INFINITINOMII AD POTENTIAM INDEFINITAM ELEVATI FORMVLA

EDIT SIMVLQVE

PRAELECTIONES SVAS INDICAT

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GOTTINGAE

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LECTVRIS

Theorema binomiale vniuersaliter demonstraui in scripto quod anno proxime elapso prodiit. Methodo qua tunc vsus sum, vidi legem qua potentia quaeuis infinitinomii formatur, facile erui & rigorose ostendi posse. IACOBVS BERNOVLLIVS Op. T. II. n. 103. art. 1. modos duos id perficiendi tradit. Sed ille quem alteri praefert quod legem progressus melius ante oculos ponat, summationes postulat; cum mea, ipsis serierum coefficientibus contineatur. Quae alii habent, vt L. B. de WOLF Analys. §. 102. DE MOIVRE Trans. n. 230; & ex eo COLSON Comm. in NEWT. Meth. Flux. p. 312. ad legem progressus accurate & generaliter demonstrandam vbi inductione contenti esse nolumus, satis idonea non videntur, ceterum facile possunt ex meis (14) deduci.

PRO-

PROBLEMA

SERIEI coefficientium datorum, $1 + \alpha z + \beta z^2 + \gamma z^3 + \delta z^4 + \varepsilon z^5 \ldots$ quaeritur potentia m;

SOL. 1) Series ipſa ſtatuatur $= 1 + y$; Sit vero alia ſeries, coefficientium quaeſitorum; $1 + Az + Bz^2 + Cz^3 + Dz^4 + Ez^5 \ldots$ quae dicatur w; & ponatur $(1+y)^m = w$; adeoque $m.(1+y)^{m-1} dy = dw$ ſeu $mwdy = (1+y)\, dw$

3) Iam $dy = (\alpha + 2\beta z + 3\gamma z^2 \ldots)\, dz$; $dw = (A + 2Bz + 3Cz^2 \ldots)\, dz$ vnde.

$$4)\ (1+y)\frac{dw}{dz} = \begin{array}{lllll} A & + A\alpha z & + A\beta z^2 & + A\gamma z^3 & + A\delta z^4 \ldots \\ + 2B & + 2B\alpha & + 2B\beta & + 2B\gamma & \\ & + 3C & + 3C\alpha & + 3C\beta & \\ & & + 4D & + 4E\alpha & \end{array}$$

$$5)\ \frac{mw\, dy}{dz} = \begin{array}{lllll} m\alpha & + 2m\beta z & + 3m\gamma z^2 & + 4m\delta z^3 & + 5m\varepsilon z^4 \ldots \\ & + mA\alpha & + 2mA\beta & + 3mA\gamma & + 4mA\delta \\ & & + mB\alpha & + 2mB\beta & + 3mB\gamma \\ & & & + mC\alpha & + 2mC\beta \\ & & & & + mD\alpha \end{array}$$

6) Aequando terminos homogeneos 4; 5; eſt

$A = m\alpha$; $B = \frac{(m-1)A\alpha + 2m\beta}{2}$; $C = \frac{(m-2)B\alpha + (2m-1)A\beta + 3m\gamma}{3}$

$$D = \frac{(m-3)C\alpha + (2m-2)B\beta + (3m-1)A\gamma + 4m\delta}{4}$$

$$E = \frac{(m-4)D\alpha + (2m-3)C\beta + (3m-2)B\gamma + (4m-1)A\delta + 5m\varepsilon}{5}$$

8) Hinc

7) Hinc profpicitur fore

$$F = \frac{(m-5)E\alpha + (2m-4)D\beta + (3m-3)C\gamma + (4m-2)B\delta + (5m-1)A\varepsilon + 6m\zeta}{5}$$

8) Vt formula generalis commode eruatur; dic feriei $1+y$ coefficientem quemuis λ, addito eius numero, vt fit $\lambda 1; = \alpha$ & $\lambda 2; = \beta$ & $\lambda n;$ fit qui pertinet ad z^n & $\lambda n-1;$ qui pertinet ad z^{n-1} & fic porro. Similiter fit L coefficiens quiuis feriei w, addito numero, vt fit $L1; = A$ & $L2; = B$ & $Ln; Ln-1;$ pertineant ad z^n; $z^{n-1};$ hoc pofito erit

$$1+y = 1 + \lambda 1; z \ldots\ldots + \lambda r; z^r \ldots\ldots + \lambda n; z^n$$

$$w = 1 + L1; z \ldots\ldots + Lr; z^r \ldots\ldots + Ln; z^n$$

$$\frac{dy}{dz} = \lambda 1; \ldots\ldots\ldots\ldots + r.\lambda r; z^{r-1} \ldots\ldots + n.\lambda n; z^{n-1}$$

$$\frac{dw}{dz} = L1; \ldots\ldots + r.Lr;.z^{r-1} \ldots\ldots + n.Ln.z^{n-1}$$

9) *Quaeritur coefficiens producti* $(1+y)\frac{dw}{dz}$ *ad* z^n?

Termini feriei $\frac{dw}{dz}$ in quibus variabilis eleuata eft ad potentiam $n; n-1; n-2; \ldots n-r; \ldots 1;$ ducti refpectiue in terminos feriei $1+y$ qui habent variabilem eleuatam ad potentiam $0; 1; 2; \ldots r; n$ praebent facta continentia z^r. Igitur fumma coefficientium horum factorum eft coefficiens quaefitus, qui adeo fit

$$1.L1.\lambda n + 2.L2;.\lambda n-1; + 3.L3;.\lambda n-2; \ldots + (n-r+1).Ln-r+1;.\lambda r \ldots\ldots + n.Ln;.\lambda 1; + (n+1).Ln+1;.1$$

10) *Quaeritur coefficiens producti* $\frac{w\,dy}{dz}$ *ad* z^n?

Componitur is eodem modo ex seriei $\frac{dy}{dz}$ coefficientibus qui pertinent ad potentias n; n-1; .... n-r .... o ductis in eos seriei w, qui pertinent ad o; 1; .... r .... n; estque adeo

$(n+1) . \lambda n+1; . 1+n. \lambda n; L1 + (n-1). \lambda n-2; . L2 ......$
$+ r. \lambda r; . Ln-r+1; ..... + 2. \lambda 2; . Ln-1; + \lambda 1; . Ln;$

11) Singulae partes coefficientis (10) multiplicentur per m; quod prodit aequetur coefficienti (9) & hinc eruatur Ln + 1; (6) fiet

$$(n+1). Ln+1; = (m-n). Ln; . \lambda 1 + (2m-n+1). Ln-1; . \lambda 2;$$
$$..... + (m. r-n+r-1). Ln-r+1; . \lambda r; . .$$
$$..... + (m. n-1). L1; . \lambda n; + m. (n+1). \lambda n+1;$$

12) Eo igitur quod hic est ad partem dextimam signi aequalitatis, diuiso per n + 1 habetur Ln + 1; per datos ab initio, seriei 1 + y, & iam inuentos, praecedentes seriei w. Q. E. I.

13) Huius ipsius coefficientis *generalis* (12) *pars generalis* est $\frac{(mr-n+r-1). Ln-r+1; . \lambda r}{m}$ quae substituendo successiue 1; 2; .... n + 1 in locum ipsius r, praebet partes coefficientis, primam, secundam, ... vltimam; Nam Lo; est = 1 Si L1 = A quia 1 praecedit coefficientem A quem seriei w primum appellaui, coefficientes vero hanc vnitatem

prae-

praecedentes, cum nulli ſint, erit L - 1; = 0; L - 2; = 0 & ſic porro.

14) Si eliminentur Seriei w coefficientes, reperitur

$B = \frac{m.(m-1)}{1.\ 2.} \alpha^2 + m\beta$

$C = \frac{m.(m-1).(m-2)}{1.\ 2.\ 3.}.\ \alpha^3 + m.(m-1).\ \alpha\beta + m\gamma$

Sed expreſſio cuiuſuis coefficientis ſeriei w per praecedentes, & legem progreſſus melius ante oculos ponit, & praxi aptior videtur.

15) Haec facile ad caſus particulares applicantur. Ita pro *trinomio* fit $\gamma = \delta = \varepsilon = 0$ hinc ex (12)

$Ln+1; = (m-n).\ Ln;\ .\alpha + (2m-n+1).\ Ln-1;\ .\ \beta$
$+ (3m-n+2)\ Ln-2,\ .\ \gamma) : (n+1)$

Hic coefficiens ipſe primus ſeu A, habet $n = 0$ & ex (13) fit $L1 = m.\alpha$ ſimiliter ſecundus & tertius reperiuntur, plane quales illos reperimus (6).

16) Alia determinatio eſt *indicis*, *manente indefinito numero partium radicis*. Ita ſi $m = 2$; coefficiens generalis quadrati infinitomii fit

$((2-n).\ Ln;\ .\ \lambda1; + (5-n)\ Ln-1;\ .\ \lambda2; + (8-n)\ Ln-2;\ .\ \lambda3\ \ldots + (3r-1-n).\ Ln-r+1;\ .\ \lambda r\ \ldots + 2.(n+1).\ \lambda n+1;) : (n+1).$

17) Pro quadrato trinomii (15) coefficiens hic generalis (16) abit in $\frac{(2-n).\ Ln;\ .\ \alpha + (5-n).\ Ln-1;\ .\ \beta}{n+1}$. Qui, in locum ipſius n poſitis ſucceſſiue 0; 1; 2; 3; 4; 5; dat

A =

$A = 2\alpha$; $B = \frac{A\alpha + 4\beta}{2} = \alpha^2 + 2\beta$, $C = \frac{0 + 3A\beta}{3} = 2\alpha\beta$;
$D = \frac{-C\alpha + 2B\beta}{4} = \beta^2$; $E = \frac{-2D\alpha + C\beta}{5} = 0$
$6F = -3.E + 0.D = 0$; deinde pro $n \geq 5$ euanescunt tam Ln; quam $Ln-1$; vnde habetur trinomii quadratum vt calculo consueto.

18. Hoc autem exemplum monstrat, quomodo pro multinomio definito, scripto integro positiuo in locum ipsius m, series abrumpatur, coefficientibus (16) euanescentibus.

19. Detur series ⊙) $\pi u^e + \alpha u^{e+w} + \beta u^{e+2w} + \gamma u^{e+3w} \ldots$ quae resoluitur in $\pi u^e . (1 + \alpha u^w + \beta u^{2w} + \gamma u^{3w} \ldots) = \pi z^{\frac{e}{w}}. (1 + \alpha z + \beta z^2 + \gamma z^3 \ldots.)$ vel $\pi z^{\frac{e}{w}}. (1 + y)$ si loco u scribatur $z^{\frac{1}{w}}$. Data vero ex (12) potentia indefinita seriei $1 + y$; datur eadem potentia ipsius $\pi z^{\frac{e}{w}}. (1 + y)$ & adeo seriei ⊙; Proinde methodus tradita complectitur series omnes in quibus exponentes potentiarum variabilis constituunt seriem arithmeticam. Notum vero est, series omnes eo reduci posse.

20. Si seriei ⊙ exponentium vnus vel plures sint negatiui, erunt e, w, ambo vel alteruter horum numerorum negatiui, & hinc determinabitur $z^{e:w}$, sed series $1 + y$ hac negatiui conditione non adficietur vt nec illa vniuersalitati methodi noceat. Haec explicare operae precium erat quia IAC. BERNOVLLIVS loco quem in praefamine citaui, dubitat an series $1 + y$ satis generalis sit, minus recte, vt iam CRAMERVS in notis monuit.

Prae-

Praelectiones studiosae iuuentuti proxima aestate V. D. offero sequentes.

I. PVBLICAS in *physicam* cel. WINKLERI quatuor singulis septimanis horis.

II. PRIVATAS in *elementa mea arithmeticae & geometriae.*

III. Alias in *Algebrae* elementa *mea* ex bibliopolio Vandenhoekiano proditura.

IIII. DISPVTATIONES philosophicas cum selectioribus ingeniis, quas hucusque summa cum voluptate institui, continuabo.

Horae e valuis indicabuntur.

Dab. Gottingae festo Paschatos A. Aerae Christianae MDCCLVIIII.

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