DISPUTATIO MATHEMATICA
DE
SUBSECANTIBUS LINEARUM SECUNDI ORDINIS.

QUAM
RECTORE
UNIVERSITATIS EBERHARDINAE CAROLINAE
MAGNIFICENTISSIMO
SERENISSIMO atque POTENTISSIMO DUCE ac DOMINO
DOMINO
CAROLO,
DUCE WIRTEMBERGIÆ ET TECCIÆ REGNANTE
rel. rel.
CONSENTIENTE AMPLISSIMO PHILOSOPHORUM ORDINE
PRÆSIDE
VIRO EXCELLENTISSIMO atque DOCTISSIMO
DOMINO
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PRAECEPTORE AC PATRONO SUO PIE DEVENERANDO
PRO GRADU MAGISTRI SIVE DOCTORIS PHILOSOPHIAE
RITE CONSEQUENDO
DIE AUGUSTI ANNI MDCCLXXIX.
PUBLICE DEFENDET
AUCTOR
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PATRONO AC FAVTORI SVO COLENDISSIMO,

QVALECVNQVE HOC STVDIORVM SVORVM SPECIMEN

EA, QVA PAR EST, ANIMI DEVOTIONE

DICARE SE SVAQVE OBSEQVIOSISSIME COMMENDARE VOLVIT

AVCTOR.

§. I.

Subiit me aliquando, generationem variarum conoidum sphæroidumque animo pensitantem, cogitatio, nonne, si tales in illis applicarentur sectiones, quales in cono regulari, novæ ea ratione prodirent determinarique possent lineæ curvæ? Sectiones omnes possibiles in corporibus, ex terra lutosa in hunc finem a me confectis, institui, sed quid harum sola contemplatione confici potuit? Cum igitur extrema harum sectionum puncta geometrice determinare aggressus essem, graviter offendi in linea extra curvam generantem indeterminate ab axe aut diametro aliqua per prolongatam chordam sive secantem desecta. Satis diu hoc in problemate hærens recordatus sum animadversionis, quam in lectione Celeberrimi *de la* CHAPELLE in methodum ejus universalem a) determinandi rationem tangentium curvarum linearum ad quantitates cognitas, feceram, ex qua mihi lux illico oriebatur.

§. II.

Etenim recte ille quidem considerat tangentes uti secantes, quorum puncta intersectionis in uno curvæ puncto confluunt, sed

 in

a) In libro ipsius (*Abhandlung von den Kegelschnitten*) e gallica lingua transposito, pag. 82, 83, 278, 400, 401.

in applicatione hujus theſeos non parum offendit. Putat ſecantis *TMD* fig. I. puncta interſectionis *M* & *D*, ſi *TMD* in *T* quaſi centro moveatur verſus ſiniſtram, ita confluere, ut *D*, quod movendo magis magisque accedit ad *M*, tandem cadat in *M*, adeoque, ducta ordinata *MP*, ſubtangens prodeat $= PT$. Sed fallit omnino; neque enim hac ratione confluent puncta, neque ſubtangens erit $= PT$, ſed potius illa in puncto aliquo intermedio *N* confluent; hæc autem erit $= TX$. Primum enim, ſi movetur *TMD* circa *T*, omnia ejus moventur puncta excepto centro *T*, adeoque etiam movetur punctum interſectionis *M*, & quidem appropinquabit verſus *D*, & *D* verſus *M*, usque dum in vertice *N* partis curvæ ſummo coincidant: deinde, ſi in *M* eſſet confluxus, ſequeretur, quod punctum *M* eſſet locus, ubi & tangens a *T* ducta tangat, & ſecans ab eodem puncto *T* ducta ſecet curvam, adeoque *TM* & tangentis & ſecantis pars ſimul eſſet, quod omnino eſt impoſſibile. Idem error in Ellipſi & Hyperbola pag. 278, 400, 401. manifeſto eſt commiſſus.

§. III.

Aſſumendum mihi potius hic videbatur centrum in puncto *M*, *b*) in quo linea ſecans *TMD* a *T* verſus dextram *A* ſe vertat, usque dum ceſſet ſecare curvam, hoc eſt, donec in tangentem tranſeat, quæ in puncto *M* erit applicata. Patebit cadere movendo *D* in *M*, eodemque tempore quieſcere *T* in *G*, adeoque, ducta ordinata *MP*, prodire ſubtangentem $= GP$. Sit $PB = d$; $AP = x$; $PT = ſ$; $TG = \xi$; Calculus hoc modo procedet:

BD²

b) Æquipolleret eademque veritas prodiret, ſi centrum aſſumeretur aut in *T*, aut in *M*, aut in *D*; ſed *M* hic magis ſe nobis præſtabit idoneum.

$BD^2 : PM^2 = x + d : x$

$BD^2 : PM^2 = (ſ + d)^2 : ſ^2$ adeoque

$x + d : x = (ſ + d)^2 : ſ^2$ ſive

$d : x = 2ſd + d^2 : ſ^2$ Ergo

$dſ^2 = 2ſdx + d^2x$

$ſ^2 = 2ſx + dx$; $R)$

Eo ipſo autem momento, quo *TMD* fiet tangens, evaneſcet d, adeoque $d = 0$, & ſimul erit $ſ = ſ - \xi$. In æquatione R igitur notatis hisce mutationibus, mutando $ſ$ in $ſ - \xi$ delendoque id, quod ductum eſt in $d = 0$; erit

$(ſ - \xi)^2 = 2x . \overline{ſ - \xi}$ adeoque

$ſ - \xi = 2x.$

§. IV.

Et hic ſponte ſe mihi obtulit linea illa indeterminate in *T* a ſecante *TMD* deſecta, in cujus determinatione hæc noſtra verſetur diſputatio.

§. V.

Sine demonſtratione patet, tangentem *GMD* in ſuperiore puncto *M* interſectionis a ſecante factæ minorem abſcindere axis prolongati partem, quam ſecans *TMD* abſcindit; majorem autem partem aſcindere tangentem *FD*, quæ ad inferius interſectionis punctum *D* ducta eſt. Hæ autem portiones axis prolongati, quas tangentes atque ordinatæ, ab eodem curvæ puncto ductæ, intercipiunt, vocantur ſubtangentes. Eodem igitur jure, quo lineæ *FB* & *GP* ſubtangentes nuncupantur, nuncupem lineas *TP* & *TB* SUBSECANTES & quidem *TB* ſubſecantem majorem, *TP* autem minorem, & ſic etiam lineam *TMD* ſecantem majorem, *TM* autem minorem, quamquam quidem TM proprie nuncupari ſecans non poteſt, tamen habet directionem ad ſecantem & majoris pars eſt: aut ut brevitati ſtudeamus, ſit Subſecans Subtangens

Subſecans		Subtangens
	Major = ſubſ. α	Maj. = ſubt. α
	Minor = ſubſ. β	Min. = ſubt. β
	Secans	Tangens
	Maj. = ſec. α	Maj. = tang. α.
	Min. = ſec. β	Min. = tang. β.

VI.

Faciamus itaque periculum, nonne determinari poſſit harum ſubſecantium ratio ad quantitatem cognitam? Sit $DB = z$; $MP = y$; $AP = x$; $AB = w$; $TG = \xi$; Parameter axis $= p$.

y ſemper minor erit, quam z; z itaque conſiderari poteſt, uti unitas, cujus partes quasdam continet y, ſive y ſemper = fractioni τϰ z, quam exprimam per $\frac{n}{m}z$; n ſcilicet $< m$ aſſumto: m & n etiam exprimant numeros irrationales; in genere qualemcunque rationem τϰ y ad το z. Conſtat eſſe $z^2 = wp$ & $y^2 = px$; ergo $px = \frac{n^2}{m^2}z^2 = \frac{n^2}{m^2}wp$, adeoque $x = \frac{n^2}{m^2}w$. Hoc eſt x ſemper = fractioni τϰ z, y exprimenti, in ſe ductæ & multiplicatæ cum abſciſſa majoris ordinatæ; ſive, ſi minor ordinata eſt pars r^{ta} majoris ordinatæ, tum minor abſciſſa eſt pars r^{2ta} majoris abſciſſæ. Determinata itaque ratione τϰ x ad w, habemus ſimul rationem ſubtangentis ad το w, quæ eſt $= 2x$ vid. §. VI. adeoque $= \frac{2n^2}{m^2}w$. Conſideremus nunc triangula ſimilia TMP & TDB erit

$z : y =$ subſ. α : subſ. β. Eſt autem $y = \frac{n}{m}z$; ergo etiam subſ. $\beta = \frac{n}{m}$ subſ. α;

$$\text{subſ. } \alpha = w + \frac{n^2}{m^2}w + \xi = \frac{m^2 + n^2}{m^2}w + \xi \quad \text{A)}$$

$$\frac{n}{m} \text{subſ. } \alpha = \frac{2n^2}{m^2}w + \xi;$$

$$n \text{ subſ. } \alpha = \frac{2n^2}{m}w + m\xi$$

$$\text{subſ. } \alpha = \frac{2n}{m}w + \frac{m}{n}\xi \quad \text{B)}$$

Subſtituto valore pro subſ. α ex æquatione A in B eſt

$$\frac{m^2 + n^2}{m^2}w + \xi = \frac{2n}{m}w + \frac{m}{n}\xi$$

$$\frac{m}{n}\xi - \xi = \frac{m^2 + n^2}{m^2}w - \frac{2n}{m}w \quad \text{ſive}$$

$$\frac{m - n}{n}\xi = \frac{(m - n)^2}{m^2}w$$

$$\xi = \frac{(m - n)n}{m^2}w.$$

Et ſic determinavimus partes conſtituentes subſ. β; nempe $\xi + 2x$. Adeoque

$$\text{subſ. } \beta = \frac{2n^2}{m^2}w + \frac{(m - n)n}{m^2}w = \frac{mn + n^2}{m^2}w$$

$$= \frac{(m + n)n}{m^2}w.$$

Subſ.

Subſ. $\alpha = PB +$ ſubſ. β; $PB = w - x = \frac{m^2 - n^2}{m^2} w$;

Itaque ſubſ. $\alpha = \frac{m^2 - n^2}{m^2} w + \frac{(m+n)n}{m^2} w$

$$= \frac{m^2 + mn}{m^2} w = \frac{m+n}{m} w.$$

§. VII.

Exemplum I.

$y = \frac{n}{m} z$; $x \frac{n^2}{m^2} w$; $\xi = \frac{\overline{m-n}.n}{m^2} w$; ſubſ. $\beta = \frac{\overline{m+n}.n}{m^2} w$; ſubſ. $\alpha = \frac{m+n}{m} w$.

Sit y	& erit x	ξ	ſubſ. β	ſubſ. α
‖	‖	‖	‖	‖
$\frac{1}{2} z$;	$\frac{1}{4} w$;	$\frac{1}{4} w = x$;	$\frac{3}{4} w = 3x$;	$\frac{3}{2} w = 6x$;
$\frac{1}{3} z$;	$\frac{1}{9} w$;	$\frac{2}{9} w = 2x$;	$\frac{4}{9} w = 4x$;	$\frac{4}{3} w = 12x$;
$\frac{1}{4} z$;	$\frac{1}{16} w$;	$\frac{3}{16} w = 3x$;	$\frac{5}{16} w = 5x$;	$\frac{5}{4} w = 20x$;
$\frac{1}{5} z$;	$\frac{1}{25} w$;	$\frac{4}{25} w = 4x$;	$\frac{6}{25} w = 6x$;	$\frac{6}{5} w = 30x$;
$\frac{1}{6} z$;	$\frac{1}{36} w$;	$\frac{5}{36} w = 5x$;	$\frac{7}{36} w = 7x$;	$\frac{7}{6} w = 42x$;
$\frac{1}{m} z$;	$\frac{1}{m^2} w$;	$(m - 1)x$;	$(m + 1)x$;	$m(m + 1)x$;
$\frac{1}{S} z$;	$\frac{1}{S^2} w$;	$(S - 1)x$;	$(S + 1)x$;	$(S^2 + S)x$.

Exemplum

Exemplum II.

Sit y	& erit x	ξ	subs. β	subs. α
$\frac{1}{2}z$;	$\frac{1}{4}w$;	$\frac{1}{4}w = x$;	$\frac{3}{4}w = 3x = (2+\frac{1}{1})x$;	$\frac{3}{2}w = 6x = (2+\frac{4}{1})x$.
$\frac{2}{3}z$;	$\frac{4}{9}w$;	$\frac{2}{9}w = \frac{x}{2}$;	$\frac{10}{9}w = \frac{5}{2}x = (2+\frac{1}{2})x$;	$\frac{5}{3}w = \frac{15x}{4} = (2+\frac{7}{4})x$.
$\frac{3}{4}z$;	$\frac{9}{16}w$;	$\frac{3}{16}w = \frac{x}{3}$;	$\frac{21}{16}w = \frac{7}{3}x = (2+\frac{1}{3})x$;	$\frac{7}{4}w = \frac{28x}{9} = (2+\frac{10}{9})x$.
$\frac{4}{5}z$;	$\frac{16}{25}w$;	$\frac{4}{25}w = \frac{x}{4}$;	$\frac{36}{25}w = \frac{9}{4}x = (2+\frac{1}{4})x$;	$\frac{9}{5}w = \frac{45x}{16} = (2+\frac{13}{16})x$.
$\frac{5}{6}z$;	$\frac{25}{36}w$;	$\frac{5}{36}w = \frac{x}{5}$;	$\frac{55}{36}w = \frac{11}{5}x = (2+\frac{1}{5})x$;	$\frac{11}{6}w = \frac{66x}{25} = (2+\frac{16}{25})x$.
$\frac{6}{7}z$;	$\frac{36}{49}w$;	$\frac{6}{49}w = \frac{x}{6}$;	$\frac{78}{49}w = \frac{13}{6}x = (2+\frac{1}{6})x$;	$\frac{13}{7}w = \frac{91}{36}w = (2+\frac{19}{36})x$.
$\frac{n}{n+1}z$;	$\frac{n^2}{(n+1)^2}$;	$\frac{x}{n}$;	$\left(2+\frac{1}{n}\right)x$;	$\left(2+\frac{3n+1}{n^2}\right)x$.

Exemplum III.

Sit y	& erit x	ξ	subs. β	subs. α
$\frac{1}{\sqrt{2}}z$;	$\frac{1}{2}w$;	$\frac{\sqrt{2}-1}{2}w$;	$\frac{\sqrt{2}+1}{2}w$;	$\frac{\sqrt{2}+1}{\sqrt{2}} = \frac{\sqrt{2}+2}{2}w$.
$\frac{\sqrt{2}}{\sqrt{3}}z$;	$\frac{2}{3}w$;	$\frac{\sqrt{6}-2}{3}w$;	$\frac{\sqrt{6}+2}{3}w$;	$\frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{6}+3}{3}w$.
$\frac{\sqrt{3}}{2}z$;	$\frac{3}{4}w$;	$\frac{2\sqrt{3}-3}{4}w$;	$\frac{2\sqrt{3}+3}{4}w$;	$\frac{\sqrt{3}+2}{2} = \frac{3{,}73205\ldots}{2} = 1{,}8660254\ldots w$.
$\frac{\sqrt{r}}{\sqrt{(r+1)}}z$;	$\frac{r}{r+1}w$;	$\frac{\sqrt{(r^2+r)}-r}{r+1}w$;	$\frac{\sqrt{(r^2+r)}+r}{r+1}w$;	$\frac{\sqrt{(r+1)}+\sqrt{r}}{\sqrt{r+1}}w = \frac{\sqrt{(r^2+r)}+r+1}{r+1}\cdot w$.

Exem-

Exemplum IV.

$$\text{Sit} = \frac{n}{m} = \frac{\sqrt{128}}{\sqrt{200}}z = \frac{\sqrt{16}\cdot\sqrt{8}}{\sqrt{25}\cdot\sqrt{8}}z = \tfrac{4}{5}z;$$ reliqua vid. Ex. II.

$$\text{Sit} = \frac{\sqrt{35}}{\sqrt{40}}z = \frac{\sqrt{7}\cdot\sqrt{5}}{\sqrt{8}\cdot\sqrt{5}}z = \frac{\sqrt{7}}{\sqrt{8}}z = \frac{\sqrt{56}}{8}z;$$ & erit

$$x = \tfrac{56}{64}w;\ \xi = \frac{(8-\sqrt{56})\sqrt{56}}{8\cdot 8}w = \frac{\sqrt{56}-7}{8}w = \frac{2\sqrt{14}-7}{8}w$$

$$\text{Subſ. } \beta = \frac{2\sqrt{14}+7}{8}w;\ \text{ſubſ. } \alpha = \frac{8+\sqrt{56}}{8}w = \frac{4+\sqrt{14}}{4}w.$$

$$\text{Sit} = \frac{\sqrt[3]{63}}{\sqrt{63}}w = \frac{63^{\frac{1}{3}}\cdot 63^{\frac{1}{2}}}{63}w = 63^{-\frac{1}{6}}\cdot w = \frac{1}{\sqrt[6]{63}}w;$$ & erit

$$x = \frac{1}{63^{\frac{1}{3}}}w = \frac{1}{\sqrt[3]{63}}w;\ \xi = \frac{\sqrt[6]{63}-1}{\sqrt[6]{63}\cdot\sqrt[6]{63}}w;$$ atque numeratore & denominatore ducto in $\sqrt[3]{63^2}$; $= \frac{\sqrt[6]{63^5}-\sqrt[3]{63^2}}{63}w.$

$$\text{Subſ. } \beta = \left(\frac{\sqrt[6]{63^5}+\sqrt[3]{63^2}}{63}\right)w;\ \text{ſubſ. } \alpha = \left(\frac{\sqrt[6]{63}+1}{\sqrt[6]{63}}\right)w$$

$$= \left(1+\frac{1}{\sqrt[6]{63}}\right)w = \left(\frac{63+\sqrt[6]{63^5}}{63}\right)w.$$

Not. Si $n = m$ aſſumimus, valorem ſubtangentis $= 2x$ invenimus. Nam ſi

$\frac{n}{m}z = \frac{1}{2}z$; & erit $x = w$; $\xi = 0$; ſubſ. $\beta = 2x$; ſubſ. $\alpha = 2x$; hoc eſt ſubtangens $= 2x$.

§. VIII.

$TA = \frac{m+n}{m}w - w = \frac{n}{m}w$. Ergo $TA : w = y \,.\, z$.

§. IX.

Et ſecundum æquationem A) §. VI. eſt ſubſ. $\alpha - \xi = \frac{m^2+n^2}{m^2}w = GB$.

§. X.

Subt. α . — Subt. . $\beta = FB - GP = 2w - \frac{2n^2}{m^2}w = \frac{(m^2-n^2).2}{m^2}w = GF + PB$. Sed $AB = AF$; & $AP = AG$; adeoque $GF = PB$; hoc eſt $\frac{\frac{(m^2-n^2)2}{m^2}w}{2} = \frac{m^2-n^2}{m^2}w = GF = PB$.

§. XI.

Subt. α — ſubſ. $\alpha = FT = \frac{m^2-n^2}{m^2}w - \frac{\overline{m-n}\,.\,n}{m^2}w = \frac{m-n}{m}w$; Adeoque $FT \,.\, TB = \frac{m-n}{m}w \times \frac{m+n}{m}w = \frac{m^2-n^2}{m^2}w^2$; Hinc $\frac{m^2-n^2}{m^2}w^2 : \frac{m^2-n^2}{m^2}w$ §. X. $= w^2 : w = w : 1 = BF^2 : 2BF =$ ſubt. α^2 : 2ſubt. α.

§. XII.

$$\text{Sec. } \alpha^2 = \left(\frac{m+n}{m}w\right)^2 + z^2 = \frac{\overline{m+n}^2 . w^2 + m^2 pw}{m^2}$$

$$= \frac{pm^2 + w . \overline{m+n}^2}{m^2} w; \text{ adeoque}$$

$$\text{Sec. } \alpha = \frac{\sqrt{(pm^2 + w\overline{m+n}^2)}}{m} w.$$

§. XIII.

$$\text{Sec. } \beta^2 = \left(\frac{n}{m}z\right)^2 + \left(\frac{\overline{m+n} . n}{m^2}w\right)^2 = \frac{n^2 m^2 pw}{m^4}$$

$$+ \frac{\overline{m+n}^2 n^2 w^2}{m^4} = \frac{(pm^2 + w . \overline{m+n}^2)n^2}{m^4} w; \text{ adeoque}$$

$$\text{Sec. } \beta = \frac{n\sqrt{(pm^2 + w . \overline{m+n}^2)}}{m^2} w.$$

§. XIV.

Brevius commodiusque rationes ſubſecantium in tabulis hisce proponam. Sit $m - n = D$; $m + n = S$.

Tabula

Tabula I.

Subſ. α : Subſ. β	$= m : n$	$= S+D : S-D$
» : ξ	$= m+n : \frac{\overline{m-n}\,.\,n}{m}$	$= S : \frac{D(S-D)}{S+D}$
» : x	$= m+n : \frac{n^2}{m}$	$= S : \frac{(S-D)^2}{2(S+D)}$
» : w	$= m+n : m$	$= S : \frac{S+D}{2}$
» $-w$: w	$= n : m$	$= S-D : S+D$
» : ſubt. β	$= m+n : \frac{2n^2}{m}$	$= S : \frac{(S-D)^2}{S+D}$
» : » α	$= m+n : 2m$	$= S : S+D$
» : z^2	$= m+n : mp$	$= S : \frac{(S+D)p}{2}$
» : y^2	$= m+n : \frac{n^2 p}{m}$	$= S : \frac{(S-D)^2 p}{2(S+D)}$
» : ſec. α^2	$= m+n : \frac{pm^2 + \overline{wm+n}^2}{m}$	
» : » β^2	$= m+n : \frac{(pm^2 + \overline{wm+n}^2).n^2}{m^3}$	
» : tang. α^2	$= m+n : m(4w+p)$	
» : » β^2	$= m+n : \frac{(4n^2 w + pm^2)n^2}{m^3}$	
» : $w - x$	$= m : m-n$	$= \frac{S+D}{2} : D$
» : ſubſ. $\alpha - w$	$= m+n : n$	$= S : \frac{S-D.}{2}$
» : ſubt. α — ſubſ. α	$= m+n : m-n$	$= S : D.$

Tabula II.

Subſ. $\beta : \xi$	$= m+n : m-n$	$= S : D$
„ $: x$	$= m+n : n$	$= S : \frac{S-D}{2}$
„ $-x : x$	$= m : n$	$= S+D : S-D$
„ : ſubt. α	$= m+n : \frac{2m^2}{n}$	$= S : \frac{(S+D)^2}{S-D}$
„ : „ β	$= m+n : 2n$	$= S : S-D$
„ $: z^2$	$= m+n : \frac{m^2p}{n}$	$= S : \frac{(S+D)^2p}{2(S-D)}$
„ $: y^2$	$= m+n : np$	$= S : \frac{(S-D)p}{2}$
„ : ſec. α^2	$m+n : \frac{pm^2+w.\overline{m+n}^2}{n}$	
„ : „ β^2	$= m+n : \frac{(pm^2+w\overline{m+n}^2)n}{m^2}$	
„ : tang. α^2	$= m+n : \frac{(4w+p)m^2}{n}$	
„ : „ β^2	$= m+n : \frac{(pm^2+4n^2w)n}{m^2}$	
„ $: w-x$	$= n : m-n$	$= \frac{S-D}{2} : D$
„ : ſubſ. $\alpha - w$	$= m+n : m$	$= S : \frac{S+D}{2}$
„ : ſubt. α — ſubſ. α	$= m+n : \frac{m.\overline{m-n}}{n}$	$= S : \frac{(S+D)D}{S-D}$

§. XV.

§. XV.

Ex quibus iterum fluunt rationes aliæ:
Collatis ſcilicet rationibus 1. & 5. Tab. I. & 3. Tab. II.; habebit ſe
ſubſ. β : ſubſ. α = ſubſ. $\alpha - w : w = x$: ſubſ. $\beta - x$; ſive
$= AT : w = x : AT$; adeoque $AT^2 = wx$.
Et coll. ultima rat. Tab. I. & 1. Tab. II. erit
ſubſ. β : ſubſ. $\alpha = \xi : FT$.

§. XVI.

Quod ſubſecantes diametrorum attinet, æquipollet, an ſubſecantem ad ordinatam majorem bd, an ad ordinatam $b\delta$ quæramus. Etenim ponamus prodire pro alterutra aut majorem aut minorem, falſitas ſi elucebit: ſi ſecans $\vartheta\mu\delta$ abſcinderet
Imo majorem diametri portionem, quam ſec. $dm\vartheta$; eſſet

$$\mu p : \delta b = p\vartheta + v : b\vartheta + v \quad \&$$

$$mp : db = p\vartheta : b\vartheta; \text{ ſed } \frac{mp}{db} = \frac{\mu p}{\delta b}$$

adeoque etiam $\frac{p\vartheta}{b\vartheta} = \frac{p\vartheta + v}{b\vartheta + v}$ eſſe deberet, quod manifeſto eſt falſum.

IIdo minorem abſcinderet; eſſet

$$\mu p : \delta b = p\vartheta - v : b\vartheta - v \quad \&$$

$$mp : db = p\vartheta : b\vartheta \text{ adeoque}$$

$\frac{p\vartheta}{b\vartheta} = \frac{p\vartheta - v}{b\vartheta - v}$ foret, quod iterum eſt abſurdum. Ergo ſi nec majorem, nec minorem alterutra poteſt abſcindere diametri portionem, vergere debent in uno eodemque diametri puncto, & dare pro utraque ordinata ſubſecantem unam eandemque.

§. XVII.

Facile, credo, mihi nunc ignoſcetur, ſi de ſubſecantibus diametrorum demonſtrationem non exhibeam; quisque enim facile perſpiciet, demonſtrationem hanc ob analogiam cum illa §. VI. plane eſſe ſupervacaneam. Formulæ itaque ac rationes propoſitæ quadrant etiam in determinandis functionibus diametrorum ad abſciſſas ipſarum majores, ſi formulas ſecantium tangentiumque excipias, quia ibi de coordinatis rectangulis, hic autem de ſcalenis eſt ſermo. Pro ſecantibus igitur quæramus formulas.

Abſciſſa ab ſit $= \mathfrak{w}$; parameter $= \pi$; ſec. $\vartheta\mu\delta = 1^{ma}$ ſec. α; $\vartheta\mu = 1^{ma}$ ſec. β; $\vartheta A d = 2^{da}$ ſec. α; $\vartheta A = 2^{da}$ ſec. β.

$$1^{ma}\text{ ſec. }\alpha^2 = (\text{ſubſ. }\alpha^2 = \vartheta b^2) + b\delta^2 - 2\vartheta b \,.\, b\delta \,.\, (-\text{ coſ. }\vartheta b\delta)$$ [c]

$$= \left(\frac{m+n}{m}\mathfrak{w}\right)^2 + \mathfrak{w}\pi - 2\left(\frac{m+n}{m}\mathfrak{w}\right)\sqrt{(\mathfrak{w}\pi)} \,.\, (-\text{ coſ. }\vartheta b\delta)$$

$$= \frac{\overline{m+n}^2 \,.\, \mathfrak{w}^2 + m^2\mathfrak{w}\pi + 2m\overline{m+n}\,\mathfrak{w}\sqrt{(\mathfrak{w}\pi)} \,.\, \text{coſ. }\vartheta b\delta}{m^2}$$

$$= \frac{\mathfrak{w}\pi \times m^2 + \mathfrak{w}\sqrt{(\mathfrak{w}\pi)}\text{ coſ. }\vartheta b\delta \times 2m\overline{m+n} + \mathfrak{w}^2 \times \overline{m+n}^2}{m^2}$$

Itaque 1^{ma} ſec. α.

$$= \frac{\sqrt{[\mathfrak{w}\pi \times m^2 + \mathfrak{w}\sqrt{(\mathfrak{w}\pi)}\text{ coſ. }\vartheta b\delta \times 2m\overline{m+n} + \mathfrak{w}^2 \times \overline{m+n}^2]}}{m}$$

2^{da} ſec. α

$$= \frac{\sqrt{[\mathfrak{w}\pi \times m^2 - \mathfrak{w}\sqrt{(\mathfrak{w}\pi)}\text{ coſ. }\vartheta b\delta \times 2m\overline{m+n} + \mathfrak{w}^2 \times \overline{m+n}^2]}}{m}$$

Adeo-

c) vid. Celeberr. Kæstner. Trigonom. 20. Saz. nr. 33. 1. Zuſaz.

Adeoque 1. ſec. $\beta = \frac{n\sqrt{(\,..m^2 + ..\,2m\,\overline{m+n} + .\,\overline{m+n}^2)}}{m^2}$

2. ſec. $\beta = \frac{n\sqrt{(\,..m^2 - ..\,2m\,\overline{m+n} + .\,\overline{m+n}^2)}}{m^2}$

Ergo ſubſ. α ; ſec. $\alpha^2 = m+n : \frac{\pi m^2 \pm \sqrt{(\mathfrak{w}\pi)}\,\text{coſ.}\,... + \mathfrak{w}\,.\,\overline{m+n}^2}{m}$

„ : „ $\beta^2 = m+n ; \frac{(\pi m^2 \pm \sqrt{\mathfrak{w}\pi}\,.\,\text{coſ.}\,... + \mathfrak{w}\,\overline{m+n}^2)n^2}{m^3}$

ſubſ. β : „ $\alpha^2 = m+n : \frac{\pi m^2 \pm \sqrt{\mathfrak{w}\pi}\,.\,\text{coſ.}\,.. + \mathfrak{w}\,\overline{m+n}^2}{n}$

„ : „ $\beta^2 = m+n : \frac{(\pi m^2 \pm \sqrt{\mathfrak{w}\pi}\,.\,\text{coſ.}\,.. + \mathfrak{w}\,\overline{m+n}^2)n}{m^2}$

§. XVIII.

Problema.

Reducere functiones diametrorum ad functiones axis.

Solutio.

Data ubicunque ſit in parabolæ curva diameter, ſitque ordinata quæcunque ad illam; tamen ſemper poterit ordinatæ huic duci parallela ex puncto *A*, fig. II. vertice parabolæ, quæ ducta exhibet ordinatam ad eandem diametrum, quamque ordinatam dirigentem voco, omnesque aliæ ordinatæ cadent, aut infra, aut ſupra *A*. Conſideremus

Imo ſi cadit ordinata *infra A*.

Sit $ap = \mathfrak{x}$; $Ap = \mathfrak{y}$; $db = \mathfrak{z}$; parameter diam. $= \pi$. ducta tangente μgC ad μ erit $\mu p : \mu A = gp : CA$; $\mu p = \frac{\mu A}{2}$;

ergo $gp = \frac{CA}{2}$; $CA = x$; ergo $gp = 2\mathfrak{x} = \frac{x}{2}$; $\mathfrak{x} = \frac{x}{4}$; Porro

$\mu p : \mu A = pV : AP$; Ergo $PV = \frac{AP}{2} = \frac{x}{2} = 2\mathfrak{x}$;

$ga = ap = \mathfrak{x} = \frac{x}{4} = \frac{n^2}{4m^2} w$; Porro

$$A\mu^2 = x^2 + y^2 = \frac{n^4}{m^4} w^2 + \frac{n^2 w p}{m^2} = \frac{(wn^2 + pm^2) n^2 . w}{m^4};$$

Itaque $\mu p^2 = Ap^2 = \mathfrak{y}^2 = \frac{(wn^2 + pm^2) n^2 . w}{4m^4}$; Ergo

$\mathfrak{y}^2 : \mathfrak{x} = \frac{wn^2 + pm^2}{m^2} : 1 = \pi : 1$; nam $\mathfrak{x} : \mathfrak{y} = \mathfrak{y} : \pi$;

adeoque $\pi = \frac{\mathfrak{y}^2}{\mathfrak{x}} = \frac{(wn^2 + pm^2) n^2 w}{4m^4} \times \frac{4m^2}{n^2 w} = \frac{n^2 w + m^2 p}{m^2}$. d)

Porro ord. β^2 : sec. β^2 = ord. α^2 : sec. α^2; sive

$$p\mu^2 . \left(\frac{\mu R}{2}\right)^2 = b\delta^2 : \left(\delta R - \frac{\mu R}{2}\right)^2$$

quae collatis §§ XII & XIII. ita exprimuntur:

d) Sed $\pi = 4x + p$; vid. DE LA CHAPELLE pag. 129, 130. adeoque $= \frac{4wn^2 + pm^2}{m^2}$? Omnino. Ibi vero ordin. μp ad punctum a, ubi diameter curvam intrat, hic autem ad illud, in quo ordin. $A\mu$ cum curva vergit, est ducta.

$$\frac{(pm^2+wn^2)n^2w}{4m^4} : \frac{(pm^2+w\overline{m+n}^2)n^2w}{4m^4} = b\delta^2 : \frac{(pm^2+w\overline{m+n}^2)w}{m^2}$$

$$- \frac{2\cdot\sqrt{(pm^2+w\overline{m+n}^2)w}}{m} \times \frac{n\sqrt{(pm^2+w\overline{m+n}^2)w}}{2m^2}$$

$$+ \frac{(pm^2+w\overline{m+n}^2)n^2w}{4m^4}$$

$$= \frac{(4m^2+n^2).(pm^2+w\overline{m+n}^2)w - 4nm\sqrt{(pm^2+..)}.(\sqrt{..m^2..})}{4m^4}$$

$$= \frac{(4m^2+n^2)(pm^2+w\overline{m+n}^2)w - 4nm(pm^2+w.\overline{m+n}^2)w}{4m^4}$$

$$= \frac{(4m^2+n^2-4nm)(pm^2+w\overline{m+n}^2)w}{4m^4}$$

$$= \frac{\overline{2m-n}^2(pm^2+w.\overline{m+n}^2)w}{4m^4} = \delta\vartheta^2 = \text{sec. } \alpha^2 \text{ diam.}$$

Et $\delta\vartheta = \frac{\sqrt{(\overline{2m-n}^2(pm^2+w\overline{m+n}^2)w}}{4m^4} = \frac{\overline{2m-n}\sqrt{(pm^2+w\overline{m+n}^2)w}}{2m^2}$

Adeoque $b\delta^2 = \frac{p\mu^2\times\delta\vartheta^2}{\mu\vartheta^2} =$

$$\frac{(pm^2+wn^2)n^2w}{4m^4}\times\frac{\overline{2m-n}^2(pm^2+w.\overline{m+n}^2)w}{4m^4}\times\frac{4m^4}{(pm^2+w\overline{m+n}^2)n^2w}$$

$$= \frac{(pm^2+wn^2)w\,\overline{2m-n}^2}{4m^4}$$

$$b\delta = \frac{\overline{2m-n}\sqrt{(pm^2+wn^2)w}}{2m^2}.$$

Porro eſt $b\delta^2 = ab \cdot \pi$; $\pi = \frac{pm^2+wn^2}{m^2}$ Ergo

$$ab = \frac{\overline{2m-n}^2(pm^2+wn^2)w}{4m^4} : \frac{pm^2+wn^2}{m^2} = \frac{\overline{2m-n}^2}{4m^2}w$$

adeoque $\mathfrak{z}^2 : \mathfrak{w} = \frac{\overline{2m-n}^2(pm^2+wn^2)w}{4m^4} : \frac{(2m-n)^2 w}{4m^2} = \pi : 1.$

Subſ. $\beta = \vartheta p = \frac{RA}{2} = \frac{n}{2m}w$

Subſ. $\beta - \mathfrak{x} = \vartheta a = \left(\frac{n}{2m} - \frac{n^2}{4m^2}\right)w = \frac{\overline{2m-n}\,n}{4m^2}w.$

Subſ. $\alpha = \vartheta b = \vartheta a + ab = \left(\frac{(2m-n)n}{4m^2} + \frac{(2m-n)^2}{4m^2}\right)w$

$$= \frac{(2m-n+n)\,\overline{2m-n}}{4m^2}w = \frac{2m-n}{2m}w$$

Subt. $\alpha = gb = 2\mathfrak{w} = \frac{\overline{2m-n}^2}{2m^2}w$

Subt. $\beta = gp = 2\mathfrak{x} = \frac{n^2}{2m^2}w$

Subſ. β — ſubt. $\beta = \vartheta g = \frac{\xi}{2} = \zeta = \frac{\overline{m-n}\,n}{2m^2}w$

tang. β^2 diam. $= \mu g^2 = \frac{1}{4}$ tang. β^2 axis $= \frac{\left(\frac{4n^4}{m^4} + \frac{pn^2}{m^2}\right)w}{4}$

$$= \frac{(4wn^2 + pm^2)n^2 w}{4m^4}$$

tang.

tang. α^2 diam. $= \delta f^2$ determinatur ſequenti modo:

$\delta R^2 : \delta\vartheta^2 = \delta Q^2 : \delta f^2$; ſive in expreſſionibus analyt.

$$\frac{(pm^2 + w\,\overline{m+n}^2)w}{m^2} : \frac{\overline{2m-n}^2\,(pm^2 + w\,.\overline{m+n}^2)w}{4m^4} = \overline{4w+pw} :$$

$$\frac{\overline{4w+pw}\,.\,\overline{2m-n}^2\,(pm^2 + w\overline{m+n}^2)w}{4m^4} \times \frac{m^2}{(pm^2 + w\overline{m+n}^2)w}$$

$$= \frac{\overline{4w+p}\;\overline{2m-n}^2\,w}{4m^2};$$ Ergo

$$\text{tang. } \alpha . = \frac{\overline{2m-n}\sqrt{\overline{4w+pw}}}{2m}.$$

Rationes propoſitas iterum ponamus in Tabulam.

Tabula III.

Rationes functionum diametrorum inter ſe, & quidem, ſi ordinata *infra* verticem axis A cadunt.

$\mathfrak{x}$	: $\mathfrak{w}$	$= n$	: $\frac{\overline{2m-n}^2}{n}$
$\mathfrak{y}$	: $\mathfrak{z}$	$= n$	: $2m - n$
$\mathfrak{x}$	: $\mathfrak{y}^2$	$= m$	: $\frac{pm^2 + wn^2}{m}$
$\mathfrak{w}$	: $\mathfrak{z}^2$	= »	»
$\mathfrak{w}$	: ſubſ. β	$= \frac{(2m-n)^2}{2m}$	: n
»	: » α	$= \frac{2m-n}{2}$	: m

w	: subt. β	$=\frac{(2m-n)^2}{2n}$	: n
„	: „ α	$=1$	: 2
„	: ζ	$=\frac{(2m-n)^2}{2n}$	: $m-n$
tang. α^2	: tang. β^2	$=(2m-n)^2$	: $\frac{(4wn^2+pm^2)n^2}{(4w+p)m^2}$
„	: subt α	$=4w+p$	: 2
„	: „ β	$=\frac{\overline{4w+p}\;\overline{2m-n}^2}{2n}$	: n
tang. β^2	: subt. α	$=\frac{(4wn^2+pm^2)n^2}{2m^2}$	: $(2m-n)^2$
„	: subt. β	$=\frac{(4wn^2+pm^2)n^2}{2m}$	: m
subt. α	: subt. β	$=\frac{(2m-n)^2}{n}$	: n
„	: subs. β	$=\frac{(2m-n)^2}{m}$	: n
„	: subs. α	$=2m-n$	: m
subs. α	: subs. β	$=2m-n$	: n
sec. α	: sec. β	$=2m-n$	: n
„	: subs. α	$=\frac{\sqrt{(pm^2+w\,\overline{m+n}^2)}}{w}$	: m
„	: „ β	$=\frac{(2m-n)\sqrt{(pm^2+w\,\overline{m+n}^2)}}{mw}$	: n

$$\text{sec.}\,\alpha \quad : \text{tang.}\,\alpha = \frac{\sqrt{(pm^2+w\overline{m+n}^2)w}}{\sqrt{(4w+p)w}} : m$$

$$\text{,,} \quad : \text{,,}\;\beta = \frac{(2m-n)\sqrt{(pm^2+w\overline{m+n}^2)w}}{\sqrt{(4w^2+pm^2)}} : n$$

$$\text{,,}\;\beta \quad : \text{subs.}\,\alpha = \frac{n\sqrt{(pm^2+w\overline{m+n}^2)w}}{(2m-n)w} : m$$

$$\text{,,} \quad : \text{,,}\;\beta = \frac{\sqrt{(pm^2+w\overline{m+n}^2)w}}{w} : m$$

$$\text{,,} \quad : \text{tang.}\,\alpha = \frac{n\sqrt{(pm^2+w\overline{m+n}^2)w}}{(2m-n)\sqrt{(4w+p)w}} : m$$

$$\text{,,} \quad : \text{,,}\;\beta = \sqrt{(pm^2+w\overline{m+n}^2)w} : \sqrt{(4wn^2+pm^2)w}$$

$$\text{subs.}\,\alpha \quad : \text{tang.}\,\alpha = w \quad : \sqrt{(4w+p)w}$$

$$\text{,,} \quad : \text{,,}\;\beta = 2m-n \quad : \frac{n\sqrt{(4wn^2+pm^2)}}{mw}$$

$$\text{subs.}\,\beta \quad : \text{tang.}\,\alpha = n \quad : \frac{(2m-n)\sqrt{(4w+p)w}}{w}$$

$$\text{,,} \quad : \text{,,}\;\beta = m \quad : \frac{\sqrt{(4wn^2+pm^2)w}}{w}.$$

IIdo si ordinata *supra* secantem seu ordinatam dirigentem cadit, aut quod idem est, si ordinata ex puncto curvæ inter axem & diametrum est ducta.

Hic major abscissa est $= AP = w$; minor $AE = x$; major ord. $P\mu = z$; minor $EO = y$, & sic etiam $p\mu = \mathfrak{z}$; $no = \mathfrak{y}$;

ap

$ap = w$; $an = x$; $T\mu =$ sec. α axis; $To =$ sec. β axis; $\mu s =$ sec. α diam.; $os =$ sec. β diam.

Nunc erit $A\mu^2 = w^2 + z^2 = (w + p)\,w$

$$p\mu^2 = \frac{(w+p)w}{4}$$

$ap = \frac{1}{4}w$; adeoque $\pi = w + p$; e) nam $\frac{p\mu^2}{ap} = \pi$;

subs. $\alpha = sp = \frac{1}{2}TA = \frac{n}{2m}w$;

$$sa = \frac{n}{2m}w - ap = \left(\frac{n}{2m} - \frac{1}{4}\right)w = \frac{2n - m}{4m}w;$$

Porro $T\mu^2 : s\mu^2 = 1 : \frac{1}{4}$; adeoque $\mu s^2 =$ sec. $\alpha^2 = T\mu^2 =$ $\frac{(pm^2 + \overline{wm+n}^2)}{4m^2}w$; vid. §. XII. $\mu s = \frac{\sqrt{(pm^2 + \overline{wm+n}^2)w}}{2m}$.

Vt nunc expressio pro ordinata *on* prodeat, consideremus triangula similia *son*, *sμp*; erit

$s\mu^2$

e) Brevioribus gauderemus expressionibus pro casu I. & generatim calculo reductionis functionum generaliori breviorique, si parametri π haberemus pro utroque casu I. & II. unam eandemque formulam $= w + p$; quod fieri posset, si uterque casus ad AP reduceretur; sed ex alia parte magis compositis uti deberemus formulis, quæ tamen hac hucusque a me adhibita ratione sunt simplicissima: exempli gratia, si ad x omnia reducerentur, esset subs. α axis $= \frac{(\overline{m-n}\,m+1)n^2 + m^2}{n^2}x$, quæ autem in relatione ad w simplicissime est $= \frac{m+n}{m}w$.

$\text{ſ}\mu^2 : \text{ſ}o^2 = \mu p^2 : no^2; \quad \mu\text{ſ}^2 = \frac{(pm^2 + w\,\overline{m+n}^2)w}{4m^2}; = \text{ſ}o^2 =$

$(To - T\text{ſ})^2 = (To - \mu\text{ſ})^2 = (\text{ſec. } \beta \text{ axis} - \text{ſec. } \alpha \text{ diam.})^2$

$$= \frac{(pm^2 + w\,\overline{m+n}^2)n^2w}{m^4} + \frac{(2m-n)^2(pm^2 + w\,\overline{m+n}^2)w}{4m^4}$$

$$- \frac{2\sqrt{(pm^2 + w\,\overline{m+n}^2)n^2w} \,.\, \sqrt{(\overline{2m-n}^2)} \,.\, (pm^2 + w\,\overline{m+n}^2)w}{m^2 \,.\, 2m^2}$$

$$= \frac{(4n^2 + \overline{2m-n}^2)(pm^2 + w\,\overline{m+n}^2)w - 4n\,\overline{2m-n}\,(pm^2 + w\,\overline{m+n}^2)w}{4m^4}$$

$$= \frac{[4n^2 + \overline{2m-5n} \,.\, \overline{2m-n}] \,.\, (pm^2 + w\,\overline{m+n}^2)w}{4m^4}$$

$$= \frac{(4n^2 + 4m^2 - 12mn + 5n^2) \,.\, (pm^2 + w\,\overline{m+n}^2)w}{4m^4}$$

$$= \frac{\overline{2m-3n}^2(pm^2 + w(m+n)^2)w}{4m^4} = \text{ſ}o^2;$$ adeoque $\text{ſ}o =$

$$\frac{\overline{2m-3n} \,.\, \sqrt{(pm^2 + w\,\overline{m+n}^2)w}}{2m^2}; \quad \mu p^2 = \frac{(w+p)w}{4};$$

Ergo $no^2 = \frac{\overline{w+p}\,w}{4} \,.\, \frac{\overline{2m-3n}^2(pm^2 + w\,\overline{m+n}^2)w}{4m^4} :$

$$\frac{(pm^2 + w \,.\, \overline{m+n}^2)w}{4m^2} = \frac{(w+p)w \,.\, (2m-3n)^2}{4m^2}$$ & $no =$

$\frac{\overline{2m-3n}\sqrt{\overline{w+pw}}}{2m}$; Porro eſt $an = \frac{no^2}{\pi} = \frac{\overline{w+pw}\,.\,\overline{2m-3n}^2}{4m^2\,.\,\overline{w+p}} =$

$\frac{\overline{2m-3n}^2}{4m^2}w.$

Subſ. β. diam. $= ſa + an = \left(\frac{\overline{2m-n}}{4m} + \frac{\overline{2m-3n}^2}{4m^2}\right)w$

$= \frac{\overline{2m-n}m + \overline{2m-3n}^2}{4m^2}w = \frac{6m^2-13mn+9n^2}{4m^2}w$

Porro $gſ = \frac{CT}{2} = \frac{m-n}{2m}w$ vid. §. XI.

ſubſ. β — ſubt. $\beta = \zeta = ſa - an = \left(\frac{\overline{2m-n}}{4m} - \frac{\overline{2m-3n}^2}{4m^2}\right)w$

$= \frac{\overline{2m-n}\,.\,m - \overline{2m-3n}^2}{4m^2}w = \frac{-2m^2+11mn-9n^2}{4m^2}w$

ſubt. $\beta = \frac{(2m-3n)^2}{2m^2}w;$

ſubt. $\alpha = gp = \frac{AC}{2} = \frac{AP}{2} = \frac{w}{2};$

tang. $\alpha^2 = \mu g^2 = \frac{\text{tang. } \alpha^2 \text{ axis}}{4} = \frac{\overline{4w+pw}}{4}; \ \mu g = \frac{\sqrt{\overline{4w+pw}}}{2};$

tang. $\beta^2 = \frac{\overline{2n-m}^2\,.\,(\overline{pm^2+wm+n}^2)w}{4m^4}$ & tang. $\beta =$

$\frac{\overline{2n-m}\sqrt{(\overline{pm^2+wm+n}^2)w}}{2m^2}$; nam $\mu p : \mu A = \mu V : \mu P; \ \mu p = \mu A$

$\frac{\mu A}{2}$; ergo $PV = Em = \frac{P\mu}{2} = \frac{z}{2}$; itaque $om\left(\frac{n}{m} - \frac{1}{2}\right)z = \frac{2n-m}{2m}\sqrt{pw}$; $om^2 = \frac{\overline{2n-m}^2}{4m^2}pw$; Porro y^2 : fec. β^2 axis $= \frac{(2n-m)^2}{4m^2}wp : o\mathit{f}^2$; five $\frac{n^2}{m^2}pw : \frac{(\overline{pm^2+wm+n})^2 n^2 w}{m^4} = \frac{(2n-m)^2 pw}{4m^2} : oS^2 = \frac{2n-m(\overline{pm^2+wm+n})^2 w}{4m^4}$.

Tabula IV.

Rationes functionum diametrorum in fe fpectatæ, & quidem fi ordinatæ *fupra* verticem axis cadunt.

𝔵	: 𝔴	$= \frac{(2m-3n)^2}{m}$	: m
"	: $𝔶^2$	= 1	: $w + p$
$𝔷^2$	: $𝔶^2$	= m	: $\frac{(m-3n)^2}{m}$
𝔷	: 𝔶	= m	: $2m - 3n$
𝔴	: fubf. β	= m	: $\frac{6m^2 - 13mn + 9n^2}{m}$
"	: " α	= m	: $2n$
"	: fubt. β	= m	: $\frac{2(2m-3n)^2}{m}$
"	: " α	= 1	: 2
"	: ζ	= m	: $\frac{-2m^2 + 11mn - 9n^2}{m}$

tang. α^2	: tang. β^2	$= m$	: $\frac{(2n-m)^2(pm^2+w\overline{m+n}^2)}{m^3(4w+p)}$
„	: ſubt. α	$= 4w+p$	: 2
„	: „ β	$= m$	: $\frac{2(2m-3n)^2}{(4w+p)m}$
tang. β^2	: „ α	$= m$	: $\frac{\overline{2n-m}^2(pm^2+w\overline{m+n}^2)}{2m^3}$
„	: ſubt. β	$= \overline{2n-m}^2$	: $\frac{\overline{2m-3n}^2\, 2m^2}{pm^2+w(m+n)^2}$
ſubt. α	: ſubt. β	$= m$	: $\frac{(2m-3n)^2}{m}$
„	: ſubſ. β	$= m$	: $\frac{6m^2-13mn+9n^2}{2m}$
„	: ſubſ. α	$= m$	: n
ſubſ. α	: ſubſ. β	$= n$	: $\frac{6m^2-13mn+9n^2}{2m}$
„	: tang. α^2	$= n$	: $\frac{(4w+p)m}{2}$
„	: tang. β^2	$= n$	: $\frac{(2n-m)^2(pm^2+w\overline{m+n}^2)}{2m^3}$

Ob nimiam formularum longitudinem plures rationes ponere nolui.

Tabula

Tabula V.

Rationes functionum diam. ad subsecantes axis pro *casu primo*, si ordinatæ cadunt *infra* A.

Subs. β axis	: subs. β diam.	$= m + n$	: $\frac{m}{2}$
"	: " α	= "	: $\frac{(2m-n)m}{2n}$
"	: subt. β	= "	: $\frac{n}{2}$
"	: " α	= "	: $\frac{(2m-n)^2}{2n}$
"	: tang. β^2	= "	: $\frac{(pm^2+4wn^2)n}{4m^2}$
"	: " α^2	= "	: $\frac{\overline{2m-n}^2(4w+p)}{4n}$
"	: subs. β — subt. β	= "	: $\frac{m-n}{2}$
"	: subs. $\beta - \mathfrak{p}$	= "	: $\frac{(2m-n)}{4}$
"	: $\mathfrak{w}$	= "	: $\frac{(2m-n)^2}{4n}$
"	: $\mathfrak{p}$	= "	: $\frac{n}{4}$
"	: $\mathfrak{z}^2$	= "	: $\frac{(2m-n)^2(pm^2+wn^2)}{4n}$
"	: $\mathfrak{y}^2$	= "	: $\frac{(pm^2+wn^2)n}{4m^2}$
Subs. α axis	: subs. β	= "	: $\frac{n}{2}$

ſubſ. α axis	: ſubſ. α diam.	$= m+n$	$: \frac{(2m-n)}{2}$
»	: ſubt. β	$=$ »	$: \frac{n^2}{2m}$
»	: » α	$=$ »	$: \frac{(2m-n)^2}{2m}$
»	: tang. β	$=$ »	$: \frac{(pm^2+4wn^2)n^2}{4m^3}$
»	: » α^2	$=$ »	$: \frac{(2m-n)^2(4w+p)}{4m}$
»	: ſubſ. β — ſubt. β	$=$ »	$: \frac{(m-n)n}{2m}$
»	: ſubſ. β — $\mathfrak{x}$	$=$ »	$: \frac{(2m-n)n}{4m}$
»	: $\mathfrak{w}$	$=$ »	$: \frac{(2m-n)^2}{4m}$
»	: $\mathfrak{x}$	$=$ »	$: \frac{n^2}{4m}$
»	: $\mathfrak{z}^2$	$=$ »	$: \frac{(2m-n)^2(pm^2+n^2w)}{4m^3}$
»	: $\mathfrak{y}^2$	$=$ »	$: \frac{(pm^2+wn^2)n^2}{4m^3}$.

Tabula VI.

Pro *caſu ſecundo*, ſi ordinatæ cadunt *ſupra A*.

ſubſ. β axis	: ſubſ. β. diam.	$= m+n$	$: \frac{6m^2-13mn+9n^2}{4n}$
»	: » α	$=$ »	$: \frac{m}{2}$

ſubſ.

subs. β axis	: subt. β diam.	$=$	$m+n$	: $\frac{(2m-3)n^2}{2n}$
»	: » α	$=$	»	: $\frac{m^2}{2n}$
»	: tang. β^2	$=$	»	: $\frac{(2m-n)^2(pm^2+\overline{wm+n}^2)}{4nm^2}$
»	: » α^2	$=$	»	: $\frac{m^2(4w+p)}{4n}$
»	: ζ	$=$	»	: $\frac{-2m^2+11mn-9n^2}{4n}$
»	: x	$=$	»	: $\frac{(2m-3n)^2}{4n}$
»	: w	$=$	»	: $\frac{m^2}{4n}$
»	: y^2	$=$	»	: $\frac{(2m-3n)^2\,\overline{w+p}}{4n}$
»	: z^2	$=$	»	: $\frac{m^2(w+p)}{4n}$
subs. α axis	: subs. β	$=$	»	: $\frac{-6m^2+13mn-9n^2}{4m}$
»	: » α	$=$	»	: $\frac{n}{2}$
»	: subt. β	$=$	»	: $\frac{(2m-3n)^2}{2m}$
»	: » α	$=$	»	: $\frac{m}{2}$
»	: tang. β^2	$=$	»	: $\frac{(2m-n)^2(pm^2+w.\overline{m+n}^2)}{4m^3}$

subs.

subs. α axis : tang. α^2 diam. $= m + n : \frac{m.(4w+p)}{4}$

„ $: \zeta$ $=$ „ $: \frac{-2m^2 + 11mn - 9n^2}{4m}$

„ $: \mathfrak{x}$ $=$ „ $: \frac{(2m - 3n)^2}{4m}$

„ $: \mathfrak{w}$ $=$ „ $: \frac{m}{4}$

„ $: \mathfrak{y}^2$ $=$ „ $: \frac{(2m - 3n)^2 \overline{w+p}}{4m}$

„ $: \mathfrak{z}^2$ $=$ $: \frac{m(w+p)}{4}$.

§. XIX.

Progredimur ad determinationem subsecantium ellipseos, Sit $AP = x$; $AT = w$, $AB = a$; $NT = z$; $MP = y = \frac{n}{m} z$; $ST =$ subs. α; $SP =$ subs. β; $OS = \xi$; Quia $y^2 : z^2 = (a-x)x : (a-w)w$ & $(a-x)x : y^2 = a : p$; vid. DE LA CHAPELLE p. 321. Ergo $y^2 = \frac{(a-x)px}{a}$ & $z^2 = \frac{(a-w)pw}{a}$.

§. XX.

Quia porro $PG : AG = AG : OG$ vid. CHAPELLE p. 280. Ergo $OG = \frac{AG^2}{PG}$; $AG = \frac{a}{2}$ & $PG = \frac{a-2x}{2}$; adeoque $OG =$ $\frac{a^2}{4} : \frac{a-2x}{2} = \frac{a}{2(a-2x)}$.

§. XXI.

Subtangens OP itaque $= OG - PG = \frac{a^2}{2(a-2x)} - \frac{a-2x}{2} =$

$$= \frac{a^2 - (a-2x)(a-2x)}{2(a-2x)} = \frac{a^2 - (a^2 - 4ax + 4x^2)}{2(a-2x)} = \frac{4ax - 4x^2}{2(a-2x)}$$
$$= \frac{(a-x)2x}{a-2x}.$$

§. XXII.

Itaque OA = ſubtangens $- x = \frac{(a-x)2x}{a-2x} - x = \frac{2ax - 2x^2 - ax + 2x^2}{a-2x} = \frac{ax}{a-2x}.$

§. XXIII.

Ob ſimilitudinem triangulorum SPM & STN eſt $y : z =$ ſubſ. β : ſubſ. α; y autem $= \frac{n}{m} z$; adeoque ſubſ. $\beta = \frac{n}{m}$ ſubſ. α & quia $y^2 : z^2 = (a-x)x : (a-w)w$; erit etiam ſubſ. $\alpha^2 : \frac{n^2}{m^2}$ ſubſ. $\alpha^2 = (a-w)w : a-x)x$, adeoque $(a-x)x = \frac{(a-w)wn^2}{m^2}$ ſive $ax - x^2 = \frac{(aw-w^2)n^2}{m^2}$ ſive $x^2 - ax = -\frac{(aw-w^2)n^2}{m^2}$; $x^2 - ax + \frac{1}{4}a^2 = \frac{1}{4}a^2 - \frac{(aw-w^2)n^2}{m^2}$; adeoque radice extracta: $x - \frac{a}{2} = -\sqrt{\left(\frac{1}{4}a^2 - \frac{(aw-w^2)n^2}{m^2}\right)}$

$$x = \frac{a}{2} - \frac{\sqrt{(a^2m^2 - (aw-w^2)4n^2)}}{2m} = \frac{am - \sqrt{[a^2m^2 - (a-w)4wn^2]}}{2m}.$$

§. XXIV.

Determinata abſciſſa x, ſubtangens $OP = \frac{(a-x)2x}{a-2x}$ in alia expreſſione prodit $= \left[a - \frac{(am - \sqrt{(a^2m^2 - (a-w)4wn^2)})}{2m}\right]$

$$\frac{(am - \sqrt{(a^2m^2 - \ldots)}}{m} : a - \frac{(am - \sqrt{(a^2m^2 - \ldots)}}{m} =$$

$$\frac{am + \sqrt{(a^2m^2\ldots)}(am - \sqrt{a^2\ldots})}{2m\sqrt{(a^2\ldots)}} = \frac{a^2m^2 - (a^2m^2 - (aw - w^2)4n^2)}{2m\sqrt{(a^3m^2\ldots)}}$$

$$= \frac{(a-w)2wn^2}{m\sqrt{a^2\ldots}}.$$

§. XXV.

$$OT = \text{ſubtang.}\ \beta + w - x = \frac{(a-w)2wn^2}{m\sqrt{a^2\ldots}} + w -$$

$$\frac{(am - \sqrt{a^2\ldots})}{2m} = \frac{(a-w)4wn^2 + (2mw - am)\sqrt{(a^2\ldots)} + a^2m^2 - (a-w)4wn^2}{2m\sqrt{a^2m^2\ldots}}$$

$$= \frac{a^2m^2 + (2w-a)m\sqrt{a^2\ldots}}{2m\sqrt{(a^2\ldots)}} = \frac{a^2m + (2w-a)\sqrt{a^2\ldots}}{2\sqrt{a^2\ldots}}.$$

§. XXVI.

Hinc ſubſ. $\alpha = OT + \xi = \frac{a^2m + (2w-a)\sqrt{a^2\ldots}}{2\sqrt{a^2\ldots}} + \xi$; A) $\frac{n}{m}$ ſubſ. α = ſubſ. $\beta = OP + \xi = \frac{(a-w)2wn^2}{m\sqrt{a^2\ldots}} + \xi$; $\frac{\text{ſubſ.}\ \alpha}{m} = \frac{(a-w)2wn}{m\sqrt{a^2\ldots}} + \frac{\xi}{n}$ ſubſ. $\alpha = \frac{(a-w)2wn}{\sqrt{a^2\ldots}} + \frac{m}{n}\xi$; B) quia æquatio $A = B$; erit etiam $\frac{(a-w)2wn}{\sqrt{a^2\ldots}} + \frac{m}{n}\xi = \frac{a^2m + (2w-a)\sqrt{a^2\ldots}}{2\sqrt{a^2\ldots}} + \xi$; $\frac{m-n}{n}\xi = \frac{a^2m + (2w-a)\sqrt{(a^2\ldots)} - (a-w)4wn}{2\sqrt{a^2\ldots}}$;

$$\xi = \frac{[a^2m - (a-w)4wn + (2w-a)\sqrt{a^2\ldots}]n}{(w-n)2\sqrt{a^2\ldots}}.$$

§. XXVII.

Determinato ξ subsecans $\beta = PS$ erit $=$ subtang. $OP + \xi$

$$= \frac{(a-w)2wn^2}{m\sqrt{a^2 \ldots}} + \frac{[a^2m - (a-w)4wn + (2w-a)\sqrt{a^2 \ldots}]n}{\overline{m-n}\,2\sqrt{a^2 \ldots}} =$$

$$\frac{\overline{a-w}\,4wn^2 . \overline{m-n} + [(a^2m - (a-w)4wn + (2w-a)\sqrt{a^2 \ldots})].mn}{\overline{m-n}\,2m\sqrt{a^2 \ldots}} =$$

$$\frac{\overline{m-n-m} . \overline{a-w} . 4wn^2 + a^2m^2n + \overline{2w-a} . mn\sqrt{a^2 \ldots}}{\overline{m-n}\,2m\sqrt{a^2 \ldots}} =$$

$$\frac{[(a^2m + \overline{2w-a}\sqrt{a^2 \ldots})m - (a-w)4wn^2]n}{\overline{m-n}\,2m\sqrt{a^2 \ldots}}.$$

§. XXVIII.

Subs. $\alpha =$ subs. $\beta + w - x$

$$= \frac{[(a^2m + \overline{2w-a}\sqrt{a^2 \ldots})m - \overline{a-w}\,4wn^2]n}{\overline{m-n}\,2m\sqrt{a^2 \ldots}} + \frac{\overline{2w-a}.m + \sqrt{a^2 \ldots}}{2m}$$

$$= \frac{\text{subs.}\,\beta + \overline{2w-a}m . \overline{m-n}\sqrt{a^2 \ldots}) + (a^2m^2 - (a-w)4wn^2)\overline{m-n}}{\overline{m-n}\,2m\sqrt{a^2 \ldots}}$$

$$= \frac{\overline{n+m-n}\,\overline{2w-a}\,m\sqrt{a^2 \ldots}) + \overline{n+m-n}\,a^2m^2 + (n-m+n)\overline{a-w}\,4wn^2}{\overline{m-n}\,2m\sqrt{a^2 \ldots}}$$

$$= \frac{\overline{2w-a}\,m\sqrt{a^2 \ldots}) + a^2m^2 - (a-w)4wn^2}{\overline{m-n}\,2\sqrt{a^2 \ldots}}.$$

§. XXIX.

Spatium suadet, subsistere in paucis hisce formulis pro subsecantibus Ellipseos ad nauseam usque extensis, nec plures, easque longiores, quærere. Reliquum esset, ut subsecantium Hyperbolæ formulas adhuc inveniremus, sed, credo, absoluturum me ab opera quemlibet esse, qui non ignorat, functiones Hyperbolæ signis modo differre a functionibus Ellipseos.

§. XXX.

De usu subsecantium haud ægre quædam superadderem, nisi temporis angustia me ita constituisset, ut de applicatione notatu digni nihil adjicere potuissem. Quod sectiones corporum §. I. memor. attinet, subsecantium adjumento adhuc nil novi reperi. Pedem itaque hic figere me oportet, de applicatione novæ hujus theseos alio forsitan tempore dicturum.

THESES

I.

Formula $\frac{r \cdot n}{2}$ hactenus pro dimensione polygoni infinite parvorum laterum, circuli inscripti, usitata (scilicet r significet radium circuli & n numerum infinite parvorum laterum) non exhibet quantitatem areæ polygoni n laterum, sed polygoni $2n$ laterum.

II.

Nam radius circuli est altitudo triangulorum polygoni circulo inscripti *secundum rigorem geometricum talis.*

III.

Orbitæ planetarum & cometarum secundum systema LAMBERTI non sunt ellipses, sed lineæ cycloidales.

IV.

Datur series nec convergens nec divergens, quam nuncupem linearem: scilicet $\frac{1}{x + x} = \frac{1}{x} - \frac{1}{x} + \frac{1}{x} - \frac{1}{x} + \ldots$ seriem nuncupo linearem.

V.

Ellipsis est sectio

1. Coni
   a) in infinito convergentis, sive Cylindri,
   b) in finito convergentis, sive regularis.

2. Conoidis

a) parabolicæ,

b) hyperbolicæ.

3. Spæroidis ellipticæ in quovis ejus puncto.

VI.

Parallelogrammi Newtoniani ad axem in peritrochio facilis est applicatio.

VII.

Non datur numerus minor, quam x^0, quicunque etiam pro x ponatur numerus.

VIII.

Axis in globo, circa se moto, quiescit.

IX.

Nec numeri, nec lineæ, nec plana, nec solida concipi possunt infinite magna.

X.

Corpora sub antlia pnevmatica in Luna tardius humi cadere debent, quam in Terra.

XI.

Corpora, quæ in superficiem Terræ verticaliter cadere nituntur, sæpius describunt lineam curvam, quod etiam accideret, si atmosphæra non esset.

XII.

Iterum, etiamsi atmosphæra mox densior, mox rarior non esset, nihilominus quotidie modo tardius modo celerius corpora superficiem terræ cadendo tangerent.

XIII.

Solem nos nunquam, & stellas quasdam tantum nonunquam in veris suis locis observamus.

XIV.

Tot dantur eodem temqore irides, quot sunt observatores.

XV.

Siquis agrum, figuram rhomboidis habentem, cujus primum latus $\frac{5}{2}$ decempedas, secundum etiam $\frac{5}{2}$ dec., tertium 4 dec., quartum totidem dec., commutat cum alio, ejusdem figuræ, cujus primum latus $\frac{5}{2}$ dec.,

dec., secundum $\frac{1}{2}$ dec. tertium 3 dec. & quartum etiam 3 dec.; commutatio erit justa.

XVI.

Dimidia fere pars incolarum Lunæ insigni caret spectaculo, Terræ scilicet discum per ætatem nunquam videns.

Animadvertenda.

Vbi expressiones $\sqrt{(\ldots)w}$ occurrunt, w &; si habet, ejus factorem, etiam sub signo $\sqrt{}$ positum intelligo. pag. 6. lin. penult. leg. §. III. pag. 11. §. VIII. leg. $= y : z$; pag. 18. lin. 4. leg. pV; ibid. lin. antepenult. leg. ord. μP; & ult. leg. cum cum curva in μ vergit; pag. 20. l. 4. leg. $= \pi : \frac{1}{w}$; ib. lin. 9. leg. $fb = 2w$; ib. lin. penult. leg. $\frac{(4n^4 w}{m^4} +$.

PRAECLARISSIMO ATQVE DOCTISSIMO

DOMINO CANDIDATO AVCTORI

PRÆSES

S. P. D.

Cum mihi dissertationem Tuam offerres, ratio valetudinis meæ non permisit, ut attente illam perlegerem, hinc TIBI, auditori meo in disciplinis, quas trado, semper solertissimo, nullam remoram objicere volui, quo minus eam publici juris faceres, qualem elaborasti, fretus Tuæ scientiæ in hoc studiorum genere, & hoc specimen abunde declarat, TE in analysi *magnos profectus fecisse: solvisti non* secuisti *nodum hujus quæstionis. Ut tangas, non feces veritatem, ex animo TIBI apprecor, & faustos in omnibus Tuis rebus, quibus dignus es, successus, ut habeas, votorum meorum & bonorum omnium summa est.* Tubingæ 25 *Aug.* 1779.

Ad §§. III & ſeqq.

Neſcio, quomodo evenit, ut æquationem $ſ^2 = 2ſx + dx$; §. III. memor. non attentius conſiderarem; cum enim ultimæ jam paginæ prelum relinquerent, demum mihi in mentem venit, valorem pro $ſ$ etiam ex hac æquatione facile poſſe erui: nam $ſ^2 - 2ſx = dx$; $ſ^2 - 2ſx + x^2 = x^2 + dx$; $ſ - x = \surd[(x + d)x]$; $ſ = x + \surd[(x + w - x)x]$; $ſ = x + \surd(wx)$; atque ſic gaudeo formulam generaliorem pro ſubſecanti TP opportuno adhuc tempore eruiſſe, quoniam altera magis habet formam arithmeticam, quæ prodit, ſi pro x ſubſtituitur valor $\frac{n^2}{m^2} w$;

ſubſ. α itaque $= x + \surd(wx) + w - x = w + \surd(wx)$

$\xi = x + \surd(wx) - 2x = \surd(wx) - x$

$AT = \surd(wx)$; $FT = w - \surd(wx)$;

ſec. $\alpha = \surd([w + x + p + 2\surd(wx)]w)$

ſec. $\beta = \surd([w + x + p + 2\surd(wx)]x)$.

Niſi temporis anguſtia vetaret, plura lubenter adderem.

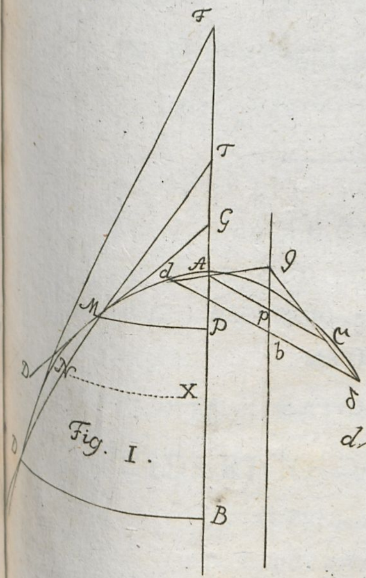

Fig. I.

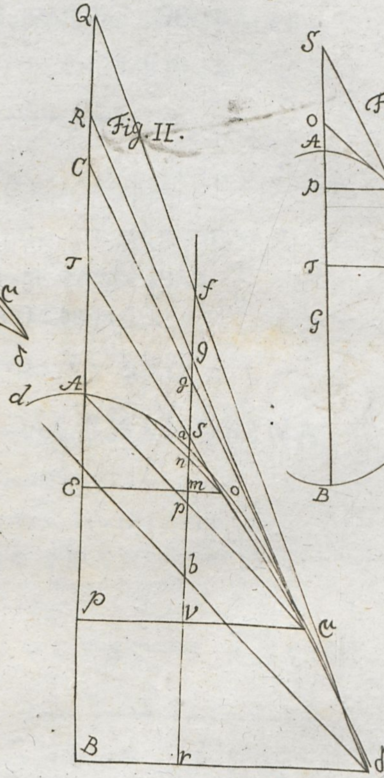

Fig. II.

Fig. III.

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DISPVTATIO MATHEMATICA
DE
SVBSECANTIBVS LINEARVM SECVNDI ORDINIS.

QVAM

RECTORE

VNIVERSITATIS EBERHARDINAE CAROLINAE

MAGNIFICENTISSIMO

ERENISSIMO atque POTENTISSIMO DVCE ac DOMINO

DOMINO

CAROLO,

DVCE WIRTEMBERGIÆ ET TECCIÆ REGNANTE

rel. rel.

CONSENTIENTE AMPLISSIMO PHILOSOPHORVM ORDINE

PRÆSIDE

VIRO EXCELLENTISSIMO atque DOCTISSIMO

DOMINO

JOHANNE KIES,

VERSITATIS ET COLLEGII ILLVSTRIS PROF. PHYS. ET MATH. P. O.
ACADEMIAE SCIENTIARVM REGIAE BORVSSICAE SODALI,

AECEPTORE AC PATRONO SVO PIE DEVENERANDO

PRO GRADV MAGISTRI SIVE DOCTORIS PHILOSOPHIAE
RITE CONSEQVENDO

DIE AVGVSTI ANNI MDCCLXXIX.

PVBLICE DEFENDET

AVCTOR

IDERICVS GVILIELMVS KOESTLIN, *Brackenhemiensis*,
ERENISSIMI STIPENDIARIVS ET MAGISTERII PHILOS. CANDIDATVS.

TVBINGÆ, LITERIS SIGMVNDIANIS.

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