# SCHOLIA
# IN LIBRUM SEXTUM ELEMENTORUM EUCLIDIS

QUORUM

*PARTEM TERTIAM*

PRÆSIDE

## CHRISTOPHORO FRIDERICO PFLEIDERER

UNIVERSITATIS ET COLLEGII ILLUSTRIS PROFESSORE PHYSICES ET MATHESEOS PUBL. ORD.

PRO CONSEQUENDO GRADU MAGISTERII

*D. SEPT. MDCCCII.*

PUBLICE DEFENDENT

CHRISTIANUS FRIDERICUS KLAIBER, *Wankheimensis*,
JOSEPHUS FRIDERICUS SEYBOLD, *Brackenheimensis*,
THEOPHILUS FRIDERICUS JAEGER, *Stuttgardianus*,
AMANDUS FRIDERICUS GÜNZLER, *Degerschlachtensis*,
CHRISTIANUS FRIDERICUS FRANCK, *Stuttgardianus*,
CANDIDATI MAGISTERII PHILOSOPHICI IN ILLUSTRI STIPENDIO THEOLOGICO.

---

*TUBINGÆ*
LITERIS SCHRAMMIANIS.

## PROPOSITIONES XVI. XVII.

§. 143.

Prior ut corollarium sistitur XIV$^{tæ}$; posterior ut corollarium prioris. Utraque immediate demonstrari potest iisdem modis, quibus XIV$^{ta}$; cujus etiam sic enunciata ad parallelogramma rectangula applicatione ambæ comprehenduntur: Parallelogrammorum rectangulorum æqualium bases sunt altitudinibus reciproce proportionales; ac vicissim æqualia sunt parallelogramma rectangula, quorum bases sunt altitudinibus reciproce proportionales.

§. 144.

Cum quodvis parallelogrammum obliquangulum æquale sit rectangulo æquealto super eadem basi (I, 35.): generatim etiam (V, 7. 11.) duo quæcunque parallelogramma æqualia habent bases altitudinibus reciproce proportionales; ac vicissim.

§. 145.

Pariter duo quælibet triangula æqualia habere bases altitudinibus reciproce proportionales; & vicissim: vel hinc per I, 41. V, 15. infertur; vel similiter de rectangulis primum triangulis, tum de ceteris ope I, 37. ex XV$^{ta}$ deducitur.

§. 146.

Proposita §. 144. sq. ex VI, 1. & ejus consectario §. 12. sic etiam inferuntur. Denotent $\left.\begin{matrix} B, A, P \\ B', A', P' \end{matrix}\right\}$ bases, altitudines & areas duorum parallelogrammorum triangulorumve; & $Q$ aream parallelogrammi trianguli ve, sub angulo quocunque super basi $B'$ cum altitudine $A$ facti.

Ita $P : Q = B : B'$ (VI, 1.)

$P' : Q = A' : A$ (§. 12.).

A

Quare

Quare si 1°. $P = P'$, ideoque $P : Q = P' : Q$ (V, 7);
erit (V, 11.) $B : B' = A' : A$.
2°. Viciſſim ſi $B : B' = A' : A$;
fit quoque (V, 11.) $P : Q = P' : Q$
& hinc (V, 9.) $P = P'$.

## §. 147.

Quodſi priores XVI^tæ & XVII^mæ partes ad Corollaria, VIII^væ in *Elementis* ac §. 93. ſeq. ſubjuncta applicantur; hæ emergunt propoſitiones.

Perpendiculo ab vertice anguli recti trianguli rectanguli in hypotenuſam demiſſo:

1°. quadratum hujus perpendiculi æquatur rectangulo ſub ſegmentis hypotenuſæ ab ipſo factis; $AD^q =$ rectg. $BD \times DC$ (*Fig.* 23.).

2°. Cujuslibet catheti quadratum æquale eſt rectangulo ſub hypotenuſa & ſub ejus ſegmento, quod catheto huic adjacet; $AB^q =$ rectg. $CB \times BD$, $AC^q =$ rectg. $BC \times CD$.

3°. Rectangulum ſub lateribus circa angulum rectum æquale eſt rectangulo ſub hypotenuſa et perpendiculo; rectg. $BA \times AC = BC \times AD$.

4°. Rectangulum ſub alterutro latere circa angulum rectum et ſub perpendiculo æquatur rectangulo ſub altero latere circa angulum rectum & ſub ſegmento hypotenuſæ, quod priori adjacet catheto; rectg. $BA \times AD = AC \times BD$, rectg. $CA \times AD = AB \times CD$.

In circulo

5°. quadratum perpendiculi ab quocunque peripheriæ puncto ad aliquam ejus diametrum ducti æquatur rectangulo ſub ſegmentis diametri ab perpendiculo factis.

6°. Cujuslibet chordæ per centrum non transeuntis quadratum æquale eſt rectangulo ſub diametro per unum chordæ extremum ducta, & ſub diametri hujus ſegmento chordæ contiguo, quod ab illa abſcindit perpendiculum in eam ex altero chordæ extremo demiſſum.

Unde porro conſequitur: perpendiculo in {hypotenuſam trianguli rectanguli / diametrum circuli} demiſſo ex {vertice anguli recti / puncto quocunque peripheriæ}, & in circulo ductis ab puncto hoc rectis ad extrema diametri;

7°. hy-

7°. $\left\{\begin{matrix}\text{hypotenuſam}\\ \text{diametrum}\end{matrix}\right\}$ eſſe ad alterutrum ipſius ſegmentum, uti quadratum $\left\{\begin{matrix}\text{hypotenuſæ}\\ \text{diametri}\end{matrix}\right\}$ ad quadratum $\left\{\begin{matrix}\text{catheti}\\ \text{chordæ}\end{matrix}\right\}$ huic ſegmento adjacentis; vel ut hujus $\left\{\begin{matrix}\text{catheti}\\ \text{chordæ}\end{matrix}\right\}$ quadratum ad quadratum iſtius ſegmenti; vel ut quadratum $\left\{\begin{matrix}\text{catheti}\\ \text{chordæ}\end{matrix}\right\}$ adjacentis alteri ſegmento ad quadratum perpendiculi. Nempe

$$\begin{aligned} BC:CD &= BC^q : BC\times CD \ (\text{§. 11.}) &&= BC^q : CA^q \\ &= BC\times CD : CD^q &&= AC^q : CD^q \end{aligned} \Bigg\} \ (\text{n°. 2. 6.})$$
$$= CB\times BD : CD\times DB \quad = BA^q : AD^q \ (\text{n°. 1. 2. 5. 6.})$$

8°. Ipſa autem $\left\{\begin{matrix}\text{hypotenuſæ}\\ \text{diametri}\end{matrix}\right\}$ ſegmenta eſſe uti quadrata $\left\{\begin{matrix}\text{cathetorum}\\ \text{chordarum}\end{matrix}\right\}$ adjacentium;

$$BD:DC = CB\times BD : BC\times CD \ (\text{§. 11.}) = BA^q : AC^q \ (\text{n°. 2. 6.}).$$

9°. Eodemque, quo n°. 8, modo oſtenditur: ex eodem peripheriæ puncto ductis diametro & duabus pluribusve chordis, perpendiculisque ab alteris harum terminis in diametrum demiſſis; quadrata chordarum eſſe ut ſegmenta ipſis adjacentia diametri.

## §. 148.

Propoſitionum §. præc. expoſitarum tres primæ, uſus in demonſtrandis X, 34. 35. 36. gratia, traduntur in *Lemmate primo ante* X, 34: & ope VI, 17. 16. ſtabiliuntur mediantibus analogiis, una ab Corollarii VI, 8. parte priori petita, ceteris ex ipſis Propoſitionibus VI, 8. 4. deductis (vid. §. 93.); tertia inſuper demonſtratur, rectangulis ſub *BA* & *AC*, *BC* & *AD* deſcriptis, colligendo ex I, 34. utrumque duplum eſſe trianguli *ABC*.

Primæ & ſecundæ demonſtratio eadem, quæ in *Lemm* I. *ante* X, 34. repetitur in *Lemmate poſt* XIII, 13. ad efficiendam præcedentis ſeptimæ (§. 147.) partem tertiam.

Ejuſdem ſeptimæ pars prima in demonſtrationibus XIII, 14. 15. 16. tanquam nota ſimpliciter ſumitur: pariterque bis in demonſtratione XIII, 18; ſemel autem, bisve, cum adjuncta ratione, ισογωνιον γαρ ἐςι τω ΒΑΓ τριγωνον τω ΓΑΔ τριγωνω: in demonſtratione XIII, 13. autem, pariter ac ſecunda pars in demonſtratione XIII, 18. ex proportione $BC : CA = AC : CD$, vel $DC : CA = AC : CB$, rurſus utrobique ab VI, 8. 4. deducta, infertur per VI, 20. Coroll. 2.

Quæ quidem, juncta iis, quæ notata fuere §. 93, nimis, quam ut auctori *Elementorum* tribui poſſint, ab methodo abludunt ipſi alias ſolenni, præmiſſas demonſtrationum ſubſequentium ſuis locis ad perpetuum deinceps uſum ſtabiliendi; nominatim etiam theorematum generaliorum ad ſpeciales caſus applicationes frequenter deinceps adhibendas, quamvis obvias & faciles, ſeorſim enunciandi, ut immediate eſſent ad uſus occurrentes paratæ (Conf. *Schol. in Lib. II.* §. 12.). Eademque cura Libris X. XIII. proſpectum haud fuiſſe, hunc præſertim oſcitantia ipſorum partim anterius, partim vix modo præmiſſorum immemore, fuiſſe tractatum, tum reliqua in illis conſpicua diligentia & ipſa doctrinarum arduitas exiſtimare prohibent; tum eo minus probabile eſt, ſi, quod *Proclus Lib. II.* p. 20. ([a]) aſſerit, Euclides της συμπασης ςοιχειωσεως τελος προεςησατο την των καλυμενων Πλατωνικων ϛχηματων συςασιν.

Vero potius ſimile eſt: ut hodienum, ita & olim communibus, tironum præcipue etiam uſibus parata fuiſſe *Elementorum* exemplaria variis modis reſpectibusque, compendii ac facilitatis gratia, forſan & arctioris ſyſtematis prætextu ([b]), truncata; ſicque plura præſertim, quæ ſolis X$^{mo}$ ac XIII$^{tio}$ in præcedentibus Libris inſervirent, his excidiſſe; ac multifariam, nec apte ſemper & uniformiter, illorum textui aſſutis Lemmatibus (conf. §. 13.) inſertisque auctariis anſam dediſſe.

§. 149.

---

(a) Conf. Lib. I. p. 7; Lib. II. p. 19. 20. ad titulum in margine: Σκοπος της γεωμετριας; & p. 21: ac Kepleri *Harmonices Mundi Lib. I. Prooemium*, p. 3. ſq.; cujus tamen aſſertum: "Euclidei operis ultimum finem, ad quem referrentur omnes omnino propoſitiones omnium Librorum, eſſe quinque corpora regularia;" præter ibi adjectam, quibusdam adhuc exceptionibus obnoxium eſt.

(b) Cujusmodi de *Elementorum* conſtitutione præcepta apud *Proclum* leguntur Lib. II. p. 21.

§. 149.

Ob $\left.\begin{array}{l} AB^q = \text{rectg.}\ CB \times BD \\ AC^q = \text{rectg.}\ BC \times CD \end{array}\right\}$ (§. 147. n°. 2.):
est $AB^q + AC^q = \text{rectg.}\ CB \times BD + BC \times CD = BC^q$ (II, 2.); conformiter I, 47$^{mæ}$. Neque hanc, vel propositionem aliquam ab ea pendentem supponunt præmissæ, quibus demonstratio asserti §. 147. n°. 2. nititur.

§. 150.

Propositio XVII. explicat identitatem constructionis problematum II, 14. VI, 13. Aequipollere nimirum docet problemata: Invenire latus quadrati rectangulo sub datis rectis *A*, *B* æqualis; & datis rectis *A*, *B* mediam proportionalem invenire.

§. 151.

Uti casus particularis Propositionis III, 35. quem sistunt assertum §. 147. n°. 5. ac solutio problematis II, 14. (vid. *Schol. in Lib. II.* §. 76.) in Libro II. & III. ope II, 5. I, 47. demonstratus, heic per III, 31. VI, 8. 17. adstruitur; ita universim Propositio III, 35. ex III, 21. VI, 4. 16. potest inferri.

In circulo enim ductis duabus quibuscunque rectis *AC*, *BD* (*Fig.* 53.) intra eum se secantibus; junctisque earum alterutris extremis *AD*, *BC*: præter triangulorum *ADE*, *BCE* angulos ad verticem *E* oppositos (I, 15.) æquales sunt eorum anguli *DAE* & *CBE*, *D* & *C* (III, 21.); ideoque $AE : ED = BE : EC$ (VI, 4.); et hinc rectg. $AE \times EC = BE \times ED$ (VI, 16.).

Duæ igitur ejusmodi rectæ in circulo se mutuo secant in partes reciproce proportionales; & rectangulum sub segmentis unius æquale est rectangulo sub segmentis alterius.

§. 152.

Similiter Propositionem III, 36. ipsiusque conversam III, 37. ex III, 32. ejusque conversa & VI, 4. 6. 17. nectere licet.

Ab puncto enim *D* extra circulum (*Fig.* 54.) ductis recta eum in *B* contingente *DB*, & quacunque ipsum secante *DCA*; ac junctis *AB*, *BC* rectis: præter communem angulum *D* triangulorum *DAB*, *DBC*, æquales

les ſunt eorum anguli $A$, $DBC$ (III, 32.), ideoque etiam $DBA$, $DCB$ (I, 32. *Coroll.*); proinde $AD : DB = BD : DC$ (VI, 4.); & hinc rectg. $AD \times DC = DB^q$ (VI, 17.).

Ab puncto igitur extra circulum ductis recta eum tangente, & altera ipſum ſecante: tangens media proportionalis eſt inter ſecantem rectam ejusque partem exteriorem; & tangentis quadratum æquatur rectangulo ſub tota ſecante ipſiusque parte exteriori.

Viciſſim ſi rectg. $AD \times DC = DB^q$, ideoque $AD : DB = BD : DC$ (VI, 17): triangulorum $DAB$, $DBC$, latera circa angulum communem $D$ proportionalia habentium, anguli $A$, $DBC$, quibus homologa latera $DB$, $DC$ ſubtenduntur, æquales ſunt (VI, 6.): quare $DB$ recta circulum in $B$ contingit (III, 32. *Conv.*).

## §. 153.

Priore caſu §. 152. in triangulis æquiangulis $DAB$, $DBC$ etiam ſunt

$$\left.\begin{array}{l} AD : DB = AB : BC = AB^q : AB \times BC \\ BD : DC = AB : BC = AB \times BC : BC^q \end{array}\right\} (\S.\ 52.\ 11.)$$

Itaque $AD : DC = AB^q : BC^q$ (V, 22.):

h. e. ab puncto extra circulum ductis recta eum tangente, & altera ipſum ſecante, ductisque in circulo rectis jungentibus punctum contactus prioris & puncta ſectionum alterius; tota ſecans recta eſt ad partem ipſius exteriorem, uti quadratum rectæ ab puncto contactus ad extremum ſecantis totius ductæ, ad quadratum rectæ jungentis punctum contactus atque alterum ſectionis punctum.

Viciſſim ſi $AD : DC = AB^q : BC^q$;

cum, ducta rectæ $AB$ parallela $CG$, etiam ſit

$$AD : DC = AB : CG = AB^q : AB \times CG \quad (\S.\ 61.\ 11.)$$

ideoque (V, 11.) $AB^q : BC^q = AB^q : AB \times CG$

(V, 9.) $BC^q = AB \times CG$

(VI, 17.) $AB : BC = BC : CG$

ac ſit angulus $ABC = BCG$ (I, 29.);

eſt angulus $A = DBC$ (VI, 6.), & hinc $DB$ circulum in $B$ contingit (III, 32. *Conv.*)

## §. 154.

Propoſita §. 152. ſq. ſic etiam enunciantur. Circa triangulum non-æquicrurum deſcripto circulo, & per verticem trianguli ducta recta circulum tangente; hæc baſi productæ ſic occurrit: ut rectangulum ſub rectis puncto huic occurſus extremisque baſis interjacentibus æquale ſit quadrato rectæ ab eodem occurſus puncto ad verticem trianguli ductæ; rectæ vero inter punctum illud occurſus ac terminos baſis in ipſa abſciſſæ eandem habeant rationem, quam crurum trianguli, quibus adjacent, quadrata.

Nempe ſi triangulo $ACB$, cujus crus $AB > BC$, circulus circumſcribitur; & hunc in $B$ contingens agitur recta $BD$: ob angulum $DBC = A$ (III, 32.) $< ACB$ (I, 18.), ideoque angulos $DBC + DCB < ACB + DCB$ h. e. (I, 13.) $<$ 2 rectis: ad partes angulorum $DBC$, $DCB$ concurrunt rectæ $AC$, $BD$ (*Lib.* I. *Ax.* 11.). Ac tum

$$\text{rectg. } AD \times DC = DB^q \quad (\text{§. } 152.\ n^o.\ 1.)$$

$$AD : DC = AB^q : BC^q \quad (\text{§. } 153.\ n^o.\ 1.)$$

Viciſſim 1°. ſi trianguli non-æquicruri $ACB$ baſis $AC$ ad partes cruris minoris $BC$ ſic producitur, ut rectangulum ſub adjecta $CD$ & ſub compoſita $DA$ ex baſi $AC$ & adjecta $CD$ æquale ſit quadrato rectæ $DB$ ab termino $D$ continuationis baſis ad trianguli verticem $B$ ductæ: recta hæc circulum triangulo circumſcriptum in $B$ contingit (§. 152. n°. 2.); & ſegmenta baſis $AD$, $DC$, terminis ipſius $A$, $C$, ac puncto $D$ intercepta, ſunt uti quadrata crurum trianguli $AB$, $BC$, ipſis adjacentium (§. 153. n°. 1.).

Atque 2°. ſi trianguli non-æquicruri $ACB$ baſis $AC$ ad partes cruris minoris ſic in $D$ usque producitur, ut ſegmenta ejus $AD$, $DC$, puncto hoc $D$ terminisque $A$, $C$ baſis intercepta, ſint ut crurum trianguli $AB$, $BC$ ipſis adjacentium quadrata: recta ab puncto $D$ ad verticem $B$ trianguli ducta circulum triangulo circumſcriptum in $B$ contingit (§. 153. n°. 2.); & rectæ $DB$ quadratum æquale eſt rectangulo ſub ſegmentis $AD$, $DC$ baſis, puncto $D$ ac terminis ejus $A$, $C$ interceptis (§. 152. n°. 1.).

Poſterior converſa eſt *Lemma* 2. *Libri* II. *Locorum planorum Apollonii.* (*Apollonius Ebene Oerter.* S. 211.)

## §. 155.

Pariter immediate per III, 21. vel 22. & VI, 4. 16. demonstratur, quod a CLAVIO (p. 310.), BAERMANNO (p. 86.), Propositioni III, 36. subjungitur Corollarium: Si a puncto $D$ extra circulum ducantur duæ quæcunque rectæ $DA$, $DF$, eum secantes (*Fig.* 55.); rectangula $AD \times DC$, $FD \times DE$ sub totis & partibus earum exterioribus æqualia esse.

Junctis enim $AE$, $FC$ rectis: ob angulos $EAD$, $CFD$ æquales (III, 21.), ac $D$ communem in triangulis $DAE$, $DFC$, sunt $AD : DE = FD : DC$ (VI, 4.); proinde rectg. $AD \times DC = FD \times DE$ (VI, 16.).

Vel junctis $AF$, $CE$ rectis: ob angulos $\begin{Bmatrix} FAC \\ FAD \end{Bmatrix} + FEC = 2$ rectis (III, 22.) $= DEC + FEC$ (I, 13.), atque $\begin{Bmatrix} AFE \\ AFD \end{Bmatrix} + ACE = 2$ rectis (III, 22.) $= DCE + ACE$ (I, 13.), sunt anguli $FAD = DEC$, $AFD = DCE$, & angulus $D$ communis in triangulis $DAF$, $DEC$; quare $AD : DF = ED : DC$ (VI, 4.), & rectg. $AD \times DC = FD \times DE$ (VI, 16.).

Ab puncto igitur extra circulum ductis duabus rectis eum secantibus; totæ secantes sunt partibus suis exterioribus reciproce proportionales; & rectangula sub totis ac partibus ipsarum exterioribus æqualia sunt.

## §. 156.

Porro analysis ac demonstratio Problematis IV, 10. propositionibus III, 36. sq. ibi nixæ, adhibitis VI, 3. 6. 17, subsidio circuli trianguli $ADC$ circumscribendi haud indigent.

Posito enim, $ADB$ (*Fig.* 56.) esse triangulum æquicrurum, cujus singuli ad basin $BD$ anguli sint dupli ejus, qui ad verticem $A$; & bifariam diviso alterutro ad basin angulo $ADB$ per rectam $DC$, unde (I, 6.) $AC = CD$ ob angulum $A = ADC$, & $CD = DB$ ob angulum $BCD = A + ADC$ (I, 32.) $= 2\,A = B$, itaque $AC = BD$: oportet, sit

$$BC : CA = BD : DA \text{ (VI, 3.)} = CA : AB \text{ (V, 7. 11.)}$$

& hinc $CA^q =$ rectg. $AB \times BC$ (VI, 17.)

Tum recta $AB$ ita in $C$ secta, ut sit $CA^q =$ rectg. $AB \times BC$ (II, 11.); & constructo triangulo $ABD$, cujus crus $AD = AB$, & basis $BD = AC$ (I, 22.): est

$BC : CA = CA : AB$ (VI, 17.) $= BD : DA$ (V, 7. 11.);
angulum igitur $ADB$ bifariam secat recta $DC$ (VI, 3.):
pariterque ob $BD^q = CA^q =$ rectg $AB \times BC$ est $CB : BD = DB : BA$ (VI, 17.);
unde, ob angulum $B$ communem, est angulus $BDC = A$ (VI, 6.):
proinde angulus $ADB = 2A$.

## §. 157.

Eodem in triangulo æquicruro $ADB$ est
rectg. $AD \times DB = BA \times AC$ (constr.) $= AC \times CB + AC^q$ (II, 3.) $= AC \times CB + DC^q$ (demonstr.):
quod universim verum esse, quodcunque sit triangulum $ABD$ (*Fig.* 57.), & quicunque ejus angulus $D$ secetur bifariam, sic ostenditur.

Recta $DC$, bifariam angulum $ADB$ secans, continuetur, donec circulo, qui triangulo $ABD$ circumscribitur, rursus in $E$ occurrat; & jungatur alterutra recta $AE$, $BE$.

Posteriore ducta, est angulus $A = E$ (III, 21.). Quare, cum etiam sit angulus $ADC = BDE$ (*hyp.*), est $AD : DC = ED : DB$ (VI, 4.); & hinc
rectg. $AD \times DB = ED \times DC$ (VI, 16.) $= DC \times CE + DC^q$ (II, 3.) $= AC \times CB + DC^q$ (III, 35. vel §. 151.).

Cujuslibet igitur trianguli $ABD$ angulo quocunque $D$ bifariam secto per rectam $DC$; rectangulum sub ejus lateribus $AD$, $DB$, angulum $D$ comprehendentibus, æquale est rectangulo sub segmentis $AC$, $CB$ tertii lateris, ab recta $DC$ factis, una cum hujus rectæ $DC$ quadrato. Rob. Simson Lib. VI. Prop. B. (p. 219. *Matthias* p. 101.).

## §. 158.

Universim pariter (quod de trianguli rectanguli lateribus circa angulum rectum §. 147. n°. 3. tradi per III, 31. *Convers.* vel III, 33. n°. 2. consequitur) rectangulum sub duobus quibuscunque lateribus cujuslibet trianguli æquale est rectangulo sub diametro circuli triangulo circumscripti & sub perpendiculo ex vertice anguli, quem latera ista comprehendunt, in latus tertium demisso.

1°. Si ipsum alterutrum latus $AB$ tertio $BC$ est perpendiculare (*Fig.* 58.): alterum $AC$ diameter est circuli triangulo circumscripti (III, 31. *Conv.* vel III, 33. n°. 2.); & propositio ad hanc identicam redit: rectg. $BA \times AC = CA \times AB$.

2°. Si uterque ad basin seu tertium latus $BC$ angulus est acutus, quicunque sit angulus $BAC$ duobus lateribus $BA$, $AC$ comprehensus (*Fig.* 59.); vel si alteruter ad basin angulus $ABC$ est obtusus (*Fig.* 60.); per verticem $A$ ducta diametro $AGE$, & juncta $BE$ recta, perpendiculoque $AD$ in basin $BC$ demisso; ob angulos $ABE$, $ADC$ rectos (III, 31. & *constr.*), atque $E = C$ (III, 21.), in triangulis $ABE$, $ADC$ est

$$BA : AE = DA : AC \quad (VI, 4.),$$

proinde rectg. $BA \times AC = EA \times AD$ (VI, 16.)

Rob. Simson Lib. VI. Prop. C. (p. 120. *Matthias* p. 101. sq.).

§. 159.

Hinc rectg. $BC \times AD : BA \times AC = BC \times AD : EA \times AD$ (§. 158. & V, 7.)
$= BC : AE$ (§. 11.)

ubi rectg. $BC \times AD$ duplum est areæ trianguli (I, 41.).

Quare duplum areæ cujusvis trianguli est ad rectangulum sub duobus ipsius lateribus, uti tertium latus ad diametrum circuli triangulo circumscripti.

§. 160.

Conversas præcedentium, præter jam (§. 152. sqq.) expositas, notemus adhuc sequentes.

Si quadratum perpendiculi $AD$ (*Fig.* 23.), quod in trianguli $ABC$ latus $BC$ angulis ipsius acutis interjacens ex vertice opposito $A$ demittitur, æquale est rectangulo sub segmentis $BD$, $DC$ lateris $BC$, ab perpendiculo $AD$ factis; seu (VI, 17.) si perpendiculum $AD$ medium proportionale est inter segmenta $BD$, $DC$ lateris $BC$: trianguli ad verticem $A$ angulus est rectus.

Quippe ob $BD : DA = AD : DC$, & angulos ad $D$ rectos, est angulus $BAD = C$ (VI, 6.); angulus igitur $BAC = C + CAD =$ recto (I, 32. *Coroll.*)

§. 161.

Pappus (*Collect. Math.* Lib. VII. Prop. 203. fol. 277.) propositioni huic adjungit: Si quadratum perpendiculi $AD$ minus fuerit rectangulo sub segmentis $BD$, $DC$ lateris $BC$, angulus $BAC$ erit obtusus (*Fig.* 61.); si majus, acutus (*Fig.* 62.)

Semicirculo enim super diametro $BC$ descripto, qui perpendiculum

DA

*DA* in puncto *E* ſecet; ac rectis *BE*, *CE* junctis: ob $ED^q =$ rectg. $BD \times DC$ (§. 147. n°. 5.), priori caſu erit $AD < ED$, & hinc angulus $BAC > BEC$ (I, 21.); poſteriori $AD > ED$, atque angulus $BAC < BEC$ (I, 21.). Rectus autem eſt angulus *BAC* (III, 31.).

§. 162.

Propoſitorum §. 160. ſq. caſus ſpecialis, quo ſegmenta *BD*, *DC* æqualia ſunt, comprehenditur etiam iis, quæ in *Schol. in Lib. II.* §. 220. traduntur.

§. 163.

Cum §. 160. oſtenſum ſit: rectum eſſe angulum *BAC* (*Fig.* 23.), quem ab extremis *B*, *C* rectæ *BC* ad extremum *A* perpendiculi *AD* ductæ *BA*, *CA* comprehendunt, ſi rectg. $BD \times DC = AD^q$, ſeu ſi $BD : DA = AD : DC$; per converſam III, 31^mæ^ conſequitur: circuli ſuper diametro *BC* deſcripti peripheriam tranſire per verticem *A* rectæ *DA* diametro *BC* inter extrema ipſius normalis, quæ media proportionalis eſt inter ſegmenta diametri *BC* puncto *D* facta, ſeu cujus quadratum rectangulo ſub ſegmentis illis eſt æquale.

§. 164.

Clavius (p. 308. ſq. 312.) per impoſſibilitatem contrarii has demonſtrat propoſitorum §. 151. 155. converſas:

1°. "Si duæ rectæ ita ſe ſecent, ut rectangulum ſub unius ſegmentis comprehenſum æquale ſit ei, quod ſub ſegmentis alterius comprehenditur, rectangulo: deſcribi poterit per quatuor illarum puncta extrema circulus; h. e. circulus, per quælibet tria puncta earum extrema deſcriptus (IV, 5.), per quartum quoque punctum tranſibit."

2°. "Si a puncto aliquo binæ lineæ rectæ finitæ egrediantur, quæ ita ſecentur in binas partes, ut rectangula ſub totis & ſegmentis prope punctum comprehenſa ſint æqualia: deſcribi poterit per extrema puncta aliorum ſegmentorum circulus; h. e. circulus, per tria puncta extrema aliorum ſegmentorum deſcriptus, per quartum etiam punctum extremum tranſibit."

Eædem vero directe per converſas propoſitionum III, 21. 22. (quas & Clavius p. 278. 279. ſq. his ſubjunxit) ſic adſtruuntur:

 1°. Sit

1°. Sit (*Fig.* 53.) rectg. $AE \times EC = BE \times ED$, seu (VI, 16.) $AE : ED = BE : EC$; cum & æquales sint anguli ad verticem $E$ triangulorum $ADE$, $BCE$ (I, 15.); est angulus $D = C$ (VI, 6.); itaque punctum $D$ ad peripheriam circuli triangulo $ABC$ circumscripti (III, 21. *Conv.*).

2°. Sit (*Fig.* 55.) rectg. $AD \times DC = FD \times DE$, seu (VI, 16.) $\begin{cases} AD : DE = FD : DC \\ AD : DF = ED : DC \end{cases}$: cum hæc latera proportionalia communem comprehendant angulum $D$ triangulorum $\begin{cases} DEA, DCF \\ DAF, DEC \end{cases}$; est angulus $DAE = DFC$ (VI, 6.), itaque punctum $A$ ad peripheriam circuli triangulo $CEF$ circumscripti (III, 21. *Convers.*); vel sunt anguli $DFA = DCE$, $DAF = DEC$ (VI, 6.); proinde anguli $\begin{cases} DFA + ACE = DCE + ACE \\ DAF + FEC = DEC + FEC \end{cases} = 2$ Rect. (I, 13.); circulus igitur potest quadrilatero $CAFE$ circumscribi (III, 22. *Conv.*).

## §. 165.

Universe autem vera non est hæc Propositionis III, 35, seu §. 151. conversa: Si (*Fig.* 53.) ad idem punctum $E$ rectæ $AC$ in circulo ductæ, ad oppositas ejus partes, duæ rectæ $EB$, $ED$ ad peripheriam sic aguntur, ut rectangulum sub ipsis æquale sit rectangulo sub segmentis $AE$, $EC$ rectæ $AC$; posteriores duæ rectæ $BE$, $ED$ jacent in directum.

Sint enim (*Fig.* 63. 64.) $AC$, $BH$ duæ rectæ in circulo, se intra eum secantes in $E$ puncto, quarum neutra per centrum $G$ circuli transeat; & recta $GE$ producta arcum $CH$, in quem incidit, sic in puncto $F$ secet, ut arcus $FH$ minor sit reliquo arcu $FC$. Tum, si ab hoc reliquo arcu abscinditur $FD = FH$, & recta $ED$ ducitur: fit $ED = EH$ (III, 27. 7.); itaque rectg. $BE \times ED = BE \times EH = AE \times EC$ (§. 151.); nec in directum jacent $BE$, $ED$.

## §. 166.

Subjungamus Problemata nonnulla, usum præcedentium declarantia.

Problema I. Quadratum describere, quod sit quadrati dati multiplum assignatum; vel pars assignata; vel quod ad quadratum datum habeat rationem inæqualitatis eo, qui §. 108. explicatur, modo datam.

Lateris $AB$ quadrati dati $ABCD$ sumatur multiplum assignatum $AE$ (*Fig.* 65.); vel (VI, 9.) pars assignata $AE$ (*Fig.* 66.): vel latus $AB$ in ratione data $M : N$ quadrati propoſiti $ABCD$ ad quadratum deſcribendum continuetur (§. 101.) ad punctum $E$ usque (*Fig.* 67.); aut ſecetur (§. 99.) in puncto $E$ (*Fig.* 68.); ſic ut ſit $M : N = BA : AE$: ac finiatur parallelogrammum rectangulum $AEFD$ ſub $EA$ & $AD$.

Erit hoc rectangulum $AEFD$ in *Fig.* 65. multiplum aſſignatum, in *Fig.* 66. pars aſſignata quadrati $ABCD$ (*Schol. in Lib.* II. §. 3.); in *Fig.* 67. 68. autem rectangulum $AEFD$ ad quadratum $ABCD$ habebit rationem $EA : AB$ (VI, 1.) datæ $N : M$ eandem (*conſtr.*).

Quadratum igitur deſcribendum rectangulo $AEFD$ æquari; proinde (VI, 17.) latus ipſius medium proportionale eſſe debet inter $EA$ & $AD$ ſeu $AB$.

Semicirculo itaque ſuper diametro $AE$ deſcripto in *Fig.* 65. 67. cui producta $BC$ in $G$ occurrit; in *Fig.* 66. 68. autem ſuper diametro $AB$ deſcripto ſemicirculo, qui rectam $EF$ in $G$ ſecat; tum juncta $AG$ recta: hæc erit latus quadrati, quod fieri jubetur (§. 126.).

## §. 167.

**Problema II.** Datam rectam in inæqualia ſic dividere, ut rectangulum ſub ſegmentis æquale ſit quadrato differentiæ ſegmentorum.

Sit $AE$ recta data (*Fig.* 65.); factaque ſit diviſio imperata in puncto $L$; h. e. ſic, ut ſit rectg. $AL \times LE = (AL - LE)^q$.

Erit
$$3\,AL \times LE = AL^q + LE^q \text{ (II, 7. \& Schol. in Lib. II. §. 88.)}$$
$$\left.\begin{matrix} 5\,AL \times LE \\ 5(AL-LE)^q \end{matrix}\right\} = (AL+LE)^q \text{ (II, 4.)} = AE^q$$
$$(AL-LE)^q = \tfrac{1}{5} AE^q.$$

Datur itaque (§. 166.) differentia ſegmentorum $AL$, $LE$. Quare, cum & ſumma ipſorum $AE$ detur, dantur ipſa ſegmenta (*Schol. in Lib.* II. §. 30. 51.).

Nempe $AB = \frac{1}{5} AE$ abſciſſa ab data $AE$, & ſemicirculo ſuper hac deſcripto, eidemque ducta in $B$ normali $BG$ ad occurſum usque ſemicirculi in $G$ (§. 166.); tum ab $AE$ abſciſſa $AK = AG$; & reſidua $KE$ bifariam in $L$ ſecta (*Schol. in Lib.* II. §. 51.): factum erit, quod jubetur.

Sic enim
$$4\,\text{Rectg.}\,AL \times LE + AK^q = AE^q \text{ (II, 8. vel Schol. in Lib. II. §. 99.)}$$
$$= 5\,AG^q \text{ (conſtr. \& §. 166.)}$$
$$= 5\,AK^q \text{ (conſtr.)}$$

Unde

Unde $4\,\text{Rectg.}\,AL \times LE = 4\,AK^q$

$\text{Rectg.}\,AL \times LE = AK^q$

$= (AL - LE)^q$, ob $LK = LE$ (*conſtr.*)

§. 168.

Problema III. Datis ſumma $AB$ laterum duorum inæqualium quadratorum (*Fig.* 69. 70.), & latere $HI$ quadrati æqualis differentiæ ipſorum quadratorum; invenire eorum latera: ſeu propoſitam rectam $AB$ in inæqualia ſic ſecare, ut quadratorum ab ſegmentis ejus factorum differentia ſit quadrato rectæ datæ $HI$ æqualis: ſeu conſtruere triangulum rectangulum, cujus unus cathetus ſit datæ rectæ $HI$; ſumma autem hypotenuſæ alteriusque catheti, datæ $AB$ æqualis: ſeu deſcribere parallelogrammum rectangulum, cujus unum latus datæ $HI$, ſumma autem diagonalis alteriusque lateris priori contigui datæ $AB$ æquetur. (*)

Facta ſit rectæ $AB$ diviſio imperata in puncto $D$, ita ut $AD > DB$.

Cum ſit $AD^q$, ac tanto majus $AD^q - DB^q < AB^q$; ut poſſit eſſe $AD^q - DB^q = HI^q$, oportet, ſit $HI^q < AB^q$, $HI < AB$.

Cum (*Fig.* 69.) bifariam in C ſecta $AB$, ſit $AD^q - DB^q = 4\,AC \times CD$ (*Schol. in Lib II.* §. 101.);

fieri debet $4\,AC \times CD = HI^q = 4\,KI^q$, bifariam in $K$ ſecta $HI$

proinde $\text{rectg.}\,AC \times CD = KI^q$

$AC : KI = KI : CD$ (VI, 17.)

Quod

(*) In *Scholiis in Lib. II. Element.* §. 136. ſqq. 179. ſq. ſoluta fuerunt Problemata: Datis {ſumma / differentia} $AB$ laterum duorum quadratorum, & latere $HI$ quadrati æqualis ſummæ ipſorum quadratorum; invenire eorum latera: ſeu propoſitam rectam $AB$ ita {ſecare / continuare}, ut {quadratorum ab ſegmentis ejus factorum ſumma æqualis ſit / quadratum adjectæ & quadratum compoſitæ ex data & adjecta ſimul æqualia ſint} quadrato rectæ datæ $HI$: ſeu conſtruere triangulum rectangulum, cujus hypotenuſa ſit datæ rectæ $HI$, {ſumma / differentia} autem cathetorum datæ $AB$ æqualis: ſeu deſcribere parallelogrammum rectangulum, cujus diagonalis datæ $HI$, ac {ſumma / differentia} laterum contiguorum datæ $AB$ æquentur. Quæ problemata algebraice ſoluta exſtant in *Lhuiliers Anleitung zur Elementar-Algebra.* I Th. §. 80. 81. 86. 88.

Quod juxta §. 125. fit: rectæ $AB$ in $C$ conſtituendo perpendicularem $CE = KI$; & $AE$ rectæ in $E$ ducendo normalem $ED$: quæ, ob $HI < AB$ (*determ.*), proinde $KI < AC$, $KI^q < AC^q$ ſeu rectg. $AC \times CB$, ideoque (§. 161. ſq.) angulum $AEB$ obtuſum, $AEC$ vero acutum (I, 17.), intra angulum $CEB$ cadet; rectam igitur $CB$ in puncto $D$ inter ejus extrema $C$, $B$ ſito ſecabit.

## §. 169.

Vel cum duorum inæqualium quadratorum differentia ſit rectangulo ſub ſumma & differentia laterum ipſorum æqualis (*Schol. in Lib. II.* §. 21. 26. 45.); h. e. (*Fig.* 70.) $AD^q - DB^q =$ rectg. $BA \times AG$, $DG = DB$ abſciſſa ab $AD$: rectangulum hoc $BA \times AG$ datæ $HI$ quadrato æquari; differentia igitur $AG$ laterum quadratorum tertia proportionalis eſſe debet datæ laterum ſummæ $AB$ et rectæ $HI$ (VI, 17.). Quæ differentia proinde, ob $HI < AB$ (*determ.*), juxta §. 124. n°. 1. invenitur: in ſemicirculo ſuper data $AB$ deſcripto aptando rectam $AF = HI$; & ex $F$ demittendo perpendiculum $FG$ in diametrum $AB$. Ab ſumma autem $AB$ ſic abſciſſa differentia $AG$, habentur latera ipſa $AD$, $DB$, bifariam in $D$ ſecando reſiduam $GB$ (*Schol. in Lib.* II. §. 51.).

Conſtructionem hanc tradunt Marinus Ghetaldus (*De reſolutione et compoſitione mathematica Libri quinque.* Romæ 1630. Lib. I. Probl. IV. p. 22. ſeq.); Jacobus de Billy (*Diophantus geometra.* Paris. 1660. Lib. I. Prop. XXVIII. p. 55. ſq.).

## §. 170.

Problema IV. Datis differentia $AB$ laterum duorum inæqualium quadratorum (*Fig.* 71. 72.), & latere $HI$ quadrati æqualis differentiæ ipſorum quadratorum; invenire eorum latera: ſeu propoſitæ rectæ $AB$ aliam in directum ſic adjicere, ut quadratum compoſitæ ex data & adjecta excedat quadratum adjectæ ſpatio æquali quadrato rectæ datæ $HI$: ſeu triangulum rectangulum conſtruere, cujus unus cathetus ſit datæ rectæ $HI$; exceſſus autem hypotenuſæ ſuper alterum cathetum, datæ $AB$ æqualis: &c.

Facta ſit continuatio imperata rectæ datæ $AB$ ad $D$ usque.

Cum ſit $AD^q - DB^q = AB^q + 2AB \times BD$ (II, 4.) $> AB^q$: ut poſſit

eſſe

esse $AD^q - DB^q = HI^q$; oportet, sit $HI^q > AB^q$, $HI > AB$. Idem inde etiam consequitur; quod in omni triangulo non æquicruro differentia crurum minor est basi seu tertio latere (I, 20. *Coroll*).

Cum (*Fig.* 71.) bifariam in $C$ secta $AB$, sit $AD^q - DB^q = 4AC \times CD$ (*Schol. in Lib.* II. §. 100.);
fieri debet $4AC \times CD = HI^q = 4KI^q$, bifariam in $K$ secta $KI$
itaque rectg $AC \times CD = KI^q$
$AC : KI = KI : CD$ (VI, 35.).

Quod juxta § 125 fit: rectæ $AB$ in $C$ erigendo perpendicularem $CE = KI$; & $AE$ rectæ in $E$ ducendo normalem $ED$: quæ, ob $HI > AB$ (*determ*), ideoque $KI > AC$, $KI^q > AC^q$ seu rectg. $AC \times CB$, proinde angulum $AEB$ acutum (§. 161. sq.), extra angulum $AEB$ cadet; ob angulum vero $A$ trianguli $ACE$ ad $C$ rectanguli acutum (I, 17.), ideoque angulos $A + AED <$ 2Rect., productæ ad has partes $AB$ in $D$ occurret (*Lib.* I. *Ax.* 11.).

## §. 171.

Vel cum duorum inæqualium quadratorum differentia sit rectangulo sub summa & differentia laterum ipsorum æqualis (*Schol. in Lib.* II. § 21. 26. 45.): rectangulum hoc datæ $HI$ æquari; proinde summa laterum quadratorum tertia proportionalis esse debet datæ laterum differentiæ $AB$ & rectæ $HI$ (VI, 17.). Quæ summa proinde, ob $HI > AB$ (*determ.*), juxta §. 124. n°. 2. invenitur: super $AB$ (*Fig.* 72), tanquam catheto, triangulum constituendo rectangulum $ABF$, cujus hypotenusa $AF$ sit datæ $HI$ æqualis; & huic $AF$ in $F$ ducendo perpendicularem $FG$, quæ desideratam $AG$ ab producta $AB$ abscindit (VI, 8. *Coroll.*). Quo facto, cum $BG$ sit excessus summæ laterum inventæ $AG$ super datam ipsorum differentiam $AB$, bifariam in $D$ secando $BG$ obtinentur ipsa latera $AD$ & $DG$ seu $DB$ (*Schol. in Lib.* II. §. 51.).

Solutionem hanc indicat Billy Lib. I. Prop, XXXI. p. 59. sq.

## §. 172.

Problema V. Datis (*Fig.* 73.) latere $AB$ quadrati æqualis summæ duorum inaequalium quadratorum, & latere $I$ quadrati æqualis differentiæ eorum; invenire ipsorum latera: seu construere triangulum rectangulum, cujus hypotenusa datæ rectæ $AB$; & differentia quadratorum cathetorum quadrato rectæ datæ $I$ sit æqualis: &c.

Cum

Cum differentia duarum magnitudinum inæqualium aggregato earum sit minor; liquet, esse debere $I^q < AB^q$, $I < AB$.

Positis $P$, $Q$ lateribus duorum quadratorum; $P > Q$:

$$\begin{aligned} \text{ut sint} \quad & P^q + Q^q = AB^q \\ & P^q - Q^q = I^q; \\ \text{esse debent} \quad 2P^q & = AB^q + I^q; \; P^q = \frac{AB^q + I^q}{2} \; (*) \\ 2Q^q & = AB^q - I^q; \; Q^q = \frac{AB^q - I^q}{2} \end{aligned}$$

Itaque rectarum $P$, $Q$ constructio ad invenienda ope I, 47. (conf. *Schol. in Lib. II.* §. 66. 70) quadratorum, quæ summæ ac differentiæ datorum $AB^q$, $I^q$ æquentur, latera; tum vero ad exhibenda juxta §. 166, quæ horum dimidia sint, quadratorum latera reducitur.

## §. 173.

Concinnior descriptio ex altero Problematis enunciato consequitur.

Vertex nimirum $C$ trianguli rectanguli $ABC$ super hypotenusa data $AB$ construendi debet esse ad peripheriam semicirculi super diametro $AB$ descripti (III, 31. *Conv.* vel III, 33. n°. 2.).

Perpendiculo $CD$ ex vertice $C$ in hypotenusam $AB$ demisso, est $AC^q - CB^q = AD^q - DB^q$ (I, 47. *Coroll.* vel *Schol. in Lib. II.* §. 226.) Fieri igitur debet $AD^q - DB^q = I^q$; ideoque inventio puncti $D$, & perpendiculi

(*) Operosiore analysi PAPPUS in Prop. 228. *Lib. VII. Collect. math.* (fol. 297.). seu in *Lemm. VIII. ad Lib. VII. & VIII. Conic. Apollonii* (*Apollonii Conicorum Libri tres posteriores — Opera Edm. Halleji* — Oxon. 1710. p. 95.) hoc sic deducit; & data summa quadratorum ex $AB$ & $BC$, (*Fig.* 74) una cum differentia eorundem, utramque ex ipsis $AB$ & $BC$ datam esse infert: Posita $BD$ ipsi $BC$ æquali, datum erit rectangulum $CAD$, quod nempe per II, 6. differentia est quadratorum ex $AB$, $BC$. Dato autem rectangulo $CAD$, ejus duplum quoque datur. Pariterque datur summa quadratorum ex $CA$ & $AD$, quippe dupla summæ quadratorum ex $AB$ & $BC$ (II, 10.). Proinde per II, 4. datum est quadratum ex $CA$ & $AD$ simul sumtis; adeoque & summa ipsarum $CA$, $AD$ data est. Hujus vero dimidia est $BA$: quare $BA$ data est; ac proinde $BC$ quoque datur.

culi $DC$ quod alter eſt locus verticis $C$, reducitur ad *Probl.* III. (§. 168. ſq.): datam rectam $AB$ in inæqualia in puncto $D$ ita ſecare, ut ſit $AD^q - DB^q =$ quadrato rectæ datæ $I$. Quod juxta §. 169. efficitur: in ſemicirculo ſuper $AB$ deſcripto rectam $AF = I$ aptando; & perpendiculo $FG$ ex $F$ in $AB$ demiſſo, rectam $BG$ biſariam in $D$ ſecando. Tum perpendiculum $DC$, hypotenuſæ $AB$ in puncto $D$ conſtitutum, verticem $C$ trianguli conſtruendi deſignat in peripheria ſemicirculi ſuper $AB$ deſcripti.

§. 174.

Biſariam in $E$ ſecta $AB$, quoque eſt $AC^q - CB^q = 2AB \times ED$ (*Schol. in Lib. II.* §. 224. ſqq.) Quare etiam, ſi in ſemicirculo ſuper $EB$ deſcripto recta $EL = \frac{1}{2}I$ aptatur, ac per $L$ diametro $EB$ normalis ducitur $DLC$, hujus cum peripheria ſemicirculi ſuper $AB$ deſcripti interſectio verticem $C$ trianguli conſtruendi determinat. Sic quippe fit $AC^q - CB^q = 4BE \times ED = 4EL^q$ (§. 147. n°. 6.) $= I^q$. Conf. Apollonii *Loca plana. Lib. II. Prop. I.*

§. 175.

Problema VI. Datis (*Fig.* 75.) latere $AB$ quadrati æqualis ſummæ duorum quadratorum, & latere $I$ quadrati æqualis rectangulo ſub ipſorum lateribus; invenire hæc latera: ſeu conſtruere triangulum rectangulum, cujus hypotenuſa datæ rectæ $AB$; & rectangulum ſub cathetis, ſeu (I, 41.) dupla area trianguli, quadrato datæ rectæ $I$ æquentur: ſeu deſcribere parallelogrammum rectangulum, cujus diagonalis datæ $AB$, & area quadrato rectæ datæ $I$ ſint æquales. (*)

Cum

(*) Problemata: conſtruere parallelogrammum rectangulum, cujus {differentia / summa} laterum contiguorum ſit datæ rectæ $AB$, area autem quadrato rectæ datæ $I$ æqualis; ſeu rectam datam $AB$ ita {continuare / ſecare}, ut rectangulum ſub {adjecta, & ſub compoſita ex data & adjecta / ſegmentis ipſius} æquale ſit quadrato rectæ datæ $I$; geometrice ſolvuntur in *Scholiis in Lib.* II. §. 66. 74. ſq; algebraice, pariter ac Problemata VI. VII, in *Lhuiliers Anleitung zur Elementar-Algebra.* §. 78 79. 85. 87. 83. 107. 108.

Cum ſumma quadratorum duarum æqualium rectarum dupla ſit parallelogrammi rectanguli ſub ipſis; ſumma autem quadratorum duarum inæqualium rectarum ſit duplo rectangulo ſub ipſis major (*Schol. in Lib.* II. §. 84.): ut fieri poſſit, quod jubetur; oportet non ſit $AB^q < 2I^q$, ſeu non ſit $I^q > \frac{1}{2}AB^q$.

Simul liquet: ſi ſit $AB^q = 2I^q$, ſeu $I^q = \frac{1}{2}AB^q$; æqualia fore latera deſiderata: utrumque igitur æquale lateri quadrati, quod ſit ipſius $AB^q$ dimidium; proinde = datæ $I$.

Quodſi $AB^q > 2I^q$: poſitis $P$, $Q$ lateribus duorum tunc inæqualium quadratorum, $P > Q$;

ob $P^q + Q^q = AB^q$

$2PQ = 2I^q$

erunt $(P+Q)^q = AB^q + 2I^q$ (II, 4.)

$(P-Q)^q = AB^q - 2I^q$ (II, 7. *Schol. in Lib.* II. §. 86.).

Quare cum quadrati $2I^q$ latus detur per I, 47. vel §. 166; rectarum $P+Q$, $P-Q$ conſtructio ad invenienda ope I, 47. quadratorum, quæ ſummæ ac differentiæ datorum $AB^q$, $2I^q$ æquentur, latera reducitur. Tum vero, inventis ſumma ac differentia rectarum $P$, $Q$; ipſæ $P$, $Q$ innoteſcunt juxta §. 30. vel 51. *Schol. in Lib.* II.

Sic ab data $AB$ abſciſſa $AC$ = datæ $I$, ipſique in $C$ ducto perpendiculo $CE = AC = I$; fit (I, 47.) $AE^q = 2AC^q = 2I^q$: & ob $AB^q > 2I^q$ (*ſupp.*) $> AE^q$, erit $AB > AE$.

Jam $AD = AE$ abſciſſa ex $AB$; & huic in $A$, $D$ conſtitutis normalibus $AF$, $DH$, quas circulus centro $A$, intervallo $AB$, deſcriptus in punctis $F$, $H$ ſecat; tum junctis $DF$, $AH$ rectis: ſunt (I, 47.) $DF^q = AF^q + AD^q = AB^q + 2I^q$, $DH^q = AH^q - AD^q = AB^q - 2I^q$.

Quare $DF = P+Q$, $DH = P-Q$. Ab $DF$ igitur abſciſſa $DK = DH$, & reſidua $FK$ bifariam in $G$ ſecta: fiunt $DG = P$, $GF = Q$ (*Schol. in Lib.* II. §. 51.).

## §. 176.

Conſtructio trianguli rectanguli $ABC$, cujus hypotenuſa datæ $AB$, rectangulum ſub cathetis $AC$, $CB$ quadrato datæ $I$ æquetur (*Fig.* 76. 77.): ob rectg. $AB \times CD = AC \times CB$ (§. 147. n°. 3.) $= I^q$ ($CD$ ex $C$ demiſſa normali in $AB$), proinde $AB : I = I : CD$ (VI, 17.), perpendiculum igitur $CD$ datum (VI, 11.); reducitur ad *Problema:* Deſcribere triangulum ad verticem rectangulum, cujus baſis atque altitudo dantur; quod æque ac alterum generalius: Deſcribere triangulum, cujus baſis, altitudo, & angulus ad verticem quicunque dantur, ope Problematum I, 31. III, 33. conſtruitur.

Trianguli nempe rectanguli, ſuper data baſi ſeu hypotenuſa $AB$, & cum altitudine data, (cui æquale perpendiculum ubicunque ſuper $AB$, ex. gr. $BE$ in $B$, conſtituitur) deſcribendi vertex $C$ erit ad occurſum peripheriæ ſemicirculi ſuper $AB$ deſcripti ac rectæ $EH$ per $E$ ipſi $AB$ parallelæ; quæ utique ſibi mutuo occurrunt, dummodo altitudo data $BE$ non ſit $\rhd \frac{1}{2}AB$: nimirum ſe invicem tangunt in puncto $C$ biſectionis ſemicirculi (*Fig.* 76.), quando $BE =$ hujus ſemidiametro $\frac{1}{2}AB$ (III, 27. I, 13. 29. 33. 34. III, 16.); quando autem $BE \lhd \frac{1}{2}AB$ (*Fig.* 77.), ſe mutuo ſecant in duobus punctis $C$, $c$ (*Schol. in Lib.* II. §. 75.), quibus ac terminis baſis $AB$ interjacent arcus ſemicirculi æquales $BC$, $Ac$ (I, 29. III, 26.).

Priore igitur caſu (*Fig.* 76.) unum triangulum æquicrurum $ABC$; altero (*Fig.* 77.) duo triangula ſcalena $ABC$, $ABc$ Problemati ſatisfaciunt; hæc vero ſitu tantum, non longitudine laterum diverſa (III, 29.).

## §. 177.

Atqui ſub determinatione *Problematis* VI. (§. 175.) $I^q$ non $\rhd \frac{1}{2}AB^q$, ſi rectis $AB$, $I$ ſit perpendiculum $BE$ tertium proportionale, juxta §. 119. ſq. ab producta $AB$ ac normali ipſi in $B$ ducta abſcindendo $BF = BG = I$, & rectam $FE$ ipſi $AG$ parallelam agendo; ob $AB \times BE = I^q$ (VI, 17) $\overset{=}{\lhd} \frac{1}{2}AB^q$, ſit $BE \overset{=}{\lhd} \frac{1}{2}AB$. Unde juxta §. 176. abſolvitur ſolutio Problematis.

Priore caſu, quo $I^q = \frac{1}{2}AB^q$ (*Fig.* 76), ab perpendiculo, datam hypotenuſam $AB$ biſariam in $D$ ſecante, $DC = DA = DB$ abſcindere; vel ſuper data $AB$ triangulum æquicrurum $ABC$, cujus crura ſingula ſint $= I$, deſcribere ſufficit. Ita enim, ſi poſterius ſit, immediate eſt $AC \times CB = \left\langle \begin{matrix} AC^q \\ CB^q \end{matrix} \right\rangle = I^q$; atque, ob $AB^q = 2I^q$ (*ſupp.*) $= AC^q + CB^q$, angulus $ACB$ ſit rectus (I, 48.). Si prius: eſt $CD^q = AD \times DB$, ideoque angulus $ACB$ rectus (§. 160. 162.); & $AC \times CB = AC^q$ (I, 4.) $= AD^q + DC^q$ (I, 47.) $= 2AD^q = \frac{1}{2}AB^q$ (*conſtr.* & *Schol. in Lib.* II. §. 13.) $= I^q$.

## §. 178.

Eo, qui §. 176. sq. traditur, modo ROB. SIMSON in Prop. 88. *Datorum Euclidis*, correctius & auctius ab ipso Glasguæ 1762. editorum, parallelogrammum rectangulum describere docet, cujus area & summa quadratorum laterum dantur. Conf. in versione eorundem Germanica (Stuttgart, 1780.) Problema 23. p. 235. sq.

## §. 179.

PROBLEMA VII. Datis (*Fig.* 78.) latere $AB$ quadrati æqualis differentiæ duorum inæqualium quadratorum, & latere $I$ quadrati æqualis rectangulo sub ipsorum lateribus; invenire hæc latera: seu construere triangulum rectangulum, cujus unus cathetus datæ rectæ $AB$, & rectangulum sub hypotenusa alteroque catheto factum quadrato rectæ datæ $I$ æquetur: seu describere parallelogrammum rectangulum, cujus unum latus datæ rectæ $AB$; rectangulum vero sub diagonali & sub altero latere sit quadrato datæ $I$ æquale.

Ad latus datum $AB$ factum sit triangulum $ABC$ in $B$ rectangulum, sic ut sit rectg. $AC \times CB = I^q$.

Hypotenusæ ejus $AC$ in $C$ normalis ducatur $CD$, productæ $AB$ in $D$ occurrens. Erit rectg. $AB \times CD = AC \times CB$ (§. 147. n°. 4.) $= I^q$; unde $AB : I = I : CD$ (VI, 17.); ideoque $CD$ datur (VI, 11.)

Sed $CD^q =$ rectg. $AD \times DB$ (§. 147. n°. 2.). Quare hoc rectangulum est quadrato rectæ datæ æquale. Proinde, cum & laterum ipsius $AD$, $DB$ differentia $AB$ detur; datur $AD$ (*Schol. in Lib. II.* §. 66.).

Ob angulum autem $ACD$ rectum (*constr.*) est punctum $C$ ad peripheriam semicirculi super diametro $AD$ descripti (III, 31. *Conv.*), idemque (*supp.*) est ad perpendiculum datæ $AB$ in $B$ erectum. Proinde datur hoc punctum $C$.

Compositio ex analysi præcedente hæc consequitur.

Primum datis $AB$, $I$ invenitur tertia proportionalis $BH$ juxta §. 125: ab perpendiculo, rectæ $AB$ in $B$ constituto, $BF = I$ abscindendo; & rectæ $AF$ in $F$ ducendo normalem $FH$, quæ productæ $AB$ in $H$ occurrit.

Tum juxta §. 66. *Schol. in Lib. II.* datæ $AB$ in directum adjicitur $BD$ sic, ut fiat rectg. $AD \times DB = BH^q$: rectam $AB$ bifariam in $E$ secancando; ab perpendiculo $FB$ abſcindendo $BG = BH$; & ab producta $AB$ abſcindendo $ED = EG$.

Quo facto ſemicirculus deſcribitur ſuper diametro $AD$; & ad punctum $C$, ubi ejus peripheria perpendiculum $BF$ ſecat, ducitur $AC$ recta.

Sic enim, juncta $CD$ recta, eſt $CD^q = AD \times DB$ (§. 147. n°. 6.)
Sed ob $AB$ bifariam in $E$ ſectam eſt rectg. $AD \times DB = ED^q - EB^q$ (II, 6.)
$= EG^q - EB^q$ (*conſtr.*)
$= BG^q$ (I, 47.)
$= BH^q$ (*conſtr.*)
Quare $CD^q = BH^q$; $CD = BH$; rectg. $AB \times CD = AB \times BH$
$= BF^q$ (§. 147. n°. 1.)
Ideoque & rectg. $AG \times CB = AB \times CD$ (§. 147. n°. 4.) eſt $= BF^q = I^q$ (*conſtr.*)

## §. 180.

Quodſi rectæ §. 179. inventæ $AC$, $CB$ ſub angulo recto junguntur; ab producta $DC$ abſcindendo $CK = CB$: obtinetur $\left\{\begin{matrix}\text{triangulum}\\ \text{parallelogrammum}\end{matrix}\right\}$ rectangulum $\left\{\begin{matrix}ACK\\ ACKL\end{matrix}\right\}$; cujus area $= \left\{\begin{matrix}\frac{1}{2} I^q\\ I^q\end{matrix}\right\}$, & differentia quadratorum $\left\{\begin{matrix}\text{cathetorum}\\ \text{laterum contiguorum}\end{matrix}\right\} = AB^q$.

Parallelogrammi, cujus area & differentia quadratorum laterum dantur, ſub quocunque angulo dato deſcribendi latera invenire EUCLIDES in *Dator.* 87. elegante analyſi, immediate huic problematis enunciato adaptata, docet; quæ vero ad parallelogrammum rectangulum applicata paulo eſt præcedente (§. 179.) prolixior, ac præter alias VI, 22dam *Elem.* ſupponit.

## §. 181.

PROBLEMA VIII. Conſtruere triangulum rectangulum; cujus dantur ſumma hypotenuſæ & unius catheti, ac ſumma alterius catheti & perpendiculi ab vertice anguli recti in hypotenuſam demiſſi.

Sit (*Fig.* 79.) $ABC$ triangulum ad $C$ rectangulum deſcribendum; in quo nempe $AB + BC =$ datæ rectæ $G$, & $AC +$ perpendiculum $CD =$ datæ $H$.

Cum

Cum (§. 93. n°. 1.) ſit $BA : AC = BC : CD$
ideoque etiam (V, 12.) $AB+BC : AC+CD = BA : AC$
proinde $AB+BC > AC+CD$, ob $BA > AC$:
ut fieri poſſit, quod jubetur, debet eſſe $G > H$.

Continuentur $AB$, $AC$, donec ſint $BF = BC$, $CE = CD$; itaque $AF = AB + BC$, $AE = AC + CD$.

Erunt (*demonſtr.*) $FA : AE = BA : AC$:
hinc $EF$, $CB$ ſunt parallelæ (§. 22.); angulus $E = ACB$ (I, 29.) eſt rectus; & datur triangulum $AEF$, cujus hypotenuſa $AF = G$, cathetus $AE = H$ dantur.

Porro ob $CB = BF$ (*conſtr.*) eſt angulus $BFC = BCF$ (I, 5.);
& ob parallelas $BC$, $FE$ (*dem.*) $BCF = CFE$ (I, 29.):
quare $BFC = CFE$.
Poſitione igitur datur recta $FC$ datum angulum $AFE$ bifariam ſecans (I, 9.); & per interſectionem ipſius cum poſitione data $AE$ datur punctum $C$; per quod ducta rectæ $EF$ parallela $CB$ finit triangulum $ABC$.

In ſemicirculo itaque ſuper recta $AF =$ datæ $G$ deſcripto aptabitur $AE$ recta $=$ datæ $H$: tum, juncta $FE$ recta, bifariam ſecabitur angulus $AFE$ per rectam $FC$; & per punctum $C$, ubi hæc rectæ $AE$ occurrit, agetur $CB$ ipſi $EF$ parallela. Erit $ABC$ triangulum deſideratum.

Quippe ob angulum $BCF = CFE$ (I, 29.) $= CFB$ (*conſtr.*) eſt $CB = BF$ (I, 6.); igitur $AB + BC = AF = G$.

Angulus $ACB = E$ (I, 29.) $=$ recto (III, 31.).

Ex $C$ in $AB$ demiſſo perpendiculo $CD$, eſt

$$BA : AC = BC : CD \quad (§. 93. n°. 1.)$$
$$= AB+BC : AC+CD \quad (V, 12.);$$

pariterque, ob parallelas $CB$, $EF$ eſt

$$BA : AC = FA : AE \quad (§ 22.):$$

igitur $AB+BC : AC+CD = FA : AE$ (V, 11.):
atque, ob $AB+BC = FA$ (*dem.*), etiam $AC+CD = AE$ (V, 14.) $= H$.

## §. 182.

Similiter, ex $AB$, $AC$ abſciſſis $BC$, $CD$; & loco 12^mæ^ quinti *Elem.* adhibita 19^na^; ſolvitur *Problema*: quo differentiæ hypotenuſæ $AB$ & catheti $BC$, atque alterius catheti $AC$ & perpendiculi $CD$, dantur.

Quodſi

Quodſi autem, alterutra ſumma §. 181. data, alterius ſummæ loco differentia datur:

ob $BA : AC = BA \times AD : CA \times AD$ (VI, 1.)

$= AC^q : CA \times AB - \begin{cases} CA \times DB \\ BC \times CD \end{cases}$ (§. 147. n°. 2.4.)

& $CA \times AB - BC \times CD = (AC \pm CD)(AB \mp BC)$ (II, 1.)

quoniam $AC \times CB = AB \times CD$ (§. 147. n°. 3.);

poſitis $AC \pm CD = H$, $AB \mp BC = G$, fieri debet

$BA : AC = AC^q : G \times H$

$2G \times AB : G \times AC = 2AC^q : G \times H$ (VI, 1. V, 4.)

$2G \times AB : 2AC^q = G \times AC : G \times H$ (V, 16)

$= AC : H$ (VI, 1.):

&, ob $(AB \mp BC)^q + AC^q = AB^q \mp 2AB \times BC + \begin{cases} BC^q + AC^q & \text{(II, 7. 4.)} \\ AB^q & \text{(I, 47.)} \end{cases}$

$= 2(AB^q \mp AB \times BC)$

$= 2(AB \mp BC)AB$ (II, 1.)

ſeu $G^q + AC^q = 2G \times AB$,

oportet, fiat

$G^q + AC^q : 2AC^q = AC : H$;

quod limites geometriæ elementaris tranſcendit.

## §. 183.

**Problema IX.** (*) Conſtruere triangulum rectangulum, cujus ſumma cathetorum datæ rectæ $H$, & perpendiculum ab vertice anguli recti in hypotenuſam demiſſum datæ $I$ æquentur (*Fig.* 80.)

Cum in triangulo $ABC$ (*Fig.* 76.) ad $C$ rectangulo & æquicruro ſit $AC + CB = 2AC$; ideoque $(AC + CB)^q = 4AC^q = 4(AD^q + CD^q) = 8CD^q$, ob $CD = AD$ (IV, 6.): in non-æquicruro autem (*Fig.* 77.) ſit

$AC^q + CB^q = AD^q + DB^q + 2CD^q$ (I, 47.)

$= 2(LB^q + LD^q + CD^q)$ bifariam in $L$ diviſa $AB$ (II, 9.)

$> 2(LB^q + CD^q)$

$> 4CD^q$, ob $LB = LC > CD$

& 2AC

(*) Problemata IX. X. XI. XII. XIII. algebraice, cum deductis inde conſtructionibus geometricis, ſoluta exſtant in Newtoni *Arithmetica univerſali*; Cap. III. inſcripto: *Quomodo quæſtiones geometricæ ad æquationem redigantur*; Probl. 6. 7. 5. 4. 3.

$$\& \quad 2AC \times CB = 2AB \times CD \text{ (§. 147. n°. 3.)}$$
$$= 4LB \times CD$$
$$> 4CD^q;$$

ideoque (II,4.) $(AC+CB)^q > 8CD^q$:
ut, quod jubetur, fieri poſſit; oportet, non ſit $H^q < 8I^q$.

Sit (*Fig.* 80) $ABC$ triangulum deſideratum; cujus catheti $AC+CB=H$, perpendiculum $CD=I$.

Ob $AB^q = AC^q + CB^q$ (I, 47.)

& $\left.\begin{matrix} 2AB \times CD \\ 2AB \times I \end{matrix}\right\} = 2AC \times CB$ (§. 147. n°. 3.)

erit $AB^q + 2AB \times I = (AC+CB)^q$ (II, 4.) $= H^q$.

Ab producta $BA$ abſcindatur $AE=EF=I$; itaque $AF=2I$.

Erit $\left.\begin{matrix} AB^q + BA \times AF \\ \text{ſeu (II, 3.) } FB \times BA \end{matrix}\right\} = H^q$.

Unde, ob datam $H$, datamque $FA=2I$ differentiam laterum $FB$, $BA$ rectanguli $FB \times BA$, innoteſcit hypotenuſa $AB$ trianguli $ABC$; ipſumque triangulum, ob datum etiam perpendiculum $CD=I$.

Nempe juxta § 66. *Schol. in Lib.* II. rectæ $AF=2I$, bifariam in $E$ ſectæ, normalis in $A$ conſtituetur $AG=H$; & ab producta $FA$ abſcindetur $EB=EG$: tum ab perpendiculo $AG$ abſciſſa $AK=I$, per $K$ parallela agetur rectæ $AB$ ad occurſum usque ſemicirculi ſuper diametro $AB$ deſcripti; & ad hunc rectæ ab punctis $A$, $B$ ducentur (§. 176.).

Reipſa autem ſemicirculum ſuper diametro $AB$ deſcriptum tangit vel ſecat recta ipſi $AB$ per $K$ parallela: cum, ob $8I^q \gtreqless H^q$ (*determ.*), h. e. $8AK^q \gtreqless AG^q$ (*conſtr.*), ideoque $9AK^q \gtreqless AG^q + AE^q$ ſeu (I, 47.) $EG^q$, $3AK^q \gtreqless EG$ ſeu $EB$, $2AK \gtreqless AB$, ſit $AK \gtreqless \frac{1}{2}AB$.

## §. 184.

Eodem redeunt Problematis hujus conſtructiones: quam GHETALDUS (l. c. *Lib.* III. *Probl.* IV. p. 110. ſqq.) ex analyſi partim geometrica, partim algebraica; & concinnior paulo, quam WOLFIUS (*Elementa Matheſeos univerſæ.* Tom. I. Halæ 1730. p. 397. ſq.) ex analyſi prolixiore mere algebraica, ſed determinatione omiſſa, deducunt. GHETALDUS determinationem, geometrice ſeorſim demonſtratam, ſic enunciat: "Tripla perpendicularis trianguli rectanguli, ab angulo recto in baſim cadens,

 non

non eſt major quam recta, cujus quadratum æquale eſt quadratis aggregati crurum & perpendicularis.”

## §. 185.

Simili, qua §. 183, ratione, loco 4tæ ſecundi *Elem.* applicando 7mam, ſolvitur *Problema* determinationi haud obnoxium: quo, præter perpendiculum ex vertice anguli recti in hypotenuſam demiſſum, differentia cathetorum trianguli rectanguli datur.

Tum quippe, datis quibuscunque $H =$ differentiæ cathetorum, $I =$ perpendiculo, ob $\left.\begin{matrix} AB^q - 2AB \times I \\ AB(AB - 2I) \end{matrix}\right\} = H^q$ (*Schol. in Lib.* II. §. 86.), datur rectangulum ſub $AB$ & $AB - 2I$, cujus differentia laterum $= 2I$: conſtructione igitur juxta §. 66. *Schol. in Lib.* II. facta, prodit $AB \triangleright 2I$, $I \triangleleft \frac{1}{2}AB$.

Problema hoc ſuo etiam more ſolvit Ghetaldus l. c. *Lib.* III. *Probl.* III. p. 109. ſq.

## §. 186.

Problema X. Conſtruere triangulum rectangulum; cujus ſumma cathetorum datæ rectæ $H$; & ſumma hypotenuſæ ac perpendiculi, in eam ab anguli recti vertice demiſſi, datæ $G$ ſit æqualis. (*Fig.* 81.).

Ob $BA : AC = BC : CD$ in triangulo $ABC$ ad $C$ rectangulo (§. 93. n°. 1.); & $BA$ maximam, $CD$ minimam quatuor rectarum (I, 19. *Coroll.*); ideoque $BA + CD \triangleright AC + CB$ (V, 25.): debet eſſe $G \triangleright H$.

Porro ob $AC^q + CB^q = AB^q$ (I, 47.)

$2AC \times CB = 2AB \times CD$ (§. 147. n°. 3.)

ideoque $(AC + CB)^q = AB^q + 2AB \times CD$ (II, 4.)

$8(AC + CB)^q = 8(AB^q + 2AB \times CD)$

ſed $(AC + CB)^q \gtreqless 8CD^q$ (§. 183.)

proinde $9(AC + CB)^q \gtreqless 8(AB + CD)^q$ (II, 4.):

debet eſſe $9H^q \gtreqless 8G^q$; ſeu oportet, non ſit $8G^q \triangleright 9H^q$.

Sit $ABC$ triangulum quæſitum: cujus catheti $AC + CB = H$; hypotenuſa $AB$ + perpendiculum $CD = G$.

Ob $AB^q + 2AB \times CD + CD^q = (AB + CD)^q$ (II, 4.) $= G^q$

& $AB^q + 2AB \times CD = (AC + CB)^q$ (*demonſtr.*) $= H^q$

manet $CD^q = G^q - H^q$

Datur

Datur itaque perpendiculum $CD$; eoque ab $G$ ablato, hypotenuſa $AB$; proinde triangulum $ABC$ (§. 176.).

Nimirum in ſemicirculo, ſuper recta $BE = G$ deſcripto, aptata recta $BF = H$ (IV, 1.); ob $EF^q = EB^q - BF^q$ (III, 31. I, 47.), erit $EF =$ perpendiculo: & ab $EB$ abſciſſa $EA = EF$, reſidua $AB$ erit hypotenuſa trianguli deſcribendi. Huic igitur in puncto $A$ ducta normali $AK = AE = EF$; rectæ ipſi $AB$ per $K$ parallelæ occurſus cum ſemicirculo ſuper $AB$ deſcripto verticem $C$ trianguli deſignabit (§. 176.): cum (*determ.*) ſit $8G^q \gtreqless H^q$, h. e. (*conſtr.*) $\left.\begin{matrix} 8EB^q \\ 8(BF^q + EF^q) \end{matrix}\right\} \gtreqless 9BF^q$; ideoque $8EF^q \gtreqless BF^q$, $9EF^q \gtreqless \left\{\begin{matrix} BF^q + EF^q \\ EB^q \end{matrix}\right\}$, $\left.\begin{matrix} 3EF \\ 3EA \end{matrix}\right\} \gtreqless EB$, $\left.\begin{matrix} 2EA \\ 2AK \end{matrix}\right\} \gtreqless AB$, $AK \gtreqless \frac{1}{2}AB$.

Conſtructionem Problematis hujus valde operoſam absque analyſi tradit THOM. SIMSON (*Treatiſe of Algebra.* London 1745. p. 400. ſq.): ejusdem ſolutionem algebraicam & compoſitionem inde deductam, ſimiles Newtonianis, expoſuerat p. 244. ſq.

## §. 187.

Quodſi dantur $AB - CD = G$, $AC - CD = H$:

$$\begin{array}{lll} \text{ob } CD < CB, \text{ ideoque } & AB - CD & > AB - CB \\ \& \ AB > AC & & > AC - CB; \\ \text{pariter oportet, ſit} & G & > H. \end{array}$$

Ceterum, 7$^{ma}$ ſecundi *Elem.* adhibita loco 4$^{tæ}$, rurſus prodit $CD^q = G^q - H^q$.

Unde, in ſemicirculo ſuper $EB = G$ deſcripto aptata $BF = H$, fit $EF = CD$; atque ipſi $BE$ in directum adjecta $EF$, invenitur $AB$; ac triangulum conſtruitur juxta §. 176: cum, ob $EF < EB$, ſit $2EF < BE + EF$, $EF < \frac{BE+EF}{2}$.

## §. 188.

Dàtis autem vel $AB + CD = G$, $AC - CB = H$;
vel $AB - CD = G$, $AC + CB = H$:

$$\begin{array}{ll} \text{ob} & \left.\begin{matrix} (AC \mp CB)^q \\ H^q \end{matrix}\right\} = AB^q \mp 2AB \times CD \ (\text{II, 4. 7. I, 47. \& §. 147. n°. 3.}) \\ \text{ſeu} & \\ & = AB(AB \mp 2CD) \ (\text{II, 1.}) \end{array}$$

$$\text{atque } AB \mp 2CD = AB - 2G + \left\{\begin{matrix} 2G \\ 2AB \pm 2CD \end{matrix}\right\} \mp 2CD$$
$$= 3AB - 2G = 3(AB - \tfrac{2}{3}G);$$



fieri

fieri debet $3AB(AB - \frac{2}{3}G) = H^q$

$$AB(AB - \tfrac{2}{3}G) = \tfrac{1}{3}H^q.$$

Juxta §. 166. igitur invento latere quadrati $= \frac{1}{3}H^q$; tum juxta §. 66. *Schol. in Lib.* II. inventis lateribus rectanguli $= \frac{1}{3}H^q$, cujus differentia laterum $= \frac{2}{3}G$: majus latus hujus rectanguli erit hypotenusa $AB$ trianguli describendi; altitudo autem $CD = \left.\begin{matrix} G - AB \\ AB - G \end{matrix}\right\}$. Priore autem casu, ob $AB > \frac{2}{3}G$, fit $CD = G - AB < \frac{1}{3}G < \frac{1}{2}AB$.

Pariterque posteriori, quo $H^q = (AC+CB)^q \geqq 8CD^q \geqq 8(AB-G)^q$

$$\begin{array}{lll}
\text{proinde} & \left.\begin{matrix} H^q \\ 3AB(AB-\frac{2}{3}G) \end{matrix}\right\} + (AB-G)^q & \geqq 9(AB-G)^q \\
\text{h. e.} & \left.\begin{matrix} 4AB^q - 2G \times AB + G^q \\ (2AB-G)^q \end{matrix}\right\} & \geqq 9(AB-G)^q \\
\text{seu} & & \\
 & 2AB - G & \geqq 3(AB-G) \\
\text{est} & 2G & \geqq AB \\
 & G & \geqq \frac{1}{2}AB \\
\text{ideoque} & CD = AB - G & \leqq \frac{1}{2}AB.
\end{array}$$

## §. 189.

Problema XI. Construere triangulum rectangulum: cujus hypotenusa sit recta data $AB$; summa autem cathetorum ac perpendiculi ex vertice anguli recti in hypotenusam demissi sit datæ rectæ $H$ æqualis (*Fig.* 82.).

Cum in triangulo $ABC$ ad $C$ rectangulo (*Fig.* 76. 77.) sit (I, 20.) $AC+CB$, tantoque magis $AC+CB+CD > AB$: primum debet esse data $H > AB$.

Porro in triangulo rectangulo æquicruro $ABC$ (*Fig.* 76.) sunt

$$\begin{array}{rl}
2(AC+CB) & = 4AC \\
2CD & = AB \\
\text{ideoque} \quad 2(AC+CB+CD) & = 4AC+AB \\
2(AC+CB+C\ \ ) - AB & = 4AC \\
(2(AC+CB+CD) - AB)^q & = 16AC^q \ (\textit{Schol. in Lib.}\ \text{II. §. 6.}) \\
 & = 8(AC^q+CB^q) = 8AB^q \ (\text{I}, 47.)
\end{array}$$

In triangulo autem rectangulo non-æquicruro $ABC$ (*Fig.* 77.), ob $2CD < AB$ (III, 3. 15.),

est

est $2(AC+CB+CD)-AB < 2(AC+CB)$;
igitur $(2(AC+CB+CD)-AB)^q < 4(AC+CB)^q$:
& $4(AC+CB)^q = 4(AC^q+CB^q)+8AC\times CB$ (II, 4.)
$= 4AB^q+8AB\times CD$ (I, 47. & §. 147. n°. 3.)
$< 8AB^q$, ob $2CD<AB$, ideoque $8AB\times CD<4AB^q$.
Quare tanto magis $(2(AC+CB+CD)-AB)^q < 8AB^q$.

Oportet igitur, ex altera parte non sit

$$(2H-AB)^q > 8AB^q$$

seu $$(H-\tfrac{1}{2}AB)^q > 2AB^q$$

h. e. $H-\frac{1}{2}AB$ major non sit diagonali quadrati, cujus latus est $AB$.

Sit jam (*Fig.* 82.) $ABC$ triangulum super data hypotenusa $AB$ construendum, in quo $AC+CB+CD=H$.

Ob $BA:AC = BC:CD$ (§. 93. n°. 1.)
$= AB+BC : AC+CD$ (V, 12.)

$$\left.\begin{matrix} BA : BA+AC \\ 2BA : 2(BA+AC) \end{matrix}\right\} = AB+BC : AB+BC+CA+CD \text{ (V, 18. 15.)}$$

est $2(BA+AC)(AB+BC) = 2BA(AB+BC+CA+CD)$ (VI, 16.)
$= EB\times BF$

in directum ipsi $BA$ adjectis $AE=AB$, $AF=BC+CA+CD=H$.

Sed $2(BA+AC)(AB+BC) = (BA+AC)(2AB+2BC)$
$= (BA+AC)(BA+AC+2BC+(BA-AC))$

$$= (BA+AC)^q+2BC(BA+AC)+\left\{\begin{matrix}(BA+AC)(BA-AC) & \\ BA^q-AC^q & \text{(II, 5. Coroll.)} \\ BC^q & \text{(I, 47.)}\end{matrix}\right.$$

$= (BA+AC+CB)^q$ (II, 4.)

Igitur $(BA+AC+CB)^q = EB\times BF$
$EB : BA+AC+CB = BA+AC+CB : BF$ (VI, 17.).

Quare, cum dentur $EB=2AB$, $BF=BA+\left\{\begin{matrix}AF\\H\end{matrix}\right.$ (*constr.*), datur trianguli perimeter $BA+AC+CB$ (VI, 13.): qua ab $BF=BA+AC+CB+CD$ ablata, manet trianguli altitudo $CD$; & trianguli descriptio reducitur ad § 176.

Datæ scilicet hypotenusæ $BA$ in directum adjectis $AE=AB$, $AF=H$; tum juxta §. 126. super $BF$ descripto semicirculo, cui in $G$ occurrit normalis rectæ $BF$ in $E$ ducta $EG$; & facta $BL=BG$: ab perpendiculo $EG$ abscindetur $EK=FL$; ac (§. 176.) per $K$ agetur rectæ $BF$ parallela, semicirculo super $AB$ descripto in $C$ occurens:

cum

cum ob $(2H - AB)^q$
h. e. (*constr.*) $(2FA - AB)^q$
seu (II, 7.) $4FA^q - 4FA \times AB + AB^q$ } $\geqq 8AB^q$ (*determ.*)
sit $4FA^q + 4FA \times AB + AB^q \geqq 8(FA \times AB + AB^q)$

h. e. (II, 4.) $(2FA + AB)^q \geqq \begin{cases} 8FB \times BA & \text{(II, 3.)} \\ 4FB \times BE & (\textit{constr.}) \\ 4BG^q & \text{(§. 147. n°. 6.)} \end{cases}$

ideoque $2FA + AB \geqq \begin{cases} 2BG \\ 2BL \end{cases}$

$\left. \begin{matrix} 2FA + 2AB \\ 2FB \end{matrix} \right\} \geqq 2BL + AB$

$2FL \geqq AB$

$\left. \begin{matrix} FL \\ FK \end{matrix} \right\} \geqq \frac{1}{2} AB.$

Reipsa sic (ductis $AC$, $CB$ ac perpendiculo $CD$), ob $BE$ bifariam in $A$ sectam (*constr.*), eique in directum adjectam $EL$,

est $EL^q + BL^q = 2AL^q + 2AB^q$ (II, o.)
Sed $BL^q = BG^q = EB \times BF$ (§. 147. n°. 6.) $= 2AB \times BF = 2BA \times AF + 2AB^q$ (II, 3.)
Quare $EL^q + 2BA \times AF + 2AB^q = 2AL^q + 2AB^q$
$EL^q + \left\{ \begin{matrix} 2BA \times AF \\ 2BA \times AL + 2BA \times LF \end{matrix} \right\} = 2AL^q$
Atqui $EL^q + 2BA \times AL = EL^q + EA \times AL = AL^q + AE^q$ (II, 7.) $= AL\ + AB^q$
& $2BA \times LF = 2AB \times EK = 2AB \times CD$ (*constr.* I, 34.)
Igitur $AL^q + AB^q + 2AB \times CD = 2AL^q$
$AB^q + 2AB \times CD = AL^q.$
Denique $AC^q + CB^q = AB^q$ (III, 31. I, 47.)
$2AC \times CB = 2AB \times CD$ (§. 147. n°. 3.)
Proinde (II, 4.) $(AC + CB)^q = AB^q + 2AB \times CD = AL^q$
$AC + CB = AL$
& ob $CD = FK = LF$
$AC + CB + CD = AF = H$ (*constr.*).

## §. 190.

Proportio $EB : BA + AC + CB = BA + AC + CB : \begin{cases} BF \\ BA + AC + CB + CD \end{cases}$
seu $2AB$ : Perimet. triang. = Perimet. : Perimet. + $CD$.
qua solutio præcedens nititur, sic etiam demonstratur.

Triangulo ad $C$ rectangulo $ABC$ (*Fig.* 83.) inscribatur circulus (IV, 4.): cujus centrum sit $S$; & qui trianguli latera $BA$, $AC$, $CB$ contingat in punctis $M$, $N$, $O$. Erunt

Erunt $AM=AN$, $BM=BO$, $CN=CO$, ang. $NCS=SCO$ (IV, 4. *Dem. et Coroll.*): igitur anguli $NCS$, $SCO$ semirecti; pariterque (I, 32.) anguli $CSN$, $CSO$, ob rectos $N$, $O$ (III, 18.); proinde $\left.\begin{matrix}CN\\CO\end{matrix}\right\}=\left\{\begin{matrix}SN\\SO\end{matrix}\right.$ (I, 6.).

Hinc $BA+AC+CB$ seu perimeter trianguli $\Big\}=\Big\{$ $2AB+2SN$ duplo aggregato hypotenusæ et radii circuli trianguli inscripti.

Atqui, ob $SM=SN=SO$, triangulum $ABC=BAS+ACS+CBS$ æquale est triangulo, cujus basis perimetro $BA+AC+CB$, altitudo rectæ $SN$ æquatur (I, 37. *Coroll.*). Aequalia igitur sunt triangula, unum sub basi $AB$ & altitudine $CD$, alterum sub basi $BA+AC+CB$ & altitudine $SN$: ideoque

$$AB : \left\{\begin{matrix}BA+AC+CB\\ \text{Perimet.}\end{matrix}\right\} = SN : CD \quad (\S.\ 145.)$$
$$2AB : \text{Perimet.} = 2SN : CD \quad (V,\ 4.)$$
$$= \left\{\begin{matrix}2AB+2SN\\ \text{Perimet.}\end{matrix}\right\} : \text{Perimet.} + CD \quad (V,\ 12.)$$

§. 191.

Paulo prolixior eâ, quæ §. 189. traditur, est constructio, quam Rob. Simson in *Appendice Operum reliquorum* (Glasguæ 1776. p. 5. sqq.) analysi sequente eruit.

Ob $BA^q = AC^q + CB^q$ (I, 47.)
& $2BA\times CD = 2AC\times CB$ (§. 147. n°. 3.)
est $BA^q + 2BA\times CD = (AC+CB)^q$ (II, 4.)

$$BA^q + 2(BA+AC+CB+CD)CD = (AC+CB)^q + 2(AC+CB+CD)CD$$
$$= (AC+CB+CD)^q + CD^q$$

seu $BA^q + 2BF\times CD = AF^q + CD^q$, facta $AF = AC+CB+CD$

Unde $\left.\begin{matrix}2BF\times CD - CD^q\\ (2BF-CD)\,CD\end{matrix}\right\} = AF^q - BA^q$
$= BF\times FE$ (II, 5. *Coroll.*), facta $AE=AB$.

Quare, cum dentur $BF$, $FE$; $CD$ erit latus minus rectanguli $=BF\times FE=FG^q$ (§. 147. n°. 6), cujus summa laterum $=2BF$; seu erit segmentum minus rectæ $=2BF$ in inæqualia sic divisæ, ut rectangulum sub segmentis æquale sit rectangulo sub datis $BF$, $FE$, seu quadrato datæ $FG$ (*Schol. in Lib.* II. §. 74. sq.).

§. 192.

Problema XII. Construere triangulum rectangulum: cujus perimeter datæ rectæ $P$; & perpendiculum, ab vertice anguli recti in hypotenusam demissum, datæ $I$ æquentur. (*Fig.* 82.)

Ob

Ob $BA \gtreqless 2CD$ in triangulo $ABC$ ad $C$ rectangulo (*Fig.* 76. 77.) & $AC+CB > 2CD$ (I, 19. *Coroll.*)
ideoque $BA+AC+CB > 4CD$: debet esse $P > 4I$.

Præterea, cum in triangulo rectangulo æquicruro $ABC$ (*Fig.* 76.) sit

$$BA+AC+CB-2CD = AC+CB = 2AC, \text{ ob } BA=2CD,$$

ideoque $(BA+AC+CB-2CD)^q = 4AC^q = 8CD^q$;
in non-æquicruro autem $ABC$ (*Fig.* 77.) sit

$$BA+AC+CB-2CD > AC+CB, \text{ ob } BA > 2CD,$$

& hinc $(BA+AC+CB-2CD)^q > (AC+CB)^q$ ac tanto magis (§. 183.) $> 8CD^q$: oportet, non sit $(P-2I)^q < 8I^q$, seu $(P-\frac{1}{2}I)^q < 2I^q$; h. e. $P-\frac{1}{2}I$ minor esse non debet diagonali quadrati, cujus latus $= I$.

Sit (*Fig.* 82.) $ABC$ triangulum desideratum: cujus perimeter $=P$; perpendiculum $CD$ ab vertice anguli recti $C$ in hypotenusam $AB$ demissum $=I$.

Ob Perim. $+ CD$ : Perimet. $=$ Perimet. : $2AB$ (§. 189 sq.), datur (VI, 11.) $2AB$, & (I, 10.) ipsa hypotenusa $AB$: ac Problema reducitur ad §. 176.

Rectæ nimirum $BL=P$ in directum adjicietur $LF=I$; rectis $BF=P+I$, $BL=P$, tertia proportionalis $BE$ invenietur juxta §. 124. n°. 1. (in semicirculo super $BF$ descripto aptando rectam $BG=BL$, & ex $G$ in ipsam $BF$ demittendo normalem $GE$); inventa $BE$ bifariam in $A$ secabitur; ab perpendiculo $EG$ abscindetur $EK=LF=I$; ac (§. 176.) per $K$ agetur rectæ $BF$ parallela, semicirculo super $AB$ descripto in $C$ occurrens: cum, ob

$$8I^q \gtreqless (P-2I)^q \text{ (determ.)}$$

h. e.

$$8LF^q \gtreqless \begin{cases}(BL-2LF)^q \text{ (constr.)} \\ BL^q - 4BL \times LF + 4LF^q \text{ (Schol. in Lib. II. §. 86. 2. 6.)},\end{cases}$$

sit

$$4(BL \times LF + LF^q) \gtreqless BL^q \text{ seu } BG^q$$

$$4BF \times FL \gtreqless FB \times BE \text{ (II, 3. \& §. 147. n°. 6.)}$$

$$4FL \gtreqless BE;\ 2FL \gtreqless AB;\ \left.\begin{matrix}FL \\ EK\end{matrix}\right\} \gtreqless \tfrac{1}{2}AB.$$

Atque ita fieri $AC+CB=AL$, proinde $BA+AC+CB=BL=P$; eodem modo, quo ad calcem §. 189, demonstratur.

## §. 193.

**Problema XIII.** Construere triangulum rectangulum; cujus perimeter sit datæ rectæ $P$; & area quadrato datæ $I$ æqualis. (*Fig.* 84.)

Ob perpendiculum $CD$ ex vertice anguli recti $C$ in hypotenusam

$AB$

$AB$ demissum $\overline{\overline{<}} \frac{1}{2}AB$ (*Fig.* 76. 77.); ideoque rectg. $AB \times CD \overline{\overline{<}} \frac{1}{2}AB^q$, & aream trianguli $ABC \overline{\overline{<}} \frac{1}{4}AB^q$ (I, 41.): debet esse $I^q \overline{\overline{<}} \frac{1}{4}AB^q$, $I < \frac{1}{2}AB$, $4I \overline{\overline{<}} 2AB$.

Sed $2AB < BA + AC + CB$, ob $AB < AC + CB$ (I, 20.). Oportet itaque, sit $4I < P$; $I < \frac{1}{4}P$.

Porro debet esse $4I + 2SN \overline{\overline{<}} 2AB + 2SN$ (*Fig.* 83.) seu $P$ (§. 190.) ideoque $4P \times I + 2P \times SN \overline{\overline{<}} P^q$

Sed $2P \times SN = 4 \times$ area trianguli $ABC$ (§. 190.) $= 4I^q$.

Necesse igitur est, sit $4P \times I + 4I^q \overline{\overline{<}} P^q$

$$\text{seu} \quad \left.\begin{matrix} P^q + 4P \times I + 4I^q \\ (P+2I)^q \end{matrix}\right\} \overline{\overline{<}} 2P^q$$

$$P + 2I \overline{\overline{<}} \text{Diagon. quadrati lateris } P.$$

Sit (*Fig.* 84.) $ABC$ triangulum ad $C$ rectangulum construendum: cujus perimeter $BA + AC + CB = P$; area $\frac{1}{2}AB \times CD = I^q$.

Ob $2AB$ : Perim. = Perim. : Perim. + $CD$ (§. 189. sq.)

ideoque $2AB \times$ Perim. + $\left\{\begin{matrix} 2AB \times CD \\ 4I^q \end{matrix}\right\} = \overline{\text{Perim.}}^q$ (VI, 17.):

erit, facto rectangulo $Q \times$ Perim. $= 4I^q$, h. e. (VI, 17.) Perim. : $2I = 2I : Q$,

$$2AB \times \text{Perim.} + Q \times \text{Perim.} = \overline{\text{Perim.}}^q$$
$$2AB + Q = \text{Perim.}; \; 2AB = \text{Perim.} - Q.$$

Quare, ob datas perimetrum & (VI, 11.) rectam $Q$, datur hypotenusa $AB$.

Qua inventa, ob $\frac{1}{2}AB \times CD = I^q$, ideoque (VI, 17.) $\frac{1}{2}AB : I = I : CD$, datur etiam (VI, 11.) altitudo $CD$: & trianguli constructio reducitur ad §. 176.

Præmissis consequenter ab recta $BL = P$ abscindetur tertia proportionalis $LE$ datis $BL$ ac $2I$, juxta §. 124. n°. 1. in semicirculo super $BL$ descripto rectam $LF = 2I$ (utpote minorem quam $BL$, ob $4I < P$ per *determ.*) aptando, atque ex $F$ ipsi $BL$ demittendo normalem $FE$: tum bifariam in $A$ secta residua $BE$, erit $AB$ hypotenusa trianguli describendi.

Qua rursus in $G$ bisecta, ab perpendiculo ei in $G$ constituto abscindetur tertia proportionalis $GK$ datis $BG$ & $I$, juxta §. 23. 119 sq. ab $GA$ dictoque perpendiculo abscindendo $GM = GN = I$, & per $M$ agendo rectæ $BN$ parallelam $MK$.

Ob $(P+2I)^q = 2P^q$ (*determ.*)

h. e. $(BL+LF)^q$ }

seu (II, 4.) $BL^q+2BL\times LF+LF^q$ } $= 2BL^q$ (*constr.*)

$2BL\times LF+\left\langle \begin{matrix} LF^q \\ BL\times LE \end{matrix} \right\rangle = BL^q$ (§. 147. n°. 6.);

igitur $2LF+LE = BL$

$2LF = BE$

$LF$ seu $2I$ }

ideoque $2GM$ } $= AB$:

erunt $\left. \begin{matrix} GM \\ GN \end{matrix} \right\} = \frac{1}{2}AB$ seu $\left\{ \begin{matrix} BG \\ GA \end{matrix} \right.$

pariterque $GK = \frac{1}{2}AB$ (V, 14.), ob $BG : GM : GN = GK$.

Semicirculo igitur super $AB$ descripto occurrit recta per K ipsi $AB$ parallela (§. 176.): ad quod occursus punctum $C$ ductæ $AC$, $BC$ rectæ finiunt triangulum $ABC$ quæsitum.

Ob $BE$ enim bifariam in $A$ sectam, ipsique in directum adjectam $EL$, est $BL\times LE + AB^q = AL^q$ (II, 6.)

Sed $BL\times LE = LF^q$ (§. 147. n°. 6.) $= 4GM^q$, ob $LF = 2I = 2GM$ (*constr.*)

$= 4BG\times GK$ (VI, 17.), ob $BG : GM = \left\langle \begin{matrix} GN \\ GM \end{matrix} \right\rangle : GK$ (*constr.* & §. 23.)

$= 2AB\times CD$ (*constr.* & I, 34.)

$= 2AC\times CB$ (§ 147. n°. 3.)

& $AB^q = AC^q + CB^q$ (III, 31. I, 47.)

Quare $2AC\times CB+AC^q+CB^q$ }

seu (II, 4.) $(AC+CB)^q$ } $= AL^q$

$AC+CB = AL$

& $BA+AC+CB = BL = P$ (*constr.*)

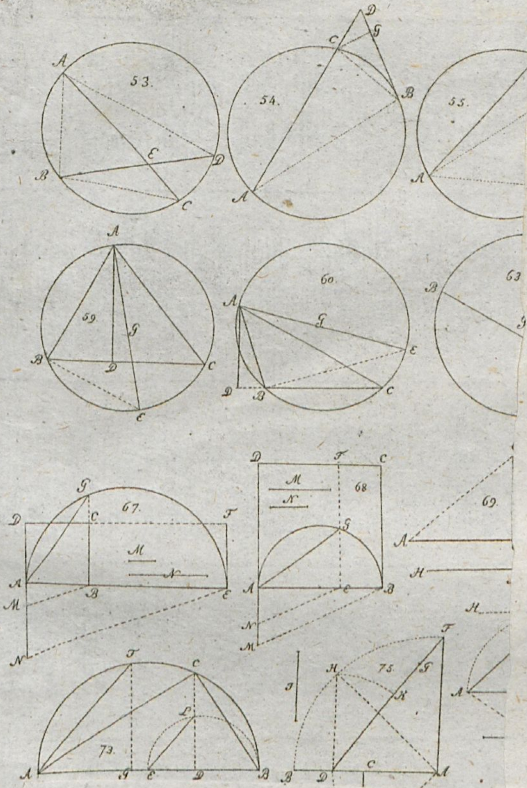
Atque ob $BG\times GK$ }

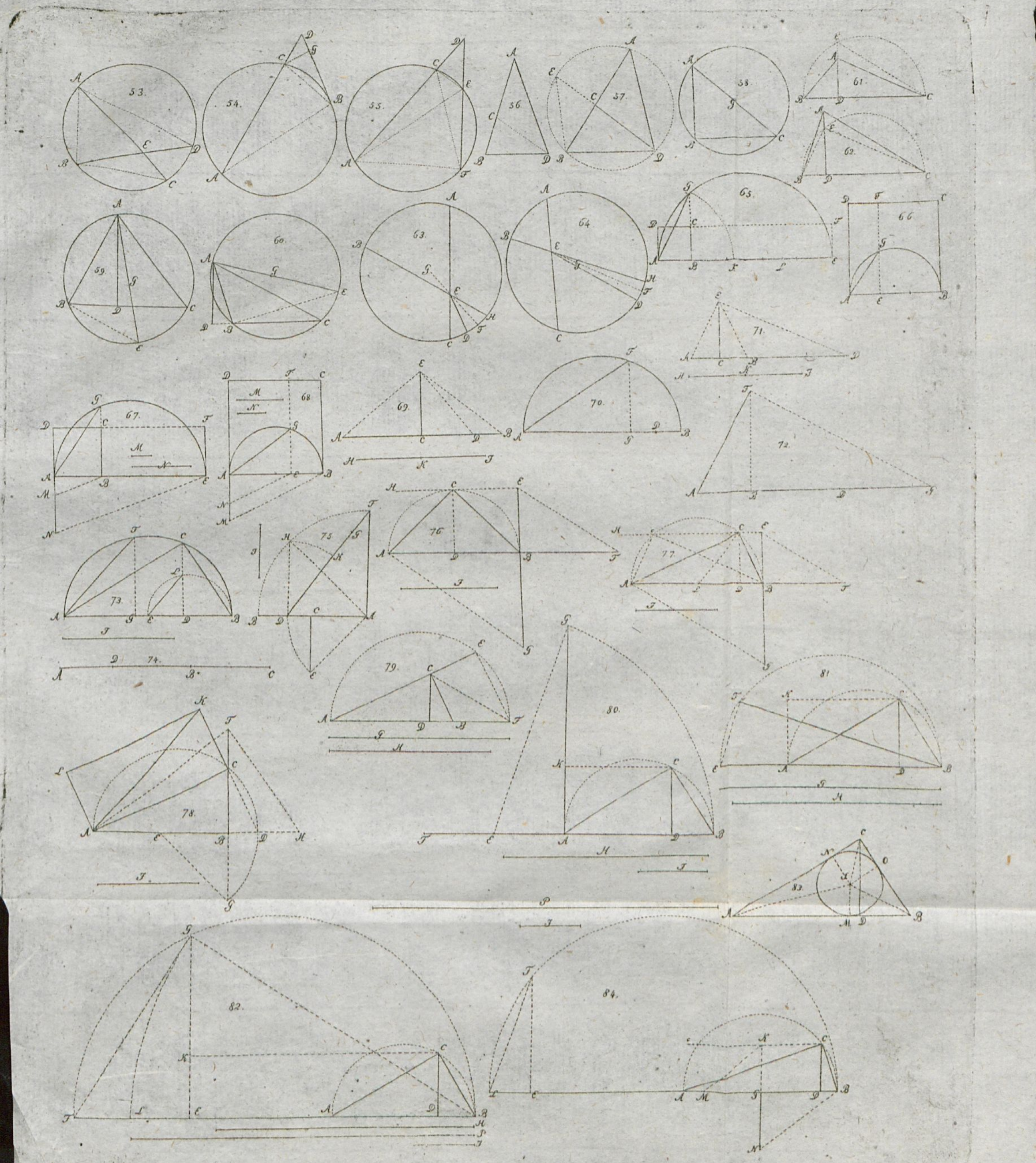
h. e. $\frac{1}{2}AB\times CD$ } $= GM^q$ (*demonstr.*) $= I^q$ (*constr.*);

trianguli $ABC$ area est $= I^q$.

## §. 194.

Ex analysi præcedente etiam consequitur Problematis XIII. compositio, omissis superfluis, satis concinna, quam THOM. SIMPSON (l. c. p. 326. sq.), demonstratione solum munitam, exhibet.


53.
54.
55.
59.
60.
63
67.
68
69.
73.
75.


53. 54. 55. 56. 57. 58. 61. 62.
59. 60. 63. 64. 65. 66.
67. 68. 69. 70. 71. 72.
73. 74. 75. 76. 77.
78. 79. 80. 81. 83.
82. 84.

# SCHOLIA
# IN LIBRUM SEXTUM ELEMENTORUM EUCLIDIS

QUORUM

*PARTEM TERTIAM*

PRÆSIDE


## CHRISTOPHORO FRIDERICO PFLEIDERER

UNIVERSITATIS ET COLLEGII ILLUSTRIS PROFESSORE PHYSICES ET MATHESEOS PUBL. ORD.


PRO CONSEQUENDO GRADU MAGISTERII

*D. SEPT. MDCCCII.*

PUBLICE DEFENDENT


CHRISTIANUS FRIDERICUS KLAIBER, *Wankheimensis*,
JOSEPHUS FRIDERICUS SEYBOLD, *Brackenheimensis*,
THEOPHILUS FRIDERICUS JAEGER, *Stuttgardianus*,
AMANDUS FRIDERICUS GÜNZLER, *Degerschlachtensis*,
CHRISTIANUS FRIDERICUS FRANCK, *Stuttgardianus*,
CANDIDATI MAGISTERII PHILOSOPHICI IN ILLUSTRI STIPENDIO THEOLOGICO.


---


*TUBINGÆ*

LITERIS SCHRAMMIANIS.