DILVCIDATIONES ANALYSEOS FINITORVM KÆSTNERIANÆ

QVAS

PRÆSIDE

IOANNE KIES

PHYSICES ET MATHESEOS PROF. P. O.

FACVLTATIS PHILOSOPHICÆ h. t. DECANO

PVBLICE TVEBITVR

Die VIII OCTOBRIS ANNI MDCCLXII.

CHRISTIANVS CVNRADVS KLEMM,

Canstadiensis.

MAGISTERII PHILOSOPHICI CANDIDATVS.

Tubingæ

TYPIS IO. AD. SIGMVNDI.

VIRIS
MERITORVM GRAVITATE, MVNERVM DIGNITATE
CONSPICVIS,
PRÆNOBILISSIMO, PERSTRENVO
ATQVE
CONSVLTISSIMO,
MAXIME REVERENDO, DIGNISSIMO
ATQVE
AMPLISSIMO,
PLVRIMVM REVERENDO atque DOCTISSIMO,
NOBILISSIMIS,
DEXTERRIMO,
PRVDENTISSIMIS atque SPECTATISSIMIS
CIVITATIS CANSTADIENSIS
LVMINIBVS, PRÆFECTIS, PATRIBVS,
PATRONIS, FAVTORIBVS SVIS
OMNI OBSERVANTIA COLENDIS
SPECIMEN HOC ACADEMICVM
PIA MENTE OFFERT
RESPONDENS.

Quum hac æſtate Elementa *Analyſeos finitorum Kæſtneriana* ſatis magno Auditorum numero explicabam, accidit ſubinde, ut evolutio diſtincta quorundam problematum, & ipſa calculi adminiſtratio nimiam horarum ſeriem inſumeret, & diutius nos teneret, quam quidem ab initio nobis erat propoſitum, conſultum duxi, in his paginis ea uberius exponere, quæ Celeberrimus *Kæſtnerus* ſuccincte, ne liber ejus in nimiam molem excreſceret, & breviter exhibuit, habebunt ita Honoratiſſimi Domini Auditores inſtrumentum quoddam utile, quo in repetendis prælectionibus meis, & ſecandis qui in profundiſſime conſcripto iſto libello obvenire ſolent nodis commode uti poſſunt, mihi vero idem labor inpoſterum inſerviet, ſi de novo iſtius *Analyſeos* interpres fuero, ut cum diſcentibus quibus hæc ſpecimina ad manus ſunt, celerius progredi liceat.

Elementa Analyseos finitorum Kæstneriana

§. 13.

$$\frac{1}{1+x} = 1 - x + x^2 - x^3 + x^4 - x^5 + \&c.$$

$$\frac{1}{x+1} = \frac{1}{x} - \frac{1}{x^2} + \frac{1}{x^3} - \frac{1}{x^4} + \frac{1}{x^5} - \&c.$$

Ergo

$$1 - x + x^2 - x^3 + x^4 - x^5 + \&c. = \frac{1}{x} - \frac{1}{x^2} + \frac{1}{x^3} - \frac{1}{x^4} + \frac{1}{x^5} - \&c.$$

§. 98.

Sit progressio geometrica

$$a \quad ea \quad e^2a \quad e^3a \quad e^4a \quad e^5a,$$

in qua terminus primus a, & exponens e, hæc series si per exponentem e unitate mulctatum i. e. per $e-1$ multiplicetur, productum erit $e^6a - a$, adeoque si $e^6a - a$ per $e-1$ dividatur, prodibit iterum series $a \; ea \; e^2a \; e^3a \; e^4a \; e^5a = \frac{e^6a - a}{e-1}$

§. 110.

$$\frac{1}{y} - \frac{1}{y^2} + \frac{1}{y^3} - \frac{1}{y^4} + \frac{1}{y^5} - \frac{1}{y^6} + \&c.$$

in hac serie exponens est $-\frac{1}{y}$ & ea abit in infinitum, hinc summa erit

$$\frac{1}{y} : 1 + \frac{1}{y} = \frac{1}{1+y}$$

§. 163.

In Figura 4. ubi DF secat lineam BC, notetur intersectio litera I, & habebitur, positis denominationibus *Kæstnerianis*

CE

$CE : EB = CF : FI.$

$\sqrt{(r^2 - \frac{1}{4}k^2)} : \frac{1}{2}k = \sqrt{(r^2 - \frac{1}{4}c^2)} : FI$, ergo

$$FI = \frac{\frac{1}{2}k \cdot \sqrt{(r^2 - \frac{1}{4}c^2)}}{\sqrt{(r^2 - \frac{1}{4}k^2)}}$$

$$DI = \frac{1}{2}c - \frac{\frac{1}{2}k\sqrt{(r^2 - \frac{1}{4}c^2)}}{\sqrt{(r^2 - \frac{1}{4}k^2)}}$$

$CE : CB = CF : CI$

$\sqrt{(r^2 - \frac{1}{4}k^2)} : r = \sqrt{(r^2 - \frac{1}{4}c^2)} : CI$ hinc

$$CI = \frac{r\sqrt{(r^2 - \frac{1}{4}c^2)}}{\sqrt{(r^2 - \frac{1}{4}k^2)}}$$

$CI : DI = CF : DG$

$$\frac{r\sqrt{(r^2 - \frac{1}{4}c^2)}}{\sqrt{(r^2 - \frac{1}{4}k^2)}} : \frac{1}{2}c - \frac{\frac{1}{2}k\sqrt{(r^2 - \frac{1}{4}c^2)}}{\sqrt{(r^2 - \frac{1}{4}k^2)}} = \sqrt{(r^2 - \frac{1}{4}c^2)} : DG$$

hinc $DG = \dfrac{\frac{1}{2}c\sqrt{(r^2 - \frac{1}{4}k^2)} - \frac{1}{2}k\sqrt{(r^2 - \frac{1}{4}c^2)}}{r}$

& $DK = \dfrac{c\sqrt{(r^2 - \frac{1}{4}k^2)} - k(\sqrt{r^2 - \frac{1}{4}c^2})}{r}$

$$DK = \frac{c\sqrt{(4r^2 - k^2)} - k\sqrt{(4r^2 - c^2)}}{2r}$$

§. 164.

$EC : BC = DG . DI$

$\frac{1}{2}\sqrt{(4r^2 - k^2)} : r = \frac{1}{2}h : \dfrac{rh}{\sqrt{(4r^2 - k^2)}}$

$EC : BE = DG : GI$

$\frac{1}{2}\sqrt{(4r^2 - k^2)} : \frac{1}{2}k = \frac{1}{2}h : \dfrac{hk}{2\sqrt{(4r^2 - k^2)}}$

$GC - GI = CI = \frac{1}{2}\sqrt{(4r^2 - h^2)} - \dfrac{hk}{2\sqrt{(4r^2 - k^2)}}$

$BC : BE = IC : IF$

$r : \frac{1}{2}k = \frac{1}{2}\sqrt{(4r^2 - h^2)} - \frac{hk}{2\sqrt{(4r^2 - k^2)}} : \frac{k\sqrt{(4r^2 - h^2)}}{4r} - \frac{hk^2}{4r\sqrt{(4r^2 - k^2)}}$

$DI + IF = DF = \frac{rh}{\sqrt{(4r^2 - k^2)}} + \frac{k\sqrt{(4r^2 - h^2)}}{4r} - \frac{hk^2}{4r\sqrt{(4r^2 - k^2)}}$

$DF = \frac{(4r^2 - k^2)h + k\sqrt{(4r^2 - h^2)(4r^2 - k^2)}}{4r\sqrt{(4r^2 - k^2)}}$

$DF = \frac{h\sqrt{(4r^2 - k^2)} + k\sqrt{(4r^2 - h^2)}}{4r}$

$2DF = c = \frac{h\sqrt{(4r^2 - k^2)} + k\sqrt{(4r^2 - h^2)}}{2r}$

§. 175.

Ex §§. 163. 164.

sit $\frac{1}{2}h = \text{sin.}\, x$, & $\sqrt{(r^2 - \frac{1}{4}h^2)} = \text{cosin.}\, x$

$\frac{1}{2}k = \text{sin.}\, y$ & $\sqrt{(r^2 - \frac{1}{4}k^2)} = \text{cosin.}\, y$

erit

$DF = \text{sin.}\,(x + y) = \frac{\frac{1}{2}h\sqrt{(r^2 - \frac{1}{4}k^2)} + \frac{1}{2}k\sqrt{(r^2 - \frac{1}{4}h^2)}}{r}$

$\text{sin.}\,(x + y) = \text{sin.}\, x \,.\, \text{cosin.}\, y + \text{sin.}\, y\, \text{cos.}\, x$

§. 201.

Elegans occurrit *reductionis* specimen

$y^2 + (ax + b)\, y + cx^2 + ex + f = y^2 + (\alpha x + \beta)\, y + \gamma x^2 + \varepsilon x + \varphi$

& fiet operatione ipsa instituta

$$\left.\begin{array}{l}(\gamma-c)^2\\ +a(a-\alpha)(\gamma-c)\\ +c(a-\alpha)^2\end{array}\right\}x^4 \left.\begin{array}{l}+2(\gamma-c)(\varepsilon-e)\\ +a(a-\alpha)(\varepsilon-e)\\ +(a-\alpha)b(\gamma-c)\\ +a(b-\beta)(\gamma-c)\\ +(a-\alpha)^2 e\\ +2(a-\alpha)(b-\beta)c\end{array}\right\}x^3 \left.\begin{array}{l}+(\varepsilon-e)^2\\ +2(\gamma-c)(\varphi-f)\\ +a(a-\alpha)(\varphi-f)\\ +(a-\alpha)b(\varepsilon-e)\\ +a(b-\beta)(\varepsilon-e)\\ +(b-\beta)b(\gamma-c)\\ +(a-\alpha)^2 f\\ +2(a-\alpha)(b-\beta)e\\ +(b-\beta)^2 c\end{array}\right\}x^2 \left.\begin{array}{l}+2(\varepsilon-e)(\varphi-f)\\ +(a-\alpha)b(\varphi-f)\\ +a(b-\beta)(\varphi-f)\\ +(b-\beta)b(\varepsilon-e)\\ +2(a-\alpha)(b-\beta)f\\ +(b-\beta)^2 e\end{array}\right\}x \begin{array}{l}+(\varphi-f)^2\\ +(b-\beta)b(\varphi-f)=0\\ +(b-\beta)^2 f\end{array}$$

§. 262.

$$\sqrt{(2+\sqrt{-3})}+\sqrt{(2-\sqrt{-3})}=\sqrt{(4+2\sqrt{7})}$$

Dem.

Sit $\sqrt{-3}=x$, erit

$\sqrt{(2+x)}+\sqrt{(2-x)}=$ quantitati ſummandæ

ponatur $\sqrt{(2+x)}=y$, & $\sqrt{(2-x)}=z$

ergo

$$\begin{array}{ll} 2+x=y^2 & \sqrt{(2-x)}=z \\ 2-x=z^2 & \sqrt{(2+x)}=y \\ \hline 4=y^2+z^2 & \sqrt{(4-x^2)}=zy \\ 4+2\sqrt{7}=y^2+2yz+z^2 & 2\sqrt{(4+3)}=2zy \end{array}$$

$\sqrt{(4+2\sqrt{7})}=y+z$, ergo ſumma iſtarum quantitatum imaginariarum eſt realis, & productum $1+\sqrt{(2+\sqrt{3})}\times 1+\sqrt{(2-\sqrt{-3})}=1+\sqrt{(4+2\sqrt{7})}+\sqrt{7}$ pariter reale.

§. 310.

§. 310.

Sit x^m, & x abeat in $x+e$, erit

$$(x+e)^m = x^m + \frac{mex^{m-1}}{1} + \frac{m.m-1}{1.2} e^2 x^{m-2}$$

Si prior series ab hac subtrahatur, habebitur

$$(x+e)^m - x^m = \frac{mex^{m-1}}{1} + \frac{m.m-1}{1.2} e^2 x^{m-2} + \frac{m.m-1.m-2}{1.2.3} e^3 x^{m-3}$$

abeat iterum x in $(x+e)$ erit

$$(x+e)^{m-1} = x^{m-1} + \frac{m-1.ex^{m-2}}{1} + \frac{m-1.m-2}{1.2.} e^2 x^{m-3}$$

$$(x+e)^{m-2} = x^{m-2} + \frac{m-2}{1} ex^{m-3} + \frac{m-2.m-3}{1.2} e^2 x^{m-4}$$

$$(x+e)^{m-3} = x^{m-3} + \frac{m-3}{1} ex^{m-4} + \frac{m-3.m-4}{1.2} e^2 x^{m-5}$$

&c. &c. &c.

$$(me)(x+e)^{m-1} = mex^{m-1} + \frac{m.m-1.e}{1} e^2 x^{m-2} + \frac{m.m-1.m-2}{1.2} e^3 x^{m-3}$$

$$\frac{m.m-1}{1.2} e^2 (x+e)^{m-2} = \frac{m.m-1.}{1.2} e^2 x^{m-2} + \frac{m.m-1.m-2}{1.2} e^3 x^{m-3}$$

$$\frac{m.m-1.m-2}{1.2.3} e^3 (x+e)^{m-3} = \frac{m.m-1.m-2}{1.2.3} e^3 x^{m-3} + \frac{m.m-1.m-2.m-3}{1.2.3} e^4 x^{m-4}$$

$$\frac{m.m-1.m-2.m-3}{1.2.3.4} e^4 (x+e)^{m-4} = \frac{m.m-1.m-2.m-3}{1.2.3.4} e^4 x^{m-4}$$

Subtrahatur hæc series à prima differentiali, erit

$$+\frac{m.m-1.m-2}{1.2.3}e^{3}x^{m-3}+\frac{m.m-1.m-2.m-3}{1.2.3.4}e^{4}x^{m-4}$$

$$+\frac{m.m-1.m-2.m-3}{1.2.3.4}e^{4}x^{m-4}+=1^{ma}$$ series differ.

$$+\frac{m-1.m-2.m-3}{1.2.3}e^{3}x^{m-4}+\frac{m-1.m-2.m-3.m-4}{1.2.3.4}e^{4}x^{m-5}$$

$$+\frac{m-2.m-3.m-4}{1.2.3}e^{3}x^{m-5}+\frac{m-2.m-3.m-4.m-5}{1.2.3.4}e^{4}x^{m-6}$$

$$+\frac{m-3.m-4.m-5}{1.2.3}e^{3}x^{m-6}+\frac{m-3.m-4.m-5.m-6}{1.2.3.4}e^{4}x^{m-7}$$

$$+\frac{m.m-1.m-2.m-3}{1.2.3}e^{4}x^{m-4}$$

$$+\frac{m.m-1.m-2.m-3}{1.2.2}e^{4}x^{m-4}+\frac{m.m-1.m-2.m-3.m-4}{1.2.2.3}e^{5}x^{m-5}$$

$$+\frac{m.m-1.m-2.m-3.m-4}{1.2.2.3}e^{5}x^{m-5}+\frac{m.m-1.m-2.m-3.m-4.m-5}{1.2.2.3.3}e^{6}x^{m-6}$$

$$\frac{m.m-1.e^2 x^{m-2}}{1} + \frac{m.m-1.m-2.e^3 x^{m-3}}{1.1}$$

abeat denuo

$$m.m-1.e^2(x+e)^{m-2} = m.m-1.e^2 x^{m-2} + \frac{m.m-1.m-2}{1} e^3 x^{m-3}$$

$$(m.m-1.m-2)\,e^3(x+e)^{m-3} = \frac{m.m-1.m-2\,e^3 x^{m-3}}{1}$$

$$\frac{m.m-1.m-2.m-3}{1.3} e^4(x+e)^{m-4} =$$

$$\frac{m.m-1.m-2.m-3}{2.2} e^4(x+e)^{m-4} =$$

$$m.m-1.m-2\,e^3 x^{m-3} \quad \left.\begin{matrix} +\frac{m.m-1.m-2.m-3}{1.1} \\ +\frac{m.m-1.m-2.m-3}{1.2} \end{matrix}\right\} e^4 x^{m-4}$$

fiat $x = x + e$

$$m.m-1.m-2\,e^3(x+e)^{m-3} = m.-1.m-2\,e^3 x^{m-3}$$

$$m.m-1.m-2.m-3\,e^4(x+e^{m-4} =$$

$$\frac{m.m-1.m-2.m-3}{1.2} e^4(x+e)^{m-4} =$$

$$m.m-1.m-2.m-3\,e^4 x^{m-4}$$

$$\left.\begin{array}{l} +\frac{m.m-1.m-2.m-3}{1.3} \\ +\frac{m.m-1.m-2.m-3}{2.2} \end{array}\right\} e^4 x^{m-4} \quad + = 2^{da}$$ series differentialis

x in $x + e$

$$+\frac{m.m-1.m-2.m-3.\ e^4 x^{m-4}}{1.2} \qquad +\frac{m.m-1.m-2.m-3.m-4\ e^5 x^{m-5}}{1.2.3}$$

$$+m.m-1.m-2.m-3\ e^4 x^{m-4} \qquad +\frac{m.m-1.m-2.m-3.m-4\ e^5 x^{m-5}}{1.2}$$

$$\frac{m.m-1.m-2.m-3\ e^4 x^{m-4}}{1.3} \qquad +\frac{m.m-1.m-2.m-3.m-4\ e^5 x^{m-5}}{1.3}$$

$$\frac{m.m-1.m-2.m-3\ e^4 x^{m-4}}{2.2} \qquad +\frac{m.m-1.m-2.m-3.m-4\ e^5 x^{m-5}}{2.2}$$

= series 3^{tia} differentialis.

$$+m.m-1.m-2.m-3\ e^4 x^{m-4}$$

$$m.m-1.m-2.m-3\ e^4 x^{m-4}$$

$$\frac{m.m-1.m-2.m-3\ e^4 x^{m-4}}{1.2}$$

+ &c. = series 4^{ta} differentialis.

Applicemus hanc theoriam ad numeros ſequentes

Diff. 2.	Diff. 1.	Quadrat.	Radices.
		3095920881	55641
2	111283	3096032164	55642
2	111285	3096143449	55643
2	111287	3096254736	55644
	111289	3096366025	55645

Differ. 2.		Differ. 1.	Quadrat.	Radices
			3095920881	55641
	8	222568	3096143449	55643
$2 . 2^2 =$	8	222576	3096366025	55645
	8	222584	3096588609	55647
		222592	3096811201	55649

Differ. 2.		Differ. 1.	Quadrat.	Radices
			3095920881	55641
	18	333855	3096254736	55644
$2 . 3^2 =$	18	333873	3096588609	55647
	18	333891	3096922500	55650
		333909	3097256409	58653

Differ. 2.		Differ. 1.	Quadrat.	Radices
			3095920881	55641
	50	556435	3096477316	55646
$2 . 5^2 =$	50	556485	3097033801	55651
	50	556535	3097590336	55656
		556585	3098146921	55661

Diff.

Diff. 4æ	Diff. 3æ	Diff. 2æ	Diff. 1æ	Biquadrat.	Radices
		110	65	16	2
1 · 3 24	48	194	175	81	3
	108	302	369	256	4
1.2.3.4 = 24	132	434	671	625	5
24	156	590	1105	1296	6
			1695	2401	7
				4096	8

Diff. 4.	Diff. 3.	Diff. 2.	Diff. 1.	Biquadrat	Radices
	960	800	240	16	2
384	1344	1760	1040	256	4
$24 \cdot 2^4 = 384$	1728	3104	2800	1296	6
384	2112	4832	5905	4096	8
		6944	10736	10000	10
			17680	20736	12
				38416	14

Diff. 4.	Diff. 3.	Diff. 2.	Diff. 1.	Biquadrat	Radices.
	4212	2862	609	16	2
1944	6156	7074	3471	625	5
$24 \cdot 3^4 = 1944$	8100	13230	10545	4096	8
1944	10044	21330	23775	14641	11
		31374	45105	38416	14
			76479	83521	17
				160000	20

Diff. 4.	Diff. 3.	Diff. 2.	Diff. 1.	Biquadrat	Radices
	28500	15950	2385	16	2
15000	43500	44450	18335	2401	7
$24 \cdot 5^4 = 15000$	58500	87950	62785	20736	12
15000	73500	146450	150735	83521	17
		219950	297185	234256	22
			517135	531441	27
				1048576	32

Diff. 3.	Diff. 2.	Diff. 1.	Cubi	Radices
			66923416	406
		495727		
	2442		67419143	407
6		498169		
	2448		67917312	408
1.2.3 = 6		500617		
	2454		68417929	409
6		503071		
	2460		68921000	410
		505531		
			69426531	411

Diff. 3.	Diff. 2.	Diff. 1.	Cubi	Radices
			66923416	406
		993896		
48	9792		67917312	408
		1003688		
	9840		68921000	410
$6 . 2^3$ = 48		1013528		
	9888		69934528	412
		1023416		
48	9936		70957944	414
		1033352		
			71991296	416

Diff. 3.	Diff. 2.	Diff. 1.	Cubi	Radices
			66923416	406
	22086	1494513		
162			68417929	409
	22248	1516599		
			69934528	412
$6 . 3^3$ = 162		1538847		
	22410		71473375	415
		1561257		
162	22572		73034632	418
		1583829		
			74618461	421

			Cubi	Radices
			66923416	406
	39360	1997584		
$6 . 4^3$ = 384			68921000	410
	39744	2036944		
384			70957944	414
	40128	2076688		
			73034632	418
		2116816		
			75151448	422

§§. 80. & seqq. aliquot problemata utilia adjiciam.

Probl.

Datis quatuor numerorum in progressione arithmetica summa a, & cuborum eorundem summa b, invenire ipsos numeros

Sol.

Sit x terminus primus, & y secundus, & erunt termini x; y; $\frac{1}{2}a - y$; $\frac{1}{2}a - x$. & ex natura progressionis arithmeticæ est $2a = \frac{1}{2}y - y + x$, ergo $3y = \frac{1}{2}a + x$ hinc progressio fiet x; $\frac{a + 2x}{6}$; $\frac{a - x}{3}$; $\frac{a - 2x}{2}$ vel x; $\frac{1}{6}a + \frac{1}{3}x$; $\frac{1}{3}a - \frac{1}{3}x$; $\frac{1}{2}a - x$. Jam summa cuborum horum terminorum debet esse $= b$.

$$x^3$$
$$\tfrac{1}{216}a^3 + \tfrac{1}{36}a^2x + \tfrac{1}{18}ax^2 + \tfrac{1}{27}x^3$$
$$\tfrac{1}{27}a^3 - \tfrac{1}{9}a^2x + \tfrac{1}{9}ax^2 - \tfrac{1}{27}x^3$$
$$\tfrac{1}{8}a^3 - \tfrac{3}{4}a^2x - \tfrac{3}{4}ax^2 - x^3$$

$$\tfrac{5}{3}ax^2 - \tfrac{5}{6}a^2x + \tfrac{1}{6}a^3 = b$$
$$x^2 - \tfrac{1}{2}ax + \tfrac{1}{10}a^2 = \frac{3b}{5a}$$
$$x = \tfrac{1}{4}a \pm \sqrt{\left(\frac{48b - 3a^3}{80a}\right)}$$

Probl.

Cognitis progressionis arithmeticæ summa terminorum a & summa cuborum eorundem b, & numero terminorum c, invenire ipsos numeros

Sol.

In serie arithmetica sequente, ubi A est terminus primus & d differentia

$$A;\ A + d;\ A + 2d;\ A + 3d;\ A + 4d;\ A + 5d;\ A + 6d = a$$
$$d + 2d + 3d + 4d + 5d + 6d = a - 7A$$
$$d(1 + 2 + 3 + 4 + 5 + 6) = a - 7A$$

Cum

Cum data ſit ſumma progreſſionis arithmeticæ $= a$, & numerus terminorum ſit $= c$, vocemus $c - 1 = n$, & primum terminum $= x$

$$\frac{a - cx}{1.2.3\ldots + n} \text{ vel } \frac{a - (n + 1)x}{1 + 2 + 3 + \ldots n} = d$$

jam vero $1 + 2 + 3 + 4 + \ldots\ldots n = n + 1 . \frac{n}{2}$ §. 90.

hinc formula pro invenienda differentia d abit in ſequentem $\frac{a - (nx + 1)x}{(nn + n) : 2}$

$= \frac{2a}{n^2 + n} - \frac{2x}{n}$, ergo progreſſio habebit formam ſequentem

$$x;\ \left(\frac{n-2}{n}\right)x + \frac{2a}{n^2+n};\ \left(\frac{n-4}{n}\right)x + \frac{4a}{n^2+n};\ \left(\frac{n-6}{n}\right)x + \frac{6a}{n^2+n};\ \left(\frac{n-8}{n}\right)x + \frac{8a}{n^2+n}$$

Jam ex forma apparet, ſeriem eſſe arithmeticam, quia quilibet terminus bis ſumtus æquivalet ſuis duobus utrinque poſitis vicinis v. g.

$$\left(\frac{(n-4)x}{n} + \frac{4a}{n^2+n}\right) \times 2 = \left(\frac{n-2}{n}\right)x + \frac{2a}{n^2+n} + \left(\frac{n-6}{n}\right)x + \frac{6a}{n^2+n}.$$

Deinde quicunque ſit terminorum numerus, ſumma ſemper eſt $= a$, namque poſito $c = 4$ adeoque $n = 3$, quatuor termini faciunt ſummam a quia terminorum æqualiter ab extremis diſtantium membra x continentia ſeſe deſtruunt, inde etiam ſequitur ſi ad cubos horum terminorum conficiendos progrediamur, ſummam omnium ubi x^3 evaneſcere,

Cubi.

$$x^3$$

$$\left(\frac{n-2}{n}\right)^3 x^3 + \frac{3(n-2)^2\, 2ax^2}{n^2(n^2+n)} + \frac{3(n-2)4a^2x}{n(n^2+n)^2} + \frac{8a^3}{(n^2+n)^3}$$

$$\left(\frac{n-4}{n}\right)^3 x^3 + \frac{3(n-4)^2\, 4ax^2}{n^2(n^2+u)} + \frac{3(n-4)16a^2x}{n(n^2+n)^2} + \frac{64a^3}{(n^2+n)^3}$$

Jam in ſummandis his terminis negligamus eos, ubi x^3 occurrit, & ſumma cuborum ſequentem æquationem producet.

$2(n-2)^2$

$$\frac{2(n-2)^2+4(n-4)^2+6(n-6)^2+8(n-8)^2+10(n-10)^2 \text{ \&c. in } 3ax^2}{n^2\ (n^2+n)} +$$

$$\frac{4(n-2)+16(n-4)+36(n-6)+64(n-8)+81(n-10) \text{ \&c. in } 3a^2x}{n(n^2+n)^2}$$

$$+\frac{8+64+216+512 \text{ \&c. in } a^3}{(n^2+n)^3} = b$$

Quæ æquatio cum ſit quadratica, facile ex ea invenietur valor primi termini x

ſcilicet

$$x^2+\frac{4(n-2)+16(n-4)+36(n-6)+64(n-8)+\text{\&c. in } ax}{2(n-2)^2+4(n-4)^2+6(n-6)^2+\text{\&c. in } (n+1)} =$$

$$\frac{(8+64+216+512+\text{\&c.}) \text{ in } -\frac{2}{3}a^2}{2(n-2)^2+4(n-4)^2+6(n-6)^2+\text{\&c. in } (n+1)^2}$$

$$+\frac{b\,(n^2\,.\,n^2+n)}{2(n-2)^2+4(n-4)^2+6(n-6)^2+\text{ in } 3a}$$

Exempl.

Quæruntur ſex numeri in progreſſione arithmetica, quorum ſumma ſit 21, ſumma cuborum 441. Hic itaque $a=21$, & $b=441$; $c=6$, unde $n=5$. hinc primum membrum rationale

$$\frac{(4.3+16.1+36.-1+64.-3+100.-5) \text{ in } -\frac{21}{2}}{(2.9+4.1+6.1+8.9+10.25) \text{ in } 6} \text{ vel}$$

$$\frac{(12+16-36-192-500) \text{ in } -\frac{21}{2}}{(18+4+6+72+250) \text{ in } 6} = \frac{-700 \times -\frac{21}{2}}{350 \times 6} \text{ vel } \tfrac{7}{2}$$

unde $(\frac{7}{2})^2 = \frac{49}{4}$ conficiet primum membrorum, quæ ſub vinculo radicali ſunt.

Quoad alterum membrum, quod sub vinculo est, habemus

$$\frac{(8+64+216+512+1000) \times -\frac{441}{3}}{(2.9+4.1+6.1+8.9+10.25) \times 36} = \frac{-264600}{12600} = -21$$

Tertium membrum quod sub vinculo est

$$\frac{441 \times 25 \times 30}{(18+4+6+72+250) \times 63} = \frac{330750}{350 \times 63} = 15.$$

hinc totum membrum irrationale erit $\surd(\frac{49}{4} - 21 + 15) = \frac{5}{2}$

Ergo terminus extremus major $\frac{7}{2} + \frac{5}{2} = 6$ & minor $\frac{7}{2} - \frac{5}{2} = 1$ differentia 5 divisa per $n = 5$ est differentia progressionis unde numeri 1.2. 3.4.5.6

2. Exempl.

Quæruntur numeri novem in progressione arithmetica quorum summa $= 39$ & summa cuborum $= 927\frac{1}{3}$ ergo $a = 39$, $b = 927\frac{1}{3}$; $n = 8$.

Membrum rationale erit

$$\frac{(4.6+16.4+36.2+64.0+100.-2+144.-4+196.-6+256.-8) \times -\frac{39}{2}}{(2.36+4.16+6.4+8.0+10.4+12.16+14.36+16.64) \times 9}$$

quod valet $\frac{-3840 \times -\frac{39}{2}}{(72+64+24+40+192+504+1024) \times 9} = \frac{74880}{17280} = 4\frac{1}{3}$

unde $(4\frac{1}{3})^2 = \frac{169}{9}$ est primum membrum sub vinculo.

Secundum membrum sub vinculo fiet

$$\frac{(8+64+216+512+1000+1728+2744+4096) \times -507}{1920 \times 81} =$$

$$\frac{-5256516}{155520} = -33\frac{4}{5}$$

Tertium sub vinculo membrum

$\frac{64 \times 927\frac{1}{3} \times 72}{1920 \times 117} = 19\frac{1}{45}$ Hinc erit integrum membrum irrationale $\surd(\frac{169}{9} - 33\frac{4}{5} + 19\frac{1}{45}) = 2$

Vnde

Vnde extremi progressionis termini erunt major $= 4\frac{1}{3} + 2$; minor $= 4\frac{1}{3} - 2$, horum ergo differentia 4 dividatur per 8, quotiens $\frac{1}{2}$ erit differentia progressionis, adeoque numeri

$$\frac{14}{6};\ \frac{17}{6};\ \frac{20}{6};\ \frac{23}{6};\ \frac{26}{6};\ \frac{29}{6};\ \frac{32}{6};\ \frac{35}{6};\ \frac{38}{6}$$

§. 105.

Invenire quinque numeros in progressione geometrica, quorum summa sit a & summa quadratorum = b

Sol.

Sint numeri x; xy; xy^2; xy^3; xy^4, erit itaque

$$a = x(1 + y + y^2 + y^3 + y^4) = \frac{x(1-y^5)}{1-y}$$

$$b = x^2(1 + y^2 + y^4 + y^6 + y^8) = \frac{xx(1-y^{10})}{1-yy} \text{ unde}$$

$$\frac{b}{aa} = \frac{(1+y^5)(1-y)}{(1-y^5)(1+y)} \text{ sit } \frac{b}{aa} = \frac{m}{n}, \ \& \ y = \frac{1-z}{1+z}$$

$$\text{hinc } \frac{m}{n} = \frac{2z}{2} \cdot \frac{2 + 20z^2 + 10z^4}{10z + 20z^3 + 2z^5} = \frac{1 + 10z^2 + 5z^4}{5 + 10z^2 + z^4}, \text{ ergo}$$

$$mz^4 + 10mz^2 + 5m = 5nz^4 + 10nz^2 + n \text{ seu}$$

$$z^4 = \frac{10(m-n)z^2 + 5m - n}{5n - m}$$

$$z^2 = \frac{5(m-n) \pm 2\sqrt{(5m^2 - 6mn + 5n^2)}}{5n - m}$$

§. 64.

Sit insula datæ perimetri, sintque duo viatores eodem tempore ex eodem perimetri loco A versus eandem plagam egredientes, & insulam datis celeritatibus

 cir-

circumeuntes, sitque celeritas præcedentis major celeritate sequentis: determinare locum concursus B, *numerum dierum itineris, itemque quoties priori, quoties posteriori insula circumeunda sit, donec se invicem assequantur.*

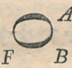
Solutio.

Sit celeritas tardioris $= c$; Perimeter insulæ $= p$

— — — celerioris $= C$; numerus dierum itineris $= x$

totum iter tardioris $= v$

Quoniam celeritates dantur per spatia eodem tempore confecta, adeoque & per itinera diurna exhiberi possunt, differentia itinerum diurnorum exprimatur per $C - c$. Porro quia tardior sequens celeriorem præeuntem nunquam assequitur, ex natura problematis, nempe per conditionem viæ in se redeuntis intelligitur, quod præcedens suæ celeritatis excessu tardiorem à tergo tandem sit assecuturus, simul ac excessus celeritatum seu itinerum diurnorum aliquoties repetitus totam insulæ perimetrum adæquaverit, eo enim *ABFA* intervallo



sub initium motus celerior à tardiore distare concipiendus est. Quoniam igitur celeritate manente eadem spatia sunt temporibus proportionalia; erit *differentia itinerum diurnorum i. e. spatium, quod celerior præ tardiore uno die lucratur, ad spatium quod toto tempore quæsito lucrandum est, ut tempus unius diei ad tempus quæsitum, quo celerior tardiorem assequitur*

est adeo $C - c : p = 1 : x$ vel

$$x = \frac{p}{C - c}$$

Cum præterea sit *tempus unius diei ad totum tempus quæsitum ut iter diurnum tardioris ad totum iter á tardiore in tempore quæsito conficiendum*, erit

$$1 : x = c : v$$

$$1 : \frac{p}{C - c} = c : v$$

$$v = \frac{cp}{C - c}$$

In

In qua expreſſione cum p denotet perimetrum inſulæ ſeu unam circuitionem, ejus coefficiens $\frac{c}{C-c}$ indicabit numerum circuitionum à tardiore factarum, cui ſi addatur unitas, prodibit $\frac{c}{C-c} + 1 = \frac{C}{C-c} =$ numero circuituum à celeriori factorum. Tandem ſi in caſibus ſpecialibus $\frac{c}{C-c}$ æquivalet numero integro, manifeſtum eſt, tum locum concurſus contingere in ipſo loco egreſſus A, ſeu eſſe diſtantiam $AB = 0$, ſi vero $\frac{c}{C-c}$ eſt numerus fractus, diviſione actu inſtituta, fractio reſidua exprimet rationem, quam habet ultima circuitio non abſoluta ad unam circuitionem integram, i. e. dabit rationem quam habet loci concurſus B à loco egreſſus A diſtantia AB ad integram perimetrum $ABFA$.

Sit p 30 milliar. $C = 15$; $c = 10$, erit $x = 6$, & numerus circuitionum à tardiore factarum $= 2$. Cadit ergo B in A, & $AB = 0$.

Sit $p = 30$; $C = 16$; $c = 10$; erit $x = 5$, & numerus circuitionum à tardiore factarum $= 1\frac{2}{3}$, hinc $AB = 20$ milliar.

§. 190.

Probl.

Invenire duos numeros x & y hujus naturæ ut $x^y = y^x$

Solut.

Ponatur $az = x$ & $y = bz$, & erit

$$(az)^{bz} = (bz)^{az}$$

$$a^b z^b = b^a z^a$$

$$a^b = b^a z^{a-b}; \quad \frac{a^b}{b^a} = z^{a-b}; \quad z = \frac{a^{b:a-b}}{b^{a:a-b}}$$

ſit

sit $a - b = m$, & erit $z = \frac{a^{a-m:m}}{(a-m)^{a:m}}$

si $m = 1$, erit $z = \frac{a^{a-1}}{(a-1)^a} = \frac{a^a}{a(a-1)^a}$

adeoque $az = \left(\frac{a}{a-1}\right)^a$ & $bz = \left(\frac{a}{a-1}\right)^{a-1}$

sit $a - 1 = n$, & hæ duæ quantitates az & bz erunt $\left(1+\frac{1}{n}\right)^n$ & $\left(1+\frac{1}{n}\right)^{1+n}$

Exempl.

sit $n = \frac{1}{2}$, erit $x = \sqrt{3}$ & $y = 3^{3:2}$

ergo debet esse

$(\sqrt{3})^{\sqrt{27}} = (\sqrt{27})^{\sqrt{3}}$, & progrediendo ad logarithmos $(\sqrt{27})\ \frac{1}{2}\ l\ 3$

$= (\sqrt{3})\ \frac{1}{2}\ l\ 27$

$(\sqrt{27})\ \frac{1}{2}\ l\ 3 = (\sqrt{3})\ \frac{3}{2}\ l\ 3$

$\sqrt{27} = 3\sqrt{3}$

sit $n = 3$, & erit $x = (\frac{4}{3})^3$ & $y = (\frac{4}{3})^4$

vel $x = \frac{64}{27}$ & $y = \frac{256}{81}$ ergo

$(\frac{64}{27})^{256:81} = (\frac{256}{81})^{64:27}$

$\frac{256}{81}\ (3l4 - 3l3) = \frac{64}{27}\ (4l4 - 4l3)$

$\frac{3.256}{81}\ (l4 - l3) = \frac{4.64}{27}\ (l4 - l3)$

$\frac{3.256}{81} = \frac{4.64}{27}$

sit

sit $n = 2$, erit $x = (\frac{3}{2})^2$ & $y = (\frac{3}{2})^3$

vel $x = \frac{9}{4}$ & $y = \frac{27}{8}$, ergo $(\frac{9}{4})^{27:8} = (\frac{27}{8})^{9:4}$ vel $(\frac{9}{4})^{27} = (\frac{27}{8})^{18}$

& $27\,(2l3 - 2l2) = 18\,(3l3 - 3l2)$ vel $2 . 27 = 3 . 18$.

Sit $x = 2$, & $y = 4$ erit $2^4 = 4^2$

§. 725.

$s = ax - bx^2 + cx^3 - dx^4 + ex^5 -$ &c.

ponatur $x = \frac{y}{1-y} = y + y^2 + y^3 + y^4 + y^5 +$ &c.

$$\begin{aligned} s = ay &+ ay^2 + ay^3 + ay^4 + ay^5 \\ &- by^2 - 2by^3 - 3by^4 \\ &\quad\quad + cy^3 + 3cy^4 \\ &\quad\quad\quad\quad - dy^4 \end{aligned}$$

$s = ay - (b-a)y^2 + (c-2b+a)y^3 - (d-3c+3b-a)y^4$ &c.

sit $x = 1$ & $y = \frac{1}{2}$, erit

$s = a - b + c - d + e - f$ &c.

$$s = \frac{a}{2} - \frac{(b-a)}{4} + \frac{c-2b+a}{8} - \frac{(d-3c+3b-a)}{16}$$

Ex.

$1 - 4 + 9 - 16 + 25 - 36$ &c.

$$\begin{matrix} 3 & & 5 & & 7 & & 9 \\ & 2 & & 2 & & 2 & \end{matrix}$$

Ergo $s = \frac{1}{2} - \frac{3}{4} + \frac{2}{8} = 0$

$$\begin{matrix} 1 - 2 + 3 - 4 + 5 - 6 \\ 1 \quad\; 1 \end{matrix}$$

ergo $s = \frac{1}{2} - \frac{1}{4} = \frac{1}{4}$

§. 13.

§. 13

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + x^5 + x^6 \ \&c.$$

$$\frac{1}{1-x} = (1+x)(1+x^2)(1+x^4)(1+x^8)(1+x^{16}) + \text{ in infin.}$$

$$1 = (1-x^2)(1+x^2)(1+x^4)(1+x^8)(1+x^{16}) \ \&c. \text{ in infin.}$$

$$1 = 1 - x^{32} \ \&c. \text{ quia } x < 1.$$

§. 690.

$$z = ay + by^2 + cy^3 + dy^4 + ey^5 \ \&c.$$

quæritur expreſſio τȣ y per z

Aſſumatur ſeries

$$y = Az + Bz^2 + Cz^3 + Dz^4 + Ez^5 \ \&c.$$

& determinentur coefficientes A, B, C, D, E &c.

erit

$$ay = aAz + aBz^2 + aCz^3 + aDz^4 + aEz^5 \ \&c.$$

$$by^2 = bA^2z^2 + 2bABz^3 + bB^2z^4 + 2bBCz^5 \ \&c.$$

$$+ 2bACz^4$$

$$cy^3 = cA^3z^3 + 3cA^2Bz^4 + 3cB^2Az^5 \ \&c.$$

$$dy^4 = + dA^4z^4 + 4dA^3Bz^5 \ \&c.$$

$$ey^5 = + eA^5z^5 \ \&c.$$

&c.

$$-z = -z$$

$$0 = \left.\begin{matrix} aA \\ -1 \end{matrix}\right\} z \left.\begin{matrix} +aB \\ +bA^2 \end{matrix}\right\} z^2 \left.\begin{matrix} +aC \\ +2bAB \\ +cA^3 \end{matrix}\right\} z^3 \left.\begin{matrix} +aD \\ +bB^2 \\ +2bAC \\ +3cA^2B \\ +dA^4 \end{matrix}\right\} z^4 \left.\begin{matrix} +aE \\ +2bBC \\ +3cB^2A \\ +4dA^3B \\ +eA^5 \end{matrix}\right\} z^5$$

Jam

Jam est $aA - 1 = 0$

$$aB + bA^2 = 0$$

$$aC + 2bAB + cA^3 = 0$$

unde oriuntur valores coefficientium ABC & in serie assumta

$$A = \frac{1}{a}$$

$$B = -\frac{bA^2}{a} = -\frac{b}{a^3}$$

$$C = -\frac{2bAB - cA^3}{a} = \frac{2b^2}{a^5} - \frac{c}{a^4}$$

$$D = -\frac{bB^2 - 2bAC - 3A^2B - dA^4}{a}.$$

Si proposita fuerit series sequens

$$z = ay + cy^3 + ey^5 + gy^7 + iy^9 \text{ \&c. erit}$$

$$y = \frac{z}{a}$$

$$-\frac{cA^3z^3}{a}$$

$$-\frac{3cA^2C - eA^5}{a}z^5$$

$$-\frac{3cC^2A - 3cEA^2 - 5eCA^4 - gA^7}{a}z^7$$

Res inde manifesta est, si in expressione antecedente omittantur quantitates, quas ingrediuntur bdf &c. tunc enim prodit hæc series v. g. $aC + 2bAB + cA^3 = 0$, quoniam secundus terminus b affectus est, negligatur ut sit $aC + cA^3 = 0$, erit $C = -\frac{cA^3}{a}$

Si denique proponatur sequens series

$$az + bz^2 + cz^3 + dz^4 \text{ \&c.} = gy + hy^2 + iy^3 + ky^4 + ly^5 \text{ \&c.}$$

& quæratur expressio τῦ z per y

$$\begin{aligned} z = {} & \frac{g}{a} \cdot y \\ & + \frac{h - bA^2}{a} y^2 \\ & + \frac{i - 2bAB - cA^3}{a} y^3 \\ & + \frac{k - bBB - 2bAC - 3cA^2B - dA^4}{a} y^4 \\ & + \frac{l - 2BC - 2bAD - 3cABB - 3cA^2C - 4dA^3B - eA5}{a} y^5 \end{aligned}$$

Sit enim $z = Ay + By^2 + Cy^3 + Dy^4 + Ey^5$ &c. erit

$$\begin{array}{lllll} az = & aAy & + aBy^2 & + aCy^3 & + aDy^4 + aEy^5 \\ bz^2 = & & + bA^2y^2 & + 2bABy^3 & + bB^2y^4 + 2bBCy^5 \\ & & & & + 2bACy^4 \\ cz^3 = & & & cA^3y^3 & + 3cA^2By^4 + 3cB^2Ay^5 \\ dz^4 = & & & & dA^4y^4 + 4dA^3By^5 \\ ez^5 = & & & & eA^5y^5 \\ \text{\&c.} & & & & \\ gy = & gy & & & \\ hy^2 = & & + hy^2 & & \\ iy^3 = & & & + iy^3 & \\ ky^4 = & & & & + ky^4 \\ ly^5 = & & & & \quad + ly^5 \end{array}$$

Ergo $gy = aAy$ & $A = \frac{g}{a}$

$hy^2 = bA^2y^2 + aBy^2$ hinc $B = \frac{h - bA^2}{a}$

$iy^3 = cA^3y^3 + 2bABy^3 + aCy^3$, hinc $C = \frac{i - cA^3 - 2bAB}{a}$

§. 294.

Methodus Newtoniana inveniendi divisores æquationum.

Proponatur æquatio $x^4 + ax^3 + bx^2 + cx + d = 0$ quæritur an habeat divisores.

Ponamus esse divisorem $x + \alpha = 0$, ita ut formula sit

$$x^4 + ax^3 + bx^2 + cx + d = (x + \alpha)(x^3 + fx^2 + gx + h)$$

ergo hæc formula est divisibilis per $x + \alpha$, quicquid ponatur pro x, ergo posito $x = 0$; d erit divisibiie per α

si ponatur $x = 1$; erit $1 + a + b + c + d$ divisibile per $1 + \alpha$

si $x = 2$; erit $16 + 8a + 4b + 2c + d$ divisibile per $2 + \alpha$

&c.

Ergo hæ expressiones cognitæ

$$\left.\begin{array}{c} d \\ 1 + a + b + c + d \\ 16 + 8a + 4b + 2c + d \\ \&c. \end{array}\right\} \text{habebunt divisores in serie arithmetica progredientes.}$$

Hoc ratiocinium secutus *Clairaut* acutissimus geometra tertiam *Algebræ* suæ partem copiose & solidissime elaboravit

§. 153.

sit radius circuli r, erit

Latus trigoni regularis circulo inscripti			$= \sqrt{(3r^2)}$
— tetragoni	——	——	$= \sqrt{(2r^2)}$
— pentagoni	——	——	$= \sqrt{\frac{5}{2}r^2 - \frac{1}{2}\sqrt{5r^2}})$
— hexagoni	——	——	$= r$
— octogoni	——	——	$= \sqrt{(2r^2 - r\sqrt{2r^2})}$
— decagoni	——	——	$= -\frac{1}{2}r + \frac{1}{2}\sqrt{5r^2}$

Sit latus polygonorum regularium l

erit radius circuli cui trigonum regulare inſcribitur $= \sqrt{\tfrac{1}{3} l^2}$

——— ——— tetragonum ——— $= \sqrt{\tfrac{1}{2} l^2}$

——— ——— pentagonum ——— $= \sqrt{\left(\frac{2l^2}{5-\sqrt{5}}\right)}$

——— ——— hexagonum ——— $= l$

——— ——— octogonum ——— $= l : \sqrt{(2-\sqrt{2})}$

——— ——— decagonum ——— $= 2l : \sqrt{5} - 1$

Sit radius ſphæræ $= r$

Latus tetraedri ſphæræ inſcripti $= 2\sqrt{\tfrac{2}{3} r^2}$

— hexaedri ——— ——— $= 2\sqrt{(\tfrac{1}{3} r^2)}$

— octaedri ——— ——— $= 2\sqrt{(\tfrac{1}{2} r^2)}$

— dodecaedri ——— ——— $= \frac{-r + r\sqrt{5}}{\sqrt{3}}$

— icoſaedri ——— ——— $= \frac{2r(\sqrt{5}-1)}{\sqrt{(10-2\sqrt{5})}}$

Sit latus corporum regularium l

erit radius ſphæræ, cui tetraedrum inſcribitur $= \tfrac{1}{2}\sqrt{\tfrac{3}{2} l^2}$

——— ——— cubus ——— $= \tfrac{1}{2}\sqrt{3 l^2}$

——— ——— octaedrum ——— $= l : \sqrt{2}$

——— ——— dodecaedrum ——— $= l\sqrt{3} : -1 + \sqrt{5}$

——— ——— icoſaedrum ——— $= l(\sqrt{10-2\sqrt{5}}) : \tfrac{1}{2}(\sqrt{5}-1)$

Element. Arithmet. Cap. IV. Probl. VIII. addantur exempla ſequentia in exercitium tyronum.

Quadrat.	Rad.
3894643909611477961	1973485219
7779575352911957243683441	2789189013479
144295118900085222589611477961	379861973485219

§. 463.

Æquatio generalis linearum ſecundi ordinis eſt

$$o = \Delta + \alpha x + \beta y + (\gamma x + \delta y)(\varepsilon x + \zeta y);$$

Δ denotat conſtantem

Ad portionis curvæ naturam inveſtigandam, quæ abſciſſæ x = infinitæ reſpondet, conſideretur membrum ultimum

$$(\gamma x + \delta y)(\varepsilon x + \zeta y)$$

Hujus vel uterque factor eſt imaginarius, vel uterque realis

I. Si uterque eſt imaginarius, curva nullum habebit ramum, qui in infinitum excurrit.

II. Si uterque factor fuerit realis, duo caſus ſunt evolvendi

a) ſi duo factores ſint inæquales

hoc caſu, facto $x = \infty$, alter factor finitum habere debet valorem, quia alias tota expreſſio non poſſet eſſe nihilo æqualis. Sit ergo $\gamma x + \delta y = A$ erit ob $x = \infty$, $\frac{y}{x} = -\frac{\gamma}{\delta}$, & $o = \Delta + (\alpha x + \beta y) + A(\varepsilon x + \zeta y)$ ubi ob Δ finitum fit $A = \frac{-\alpha x - \beta y}{\varepsilon x + \zeta y} = -\frac{\alpha\delta + \beta\gamma}{\varepsilon\delta - \zeta\gamma}$, natura hujus rami in infinitum excurrentis exprimitur hac æquatione $\gamma x + \delta y = \frac{\beta\gamma - \alpha\delta}{\delta\varepsilon - \gamma\zeta}$ quæ eſt pro linea recta, quæ ergo in infinitum producta cum curva confunditur, ideoque ejus eſt aſymptotos. Similiter alter factor $\varepsilon x + \zeta y$ monſtrabit aſymptoton, cujus erit æquatio $\varepsilon x + \zeta y = \frac{\beta\varepsilon - \alpha\zeta}{\gamma\zeta - \delta\varepsilon}$.

b) Si ambo factores ſint inter ſe æquales, ſeu $\varepsilon = \gamma$; & $\zeta = \delta$ fiet $o = \Delta + (\alpha x + \beta y) + (\gamma x + \delta y)^2$ & ob evaneſcens Δ præ in-

infinito, fiet $(\gamma x + \delta y)^2 + (\alpha x + \beta y) = 0$, quæ eſt æquatio ad parabolam, & oſtendit, curvam in infinitum eſſe parabolam. Erit ergo tota curva parabola.

Æquatio generalis pro lineis tertii ordinis,

$$0 = \Delta + (\alpha x + \beta y) + (\gamma x + \delta y)(\varepsilon x + \zeta y) + (\eta x + \theta x)(\iota x + \varkappa y)(\lambda x + \mu y)$$

quomodo curvæ portiones in infinitum abeuntes ſint comparatæ, ita definietur. Sumatur membrum ultimum in ſuos factores ſimplices reſolutum $(\eta x + \theta y)(\iota x + \varkappa y)(\lambda x + \mu y)$

I Vel erunt duo factores imaginarii, uniusque $\eta x + \theta y$ realis, debebit facto x vel $y = \infty$ hic factor eſſe finitus, ut à præcedente membro infinito hoc membrum tolli poſſit. Sit ergo $\eta x + \theta y = A$, erit $(\gamma x + \delta y)(\varepsilon x + \zeta y) + A(\iota x + \varkappa y)(\lambda x + \mu y) = 0$, hincque ob $\frac{x}{y} = -\frac{\theta}{\eta}$ erit $A = \frac{-(\delta\eta - \gamma\theta)(\zeta\eta - \varepsilon\theta)}{(\varkappa\eta - \iota\theta)(\mu\eta - \lambda\theta)}$ unde habetur æquatio pro aſymptoto una.

II. Sint omnes tres factores membri ultimi reales iique inter ſe

a) inæquales omnes, unusquisque præcedente modo tractatus dabit unam aſymptoton, unde curva habebit tres aſymptotos & ſex ramos in infinitum excurrentes,

b) ſint duo factores æquales, nempe $\iota = \eta$ & $\varkappa = \theta$, tertius factor $\lambda x + \mu y$ unam præbebit aſymptoton. Factores autem æquales poſito $x = \infty$ ponantur $(\eta x + \theta y)^2 = P$, erit $P(\lambda x + \mu y) + (\gamma x + \delta y)(\varepsilon x + \zeta y) + (\alpha x + \beta y) + \Delta = 0$.

α) Si membrum $(\gamma x + \delta y)(\varepsilon x + \zeta y)$ penitus deſit, fiet $P(\lambda x + \mu y) + \alpha x + \beta y = 0$ & $P = 0$, & ob $\frac{x}{y} = -\frac{\theta}{\eta}$ erit $P = -\frac{\alpha\theta - \beta\eta}{\lambda\theta + \mu\eta}$ ac propter duplicem valorem $\sqrt{P}$ curva tres habebit aſymptotas & ſex ramos in infinitum excurrentes.

β) Si membrum ſecundum non deſit, vel alter factor $\gamma x + \delta y$ eſt æqualis ipſi $\eta x + \theta y = \sqrt{P}$, vel neuter ipſi eſt æqualis

1) ſit $\eta x + \theta y = \gamma x + \delta y = \sqrt{P}$, erit $P(\lambda x + \mu y) + (\varepsilon x + \zeta y)$

$\sqrt{P}$

$\sqrt{P} + (\alpha x + \beta y) = 0$, & ob $\frac{y}{x} = -\frac{\eta}{\theta}$ erit $P(\lambda\theta - \mu\eta) + (\varepsilon\theta - \zeta\eta)\sqrt{P} + \alpha\theta - \beta\eta = 0$ unde duplex pro $\sqrt{P}$ nascitur valor, ex quo curva tres habebit asymptotas. 2) Si uterque factor membri secundi fuerit æqualis ipsi $\eta x + \theta y$, tum fiet $P(\lambda x + \mu y) + P + \alpha x \pm \beta y = 0$, hic P unicum habet valorem, unde curva habebit duas asymptotas. 3) Si neuter factor membri secundi fuerit æqualis $\eta x \pm \theta y$, tunc erit $P(\lambda x \pm \mu y) \pm (\gamma x \pm \delta y)(\varepsilon x \pm \zeta y) = 0$ & ob $\frac{y}{x} = -\frac{\eta}{\theta}$ erit $P = \frac{-(\gamma\theta - \delta y)(\varepsilon\theta - \zeta\eta)}{\lambda\theta - \mu\eta}\,\frac{x}{\theta}$ ideoque curva hoc casu præter unam asymptoton habebit duos ramos parabolicos in infinitum excurrentes

c) sint omnes tres factores membri supremi æquales & ponatur $(\eta x \pm \theta y)^3 = P^3$ erit $P^3 \pm (\gamma x \pm \delta y)(\varepsilon x \pm \zeta y) \pm (\alpha x \pm \beta y) = 0$.

α) desit membrum secundum, erit $P^3 \pm \alpha x \pm \beta y = 0$ ideoque $P^3 = -\alpha x - \beta y = (-\alpha\theta - \beta\eta)\,\frac{x}{\theta} = (\eta x \pm \theta y)^3$ & habebit curva duos ramos parabolicos secundi gradus in infinitum abeuntes.

β) Non desit membrum secundum, sit autem 1) ejus uterque factor $\eta x \pm \theta y = P$, erit iterum $P^3 \pm P^2 \pm \alpha x \pm \beta y = 0$, ideoque P^3 infinitum primi gradus & propterea $P^3 \pm \alpha x \pm \beta y = 0$, unde curva erit eadem quæ lit. α. 2) Sit unius tantum factor $\varepsilon x \pm \zeta y = \eta x \pm \theta y = P$, erit $P^3 \pm P(\gamma x \pm \delta y) \pm \alpha x \pm \beta y = 0$, erit ergo vel P finitum adeoque $P = -\frac{(\alpha x \pm \beta y)}{\gamma x \pm \delta y}$ vel P^2 = infinito unius dimersionis, ut sit $P = \sqrt{} - (\gamma\theta - \delta\eta)\,\frac{x}{\theta} = \eta x \pm \theta y$ un-

de

de curva duos habebit ramos parabolicos, illo casu autem unam asymptoton. 3) Sit neuter factor membri secundi ipsi $\eta x + \theta y$ æqualis, erit $P^3 + (\gamma x + \delta y)(\varepsilon x + \zeta y) = 0$ & $P^3 = -(\gamma\theta - \delta\eta)(\varepsilon\theta - \zeta y)\frac{xx}{\theta\theta} = (\eta x + \theta y)^3$ ex quo curva nullam habebit asymptoton at duos ramos parabolicos in infinitum excurrentes speciei $y^3 = ax^2$

Enumeratio generum curvarum trium dimensionum

I. Curvæ duobus ramis asymptoticis in infinitum excurrentes.

II. Curvæ duobus ramis parabolicis in infinitum abeuntes speciei $yy = ax$

III. Curvæ duobus ramis parabolicis in infinitum excurrentes, speciei $y^3 = a^2x$

IV. Curvæ duobus ramis parabolicis in infinitum abeuntes speciei $y^3 = ax^2$

V. Curvæ quatuor ramis asymptoticis in infinitum excurrentes.

VI. Curvæ quatuor ramis duobus asymptoticis & duobus parabolicis (speciei $yy = ax$) in infinitum excurrentes.

VII. Curvæ sex ramis asymptoticis in infinitum abeuntes.

§. 665.

Duplex est hujus æquationis & aliarum similium finis: $y^3 + a^2y - 2a^3 + axy - x^3 = 0$. Vnus est, ut y per seriem eo magis convergentem determinetur, quo minor quantitas x assumitur; alter, ut y exprimatur eo exactius, & eo convenientius veritati, quo major x assumitur. In primo casu quæstio est, invenire y, si $x =$ infinite parvo, in secundo si $x =$ infinite magno. In utroque casu plures termini æquationis præ reliquis evanescnnt, & ii saltem, qui residui sunt, considerantur. Ita in æquatione proposita, si x est infinite parvum, terminus x^3 evanescit præ

præ $2a^3$, & axy præ aay ita ut supersit sequens æquatio $y^3 + aay - 2a^3 = 0$ unde invenitur $y = a$ reliquis radicibus futuris imaginariis. Ergo si x est quantitas admodum parva, concluditur $y =$ seriei convergenti, cujus primum membrum a & reliqui termini sint dignitates τ8 x, quarum exponentes crescant, adeoque sumetur $y = a + Ax + Bx^2 + Cx^3$ &c. quæ series substituta in æquatione data exhibet

$$\left.\begin{array}{llllll} y^3 = a^3 & + 3aaAx & + 3aaBx^2 & + 3aaC & & \\ & & + 3aAA & + 6aAB & x^3 & \\ & & & + A^3 & & \\ & & & & & \text{\&c.} \\ a^2y = a^3 & + aaAx & + aaBx^2 & + aaCx^3 & & \\ axy = & aax & + aAx^2 & + aBx^3 & & \\ -2a^3 = -2a^3 & & & & & \\ -x^3 = & & & -x^3 & & \end{array}\right\} = 0$$

$$4aaA + aa = 0, \text{ unde } A = -\tfrac{1}{4}$$

$$\frac{3a}{16} + 4a^2B - \tfrac{1}{4}a = 0, \text{ hinc } B = \frac{1}{64a}$$

$$4a^2C - \frac{131}{128} = 0, \text{ quare } C = \frac{131}{512a^2}$$

Erit ergo $y = a - \frac{x}{4} + \frac{x^2}{64a} + \frac{131x^3}{512a^2}$ quæ series eo veritati propiorem valorem τ8 y præbet quo minor est quantitas x.

In secundo casu ubi y quæritur, si x sit quantitas notabilis, ponatur $x = \infty$, & in æquatione proposita terminus $2a^3$ evanescet præ x^3 & aay præ axy hinc æquatio erit $y^3 + axy - x^3 = 0$, jam judicandum est an y sit infinitum ejusdem vel superioris vel inferioris ordinis quam x, hic statim apparet, quod y non sit ordinis superioris, namque esse deberet $y^3 + axy > x^3$ neque etiam ordinis inferioris, quia alias foret $y^3 + axy$

 $< x^3$

$< x^3$ adeoque y eſt ejusdem ordinis cujus eſt x, quare terminus axy evaneſcit præ duobus reliquis, ita ut ſit $y^3 - x^3 = 0$, hinc $y = x$ valor verus τοῦ y ſi $x = \infty$ ſi vero x non ſumatur pro quantitate infinite magna, ſed ſaltem admodum magna, adjiciendi adhuc ſunt quidam termini vel poteſtates quantitatis x quarum exponentes decreſcunt, ita ut quilibet conſequens evaneſcat præ antecedente. Erit itaque forma æquationis

$$y = x + A + \frac{B}{x} + \frac{C}{x^2} + \frac{D}{x^3} + \text{\&c. hinc æquatio propoſita abibit}$$

$$\left.\begin{array}{lllll}
y^3 = & x^3 + & 3Axx & + 3Bx & + 3C \\
 & & & + 3A^2 & + 6AB \\
 & & & & + A^3 \\
a^2y = & & & + a^2x & + a^2A \\
axy = & & ax^2 & + aA & + aB \\
-x^3 = & -1 & & & \\
-2a^3 = & & & & -2a^3
\end{array} \text{\&c.} \right\} = 0$$

$$3A + a = 0, \text{ hinc } A = -\frac{a}{3}$$

$$3B + \frac{a^2}{3} + a^2 - \frac{a^2}{3} = 0 \;\&\; B = -\frac{a^2}{3}$$

$$3C + \frac{2a^3}{3} - \frac{a^3}{27} - 2a^8 - \frac{a^3}{3} - \frac{a^3}{3} = 0 \;\&\; C = \frac{55}{81}a^3$$

Cum hæc procedendi & judicandi methodus in æquationibus maxime compoſitis perquam difficilis fieri poſſit, Newtonus ope ſui parallelogrammi eam faciliorem reddere annisus eſt.

Jam

x^3	x^3y	x^3y^2	x^3y^3
x^2	x^2y	x^2y^2	x^2y^3
x	xy	xy^2	xy^3
1	y	y^2	y^3

Jam notandum eſt, ſi x eſt quantitas infinite parva, in qualibet columna termini ſuperiores evaneſcunt præ inferioribus, & vice verſa ſi x eſt quantitas infinite magna, termini inferiores evaneſcunt præ ſuperioribus. Terminis itaque æquationis alicujus propoſitæ ita diſpoſitis, in utroque caſu ſtatim apparebit quinam termini negligi poſſint, ut primus ſeriei formandæ terminus inveniatur: in primo caſu inferiores termini ſerierum verticalium, in ſecundo ſuperiores adhibentur, qui etiam homogenei conſiderantur. Conſiderandum adhuc eſt ſi v. g. x^3 & xy^3 ſint termini homogenei, & ducatur linea recta à x^3 ad xy^3 omnes termini ex una parte hujus lineæ erunt infinite majores, ex altera infinite minores, ſed ſi linea recta per centrum alicujus quadrati adhuc tranſiret, terminus ejus etiam foret homogeneus. v. g. ſi recta à x^3y ad xy^3 duceretur tranſiret ea per x^2y^2 qui adeoque foret homogeneus τοῖς x^3y & xy^3, adeoque nec infinite major nec infinite minor illis.

	x^6y					
x^5			x^5y^3			
				x^4y^4	x^4y^5	
x^3		x^3y^2				
x^2						x^2y^6
	xy			xy^4		
1		y^2				

Sit e. g. propoſita æquatio $a + bx^2 + cxy + dy^2 + ex^3 + fx^5 + gx^3y^2 + hxy^4 + ix^6y + kx^5y^3 + lx^4y^4 + mx^5y^6 + nx^4y^5 = 0$; diſpoſitis terminis in parallelogrammo uti vides. Si igitur quæratur valor τοῦ y, ſi $x = 0$ infin. parv. non niſi termini inferiores conſiderantur, hinc duæ ſunt ſolutiones, prima ſumitur

$a + dy^2$

$a + dy^2 = 0$, & valor τ8 $y = \sqrt{-\frac{a}{d}}$; secunda solutio $dy^2 + bxy^4 + mx^2y^6 = 0$, hinc vel $y = 0$, vel $d + bxy^2 + mx^2y^4 = 0$ i. e. $y = \frac{\sqrt{(-b + \sqrt{(b^2 - 4dm)})\, x^{-\frac{1}{2}}}}{\sqrt{2m}}$. Tres igitur casus sunt evolvendi

1) $y = A + Bx + Cx^2 +$ &c.

2) $y = Ax + Bx^2 + Cx^3 +$ &c.

3) $y = Ax^{-\frac{1}{2}} + Bx^{\frac{1}{2}} + Cx^{\frac{3}{2}}$ &c.

accidere potest, ut hæ series crescant x^2, x^3, quod in quolibet casu facile animadvertitur. Sed si quæratur valor τ8 y, si x est admodum magna quantitas, ponatur $x = \infty$, recta à terminis superioribus ad inferiores ducitur, unde tres nascuntur æquationes

1) $fx^5 + ix^6y = 0$, unde $y = \frac{-f}{ix}$, & series ipsa $y = \frac{-f}{ix} + \frac{B}{x^2} + Cx^{-3} + Dx^{-4}$ &c.

2) $ix^6y + kx^5y^3 + nx^4y^5 = 0$, hinc vel $y = 0$, vel $ix^2y + kxy^3 + ny^4 = 0$, & $y = Ax^{\frac{1}{2}}$, & ipsa series $y = Ax^{\frac{1}{2}} + Bx^{-\frac{1}{2}} + Cx^{-\frac{3}{2}}$

3) $nx^4y^5 + mx^2y^6 = 0$ vel $nx^2 + my = 0$, hinc $y = -\frac{nx^2}{m}$ & ipsa series $y = -\frac{nx^2}{m} + Ax + B + \frac{C}{x} + \frac{D}{x^2}$ &c.

Hæ resolutiones quam maxime locum habent, si forma lineæ curvæ investigatur, ubi abscissa est vel infinite magna vel infinite parva.

THESES.

THESES.

I.

In magnete dantur interstitia eo modo disposita ut ductus forment, qui subtiliori materiæ transitum secundum determinatam directionem concedunt, secundum aliam recusant.

II.

Eodem modo potest ferrum disponi, ita ud id loco magnetis naturalis possit adhiberi.

III.

Vapores sunt bullulæ pellicula aquea constantes & interne aere repletæ.

IV.

Hinc elevatio vaporum bene ex notissimo principio hydrostatico explicari potest: *Levius fluidum ascendit in specifice graviori seu densori. gravius descendit in leviore seu rariore.*

V.

Ros male inter meteora refertur.

VI.

Cur quidam lacus in Italia tempestate pluvia evacuati aqua sint, ex doctrina de siphonibus, quorum effluxum pressio aeris efficit, bene illustratur.

VII.

Abſurdiſſima eſt hypotheſis Abbatis de *Brancas*, qui ſtellas fixas à ſole noſtro illuminari contendit.

VIII.

Apparitiones ſtellarum variabilium optime explicantur, ſi eadem earum figura, quæ lentibus eſt, eſſe ſupponitur.

IX.

Ratio cur radii ſolares magis ad perpendiculum refringantur in medio denſiore quam rariore, eſt quia ab iſto magis attrahuntur quam ab hoc.

X.

Hinc celeritas lucis in diverſis mediis eſt in ratione inverſa denſitatis mediorum, id quod bene demonſtratur methodo, qua MAUPERTUIS utitur.

XI.

Abſurde aſſumunt Platonici & Galenus radios quibus objecta videmus, ex oculo videntis promanare.

XII.

Sit f locus imaginum in ſpeculo ſphærico cujus radius r, & d diſtantia objecti ante ſpeculum, erit $f = \frac{dr}{2d-r}$ ſi $r = \infty$ habetur ſpeculum planum, quare hic $f = -d$.

XIII.

Non datur methodus cujus ope exacte dolia ad aliquam partem evacuata menſurari poſſint.

XIV.

Lunæ jure atmosphæra videtur tribui.

XV.

Motus rotatorius corporum cœlestium circa suos axes bene ex observatione macularum in earum superficiebus positarum concluditur.

XVI.

Cometæ non differunt à planetis, nisi quod eorum orbitæ sint aliæ sectiones conicæ.

XVII.

Si terra 18es celerius circa axem volveretur, ad atomum redigeretur.

XVIII.

Si gravitas assumitur proportionata distantiæ à centro, & vis centrifuga æqualis gravitati, terra esset planum circulare, est enim hoc casu

Diameter æquatoris ad axem revolutionis ut $\sqrt{p}$ ad $\sqrt{(p-f)}$ denotante p gravitatem, f vim centrifugam.

XIX.

Systema vulgare Briggianum reliquis possibilibus jure præfertur.

XX.

Perfectum vacuum effici non potest.

XXI.

Fulgur & tonitru bene ab electricitate explicantur.

XXII.

Capite majus ſpatium ambulando abſolvimus quam pedibus.

XXIII.

x^0 eſt = 1, quicunque numerus pro x ponatur.

XXIV.

Cur tempeſtate nebuloſa Mercurius in Barometro nonnunquam aſcendat, probabiliſſime explicuit HALLEIUS cauſam hujus phænomeni à ventis horizontaliter flantibus deducens.

XXV.

Cauſſa fractionis radii ad perpendiculum eſt attractio medii denſioris.

XXVI.

Remi bene inter vectes referuntur. Vid. Phyſ. KRAFFT. P. II. §. 32.

XXVII.

Libra male inter machinas ſimplices refertur.

F I N I S.

DILVCIDATIONES ANALYSEOS FINITORVM KÆSTNERIANÆ

QVAS

PRÆSIDE

IOANNE KIES

PHYSICES ET MATHESEOS PROF. P. O.

FACVLTATIS PHILOSOPHICÆ h. t. DECANO

PVBLICE TVEBITVR

Die VIII OCTOBRIS ANNI MDCCLXII.

CHRISTIANVS CVNRADVS KLEMM,

Canſtadienſis.

MAGISTERII PHILOSOPHICI CANDIDATVS.

Tubingæ

TYPIS IO. AD. SIGMVNDI.