SCHOLIA
IN LIBRUM SECUNDUM ELEMENTORUM EUCLIDIS

QUORUM

PARTEM TERTIAM

PRÆSIDE

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UNIVERSITATIS ET COLLEGII ILLUSTRIS PROFESSORE PHYSICES ET MATHESEOS PUBL. ORD.

PRO CONSEQUENDO GRADU MAGISTERII

D. SEPT. MDCCIC.

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PROPOSITIONES XII. XIII.

§. 193.

Austin (p. 39. sq.) tres ultimas Libri ſecundi Propoſitiones partem conſtituere monet ab ceteris ſejunctam, pariter ac tres ultimas primi Libri, ſed non æque ac has in Elementis neceſſariam; nec omnino, præſertim quod ad priores duas attineat, valde utilem: ideoque eas ſucceſſorum Euclidis imprudenti glorioſove Elementa ſuis theoriis augendi ſtudio tribuendas eſſe cenſet. Sed, præter nexum argumenti Propoſitionum XII. XIII. cum I, 47. jam §. 79. indicatum, pronam ipſarum ex hac & II, 4. 7. conſequentiam, & ad plures applicationes inſignes uſum, quarum exempla quædam infra ſubjungemus: re ipſa etiam Propoſitionis XII, 17. demonſtratio priorem ſupponit; & Datorum Propoſitiones 64. 65. in textu Græco Claudii Hardy (Ευκλειδȣ Δεδομενα. Paris. 1625.) & Dav. Gregorii, 74. 75. in verſionibus Rob. Simson & Schwab, illis nituntur.

§. 194.

Perpendiculum AD, quod (*Fig.* 46.) in trianguli obtuſanguli ABC latus alterum BC circa angulum obtuſum B ex vertice trianguli oppoſito A demittitur, extra triangulum cadere ad partes anguli ABH deinceps poſiti obtuſo ABC; id autem, quod (*Fig.* 47.) in latus quodcunque BC trianguli acutanguli ABC ex vertice oppoſito A demittitur, cadere intra triangulum; ultro (idem in demonſtrationibus Propoſitionum XII. XIII. p. 62. ſq. monente Claud. Richardo) conſequitur ex Propoſitionis I, 17. Corollario, quod nominatim illi adjungunt Clavius (p. 66.), Richardus (p. 30.), Barrow (p. 18.), Bærmannus (p. 18.): Ab eodem extra rectam aliquam puncto ductis ad eam duabus rectis, una perpendiculari, altera obliqua; perpendiculum ad eas obliquæ partes cadit, ad quas hæc cum recta propoſita efficit angulum acutum.

 Quare

Quare mireris, Clavium in Scholiis ad II, 12. 13. (p. 192. ſq. 199.) eadem, confutatione caſuum reliquorum ad I, 17. 16. reducta, prolixe adſtruere.

§. 195.

Coetsius (p. 222. ſq. 229. ſq.), tertiis, quas tradit, Propoſitionum XII. XIII. demonſtrationibus, primum in triangulo obtuſangulo quadratum lateris, quod angulo obtuſo opponitur, ſuperare ſummam quadratorum reliquorum laterum; in triangulo autem acutangulo quadratum cujuslibet lateris minus eſſe quadratis reliquorum laterum ſimul, ſic oſtendit.

Sit (*Fig.* 48.) $A'BC$ triangulum ad B obtuſangulum, ABC acutangulum; ſintque, ut eadem figura dicendis adhiberi poſſit, eorum latera $A'B$, AB æqualia. Lateri ipſorum communi BC ad extremum ejus B conſtituto perpendiculo $BE = BA' = BA$; & juncta CE recta: eſt $EC^q = EB^q + BC^q$ (I, 47.) $= \left\{\begin{matrix} A'B^q \\ AB^q \end{matrix}\right\} + BC^q$. Atqui ob $EB = A'B = AB$, & BC latus commune; ſed angulum $A'BC > EBC$, contra $ABC < EBC$ (ſupp. & conſtr.); eſt (I, 24.) $A'C > EC$, ſed $AC < EC$. Quare $A'C^q > A'B^q + BC^q$, $AC^q < AB^q + BC^q$.

§. 196.

Tum demonſtrationes Euclideas Propoſitionum XII. XIII. ad aſſerendos, quos eæ aſſignant, exceſſus hinc ipſius $A'C^q$ ſuper EC^q ſeu $A'B^q + BC^q$, inde ipſius EC^q ſeu $AB^q + BC^q$ ſuper AC^q, perpendiculis $A'D'$, AD ex A', A in BC demiſſis, Coetsius ita applicat:

1°. $A'C^q = A'D'^q + D'C^q$ (I, 47.) $= A'D'^q + D'B^q + BC^q + 2\,\text{rectg.}\,CBD'$ (II, 4.)

$$\left.\begin{matrix} = \left\{\begin{matrix} A'B^q \\ EB^q \end{matrix}\right\} + BC^q + 2\,\text{rectg.}\,CBD' \\ = EC^q + 2\,\text{rectg.}\,CBD' \end{matrix}\right\} \text{(I, 47.)}$$

2°. $EC^q = \left\{\begin{matrix} EB^q \\ AB^q \end{matrix}\right\} + BC^q$ (I, 47.) $= AD^q + BD^q + BC^q$ (I, 47.)

$= AD^q + CD^q + 2\,\text{rectg.}\,CBD$ (II, 7.)

$= AC^q + 2\,\text{rectg.}\,CBD$ (I, 47.)

§. 197.

AMBROS. RHODIUS (*Euclidis Elementorum Libri* XIII. Edit. 2da. Witebergæ 1634. p. 56. ſq.) in figura, quam ad declarandam & demonſtrandam Propoſitionem XII. adhibet, deſcripta ſiſtit quadrata rectarum AD, CD (*Fig.* 46.), & poſterius eodem modo, quo in demonſtratione Propoſitionis IV, diviſum; pariterque in figura Propoſitioni XIII. adſcripta quadratum lateris BC conſtructum ac diviſum exhibet (*Fig.* 47.): ſed poſterioris tantum Propoſitionis demonſtrationem, nec prorſus diſtincte, ad figuras ita deſcriptas applicat.

CLAUD. RICHARDUS (p. 62. ſq.), GUARINUS (*Euclides adauctus & methodicus.* Aug. Taurin. 1671. p. 60. ſq.), *Euclidis Elementa* Lond 1678. (p. 95. ſq.), COETSIUS (demonſtrationibus ſecundis p. 220. ſq. 227. ſq.), iisdem conſtructionibus adhibitis, Propoſitiones XII. XIII. "clarius & quaſi ad oculum," ut COETSIUS ait, hoc fere modo demonſtratas ſiſtunt:

1°. In trianguli ad B obtuſanguli ABC (*Fig.* 46.) latus alterutrum BC circa angulum obtuſum ex vertice oppoſito A demiſſo perpendiculo AD (quod per §. 194. in latus CB productum ad partes anguli ABH deinceps poſiti obtuſo ABC cadit); tum rectarum AD, CD deſcriptis quadratis AH, CG; atque hoc eodem modo, quo in demonſtr. Prop. IV, diviſo:

eſt $\underline{AC^q = AH + CG\,(\text{I}, 47.) = AH + BM + NF + CL + LG}$

Atqui ob $BM = BD^q$ (Dem. II, 4.) eſt $AH + BM = AB^q$ (I, 47.)

Porro $NF = BC^q$ (Dem. II, 4.)

& $\underline{\text{rectg. } CL = LG\,(\text{I}, 43.)\text{; itaque } CL + LG = 2CL = 2\text{rectg. } CBD\text{, ob } BL = BD\,(\text{Dem. II}, 4.)}$

Igitur $AC^q = AB^q + BC^q + 2\text{rectg. } CBD.$

2°. In trianguli acutanguli ABC (*Fig.* 47.) latus quodcunque BC ex vertice oppoſito A demiſſo perpendiculo AD (quod intra triangulum cadit, cum per §. 194. ad partes utriusque anguli acuti ABC, ACB jacere debeat); tum rectarum AD, BC deſcriptis quadratis DI, CF; atque hoc rurſus uti in demonſtr. Prop. IV. diviſo; denique ſuper recta $FG = BD$ (I, 34.) conſtructo (ſimiliter ac in Propoſitionis VII. demonſtratione §. 81.) quadrato FO, quod igitur $= BD^q = DL$ (Dem. II, 4.):

$$\begin{array}{lll}
\text{sunt} & AB^q & = DI + FO \text{ (I, 47. \& dem.)} \\
& BC^q & = NG + CL + LG \\
\text{itaque} & AB^q + BC^q & = NG + DI + CL + LG + FO \\
& & = NG + DI + 2CL;\ \text{quia } LG = CM \text{ (I, 43.), \& } FO = DL \text{ (dem.)} \\
\text{Sed} & AC^q & = NG + DI \text{ (I, 47.), ob } NG = CD^q \text{ (Dem. II, 4.)} \\
\text{Ergo} & AB^q + BC^q & = AC^q + 2CL \\
& & = AC^q + 2\,\text{rectg.}CBD,\ \text{ob } BE = BD \text{ (Dem. II, 4.)}
\end{array}$$

§. 198.

Cum, demisso in trianguli ABC acutanguli (*Fig.* 50.), vel ad B obtusanguli (*Fig.* 49.) latus alterum AB circa angulum B perpendiculo CE ex vertice opposito C, iisdem modis, quibus in Propositionum XII. XIII. demonstrationibus Euclideis, vel §. 195. sq. 197, evincatur esse

$$\begin{array}{lll}
(\textit{Fig. } 49.) & AC^q & = AB^q + BC^q + 2\,\text{rectg.}ABE; \\
(\textit{Fig. } 50.) & AB^q + BC^q & = AC^q + 2\,\text{rectg.}ABE;
\end{array}$$

sequitur, utrobique esse rectg.CBD = rectg.ABE; quod eodem modo infert Cel. GILBERT (p. 292. sq.).

§. 199.

Idem immediate sic adstruitur:

1°. (*Fig.* 49.) $AD^q + CD^q = CE^q + AE^q$; quia utraque summa $= AC$ (I, 47.)

$$\begin{array}{lll}
\text{Proinde ob} & \left.\begin{array}{l} CD^q = BD^q + BC^q + 2\,\text{rectg.}CBD \\ AE^q = BE^q + AB^q + 2\,\text{rectg.}ABE \end{array}\right\} & \text{(II, 4.)} \\
\text{est} & AD^q + BD^q + BC^q + 2CBD = CE^q + BE^q + AB^q + 2ABE & \\
\text{Atqui} & \left.\begin{array}{l} AD^q + BD^q = AB^q \\ BC^q = CE^q + BE^q \end{array}\right\} & \text{(I, 47.)} \\
\text{Demtis igitur utrinque æqualibus, manet} & 2CBD = 2ABE & \\
\text{\& hinc} & \text{rectg.}CBD = \text{rectg.}ABE. &
\end{array}$$

2°. (*Fig.* 50.) $CE^q + AE^q = AD^q + CD^q$; quia utraque summa $= AC^q$ (I, 47.)

$$\begin{array}{lll}
\text{Quare etiam} & CE^q + AE^q + 2ABE + 2CBD = AD^q + CD^q + 2CBD + 2ABE & \\
\text{Sed} & \left.\begin{array}{l} AE^q + 2ABE = BE^q + AB^q \\ CD^q + 2CBD = BD^q + CB^q \end{array}\right\} & \text{(II, 7.)} \\
\text{Igitur} & CE^q + BE^q + AB^q + 2CBD = AD^q + BD^q + CB^q + 2ABE & \\
\text{Atqui} & \left.\begin{array}{l} CE^q + BE^q = CB^q \\ AB^q = AD^q + BD^q \end{array}\right\} & \text{(I, 47.)}
\end{array}$$

Utrinque ergo demtis æqualibus, rurſus manet

$$2CBD = 2ABE$$

Unde $$\text{rectg.}\,CBD = \text{rectg.}\,ABE.$$

§. 200.

Hisce ſuppoſitis, quæ vero ab Propoſitionibus III, 31. 35. 36. repetit, Gregorius a S. Vincentio (p. 31. ſq.) Propoſitiones XII. XIII. "eo", ut ait, "diſcurſu, quo Pythagoras 47mam primi Elementorum," demonſtrat; quem etiam Gilbert (p. 296. ſq.) exponit. Nempe tam ſuper trianguli ad B obtuſanguli (*Fig.* 51.), quam ſuper trianguli acutanguli (*Fig.* 52.) ABC ſingulis lateribus conſtructis quadratis $ACFG$, $ABKL$, $BCHI$; tum in latera BC, AB circa angulum B ab extremis A, C lateris oppoſiti AC demiſſis perpendiculis AD, CE; iisdemque productis, donec quadratorum ſuper BC, AB factorum lateribus HI, LK ipſis, vel productis, occurrant in punctis N, Q; porro per verticem anguli B acta lateribus AG, CF quadrati ab AC parallela BOM; denique junctis BF, BG, AH, CL rectis: ſunt, æque ac in demonſtratione I, 47.

$$\begin{array}{l} \text{rectg.}\,AGMO = 2\Delta AGB \\ \text{rectg.}\,CFMO = 2\Delta CFB \end{array}{\scriptstyle (I,41.)} \begin{array}{l} = 2\Delta ALC \\ = 2\Delta CHA \end{array}{\scriptstyle (I,4.)} \begin{array}{l} = \text{rectg.}\,ALQE \\ = \text{rectg.}\,CHND \end{array}{\scriptstyle (I,41.)}$$

itaque

$$\begin{aligned} \text{quadrat.}\,ACFG &= \text{rectg.}\,ALQE + \text{rectg.}\,CHND \\ &= \text{quadr.}\,AK + \text{quadr.}\,CI + \text{rectg.}\,EK + \text{rectg.}\,DI\ (\textit{Fig.}\ 51.) \\ &= \text{quadr.}\,AK + \text{quadr.}\,CI - \text{rectg.}\,EK - \text{rectg.}\,DI\ (\textit{Fig.}\ 52.) \end{aligned}$$

$$\text{h. e.}\quad \left.\begin{aligned} AC^q &= AB^q + BC^q + \text{rectg.}\,ABE + \text{rectg.}\,CBD\ (\textit{Fig.}\ 51.) \\ &= AB^q + BC^q - \text{rectg.}\,ABE - \text{rectg.}\,CBD\ (\textit{Fig.}\ 52.) \end{aligned}\right\}\text{ob}\ \begin{array}{l} BK = AB \\ BI = CB \end{array}(\text{conſtr.})$$

Quare, cum utrobique ſit rectg. CBD = rectg. ABE (§. 199.),

$$\begin{aligned} AC^q &= AB^q + BC^q + \begin{cases} 2\text{rectg.}\,CBD \\ 2\text{rectg.}\,ABE \end{cases} (\textit{Fig.}\ 51.) \\ &= AB^q + BC^q - \begin{cases} 2\text{rectg.}\,CBD \\ 2\text{rectg.}\,ABE \end{cases} (\textit{Fig.}\ 52.) \end{aligned}$$

§. 201.

Propoſitionis XIII. demonſtratio Euclidea, æque ac eæ, quæ exhibentur §. 195. ſq. 197, pariter valent, etiamſi angulus BAC (*Fig* 47. 48.), ex cujus vertice perpendiculum demittitur in latus oppoſitum BC,

quod

quod angulo acuto ABC adjacet, rectus ſit, vel obtuſus. Cum enim uterque angulus lateri BC adjacens tum ſit acutus (I, 17.): perpendiculum AD intra triangulum in latus BC incidit (§. 194. 197. nº. 2.); & reliqua perſtant, quibus evincitur eſſe

$$AB^q + BC^q = AC^q + 2\text{rectg. ſub } CB \ \& \ BD.$$

Eædemque demonſtrationes, ob paritatem rationis, efficiunt: tam in triangulo acutangulo, quam in triangulo ad A rectangulo vel obtuſangulo, ABC eſſe

$$AC^q + BC^q = AB^q + 2\text{rectg. ſub } BC \ \& \ CD.$$

Quare in triangulis etiam rectangulis & obtuſangulis quadratum utriusque lateris ſubtendentis angulum acutum minus eſt quam quadrata ſimul laterum comprehendentium hunc angulum acutum, rectangulo bis contento ſub latere circa eundem angulum acutum, quod angulo recto vel obtuſo opponitur, & ſub ſegmento ejus, quod perpendiculo, in illud ex vertice anguli recti vel obtuſi demiſſo, ac vertici anguli acuti, de quo agitur, interjacet.

§. 202.

Quod §. 201. de ambitu demonſtrationis Euclideæ Propoſitionis XIII. obſervatur, monent Isaacus Monachus in Scholio ad hanc Propoſitionem relato, quod & apud *Commandinum* (Fol. 35.) exſtat, Campanus (p. 50. ſq.), Peletarius (p. 105. ſq.), Clavius (p. 199.), alii.

§. 203.

Scheubelius in Admonitione Propoſitioni XIII. ſubjuncta (p. 161.), ubi eam de omnibus, cujuscunque generis fuerint, triangulis intelligi poſſe dicit, & exemplis numericis declarat, in exemplo pro triangulo rectangulo eum quoque complectitur caſum, quo perpendiculum in latus angulo acuto, de quo agitur, & recto interjacens ex vertice trianguli oppoſito demitti, ſeu cum altero trianguli circa angulum rectum latere coincidere ſumitur.

Commandinus deinde in Commentario ad Propoſitionem XIII. (Fol. 35.) reſtrictionem propoſiti §. 201. de triangulis rectangulis & obtuſangulis ad latus angulo recto vel obtuſo oppoſitum, perpendiculumque in hoc demiſſum, haud neceſſariam eſſe demonſtravit: quod alii post ipsum vel iisdem modis, vel & aliis, in triangulis nempe obtusangulis, ostenderunt.

§. 204.

Nempe sit primum (*Fig.* 53.) triangulum ABC ad A rectangulum. Tunc cum latere ipsius CA coincidit perpendiculum, quod in latus BA circa angulum acutum B, adjacens recto A, ex trianguli vertice opposito C demittitur: & recta vertice anguli B atque perpendiculo hoc intercepta est ipsa BA; atque rectangulum sub hac recta & sub latere BA fit ipsius BA quadratum.

Atqui ob $BC^q = AC^q + AB^q$ (I, 47.)
est $BC^q + AB^q = AC^q + 2AB^q$; seu $AC^q = BC^q + AB^q - 2AB^q$.
(COMMANDINUS fol. 35.b; CLAVIUS p. 199; ROB. SIMSON p. 68; AUSTIN p. 39.)

§. 205.

Sit dein (*Fig.* 54.) triangulum ABC ad A obtusangulum: proinde, quod in latus BA circa angulum acutum B, sed adjacens obtuso BAC, ex trianguli vertice opposito C demittitur perpendiculum CE, extra triangulum cadat ad partes anguli CAH deinceps positi obtuso BAC (§. 194.); & ab latere producto BA verticem inter anguli B, cui latus AC opponitur, abscindat rectam BE. Tum

1°. $BC^q = CE^q + BE^q$ (I, 47.)
Igitur $BC^q + AB^q = CE^q + BE^q + AB^q$
Atqui $BE^q = AE^q + 2\text{rectg.}\,BAE + AB^q$ (II, 4.)
Ergo $BC^q + AB^q = CE^q + AE^q + 2\text{rectg.}\,BAE + 2AB^q$
Sed $CE^q + AE^q = AC^q$ (I, 47.)
& $\text{rectg.}\,BAE + AB^q = \text{rectg.}\,ABE$ (II, 3.)
Proinde $BC^q + AB^q = AC^q + 2\text{rectg.}\,ABE$;
seu $AC^q = BC^q + AB^q - 2\text{rectg.}\,ABE$.
(COMMANDINUS fol. 35; CLAVIUS p. 199. sq.)

2°. Vel $BC^q = AC^q + AB^q + 2\text{rectg.}\,BAE$ (II, 12.)
Quare $BC^q + AB^q = AC^q + 2AB^q + 2\text{rectg.}\,BAE$
$= AC^q + 2\text{rectg.}\,ABE$ (II, 3.)
(CLAVIUS p. 200; WHISTON Demonstr. 1. p. 60; ROB. SIMSON p. 68.)

3°. Vel

3°. Vel $BC^q + AB^q = CE^q + BE^q + AB^q$ uti n° 1.

$$\begin{array}{ll} \text{sed} & BE^q + AB^q = AE^q + 2\,\text{rectg.}\ ABE\ (\text{II}, 7.) \\ \text{Hinc} & BC^q + AB^q = \left\{\begin{matrix} CE^q + AE^q \\ AC^q \end{matrix}\right\} + 2\,\text{rectg.}\ ABE \end{array}$$

(WHISTON Demonſtr. 2. p. 60; AUSTIN p. 38; GILBERT p. 288. ſq.)

Atque ita hic caſus eodem prorſus modo, quo alter in textu Elementorum demonſtratur.

4°. Idem, deſcriptis rectarum CE, BE quadratis CH, EG; atque hoc uti in demonſtratione Propoſitionis IV. diviſo; & ſuper recta $GN = BA$ (I, 34.) conſtructo quadrato GO, quod igitur $= BA^q = AK$ (Demonſtr. II, 4.); æque ac §. 197. n° 2. delineatum ſiſtitur.

$$\begin{array}{lll} \text{Scilicet} & BC^q & = CH + EG\ (\text{I}, 47.) \\ & & = CH + NM + EK + KN \\ & & = AC^q + EK + KN\ (\text{I},47.)\ \text{ob}\ NM = AE^q\ (\text{Dem.II},4.) \\ & AB^q & = GO \\ \text{Igitur} & BC^q + AB^q & = AC^q + EK + KN + GO \\ & & = AC^q + 2EK\ \text{ob}\ KN = EL\ (\text{I},43.)\ \&\ GO = AK\ (\text{dem.}) \\ & & = AC^q + 2\,\text{rectg.}\ \text{ſub}\ AB\ \&\ BE,\ \text{ob}\ BK = AB\ (\text{Dem.II},4.) \end{array}$$

§. 206.

In triangulis rectangulis enunciatum Propoſitionis XIII. (omiſſa determinatione ſitus perpendiculi intra triangulum) valere de lateribus angulos acutos ſubtendentibus, utrumvis laterum angulos hos comprehendentium ſumatur pro baſi dupli rectanguli, quo ſumma quadratorum laterum circa angulum acutum excedat quadratum lateris eidem angulo oppoſiti; ex conſtructione ipſa etiam conſequitur, quam EUCLIDES demonſtrandæ Propoſitioni I, 47. adhibet. Per eam quippe, exiſtente ABC (*Fig.* 53.) triangulo ad A rectangulo,

$$\begin{array}{llllll} \text{cum ſint} & \text{rectg.}\ BM = 2\Delta BFA\ (\text{I},41.) & = 2\Delta BCK\ (\text{I},4.) & = \text{quadr.}\ BL\ (\text{I},41.) & = AB^q\ (\text{conſtr.}) \\ & \text{rectg.}\ CM = 2\Delta CEA & = 2\Delta CBI & = \text{quadr.}\ CH & = AC^q \\ \& & BC^q = \text{quadr.}\ BE & = \text{rectg.}\ BM + CM\ (\text{conſtr.}) & & \end{array}$$

$$\begin{array}{lll} \text{eſt } 1°. & BC^q & = \left\{\begin{matrix} CM \\ CH \end{matrix}\right\} + BM = CH + BL \\ & AB^q & = \qquad BM = \qquad BL \\ \text{Itaque} & BC^q + AB^q & = CH + 2BM = CH + 2BL \\ & & = AC^q + 2CBD = AC^q + 2AB^q. \end{array}$$

2°. BC^q

$$2^{\circ}.\quad BC^q = \left\{\begin{matrix} BM \\ BL \end{matrix}\right\} + CM \quad = BL + CH$$

$$AC^q = \quad CM \quad = \quad CH$$

$$\text{Igitur } BC^q + AC^q = BL + 2CM \quad = BL + 2CH$$

$$= AB^q + 2BCD \quad = AB^q + 2AC^q.$$

§. 207.

Pariter constructio GREGORII A S. VINCENTIO exposita §. 200. utrumque efficit in triangulis obtusangulis. Nempe (*Fig.* 51.)

$$1^{\circ}.\quad AC^q = CM + AM$$

$$= CN + AM \text{ (§. 200.)}$$

$$= CI + BN + AM$$

$$= CI + BQ + AM \text{ (§. 199. n}^{\circ}\text{. 1.)}$$

$$\& \quad AB^q = \quad AK$$

$$\text{Quare } AC^q + AB^q = CI + AQ + AM$$

$$= \text{quadr.} CI + 2\text{rect.} \left\{\begin{matrix} AM \\ AQ \end{matrix}\right. \text{ (§. 200.)}$$

$$= BC^q + 2\text{rectg. sub} \left\{\begin{matrix} CA \;\&\; AO \\ BA \;\&\; AE. \end{matrix}\right.$$

$$2^{\circ}.\quad AC^q = AM + CM$$

$$= AQ + CM \text{ (§. 200.)}$$

$$= AK + BQ + CM$$

$$= AK + BN + CM \text{ (§. 199. n}^{\circ}\text{. 1.)}$$

$$\& \quad BC^q = \quad CI$$

$$\text{Proinde } AC^q + BC^q = AK + CN + CM$$

$$= \text{quadr.} AK + 2\text{rectg.} \left\{\begin{matrix} CM \\ CN \end{matrix}\right. \text{ (§. 200.)}$$

$$= AB^q + 2\text{rectg. sub} \left\{\begin{matrix} AC \;\&\; CO \\ BC \;\&\; CD. \end{matrix}\right.$$

§. 208.

Universim itaque in omni triangulo quadratum ex quolibet latere angulum acutum subtendente minus est quam quadrata simul ex lateribus hunc angulum comprehendentibus, rectangulo contento bis ab utrolibet laterum, quæ sunt circa angulum illum acutum, & ab recta linea ad eundem

eundem angulum acutum in latere hoc intercepta ab perpendiculo, quod ex trianguli vertice oppoſito in iſtud demittitur.

§. 209.

Propoſitiones XII. XIII. earumque expoſitiones in textu Græco, quem habemus, ſic enunciantur:

XII. Εν τοις αμβλυγωνιοις τριγωνοις το απο της την αμβλειαν γωνιαν υποτεινουσης πλευρας τετραγωνον μειζον εςι των απο των την αμβλειαν γωνιαν περιεχουσων πλευρων τετραγωνων, τω περιεχομενω δις υποτε μιας των περι την αμβλειαν γωνιαν, εφ ην εκβληθεισαν η καθετος πιπτει, και της απολαμβανομενης εκτος υπο της καθετου προς τη αμβλεια γωνια.

Εςω (*Fig.* 55.) αμβλυγωνιον τριγωνον το ΑΒΓ, αμβλειαν εχον την υπο ΒΑΓ γωνιαν, και ηχθω απο του Β σημειου επι την ΓΑ εκβληθεισαν καθετος η ΒΔ· λεγω, οτι το απο της ΒΓ τετραγωνον μειζον εςι των απο των ΒΑ, ΑΓ τετραγωνων, τω δις υπο των ΓΑ, ΑΔ περιεχομενω ορθογωνιω.

XIII. Εν τοις οξυγωνιοις τριγωνοις το απο της την οξειαν γωνιαν υποτεινουσης πλευρας τετραγωνον ελαττον εςι των απο των την οξειαν γωνιαν περιεχουσων πλευρων, τω περιεχομενω δις υποτε μιας των περι την οξειαν γωνιαν, εφ ην η καθετος πιπτει, και της απολαμβανομενης εντος υπο της καθετου προς τη οξεια γωνια.

Εςω (*Fig.* 56.) οξυγωνιον τριγωνον το ΑΒΓ, οξειαν εχον την προς τω Β γωνιαν, και ηχθω απο του Α σημειου επι την ΒΓ καθετος η ΑΔ· λεγω, οτι το απο της ΑΓ τετραγωνον ελαττον εςι των απο των ΓΒ, ΒΑ τετραγωνων, τω δις υπο των ΓΒ, ΒΔ περιεχομενω ορθογωνιω.

Et, quæ ipſis reſpondent, *Datorum* Propoſitiones 64. 65. (74. 75.) ita habent:

LXIV. Εαν τριγωνον αμβλειαν εχη γωνιαν δεδομενην· ω μειζον δυναται η την αμβλειαν γωνιαν υποτεινουσα πλευρα των την αμβλειαν γωνιαν περιεχουσων πλευρων, εκεινο το χωριον προς το τριγωνον λογον εχει δεδομενον.

Εςω (*Fig.* 55.) τριγωνον αμβλυγωνιον το ΑΒΓ, αμβλειαν εχον γωνιαν την υπο ΒΑΓ δεδομενην, και διηχθω επ ευθειας της ΑΓ ευθεια η ΑΔ, και ηχθω απο του Β επι την ΔΓ καθετος η ΒΔ· λεγω, οτι ο μειζον εςι το απο της ΒΓ τȣ απο των ΑΒ, ΑΓ, τȣτ εϛι το δις υπο των ΔΑ, ΑΓ, εκεινο το χωριον προς το ΑΒΓ τριγωνόν λογον εχει δεδομενον.

LXV. Εαν τριγωνον οξειαν εχη γωνιαν δεδομενην· ω ελασσον δυναται η την οξειαν γωνιαν υποτεινȣσα πλευρα των την οξειαν γωνιαν περιεχȣσων πλευρων, εκεινο το χωριον προς το τριγωνον λογον εξει δεδομενον.

Εϛω (*Fig.* 56.) τριγωνον οξυγωνιον το ΑΒΓ, οξειαν εχον γωνιαν δεδομενην την υπο ΑΒΓ, και ηχθω απο τȣ Α επι την ΒΓ καθετος η ΑΔ: λεγω, ωτι ω ελασσον εϛι το απο της ΑΓ των απο των ΑΒ, ΒΓ, τȣτ εϛι το δις υπο των ΓΒ, ΒΔ, προς το ΑΒΓ τριγωνον λογον εχει δεδομενον.

§. 210.

Auctorem Elementorum arctioribus, quam demonſtratio jubet, limitibus enunciatum Propoſitionis XIII. conſtrinxiſſe, atque ita mancam reliquiſſe doctrinam Propoſitionibus I, 47. II, 12. 13. traditam, haud veroſimile eſt.

Neque vero probabilis eſt Isaaci Monachi interpretatio, ſtatuentis l. c: "Quoniam in definitionibus Libri primi (Euclides) docet, tri-„ gonum oxygonium eſſe, quod tres habeat acutos angulos; ſciendum„ eſt, quod hoc in loco illud non ſic intelligat; ſed omnia triangula ap-„ pellat oxygonia, quia omnia habent acutos angulos, etſi non omnes,„ tamen ad minimum duos."

§. 211.

Quidam interpretes, omiſſa trianguli *acutanguli* mentione, reliqui Propoſitionis XIII. enunciati atque expoſitionis verſionem ita concinnarunt, ut eo, qui §. 201. indicatur, reſpectu in triangula rectangula & obtuſangula quadrarent.

Sic Peletarius (p. 105. ſq.) Propoſitionem XIII. vertit: "In triangulis, quod ex latere alterum acutorum angulorum ſubtendente„ fit quadratum, tanto minus eſt duorum reliquorum laterum quadratis,„ quantum eſt id, quod bis continetur ſub illo, in quod perpendicularis„ introrſum cadit, & ea ipſius parte, quæ perpendiculari anguloque acu-„ to interjacet." Ac, ſubjunctis quibusdam, ad ea, quæ §. 201. ſq. commemorantur, pertinentibus, pergit: "Sit igitur triangulum *ABC* (*Fig.* 47.), cujus duo anguli *B* & *C* acuti, quantuscunque ſit angulus *A*.„ Ab angulo *A* demitto perpendiculum in latus *BC*:" &c.

Eundem sensum versionibus suis exprimunt De Chales (*Cursus mathemat. Tom. I.* Lugd. 1674. p. 23.), & Ximenes (*I sei primi Elementi della geometria piana.* In Venezia 1752. p. 126.)

Et cel. Lorenz (*Euklids Elemente, fünfzehn Bücher. Zweyte Ausgabe Halle 1798.*), præmissa (p. XXXI.) observatione: *Des 2. Buchs 13. S. ist im Texte unrichtig bloss auf spitzwinklige Triangel eingeschränkt, und daher in der Uebersetzung allgemein ausgedruckt worden; weshalb die kleine Erinnerung: Man nehme an u. s. w. dem Beweise voran zu schicken war;* Propositionem XIII. ita reddit: (*Fig* 47) *In jedem Triangel* ABC *ist das Quadrat der einem von den spitzen Winkeln*, B, *gegenüberliegenden Seite*, AC, *kleiner als die Quadrate der diesen Winkel einschliessenden Seiten*, AB, BC, *und zwar um das zwiefache Rectangel, welches unter einer der einschliessenden Seiten*, BC, *auf welche der Perpendikel*, AD, *fällt, und dem innern, zwischen dem Perpendikel und dem spitzen Winkel*, B, *liegenden Abschnitte*, BD, *enthalten ist.*

Man nehme an, dass B *und* C *spitze Winkel sind;* A *mag ein spitzer, oder rechter, oder stumpfer Winkel seyn: dass folglich der Perpendikel*, AD, *von* A *auf* BC, *innerhalb des Triangels*, ABC, *falle.*

§. 212.

Genuinam non esse textus Græci Propositionis XIII. ad triangula acutangula restrictionem, inde etiam suspiceris: quod in expositione ejus verba: Εςω οξυγωνιον τριγωνον το ΑΒΓ οξειαν εχον την προς το Β γωνιαν, mera sunt tavtologia (monente Austin p. 37.), vel rectius absonam præ se ferunt determinationem: a quo vitio nec ipsum generale Propositionis enunciatum eique respondens conclusio libera sunt.

Accedit, quod in Propositionis 65. (75.) *Datorum* enunciato generatim sumitur: Εαν τριγωνον οξειαν εχη γωνιαν δεδομενην.

Expositio quidem ejus adjunctam rursus habet ad triangulum acutangulum restrictionem. Sed eadem, quin magis adhuc otiosa laborat vocis οξειαν adjectione, qua Propositionis II, 13. expositio; unde, contra ipsius propositi tenorem, translata censeri debet.

Scopus enim, quo Propositiones 64. 65. (74. 75.) *Datorum* tendunt, haud permittit, ut posterior ad triangula acutangula restringatur. Is quippe, ut problematibus, quibus inserviunt, (cujusmodi est, quod cl. Camerer in *Append.* 2. versionis suæ *Locorum planorum Apollonii* p. 439. 443. tractat) Propositiones illæ reipsa sufficiant, arctior constitui nequit, quam ut, trian-

trianguli cujuslibet dato angulo obliquo, rationem trianguli ad differentiam quadrati lateris angulo dato oppoſiti & ſummæ quadratorum laterum eundem angulum comprehendentium dari oſtendant.

Hoc autem poſito, Propoſitio II, 13. *Elementorum*, qua *Datorum* 65ta (75ta) nititur, pariter ad omnis generis triangula patere debet.

§. 213.

Jam fundandæ poſteriori ſufficit quidem ea prioris ad triangula rectangula atque obtuſangula extenſio, quæ §. 201. 211. docetur.

Veruntamen Propoſitionis 65. (75.) *Datorum* expoſitioni, ut Propoſitio II, 13. ſic concepta legitime applicetur, inſeri aliquid, ex. gr. *και ετεραν την προς τω Γ γωνιαν εχον οξειαν*, ante verba, *και ηχθω*, deberet.

Præterea enunciati Propoſitionis XII. verba: *υπο τε μιας των περι την αμβλειαν γωνιαν, εφ ην εκβληθεισαν η καθετος πιπτει, και της απολαμβανομενης εκτος υπο της καθετε προς τη αμβλεια γωνια*, conſequentiam manifeſto involvunt ſuppoſiti: *εν τοις αμβλιγωνιοις τριγωνοις*. Contra in verſionibus XIIItiæ §. 211. recenſitis verba ejus: *υπο τε μιας των περι την οξειαν γωνιαν, εφ ην η καθετος πιπτει, και της απολαμβανομενης εντος υπο της καθετε προς τη οξεια γωνια*, conditionis loco ſumi videas, qua latus trianguli acuto angulo adjacens, in quod ducendum ſit perpendiculum, determinetur.

Et, ſi non *Datorum* Propoſitio 65. (75.), aliæ certe Propoſitionis II, 13. applicationes univerſum ejus §. 204. ſeqq. ſtabilitum ambitum requirunt. Vid. §. 225. 227. ſq. 232. 233.

§. 214.

Expuncta igitur etiam voce *εντος*, & expreſſione: *εφ ην η καθετος πιπτει*, latius ſumta, neque ad caſum intra lateris hujus terminos reſtricta; alii interpretes Propoſitionem XIII. concluſioni §. 208. conformiter enunciarunt.

Sic Tacquet (p. 67. ſq.): "In triangulo quocunque quadratum„ lateris acuto angulo oppoſiti a quadratis laterum reliquorum exceditur„ rectangulo bis, quod continetur ſub latere alterutro acutum angulum„ comprehendentium, in quod cadit perpendicularis ab oppoſito, & ſub„ intercepta inter perpendicularem & acutum angulum." Vulgarem tamen demonſtrationem ſolam tradidit, atque ad triangula non-acutangula

gula eo, qui §. 201. notatur, modo extendi ſchemate adjuncto indicavit; nullaque caſus §. 204. facta mentione, ſub Corollarii titulo tantum addidit: "Vera eſt propoſitio, licet perpendicularis cadat extra triangu-„lum. Demonſtratio fere eadem eſt."

Rob. Simson pronunciat, & omnimode (vid. §. 204. ſq.) demonſtrat (p. 67. ſq.): "In omni triangulo quadratum ex latere acutum an-„gulum ſubtendente minus eſt quam quadrata ex lateribus angulum „illum acutum continentibus, rectangulo contento bis ab uno laterum, „quæ ſunt circa acutum angulum, in quod perpendicularis cadit, & re-„cta linea intercepta a perpendiculari ad angulum acutum;" ſubjuncto tantum (p. 348.) monito: "Propoſitio hæc enunciatur ſolummodo de „triangulis acutangulis, quamvis vera ſit de omni triangulo. Adjecta „igitur eſt demonſtratio in reliquis caſibus."

Cum Rob. Simson conſentit Austin (p. 37. ſq.): niſi quod in enunciato propoſitionis habet: "in quod, productum, ſi opus eſt, perpen-„dicularis cadit;" & caſum, ad quem adjectum hoc ſpectat, aliter demonſtrat (§. 205. n°. 3.).

§. 215.

Hac caſum §. 205. evincendi methodo Propoſitionis generalis §. 208. 214. demonſtratio ad duo membra redigitur (Austin p. 37. ſqq.): quorum unum (§. 204.) cum Propoſitione I, 47. fere coincidit; alterum diverſa duo ſchemata ſupponit, ad quæ eadem argumentatio applicatur.

Omiſſio membri prioris, ut facilis & I, 47.ma jam contenti (cujusmodi exempla etiam demonſtrationes Propoſitionum I, 7. 24. 35. III, 20. offerunt), ac figuræ ad caſum §. 205. pertinentis, (utriusque omiſſionis, vel ſimilis tantum poſteriori, exempla recenſentur §. 226. 228.) anſam præbuiſſe conjici poteſt Propoſitionem XIII. ad triangula acutangula reſtringendi, atque ad præcedentis XIImæ enunciatum ex oppoſito conformandi.

§. 216.

Expunctis tamen ſolum verbis: τοις οξυγωνιοις, οξυγωνιον; ac demonſtrationis initio (additis, quæ uncis incluſa ſiſtuntur) ad *Fig.* 56. & ſupplendam 57. ſic relato: Επι γαρ (ητοι) ευθεια η ΓΒ τετμηται ως ετυχε κατα το Δ (η ευθεια η ΔΒ κατα το Γ); cetera ſervari poſſe videndentur; dummodo verba: εφ ην η καθετος πιπτει, της απολαμβανομενης εντός πρός τη οξεια γωνια, iisque oppoſita Propoſitionis XII: Εφ ην εκβληθεισαν η καθετος πιπτει, της απολαμβανομενης εκτος προς τη αμβλεια γωνια, ſitum perpendiculi ac ſegmenti per illud abſciſſi diverſum relate ad angulum ipſiusque verticem, non ad triangulum, indicare cenſeantur.

§. 217.

Propoſitione XIII. alterutro modo §. 201. 211. vel §. 208. 214. ad triangula quæcunque extenſa, ex ipſa & ex I, 47. II, 12. facile per demonſtrationes apagogicas inferuntur earum converſæ. Nempe

1°. Si in triangulo ABC (*Fig.* 53.) eſt $BC^q = AB^q + AC^q$: angulus BAC, qui lateri BC opponitur, nequit eſſe obtuſus vel acutus; cum ita foret $BC^q >$ vel $< AB^q + AC^q$ (II, 12. 13.); proinde rectus eſt angulus BAC.

2°. Si in triangulo ABC (*Fig* 58.) eſt $BC^q > AB^q + AC^q$; obtuſus ſit, oportet, quem latus BC ſubtendit, angulus BAC: quippe nec rectus eſſe poteſt, nec acutus; quia tum foret $BC^q =$ vel $< AB^q + AC^q$ (I, 47. II, 13.)

3°. Si in triangulo ABC (*Fig.* 59.) eſt $BC^q < AB^q + AC^q$; acutus erit, qui lateri BC opponitur, angulus BAC: utpote qui nec poteſt eſſe rectus, nec obtuſus; cum ſic eſſe deberet $BC^q =$ vel $> AB^q + AC^q$ (I, 47. II, 12.)

§. 218.

Poſteriores duas converſas Clavius (p. 193. 200. ſq.) directe ejusdem conſtructionis ope demonſtrat, quam Euclides ſtabiliendæ primæ adhibet in Propoſitione I, 48.

Nimirum ſi $BC^q > AB^q + AC^q$ (*Fig.* 58.), vel $< AB^q + AC^q$ (*Fig.* 59.): alterutri circa angulum BAC lateri AC in puncto A conſtituto perpendiculo $AD =$ alteri circa angulum A lateri AB, & juncta CD recta, ob $CD^q = AD^q + AC^q$ (I, 47.) $= AB^q + AC^q$ (conſtr.), erit priori caſu (*Fig.* 58.) $BC^q > CD^q$, $BC > CD$; poſteriori (*Fig.* 59.) $BC^q < CD^q$, $BC < CD$. Sed $AB = AD$ (conſtr.), & $AC = AC$, in triangulis ABC, ADC. Itaque (I, 25.) priori caſu (*Fig.* 58.) angulus $BAC > CAD$; poſteriori (*Fig.* 59.) angulus $BAC < CAD$; h. e. (conſtr.) priori caſu (*Fig.* 58.) angu-

angulus BAC major est recto, seu obtusus; posteriori (*Fig.* 59.) minor est recto, seu acutus.

Demonstrationem hanc casus posterioris etiam indicat ISAACUS MONACHUS in Scholio supra §. 202. citato.

§. 219.

Triangulum scalenum igitur erit $\left\{\begin{array}{l}\text{rectangulum}\\ \text{obtusangulum}\\ \text{acutangulum}\end{array}\right\}$, si quadratum maximi ejus lateris quadratis simul reliquorum duorum laterum est $\left\{\begin{array}{l}\text{æquale}\\ \text{majus}\\ \text{minus}\end{array}\right\}$ (§. 217. sq.): ultimo scilicet casu maximus trianguli angulus (I, 18.) acutus erit; tantoque magis ceteri.

Et triangulum æquicrurum, cujus basis utroque crurum major est, (quodsi enim singula crura basi majora sunt, non potest non esse acutangulum per I, 5. 17. 18.), erit $\left\{\begin{array}{l}\text{rectangulum}\\ \text{obtusangulum}\\ \text{acutangulum}\end{array}\right\}$, si quadratum basis duplo quadrato unius cruris est $\left\{\begin{array}{l}\text{æquale}\\ \text{majus}\\ \text{minus}\end{array}\right.$.

Sic comparando invicem quadrata laterum trianguli, cujus speciei sit ratione habita angulorum, erui potest: quod comparatio ipsorum laterum non præstat, nisi vel omnia æqualia sint; vel duo æqualia, ac singula majora tertio.

§. 220.

Si recta, a vertice trianguli ad punctum, quo basis bifariam secatur, ducta, æqualis est semissi basis; angulus ad verticem trianguli est rectus: contra $\left\{\begin{array}{l}\text{obtusus}\\ \text{acutus}\end{array}\right\}$ est angulus ad verticem, si recta ab eo ad punctum bisectionis basis acta semisse basis est $\left\{\begin{array}{l}\text{minor}\\ \text{major}\end{array}\right.$.

Bifariam in G secta trianguli ABC (*Fig.* 60. 61. 62.) basi BC, jungatur AG recta; & versus E producatur alterutrum crus BA anguli ad verticem BAC. Tum

1°. ſi $AG=\begin{cases}BG\\CG\end{cases}$ (*Fig.* 60.): ſunt anguli $GAB=B, GAC=C$ (I, 5.); itaque angulus $BAC=B+C=EAC$ (I, 32.), proinde rectus.

2°. Si $BG, CG \left\{\begin{matrix}>\\<\end{matrix}\right\} AG$ (*Fig.* 61. 62.): ſunt anguli $GAB \left\{\begin{matrix}>\\<\end{matrix}\right\} B$, $GAC \left\{\begin{matrix}>\\<\end{matrix}\right\} C$ (I, 18.); igitur angulus $BAC \left\{\begin{matrix}>\\<\end{matrix}\right\} B+C$ ſeu (I, 32.) EAC; ideoque $\begin{cases}\text{obtuſus}\\\text{acutus}\end{cases}$ (I, 13.).

§. 221.

Viciſſim recta a vertice trianguli ad punctum, quo biſariam ſecatur baſis, ducta ſemiſſi baſis æqualis eſt, ſi trianguli angulus ad verticem eſt rectus: eademque recta ſemiſſe baſis eſt $\left\{\begin{matrix}\text{minor}\\\text{major}\end{matrix}\right\}$, ſi angulus ad verticem trianguli eſt $\begin{cases}\text{obtuſus}\\\text{acutus}\end{cases}$. Nempe

1°. Si angulus BAC eſt rectus (*Fig.* 60.); non poteſt eſſe $AG \left\{\begin{matrix}<\\>\end{matrix}\right\}$ ſemiſſibus baſis BG, CG: alioquin foret angulus BAC $\begin{cases}\text{obtuſus}\\\text{acutus}\end{cases}$ (§. 220. n°. 2.)

2°. Si angulus BAC eſt obtuſus (*Fig.* 61.): nequit eſſe $AG \left\{\begin{matrix}=\\>\end{matrix}\right\} BG, CG$; cum ſic foret angulus BAC $\begin{cases}\text{rectus}\\\text{acutus}\end{cases}$ (§. 220.)

3°. Si angulus BAC eſt acutus (*Fig.* 62.): non erit $AG \left\{\begin{matrix}=\\<\end{matrix}\right\} BG, CG$; quod efficeret angulum BAC $\begin{cases}\text{rectum}\\\text{obtuſum.}\end{cases}$ (§. 220.)

§. 222.

Quæ a vertice trianguli æquicruri ad punctum, quo biſariam ſecatur baſis, ducitur recta, ad angulos rectos baſi inſiſtit (I, 8.).

Quæ autem a trianguli non æquicruri vertice ad punctum biſectionis baſis agitur recta, oblique in baſin incidit; ita ut angulum obtuſum cum ea efficiat ad partes cruris majoris; acutum, ad partes cruris minoris (I, 25. 13.).

§. 223.

In triangulo igitur non æquicruro, cujus anguli ad basin ambo sunt acuti, perpendiculum ab vertice in basin demissum hanc in inæqualia secat: sic, ut majus segmentum adjaceat cruri majori; minus, minori. (§. 222. 194.)

Idem etiam consequitur ex I, 47.

§. 224.

In quocunque triangulo non æquicruro differentia quadratorum crurum æqualis est duplo rectangulo sub basi & sub ejus segmento duobus intercepto punctis, quorum uno bifariam secatur basis, altero in ipsam incidit perpendiculum ex vertice trianguli super eam demissum.

Sint (*Fig.* 63. 64. 65.) ABC triangula non æquicrura; nominatim sit $AC > AB$: sintque eorum bases BC bifariam sectæ in punctis G.

1°. Si rectus est angulus B, qui cruri majori AC opponitur (*Fig.* 63.): est

$AC^q = AB^q + BC^q$ (I, 47.) $= AB^q +$ 2rectg.sub GB & BC (§. 8.)

2°. Si angulus ABC est obtusus (*Fig.* 64.): est

$AC^q = AB^q + BC^q +$ 2rectg.sub BD & BC (II, 12.)

$= AB^q +$ 2rectg.sub GB & $BC +$ 2rectg.sub BD & BC (§. 8.)

$= AB^q +$ 2rectg.sub GD & BC (II, 1.)

3°. Si angulus B est acutus (*Fig.* 65.): est

$AC^q +$ 2rectg.DB&$BC = AB^q + BC^q$ (II, 13.)

$= AB^q +$ 2rectg.sub GB & BC (§. 8.)

$= AB^q +$ 2rectg.sub GD & $BC +$ 2rectg.sub DB & BC (II, 1.)

itaque

$AC^q = AB^q +$ 2rectg.sub GD & BC.

Proinde $AC^q - AB^q = \begin{cases} \text{2rectg.sub } GB \text{ \& } BC \text{ (Fig. 63.)} \\ \text{2rectg.sub } GD \text{ \& } BC \text{ (Fig. 64. 65.)} \end{cases}$

§. 199.

Cum angulus C (*Fig.* 63. 64. 65.), qui lateri $AB < AC$ opponitur, acutus sit (I, 18. 17.): generatim juxta §. 208. 214. est

$AC^q + BC^q = AB^q +$ 2rectg.sub BC & CD

seu (§. 8. & II, 1.) $AC^q + 2BC \times CG = AB^q + 2BC \times CG + 2BC \times GD$

Igitur $AC^q = AB^q + 2BC \times GD$

& $AC^q - AB^q =$ 2rectg.sub BC & GD.

Huc

Huc redeunt demonstrationes WHISTONI (p. 61. sq.), GILBERTI (p. 295.)

§. 226.

Duobus casibus posterioribus §. 224. (*Fig.* 64. 65.) sunt

$AC^q = CD^q + AD^q$ (I, 47.)

$AB^q = BD^q + AD^q$

Unde $AC^q - AB^q = CD^q - BD^q$.

Porro $CD^q - BD^q =$ rectg. sub $CD + BD$ & $CD - BD$ (§. 21. 26. 45.)

& (*Fig.* 64.) sunt $CD + BD = 2GD$ (§. 55.), $CD - BD = BC$

(*Fig.* 65.) $CD + BD = BC$, $CD - BD = 2GD$ (§. 30.)

Quare utroque casu $CD^q - BD^q =$ 2rectg. sub BC & GD. (§. 3.); quod, vi ejusdem §. 3, consentit cum ostensis §. 100. 107: & pariter $AC^q - AB^q =$ 2rectg. sub BC & GD.

Utrumque assertum $AC^q - AB^q = CD^q - BD^q =$ 2rectg. sub BC & GD complectuntur, & posterius ex priori alterutro modo heic indicato inferunt PAPPI Lemma 2. vel 1. in *Apollonii Locorum planorum Lib.* II. (*Collect. math.* fol. 232. b. sq. *Apollonius von Pergen Ebene Oerter.* Leipzig 1796. p. 13. sq. 27. sq. 208. sq.), ac GILBERT (p. 304. sq.)

Lemma PAPPI de triangulo universim enunciatur. Schema ei adjunctum nonnisi casum nostræ *Fig.* 65. exhibet; demonstratio tamen æque ad *Fig.* 64. pertinet. Ac Propositio I. Lib. II. *Apollonii*, quæ posteriori hujus Lemmatis parte nititur, eam non solum ad utramque *Fig.* 64. 65, sed & ad casum *Fig.* 63. extensam requirit.

§. 227.

Cujusvis trianguli quadrata duorum crurum simul dupla sunt quadratorum dimidiæ basis, ac rectæ ab vertice trianguli ductæ ad punctum, quo basis bifariam secatur.

Triangulorum ABC (*Fig.* 63. 64. 65. 66.) bases BC bifariam in punctis G secentur; & ab verticibus A ad puncta G ducantur rectæ AG.

1°. Cum in triangulo æquicruro (*Fig.* 66.) recta AG sit basi perpendicularis (§. 222.); ideoque sint $AC^q = AB^q = \left\{\begin{matrix} GC^q \\ GB^q \end{matrix}\right\} + AG^q$ (I, 47.): est $AC^q + AB^q = 2(GB^q + AG^q)$.

2°. Sint

2°. Sint triangulorum ABC (*Fig.* 63. 64. 65.) crura inæqualia, nempe $AC > AB$.

Cum in his recta AG oblique in basin BC incidat (§. 222.): sit

α) ipsum crus minus AB basi BC perpendiculare (*Fig.* 63.)

$$\begin{array}{lll} \text{Tum} & AC^q = AG^q + GC^q + 2\,\text{rectg sub}\,CG\,\&\,GB & (\text{§. 222. \& II, 12.}) \\ & = AG^q + GB^q + 2GB^q & \text{ob } CG = GB \text{ (constr.)} \\ \text{Itaque} & AC^q + AB^q = AG^q + AB^q + GB^q + 2GB^q & \\ & = 2AG^q + 2GB^q & \text{ob } AB^q + GB^q = AG^q \text{ (I, 47.)} \\ \text{Seu} & AC^q = AB^q + BC^q\,(\text{I, 47.}) = AB^q + 4GB^q & (\text{constr. \& §. 13.}) \\ \text{Quare} & AC^q + AB^q = 2AB^q + 4GB^q = 2(AB^q + GB^q) + 2GB^q = 2AG^q + 2GB^q & (\text{I, 47.}) \end{array}$$

β) Si etiam crus minus AB basi BC oblique insistit (*Fig.* 64. 65.): ex vertice A demittatur in basin BC perpendiculum AD; quod ad partes anguli acuti AGB cadet (§. 222. 194.)

Tunc, cum angulus AGC sit obtusus, AGB acutus (§. 222.), sunt

$$\begin{array}{ll} AC^q = AG^q + CG^q + 2\,\text{rectg. sub}\,CG\,\&\,GD & (\text{II, 12.}) \\ = AG^q + GB^q + 2\,\text{rectg. sub}\,GB\,\&\,GD & \text{ob } CG = GB \text{ (constr.)} \\ AB^q = AG^q + GB^q - 2\,\text{rectg. sub}\,GB\,\&\,GD & (\text{II, 13. \& §. 205.}) \end{array}$$

Quare $AC^q + AB^q = (2AG^q + GB^q)$.

$$\text{Seu} \quad \begin{array}{l} AC^q = AD^q + CD^q \\ AB^q = AD^q + BD^q \end{array} \quad (\text{I, 47.})$$

$$\begin{array}{lll} \text{Igitur} & AC^q + AB^q = 2AD^q + CD^q + BD^q & \\ & = 2AD^q + 2GD^q + 2GB^q & (\text{II, 9. 10.}) \\ & = 2AG^q + 2GB^q & (\text{I, 47.}) \end{array}$$

§. 228.

Propositionem §. præced. traditam exhibent PAPPI Lemma 4. vel 6. in *Apollonii Locorum planorum Lib.* II. (*Collect. math.* Fol. 234. b. sq. *Apollonius von Pergen Ebene Oerter.* p. 15. 29. 234. sq.), SERENUS (*De coni sectione.* Prop. 16. p. 46. sq.), CLAVIUS (p. 208. sq.), GREGORIUS A S. VINCENTIO (p. 28. sq.), FRANCISC. A SCHOOTEN (*Exercitationum mathematicarum Libri* V. Lugd. Bat. 1657. *Proposit. geometr.* Prop. 18. p. 62. sq.), VIVIANI (*De Locis solidis.* Lib. III. Prop. 6. p. 54. sq.), WHISTON (p. 62.), VAN SWINDEN (p. 78.), GILBERT (p. 307. sqq.).

Pappus ac *Whiston* casum n° 2. β. *Fig.* 65. solum, hic priori, ille posteriori methodo demonstratum sistunt: propositum tamen generatim enun-

enunciant; quod & supponit *Apollonii* Lib. II. Propositio V, cui apud *Pappum* illud inservit.

Clavius & *Franc. a Schooten* omnes recensent ac demonstrant casus: ille casum n° 2. α. modo posteriori, hic priori; alterum n° 2. β. uterque modo posteriori.

Gregorius, casu n° 1. omisso, casum n° 2. α. adstruit methodo priori: & casum n° 2. β. in duos dirimit; quorum priorem (*Fig.* 63.) pariter modo priori; alterum (*Fig.* 65.) posteriori stabilit.

Serenus, *Viviani*, *Gilbert*, hic exposito, illi strictim indicato & ad I, 47. remisso casu n° 1, duos casus n° 2. una demonstratione complectuntur: *Viviani* quidem ac *Gilbert* priori modo juxta §. 204. sqq., cui hic & alteram casus n° 2. β. demonstrationem subjungit; *Serenus* autem sequenti methodo simili priori n° 2. β.

Cum (*Fig.* 67. 68. 69.) ob $AC > AB$ (supp.) sit angulus AGC obtusus, AGB acutus (§. 222.): ad hujus partes cadit perpendiculum BF ex puncto B in rectam AG demissum, quicunque sint anguli ABC, BAG; & perpendiculum CK ex puncto C in eandem AG demissum in ipsam ultra G productam incidit ad partes anguli CGK deinceps positi obtuso AGC (§. 194.): atque ob $GC = GB$ (constr.), & angulos $K = BFG$ (constr.), $CGK = BGF$ (I, 15.), est $GK = GF$ (I, 26.)

Proinde $AC^q = AG^q + GC^q + 2$ rectg. sub AG & GK (II, 12.)
$= AG^q + GB^q + 2$ rectg. sub AG & GF
$AB^q = AG^q + GB^q - 2$ rectg. sub AG & GF (§. 208. 214.)
& $AC^q + AB^q = 2(AG^q + GB^q)$.

Textui *Sereni*, præter figuras nostris 66. 67. respondentes, adjuncta est tertia, angulum ABC (*Fig.* 67.) sistens obtusum; eo itaque saltem modo extensam supponens Propositionem II, 13. qui §. 210. 211. indicatur: quod tamen ad propositum universim evincendum haud sufficit.

§. 229.

Casum n° 1. (*Fig.* 66.) Propositionis §. 227, quo

$$AC^q + AB^q \text{ seu } 2AB^q = 2(AG^q + GB^q),$$

$$AB^q = AG^q + GB^q = AG^q + \text{rectg.} BGC,$$

etiam exprimit assertum: Trianguli isoscelis quadratum cruris æquari quadrato rectæ ab vertice trianguli ductæ ad punctum bisectionis basis, & rectangulo sub segmentis basis, in quæ puncto illo dividitur, simul.

Quod

Quod assertum perstat, quodcunque basis punctum M loco puncti bisectionis G accipiatur. Etenim

$$\begin{aligned} AB^q &= AG^q + GB^q \text{ (I, 47.)} \\ &= AG^q + GM^q + \text{rectg. } BMC \text{ (II, 5.)} \\ &= AM^q + \text{rectg. } BMC \text{ (I, 47.)} \end{aligned}$$

Sed si punctum M sumitur in basi BC producta (*Fig.* 70.): fit

$$\begin{aligned} AB^q &= AG^q + GB^q \text{ (I, 47.)} \\ &= AG^q + GM^q - \text{rectg. } BMC \text{ (II, 6.)} \\ &= AM^q - \text{rectg. } BMC \text{ (I, 47.)} \end{aligned}$$

seu $AB^q + \text{rectg. } BMC = AM^q$.

Trianguli igitur isoscelis quadratum cruris excedit quadratum rectæ ab vertice trianguli ad punctum quodcunque ipsius basis ductæ, rectangulo sub segmentis basis, quæ punctum hoc dirimit: sed quadratum rectæ ab vertice trianguli ad quodlibet basis continuatæ punctum ductæ excedit quadratum cruris, rectangulo sub rectis, quæ pariter puncto illi & extremis basis interjacent.

Generatim differentia quadratorum cruris trianguli isoscelis & rectæ ab vertice ejus ductæ ad punctum quodcunque basis ipsius, vel productæ, æqualis est rectangulo sub rectis, quæ puncto illi & extremis basis interjacent.

Lemma hoc ROB. SIMSON præmittit Propositioni 76. *Datorum* (p. 107. sq.). Priorem ejus partem demonstrationi primæ ejusdem Propositionis, apud ipsos 67mæ, ex veteri Scholiaste subjungit CLAUD. HARDY (p. 121. sq.); in Nota annectit DAV. GREGORIUS (p. 503.) Eandem Propositionem tradit GILBERT (p. 351. 357.).

§. 230.

Quodsi autem ab trianguli non æquicruri ABC (*Fig.* 71. 72.) vertice A ducitur AM recta ad punctum quodcunque M basis BC, ab puncto bisectionis ejus G diversum; bifariam vero in L secatur AM, & jungitur GL recta: quadrata crurum trianguli simul excedunt quadratum rectæ AM & quadruplum quadratum rectæ GL, duplo rectangulo sub segmentis BM & MC. Et si ad basis productæ punctum quodcunque M ab vertice A recta AM agitur (*Fig.* 73. 74.), pariterque in L bifariam secatur; ac recta GL jungitur: quadratum rectæ AM & quadruplum rectæ

GL

GL quadratum ſimul excedunt ſummam quadratorum crurum trianguli duplo ſub BM & MC rectangulo.

Nempe, ducta AG recta, ſunt

$$AC^q + AB^q = 2AG^q + 2GB^q \quad (\S.\ 227.\ \text{ſq.})$$
$$= 2AG^q + 2GM^q + 2BMC (Fig. 71. ſq. II,5.) = 2AG^q + 2GM^q - 2BMC (Fig. 73 ſq. II,6)$$
$$= 4AL^q + 4GL^q + 2BMC \qquad = 4AL^q + 4GL^q - 2BMC \quad (\S.\ 227.\ \text{ſq.})$$
$$= AM^q + 4GL^q + 2BMC \qquad = AM^q + 4GL^q - 2BMC \quad (\S.\ 13.)$$

§. 231.

Eadem demonſtratio applicatur ad triangulum æquicrurum (*Fig.* 66. 70.) Tum vero, ob angulum AGM rectum (§. 222.), eſt $GL = AL = ML$ (§. 221.); $2GL = AM$; $4GL^q = AM^q$ (§. 13.) Unde (§. 230.)

$$AC^q + AB^q \text{ ſeu } 2AB^q = 2AM^q + 2BMC \ (Fig.\ 66.) = 2AM^q - 2BMC \ (Fig.\ 70.)$$
$$AB^q = AM^q + BMC \qquad = AM^q - BMC$$

conformiter cum §. 229.

§. 232.

Si in trianguli æquicruri alterutrum crus perpendicularis demittitur ex vertice oppoſiti anguli ad baſin; rectangulum ſub hoc crure & ſub ejus ſegmento, quod baſi ac perpendiculo interjacet, dimidium eſt quadrati baſis. (Pappi Theor. I. ſubjunctum Prop. XXV. Lib. V. *Collect. math.* fol. 89.)

Ob angulos ad baſin BC trianguli æquicruri ABC (*Fig.* 75. 76. 77.) acutos (I, 5. 32.), ad partes anguli C cadit perpendiculum BD in crus AC demiſſum ex vertice B anguli oppoſiti ad baſin (§. 194.): idemque cum altero crure BA coincidit, ſi trianguli ad verticem A angulus eſt rectus (*Fig.* 75.); intra triangulum in crus ipſum CA cadit, ſi angulus A eſt acutus (*Fig.* 76.); extra triangulum in productum crus CA incidit (*Fig.* 77), ſi trianguli ad verticem angulus BAC eſt obtuſus. (§. 194.)

Primo caſu (*Fig.* 75.) eſt $BC^q = AB^q + AC^q$ (I, 47.) $= 2CA^q$; ideoque $CA^q = \frac{1}{2}BC^q$.

Secundo caſu (*Fig.* 76.) eſt $BC^q + 2CAD = AB^q + AC^q$ (II, 13.) $= 2CA^q = 2CAD + 2ACD$ (II, 2):
igitur $BC^q = 2$ rectg. ACD; rectg. $ACD = \frac{1}{2}BC^q$.

Tertio caſu (*Fig.* 77.) eſt $BC^q = AB^q + AC^q + 2CAD$ (II, 12.) $= 2(AC^q + CAD) = 2ACD$ (II, 3.);
itaque rectg. $ACD = \frac{1}{2}BC^q$.

Ita

Ita *Pappus* apud *Commandinum.* Succinctius & generatim (*Fig.* 75. 76. 77.), ob angulum C acutum est

$$AB^q + 2ACD = BC^q + AC^q \quad (\text{§. } 208.\ 214.)$$

&, ob $AB = AC$, $\quad 2ACD = BC^q$; $\quad$ rectg. $ACD = \frac{1}{2} BC^q$.

§. 233.

Sit 1°. $ABCD$ (*Fig.* 78.) parallelogrammum rectangulum, cujus jungantur diagonales AC, BD.

Erit
$$AC^q = AB^q + BC^q$$
$$BD^q = \left.\begin{matrix} AB^q \\ CD^q \end{matrix}\right\} + DA^q \quad (\text{I, } 47.)$$

ideoque $AC^q + BD^q = AB^q + BC^q + CD^q + DA^q = 2(AB^q + BC^q)$.

Posterius immediate etiam inde consequitur: quod parallelogrammi rectanguli diagonales invicem sunt æquales (I, 34. 4.).

2°. Sit $ABCD$ (*Fig.* 79.) parallelogrammum obliquangulum; cujus igitur duo anguli, qui eidem lateri, ex. gr. AB, adjacent, & qui (I, 29.) simul valent duos rectos, sunt unus DAB acutus, alter CBA obtusus. Ductis ejus diagonalibus AC, BD (quarum prior, quæ vertices angulorum acutorum parallelogrammi jungit, seu obtusis ejus angulis opponitur, altera major est per I, 34. 24.); in latus parallelogrammi quodlibet AB, si parallelogrammum est æquilaterum, & in laterum contiguorum majus AB, si parallelogrammum non est æquilaterum, ab extremis D, C lateris oppositi CD demittantur perpendicula DE, CF: quorum prius, ob angulum $DBA \left\{\begin{matrix} = \\ < \end{matrix}\right\} DAB$ (supp. & I, 5. 18.), ideoque acutum (I, 17. Cor.), pariterque (supp.) acutum DAB, intra triangulum ABD; alterum, ob angulum CBA obtusum, extra triangulum ABC cadet (§. 194.): ita ut sit $BF = AE$ (I, 26.); ob $BC = AD$ (I; 34.), atque angulos $CBF = DAE$ (I, 29.), & $F = AED$ rectos (constr.).

Igitur
$$AC^q = AB^q + BC^q + 2\,\text{rectg. sub } AB \ \& \ BF \quad (\text{II, } 12.)$$
$$BD^q = AB^q + AD^q - 2\,\text{rectg. sub } BA \ \& \ AE \quad (\text{§. } 211.)$$
$$= CD^q + DA^q - 2\,\text{rectg. sub } AB \ \& \ BF \quad (\text{I, } 34.\ \&\ \text{dem.})$$

atque $AC^q + BD^q = AB^q + BC^q + CD^q + DA^q = 2(AB^q + BC^q)$.

Proinde in omni parallelogrammo quadrata diagonalium æqualia sunt quadratis laterum simul; seu dupla sunt quadratorum duorum laterum contiguorum.

Et

Et eo, qui heic traditur, modo (ſimpliciter tamen pro BD^q in ſubſidium vocata II, 13. nec indicata lateris AB, in quod perpendicula demittuntur, determinatione; quæ demum, ad quæcunque triangula juxta §. 208. 214. extenſa Propoſitione II, 13, ſuperflua fit) propoſitum de parallelogrammis demonſtrant LAGNY (*Sur une propoſition de geometrie elementaire. Mem. de l'Acad. de Sc. de Paris.* Année 1707. Amſt. p. 412. ſq.), 1706
BÆRMANNUS (p. 55.) p. 319

§. 234.

Idem GREGORIUS A S. VINCENTIO (p. 33.), VIVIANI (*De Locis ſolidis.* Lib. III. p. 110. ſq.), GILBERT (p. 313.), inferunt ex Propoſitione demonſtrata §. 227. ſq.

Nempe cum diagonales parallelogrammi $ABCD$ (*Fig.* 78. 79.) ſe mutuo bifariam ſecent (quod per I, 26. colligitur ex triangulis ABG, CDG; in quibus $AB=CD$ (I, 34.), & (I, 29.) anguli $GAB=GCD$, $GBA=GDC$): ſunt

$$AB^q+BC^q=2GB^q+2GA^q$$
$$CD^q+DA^q=2\left\{\begin{matrix}GD^q\\GB^q\end{matrix}\right\}+2GA^q \quad (\text{§. 227. ſq.})$$

Quare $AB^q+BC^q+CD^q+DA^q=4GB^q+4GA^q=BD^q+AC^q$ (§. 13.)

§. 235.

Viciſſim propoſitum §. 227. de triangulis ex propoſitione §. 233. de parallelogrammis demonſtrata poteſt deduci.

Trianguli enim cujuscunque ABD (*Fig.* 78. 79.) baſi BD bifariam in puncto G ſecta; tum ab trianguli vertice A ad punctum G ducta AG recta, eaque continuata, donec ſit $GC=GA$; & ab extremis B, D baſis ad punctum C ductis BC, DC rectis: ob $GB=GD$, $GA=GC$ (conſtr.), & angulum $AGB=CGD$ (I, 15.); ſunt (I, 4.) $AB=CD$, & angulus $CAB=ACD$; igitur rectæ AB, CD parallelæ (I, 27.), & æquales (dem.); proinde (I, 33.) quadrilaterum $ABCD$ eſt parallelogrammum.

Et hinc $2(AB^q+AD^q)=AC^q+BD^q$ (§. 233.) $=4AG^q+4GB^q$ (conſtr. & §. 13.)

ideoque $AB^q+AD^q=2AG^q+2GB^q$.

§. 236.

Cum parallelogrammi utraque diagonalis alteram ſecet bifariam (§. 234.); & viciſſim quadrilaterum, cujus ambæ diagonales ſe mutuo bifariam secant, sit parallelogrammum (§. 235.); sequitur: quadrilateri non parallelogrammi, seu trapezii, ambas diagonales se mutuo bifariam non secare.

1°. Bifariam in E secet trapezii $ABCD$ diagonalis BD ipsa (*Fig.* 80.), vel producta (*Fig.* 82.), alteram AC; ipsa autem BD bifariam secta sit in puncto F.

$$\begin{aligned} \text{Erunt} \quad & AB^q+BC^q = 2AE^q+2BE^q \\ & CD^q+DA^q = 2AE^q+2DE^q \quad (\text{§. 227. sq. 235.}) \\ \text{Igitur} \quad & AB^q+BC^q+CD^q+DA^q = 4AE^q+2(BE^q+DE^q) \\ \text{Sed} \quad & BE^q+DE^q = 2DF^q+2EF^q \quad (\text{II, 9. 10.}) \\ \text{Quare} \quad & AB^q+BC^q+CD^q+DA^q = 4AE^q+4DF^q+4EF^q \\ & = AC^q+BD^q+4EF^q \quad (\text{§. 13.}) \end{aligned}$$

2°. Neutra trapezii $ABCD$ diagonalis ipsa (*Fig.* 81.), vel producta (*Fig.* 83.) alteram secet bifariam; seu ab puncto, in quo diagonales AC, BD se mutuo secant, diversa sint puncta E, F, quibus bifariam secantur ipsæ AC, BD; rectæ igitur BE, DE triangulum constituant super diagonali BD, intra quod cadit EF recta.

$$\begin{aligned} \text{Tum rursus} \quad & AB^q+BC^q = 2AE^q+2BE^q \\ & CD^q+DA^q = 2AE^q+2DE^q \quad (\text{§. 227. sq. 235.}) \\ \text{Quare} \quad & AB^q+BC^q+CD^q+DA^q = 4AE^q+2(BE^q+DE^q) \\ \text{Sed pariter} \quad & BE^q+DE^q = 2BF^q+2EF^q \quad (\text{§. 227. sq. 235.}) \end{aligned}$$

Etiamnunc itaque

$$\begin{aligned} AB^q+BC^q+CD^q+DA^q &= 4AE^q+4BF^q+4EF^q \\ &= AC^q+BD^q+4EF^q \quad (\text{§. 13.}) \end{aligned}$$

In quovis igitur trapezio, seu quadrilatero non parallelogrammo, quadrata laterum simul æqualia sunt quadratis diagonalium & quadruplo quadrato rectæ interceptæ punctis, quibus diagonales bifariam secantur.

§. 237.

Theorema hoc, quod singularem proprietatem omnium quadrilaterorum notatu maxime dignam complecti jure prædicat, & nusquam adhuc neque prolatum, neque demonstratum esse profitetur LEONH. EULERUS in Dissertatione inscripta: *Variae demonstrationes geometricae*, ac *Novis Commentariis Acad. sc. Petrop.* Tom. I. ad annum 1747 & 1748. (Petrop. 1750.

1750.) inserta, ipse (§. 25. sqq. p. 63. sqq.) ab parallelogrammorum proprietate §. 233. sq. ostensa sic fere deducit:

Circa trapezii $ABCD$ (*Fig.* 84.) latera contigua AB & AD, BC & CD, compleantur parallelogramma $ABKD$, $BCDL$; & præter trapezii ac parallelogrammorum diagonales AC, BD, AK, CL, ducantur rectæ AL, CK.

Ita erunt anguli $CDB = DBL$, $KDB = DBA$ (I, 29.); ideoque $CDK = ABL$. Porro sunt $DK = AB$, $DC = BL$ (I, 34.) Quare (I, 4.) $CK = AL$, & angulus $CKD = LAB$. Sed & angulus $DKA = BAK$ (I, 29.) Igitur angulus $CKA = LAK$; ideoque (I, 27.) CK, AL sunt parallelæ. Quare, cum & æquales sint (demonstr.), $ALKC$ pariter est parallelogrammum (I, 33.)

Igitur $2(BC^q + CD^q) = BD^q + CL^q$
$CL^q + AK^q = 2(AC^q + CK^q)$ (§. 233. sq.)

Unde $2(BC^q + CD^q) + AK^q = BD^q + 2(AC^q + CK^q)$

Sed & $2(AB^q + AD^q) = BD^q + AK^q$ (§. 233. sq.)

Ergo $2(AB^q + AD^q) + 2(BC^q + CD^q) = 2BD^q + 2(AC^q + CK^q)$

$$AB^q + BC^q + CD^q + DA^q = BD^q + AC^q + CK^q$$

h. e. summa quadratorum laterum trapezii excedit summam quadratorum diagonalium ejus quadrato rectæ CK, qua trapezii $ABCD$ & parallelogrammi $ABKD$, tria puncta D, A, B communia habentium, puncta diversa C, K junguntur, atque, ut ait *Eulerus* (§. 26. p. 65.), discrimen trapezii a parallelogrammo exponitur.

Porro per punctum F, in quo parallelogrammi $ABKD$ diagonales AK, BD se mutuo secant, idque bifariam (§. 234.), per quod proinde etiam transit altera parallelogrammi $BCDL$ diagonalis CL (§. 234.), rectis AC, CK parallelæ agantur FM, FE; itaque describatur parallelogrammum $FMCE$, cujus latera $EC = FM$, $EF = CM$ (I, 34.). Cum, ob $AF = FK$ (§. 234.), & angulos $FAE = KFM$, $AFE = FKM$ (I, 29.), etiam sint (I, 26.) $AE = FM$, $EF = MK$: erunt $AE = EC$ (h. e. AC bifariam in E secabitur); & $CM = MK = EF$, adeoque $CK = 2EF$, $CK^q = 4EF^q$ (§. 13.): & $AB^q + BC^q + CD^q + DA^q = BD^q + AC^q + 4EF^q$.

Ejusdem propositionis, ab *Eulero* sine subjuncta demonstratione cum ipso communicatæ, demonstrationem analytico-trigonometricam eodem in Tomo *Commentar. Petrop.* (p. 131. sq.) dedit b. noster KRAFFTIUS.

Demonstrationem *Euleri* etiam exponunt cel. WUCHERER (*Einige geometri-*

metrische Säze. Progr. 1780. §. 30. ff. *Kleine Schriften.* Carlsruhe 1799. S. 133. f.), ac GILBERT (p. 314. ſq.).

§. 238.

In quadrilateris igitur ſumma quadratorum laterum vel adæquat (nempe in parallelogrammis §. 233. ſq.), vel excedit (nimirum in nonparallelogrammis §. 236. ſq.) ſummam quadratorum diagonalium: nunquam ea eſt minor; ſeu nullum exhiberi poteſt quadrilaterum, in quo ſumma quadratorum laterum minor ſit quam ſumma quadratorum diagonalium. (EULER l. c. p. 65.)

§. 239.

In trapezio, cujus duo latera oppoſita ſunt parallela, latera hæc inæqualia ſunt; & neutra diagonalis alteram ſecat bifariam.

Quippe 1°. trapezii *ABCD* (*Fig.* 85.), cujus latera *AB*, *CD* parallela ſunt, eadem latera æqualia non ſunt; alioquin *ABCD* foret parallelogrammum (I, 33.)

2°. Nec puncto *G*, ubi ſe mutuo ſecant diagonales, alterutra earum bifariam ſecabitur. Ob angulos enim $AGB = DGC$ (I, 15.), $GAB = GCD$, $GBA = GDC$ (I, 29.), contra n^rum^ 1. foret (I, 26.) $AB = CD$, ſi vel AG eſſet $= CG$, vel $BG = DG$.

§. 240.

Quæ in trapezio, cujus duo latera parallela ſunt, iisdem lateribus per punctum biſectionis unius diagonalis parallela agitur recta, bifariam quoque ſecat alteram diagonalem; ipſaque recta, punctis, quibus diagonales bifariam ſecantur, intercepta, eſt ſemidifferentia laterum parallelorum trapezii.

Trapezii *ABCD* (*Fig.* 85.) parallela ſint latera *AB*, *CD*; & $AB > CD$. Bifariam in *E* ſecetur altera ejus diagonalis *AC*; & per punctum *E* agatur lateribus *AB*, *CD* parallela *EK*, alterutri *BC* reliquorum laterum occurrens in *K*, diagonalem vero *BD* ſecans in puncto *F*: per quod, pariter ac per *E*, ducantur lateri *BC* parallelæ *FI*, *EL*.

Erunt (I, 34.) $EK = BL$, $FK = CI$, $EL = BK$, $FI = CK$.

Porro ob $AE = CE$ (conſtr.), & angulos $EAL = CEK$, $AEL = ECK$ (I, 29.), ſunt (I, 26.) $AL = EK$, $EL = CK$.

Igitur

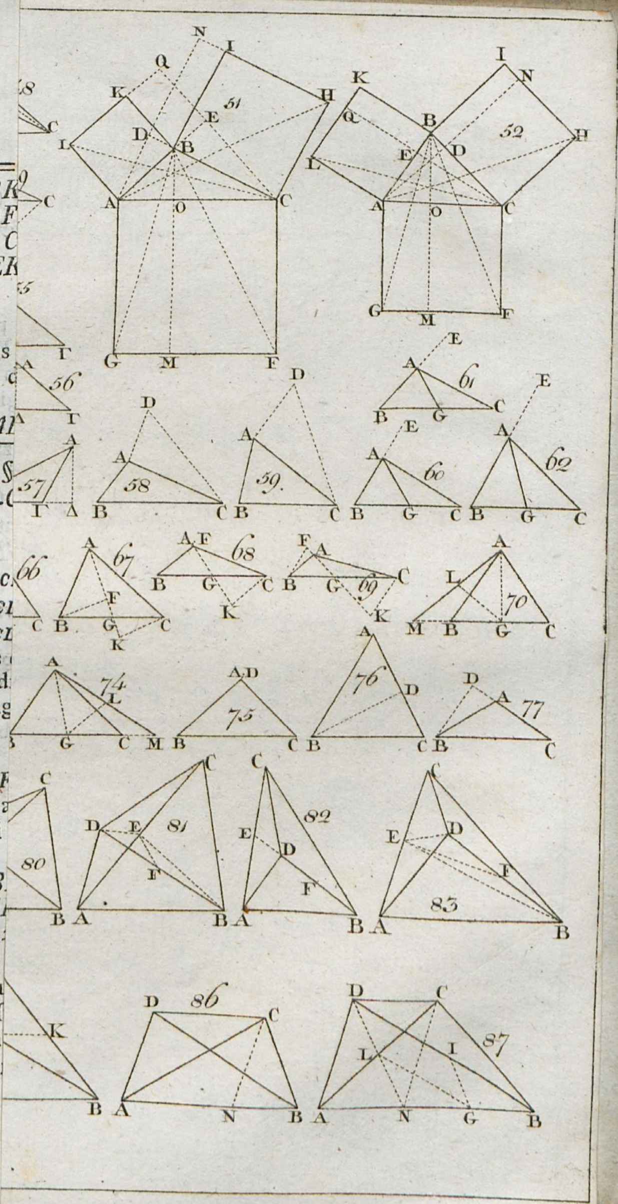
Igitur etiam $AL =$
Sed & anguli FBK
ſunt (I, 26.) $BF = DF$
Unde, ob $FK = C$
Denique $EF = EK$

In trapezio, cujus
æqualia ſunt quadratis d
rallelorum.
Nam ob $EF = AI$
$4EF^q = (AB - CD)^q$ (§
$= AC^q + BD^q + (AB - C$

Ob AB^q $+C$
porro fit $BC^q + DA^q + 2AB \times CL$
$BC^q + DA^q + 2AB \times CL$
h. e. in trapezio, cujus
ſimul æqualia ſunt quad
teribus parallelis rectang

Trapezii $ABCD$ (F
tera duo AD, BC ſint
Per C ducta lateri
$CN = AD = CB$.
Itaque angulus CB
ob $BC = AD$, $AB = A$
$2AC^q = 2BC^q +$
$AC^q = BC^q +$

Trapezii igitur, cuju
lia, diagonales invicem
æquale eſt rectangulo ſu
laterum non-parallelorum



Igitur etiam $AL = BL$; & hinc $EK = \frac{1}{2}AB$: atque EL ſeu $BK = FI$.
Sed & anguli $FBK = DFI$, $BFK = FDI$ (I, 29.). Pariter itaque ſunt (I, 26.) $BF = DF$, $FK = DI$.
Unde, ob $FK = CI$ (dem.), etiam $CI = DI$, & $FK = \frac{1}{2}CD$.
Denique $EF = EK - FK = \frac{1}{2}AB - \frac{1}{2}CD = \frac{AB - CD}{2}$.

§. 241.

In trapezio, cujus duo latera ſunt parallela, quadrata laterum ſimul æqualia ſunt quadratis diagonalium, & quadrato differentiæ laterum parallelorum.

Nam ob $EF = \frac{AB - CD}{2}$ (§. 240.); ideoque $2EF = AB - CD$, $4EF^q = (AB - CD)^q$ (§. 13.): fit (§. 236. ſq.) $AB^q + BC^q + CD^q + DA^q = AC^q + BD^q + (AB - CD)^q$.

§. 242.

Ob $AB^q + CD^q = 2$ rectg. ſub AB & $CD + (AB - CD)^q$ (II, 7. & §. 84.)
porro fit $BC^q + DA^q + 2AB \times CD + (AB - CD)^q = AC^q + BD^q + (AB - CD)^q$ (§. 241.)
$BC^q + DA^q + 2AB \times CD = AC^q + BD^q$:
h. e. in trapezio, cujus duo latera ſunt parallela, quadrata diagonalium ſimul æqualia ſunt quadratis laterum non-parallelorum & duplo ſub lateribus parallelis rectangulo.

§. 243.

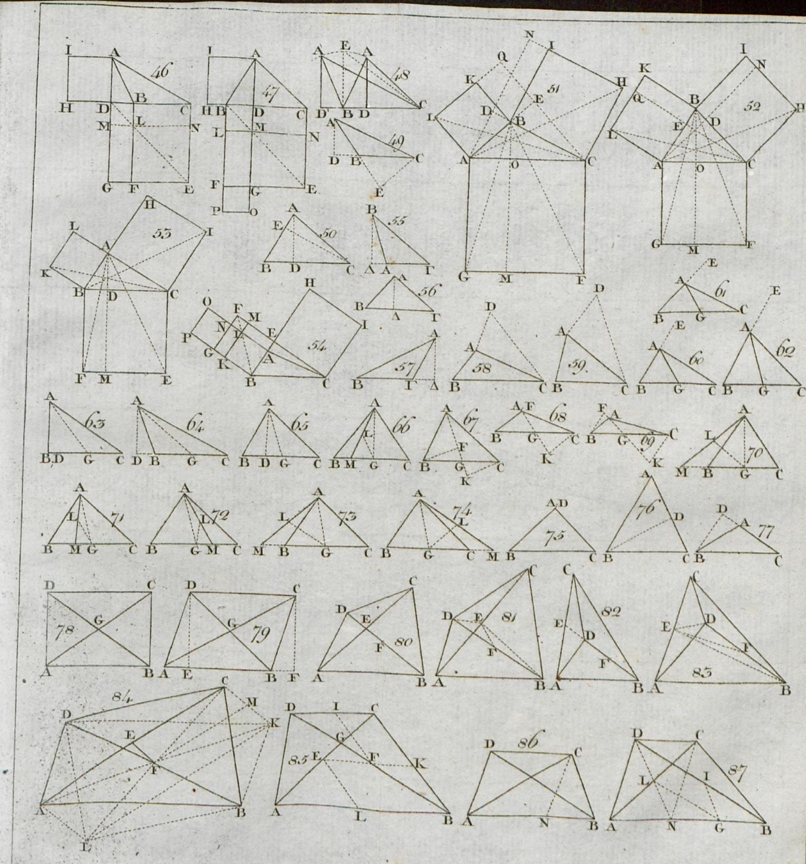
Trapezii $ABCD$ (*Fig.* 86.) latera AB, CD ſint parallela; atque altera duo AD, BC ſint æqualia: & ſit $AB > CD$.

Per C ducta lateri AD parallela CN; ſunt (I, 34.) $AN = CD$, $CN = AD = CB$.

Itaque angulus $CBN = CNB$ (I, 5.) $= DAB$ (I, 29.). Et hinc, ob $BC = AD$, $AB = AB$, eſt $AC = BD$ (I, 4.):
$2AC^q = 2BC^q + 2$ rectg. ſub AB & CD (§. 242.)
$AC^q = BC^q +$ rectg. ſub AB & CD.

Trapezii igitur, cujus duo latera ſunt parallela, & altera duo æqualia, diagonales invicem ſunt æquales; & cujuslibet diagonalis quadratum æquale eſt rectangulo ſub lateribus parallelis trapezii & quadrato unius laterum non-parallelorum ſimul.

§. 244.



§. 244.

Ob triangulum BNC isosceles, & $AN = CD$ (dem. §. 243.), immediate etiam ex parte posteriori Propositionis §. 229. consequitur esse
$$AC^q = BC^q + 2\,\text{rectg. sub } CD \,\&\, AB.$$

§. 245.

Pariter propositum §. 242. potest ad partem posteriorem Propositionis §. 230. reduci. In trapezio enim $ABCD$ (*Fig.* 87.), cujus latera AB, CD sunt parallela, & $AB > CD$, per punctum C ducta CN lateri AD parallela: fit $ANCD$ parallelogrammum; cujus igitur latera $AN = CD$, $CN = AD$ (I, 34.); ac diagonales AC, DN se mutuo bifariam secant in L (§. 234.). Per punctum L agatur alteri trapezii $ABCD$ diagonali BD parallela LG; & per punctum G, ubi hæc lateri AB occurrit, recta GI parallela rectæ DN. Erunt (I, 34.) $GL = DI$, $GI = DL$. Quare, ob $DL = NL$ (dem.), etiam $GI = NL$. Præterea sunt anguli $IGB = LNG$, $IBG = LGN$ (I, 29.) Ideoque (I, 26.) $BG = GN$, $BI = GL$. Sed & $DI = GL$ (dem.). Ergo $BD = 2\,GL$.

Trianguli igitur BNC basin BN bifariam dividit punctum G: & ad basis hujus BN productæ punctum A ab trianguli vertice C ducta est AC recta, quæ pariter bifariam dividitur puncto L: proinde est (§. 230.)
$$AC^q + 4\,GL^q = BC^q + CN^q + 2\,\text{rectg.}\,NAB.$$
Sed & $2\,GL = BD$, $CN = DA$, $NA = CD$ (dem.)
Ergo $$AC^q + BD^q = BC^q + DA^q + 2\,\text{rectg. sub}\,CD \,\&\, AB.$$

§. 244.

Ob triangulum BNC isosceles, & $AN = CD$ (dem. §. 243.), immediate etiam ex parte posteriori Propositionis §. 229. consequitur esse

$$AC^q = BC^q + 2\text{rectg. sub } CD \,\&\, AB.$$

§. 245.

Pariter propositum §. 242. potest ad partem posteriorem Propositionis §. 230. reduci. In trapezio enim $ABCD$ (*Fig.* 87.), cujus latera AB, CD sunt parallela, & $AB > CD$, per punctum C ducta CN lateri AD parallela: fit $ANCD$ parallelogrammum; cujus igitur latera $AN = CD$, $CN = AD$ (I, 34.); ac diagonales AC, DN se mutuo bifariam secant in L (§. 234.). Per punctum L agatur alteri trapezii $ABCD$ diagonali BD parallela LG; & per punctum G, ubi hæc lateri AB occurrit, recta GI parallela rectæ DN. Erunt (I, 34.) $GL = DI$, $GI = DL$. Quare, ob $DL = NL$ (dem.), etiam $GI = NL$. Præterea sunt anguli $IGB = LNG$, $IBG = LGN$ (I, 29.) Ideoque (I, 26.) $BG = GN$, $BI = GL$. Sed & $DI = GL$ (dem.). Ergo $BD = 2GL$.

Trianguli igitur BNC basin BN bifariam dividit punctum G: & ad basis hujus BN productæ punctum A ab trianguli vertice C ducta est AC recta, quæ pariter bifariam dividitur puncto L: proinde est (§. 230.)

$$AC^q + 4GL^q = BC^q + CN^q + 2\text{rectg.} NAB.$$

Sed & $2GL = BD$, $CN = DA$, $NA = CD$ (dem.)

Ergo $AC^q + BD^q = BC^q + DA^q + 2\text{rectg. sub } CD \,\&\, AB.$

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