DE THEOREMATE TAYLORIANO,

SIVE DE

LEGE GENERALI, SECUNDUM QUAM FUNCTIONES MUTENTUR, MUTATIS A QUIBUS PENDEANT VARIABILIBUS.

DISSERTATIO

QUAM

PRO LICENTIA RITE OBTINENDA

PUBLICE TUEBITUR AUCTOR

IACOBUS SIGISMUNDUS BECK

PHILOSOPHIAE DOCTOR ET LIBERALIUM ARTIUM MAGISTER,

RESPONDENTE

FRIDERICO THEODORO POSELGER

ELBINGA-BORUSSO, IURIUM CULTORE,

ANNO MDCCXCI. DIE XVI. APRILIS,

HORIS LOCOQUE SOLITIS.

HALAE,

TYPIS FRANCKIANIS.

VIRO SUMME VENERANDO

CAROLO THEODORO WUNDSCH

PASTORI ET RECTORI SCHOLAE MARIAEBURGENSIS

HAS

INGENII PRIMITIAS

OB

MAXIMA IN SE COLLOCATA BENEFICIA

GRATO, PIOQUE ANIMO

consecrat

A U C T O R.

PROOEMIUM.

Demonstrationem profero theorematis, quod certe magni momenti inuentum putandum est. Illud enim problema resoluit: Quid fit ex y, quae functio est quantitatis x, si $x \pm b$ pro x ponitur? Basis ejusdem est theorema quod Newtonus inuenit, omnem functionem quantitatis x, reduci posse ad formam: $Ax^0 + Bx + Cx^2 + Dx^3 + Ex^4$ etcet. cui vt mihi videtur, ob vtilitatem non multum cedit. Cujus theorematis magnum momentum vel tiro in mathesi suspicari potest. Vt vero eo magis ipsius attentionem excitem, liceat mihi obseruare, niti hoc theoremate theoriam maximi et minimi, vtilioresque regulas ex illo profluere partim ad inueniendos logarithmos systematis cujuslibet, partim ad detegendas radices numerorum et radices aequationum. Qui scire cupit illius vsum, institutiones euoluat calculi differentialis Euleri, cujus pars posterior praecipue, theorematis hujus applicationem continet. At fortasse quis ex me quaerat, cur theorema illud Taylorianum dixerim. Notae sunt rixae, quae Taylorus habuit cum Mathematicis sui temporis et praecipue cum Ioanne Bernoullio. Plagii illum accusauerunt, qui inuentiones Germanorum vt suas protulerit. Quae accusatio maxime patet ex integrali $n\,dz$ vel ex $\int n\,dz = nz - \frac{z^2}{1.2}\frac{dn}{dz} + \frac{z^3}{1.2.3}\frac{d^2n}{(dz)^2} - \frac{z^4}{1.2.3.4}\frac{d^3n}{(dz)^3}$ etc. (en expressionem illi theorematis nostri fere similem) quod Leibnitius publicauit et post eum quamuis alio modo Ioan. Bernoullius in Actis Eruditorum. Hinc accusauerunt eum obscuritatis, qua plagium occultare voluerit. De hoc autem theoremate cujus demonstrationem exhibeo, persuasum mihi est, Taylorum illius esse inuentorem. Ejus enim in nullo libro ante Taylori methodum incrementorum directam et inuersam A. MDCCXV editam, qua hoc theorema continetur, vestigia reperiuntur. Itaque L' Huilier in libro: Exposition élémentaire des principes des calculs supérieurs, hoc theorema a Tayloro euolutum esse dicit. Taylorus demonstrationem addidit theoremati. Quo modo discrepet a nostra, in fine libelli observari poterit.

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§. 1.

Sit y functio variabilis x; quae quidem variabilis ſi augeatur vel minuatur quantitate Δx, mutetur functio y in y'; ſi porro x creſcat vel decreſcat quantitate $2\Delta x$ fiat y'' ex y. Similiterque deſignet y''' valorem mutatum functionis y, ſi pro x ponatur $x + 3\Delta x$ vel $x - 3\Delta x$ et generaliter fiat y^N ex y ſi pro x ponatur $x \pm n\Delta x$. Sufficiet autem variabilem concipere creſcentem tantum, quandoquidem quae de incremento functionis per creſcentem variabilem auctae demonſtrantur, vltro ad ejus decrementum per decreſcentem variabilem ortum, applicari poſſunt, modo Δx ponatur negatiuum. Iam vero primo perſpicuum eſt y'' etiam ex y' naſci, poſito in y', $x + \Delta x$ pro x. Nam poſito $x + 2\Delta x$ pro x in functione y, vno actu fit eadem quantitas, quae poſito $x + \Delta x$ in y, et iterum poſito $x + \Delta x$ in y', per duas mutationes oritur. Et generaliter: ſive in y^{N-I}, $x + \Delta x$ pro x ponatur, ſiue in y^{N-II}, $x + 2\Delta x$, ſiue in y^{N-III}, $x + 3\Delta x$ etc. ponatur, orietur eadem quantitas y^N, quae vno actu naſcitur poſito $x + n\Delta x$ pro x in functione y.

§. 2.

Sit

I.	II.	III.
$y' - y = \Delta y$	$\Delta y' - \Delta y = \Delta^2 y$	$\Delta^2 y' - \Delta^2 y = \Delta^3 y$
$y'' - y' = \Delta y'$	$\Delta y'' - \Delta y' = \Delta^2 y'$	$\Delta^2 y'' - \Delta^2 y' = \Delta^3 y'$
$y''' - y'' = \Delta y''$	$\Delta y''' - \Delta y' = \Delta^2 y''$	$\Delta^2 y''' - \Delta^2 y'' = \Delta^3 y''$
etc.	etc.	etc.

Facile perspicitur, $\Delta y'$ oriri ex Δy, si in Δy ponatur $x + \Delta x$ pro x. Nam $\Delta y = y' - y$ et $\Delta y' = y'' - y'$. Sed cum y'' fiat ex y', posito $x + \Delta x$ pro x in y', similiter ac y' ex y, posito $x + \Delta x$ pro x in y, orietur $y'' - y'$ ex $y' - y$, hoc est, orietur $\Delta y'$ ex Δy, posito pro x, $x + \Delta x$ in Δy. Eodem modo demonstramus, $\Delta^2 y'$ fieri ex $\Delta^2 y$, si in $\Delta^2 y$, pro x ponatur $x + \Delta x$. Cum enim $\Delta^2 y = \Delta y' - \Delta y$, et $\Delta^2 y' = \Delta y'' - \Delta y'$; porro $\Delta y''$ et $\Delta y'$ nascantur ex $\Delta y'$ et Δy posito $x + \Delta x$ pro x, perspicuum est $\Delta y'' - \Delta y'$, seu $\Delta^2 y'$ oriri ex $\Delta y' - \Delta y$, seu ex $\Delta^2 y$, si in $\Delta^2 y$ ponas $x + \Delta x$ pro x. Generaliter: ponendo $x + \Delta x$ pro x, in $\Delta^n y$, producitur $\Delta^n y'$.

§. 3.

Functionis per successiua incrementa ipsius x successiue auctae, valores omnes componi ex differentiis §. 2. indicatis possunt secundum formulam generalem hoc modo expressam, vt sit,

$$y_N = y + n\Delta y + \frac{n(n-1)}{1.\ 2.}\Delta^2 y + \frac{n(n-1)(n-2)}{1.\ 2.\ 3}\Delta^3 y + \frac{n(n-1)(n-2)(n-3)}{1.\ 2.\ 3.\ 4}\Delta^4 y - - - + \Delta^n y$$

Regula data valet pro $n=1$, $n=2$, $n=3$. Est enim

$$y' = y + \Delta y$$

$$y'' = y + 2\Delta y + \Delta^2 y$$

$$y''' = y + 3\Delta y + 3\Delta^2 y + \Delta^3 y$$

Iam vero si illa regula valeat pro numero r vt sit

$$y^R = y + r\Delta y + \frac{r(r-1)}{1.\,2}\Delta^2 y + \frac{r(r-1)(r-2)}{1.\,2.\,3}\Delta^3 y + \frac{r(r-1)(r-2)(r-3)}{1.\,2.\,3.\,4.}\Delta^4 y \;——\; + \Delta^r y$$

demonstrari potest, eandem omnino valere pro numero $r+1$, proptereaque pro quolibet numero n. Ponatur $R+I=N$ et $r+1=n$ et sit

$$y^R = y + r\Delta y + \frac{r(r-1)}{1.\,2}\Delta^2 y + \frac{r(r-1)(r-2)}{1.\,2.\,3}\Delta^3 + y\frac{r(r-1)(r-2)(r-3)}{1.\,2.\,3.\,4}\Delta^4 y \;——\; + \Delta^r y$$

Ex hoc orietur y^{R+I} si ponas in y^R, y' pro y, $\Delta y'$ pro Δy, $\Delta^2 y'$ pro $\Delta^2 y$, —— $\Delta^r y'$ pro $\Delta^r y$.

Unde erit

$$y^{R+I} = y' + r\Delta y' + \frac{r(r-1)}{1.\,2}\Delta^2 y' + \frac{r(r-1)(r-2)}{1.\,2.\,3.}\Delta^3 y' + \frac{r(r-1)(r-2)(r-3)}{1.\,2.\,3.\,4}\Delta^4 y' \;——\; + \Delta^r y'$$

Sed eſt

$$y' = y + \Delta y$$

$$\Delta y' = \Delta y + \Delta^2 y$$

$$\Delta^2 y' = \Delta^2 y + \Delta^3 y$$

etc.

$$\Delta^r y' = \Delta^r y + \Delta^{r+1} y$$

Ergo

$$y^{R+1} = y + (r+1)\Delta y + \frac{(r+1)r}{1.\ 2}\Delta^2 y + \frac{(r+1)r(r-1)}{1.\ 2.\ 3}\Delta^3 y + \frac{(r+1)r(r-1)(r-2)}{1.\ 2.\ 3.\ 4}\Delta^4 y + \text{---} + \frac{(r+1)r(r-1)\text{---}}{1.\ 2.\ 3\ \text{---}}\frac{(r-(r-2))}{r}\Delta^r y + \Delta^{r+1} y$$

$$= y^N = y + n\Delta y + \frac{n(n-1)}{1.\ 2}\Delta^2 y + \frac{n(n-1)(n-2)}{1.\ 2.\ 3}\Delta^3 y \text{---} + \frac{n(n-1)(n-2)\text{---}(n-(n-2))}{1.\ 2.\ 3.\ \text{---}(n-1)}\Delta^{n-1} y + \Delta^n y$$

§ 4.

Qualiscunque functio quantitatis x concipiatur eſſe y, ſiue algebraica ſiue transcendens, ſiue ex algebraicis et transcendentibus functionibus utcunque compoſita, ſemper y exprimi poterit per ſeriem hujus formae, ſiue finitam ſiue infinitam: $\mathfrak{A} + \mathfrak{B}x + \mathfrak{C}x^2 + \mathfrak{D}x^3 + \mathfrak{E}x^4 +$ etc. Nam ſi x contineatur forma $\frac{a + bx + cx^2 + \cdots}{a + bx + cx^2 + \cdots}$ haec in ſeriem recurrentem ſemper mutari poterit. Si y vero ſit functio transcendens ipsius x, reduci ad expressionem indicatam pariter poterit. Est e. g. log. nat. $(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} +$ etc. sin. $x = x - \frac{x^3}{1.2.3.} + \frac{x^5}{1.2.3.4.5.}$ etc. Similiterque respectu caeterarum functionum transcendentium res sese habet. Et cum $x^n = 1 + nlx + \frac{n^2}{2}(lx)^2 + \frac{n^3}{1.2.3}(lx)^3 + \frac{n^4}{1.2.3.4}(lx)^4$ etc. clarum est, terminos formae $M x^{\frac{m}{n}}$, quibus potestates quantitatis x sunt fractae, quoque exprimi posse, per seriem finitam aut infinitam formae

$\mathfrak{A} + \mathfrak{B} x + \mathfrak{C} x^2 \mathfrak{D} x^3 + - -$

§ 5.

Concipiatur jam y esse functio per generalem hanc seriem expressa, cujus seriei termini, si posito $x + \Delta x$ loco x, secundum theorema binomiale euoluantur et ab orta hoc modo functione y', si y subtrahatur, differentia Δy hac semper forma continebitur

$$\Delta y = A \Delta x + B (\Delta x)^2 + C (\Delta x)^3 + D (\Delta x)^4 + - -$$

$$\text{et } \frac{\Delta y}{\Delta x} = A + B \Delta x + C (\Delta x)^2 + D (\Delta x)^3 + - -$$

vbi coefficientes A, B, C etc. vtcunque constantes et variabilem contineant, nulla in sese complectentur Δx. Quum eodem modo, quo Δy ex y oritur, etiam $\Delta^2 y$ ex Δy oriatur, et $\Delta^3 y$ ex $\Delta^2 y$ et cet. patet, simili etiam modo ac $\frac{\triangle y}{\triangle x}$ expressa est, exprimi posse $\frac{\triangle^2 y}{(\triangle x^2)}$, $\frac{\triangle^3 y}{(\triangle x)^3}$ etc.

Fiet hoc modo,

$$\frac{(\triangle^2 y)}{(\triangle x)^2} = A' + B' \triangle x + C' (\triangle x)^2 + D' (\triangle x)^3 + - -$$

$$\frac{(\triangle^3 y)}{(\triangle x)^3} = A'' + B'' \triangle x + C'' (\triangle x)^2 + D'' (\triangle x)^3 + - -$$

etc.

§ 6.

Quantitas constans semper major quantitate altera variabili, quae tamen variabilis superare potest omnem quantitatem constante illa minorem est limes istius variabilis quoad magnitudinem. Sic in aequatione $y = A - Bx$ erit A quantitatis y limes quoad magnitudinem. Quantitas constans semper minor quantitate altera variabili, quae vero variabilis minor fieri potest omni quantitate constantem excedente, dicitur limes variabilis quoad paruitatem. Sic in aequatione $y = A + Bx$ est A quantitatis y limes quoad paruitatem.

§ 7.

Ratio constans semper major ratione quadam variabili dicitur ejus rationis variabilis limes, quoad magnitudinem, velut si $A - Bx$ exponens sit rationis $\frac{y}{x}$. Quia $\frac{y}{x} = A - Bx$, et A est quantitatis $A - Bx$, limes quoad magnitudinem; eadem $\frac{A}{1}$ limes erit rationis $\frac{y}{x}$ quoad magnitudinem. Ratio constans semper minor ratione quadam variabili, limes dicitur ejus rationis variabilis quoad magnitatem. Sic si $\frac{y}{x} = A + Bx$; erit A limes rationis $\frac{y}{x}$ quoad paruitatem.

§ 8.

Terminus quilibet aequationis $y = A + Bx + Cx^2 + Dx^3$ — — decrescente x, major fieri potest quam summa omnium terminorum illum sequentium. Ostendamus e. g. terminum Bx superare summam reliquorum omnium posse. Nam cum x quemlibet valorem induere posse supponatur, sit $Bx > 2Cx^2$, $Cx^2 > 2Dx^3$, $Dx^3 > 2Ex^4$ etc. Unde sequitur quantitatem x minorem poni debere quantitatibus $\frac{B}{2C}$, $\frac{C}{2D}$, $\frac{D}{2E}$, $\frac{E}{2F}$ etc. Terminorum summa progressionis geometricae, cujus terminus primus $= \frac{1}{2}$, exponens $= \frac{1}{2}$ et terminorum numerus $= n$ est $1 - \frac{1}{2^n}$ qui numerus semper est minor unitate. Ideoque si in serie $A + Bx + Cx^2 + Dx^3 +$ — — omnis terminus istum Bx sequens, antecedentis sui dimidio aequetur, summa terminorum Bx sequentium minor esse debet quantitate Bx. Ergo a fortiori concludimus majorem esse Bx quantitatem quam summa omnium terminorum eum sequentium, quorum quisque dimidio antecedentis minor existat.

§ 9.

In aequatione $\frac{\triangle y}{\triangle x} = A + B \triangle x + (\triangle x)^2 + D(\triangle x)^3 +$ — — cujus termini omnes ſint affirmatiui, minuendo quantitatem $\triangle x$ effici poteſt $B \triangle x$ major quam ſumma omnium terminorum ſequentium. Quo facto erit $A + 2 B \triangle x > (A + B \triangle x + C(\triangle x)^2 + D(\triangle x)^3 + — —)$ At limes quantitatis $A + 2 B \triangle x$ quoad paruitatem eſt A. Ergo a fortiori concludimus A limitem eſſe quantitatis $A + B \triangle x + C(\triangle x)^2 + — —$ hoc eſt rationis $\frac{\triangle y}{\triangle x}$ cujus exponens eſt $A + B \triangle x + C(\triangle x)^2 + D(\triangle x)^3 + — —$

§ 10.

Si aequationis $\frac{\triangle y}{\triangle x} = A + B \triangle x + C(\triangle x)^2 + D(\triangle x)^3 + — —$ omnes termini exceptis A et $B \triangle x$ ſint negatiui, $B \triangle x$ major fieri poteſt quam ſumma ſequentium $C(\triangle x)^2 + D(\triangle x)^3 + E(\triangle x)^4 + — —$ Quo facto $B \triangle x + C(\triangle x)^2 + D(\triangle x)^3 + —$ erit quantitas affirmatiua, et $A + B \triangle x > (A + B(\triangle x) + C(\triangle x)^2 + D(\triangle x)^3 + —)$ Sed quantitatis $A + B \triangle x$ limes quoad paruitatem eſt A. Quapropter A omnino erit quantitatis $A + B \triangle x + C(\triangle x)^2 + —$ limes quoad paruitatem. Sed $A + B \triangle x + C(\triangle x)^2 + —$ exponens eſt rationis $\frac{\triangle y}{\triangle x}$. Ideo rationis $\frac{\triangle y}{\triangle x}$ limes quoad paruitatem eſt A.

§ 11.

Sint aequationis $\frac{\triangle y}{\triangle x} = A + B \triangle x + C(\triangle x)^2 + -$ excepto A omnes termini negatiui. Minuta quantitate $\triangle x$ efficitur $B \triangle x > (C(\triangle x)^2 + D(\triangle x)^3 + -)$ Tum erit $A + 2 B \triangle x < A + B \triangle x + C(\triangle x)^2 + D(\triangle x)^3 + -$ ſed quantitatis $A + 2 B \triangle x$ limes quoad magnitudinem eſt A. Ergo A limes quoad magnitudinem erit quantitatis $A + B \triangle x + C(\triangle x)^2 + - -$ Quae quantitas cum ſit exponens rationis $\frac{\triangle y}{\triangle x}$, iſtius utique rationis limes quoad magnitudinem eſt A.

§ 12.

Si in $\frac{\triangle y}{\triangle x} = A + B \triangle x + C(\triangle x)^2 + D(\triangle x)^3 + -$ negatiua ſit quantitas $B \triangle x$, ſed termini eam ſequentes omnes ſint affirmatiui, minuendo $\triangle x$ poterit $B \triangle x$ iterum major fieri quam ſumma $C(\triangle x)^2 + D(\triangle x)^3 + E(\triangle x)^4 + -$ Quo facto erit $B \triangle x + C(\triangle x)^2 + D(\triangle x)^3 + -$ quantitas negatiua. Sed quoniam quantitatis $A + B \triangle x$ limes quoad magnitudinem eſt A; patet A eſſe quantitatis $A + B \triangle x + C(\triangle x)^2 + D(\triangle x)^3 + -$ ideoque et rationis $\frac{\triangle y}{\triangle x}$ cujus exponens eſt $A + B \triangle x + C(\triangle x)^2 + -$ limitem quoad magnitudinem.

§ 13.

Denique ſit $B \triangle x$ quantitas negatiua, at terminorum eam ſequentium ſint alii affirmatiui, alii negatiui. Minuendo $\triangle x$ effici poterit, ut ſit $B \triangle x > C(\triangle x)^2 + D(\triangle x)^3 + - -$ affirmatiue poſitis. Quamobrem ſiue $C(\triangle x)^2 + D(\triangle x)^3 + E(\triangle x)^4 + -$ ſumma poſitiua ſiue negatiua ſit quantitas, in utroque caſu eſt $A + 2 B \triangle x < (A + B \triangle x + C(\triangle x)^2 + - -)$ Iam vero cum A ſit limes quantitatis $A + 2 B \triangle x$, eo magis limes erit ſummae $A + B \triangle x + C(\triangle x)^2 + - -$ quae exponens eſt rationis $\frac{\triangle y}{\triangle x}$. Ergo A rationis $\frac{\triangle y}{\triangle x}$ erit quoad magnitudinem limes.

§ 14.

Facile quae hic expoſita ſunt de limite rationis $\frac{\triangle y}{\triangle x}$ applicari ad rationes $\frac{\triangle^2 y}{(\triangle x)^2}$, $\frac{\triangle^3 y}{(\triangle x)^3}$ etc. poterunt, quarum itaque limites ſiue quoad magnitudinem, ſiue quoad paruitatem, erunt A^{I}, A^{II}, A^{III}, A^{IV} etc. Rationum $\frac{\triangle y}{\triangle x}$, $\frac{\triangle^2 y}{(\triangle x)^2}$, $\frac{\triangle^3 y}{(\triangle x)^3}$ etc. ſuus quisque limes denotatur mutata litera $\triangle$ in d, ſitque adeo $\frac{d y}{d x} =$ limes $\frac{\triangle y}{\triangle x}$, $\frac{d^2 y}{(d x)^2} =$ limes $\frac{\triangle^2 y}{(\triangle x)^2}$, $\frac{d^3 y}{(d x)^3} =$ lim. $\frac{\triangle^3 y}{(\triangle x)^3}$ etc.

§. 15.

Sit $n \triangle x = b$, ideoque $n = \frac{b}{\triangle x}$. Si in formula §. 3. hoc modo expressa

$$y^N = y + n \triangle y + \frac{n(n-1)}{1. \ 2} \triangle^2 y + \frac{n(n-1)(n-2)}{1. \ 2. \ 3} \triangle^3 y + \frac{n(n-1)(n-2)(n-3)}{1. \ 2. \ 3. \ 4} \triangle^4 y - - + \triangle^n y$$

loco numeri n ponatur $\frac{b}{\triangle x}$, erit

$$y^N = y + \frac{b}{\triangle x} \triangle y + \frac{\frac{b}{\triangle x}\left(\frac{b}{\triangle x} - 1\right)}{1. \ 2} \triangle^2 y + \frac{\frac{b}{\triangle x}\left(\frac{b}{\triangle x} - 1\right)\left(\frac{b}{\triangle x} - 2\right)}{1. \ 2. \ 3} \triangle^3 y + \frac{\frac{b}{\triangle x}\left(\frac{b}{\triangle x} - 1\right)\left(\frac{b}{\triangle x} - 2\right)\left(\frac{b}{\triangle x} - 3\right)}{1. \ 2. \ 3. \ 4} \triangle^4 y + - - + \triangle^n y$$

$$= y + b \frac{\triangle y}{\triangle x} + \frac{b(b - \triangle x)}{1. \ 2} \frac{\triangle^2 y}{(\triangle x)^2} + \frac{b(b - \triangle x)(b - 2 \triangle x)}{1. \ 2. \ 3} \frac{\triangle^3 y}{(\triangle x)^3} + \frac{b(b - \triangle x)(b - 2 \triangle x)(b - 3 \triangle x)}{1. \ 2. \ 3. \ 4} \frac{\triangle^4 y}{(\triangle x)^4} + - - + \triangle^n y$$

§. 16.

Quoniam $b = n \triangle x$, patet incrementum $\triangle x$ eo majus fieri quo minor est numerus n, manente b constante. Sed cum y^N indicet quantitatis y valorem, eum quem induat, posito in ea $x + b$ pro x non mutabitur quantitatis y^N valor, quantuscunque pro n sumatur numerus integer.

§. 17.

Series terminorum functionis y sit finita, sintque in ea functiones quantitatis x formae $L x^m$ vbi L denotet numerum quendam constantem et m integrum; patet si maximus exponens quantitatis x in functione y sit n, tum maximum exponentem quantitatis x in functione $\triangle y$, fore $n - 1$. Nam transeunte y in y' mutatur x^n in

$$x^n + n x^{n-1} \triangle x + \frac{n(n-1)}{1.\ 2} x^{n-2} (\triangle x)^2 + \frac{n(n-1)(n-2)}{1.\ 2.\ 3} x^{n-3} (\triangle x)^3 + - - + (\triangle x)^n$$

Primus autem hujus summae terminus x^n, quia etiam in quantitate y continetur, facta ipsius y ab y' subtractione, tollitur, quo manet

$$n x^{n-1} \triangle x + \frac{n(n-1)}{1.\ 2} x^{n-2} (\triangle x)^2 + \frac{n(n-)(n-2)}{1.\ 2.\ 3} x^{n-3} (\triangle x)^3 + - - (\triangle x)^n$$

quae quantitas pars erit quantitatis $\triangle y$, ita quidem, ut ejus maxima potestas futura sit x^{n-1}. Eodem ratiocinio continuato facile perspicitur, fore x^{n-2}, potestatem maximam quantitatis x in $\triangle^2 y$, x^{n-3} in $\triangle^3 y$, x^{n-4} in $\triangle^4 y$ etc.

§. 18.

Consideratis igitur hisce rationum expressionibus:

$$\frac{\Delta y}{\Delta x} = A + B\,\Delta x + C\,(\Delta x)^2 + D(\Delta x)^3 + - - + I(\Delta x)^{n-1}$$

$$\frac{\Delta^2 y}{(\Delta x)^2} = A' + B'\,\Delta x + C'(\Delta x)^2 + D'(\Delta x)^3 + - - + H'(\Delta x)^{n-2}$$

$$\frac{\Delta^3 y}{(\Delta x)^3} = A'' + B''\,\Delta x + C''(\Delta x)^2 + D''(\Delta x)^3 + - - + G''\,(\Delta x)^{n-3}$$

etc.

$$\frac{\Delta^n y}{(\Delta x)^n} = A^{N-1}$$

cum appareat quantitatis x exponentes, via ostensa (§. 17.) in qualibet differentia semper vna vnitate minui; necesse est, si n sit finitus numerus vt tandem deveniatur ad talem differentiam $\Delta^n y$ cui x amplius non insit et quae producto aequetur constantis cujusdam in potestatem $(\Delta x)^n$. Tum vero erit $\Delta^n y' = \Delta^n y$ et $\Delta^n y' - \Delta^n y = 0 = \Delta^{n+1} y$.

§. 19.

Sit iterum n (puta n in expressione $n\,\Delta\,x = b$) exponens maximae potestatis quantitatis x in serie finita functionis y: erit, posito in functione y, pro x, $x + n\,\Delta\,x$

$$y^N = y + b\,\frac{\Delta y}{\Delta x} + \frac{b\,(b - \Delta x)}{1.\quad 2}\,\frac{\Delta^2 y}{(\Delta x)^2} + \frac{b(b - \Delta x)\,(b - 2\,\Delta x)}{1.\quad 2.\qquad 3}\,\frac{\Delta^3 y}{(\Delta x)^3} +$$

$$- - + \frac{b(b - \Delta x)\,(b - 2\,\Delta x) - - (b - (n-1)\,\Delta x)}{1\quad 2.\qquad 3 - - - n}\,\frac{\Delta^n y}{(\Delta x)^n}$$

Formula haec quae generalem exprimit legem mutationum, quas ſubit y, creſcente x, eadem manet, neque minimam patitur alterationem, etiam ſi pro n numerus major integer qui m ſit, ducatur in Δx vt ſit $m \Delta x = b$. Nempe ſi ponatur pro x in functione y, $x + m \Delta x$, erit

$$y^M = y + m \Delta y + \frac{m(m-1)}{1.\ 2.} \Delta^2 y + \frac{m(m-1)(m-2)}{1.\ 2.\ 3} \Delta^3 y + — — + \Delta^m y$$

$$= y + b\frac{\Delta y}{\Delta x} + \frac{b(b-\Delta x)}{1.\ 2} \frac{\Delta^2 y}{(\Delta x)^2} + \frac{b(b-\Delta x)(b-2\Delta x)}{1.\ 2.\ 3} \frac{\Delta^3 y}{(\Delta x)^3} + — — + \Delta^m y$$

At $\Delta^{n+1} y = 0$, $\Delta^{n+2} y = 0$, — — $\Delta^m y = 0$. Ergo

$$y^M = y + m \Delta y + \frac{m(m-1)}{1.\ 2} \Delta^2 y + \frac{m(m-1)(m-2)}{1.\ 2.\ 3} \Delta^3 y + — — — + \frac{m(m-1)(m-2) — — (m-(n-1))}{1.\ 2.\ 3 — n} \Delta^n y$$

quae expreſſio poſito $\frac{b}{\Delta x}$ pro m, mutatur in

$$y + b\frac{\Delta y}{\Delta x} + \frac{b(b-\Delta x)}{1.\ 2} \frac{\Delta^2 y}{(\Delta x)^2} + \frac{b(b-\Delta x)(b-2\Delta x)}{1.\ 2.\ 3} \frac{\Delta^3 y}{(\Delta x)^3} — — + \frac{b(b-\Delta x)(b-2\Delta x) — (b-(n-1)\Delta x)}{1.\ 2\ 3 — — n} \frac{\Delta^n y}{(\Delta x)^n}$$

§. 20.

Si expreſſionis hujus termini reſoluantur ducendo

$A + B \Delta x + C(\Delta x)^2 + — — — + I(\Delta x)^{n-1}$ hoc eſt

$\frac{\Delta y}{\Delta x}$ in b; $A' + B'\Delta x + C'(\Delta x)^2 + \text{---}\ \text{---}\ \text{---} + H'\Delta x^{n-2}$

hoc est $\frac{\Delta^2 y}{(\Delta x)^2}$ in $\frac{b(b-\Delta x)}{1.\ 2}$; $A'' + B''\Delta x + C''(\Delta x)^2 \text{---} + G''(\Delta x)^{n-3}$

siue $\frac{\Delta^3 y}{(\Delta x)^3}$ in $\frac{b(b-\Delta x)}{1.\ 2.}\frac{(b-2\Delta x)}{3}$ etc. fiet.

$$y^N = y + bA + bB\Delta x + bC(\Delta x)^2 + \text{---}\ \text{---} + bI(\Delta x)^{n-1}$$

$$+\frac{b^2}{2}A' + \frac{b^2}{2}B' + \frac{b^2}{2}C' + \text{---}\ \text{---} + \frac{b^2}{2}H'(\Delta x)^{n-2}$$

$$-\frac{b}{2}A' - \frac{b}{2}B' \text{---}\ \text{---}\ \text{---} \frac{b}{2}H'(\Delta x)^{n-1}$$

$$+\frac{b^3}{6}A'' + \frac{b^3}{6}B'' + \frac{b^3}{6}C'' + \text{---}\ \text{---} + \frac{b^3}{6}G''(\Delta x)^{n-3}$$

$$-\frac{3}{6}b^2A'' - \frac{3}{6}b^2B'' \text{---}\ \text{---}\ \text{---} \frac{3}{6}b^2G''(\Delta x)^{n-2}$$

$$+\frac{2}{6}bA'' + \text{---}\ \text{---} + \frac{2}{6}G''(\Delta x)^{n-1}$$

etc.

Sit breuitatis gratia

$$+bA + \frac{b^2}{1.2}A' + \frac{b^3}{1.2.3}A'' + \frac{b^4}{1.2.3.4}A''' \text{---}\ \text{---}\ \text{---} = c$$

$$+bB + \frac{b^2}{1.\ 2}B' - \frac{b}{2}A' + \frac{b^3}{1.\ 2.\ 3}B'' - \frac{3b^2}{6}A'' \text{---} = d$$

$$+bC + \frac{b^2}{1.2}C' - \frac{b}{2}B' + \frac{b^3}{1.2.\ 3}C'' - \frac{3b^2}{1.2.3}B'' + \frac{b}{1.2.3.}A'' \text{---} = e$$

etc.

Erit $y^N = y + c + d\Delta x + e(\Delta x)^2 + f(\Delta x)^3 - - + k(\Delta x)^{n-1}$

Eandem vero formam etiam y^M induere, ſi ponas $m\Delta x = b$ ſumto m majore quam maximus exponens dignitatum ipſius x in functione y, §. 19. demonſtrauimus. Si $m > n$, erit Δx in $m\Delta x$, minor incremento Δx in $n\Delta x$, dum $m\Delta x = n\Delta x = b$. Denotetur $'\Delta x$ incrementum quod quantitas x capit in y^M, ſigno Δx manente pro incremento quod x capit in y^N, vt itaque ſit $m'\Delta x = n\Delta x$ ſequitur eſſe

$y + c + d\Delta x + e(\Delta x)^2 - - - + k(\Delta x)^{n-1} =$

$y + c + d'\Delta x + e('\Delta x)^2 + f('\Delta x)^3 - - + k('\Delta x)^{n-1}$

Ergo $d\Delta x + e(\Delta x)^2 + f(\Delta x)^3 - - + k(\Delta x)^{n-1} =$

$d'\Delta x + e('\Delta x)^2 + f('\Delta x)^3 - - + k('\Delta x)^{n-1}$

At aequatio haec poſterior conſiſtere non poteſt niſi ſit $d = 0$, $e = 0$, $f = 0$, $k = 0$. Si enim $m'\Delta x = n\Delta x = b$ et $m > n$ erit $'\Delta x < \Delta x$. Diuiſo membro primo per Δx et ſecunda per $'\Delta x$, aequationis $d\Delta x + e(\Delta x)^2 + f(\Delta x)^3 - + k(\Delta x)^{n-1} = d'\Delta x + e('\Delta x)^2 + f('\Delta x)^3 + - + k('\Delta x)^{n-1}$ erit

$d + e\Delta x + f(\Delta x)^2 - - + k(\Delta x)^{n-2} < (d + e'\Delta x + f('\Delta x)^2 + - - + k('\Delta x)^{n-2}$ et $e\Delta x + f(\Delta x)^2 + - - + k(\Delta x)^{n-2} < e'\Delta x + f('\Delta x)^2 + - - + k('\Delta x)^{n-2}$

Et iterata diuiſione eo magis erit

$e + f\Delta x + - - + k(\Delta x)^{n-3} < e + f'\Delta x - - + k('\Delta x)^{n-3}$

vel $f\Delta x + g(\Delta x)^2 + - - + k(\Delta x)^{n-3} < f'\Delta x + g('\Delta x)^2 - - + k('\Delta x)^{n-3}$

Continuata diuisione et subtractione terminorum, jam a Δx liberatorum, tandem devenitur ad $k > k$. Quod quidem absurdum est si k esse debeat quantitas quaedam finita. Necesse ergo est vt sit $k = 0$. Sed tum erit

$$d\Delta x + e(\Delta x)^2 + f(\Delta x)^3 - - + i(\Delta x)^{n-2} =$$
$$d'\Delta x + e('\Delta x)^2 + f('\Delta x)^3 - - + i('\Delta x)^{n-2}$$

Itaque ut antea concludimus esse $i = 0$. Conclusio eadem valet de ceteris h, g, f, e et d quae omnia nihilo aequari debent. Ex demonstratis sequitur si y summa sit functionum x formae $I x^m$, vbi I quantitatem constantem et m numerum quemlibet integrum et affirmatiuum designet, quantitatis y valorem mutatum posito $x + b$ pro x semper vnum eundemque esse nempe $y + c$. Et quoniam

$$c = bA + \frac{b^2}{1.2}A' + \frac{b^3}{1.2.3}A'' + \frac{b^4}{1.2.3.4}A''' - - \frac{+b^n}{1.2.3. - n}A^{N-1} \text{ et}$$

$$A = \text{limes rat. } \frac{\Delta y}{\Delta x} = \frac{dy}{dx}, A' = \frac{d^2y}{(dx)^2}, A'' = \frac{d^3y}{(dx)^3} \text{ etc. } A^{N-1} = \frac{d^ny}{(dx)^n}$$

erit valor mutatus functionis y, seriem functionum x finitam expressae

$$= y + b\frac{dy}{dx} + \frac{b^2}{1.2}\frac{d^2y}{(dx)^2} + \frac{b^3}{1.2.3}\frac{d^3y}{(dx)^3} + \frac{b^4}{1.2.3.4}\frac{d^4y}{(dx)^4} + - - - + \frac{b^n}{1.2 - n}\frac{d^ny}{(dx)^n}$$

§. 21.

Sit series terminorum summam functionis y constituens infinita, nempe $y = Ax^k + Bx^l + Cx^m + Dx^o + Ex^p - - -$ Ponatur s =

Ax^k, $t = Bx^l$, $u = Cx^m$, $w = Dx^o$, $z = Ex^p$ et ita porro. Functionis s, t, u, w, z etc. de genere funt functionis y §. 21. Si litera c^I idem denotet refpectu functionis s, c^{II} idem refpectu functionis t, c^{III} idem refpectu functionis u et fic porro, quod c denotauerit refpectu functionis y §. 21; patet fi $x + b$ ponatur pro x in s, t, u, w, z, etc. valores tranfire s. in $s + c^I$, t in $t + c^{II}$, u in $u + c^{III}$, w in $w + c^V$, z in $z + c^V$ etc. Sed cum $y = s + t + u + w + z + --$ valor mutabitur ipfius y in $y + c^I + c^{II} + c^{III} + c^{IV} + c^V + -$ Ex hoc quoque eft perfpicuum valores pofito pro x in s, t, u, w, z etc. mutatos functionum quarum aggregatum eft y futuros effe:

$$s + b\frac{dy}{dx} + \frac{b^2}{1\ 2}\frac{d^2s}{(dx)^2} + \frac{b^3}{1.2.3}\frac{d^3s}{(dx)^3} + --- + \frac{b^k}{1.2.3 -- k}\frac{d^ks}{(dx)^k};$$

$$t + b\frac{dt}{dx} + \frac{b^2}{1.2}\frac{d^2t}{(dx)^2} + \frac{b^3}{1.2.3}\frac{d^3t}{(dx)^3} + -- \frac{b^l}{1.2.3 - l}\frac{d^lt}{(dx)^l};$$

$$u + b\frac{du}{dx} + \frac{b^2}{1.2}\frac{d^2u}{(dx)^2} + \frac{b^3}{1.2.3}\frac{d^3u}{(dx)^3} + -- + \frac{b^m}{1.2.3. - m}\frac{d^mu}{(dx)^m}$$

etc.

At quoniam $y = s + t + u + w + z + ---$

erit $\Delta y = \Delta s + \Delta t + \Delta u + \Delta w + \Delta z + ---$

$\Delta^2 y = \Delta^2 s + \Delta^2 t + \Delta^2 u + \Delta^2 w + \Delta^2 z + ---$

$\Delta^3 y = \Delta^3 s + \Delta^3 t + \Delta^3 u + \Delta^3 w + \Delta^3 z + ---$

etc.

Porro erit $\frac{\Delta y}{\Delta x} = \frac{\Delta s}{\Delta x} + \frac{\Delta t}{\Delta x} + \frac{\Delta u}{\Delta x} + \frac{\Delta w}{\Delta x} + \frac{\Delta z}{\Delta x} + ---$

$$\frac{\Delta^2 y}{(\Delta x)^2} = \frac{\Delta^2 s}{(\Delta x)^2} + \frac{\Delta^2 t}{(\Delta x)^2} + \frac{\Delta^2 u}{(\Delta x)^2} + \frac{\Delta^2 w}{(\Delta x)^2} + \frac{\Delta^2 z}{(\Delta x)^2} + -$$

$$\frac{\Delta^3 y}{\Delta x^3} = \frac{\Delta^3 s}{\Delta x^3} + \frac{\Delta^3 t}{\Delta x^3} + \frac{\Delta^3 u}{\Delta x^3} + \frac{\Delta^3 w}{\Delta x^3} + \frac{\Delta^3 z}{\Delta x^3} + ---$$

etc.

Ergo etiam $\frac{dy}{dx} = \frac{ds}{dx} + \frac{dt}{dx} + \frac{du}{dx} + \frac{dw}{dx} + \frac{dz}{dx} + ---$

$$\frac{d^2 y}{dx^2} = \frac{d^2 s}{dx^2} + \frac{d^2 t}{dx^2} + \frac{d^2 u}{dx^2} + \frac{d^2 w}{dx^2} + \frac{d^2 z}{dx^2} + ---$$

$$\frac{d^3 y}{dx^3} = \frac{d^3 s}{dx^3} + \frac{d^3 t}{dx^3} + \frac{d^3 u}{dx^3} + \frac{d^3 w}{dx^3} + \frac{d^3 z}{dx^3} + ---$$

etc.

Sed eſt $c' = b\frac{ds}{dx} + \frac{b^2}{1.2}\frac{d^2 s}{dx^2} + \frac{b^3}{1.2.3}\frac{d^3 s}{dx^3} -- \frac{+ b^k}{1.2.\text{---}k}\frac{d^k s}{dx^k}$

$$c'' = b\frac{dt}{dx} + \frac{b^2}{1.2}\frac{d^2 t}{dx^2} + \frac{b^3}{1.2.3} + -- + \frac{b^l}{1.2.\text{---}l}\frac{d^l t}{dx^l}$$

$$c''' = b\frac{dz}{dx} + \frac{b^2}{1.2}\frac{d^2 z}{dx^2} + \frac{b^3}{1.2.3}\frac{d^3 z}{dx^3} + -- + \frac{b^m}{1.2.\text{--}m}\frac{d^m z}{dx_m}$$

etc.

Ergo $c^{I} + c^{II} + c^{III} + c^{IV} + -- = \frac{dy}{dx} + \frac{b^2}{1.2}\frac{d^2 y}{dx^2} + \frac{b^3}{1.2.3}\frac{d^3 y}{dx^3} + -$

Et cum valor mutatus functionis y ſit $= y + c^{I} + c^{II} + c^{III} + --$ erit etiam in eo caſu, in quo ſeries functionum x in y aſſumta infinita ſit, poſito $x + b$ pro x functio $= y + \frac{b\,dy}{dx} + \frac{b^2}{1.2}\frac{d^2 y}{dx^2} + \frac{b^3}{1.2.3}\frac{d^3 y}{dx^3} + -$

*** *** ***

Habes, Lector Beneuole, demonstrationem theorematis quod ab inuentore, Taylorio, Taylorianum dicitur. Quum ex eo, quasi ex fonte haud infoecundo, problematum difficillimorum resolutiones scateant, quae nisi inde hauriantur, remotiores sunt, non alienum videbatur fontem hunc, adire, et cum in Taylorii libro satis absconditus sit, magis conspicuum reddere. Idque feci, duce l' Huilier, qui in libro, auspiciis Apollinis scripto: Exposition élémentaire des principes des calculs supérieurs, qui a remporté le prix proposé par l' Académie Royale des sciences et belles-lettres pour l' année 1786 praeclare hoc theorema demonstrauit, sed vt mihi videtur non satis euidenter. Ejus enim demonstratio in qua $dx = o$, et tamen $ndx = b$ ponitur, quo non discrepat a Taylorii demonstratione, aliquid absurdi inuoluere videtur. Itaque alio, quam quo L' Huilier vtitur modo ostendere sum conatus, fieri $d = o$; $e = o$, $f = o$ (§. 21.) in functione y, siue per finitam siue per infinitam seriem sit expressa. Hinc fortasse, quae praestiti parui momenti sunt, rogo Te itaque ut primitias has beneuolo animo accipere velis.

THESES.

I. Logica pura eſt doctrina analytica.

II. Logarithmi numerorum negatiuorum ſunt quantitates imaginariae, quae vero diſcrepent a quantitatibus imaginariis formae $\sqrt{-1}$.

III. Matheſis eſt diſciplina, quae conceptuum conſtructione conficitur.

IV. Analyſis finitorum et infinitorum non eſt ſcientia analytica.

V. Dubitari poteſt, vtrum arithmetica axiomata habeat.

VI. In dijudicanda quaeſtione vtrum metaphyſica a Leibnitii temporibus vsque ad Kantium progreſſus fecerit, omnia in definitione metaphyſices poſita eſſe, videntur.

VIII.. Infiniti magni et parui conceptus plane infoecundus eſt in matheſi.

VII. Quies corporum non definiri poteſt per defectum motus.

IX. Syntheſis intellectualis, ſyntheſis ſpecioſa, et ſyntheſis apprehenſionis iidem mentis actus ſunt, qui ſolummodo reſpectu varii quod ad apperceptionis vnitatem cogunt, a ſe inuivem diſcrepant.

X. Synthietica et analytica vnitas eandem originariae apperceptionis vnitatem denotant. Synthetica illa eſt qua (ſynthetice) varium quoddam ad conceptum cogitur; at ſi eadem vnitas quae varium complectitur, iterum invenitur in partibus varii, vnitas analytica eſt.

DE

...TE TAYLORIANO,

...SIVE DE

...ECUNDUM QUAM FUNCTIONES
...TATIS A QUIBUS PENDEANT
...ARIABILIBUS.

...SERTATIO

QUAM

...IA RITE OBTINENDA

...TUEBITUR AUCTOR

...GISMUNDUS BECK

... ET LIBERALIUM ARTIUM MAGISTER,

...SPONDENTE

...HEODORO POSELGER

...USSO, IURIUM CULTORE,

...CXCI. DIE XVI. APRILIS,

...OCOQUE SOLITIS.

...HALAE,

...RANCKIANIS.