MEMORIAM
BESTVCHEFFIANAM
SOLEMNI ORATIONE

DIE XXX IVNII cIↄIↄCCXCV

HORA IX.

IN AVDITORIO PHILOSOPHORVM

CELEBRANDAM

INDICVNT

DECANI SENIORES
CETERIQVE ASSESSORES QVATVOR
FACVLTATVM IN ACADEMIA LIPSIENSI.

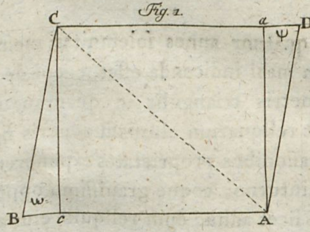
Inest disquisitio II.
de figuris rectilineis quadrangulis Isoperimetris.

Cum ante quatuor annos ſolemnitas memoriae Beſtuſcheffianae ſacra mihi indicenda eſſet, coepi de figuris planis rectilineis iſoperimetris triangulis ac quadrangulis eo animo commentari, vt et reliquarum eiusdem generis figurarum polygonalium datis occaſionibus proprietates exponerem: ab eo vero tempore morbo diuturno, eoque grauiſſimo oppreſſus nunc demum reconualescens licet adhuc cum reliquiis eius ac prouectioris aetatis incommodis mihi conflictandum ſit, audeo tamen telam inceptam pertexere. Expoſui in proluſione I. palmarias triangulorum planorum iſoperimetrorum proprietates, neque habeo, quod iis addam: quod vero quadrilateras concernit, ſtrictius §. 8 et 9 ac breuius de iis egi, quam vt aequieſcam, inprimis cum mihi, poſtquam iam edita erat, contigit in commentariis Academiae Suecicae ab ill. Kaeſtnero in idioma germanicum conuerſis ſub titulo: *der Koenigl. Svediſchen Akademie der Wiſſenſchafften Abhandlungen*, legere cum in notis a Kaeſtnero adiectis Tom. III. tum a Klingenſtirna Prof. Vpſal. Cl. Tom. IV. occaſione diſſertationis ab Eluio iis inſertae: de figura circulo inſcripta maiore qualibet figura circulo non inſcripta, iisdem lateribus et eodem ordine diſpoſitis, monita ac deſiderata; nihil itaque reliqui faciendum duxi, quam vt iis ſaltem in figuris quadrilateris ſatis facerem. Elegi itaque ex ill. poſt fata Lamberti Tetragonometria Tom. II. *der Beyträge zum Gebrauch der Mathematik* propoſitionem ex quatuor primariis primam, eamque leuiſſimam, eaque huic theoriae lucem affundere conaturus ſum.

 §. 1.

§. 1.

Sumtis I. in figura dectilinea quadrilatera ABCD fig. 1.



Omnibus lateribus AB = a . BC = b CD = d . AD = c et duobus Angulis diagonaliter oppositis B = ω et D = ψ inuenire aequationem et ex ea quodlibet datorum spectatum vt incognitum ex quinque reliquis definire

Ducantur Diagonalis inter reliquos angulos A et C et perpendicula Aa et Cc ex punctis A et C . in latera opposita, erit per princ. Trig.

$$Da = c \cos\psi \ . \ Bc = b \cos\omega.$$

ac praeterea ex 13, II Elem. Euclidis

$$AB^2 + CB^2 - 2 AB . Bc = AD^2 + CD^2 - 2 CD . Da$$

h. e. $a^2 + b^2 - 2ab \cos\omega = c^2 + d^2 - 2cd . \cos\psi$

Consequenter erit

$$a^2 - 2a\,b \cos\omega = c^2 + d^2 - 2cd \cos\psi - b^2$$

$$a^2 - 2ab \cos\omega + b^2 \cos\omega^2 = c^2 - d^2 - 2cd \cos\psi - b^2 + b^2 \cos\omega^2$$

$$\text{sed} - b^2 + b^2 \cos\omega^2 = - b^2 (1 - \cos\omega^2 = - b^2 \sin b^2$$

$$\text{hinc}\ a^2 - 2ab \cos\omega + b^2 \cos\omega^2 = c^2 + d^2 - 2cd \cos\psi - b \sin\omega^2$$

itaque

itaque extrahendo radicem et reducendo erit

$$a = \pm b \cos\omega + \sqrt{(c^2 + d^2 \mp 2cd \cos\psi - b^2 \sin\omega^2)}$$
$$b = \pm a \cos\omega + \sqrt{(c^2 + d^2 \mp 2cd \cos\omega - a^2 \sin\omega^2)}$$
$$d = \pm c \cos\psi + \sqrt{(a^2 + b^2 \mp 2ab \cos\omega - c^2 \sin\psi^2)}$$
$$c = \pm d \cos\psi + \sqrt{(a^2 + b^2 \mp 2ab \cos\omega - d^2 \sin\psi^2)}$$

Angulorum datorum erit

$$\cos\omega = \frac{a^2 + b^2 - c^2 - d^2 \pm 2cd \cos\psi}{2ab}$$ atque

$$\cos\psi = \frac{c + d - a^2 - b^2 \pm 2ab \cos\omega}{2cd}$$

§. 2.

Suppositis iis II. quae §. antecedenti sumta, determinare quadrilateram figuram rectilineam, quae sub eadem perimetro maximam complectatur Aream earum, quae describi possunt.

Cum Area Δ ABC $= \frac{ab \sin\omega}{2}$

atque Area Δ ABC $= \frac{cd \sin\psi}{2}$

Erit Area quadrilateri $= \frac{ab \sin\omega + cd \sin\psi}{2}$

differ. erit Area $= \frac{ab \cos\omega\, d\omega + cd \cos\psi\, d\psi}{2}$

Erit itaque differentiendo

$$(ab \cos\omega\, d\omega + cd \cos\psi\, d\psi) : 2$$

Cum vero sit in Δ ABC, $AC^2 = a^2 + b^2 - 2ab \cos\omega$

atque in Δ ABC, $AC^2 = c^2 + d^2 - 2cd \cos\psi$

erit $a^2 + b^2 - 2cb \cos\omega = c^2 + d^2 - 2cd \cos\psi$ atque cum hac aequatione contineatur ratio angulorum, differentiando dabit:

$$ab \sin\omega \, d\omega = cd \sin\psi \, d\psi \text{ erit etiam}$$
$$ab \cos\omega \, d\omega = - cd \cos\psi \text{ per nat. Maxim.}$$

diuidendo fit

$$\frac{\sin\omega}{\cos\omega} = \frac{-\sin\psi}{\cos\psi}$$

multipl.

$$\sin\omega \, . \cos\psi = - \sin\psi \cos\omega$$

$$\text{h. e. } \sin\omega \cos\psi + \sin\psi \cos\omega = 0 = \sin\omega + \sin\psi = 0$$

$$\text{Ergo } \sin\omega + \sin\psi = 180 = 2R.$$

ac cum formulae $ab \cos\omega \, d\omega \pm cd \cos\psi \, d\psi$ differentiale vti notum est, fit negatiuum; certe concludere possumus, quadrilaterum ex datis constructum circulo, quod inscribi post, esse maximae capacitatis. conf. Germ. illustr. Kastneri prop. 22. Corollario 9.

§. 3.

In antecedenti §. I. demonstratum est . . . . . . esse:

$$\cos\omega = (a^2 + b^2 - c^2 - d^2 \pm cd \cos\psi.) : 2cd$$

sed in casu Maximi quadrilateri h. e. circulo inscriptibilis est

$$\psi = 180 - \omega.$$

$$\text{Ergo } \cos\psi = \cos(180 - \cos\omega.)$$
$$= - \cos\omega.$$

Ergo facta substitutione:

$$\text{erit } 2ab \cos\omega \pm 2cd \cos\omega = (a^2 + b^2 - c^2 - d^2)$$

$$\cos\omega = \frac{(a^2 + b^2 - c^2 - d^2)}{2\,(ab \pm cd.)}$$

$$\cos\psi = \frac{c^2 + d^2 - a^2 - b^2}{2\,(cd \pm ab.)}$$

§. 4.

Aeque facile (III) in quadrilateris rectilineis figuris ſupra memoratae conditionis ac circulo inſcriptibilibus determinabitur Area earum.

Sit Circulo inſcriptum quadrilaterum ABCD, ſint $AB = a$ $BC = b$. $CD = d$. $AD = c$. $B = \omega$, $D = \psi$ diametraliter oppoſiti anguli: $AC = x$ diagonalis, erit igitur.

Area quadrilari,

$$\tfrac{1}{2} ab \sin \omega + \tfrac{1}{2} cd \sin \psi \ldots$$

Cum vero eſt in circulo inſcriptum: neceſſario erit $\psi = (180 - \omega.)$

$$\sin \psi = (0 - \sin \omega). - 1.) = \sin \omega.$$

$$\text{Ergo } \frac{ab + cd.}{2} (\sin \omega.)$$

$$\text{Sinus } \omega \text{ vero} = \sqrt{(1 - \cos \omega^2)}$$

$$\text{et } \cos \omega^2 = \frac{(a^2 + b^2 - c^2 - d^2)^2}{4 (ab + cd)^2} \quad (\text{§. } 3.)$$

$$\text{hinc } \sqrt{(1 - \frac{(a^2 + b^2 - c^2 - d^2)^2}{4 (ab + cd)^2}}$$

Ergo reductis ad eandem denominationem quantitatibus erit

$$\text{Area quadrilateri} =$$

$$\frac{ab + cd}{2} . \frac{\sqrt{(4 (ab + cd)^2 - (a^2 + b^2 c^2 - d^2)^2}}{2 . (ab + cd.)}$$

diuiſione facta formula areae mutabitur in ſequentem:

$$\frac{\sqrt{(4 (ab + cd)^2 - (a^2 + b^2 - c^2 - d^2))^2}}{4}$$

$$= \tfrac{1}{4} \sqrt{((2ab + 2cd)^2 + (a^2 + b^2 - c^2 - d^2) . (2ab + 2cd) - (a^2 + b - c^2 - d^2))}$$

$$= \tfrac{1}{4} \sqrt{((a + b^2 + 2ab - (c^2 + d^2 - 2cd) . (c^2 + d^2 + 2cd) - (a^2 + b^2 - 2ab))}$$

$$= \tfrac{1}{4} \sqrt{((a + b)^2 - (c - d)^2) . (c + d)^2 - (a - b)^2))}$$

$$= \tfrac{1}{4} \sqrt{((a + b + c - d) . (a + b + d - c) . (a + d + c - b) . (b + c + d - a))}$$

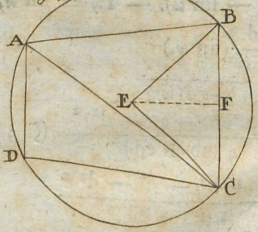
ſint

ſint latera aequalia h. e. ſit quadratum. Erit Area $= \frac{1}{4} \sqrt{2a . 2a . 2a . 2a} = \frac{4 a^2}{4} = a^2$

§. 5.

*Reſtat adhuc (IV) in quadrilatera circulo inſcripta figura ex ante aſſumptis ſex partibus diameter determinanda, ſequar itaque viam in commentariis acad. Suecicae ſupra citatis proſcriptam.

Fig. 2.



ſint in figura hac $AB = a$. $BC = b$. $CD = d$. $AD = c$. atque E. centrum BE radius $= r$ ac Diagonalis $= x$

omnium primo ex ill. Kaeſtneri Trigonom. 20 prop. Corol. 15. eſt

$$\text{Angulus } A = \frac{\sqrt{(a+b+x)(a+b-x).(a+x-b).(b+x-a)}}{2ax}$$

ac praeterea Angulus BEF = ang. BAC.

erit itaque ſin Totus : $r =$ ſin FEB : $\frac{1}{2}$ b.

h. e. $1 : r =$ ſinus A : $\frac{1}{2}$ b.

Conſequenter

r =

$$r = \frac{abx}{\sqrt{(a + b +).(a + b - x).(a + x - b)(b + x - a)}}$$

vt vero quadrilateri obtineatur radius neceſſario diagonalis x ex vtroque triango determinanda atque ita procedendum: ex triangulis ACB atque ADC eruatur AC^2 proinde quadrando erit

$$\frac{a^2b^2x^2}{(a+b+x).(a+b-x).(a+x-b)(b+x-a)} = \frac{c^2d^2x}{(c+d+x).(c+d-x).(c+x-d)(d+x-c)}$$

$$\frac{a^2b^2}{((a+b)^2 - x^2).(x^2 - (a+b)^2} = \frac{c^2d^2}{((c+d)^2 - x^2).(x^2 - (c+d)^2)}$$

haec probe reducta atque extracta vtrinque radice erit

$$\frac{ab}{(a+b)^2 - x^2} = \frac{cd}{(c+d)^2 - x^2}$$

erit itaque

$$ab((c+d)^2 - x^2)) = cd((a+b)^2 - x^2))$$

h. e. $ab(c+d)^2 - abx^2 = cd(a+b)^2 - cdx^2$

$$cdx^2 - abx^2 = cd(a+b)^2 - ab(c+d)^2$$

tandem vero

$$x^2 = \frac{cd(a^2 + b^2) - ab(c^2 + d^2)}{cd - ab.}$$

itaque cum radii Quadratum $= r^2 =$

$$\frac{a^2b^2x^2}{((a+b)^2 - x^2) r (x^2 - a^2 + b^2)^2)} = \frac{c^2d^2x^2}{((c+d)^2 - x^2).x^2 - (c+d)^2)}$$

ſubſtituto pro x^2 inuento valore in alterutrum membrum aequationis erit

$$r^2 = \frac{c^2d^2(cd(a^2+b^2) - ab(c^2+d^2) : (cd - ab)}{\left((c+d)^2 - \frac{cd(a^2+b^2) - ab(c^2+d^2)}{(cd - ab)}\right).\left(\frac{(cd(a^2+b^2) - ab(c^2+d^2)}{(cd - ab)} - (c+d)^2\right)}$$

 erit

erit reductis quantitatibus ad eandem denominationem ac diuisis

$$\text{erit } r^2 = \frac{c^2 d^2 \left((cd - ab) . cd (a^2 + b^2) - ab (c^2 + d^2)\right)}{\left((c + d)^2 . (cd - ab) - cd (a^2 + b^2) + ab (c^2 + d^2)\right)^2}$$

denominatoris membris in ſe multiplicatis ac rite ordinatis: denominatur erit

$$\left(\left.\begin{matrix} c^3 & . \\ -2abc & . \\ -ca^2 & . \\ -cb^2 & . \end{matrix}\right\} d. \quad + 2c^2d^2 + cd^3 \right)^2$$

$$\text{Eritque denominator} = c^3d + 2c^2d^2 + cd^3 - (a + b)^2 \, cd)^2$$
$$= \left((c + d)^2 \, cd - (a + b)^2 . cd\right)^2$$
$$= \left((c + d)^2 - (a + b)\right)^2 . c^2d^2$$

hinc

$$r^2 = \frac{c^2 d^2 . \left((cd - ab) . (cd (a^2 + b^2) - ab (c^2 + d^2)\right)}{\left((c + d)^2 - (a + b)^2\right)^2 . c^2 d^2}$$

diuiſione facta et radice extracta erit

$$r = \frac{\sqrt{cd - ab} . cd (a^2 + b^2) - ab (c^2 + d^2)}{(c + d)^2 - (a + b)^2}$$

$c = d = a = b$. h. e. fiat Quadrilaterum datum quadratum: erit

$$r = \frac{\sqrt{(cd - ab)} . (cd - ab) . a^2 + b^2}{(c + d)^2 - (a^2 + b^2)}$$

$$r = \frac{(cd - ab) . \sqrt{a^2 + b^2}}{2 (cd - ab) + (c^2 + d^2) - (a^2 + b^2} = \sqrt{\frac{a^2 + b^2}{2}}$$

Animus quidem erat figurarum polygonalium iſoperimetrarum indolem exponere, excreuit vero, licet quam plurima calculi membra, ea ex cauſa omiſerim, conſideratio figurarum quadrangularum in eam molem; vt cogar in aliam occaſionem, quam mihi

mihi officii ratio, quod hoc ſemeſtri gero, offeret et qua proprio nomine ſum ſcripturus, differre. Propero itaque Vobis, ciues Academici indicere orationem in memoriam illuſtriſſimae Com: IOHANNAE HENRICAE LVDOVICAE DE BESTVSCHEFF-RVMIN e nobiliſſima gente de CARLOWITZ. Habebit vero eam Generoſiſſimus Vir iuuenis

CVRT. AVGVSTVS ALEXANDER
DE CARLOWITZ

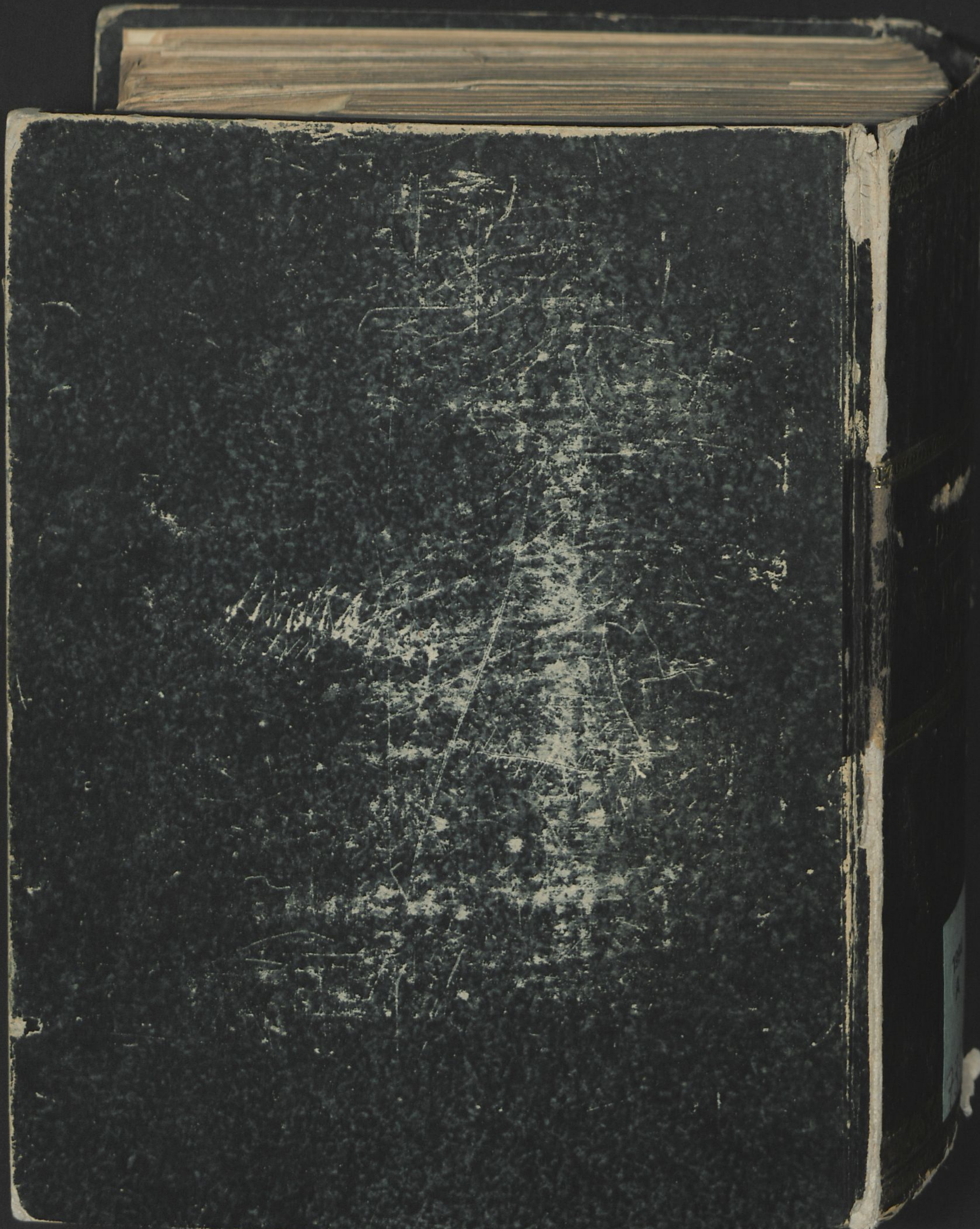
dicturus nempe: Cur amor in patriam, *patriotismum* dicunt, hodie nomine tantum floreat, re vero extinctus ſit, et inſignem Municentiam illuſtriſſimae Matronae erga hanc Academiam, Seniorem eius illuſtrem, et erga alios litterarum cultores digniſſimis proſequetur laudibus.

Omni vero obſeruantia rogamus Vos, RECTOR MAGNIFICE COMITES ILLVSTRISSIMI, PROCERES VTRIVSQVE REIPVBLICAE, grauiſſimi, commilitones generoſiſſimi atque humaniſſimi vt dicta die beneuole conueniatis atque generoſiſſimum Oratorem praeſentia Veſtra ornetis. Quam Veſtram beneuolentiam nos perpetuo cultu proſecuturos et quauis occaſione demerituros pollicemur.

P. P. Dom. IV. p. Feſtum Trinit. A. R. S. cIↄIↄccxcv.

LIPSIAE

EX OFFICINA KLAVBARTHIA.



MEMORIAM BESTVCHEFFIANAM

SOLEMNI ORATIONE

DIE XXX IVNII cIↄIↄccxcv

HORA IX.

IN AVDITORIO PHILOSOPHORVM

CELEBRANDAM

INDICVNT

DECANI SENIORES

CETERIQVE ASSESSORES QVATVOR

FACVLTATVM IN ACADEMIA LIPSIENSI.

Inest disquisitio II.

de figuris rectilineis quadrangulis Isoperimetris.

22

21

1795