f}{mr + ms + 2nr} \left(\frac{(mr + 2nr)\, b^{\frac{2n + m}{n}}}{b^{\frac{2n + m}{n}} - y^{\frac{2n + m}{n}}} - \frac{ms\, y^{\frac{ms}{nr}}}{b^{\frac{ms}{nr}} - y^{\frac{ms}{nr}}} \right) \quad \text{(IV.}$$

id quod apparet e §. 2. II. poſito $\alpha = nr$; $\beta = mr + nr$; $\gamma = ms$.

COROLLARIVM I.

Ex formula I. et III. ſequitur

poſito $x = o$, $\quad l h' = \frac{2n + m}{2n + 2m} a \quad$ et $\quad h' k = \frac{mr + 2nr}{mr + ms + 2nr} k$,

et poſito $x = a$, $\quad LH' = a \quad$ et $\quad H'K = k$,

uti ex formula I. §. 2. apparet, ſi ſcilicet loco α; β; γ, ſcribatur m; $2n + m$; o in formula LH' et mr; $mr + 2nr$; ms in formula $H'K$.

Ex formulis autem II. IV. poſito $y = b$, ſequitur §. 2. III.

$$H'\sigma = \tfrac{1}{2} c \qquad \text{et} \qquad H'\vartheta = \tfrac{1}{2} f,$$

ſi ſcilicet pro literis α; β; γ ſcribatur m; $n + m$; m in formula II. in IV. vero nr; $nr + mr$; ms. De his diſtantiarum $H'\sigma$ et $H'\vartheta$ valoribus ea conferantur, quae §. 4. Coroll. I. monuimus.

Exemplum 1. Ex cono recto ortum ſit ſolidum αPQS Fig. 8. eo, quem ſupra deſcripſimus modo, elementis quidem ita traductis, ut axis gravitatis tranſierit in parabolam Neilianam, quae axem habeat $\alpha\varepsilon$; erit

$$\begin{matrix} \beta\gamma : \mu\nu = \alpha\varepsilon^{1} : \alpha\zeta^{1} \\ \varepsilon\delta : \zeta\varrho = \alpha\varepsilon^{\frac{3}{2}} : \alpha\zeta^{\frac{3}{2}} \end{matrix} \qquad \text{ergo} \qquad \begin{matrix} m = 1;\ n = 1, \\ r = 2;\ s = 3, \end{matrix}$$

Itaque $\quad H'\sigma = \frac{c}{4}\left(\frac{3b^3}{b^3 - y^3} - \frac{y}{b - y}\right) \qquad H'\vartheta = \frac{f}{3}\left(\frac{2b^3}{b^3 - y^3} - \frac{y^{\frac{3}{2}}}{b^{\frac{3}{2}} - y^{\frac{3}{2}}}\right)$

Exemplum 2. In ſolido αPQS fig. 7. hae proportiones occurrant

$$\begin{matrix} \beta\gamma : \mu\nu & = & \alpha\varepsilon^{\frac{1}{2}} : \alpha\zeta^{\frac{1}{2}}, \\ \delta\varepsilon : \zeta\varrho & = & \alpha\varepsilon^{\frac{1}{2}} : \alpha\zeta^{\frac{1}{2}}, \end{matrix}$$

tum αPQS exhibet pyramidoides, cuius axis gravitatis in parabolam Apollonianam tranſiit, cuiusque ſectiones pqs, PQS eandem, quam paraboloidis ſectiones ſervant rationem. Ex proportionibus vero allatis ſequitur $m = 2$; $n = 1$ et $r = 2$ et $s = 1$ ergo reperiuntur ex formulis II. et IV. pro centro gravitatis partis variabilis $PQSpqs$ diſtantiae

$$H'\sigma = \frac{c}{3}\left(\frac{2b^4}{b^4 - y^4} - \frac{y^2}{b^2 - y^2}\right) = \frac{c}{3}\left(1 + \frac{b^2}{b^2 + y}\right)$$

$$H'\vartheta = \frac{f}{5}\left(\frac{4b^4}{b^4 - y^4} - \frac{y}{b - y}\right)$$

et pro integro ſolido αPQS ex Corollario I.

$$h`l = \tfrac{2}{3} a \qquad \text{et} \qquad h`k = \tfrac{4}{5} k$$

Quando baſes PQS et pqs ſunt circuli, ſolidum αPQS ex paraboloide Apolloniano integro, alias vero ex illius parte baſibus reſpondente originem traxiſſe in aperto eſt.

§. 7.

Comparantur diſtantiae centri gravitatis in ſolidis et figuris, quibus axis aut curva gravitatis communis eſt.

Si formulae diſtantiarum centri gravitatis in figuris planis §. 4. repertae iis conferantur, quas §. 6. pro diſtantiis centri gravitatis in ſolidis reperimus, miram formulis iſtis ſimilitudinem eſſe animadvertetur, quum ſolida et figurae ſive axin, ſive curvam gravitatis communem habent. Eſt enim pro figuris §. 4. fig. 5.

$$\text{I.} \qquad G`L = \frac{m+n}{n+2m} \left(\frac{a^{\frac{n+2m}{m}} - x^{\frac{n+2m}{m}}}{a^{\frac{n+m}{m}} - x^{\frac{n+m}{m}}} \right)$$

$$\text{II.} \qquad G`K = \frac{mr+nr}{mr+ms+nr} \, \frac{k}{a^{\frac{s}{r}}} \left(\frac{a^{\frac{mr+ms+nr}{mr}} - x^{\frac{mr+ms+nr}{mr}}}{a^{\frac{m+n}{m}} - x^{\frac{m+n}{m}}} \right)$$

$$\text{III.} \qquad G`\sigma = \frac{c}{n+2m} \left(\frac{(n+m)\, b^{\frac{n+m}{n}}}{b^{\frac{n+m}{n}} - y^{\frac{n+m}{m}}} - \frac{m\, y^{\frac{m}{n}}}{b^{\frac{m}{n}} - y^{\frac{m}{n}}} \right)$$

$$\text{IV.} \qquad G`\vartheta = \frac{f}{mr+ms+nr} \left(\frac{(mr+nr)\, b^{\frac{m+n}{n}}}{b^{\frac{m+n}{n}} - y^{\frac{m+n}{n}}} - \frac{ms\, y^{\frac{ms}{nr}}}{b^{\frac{ms}{nr}} - y^{\frac{ms}{nr}}} \right)$$

et pro ſolidis §. 6. fig. 7.

$$\text{I.}\quad H`L = \frac{m+2n}{2n+2m}\left(\frac{a^{\frac{2n+2m}{m}} - x^{\frac{2n+2m}{m}}}{a^{\frac{2n+m}{m}} - x^{\frac{2n+m}{m}}}\right)$$

$$\text{II.}\quad H`K = \frac{mr+2nr}{mr+ms+2nr}\,\frac{k}{a}\left(\frac{a^{\frac{mr+ms+2nr}{mr}} - x^{\frac{mr+ms+2nr}{mr}}}{a^{\frac{m+2n}{m}} - x^{\frac{m+2n}{m}}}\right)$$

$$\text{III.}\quad H`\sigma = \frac{c}{2n+2m}\left(\frac{(2n+m)\, b^{\frac{2n+m}{n}}}{b^{\frac{2n+m}{n}} - y^{\frac{2n+m}{n}}} - \frac{m\, y^{\frac{m}{n}}}{b^{\frac{m}{n}} - y^{\frac{m}{n}}}\right)$$

$$\text{IV.}\quad H`\vartheta = \frac{f}{mr+ms+2nr}\left(\frac{(mr+2nr)\, b^{\frac{m+2n}{n}}}{b^{\frac{m+2n}{n}} - y^{\frac{m+2n}{n}}} - \frac{ms\, y^{\frac{ms}{nr}}}{b^{\frac{ms}{nr}} - y^{\frac{ms}{nr}}}\right)$$

Formulae $H`L$, $H`K$ ex formulis $G`L$, $G`K$ ſine mora deducuntur, ſcripto $2n$ loco n. De caeteris videbimus infra. Quando ſcilicet ſolida et figurae planae communem habent gravitatis axem, ſeu curvam communesque abſciſſas, eademque ſectionum eſt ratio in illis, quae ordinatarum in his, concidunt in idem punctum gravitatis centra ſolidorum et figurarum. Exempla parata praebent paraboloide Apollonian ı et triangula. Huius rei cauſſa eſt haec: Tam ſolidis, quam figuris pon.lus tribuitur pro ſua magnitudine, itaque in indagando centro gravitatis elementa gravia, neque ſolida ſint, neque areae cuiusdam partes, ſed tantum, quae ſit magnitudinis elementorum, quaeque momentorum ſuorum ratio, quaeritur. Itaque ſi prior ratio in ſolidis eadem eſt, quae in figuris, ſolida quoque et figurae eadem centra gravitatis habere oportet, dum communes ſint axes ſeu curvae gravitatis, quippe quo efficitur, ut poſterior quoque eadem ſit ratio. In quaerendo igitur gravitatis centro ſolidi alicuius ſic prorſus agendum eſt, quaſi centrum gravitatis in figura reperienda eſſet. Haec vero ratiocinia non modo ad firmandum illum priorum formularum conſenſum explicandumque valent, ſed eorum quoque auxilio poſteriores pro ſolidis prolatas formulas $H`\sigma$, $H`\vartheta$ ex formulis $G`\sigma$, $G`\vartheta$ deducere poterimus. Namque cum tantum magnitudo et diſpoſitio elementorum gravium hoc reſpiciatur loco, ſectiones B, T in calculo inſtituendo ita tractandae ſunt, quaſi ordinatas exprimerent, et hac ratione ex formulis $G\grave{}\sigma$, $G\vartheta$ formulae pro diſtantiis $H\grave{}\sigma$ et $H\grave{}\vartheta$ deducentur, ſi ſcribitur Υ; B; $2n$ loco y; b; n nempe

$$H\grave{}\sigma = \frac{c}{2n+2m}\left(\frac{(2n+m)\,B^{\frac{2n+m}{2n}}}{B^{\frac{2n+m}{2n}} - \Upsilon^{\frac{2n+m}{2n}}} - \frac{m\,\Upsilon^{\frac{m}{2n}}}{B^{\frac{m}{2n}} - \Upsilon^{\frac{m}{2n}}}\right) \text{ et}$$

$$H\grave{}\vartheta = \frac{f}{mr+ms+2nr}\left(\frac{(mr+2nr)\,B^{\frac{m+2n}{2n}}}{B^{\frac{m+2n}{2n}} - \Upsilon^{\frac{m+2n}{2n}}} - \frac{ms\,\Upsilon^{\frac{ms}{2nr}}}{B^{\frac{ms}{2nr}} - \Upsilon^{\frac{ms}{2nr}}}\right)$$

Ex his formulis ut amoueantur B et Υ, ſubſtituatur ex proportione $B:\Upsilon = b^2:y^2$ §. 6. valor ipſius $\Upsilon = \frac{By^2}{b^2}$, tum formulae III. et IV. pro ſolidis exhibitarum prodibunt. Neque eſt, quid miremur in prioribus formulis $H\grave{}L$, $H\grave{}K$ non requiri ſimilem ſubſtitutionem, cum deriventur ex formulis $G\grave{}L$ et GK. In his enim diſtantiae centri gravitatis, non per elementa gravia, ſed per abſciſſas ſolido et figurae communes exprimuntur. Sequitur ex his, quae protulimus, formularum ope, quae, centro gravitatis in aliquo ſolido inveſtigato, repertae ſunt, formulas reperiri poſſe, quae centrum gravitatis in eius ſectione indicant, cui ſive axis ſive curva gravitatis cum ipſo ſolido communis eſt et vice verſa. Haec vero communio axis, ſeu curvae gravitatis poſcit, ut axis, ſeu curva gravitatis ſolidi ordinatas ſectionis bipartiatur, alias axis, aut curva gravitatis ſolido et ſectioni non eſt communis, veluti fig. 7. ubi $\alpha\varrho\delta$ ſolidi αPQS, non ſectionis $\alpha\beta\gamma$ curva grauitatis eſt. At ſi $\beta\gamma$ parallela eſſet lateri PQ; tum $\alpha\varrho\delta$ communis ſolido et ſectioni foret, veluti fig. 11. axis gravitatis $\alpha\delta$ Pyramidi et triangulo $\alpha\beta\gamma$ communis eſt, cum $\beta\gamma$ ſit lateri baſeos parallela.

§. 8.

Quaeritur centrum gravitatis, quum curva gravitatis tranſit in rectam.

I. *De figuris planis.* Curva gravitatis §. 4. in rectam $\alpha\varrho\delta$ fig. 9. transeunte, centra gravitatis g, G figurae $\alpha\beta\gamma$ et partis eius variabilis $\mu\nu\beta\gamma$ puncta ſunt rectae $\alpha\varrho\delta$ magisque expeditum eſt centrorum G et g diſtantias ab α quaeri, quam priorem methodum adhiberi. Ponatur angulus $\alpha\delta\gamma = \varphi$, quem diameter gravitatis $\alpha\delta$ et basis $\beta\gamma$ includunt et erit:

$$\begin{aligned} \alpha\varepsilon &= a = \alpha\delta \,\text{Sin.}\varphi \\ \alpha\zeta &= x = \alpha\varrho \,\text{Sin.}\varphi \\ \varrho\iota &= c = \varrho\delta \,\text{Sin.}\varphi \\ GL &= \alpha G \,\text{Sin.}\varphi \\ G\sigma &= \varrho G \,\text{Sin.}\varphi \end{aligned}$$

itaque $$\alpha G = \frac{GL.}{\text{Sin.}\varphi} = \frac{m+n}{(2m+n)\,\text{Sin.}\varphi} \left(\frac{a^{\frac{2m+n}{m}} - x^{\frac{2m+n}{m}}}{a^{\frac{m+n}{m}} - x^{\frac{m+n}{m}}} \right)$$

secundum formulam I. §. 4. Substitutis vero pro a et x valoribus modo propositis et deletis Sinuum potentiis, quae factorem terminorum communem constituunt, in figurae $\alpha\beta\gamma$ parte variabili $\mu\nu\beta\gamma$ centri gravitatis G distantia ab α reperitur:

$$\alpha G = \frac{m+n}{(2m+n)\,\text{Sin.}\varphi} \left(\frac{\alpha\delta^{\frac{2m+n}{m}} - \alpha\varrho^{\frac{2m+n}{m}}}{\alpha\delta^{\frac{m+n}{m}} - \alpha\varrho^{\frac{m+n}{m}}} \right) \qquad \text{(I.}$$

Est $G\varrho = \frac{G\sigma}{\text{Sin.}\varphi}$ ideoque in formula III. §. 4. loco c ipsius valore $\varrho\delta$ Sin. φ substituto, centri G distantia ab ϱ deprehenditur

$$\varrho G = \frac{\varrho\delta}{n+2m} \left(\frac{(n+m)\, b^{\frac{n+m}{m}}}{b^{\frac{n+m}{n}} - y^{\frac{n+m}{n}}} - \frac{m\, y^{\frac{m}{n}}}{b^{\frac{m}{n}} - y^{\frac{m}{n}}} \right) \qquad \text{(II.}$$

Sequitur ex I., $\alpha\varrho$ evanescente, centri gravitatis g in integra figura $\alpha\beta\gamma$ distantia ab α

$$\alpha g = \frac{(m+n)\;\alpha\delta}{(2m+n)\,\text{Sin.}\varphi} \qquad \text{(III.}$$

Exemplum. Exhibeat $\alpha\beta\gamma$ fig. 9. parabolam Apollonianam, cuius axem BT secet $\beta\gamma$ sub angulo arbitrario $BT\gamma = \varphi$ et quaerantur centra gravitatis G; g figurae $\alpha\beta\gamma$ et partis variabilis $\mu\nu\beta\gamma$.

Sit F focus et parabolae et ducantur parallelae binae, αL et $\alpha\delta$, haec quidem axi BT e medio δ baseos $\beta\gamma$, illa vero basi $\beta\gamma$, per punctum α. Tum αL parabolam tanget eſt atque $\alpha\delta$ tam parabolae, quam gravitatis diameter erit, quippe parallela $\alpha\delta$ ordinatas tangenti αL parallelas, velut $\beta\gamma$ et $\mu\nu$, bipartitur omnes. Sed conſtat eſſe $y = \mu\nu = (4AF\alpha\varrho)^{\frac{1}{2}}$

itaque $y : b = \alpha\varrho^{\frac{1}{2}} : \alpha\delta^{\frac{1}{2}} = x^{\frac{1}{2}} : a^{\frac{1}{2}}$

ex qua ſequitur $m = 2;\ n = 1$, nec non:

$$\alpha G = \tfrac{3}{5}\left\{\frac{\alpha\delta^{\frac{5}{2}} - \alpha\varrho^{\frac{5}{2}}}{\alpha\delta^{\frac{3}{2}} - \alpha\varrho^{\frac{3}{2}}}\right\}$$

$$\varrho G = \frac{\varrho\delta}{5}\left\{\frac{3\,b^3}{b^3 - y^3} - \frac{2\,y^2}{b^2 - y^2}\right\}$$

$$\alpha g = \tfrac{3}{5}\,\alpha\delta$$

II. *De ſolidis.* Curva gravitatis ſolidi alicuius in rectam transeunte, ſectionem ſolidi, plano per diametrum gravitatis poſito, ortam exhibeat. Diſtantiae vero centrorum gravitatis H et h ſolidi integri et partis variabilis, quibus ſectiones $\alpha\beta\gamma$, et $\mu\nu\beta\gamma$ reſpondent ex formulis I. II. III., paullo ſuperius, cum de figuris ageretur, propoſitis, ea, quam §. 7. oſtendimus, methodo derivari poſſunt. Subſtitutis ſcilicet $2n$; Υ; B; pro n; y; b reperitur:

$$\alpha H = \frac{m + 2n}{(2m + 2n)\,\text{Sin.}\varphi}\left(\frac{\alpha\delta^{\frac{2m+2n}{m}} - \alpha\varrho^{\frac{2m+2n}{m}}}{\alpha\delta^{\frac{m+2n}{m}} - \alpha\varrho^{\frac{m+2n}{m}}}\right) \qquad \text{(I.}$$

$$\varrho H = \frac{\varrho\delta}{2m + 2n}\left(\frac{(2n + m)\,B^{\frac{2n+m}{m}}}{B^{\frac{2n+m}{2n}} - \Upsilon^{\frac{2n+m}{2n}}} - \frac{m\,\Upsilon^{\frac{m}{2n}}}{B^{\frac{m}{2n}} - \Upsilon^{\frac{m}{2n}}}\right)$$

ſive introducto valore $\frac{By^2}{b^2}$ areae Υ,

$$\varrho H = \frac{\varrho\delta}{2m + 2n}\left(\frac{(2n + m)\,b^{\frac{2n+m}{n}}}{b^{\frac{2n+m}{n}} - y^{\frac{2n+m}{n}}} - \frac{m\,y^{\frac{m}{n}}}{b^{\frac{m}{n}} - y^{\frac{m}{n}}}\right) \qquad \text{(II.}$$

et ex I. evanescente $\alpha\varrho$, $\alpha h = \frac{(m + 2n)\,\alpha\delta}{(2n + 2m)\,\text{Sin.}\varphi}$. (III.

Expeditius formula II. pro αH ex formula distantiae αG superius proposita reperta esset, substituto $2n$ loco n ubique, nisi in denominatoribus exponentium in quibus n non cum $2n$ commutandum est.

Exemplum 1. Neiliani paraboloidis elementa ita translocentur, eo, quem supra §. 6. descripsimus modo, ut oriatur solidum $\alpha\beta\gamma\psi$ fig. 10. et axis recta quidem maneat, vertatur tamen in obliquam $\alpha\varrho\delta$, erit:

$$\mu\nu : \beta\gamma = \alpha\varrho^{\frac{3}{2}} : \alpha\delta^{\frac{3}{2}} = x^{\frac{3}{2}} : a^{\frac{3}{2}}, \text{ itaque } m = 2;\ n = 3, \quad \text{et}$$

$$\alpha H = \tfrac{4}{5}\left(\frac{\alpha\delta^5 - \alpha\varrho^5}{\alpha\delta^4 - \alpha\varrho^4}\right)$$

$$\varrho H = \frac{\varrho\delta}{5}\left(\frac{4b^{\frac{8}{3}}}{b^{\frac{8}{3}} - y^{\frac{8}{3}}} - \frac{y^{\frac{2}{3}}}{b^{\frac{2}{3}} - y^{\frac{2}{3}}}\right)$$

$$\alpha h = \tfrac{4}{5}\,\alpha\delta$$

Exemplum 2. Pyramidis obliquae fig. 11. haec est natura

$$\mu\nu : \beta\gamma = \alpha\varrho : \alpha\varepsilon$$

i. e.

$$y : b = \frac{x^1}{\text{Sin.}\,\varphi} : \frac{a^1}{\text{Sin.}\,\varphi}$$

ergo $m = n = 1$ et

$$\alpha H = \tfrac{3}{4}\left(\frac{\alpha\delta^4 - \alpha\varrho^4}{\alpha\delta^3 - \alpha\varrho^3}\right)$$

$$\varrho H = \frac{\varrho\delta}{4}\left(\frac{3b^3}{b^3 - y^3} - \frac{y}{b - y}\right)$$

$$\alpha h = \tfrac{3}{4}\,\alpha\delta.$$

Notetur diametrum gravitatis solidi obliqui non communem esse sectioni, quae oritur, plano per hanc diametrum posito, nisi sectionis elementa, seu ordinatas in partes aequales dividat, quibus colligitur communem esse semper, quando basis est circulus.

§. 9.

Propositorum latius, atque hic ostendimus, usus patet et quaestiones similes iis, quas de parabolarum familia habuimus, ad formulam sectionum conicarum generalem aliasque ad curvas transferri possunt, sed istae disquisitiones, licet operae pretium soluturae, longius tamen nos abducerent, quam libelli huius sive postulet ratio, sive fines patiantur.

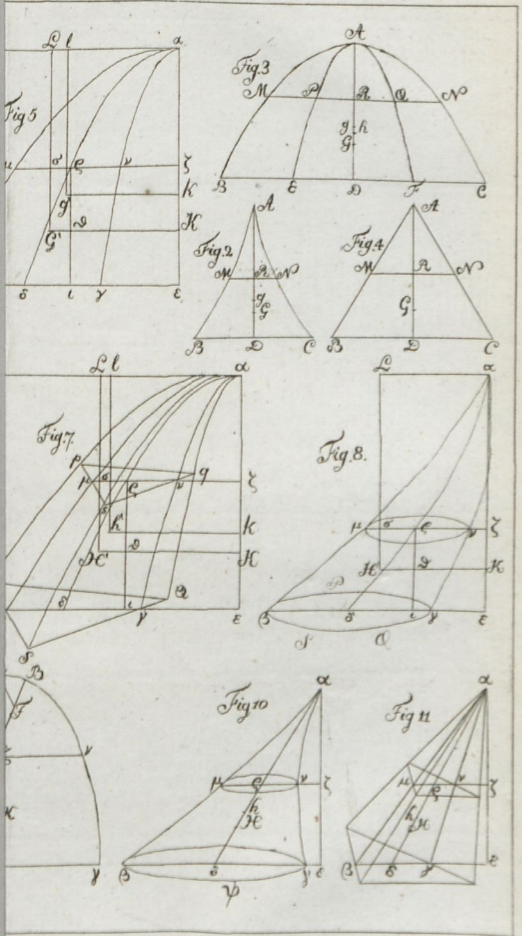


Fig.5
Fig.3
Fig.2
Fig.4
Fig.7
Fig.8.
Fig.10
Fig.11

THESES.

I. *Sylvas ſuas excidere poſſeſſoribus non quocunque tempore licet.*

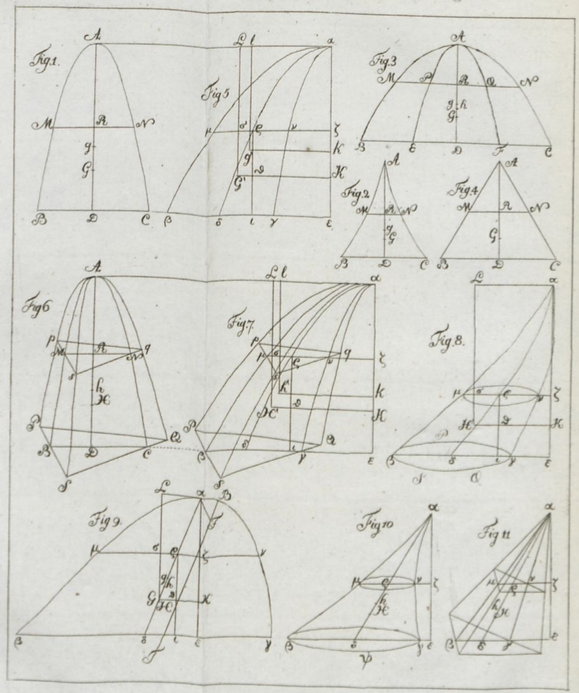
II. *In computando interuſurio legibus uſurae uſurarum nequeunt prohiberi.*

III. *Apparatibus literariis aliter a privato homine, aliter a republica proſpiciendum eſt.*

IV. *Atomicum ſyſtema poſtponendum eſt dynamico.*

V. *Vera interdum ex falſis ſequi videntur, nunquam ſequuntur.*

VI. *Gravioribus incommodis aliae eruditionis partes laborant, quam Matheſis.*



CORRIGENDA.

Pag.	Lin.	Errata	Corrige
3.	2.	$y^{\frac{\beta}{\ }}$	$\gamma y^{\frac{\gamma}{\alpha}}$
3.	6.	$\gamma y^{\frac{\beta}{\alpha}}$	$\gamma y^{\frac{\gamma}{\alpha}}$
3.	4. a fine	$(\alpha\ \beta+\gamma) b^{\frac{\gamma}{\alpha}} y^{\frac{\beta-\gamma}{\alpha}}$	$(\alpha+\beta-\gamma) b^{\frac{\gamma}{\alpha}} y^{\frac{\beta-\gamma}{\alpha}}$
4.	16.	G et g	g, G
6.	8. a fine	$x^{\frac{m}{n}}$	$y^{\frac{m}{n}}$
6.	5. a fine	$b^{\frac{n+m}{m}}$	$b^{\frac{n+m}{n}}$
7.	3.	RD	AD
9.	11. a fine	αL	$\alpha \varepsilon$
10.	8. a fine	denique	provenit denique
10.	5. a fine	dimidio	dimidia
11.	3. a fine	$G`k$	$G`K$
15.	13.	H	AH
15.	3. a fine	$\alpha : x'$	$\alpha^1 : x^1$
16.	9.	ab	Ab
16.	11.	$y=a$	$y=b$
16.	7. a fine	figuris	figuris planis
18.	3. a fine	$x=o$	$x=a$
19.	11.	$\varrho\iota=f$	$\delta\iota=f$
19.	6. a fine	$H`k$	$H`K$

Caeterum monemus in libello nostro *de Ellipseos Evolutis et Aequidistantibus* p. 7. su finem excidisse

$$LE=\frac{a^2-b^2}{b}=e=VD$$

CORRIGENDA.

Pag.	Lin.	Errata	Corrige
3.	2.	$y^{\frac{\beta}{}}$	$\gamma y^{\frac{\gamma}{\alpha}}$
3.	6.	$\gamma y^{\frac{\beta}{\alpha}}$	$\gamma y^{\frac{\gamma}{\alpha}}$
3.	4. a fine	$(\alpha\ \beta+\gamma)\, b^{\frac{\gamma}{\alpha}}\, y^{\frac{\beta-\gamma}{\alpha}}$	$(\alpha+\beta-\gamma)\, b^{\frac{\gamma}{\alpha}}\, y^{\frac{\beta-\gamma}{\alpha}}$
4.	16.	G et g	g, G
6.	8. a fine	$x^{\frac{m}{n}}$	$y^{\frac{m}{n}}$
6.	5. a fine	$b^{\frac{n}{m}+\frac{m}{m}}$	$b^{\frac{n}{n}+\frac{m}{n}}$
7.	3.	*RD*	*AD*
9.	11. a fine	αL	$\alpha\varepsilon$
10.	8. a fine	denique	provenit denique
10.	5. a fine	dimidio	dimidia
11.	3. a fine	*Gk*	*GK*
15.	13.	*H*	*AH*
15.	3. a fine	$\alpha : x'$	$\alpha^1 : x^1$
16.	9.	*ab*	*Ab*
16.	11.	$y=a$	$y=b$
16.	7. a fine	figuris	figuris planis
18.	3. a fine	$x=o$	$x=a$
19.	11.	$\varrho\iota=f$	$\varepsilon\iota=f$
19.	6. a fine	*H'k*	*H'K*

Caeterum monemus in libello nostro *de Ellipseos Evolutis et Aequidistantibus* p. 7. sub finem excidisse

$$LE=\frac{a^2-b^2}{b}=e=VD$$

EXPOSITIO
QVARVNDAM FORMVLARVM
DE
CENTRO GRAVITATIS

QVAM
AMPLISSIMI PHILOSOPHORVM ORDINIS
AVCTORITATE
PRO LOCO
IN EO RITE OBTINENDO
D. XXI. AVGVSTI MDCCXCIX
H. L. Q. C.
PVBLICE DEFENDET

MAVRICIVS DE PRASSE
PHIL. DOCT. ET IN VNIVERS. LIPS. MATH. PROF. PVBL. ORDIN.
RESPONDENTE
TOUSSAINT DE CHARPENTIER
EQVITE FRIBERGA - MISNICO

ADIECTA EST TABVLA AENEA.

LIPSIAE
IMPRESSIT CAROLVS TAVCHNITZ.