Amalgams for the O'Nan Sporadic Group

Dissertation

zur Erlangung des akademischen Grades

doctor rerum naturalium (Dr. rer. nat.)

vorgelegt der

Mathematisch-Naturwissenschaftlich-Technischen Fakultät
der Martin-Luther-Universität Halle-Wittenberg

von Herrn Harald Gottschalk

geboren am 13.05.1968 in Bad Arolsen

Gutachter:

1. Prof. Dr. Gernot Stroth, Martin-Luther-Universität Halle-Wittenberg

2. PD Dr. Wolfgang Lempken, Universität Gesamthochschule Essen

3. Prof. Dr. Francis Buekenhout, Université Libre de Bruxelles

Halle (Saale), den 21. März 2002

urn:nbn:de:gbv:3-000003255
[http://nbn-resolving.de/urn/resolver.pl?urn=nbn%3Ade%3Agbv%3A3-000003255]

dem Andenken meines Bruders Hansjörg gewidmet

Acknowledgements. *First of all I would like to thank my advisor Gernot Stroth for many stimulating and helpful discussions and his permanent interest during my work on this thesis. I also thank my colleagues Barbara Baumeister and Alexander Stein for their suggestions and advice. I wish to thank Francis Buekenhout and the staff of the Service de Géometrie at the Université Libre de Bruxelles where I had the opportunity to stay for their support and hospitality. Moreover I would like to thank Wolfgang Lempken of the Institut für experimentelle Mathematik at the Universtät GHS Essen where computer calculations for this thesis have been carried out. For proofreading I thank my colleague Bettina Wilkens.*

Not to forget, I thank my familiy and friends, especially Annika Thum and Daniela Siebert, for their kind support from 'real life'.

Contents

Chapter 1

Introduction and preliminaries

1.1 Introduction

J. Tits' theory of buildings for groups of Lie type (see e.g. [30] or Chapter 11 of [3] for a survey) and their more general companions, the diagram geometries mainly developed by F. Buekenhout (see e.g. various Chapters of [3] or [25] for a survey) provide a major tool for understanding the interplay between groups and geometries. Since then, various traces have been pursued. For example, given a particular group G, classify all geometries (under certain assumptions) for G (see e.g. [7], also their list of references), or classify all geometries and their automorphism groups having a diagram of a certain type (see e.g. [14]), or use a particular geometry to characterize its automorphism group. In this last branch various (computer-free) existence and uniqueness proofs for sporadic groups have been completed in the past few years (see [1], [16], [28] and [29] for computer-free proofs or [19]). Such proofs follow a two step program:

- Prove that the geometry is simply connected (giving the uniqueness) and
- construct a suitable and faithful representation of the amalgam of the geometry in some $GL(V)$ (giving the existence).

The origin of this thesis was to give such an existence and uniqueness proof for the sporadic O'Nan group ($O'N$) - discovered by M. O'Nan in [24] - using the two known flag-transitive geometries for that group. The importance of such a construction is simply given by the fact that the group $O'N$ is the only sporadic group which has not been contructed computer-freely up to now.

Unfortunately, this original attempt failed. The main reason for this lies in the subgroup structure of $O'N$. Its maximal subgroups are themselves quite small or their maximal subgroups are small. This leads to e.g. to the fact that, if one tries to get the point-line graph of a geometry under control, one may have good control over its points (in the sense that we have a small permutation degree) but for the lines there is complete wilderness. Also the representation which was chosen suits the geometry but the module carries no further structure like a form respected by $O'N$.

Nevertheless, this thesis presents some new results. The following will be proved:

- The amalgams of the geometries of Buekenhout and of Ivanov and Shpectorov for the group $O'N$ are uniquely determined by their diagram and residues of rank $n-1$.
- The Buekenhout geometry for $O'N$ is simply connected.
- The 3-fold cover for $3O'N$ of the Ivanov-Shpectorov geometry for $O'N$ is its universal cover.
- Every completion of the amalgam of the Buekenhout geometry has an irreducible 154-dimensional $GF(3)$-module.
- Every completion of that amalgam is also a completion of the amalgam related to the Ivanov-Shpectorov geometry.

The proofs of the first result are computer-free. The proofs for the simply connectedness of the geometries for $O'N$, resp. $3O'N$ involve computer use for coset enumeration. The computer is also involved (but just in a small way) in the construction of the representation. Furthermore this construction does not make use of the fact that the universal completion of the amalgam related the Buekenhout geometry is $O'N$. This is also not used in the last chapter of the thesis proving that any completion of this amalgam also acts on the Ivanov-Shpectorov geometry.

1.2 Preliminaries

For the basic definitions for (coset) geometries, diagrams and (universal) coverings we refer to [25], [5] or [3]. We will briefly state the most important group theoretic tools for this thesis.

Definition 1.2.1 [10] Let I be a finite set. An *amalgam* $\mathcal{A}$ consists of a family $(G_J)_{J\subset I}$ of groups and a family of group homomorphism $\delta_{JK} : G_J \to G_K$ for every pair $J, K \subset I$ with $K \subset J$ satisfying the following conditions:

1. For all $J, K, L \subset I$ with $L \subset K \subset J$ the composite $\delta_{JK}\delta_{KL}$ equals δ_{JL}.
2. We have $\delta_{JJ} = id$ for every $J \subset I$.

We shall give an easy example for this definition. Let Γ be some geometry of rank n and $C = \{x_1, x_2, \dots x_n\}$ be a chamber in Γ. Let G be a flag-transitive group of type preserving automorphisms of Γ and set for $J \subset \{1, 2, \dots, n\} =: I$ the group $G_J := G_{\{x_i : i \in J\}}$. Then $(G_J)_{J\subset I}$ forms an amalgam with δ_{JK} being the inclusion mapping for all $J, K \subset I$ with $K \subset J$.

Definition 1.2.2 [10] A *completion* of an amalgam $\mathcal{A}$ is a group G and a familiy of homorphisms $\eta_J : G_J \to G$ for all $J \subset I$ such that:

1. $\eta_J = \delta_{JK}\eta_K$ for all $K \subset J$ and
2. $G :=< G_J\eta_J : J \subset I >$.

For two completions G and $\hat{G}$ with homomorphisms η_J and $\hat{\eta}_J$ of $\mathcal{A}$ a *morphism of completions* is a homomorphism $\psi : G \to \hat{G}$ such that $\hat{\eta}_J = \eta_J\psi$ for all $J \subset I$. A completion of $\mathcal{A}$ is called *universal* if and only if there is a unique morphism of completions from it to any given completion.

Returning to our above example of the amalgam $(G_J)_{J\subset I}$ of a flag-transitive geometry Γ, we see that G is a completion of $(G_J)_{J\subset I}$ if and only if $G =< G_1, G_2, \ldots, G_n >$ which holds if Γ is connected.

The existence of the universal completion of an amalgam is ensured by the following (see e.g. [10]):

Proposition 1.2.3 *Let $\mathcal{A}$ be an amlgam. Then $\mathcal{A}$ has a universal completion (possibly infinite), unique up to isomorphism of completions.*

□

The next proposition establishes a connection between universal covers of flag-transitive geometries and the universal completions of the related amalgams:

Proposition 1.2.4 *[26], [31] Let Γ be a geometry and let $G \leq Aut\Gamma$ be flag-transitive. Denote by $\mathcal{A}$ the amalgam of maximal parabolic subgroups associated with the action of G on Γ and by $U(\mathcal{A})$ the universal completion of $\mathcal{A}$. Then $\Gamma(U(\mathcal{A}), \mathcal{A})$ is the universal cover of $\Gamma(G, \mathcal{A})$.*

□

In the following, we describe a technique to determine the universal completion $U(\mathcal{A})$ of some amalgam $\mathcal{A} = (G_J)_{J\subset I}$ (see [27], also [14] or [25]) in terms of generators and relations.

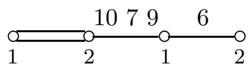
For every $i \in I$ denote by $\mathcal{X}_i$ a set of generators and by $\mathcal{R}^i_{xy}$ a set of relations between the elements of $\mathcal{X}_i$ such that $G_i \simeq< \mathcal{X}_i : \mathcal{R}^i_{xy} >$. Put moreover $\mathcal{X}_U := \bigcup_{i\in I} \mathcal{X}_i$ and $\mathcal{R}^U_{xy} := \bigcup_{i\in I} \mathcal{R}^i_{xy}$. Then we find

$$U(\mathcal{A}) \simeq< \mathcal{X}_U : \mathcal{R}^U_{xy} >.$$

Note that every relation in $\mathcal{R}^U_{xy}$ holds in at least one of the groups G_i.

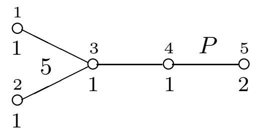
1.2.1 The known geometries for the O'Nan sporadic group

In his 1985-paper [4], Buekenhout gives the diagram of a rank four geometry Γ admitting $O'N$ as a flag-transitive automorphism group. This geometry, no. (102) in the notation of [4], can be constructed from two geometries of rank three for the groups $L_3(7) : \mathbb{Z}_2$ (no. (100) of the list in [4]) and for $\mathbb{Z}_4 L_3(4) : \mathbb{Z}_2$ (no. (101) in [4]). Note that the latter group is the centralizer of an involution in $O'N$. The Buekenhout diagram for the $O'N$-geometry is the following:



If we denote the maximal parabolic subgroups of this geometry by G_1, G_2, G_3 and G_4, from the left to the right of the diagram nodes, we have $G_1 \simeq L_3(7) : \mathbb{Z}_2$ and $G_4 \simeq \mathbb{Z}_4 L_3(4) : \mathbb{Z}_2$. Note that the involutions in $G_4 - G_4'$ are unitary. This will be sometimes indicated by writing $G_4 \simeq \mathbb{Z}_4 L_3(4) : 2_1$ following the notation of [8] or [22].

In 1986, a second geometry, now of rank five, was found by A. Ivanov and S. Shpectorov [17] admitting the group $O'N$ as a flag-transitive automorphism group. This geometry involves the Petersen graph as a residue of rank two. Its diagram is the following:



The maximal parabolics are $G_1 \simeq J_1$, $G_2 \simeq M_{11}$ and $G_5 \simeq (\mathbb{Z}_4 * Q_8 * Q_8) : A_5$ the latter group being a maximal parabolic of $\mathbb{Z}_4 L_3(4)$. This geometry also admits a 3-fold cover with automorphism group $3O'N$ such that its center acts as a deck transformation group (see e.g. [18]).

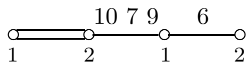
For both geometries it is not known whether they are simply connected, resp. the 3-fold cover is universal. This will be shown in this thesis using the technique to determine the universal completions of amalgams which was described above.

Chapter 2

Generators and relations for the Buekenhout geometry

2.1 The geometry of Buekenhout for the O'Nan group

In this section we give the corresponding amalgam for Buekenhout's geometry [4]. To recall, this is a geometry of rank four having the following Buekenhout diagram.



We denote the maximal parabolics of this geometry by G_1, G_2, G_3 and G_4, read from the left to the right of the diagram nodes. In [4], we see that $G_1 \simeq L_3(7) : \mathbb{Z}_2$ and $G_4 \simeq \mathbb{Z}_4 L_3(4) : \mathbb{Z}_2$. The complete amalgam of the residue Γ_4 for $G_4/K_4 \simeq L_3(4) : \mathbb{Z}_2$ (where $K_4 = Z(G_4')$ denotes the kernel of the action of G_4) is given in [11]. Since $G_{14} = G_1 \cap G_4$ is the centralizer of an inner involution of G_1, we get $G_{14} \simeq (\mathbb{Z}_4 * SL_2(7)) : \mathbb{Z}_2$ (see e.g. [8]). Futhermore, since any outer automorphism of G_4' in G_4 inverts the center of G_4', we obtain $B = G_{1234} \simeq \mathbb{Z}_2 \times D_8$. Then [11] implies $G_{34} \simeq (\mathbb{Z}_3^2 : \mathbb{Z}_4) \times D_8$, $G_{134} \simeq S_3 \times D_8$ and $G_{234} \simeq \mathbb{Z}_4 \times D_8$. In all these groups the direct factor D_8 is generated by K_4 and an automorphism of G_4. From [4] we draw that G_{12} must be the centralizer in G_1 of an outer involution, thus, $G_{12} \simeq \mathbb{Z}_2 \times PGL_2(7)$ (see e.g. [8]). Since we have $|G_{124}| = 2^5$, we have therefore $G_{124} \in Syl_2(G_{12})$, hence $G_{124} \simeq \mathbb{Z}_2 \times D_{16}$. This implies that the structure of G_{24} can be described as $G_{24} \simeq (\mathbb{Z}_4 \times D_8) : \mathbb{Z}_2$. By [4], we hold that $G_{13} \simeq S_3 \times S_4$ where the direct factor S_4 is not contained in G_1'. Now G_2 is described in [4] as a group of shape $([2] \times PGL_2(7))2$. Since $G_{12} \leq G_2 \geq G_{24}$, we can describe the structure of $G_2 \simeq (\mathbb{Z}_4 \times L_2(7)) : \mathbb{Z}_2$. Then G_2 acts on a geometry which is the direct sum of a folded projective plane of order two with an isolated node. Thus $|G_2 : G_{23}| = 14$ and $G_{23} \simeq \mathbb{Z}_4 \times S_4$. Now the geometry for G_3 is also some direct sum of a 3×3-grid and some isolated node implying $G_3 \simeq (\mathbb{Z}_3^2 : \mathbb{Z}_4) \times S_4$ because of its subgroups.

We will construct a presentation of the universal completion of the amalgam $\mathcal{A}$ of the Buekenhout geometry and establish that this completion is the group $O'N$. In order to achieve this, we derive a representation for G_1, G_2, G_3 and G_4. Since the residues for G_2 and G_3 are direct sums of geometries, we get $G_2 =< G_{12}, G_{23}, G_{24} >$ and $G_3 =< G_{13}, G_{23}, G_{34} >$. By a result of Ronan [27], the residues for G_1 and G_4 have an infinite universal cover. Therefore we have to identify the groups $L_3(7) : \mathbb{Z}_2 \simeq G_1 \leq< G_{12}, G_{13}, G_{14} >$ and $\mathbb{Z}_4 L_3(4) : \mathbb{Z}_2 \simeq G_4 \leq< G_{14}, G_{24}, G_{34} >$. This will be done by giving a canonical presentation for these groups and indentifications of the canonical generators as words in the geometric generators for G_1 and, respectively, the geometric generators as words in the canonical ones for G_4.

2.2 Generators and relations for Buekenhout's geometry

In this section we draw generators and relations from the amalgam of Buekenhout's geometry for the O'Nan group.

2.2.1 A presentation for the Borel subgroup

The Borel subgroup B of the Buekenhout geometry is isomorphic to $\mathbb{Z}_2 \times D_8$. We give a presentation of $B =< z, X, x >$ such that the generator z will become the center of the group $G_4' \simeq \mathbb{Z}_4 L_3(4)$ and X become an automorphism of that group. This implies the following relations:

$$z^4 = X^2 = 1, z^X = z^{-1}.$$

We construct a second subgroup D_8 of B and the amalgamation via

$$x^4 = 1, x^X = x^{-1}, x^2 = z^2, [x, z] = 1.$$

Then we get

$$\mathbb{Z}_2 \times D_8 \simeq B =< z, X, x \mid z^4 = X^2 = 1, x^2 = z^2, [z, x] = 1, z^X = z^{-1}, x^X = x^{-1} >,$$
$$Z(B) =< z^2, z^{-1}x >.$$

2.2.2 Presentations for the minimal parabolic subgroups

2.2.2.1 $G_{123} \simeq \mathbb{Z}_2 \times S_4$

We add a generator Z of order four. In order to produce a group of shape S_4, we set

$$Z^4 = 1, Z^{x^2} = Z^{-1}, x^{Z^2} = x^{-1}, (xZ)^3 = 1.$$

We see that $< x^2, Z^2 > \trianglelefteq < x, Z >$, $< x^2, Z^2, xZ > \simeq A_4$ and $(xZ)^{x^2 Z^{-1}} = ZxZ^2 = Z^{-1}x^{-1}$. To generate $\mathbb{Z}_2 \times S_4$ with center $< z^{-1}x >$, we generate the second S_4 via

$$(z^2)^{xZ} = z^{-1}X, [Z, z^{-1}x] = 1.$$

Then the first relation implies $z^2Z^2 = z^{-1}X$, hence an indentification $Z^2 = zX$. The second relation implies $Z^{z^{-1}} = Z^{x^{-1}}$ and from both relations we hold $(xZ)^X = (xZ)^{z^{-1}Z^2} = x^2Zx = Z^{-1}x^{-1}$. Therefore we get a presentation of G_{123} as follows:

$$\mathbb{Z}_2 \times S_4 \simeq G_{123} =< z, X, x, Z \mid z^4 = X^2 = Z^4 = 1, x^2 = z^2, [z, x] = 1, z^X = z^{-1}, x^X = x^{-1}, Z^{x^2} = Z^{-1}, x^{Z^2} = x^{-1}, (xZ)^3 = 1, (z^2)^{xZ} = z^{-1}X, [Z, z^{-1}x] = 1 >$$

2.2.2.2 $G_{124} \simeq \mathbb{Z}_2 \times D_{16}$

We add a generator Y which also will become an automorphism of G_4' such that $Z(G_{124}) = Z(B)$. Therefore we generate a subgroup D_{16} as follows:

$$(XY)^8 = Y^2 = 1.$$

Then we identify B inside G_{124} and generate a second $\mathbb{Z}_2 \times D_8$ by

$$(XY)^2 = x, z^Y = z^{-1}.$$

Thereby we hold a presentation of G_{124} via

$$\mathbb{Z}_2 \times D_{16} \simeq G_{124} =< z, X, Y, x \mid z^4 = X^2 = Y^2 = 1, x^2 = z^2, [z, x] = 1, z^X = z^{-1}, x^X = x^{-1}, (XY)^2 = x, z^Y = z^{-1} >, \ Z(G_{124}) = Z(B).$$

2.2.2.3 $G_{134} \simeq S_3 \times D_8$

We add a generator ρ of order three. Furthermore the amalgam implies that $< X, z > \simeq D_8$ is the (unique) direct factor of G_{134} of shape D_8, thus

$$\rho^3 = 1, [\rho, z] = [\rho, X] = 1.$$

Therefore we set

$$\rho^x = \rho^{-1}$$

and get a presentation of G_{134}:

$$S_3 \times D_8 \simeq G_{134} =< z, X, x, \rho \mid z^4 = X^2 = \rho^3 = 1, x^2 = z^2, [z, x] = 1, z^X = z^{-1}, x^X = x^{-1}, [\rho, z] = [\rho, X] = 1, \rho^x = \rho^{-1} >.$$

2.2.2.4 $G_{234} \simeq \mathbb{Z}_4 \times D_8$

Again, $< X, z >$ is a direct factor D_8 of G_{234} and the square of the new generator a is in $Z(B) - \{z^2\}$. Thus

$$a^2 = z^{-1}x, [a,z] = [a,X] = [a,x] = 1$$

which gives the following presentation for G_{234}:

$$\mathbb{Z}_4 \times D_8 \simeq G_{234} =< z, X, x, a \mid z^4 = X^2 = 1, x^2 = z^2, [z,x] = 1, z^X = z^{-1}, x^X = x^{-1}, a^2 = z^{-1}x, [a,z] = [a,X] = [a,x] = 1 >.$$

2.2.3 Presentations for the groups generated by two minimal parabolics

2.2.3.1 $< G_{123}, G_{124} >\simeq \mathbb{Z}_2 \times PGL_2(7)$

The group $< G_{123}, G_{124} >$ acts on the folded projective plane of order two. Thus we add the Weyl relation as

$$(ZZ^Y)^3 = 1$$

and get a presentation as:

$$\mathbb{Z}_2 \times PGL_2(7) \simeq< G_{123}, G_{124} >=< z, X, x, Z, Y \mid z^4 = X^2 = Z^4 = Y^2 = 1, x^2 = z^2, [z,x] = 1, z^X = z^{-1}, x^X = x^{-1}, Z^{x^2} = Z^{-1}, x^{Z^2} = x^{-1}, (xZ)^3 = 1, (z^2)^{xZ} = z^{-1}X, [Z, z^{-1}x] = 1, (XY)^2 = x, z^Y = z^{-1}, (ZZ^Y)^3 = 1 >.$$

2.2.3.2 $< G_{123}, G_{134} >\simeq S_3 \times S_4$

The direct factor of a group $S_3 \times S_4$ of shape S_4 is uniquely determined. We proceed as follows. The group $< G_{123}, G_{134} >$ contains $G_{134} \simeq S_3 \times D_8$ with the uniquely determined direct factor $< z, X >\simeq D_8$. Thus we choose $< z, X, xZ >\simeq S_4$ as direct factor. This implies

$$[\rho, xZ] = 1$$

and yields the following presentation:

$$S_3 \times S_4 \simeq< G_{123}, G_{134} >=< z, X, x, Z, \rho \mid z^4 = X^2 = Z^4 = \rho^3 = 1, x^2 = z^2, [z,x] = 1, z^X = z^{-1}, x^X = x^{-1}, Z^{x^2} = Z^{-1}, x^{Z^2} = x^{-1}, (xZ)^3 = 1, (z^2)^{xZ} = z^{-1}X, [Z, z^{-1}x] = 1, [\rho, z] = [\rho, X] = 1, \rho^x = \rho^{-1}, [\rho, xZ] = 1 >.$$

Therefore the involution X is not contained in G_1' because it is contained in the direct factor S_4 but not in its commutator subgroup.

2.2.3.3 $< G_{123}, G_{234} >\simeq \mathbb{Z}_4 \times S_4$

We construct $< G_{123}, G_{234} >$ such that a is contained in its center. Therefore we add

$$[a, Z] = 1$$

and hold:

$$\mathbb{Z}_4 \times S_4 \simeq< G_{123}, G_{234} >=< z, X, x, Z, a \mid z^4 = X^2 = Z^4 = a^4 = 1, x^2 = z^2, [z, x] = 1, z^X = z^{-1}, x^X = x^{-1}, Z^{x^2} = Z^{-1}, x^{Z^2} = x^{-1}, (xZ)^3 = 1, (z^2)^{xZ} = z^{-1}X, [Z, z^{-1}x] = 1, a^2 = z^{-1}x, [a, z] = [a, X] = [a, x] = 1, [a, Z] = 1 >.$$

2.2.3.4 $< G_{124}, G_{134} >\simeq (\mathbb{Z}_4 * SL_2(7)) : \mathbb{Z}_2$

The group $< G_{124}, G_{134} >$ acts on the Coxeter graph for $PGL_2(7)$. This graph is defined as follows: The vertices are the anti-flags of the projective plane $PG(2, 2)$ and two anti-flags are called adjacent if the union of their point sets covers the plane. Therefore it is clear that the stabilizer of an anti-flag and the stabilizer of an edge generate a Singer cycle of the plane. Thus we add

$$(\rho z^{-1}XY)^7 = 1$$

yielding the presentation

$$(\mathbb{Z}_4 * SL_2(7)) : \mathbb{Z}_2 \simeq< G_{124}, G_{134} >=< z, X, x, Y, \rho \mid z^4 = X^2 = Y^2 = \rho^3 = 1, x^2 = z^2, [z, x] = 1, z^X = z^{-1}, x^X = x^{-1}, (XY)^2 = x, z^Y = z^{-1}, \rho^3 = 1, [\rho, z] = [\rho, X] = 1, (\rho z^{-1}XY)^7 = 1 >.$$

2.2.3.5 $< G_{124}, G_{234} >\simeq (\mathbb{Z}_4 \times D_8) : \mathbb{Z}_2$

We have to add a relation such that Y also acts on $Z(G_{234})$, hence

$$a^Y = a^{-1}.$$

This yields a presentation

$$(\mathbb{Z}_4 \times D_8) : \mathbb{Z}_2 \simeq< G_{124}, G_{234} >=< z, X, Y, x, a \mid z^4 = X^2 = 1, x^2 = z^2, [z, x] = 1, z^X = z^{-1}, x^X = x^{-1}, (XY)^2 = x, z^Y = z^{-1}, a^2 = z^{-1}x, [a, z] = [a, X] = [a, x] = 1, a^Y = a^{-1} >.$$

2.2.3.6 $< G_{134}, G_{234} >\simeq (\mathbb{Z}_3^2 : \mathbb{Z}_4) \times D_8$

For this group it is easy to see that we have to add

$$[\rho, \rho^a] = 1$$

to get a presentaion as

$$(\mathbb{Z}_3^2 : \mathbb{Z}_4) \times D_8 \simeq < G_{134}, G_{234} >=< z, X, x, \rho \mid z^4 = X^2 = \rho^3 = 1, x^2 = z^2, [z, x] = 1, z^X = z^{-1}, x^X = x^{-1}, [\rho, z] = [\rho, X] = 1, \rho^x = \rho^{-1}, a^2 = z^{-1}x, [a, z] = [a, X] = [a, x] = 1, [\rho, \rho^a] = 1 >.$$

2.3 Canonical generators and relations for $L_3(7) : 2$

In this section we give a presentation of $L_3(7) : \mathbb{Z}_2$ using the projective plane $PG(2, 7)$. We will call these generators and relations *canonical*.

2.3.1 A presentation for a maximal parabolic subgroup for $PG(2, 7)$

It is well known that the stabilizer of a point or a line of $PG(2, 7)$ in $L_3(7)$ is a maximal subgroup of shape $\mathbb{Z}_7^2 : (SL_2(7) : \mathbb{Z}_2)$ (see e.g. [8]). We will construct a presentation of such a group.

We set $\nu := \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$, $\rho := \begin{pmatrix} 2 & 0 \\ 0 & -3 \end{pmatrix}$ and $x := \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ as matrices over $GF(7)$. Then we have $< \nu, \rho, x >\simeq SL_2(7)$ with $x^2 \in Z(< \nu, \rho, x >)$.

Lemma 2.3.1 $G :=< \nu, \rho, x \mid \nu^7 = \rho^3 = x^4 = 1, \nu^\rho = \nu^4, \rho^x = \rho^{-1}, (\nu x)^3 = x^2, [\nu, x^2] = 1 >\simeq SL_2(7)$.

Proof. We set $B :=< \nu, \rho, x^2 >\simeq \mathbb{Z}_2 \times (\mathbb{Z}_7 : \mathbb{Z}_3)$ and $N :=< x >\simeq \mathbb{Z}_4$ and show that these groups provide a rank one BN-pair for G. Therefore it remains to show that $G = BNB$. Since $x^2 \in B$, we need to prove that $G = B \cup BxB$. The claim follows if $B \cup BxB$ is a subgroup of G because $x, \nu, \rho \in B \cup BxB$. Thus we need to show that for any $a, b \in BxB$ we have $ab^{-1} \in BxB$. Let $a = b_1xb_2$, $b = b_3xb_4$. Then $ab^{-1} = b_1xb_2b_4^{-1}x^{-1}b_3^{-1} = b_1xb_2'xb_3'$ with $b_2' = b_2b_4^{-1}$ and $b_3' = x^2b_3^{-1}$. Now x normalizes $< \rho, x^2 >$, thus we only need to prove $x\nu^ix \in BxB \cup B$ for any power of ν. Since $\nu^\rho = \nu^4$ and $x\nu^{-1}x = (x\nu x)^{-1}$, we only have to verify $x\nu x \in BxB$. But we have $(\nu x)^3 = x^2$, hence $x\nu x = \nu^{-1}x\nu^{-1}$ and the lemma is proved. □

Clearly, an involutory automorphism of our (matrix) group $SL_2(7)$ is $i := \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$. This provides a presentation of $SL_2(7) : \mathbb{Z}_2$ as

$$SL_2(7):\mathbb{Z}_2 \simeq <\nu,\rho,x,i \mid \nu^7=\rho^3=x^4=i^2=1, \nu^\rho=\nu^4, \rho^x=\rho^{-1}, (\nu x)^3 = x^2, [\nu,x^2]=1, [\rho,i]=1, x^i=x^{-1}, \nu^i=\nu^{-1}>.$$

We add a natural module $\mathbb{Z}_7^2 \simeq <v_1,v_2>$ to this group via the identification $v_1 \sim (1,0)$ and $v_2 \sim (0,1)$. Thus

$$\mathbb{Z}_7^2:(SL_2(7):\mathbb{Z}_2) \simeq <\nu,\rho,x,i,v_1,v_2 \mid \nu^7=\rho^3=x^4=i^2=v_1^7=v_2^7=1, \nu^\rho=\nu^4, \rho^x = \rho^{-1}, (\nu x)^3=x^2, [\nu,x^2]=1, [\rho,i]=1, x^i=x^{-1}, \nu^i=\nu^{-1}, [v_1,v_2]=[v_1,\nu]=1, v_2^\nu = v_1v_2, v_1^\rho=v_1^2, v_2^\rho=v_2^4, v_1^x=v_2, v_2^x=v_1^{-1}, [v_1,i]=1, v_2^i=v_2^{-1}>.$$

2.3.2 Adding the graph automorphism

We add a graph automorphism u to the presentation of $\mathbb{Z}_7^2:(SL_2(7):\mathbb{Z}_2)$ obtained in the previous subsection such that $<\nu,\rho,x,i,v_1,v_2> \cap <\nu,\rho,x,i,v_1,v_2>^u$ is a Borel group of $L_3(7)$. Thus we construct u such that u normalizes $\mathbb{Z}_7^{1+2} \simeq <v_1,v_2,\nu>$ and maps $<v_1,v_2>$ to $<v_1,\nu>$. Moreover we can choose u to fulfill $[u,\rho]=1$ and $(x^2)^u = x^2 i$. Then it remains to add the Weyl relation, e. g. $(xx^u)^3=1$. Thus we get

$$L_3(7):\mathbb{Z}_2 \simeq <\nu,\rho,x,i,v_1,v_2,u \mid \nu^7=\rho^3=x^4=i^2=v_1^7=v_2^7=u^2=1, \nu^\rho=\nu^4, \rho^x = \rho^{-1}, (\nu x)^3=x^2, [\nu,x^2]=1, [\rho,i]=1, x^i=x^{-1}, \nu^i=\nu^{-1}, [v_1,v_2]=[v_1,\nu]=1, v_2^\nu = v_1v_2, v_1^\rho=v_1^2, v_2^\rho=v_2^4, v_1^x=v_2, v_2^x=v_1^{-1}, [v_1,i]=1, v_2^i=v_2^{-1}, v_1^u=v_1^{-1}, v_2^u=\nu, \nu^u = v_2, [\rho,u]=1, (x^2)^u=x^2i, (xx^u)^3=1>.$$

2.3.3 3×3-matrices for the canonical generators

In this subsection we identify our canonical generators with the corresponding 3×3-matrices in $L_3(7)$. These matrices will be used to identify the geometric generators as words in the canonical ones.

Our construction from the above subsections suggests the following identification: $\nu = \begin{pmatrix} 1&0&0\\ 0&1&0\\ 0&1&1 \end{pmatrix}$, $\rho = \begin{pmatrix} 1&0&0\\ 0&2&0\\ 0&0&-3 \end{pmatrix}$, $x = \begin{pmatrix} 1&0&0\\ 0&0&1\\ 0&-1&0 \end{pmatrix}$ and $i = \begin{pmatrix} -1&0&0\\ 0&1&0\\ 0&0&-1 \end{pmatrix}$.

Set $v_1 := (a_{ij})$. We construct v_1 and v_2 such that $<v_1,v_2,\nu,\rho,x,i>$ fixes the vector $(1,0,0)$. Thus $a_{12}=a_{13}=0$. Using the relations $[\nu,v_1]=[i,v_1]=1$, we hold $v_1 = \begin{pmatrix} a_{11}&0&0\\ 0&a_{22}&0\\ a_{31}&0&a_{33} \end{pmatrix}$. Proceeding with $v_1\rho = \rho v_1^2$, we get $a_{ii}=a_{ii}^2$, hence $a_{ii}=1$ (for $i=1,2,3$), and a_{31} to be some non-zero element in $GF(7)$. Therefore we set $v_1 = \begin{pmatrix} 1&0&0\\ 0&1&0\\ 1&0&1 \end{pmatrix}$. By $v_1^x = v_2$, this implies $v_2 = \begin{pmatrix} 1&0&0\\ -1&1&0\\ 0&0&1 \end{pmatrix}$.

2.4 Identification of the geometric generators

In this section we identify the geometric generators for $L_3(7):\mathbb{Z}_2$ as words in the canonical generators. For this purpose we use the relations as given in 2.3.2 and Section 2.2 as well as the matrices obtained in the previous subsection. Since u cannot be expressed as a 3×3-matrix we also use $(x^2)^u = x^2 i = \begin{pmatrix} 0&1&0\\ -1&0&0\\ 0&0&1 \end{pmatrix}$ and $\rho^u = 2\rho = \begin{pmatrix} 2&0&0\\ 0&-3&0\\ 0&0&1 \end{pmatrix}$ since we compute inside $SL_3(7)$ using the matrices.

Set $w_1 := v_1^{-1}$, $w_2 := \nu$ and $\omega := v_2$. Then x^u acts in the same way on $< w_1, w_2, \omega >$ as x on $< v_1, v_2, \nu >$. Therefore x^u has to fulfill $w_1^{x^u} = w_2$, $w_2^{x^u} = w_1^{-1}$, $(\rho^u)^{x^u} = (\rho^u)^{-1}$ and $(x^2)^u = x^2 i$. Set $x^u = (a_{ij})$. Then we use these relations and compute $x^u = \begin{pmatrix} 0&-1&0\\ 1&0&0\\ 0&0&1 \end{pmatrix}$.

2.4.1 The generators z, x and ρ

We construct z such that $z^2 = t := x^2$ because $t \in Z(< \nu,\rho,x,i >)$ and the group $M :=< z, X, Y, \rho >$ will be the centralizer of z^2 in $L_3(7):\mathbb{Z}_2$. Set $z' := ut$. Then $(z')^2 = utut = i$ and $o(z') = 4$. Moreover $x^i = x^{-1}$ leads to $i^x = ti$ and therefore we set $z := (z')^{xu} = (ut)^{xu}$. This implies $M =< \nu,\rho,x,i,z >$. We set the *geometrical* generator ρ to be the *canonical* ρ because, using $[u,\rho] = 1$ and $\rho^x = \rho^{-1}$, we find $[z,\rho] = 1$, also we use the canonical x as the geometric generator x .

2.4.2 The generator X

Set $L :=< v_1, v_2, \nu, \rho, x, i, u >' \simeq L_3(7)$. Then we get $M \cap L =< \nu,\rho,x,i >$ and $C_{M\cap L}(\rho) =< \rho,t,i >$. Therefore we have $C_M(\rho) =< \rho,z,i > \simeq \mathbb{Z}_3 \times D_8$. Furthermore we must find $X \in < \rho,z,i >$ since $[\rho,X] = 1$. By 2.2.3.2, $X \notin L$. Thus we set $X := zi$ since $z^i = z^{-1}$ and $z^X = z^{-1}$ has to hold.

2.4.3 The generator Y

We construct $Y \in M \cap L$ such that $x = (XY)^2$. Since $X \notin L$, we must find $XY \notin L$. We proceed as follows. From the relations between the geometric generators we draw $[XY, z] = 1$. Thus we construct some element $e \in M' \simeq SL_2(7)$ such that $e^2 = x^{-1}$. Then $ez \notin L$ and $(ez)^2 = x^{-1}x^2 = x$. We compute e inside $L_2(7) \simeq < \nu,\rho,x \mid \nu^7 = \rho^3 = x^2 = 1, \rho^x = \rho^{-1}, \nu^\rho = \nu^4, (\nu x)^3 = 1 >$. Hence we can identify $\nu = (1234567)$, $\rho = (253)(467)$ and $x = (23)(47)$. Then $\nu^3 = (1473625)$, $x^{\nu^3} = (37)(56)$ and $xx^{\nu^3} =$

(2743)(56). Furthermore $(xx^{\nu^3})^2 = (24)(37)$, $x^\nu = (34)(51)$ and $((xx^{\nu^3})^2)^{x^\nu} = x$. Therefore we set $e := (xx^{\nu^3})^{x^\nu}$. Using our 3×3-matrices, we get $e = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -2 & 2 \\ 0 & -2 & -2 \end{pmatrix}$ with $e^2 = x^{-1}$. Thus our 'candidates' for Y are $Y_j = zi(ez)^j = tie^j$, $j = 1, 5$ and $t = z^2$. For $j = 5$ we get $Y_5 = ie$. Using our matrices, we get $Y_1 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & 2 & -2 \\ 0 & -2 & -2 \end{pmatrix}$ and $Y_5 = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -2 & 2 \\ 0 & 2 & 2 \end{pmatrix}$.

Now Y must fulfill the relation $(\rho z^{-1}XY)^7 = 1$. With $z^{-1}X = i$ we compute $\rho iY_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -3 & 3 \\ 0 & 1 & 1 \end{pmatrix}$ and $(\rho iY_1)^7 = t$ since $\begin{pmatrix} -3 & 3 \\ 1 & 1 \end{pmatrix}^7 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$. Thus for Y_5 we get $\rho iY_5 = t\rho iY_1$. Therefore we set $Y = ie = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -2 & 2 \\ 0 & 2 & 2 \end{pmatrix}$.

2.4.4 The generator Z

We set $H :=< z, X, Y, Z >$. Then $H \cap L$ must be isomorphic to $PGL_2(7)$. Since $Z(H) =< z^{-1}x >$ has to hold, we compute the centralizer of $z^{-1}x$ in L. Since $x^Y = x^{-1}$, we have $D_{16} \simeq < Y, i, x > \leq H \cap L$. Using the relations between the canonical generators, we find $z^{-1}x = (ux)^3$. Then we get $[x^u, (ux)^3] = (xu)^2 t(ux)^2 = xtuiiux = 1$. Thus $H \cap L =< Y, i, x, x^u >$. Furthermore the relations imply $[u, (ux)^3] = (xu)^2 t(ux)^2 = 1$ so $Y^u \in H \cap L$. We set $Z' := (iY^u)^2 = (e^u)^2$. Again, we use our relations between the canonical generators and get $x^\nu u = uv_2^{-1}x^u v_2$ and $\nu^3 u = uv_2^3$, thus $e^u = (xx^{\nu^3})^{x^\nu u} = (x^u x^{uv_2^3})^{v_2^{-1}x^u v_2}$. Using the matrices, we get $e^u = \begin{pmatrix} -1 & -1 & 0 \\ 2 & -2 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ and $Z' = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix} = (x^{-1})^u$.

Then we compute (using the matrices) $o(Z'(Z')^Y) = 3$. Since $x^i = x^{-1}$, we obtain $(Z')^{x^2} = ((x^{-1})^u)^t = x^u = (Z')^{-1}$. Furthermore $xZ' = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}$, thus $o(xZ') = 3$. Moreover $x^{(Z')^2} = x^{ti} = x^i = x^{-1}$ and $(Z')^2 = (x^2)^u = ti = z^2 i = zX$ since $X = zi$. Then we compute, using the matrices, $[\rho, xZ] = \begin{pmatrix} -3 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & -3 \end{pmatrix}$, so $[\rho, xZ] = 1$ in L. Therefore we set $Z := Z'$.

2.5 Identification of the canonical generators

In this section we identify our canonical generators for $L_3(7) : \mathbb{Z}_2$ as words in the geometric ones. For this purpose, we use the matrices obtained in the previous section.

2.5.1 The generators ρ, x and i

Since the geometric ρ and the canonical ρ coincide, we keep the geometric generator ρ. The previous section implies to set $x = (XY)^2$ and $i = z^{-1}X$.

2.5.2 The generator ν

Using the matrices of the previous section, we obtain $((\rho iY)^2)^{i(\rho^{-1})^Y} = \begin{pmatrix} 1\,0\,0 \\ 0\,1\,0 \\ 0\,1\,1 \end{pmatrix}$. Thus we get $\nu = ((\rho z^{-1}XY)^2)^{z^{-1}X(\rho^{-1})^Y}$ since $i = z^{-1}X$.

2.5.3 The generator u

We have the relation $z = (ut)^{xu}$. Thus we get $z = ux^{-1}utxu = uxuiux = x^u ixu = x^u x^{-1}ui$. Therefore we obtain $X = zi = x^u x^{-1}u$. Since $x^u = Z^{-1}$, we find $X = Z^{-1}x^{-1}u = (zZ)^2u$ or $u = xZX = (XY)^2ZX$.

2.5.4 The generators v_1 and v_2

Because of our canonical relations, we set $v_2 = \nu^u$ and $v_1 = v_2^{-1}v_2^{\nu}$.

2.5.5 The presentation for $L_3(7) : \mathbb{Z}_2$

Putting together the geometric relations, the identification of the canonical generators and the canonical relations, we get the following presentation of $G_1 \simeq L_3(7) : \mathbb{Z}_2$ of the amalgam of the Buekenhout geometry:

$$L_3(7) : \mathbb{Z}_2 \simeq G_1 =< z, X, Y, Z, \rho, x, v_1, v_2, \nu, u, i \mid z^4 = X^2 = Y^2 = Z^4 = \rho^3 = 1, x^2 = z^2, z^X = z^{-1}, x^X = x^{-1}, [x, z] = [z^-1x, Z] = 1, x^{Z^2} = x^{-1}, Z^{x^2} = Z^{-1}, (xZ)^3 = 1, (z^2)^{xZ} = z^{-1}X, x = (XY)^2, z^Y = z^{-1}, [z, \rho] = [X, \rho] = 1, \rho^x = \rho^{-1}, (ZZ^Y)^3 = 1, [\rho, xZ] = 1, (\rho z^{-1}XY)^7 = 1, i = z^{-1}X, \nu = ((\rho z^{-1}XY)^2)^{z^{-1}X(\rho^{-1})^Y}, u = xZX, v_2 = \nu^u, v_1 = v_2^- 1v_2^{\nu}, \nu^{\rho} = \nu^4, [\rho, i] = 1, (\nu x)^3 = x^2, \nu^i = nu^{-1}, [\nu, x^2] = 1, x^i = x^{-1}, v_1^u = v_1^{-1}, v_2^u = \nu, \nu^u = v_2, [\rho, u] = 1, (x^2)^u = x^2 i, (xx^u)^3 = 1 >.$$

2.6 Canonical generators and relations for $L_3(4):2_1$

In this section we construct a presentation of $L_3(4)$ extended by a unitary polarity using the projective plane of order four. Again, we will call these generators and relations the *canonical* ones.

2.6.1 A presentation for a maximal parabolic subgroup for $PG(2,4)$

We give a presentation of a point or line stabilizer of $PG(2,4)$ in $L_3(4)$. This is a group of shape $\mathbb{Z}_2^4 : A_5$ where its O_2 is the natural $L_2(4)$-module.

We set $\tilde{a} := \begin{pmatrix} 1 & 0 \\ \xi & 1 \end{pmatrix}$, $\rho_n := \begin{pmatrix} \xi^2 & 0 \\ 0 & \xi \end{pmatrix}$ and $\tau := \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ as matrices over $GF(4)$ where $<\xi>:= GF(4)^*$. Then clearly, $A_5 \simeq L_2(4) \simeq <\tilde{a}, \rho_n, \tau>$. This leads to the following presentation:

$$A_5 \simeq <\tilde{a}, \rho_n, \tau \mid \tilde{a}^2 = \rho_n^3 = \tau^2 = 1, (\rho_n\tilde{a})^3 = 1, \rho_n^\tau = \rho_n^{-1}, (\tau\rho_n\tilde{a})^5 = 1>$$

where $A_4 \simeq <\tilde{a}, \rho_n>$ (see e.g. [9]), $S_3 \simeq <\rho_n, \tau>$ and $(\tau\rho_n\tilde{a})^5 = 1$ is a $(2,3,5)$-relation.

We add a natural module $\mathbb{Z}_2^4$ by the following. We identify involutions f_1, f_2, f_3 and f_5 via $f_1 \sim (1,0)$, $f_2 \sim (0,1)$, $f_3 \sim (\xi, 0)$ and $f_5 \sim (0, \xi)$. Using this identification, we get a presentation of a maximal parabolic subgroup of $L_3(4)$ as

$$\begin{gathered}\mathbb{Z}_2^4 : A_5 \simeq <\tilde{a}, \rho_n, \tau, f_1, f_2, f_3, f_5 \mid \tilde{a}^2 = \rho_n^3 = \tau^2 = f_1^2 = f_2^2 = f_3^2 = f_5^2 = 1(\rho_n\tilde{a})^3 = \\ 1, \rho_n^\tau = \rho_n^{-1}, (\tau\rho_n\tilde{a})^5 = 1[f_1,f_2] = [f_1,f_3] = [f_1,f_5] = 1, [f_2,f_3] = [f_2,f_5] = 1, [f_3,f_5] = \\ 1, f_1^{\rho_n} = f_1f_3, f_2^{\rho_n} = f_5, f_3^{\rho_n} = f_1, f_5^{\rho_n} = f_5f_2, f_1^\tau = f_2, f_3^\tau = f_5, [f_1,\tilde{a}] = [f_3,\tilde{a}] = \\ 1, f_2^{\tilde{a}} = f_2f_3, f_5^{\tilde{a}} = f_1f_3f_5>.\end{gathered}$$

2.6.2 Adding the unitary polarity

We have to add a unitary polarity β. This is a polarity of $PG(2,4)$ such that every point is incident with its polar. Group theoretically β is the product of the field automorphism of $GF(4)$ with a graph automorphism of $PG(2,4)$. Then we have $C_{L_3(4)}(\beta) \simeq \mathbb{Z}_3^2 : Q \simeq U_3(2)$. Thus, if $S \in Syl_2(L_3(4))$ is normalized by β, then β acts non-trivially on $Z(S)$ and therefore β does not centralize an element of order three in $N_{L_3(4)}(S)$. Setting $N :=< N_{L_3(4)}(S), \beta>$, thus $N/S \simeq S_3$ and S has a complement in N. Thus there exists an element d of order three in N with $d^\beta = d^{-1}$.

We construct β such that β nomalizes $B :=< f_1, f_2, f_3, f_5, \tilde{a}, \rho_n>$. We set $f_4 := \tilde{a}\tilde{a}^{\rho_n}$. Then $S =< f_1, f_2, f_3, f_4, f_5, \tilde{a}>= O_2(B) \in Syl_2(L_3(4))$ with $Z(S) =< f_1, f_3>$. We set $\rho_n^\beta := \rho_n^{-1}$. Furthermore we define the remaining β-relations such that they respect the other relations, i.e., if $\mathcal{R}(x,y)$ holds for $x, y \in \{f_1, f_2, f_3, f_5, \tilde{a}, \rho_n, \tau\}$, then also $\mathcal{R}(x^\beta, y^\beta)$ is true. We identify our generators in the following way with 3×3-matrices:

$\tilde{a} = \begin{pmatrix} 1&0&0 \\ 0&1&0 \\ 0&\xi&1 \end{pmatrix}$, $\rho_n = \begin{pmatrix} 1&0&0 \\ 0&\xi^2&0 \\ 0&0&\xi \end{pmatrix}$, $\tau = \begin{pmatrix} 1&0&0 \\ 0&0&1 \\ 0&1&0 \end{pmatrix}$. Now $< B, \tau >$ should fix the vector $(1,0,0)$. Then our relations suggest the following: $f_1 = \begin{pmatrix} 1&0&0 \\ 0&1&0 \\ 1&0&1 \end{pmatrix}$, $f_3 = \begin{pmatrix} 1&0&0 \\ 0&1&0 \\ \xi&0&1 \end{pmatrix}$, $f_2 = \begin{pmatrix} 1&0&0 \\ 1&1&0 \\ 0&0&1 \end{pmatrix}$ and $f_5 = \begin{pmatrix} 1&0&0 \\ \xi&1&0 \\ 0&0&1 \end{pmatrix}$.

Since β centralizes an element in $< f_1, f_3 >$, we set w.l.o.g. $[f_1, \beta] = 1$ and therefore $f_3^\beta = f_1^{\rho_n^{-1}\beta} = f_1^{\beta\rho_n} = f_1 f_3$. Now $f_2^\beta \in < f_1, f_3, f_4, \tilde{a} > - < f_1, f_3 >$. Furthermore we want $(f_2^\beta)^{\tilde{a}^\beta} = f_2^\beta f_3^\beta = f_2^\beta f_1 f_3$ to hold. Since $Z(< f_1, f_2, f_3, f_4, f_5, \tilde{a} >) = < f_1, f_3 >$, we only need to check this relation for $f_2^\beta \in < f_4, \tilde{a} >$. Assume that $f_2^\beta = \tilde{a}$. Then $\tilde{a}^\beta = f_2$ and $f_2^\beta (f_2^\beta)^{\tilde{a}^\beta} = \tilde{a}\tilde{a}^{f_2} = f_3 \neq f_1 f_3$. Assume $f_2^\beta = f_4\tilde{a}$. Then $f_5^\beta = f_2^{\beta\rho_n^{-1}} = \tilde{a}$, thus $\tilde{a}^\beta = f_5$. This yields $f_2^\beta (f_2^\beta)^{\tilde{a}^\beta} = f_4\tilde{a}(f_4\tilde{a})^{f_5} = f_1 \neq f_1 f_3$. Hence we get $f_2^\beta \in f_4 < f_1, f_3 >$ and we set w.l.o.g. $f_2^\beta := f_3 f_4$. Thereby $f_4^\beta = f_1 f_2 f_3$, $f_5^\beta = f_2^{\beta\rho_n^{-1}} = (f_3 f_4)^{\rho_n^{-1}} = f_1 f_3 f_4 \tilde{a}$. This gives $\tilde{a}^\beta = f_1 f_1 f_3 f_1 f_2 f_3 f_5 = f_1 f_2 f_5$. Adding the Weyl relation $(\tau\tau^\beta)^3 = 1$ leads to the following presentation:

$$L_3(4):2_1 \simeq < \tilde{a}, \rho_n, \tau, f_1, f_2, f_3, f_4, f_5, \beta \mid \tilde{a}^2 = \rho_n^3 = \tau^2 = f_1^2 = f_2^2 = f_3^2 = f_5^2 = \beta^2 = 1, (\rho_n\tilde{a})^3 = 1, \rho_n^\tau = \rho_n^{-1}, (\tau\rho_n\tilde{a})^5 = 1, [f_1, f_2] = [f_1, f_3] = [f_1, f_5] = 1, [f_2, f_3] = [f_2, f_5] = 1, [f_3, f_5] = 1, f_1^{\rho_n} = f_1 f_3, f_2^{\rho_n} = f_5, f_3^{\rho_n} = f_1, f_5^{\rho_n} = f_5 f_2, f_1^\tau = f_2, f_3^\tau = f_5, [f_1, \tilde{a}] = [f_3, \tilde{a}] = 1, f_2^{\tilde{a}} = f_2 f_3, f_5^{\tilde{a}} = f_1 f_3 f_5, f_4 = \tilde{a}\tilde{a}^{\rho_n}, [f_1, \beta] = 1, f_3^\beta = f_1 f_3, \rho_n^\beta = \rho_n^{-1}, \tilde{a}^\beta = f_5 f_2 f_1, f_2^\beta = f_3 f_4, f_5^\beta = \tilde{a} f_1 f_3 f_4, (\tau\tau^\beta)^3 = 1 >,$$

where $P_1 := < f_1, f_2, f_3, f_5, \tilde{a}, \rho_n, \tau > \simeq \mathbb{Z}_2^4 : A_5$, $P_2 := P_1^\beta$ with $B := P_1 \cap P_2 \simeq \mathbb{Z}_2^{2+4} : \mathbb{Z}_3$, generate the commutator subgroup $L_3(4)$ because of the Weyl relation and β is a unitary polarity by construction, so $< P_1, \beta > \simeq L_3(4) : 2_1$.

2.7 Generators and relations for $\mathbb{Z}_4 L_3(4) : 2_1$

In this section we add a central generator z of order four to obtain a presentation of $\mathbb{Z}_4 L_3(4) : \mathbb{Z}_2$ such that its factor group modulo $< z >$ satisfies the relations of the previous section and we keep $< \tau, \rho_n, \tilde{a} > \simeq A_5$ as a complement of $O_2(P_1)$ in the maximal parabolic P_1. First we prove the following lemma:

Lemma 2.7.1 *Let $G \simeq \mathbb{Z}_4 L_3(4)$, $S \in Syl_2(G)$, $\bar{S} := S/Z(G)$. Then the preimage of $Z(\bar{S})$ is abelian.*

Proof. If P is a maximal parabolic of G, then $O_2(P) \simeq \mathbb{Z}_4 * Q_8 * Q_8$. Hence, if we assume that the preimage C of $Z(\bar{S})$ is non-abelian, we have that $C \simeq \mathbb{Z}_4 * Q_8$.

Let $< z >:= Z(G)$, $C =:< z, a, b >$ with $< a, b >\simeq Q_8$. Then we can choose involutions $i, j, k, l \in C_S(< a, b >)$ with $< i, j >\simeq D_8 \simeq< k, l >$, i.e. $< z, a, b, i, j >$ and $< z, a, b, k, l >$ are the preimages of the two elementary abelian subgroups of order 16 in $\bar{S}$. W.l.o.g., we have $i^k = iaz$ and $i^l = ibz^{\nu}$ where $\nu \in \{-1, 1\}$. Thus $i^{kl} = (iaz)^l = i^l az = ibaz^{\nu+1}$. In both cases we hold $(ibaz^{\nu+1})^2 = (ba)^2 \neq 1$, a contradiction. $\square$

To construct a presentation of $\mathbb{Z}_4L_3(4) : 2_1$, we start by setting $z^4 = 1$, $[z, r] = 1$ for $r \in \{f_1, f_2, f_3, f_5, \tilde{a}, \tau, \rho_n\}$, $z^{\beta} = z^{-1}$ and $f_i^2 = z^2$ for $i \in \{1, 2, 3, 5\}$. The above lemma yields $[f_1, f_3] = 1$ and, since $< z, f_1, f_3 >^{\tau} / < z >=< z, f_2, f_5 > / < z >$, $[f_2, f_5] = 1$. Now we must find $f_1^{\beta} = f_1z^{\epsilon}$, $\epsilon \in \{0, 2\}$ and $f_3^{\beta} = f_1f_3z^{\delta}$, $\delta \in \{-1, 1\}$ because $o(f_1f_3) = 2$. Then $f_3 = f_3^{\beta^2} = (f_1f_3z^{\delta})^{\beta} = f_1z^{\epsilon}f_1f_3z^{\delta}z^{-\delta} = f_1z^{\epsilon}f_1f_3$, hence $f_1^{\beta} = f_1^{-1}$. Furthermore $f_1^{\rho_n} = f_1f_3z^{\gamma}$, $\gamma \in \{-1, 1\}$ and $f_3^{\rho_n} = f_1z^{\epsilon}$, $\epsilon \in \{0, 2\}$. Using the relation $\rho_n^{\beta} = \rho_n^{-1}$, we hold $f_1^{-1}z^{\epsilon} = f_3^{\rho_n\beta} = f_3^{\beta\rho_n^{-1}} = (f_1f_3z^{\delta})^{\rho_n^{-1}} = f_1^{-1}z^{\gamma+\delta}$, thus $\epsilon = \gamma + \delta$. It is easy to see that all relations are consistent for any choice of γ and δ, so we set w.l.o.g. $f_3^{\beta} = f_1f_3z^{-1}$, $f_1^{\rho_n} = f_1f_3z^{-1}$ and thereby $f_3^{\rho_n} = f_1^{-1}$.

For the τ-relations, we get $f_1^{\tau} = f_2z^{\epsilon}$ and $f_3^{\tau} = f_5z^{\gamma}$, where $\epsilon, \gamma \in \{0, 2\}$. Moreover we have $f_2^{\rho_n} = f_5z^{\delta}$, $\delta \in \{0, 2\}$, and $f_5^{\rho_n} = f_2f_5z^{\nu}$, $\nu \in \{-1, 1\}$. Using $\rho_n^{\tau} = \rho_n^{-1}$, we find $f_2z^{\epsilon}f_5z^{\gamma}z^{-1} = (f_1f_3z^{-1})^{\tau} = f_1^{\rho_n\tau} = f_1^{\tau\rho_n^{-1}} = (f_2z^{\epsilon})^{\rho_n^{-1}} = f_2f_5z^{\nu}z^{\delta}z^{\epsilon}$. Thus we hold $\gamma - 1 = \nu + \delta$. Also $f_2^{-1}z^{\epsilon} = (f_1^{-1})^{\tau} = f_3^{\rho_n\tau} = f_3^{\tau\rho_n^{-1}} = (f_5z^{\gamma})^{\rho_n^{-1}} = (f_2f_5z^{\nu})^{\rho_n}z^{\gamma} = f_5z^{\delta}f_2f_5z^{2\nu}z^{\gamma}$. This yields $\epsilon + 2 = \delta + \gamma$. Furthermore $f_2f_5z^{\nu}z^{\delta} = (f_5z^{\delta})^{\rho_n} = f_3^{\tau\rho_n} = f_3^{\rho_n^{-1}\tau} = (f_1^{-1})^{\rho_n\tau} = (f_1f_3z)^{\tau} = f_2z^{\epsilon}f_5z^{\gamma}z$. Hence $\nu + \delta = \epsilon + \gamma + 1$. Using the above equation, we get $\gamma - 1 = \epsilon + \gamma + 1 \Leftrightarrow 2 = \epsilon$. This gives $\delta + \gamma = 0 \Leftrightarrow \delta = \gamma$ and $\gamma - 1 = \delta + \nu \Leftrightarrow \nu = -1$. Thus $f_1^{\tau} = f_2^{-1}$ and $f_5^{\rho_n} = f_2f_5z^{-1}$. We can also see that the relations are consistent for any choice of γ, so we set w.l.o.g. $f_3^{\tau} = f_5$ which implies $f_2^{\rho_n} = f_5$.

Now we settle the relations $[f_i, f_j]$. Since the O_2 of a maximal parabolic in $\mathbb{Z}_2L_3(4)$ is elementary abelian, we hold that $[f_i, f_j] \in \{1, z^2\}$. Moreover $\mathbb{Z}_4 * Q_8 * Q_8$ has no abelian subgroup of order 2^5. Therefore we obtain that at least one of $[f_1, f_2]$ and $[f_2, f_3]$, one of $[f_1, f_2]$ and $[f_1, f_5]$, one of $[f_1, f_5]$ and $[f_3, f_5]$, and one of $[f_3, f_5]$ and $[f_2, f_3]$ equals z^2. Furthermore we get $[f_2, f_3] = [f_2^{\rho_n}, f_3^{\rho_n}] = [f_5, f_1^{-1}] = [f_1, f_5]$ and $[f_3, f_5] = [f_3^{\rho_n}, f_5^{\rho_n}] = [f_1^{-1}, f_2f_5z^{-1}] = [f_1, f_2f_5] = [f_1, f_5][f_1, f_2]^{f_5} = [f_1, f_5][f_1, f_2]$. This yields three possibilities: If $[f_3, f_5] = 1$, then $[f_1, f_5] = [f_1, f_2] = [f_2, f_3] = z^2$. If $[f_1, f_2] = 1$, then $[f_1, f_5] = [f_3, f_5] = [f_2, f_3] = 1$. And, if $[f_3, f_5] = [f_1, f_2] = z^2$, then $[f_1, f_5] = [f_2, f_3] = z^2$. It is easy to see that all three possibilities are consistent with the relations obtained so far and in all cases we find $< z, f_1, f_3, f_2, f_5 >\simeq \mathbb{Z}_4 * Q_8 * Q_8$. Therefore we choose w.l.o.g. $[f_3, f_5] = 1$ yielding $[f_1, f_5] = [f_1, f_2] = [f_2, f_3] = z^2$.

Adding the $\tilde{a}$-relations, we see that $f_1^{\tilde{a}} = f_1z^{\epsilon}$, $f_3^{\tilde{a}} = f_3z^{\gamma}$, $f_2^{\tilde{a}} = f_2f_3z^{\delta}$, where $\epsilon, \gamma, \delta \in \{0, 2\}$, and $f_5^{\tilde{a}} = f_1f_3f_5z^{\nu}$, with $\nu \in \{-1, 1\}$ since $(f_1f_3f_5)^2 = 1$. Now $f_2 = f_2^{\tilde{a}^2} = (f_2f_3z^{\delta})^{\tilde{a}} = f_2f_3f_3z^{\gamma}z^{2\delta} = f_1f_3f_3z^{\gamma}$, hence $f_3^{\tilde{a}} = f_3^{-1}$. This implies

$f_5 = f_5^{\tilde{a}^2} = (f_1f_3f_5z^{\nu})^{\tilde{a}} = f_1^{\tilde{a}}f_3^{-1}f_1f_3f_5z^{2\nu}$, hence $[f_1,\tilde{a}] = 1$ since $z^{2\nu} = z^2$. Again, we can see that the relations are consistent for each choice of δ and ν, so w.l.o.g. we set $f_2^{\tilde{a}} = f_2f_3$ and $f_5^{\tilde{a}} = f_1f_3f_5z^{-1}$.

For the remaining relations we get $f_4 = \tilde{a}\tilde{a}^{\rho_n}z^{\delta}$, with $\delta \in \{-1,1\}$, and $f_2^{\beta} = f_3f_4z^{\epsilon}$, where $\epsilon \in \{-1,1\}$ since $(f_3\tilde{a}\tilde{a}^{\rho_n})^2 = z^2$. Then we hold $f_4^{\beta} = (f_3^{-1})^{\beta}f_2z^{\epsilon} = f_1f_3f_2z^{1+\epsilon}$. Moreover $f_5^{\beta} = f_2^{\rho_n\beta} = f_2^{\beta\rho_n^{-1}} = (f_3f_4)^{\rho_n^{-1}}z^{\epsilon} = f_1f_3z\tilde{a}^{\rho_n^{-1}}\tilde{a}z^{\delta+\epsilon} = f_1f_3\tilde{a}\tilde{a}^{\rho_n}z^{\delta}\tilde{a}z^{1+\epsilon} = \tilde{a}f_1f_3f_4z^{\epsilon-1}$. Thus for $\epsilon = 1$, we hold $f_5^{\beta} = \tilde{a}f_1f_3f_4$, and $f_5^{\beta} = \tilde{a}f_1f_3f_4z^2$ if $\epsilon = -1$. Let us abreviate this as $f_5^{\beta} = \tilde{a}f_1f_3f_4z^{\gamma}$. Then $\tilde{a}^{\beta} = f_5(f_4^{-1})^{\beta}(f_3^{-1})^{\beta}f_1z^{\gamma} = f_5z^{-\epsilon}f_2^{-1}f_3^{\beta}(f_3^{-1})^{\beta}f_1z^{\gamma} = f_5f_2f_1z^{\gamma-\epsilon-2}$. Thus if $\epsilon = 1$, we get $\gamma-\epsilon-2 = -3$, and, if $\epsilon = -1$, we get $\gamma-\epsilon-2 = 1$. Therefore we obtain $\tilde{a}^{\beta} = f_5f_2f_1z$ in both cases. Now $f_1f_3f_2zz^{\epsilon} = f_4^{\beta} = (\tilde{a}\tilde{a}^{\rho_n}z^{\delta})^{\beta} = \tilde{a}^{\beta}\tilde{a}^{\beta\rho_n^{-1}}z^{-\delta} = f_1f_3f_2zz^{-\delta}$. Thereby we hold $\delta = -\epsilon$. Moreover one can check that both choices for ϵ are consistent with all relations obtained so far. Therefore we set $f_2^{\beta} = f_3f_4z^{-1}$ implying $f_4 = \tilde{a}\tilde{a}^{\rho_n}z$ and $f_5^{\beta} = \tilde{a}f_1f_3f_4z^2$.

As a summary, we obtain the following presentation of $\mathbb{Z}_4L_3(4):2_1$:

$$\mathbb{Z}_4L_3(4):2_1 \simeq < z,\tilde{a},\rho_n,\tau,f_1,f_2,f_3,f_4,f_5,\beta \mid \tilde{a}^2 = \rho_n^3 = \tau^2 = \beta^2 = z^4 = 1, f_1^2 = f_2^2 = f_3^2 = f_5^2 = z^2, (\rho_n\tilde{a})^3 = 1, \rho_n^{\tau} = \rho_n^{-1}, (\tau\rho_n\tilde{a})^5 = 1, [f_1,f_3] = [f_2,f_5] = [f_3,f_5] = 1, [f_1,f_2] = [f_1,f_5] = [f_2,f_3] = z^2, [f_1,z] = [f_2,z] = [f_3,z] = [f_5,z] = 1, [\rho_n,z] = [\tau,z] = [\tilde{a},z] = 1, z^{\beta} = z^{-1}, f_1^{\rho_n} = f_1f_3z^{-1}, f_2^{\rho_n} = f_5, f_3^{\rho_n} = f_1^{-1}, f_5^{\rho_n} = f_2f_5z^{-1}, f_1^{\tau} = f_2^{-1}, f_3^{\tau} = f_5, [f_1,\tilde{a}] = 1, f_3^{\tilde{a}} = f_3^{-1}, f_2^{\tilde{a}} = f_2f_3, f_5^{\tilde{a}} = f_1f_3f_5z^{-1}, f_4 = \tilde{a}\tilde{a}^{\rho_n}z, f_1^{\beta} = f_1^{-1}, f_3^{\beta} = f_1f_3z^{-1}, \rho_n^{\beta} = \rho_n^{-1}, \tilde{a}^{\beta} = f_5f_2f_1z, f_2^{\beta} = f_3f_4z^{-1}, f_5^{\beta} = \tilde{a}f_1f_3f_4z^2, (\tau\tau^{\beta})^3 = 1 >.$$

2.8 Identification of the geometric generators

In this section we identify the geometric generators for $\mathbb{Z}_4L_3(4):\mathbb{Z}_2$ as words in the canonical generators. For this purpose we use the relations given above. Again, we have that $M := < z,\rho,X,Y > \simeq (\mathbb{Z}_4 * SL_2(7)):\mathbb{Z}_2$. We start with a maximal parabolic $P = < z,f_1,f_3,f_2,f_5,\tau,\rho_n,\tilde{a} >$ of $\mathbb{Z}_4L_3(4)$. We set $B := < z,f_1,f_2,f_3,f_5,\rho_n,\tilde{a} >$ and $B_{\beta} := < B,\beta >$. W.l.o.g., we can assume that $\bar{P} := P \cap M$ is the preimage of a maximal parabolic of $< M',z > / < z > \simeq L_3(2)$, hence $\bar{P} \simeq \mathbb{Z}_4 * GL_2(3)$.

2.8.1 The generators z, $(XY)^2$ and a

Of course, we identify the *geometric* generator z with the *canonical* z. Also we set w.l.o.g. $(XY)^2 := f_1$. Since we must have $[a,(XY)^2] = [a,f_1] = 1$, we can assume that $a \in C_B(f_1)$. Therefore we can set $a := \tilde{a}f_2f_5$, because $(\tilde{a}f_2f_5)^2 = f_2f_3\tilde{a}f_5\tilde{a}f_2f_5 = f_2f_3f_1f_3f_5z^{-1}\tilde{a}^2f_2f_5 = f_2f_1f_5^{-1}f_2f_5z^{-1} = f_2f_1f_2z^{-1} = z^{-1}f_1 = z^{-1}(XY)^2$. Since $z \in Z(B)$, we also have $[z,a] = 1$.

2.8.2 The generators X and Y

Set $\bar{X} := \beta^{f_3 f_5}$. Then $z^{\bar{X}} = z^{-1}$ and $f_1^{\bar{X}} = f_1^{f_5 \beta f_3 f_5} = (f_1^{-1})^{\beta f_3 f_5} = f_1^{f_5} = f_1^{-1}$. Using our relations, we hold $a^{\bar{X}} = (f_1^{-1} f_3 \tilde{a} z f_2^{-1} f_5)^{\beta f_3 f_5} = (f_3^{-1} f_5 f_2 f_3 f_1 \tilde{a} f_1 f_3 z^2)^{f_3 f_5}$ since $f_1^{f_4} = f_1^{-1}$. Now $\tilde{a} f_1 f_3 = f_1 f_3 \tilde{a} z^2$, hence we obtain $a^{\bar{X}} = (f_3^{-1} f_5 f_2 \tilde{a})^{f_3 f_5} = f_3^{-1} f_5 f_2^{-1} f_1^{-1} f_3 \tilde{a} z = f_5 f_2 f_1^{-1} \tilde{a} z = \tilde{a} f_1 f_3 f_5 f_2 f_3 f_1^{-1} = \tilde{a} f_2 f_5 = a$. Thus we can set $X := \bar{X}$.

Setting $\bar{Y} := a\beta$, we get $a^{\bar{Y}} = a^{\beta} = f_5 f_2 f_1 f_3 \tilde{a} f_1 f_3 z^2 = f_5 f_2 \tilde{a} = a^{-1}$ and $z^{\bar{Y}} = z^{-1}$. Also we get $X\bar{Y} = f_3 f_5 \beta f_3 f_5 a \beta = f_3 f_5 f_4 z^{-1} (f_2 f_5)^{\tilde{a}} = f_3 f_5 f_4 f_2 f_1 f_5 = f_3 f_4^{f_5} f_2 f_1$. Using $[f_4, f_5] = f_3 z$, we get $X\bar{Y} = f_3 f_4 f_3 f_2 f_1 z = f_4 f_2 f_1 z^{-1}$ since $[f_3, f_4] = 1$. This implies $(X\bar{Y})^2 = f_4 f_2 f_1 f_4 f_2 f_1^{-1} = f_4 f_2 f_1 f_2 f_4$ because we have $[f_2, f_4] = f_1^{-1}$. Thus we find $(X\bar{Y})^2 = f_4 f_2 f_2^{-1} f_1 f_4 = f_4 f_1 f_4 = f_1$ since $[f_1, f_4] = z^2$. Therefore we set w.l.o.g. $Y := \bar{Y}$. Using the results obtained so far, we also see that $f_3 = [a, f_2]$.

2.8.3 The generator ρ

In order to identify our generator ρ, we construct the subgroup M explicitly using the canonical and geometric generators obtained so far.

We start to construct $\bar{P}$. Since $< f_1, f_2 > \simeq Q_8$, we can assume that $< z, f_1, f_2 > = O_2(\bar{P})$. Furthermore $f_1^{f_4} = f_1^{-1}$ and $[f_2, f_4] = f_1^{-1}$, thus $f_2^{f_4} = f_2 f_1^{-1}$. Also, τ normalizes $< z, f_1, f_2 >$. Therefore we can assume that $< z, f_1, f_2, f_4 > \in Syl_2(\bar{P})$ and $< z, f_1, f_2, \tau > \in Syl_2(\bar{P})$.

We see that $(f_4\tau)^3 = (\tilde{a}\tilde{a}^{\rho_n}\tau)^3 z^{-1}$. Using $(\tilde{a}\rho_n\tau)^3 = 1$, we hold $(\tilde{a}\tilde{a}^{\rho_n}\tau)^3 = (\tilde{a}\tau)^2 \rho_n \tilde{a} \rho_n \tau \tilde{a} \rho_n^{-1} \tilde{a} \rho_n \tau = (\tilde{a}\tau)^2 \tau \rho_n \tilde{a} \rho_n \tilde{a} \tau \rho_n^{-1} \tilde{a} = \tilde{a}\tau\tilde{a}\rho_n\tilde{a}\rho_n\tilde{a}\tau\rho_n^{-1}\tilde{a} = \tilde{a}\tau(\tilde{a}\rho_n)^3\tau\tilde{a} = 1$ because $(\tilde{a}\rho_n)^3 = 1$. Thus $o(f_4\tau) = 12$ and $(f_4\tau)^3 = z^{-1}$. So we set $\rho_1 = (f_4\tau)^4 = f_4 \tau z^{-1}$ and get $f_1^{\rho_1} = f_2$, $f_2^{\rho_1} = f_1^{-1} f_2$ and $f_4 \in \bar{P} =< z, f_1, f_2, \tau, \rho_1 >$.

Moreover $< z, f_1 >^X =< z, f_1 >$ and $f_2^X = f_3 f_4^{f_5} z = f_3 f_4 f_3 z^2 = f_4$. Thus $< z, f_1, f_2, f_4 > \leq \bar{P} \cap \bar{P}^X$. Since $[\tau, f_3 f_5] = 1$, we get $o(\tau\tau^X) = o(\tau\tau^{\beta}) = 1$ and $\tau^X \notin \bar{P}$. This implies that $< z, f_1, f_2, \tau, \rho_1, X > \simeq M$. Using our relations, we get $YX = f_2 f_1 f_5 f_4 f_3 f_5 = f_2 f_1 f_4 f_5 f_3^2 f_5 z = f_2 f_1 f_4 z$. Thereby we see that we have $Y \in< z, f_1, f_2, \tau, \rho_1, X >$, so w.l.o.g. $M =< z, f_1, f_2, \tau, \rho_1, X >$.

Now our generator $\rho \in C_M(X)$. Since $f_1^{\rho_1} = f_2$, we get that f_1 inverts $(\rho_1^{-1})^{X\tau} =: \bar{\rho}$. Then $[z, \bar{\rho}] = 1$ and $\bar{\rho}^{f_1} = \bar{\rho}^{-1}$. Also, $\bar{\rho}^X = (\tau f_4^{-1} z)^{X\tau X} = (\tau f_4^{-1})^{X\tau X} z$. We find $(f_4^{-1})^{X\tau X} = (f_2^{-1})^{\tau X} = f_1^{-1}$ and $\tau^{X\tau X} = (X\tau)^3 X$. So, since $(X\tau)^6 = 1$, $\bar{\rho}^X = (X\tau)^3 X f_1^{-1} z = (\tau X)^3 X f_1^{-1} z = (\tau X)^2 \tau f_1^{-1} z = (\tau X)(\tau X) f_2 \tau z = (\tau X)(\tau f_4 z^{-1})(X\tau) = (\tau X)(\tau f_4^{-1} z)(X\tau) = \bar{\rho}$. Therefore we hold $[\bar{\rho}, X] = 1$.

Now a and $\bar{\rho}$ centralize X, thus $< z, \bar{\rho}, a, X >$ is isomorphic to a subgroup of $(\mathbb{Z}_3^2 : Q_8) \times D_8$ which is the preimage of the centralizer of X in $L_3(4) : 2_1$. This implies $[\bar{\rho}, \bar{\rho}^a] = 1$.

The relation $(\bar{\rho} X Y z^{-1})^7 = 1$ holds if and only if $(\bar{\rho} X Y)^7 = z^{-1}$. Since $XY = f_4 f_2 f_1 z^{-1}$, we get $\bar{\rho} X Y = (\tau f_4^{-1} z)^{X\tau} f_4 f_2 f_1 z^{-1} = (\tau X)^2 \tau f_1 f_4 f_2 f_1 z^2 = (\tau X)^2 \tau f_4 f_2$. This implies the equalities $(\bar{\rho} X Y)^2 = ((\tau X)^2 \tau f_4 f_2)^2 = (\tau X)^2 \tau f_4 f_2 \tau (X\tau)^2 f_4 f_2 =$

$(\tau X)^2\tau f_4 f_2 \tau z^{-1} f_1^{-1}\tau^{X\tau X} f_1^{-1}(X\tau)^2 f_2$. The last equality holds because $(\tau f_4)^3 = z^{-1}$ implies $f_4^\tau = z^{-1}f_4^{-1}\tau f_4^{-1}$ yielding $f_4^{\tau X} = z f_2^{-1}\tau^X f_2^{-1}$ and $f_4^{(\tau X)^2} = z^{-1}f_1^{-1}\tau^{X\tau X} f_1^{-1}$. Using the fact $(X\tau)^2 f_2 = f_4^{-1}(X\tau)^2$, we obtain the following identities: $(\bar\rho XY)^2 = (\tau X)^2(\tau f_4 f_2 \tau z^{-1} f_1^{-1}\tau^{X\tau X} f_1^{-1} f_4^{-1})(X\tau)^2 = (\tau X)^2(f_4^{-1}\tau f_4^{-1}\tau^{X\tau X} f_1^{-1} f_4^{-1})(X\tau)^2 = (\tau X)^2(z^{-1}\tau f_4^{-1}(\tau X)^4 f_1^{-1} f_4^{-1})(X\tau)^2 = (\tau X)^2(z^{-1}\tau f_4^{-1}(X\tau)^2 f_1^{-1} f_4^{-1})(X\tau)^2$. This implies that for $(\bar\rho XY)^3$ we hold the following equalities: $(\bar\rho XY)^3 = (\bar\rho XY)^2\bar\rho XY = (\tau X)^2(z^{-1}\tau f_4^{-1}(X\tau)^2 f_1^{-1} f_4^{-1})\tau f_4 f_2 = (\tau X)^2(\tau f_4^{-1}(X\tau)^2 f_1^{-1} f_4^{-1}\tau f_2) = (\tau X)^2(\tau f_4^{-1}(X\tau) f_4 f_2^{-1} f_1)(X\tau) = (\tau X)^2(\tau X f_2^{-1}\tau f_4 f_2^{-1} f_1)(X\tau) = (\tau X)^3\tau(f_4 f_2)(X\tau) = (\tau X)^4 f_2 f_4 \tau = (X\tau)^2 f_2 f_4 \tau$.

We have $(\bar\rho XY)^7 = z^{-1}$ if and only if we find for $(\bar\rho XY)^6$: $(\bar\rho XY)^6 = z^{-1} Y X \bar\rho^{-1} = z^{-1} f_2 f_1 f_4 z (f_4 \tau z^{-1})^{X\tau} = f_2 f_4 (\tau X)^2 \tau z^{-1}$. Conversely, we obtain that $(\bar\rho XY)^6 = ((X\tau)^2 f_2 f_4 \tau)^2 = (f_2 f_4)^{(\tau X)^2}(f_2 f_4)^\tau = f_4^{-1} f_1^{-1}\tau^{X\tau X} f_4^{-1}\tau f_4^{-1} = f_4^{-1} f_1^{-1}(\tau X)^2\tau f_4^{-1}\tau f_4^{-1} = f_4^{-1} f_1^{-1}(\tau X)^2 f_4 \tau z = f_4 f_2 f_1 (\tau X)^2 \tau z$. Together with $[f_4, f_2] = f_1$, this gives $(\bar\rho XY)^6 = f_2 f_4 (\tau X)^2 \tau z^{-1}$. Therefore we can set $\rho = \bar\rho$.

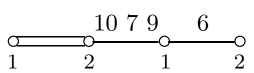
2.8.4 The presentation for $\mathbb{Z}_4 L_3(4) : \mathbb{Z}_2$

Putting together the geometric relations, the identification of the canonical generators and the canonical relations, we get the following presentation of the group $G_4 \simeq \mathbb{Z}_4 L_3(4) : \mathbb{Z}_2$ of the amalgam of the Buekenhout geometry:

$$\mathbb{Z}_4 L_3(4) : \mathbb{Z}_2 \simeq G_1 =< f_1, f_2, f_3, f_4, f_5, \tilde a, \rho_n, \tau, \beta, z, X, Y, \rho, a, x \mid z^4 = X^2 = Y^2 = \rho^3 = 1, x^2 = z^2, z^X = z^{-1}, x^X = x^{-1}, [x, z] = 1, x = (XY)^2, z^Y = z^{-1}, [z, \rho] = [X, \rho] = 1, \rho^x = \rho^{-1}, (\rho z^{-1} XY)^7, [X, a] = 1, a^2 = z^{-1}(XY)^2, [a, z] = 1, a^Y = a^{-1}, [x, a] = 1, [\rho, \rho^a] = 1, f_1 = (XY)^2, a = \tilde a f_2 f_5, X = \beta^{f_3 f_5}, Y = a\beta, \rho = (\tau f_4^{-1} z)^{X\tau}, [f_1, z] = [f_2, z] = [f_3, z] = [f_5, z] = [\tilde a, z] = [\rho_n, z] = [\tau, z] = 1, z^\beta = z^{-1}, f_1^2 = z^2, f_2^2 = z^2, f_3^2 = z^2, f_5^2 = z^2, [f_1, f_2] = z^2, [f_1, f_3] = 1, [f_1, f_5] = z^2, [f_2, f_3] = z^2, [f_2, f_5] = 1, [f_3, f_5] = 1, \tilde a^2 = \rho_n^3 = \tau^2 = 1, (\tilde a \rho_n)^3 = 1, \rho_n^\tau = \rho_n^{-1}, (\tau \rho_n \tilde a)^5 = 1, f_4 = z\tilde a \tilde a^{\rho_n}, f_1^{\rho_n} = f_1 f_3 z^{-1}, f_2^{\rho_n} = f_5, f_3^{\rho_n} = f_1^{-1}, f_5^{\rho_n} = f_5 f_2 z^{-1}, f_1^\tau = f_2^{-1}, f_3^\tau = f_5, [f_1, \tilde a] = 1, f_3^{\tilde a} = f_3^{-1}, f_2^{\tilde a} = f_2 f_3, f_5^{\tilde a} = f_1 f_3 f_5 z^{-1}, \beta^2 = 1, \rho_n^\beta = \rho_n^{-1}, \tilde a^\beta = f_5^{-1} f_2^{-1} f_1 z, f_1^\beta = f_1^{-1}, f_3^\beta = f_1 f_3 z^{-1}, f_2^\beta = f_3 f_4 z^{-1}, f_5^\beta = \tilde a f_1 f_3 f_4 z^2, (\tau\tau^\beta)^3 = 1 >.$$

2.9 A presentation for the amalgam of Buekenhout's geometry

In this section we put together all information obtained so far in order to give a presentation for the universal completion of Buekenhout's geometry Γ for the O'Nan group. We recall that this geometry has the following Buekenhout diagram:



We denote the maximal parabolics of this geometry by G_1, G_2, G_3 and G_4, read from the left to the right of the diagram nodes, with $G_1 \simeq L_3(7) : \mathbb{Z}_2$ and $G_4 \simeq \mathbb{Z}_4 L_3(4) : \mathbb{Z}_2$. Since the residues for G_2 and G_3 are just direct sums of geometries, we do not have to add further relations and obtain that for the universal completion U of the amalgam of this geometry we have $U =< G_1, G_4 \mid [a, Z] = 1 >$. Moreover the following holds:

Theorem 2.9.1 *Let Γ' be a flag-transitive geometry with the same Buekenhout diagram as Γ. Assume furthermore that for a flag-transitive automorphism group H we have $H_1/K_1 \simeq L_3(7) : \mathbb{Z}_2$ and $H_4/K_4 \simeq L_3(4) : 2_1$ where K_i denotes the kernel of the action of H_i on the corresponding residue. Then $K_1 = 1$ and $K_4 \simeq \mathbb{Z}_4$.*

Proof. The hypothesis of the theorem immediately implies that $K_1 \lhd K_4 \leq H_1$ and $K_4/K_1 \simeq \mathbb{Z}_4$ since $H_{14}/K_1 \simeq (\mathbb{Z}_4 * SL_2(7)) : \mathbb{Z}_2$. Therefore we have that K_i must be a 2-group for $i = 1, 4$ since $O^2(K_4) \leq K_1$. The assertion follows if $K_1 = 1$. Let us assume that $K_1 \neq 1$.

We prove that $Z(K_4) \leq Z(K_1)$. Assume $Z(K_4) \not\leq K_1$. Then there exists some $1 \neq x \in Z(K_4) - K_1$ and therefore there exists some $1 \neq \bar{x} \in K_4/K_1$. Since $[x, K_1] = 1$, we get $[< x^{H_1} >, K_1] = 1$. Thus we find at least $[U, K_1] = 1$ where $UK_1/K_1 = (H_1/K_1)' \simeq L_3(7)$. Set $U :=< x^{H_1} >$ and $U_{14} := H_{14} \cap U$. Then $U_{14}K_4/K_4 \simeq L_2(7) : \mathbb{Z}_2$. Since K_4/K_1 is centralized by $(H_{14}/K_1)'$ (recall that $H_{14}/K_1 \simeq (\mathbb{Z}_4 * SL_2(7)) : \mathbb{Z}_2$), we hold $[U'_{14}, K_4] = 1$. Now $C_{H_4}(K_4) \lhd H_4$ and $(U_{14}K_4/K_4)' \leq (H_4/K_4)' \simeq L_3(4)$. Together with the fact that H_{14}/K_4 contains an automorphism of $(H_4/K_4)'$ this implies $K_1 \lhd H_4$, a contradiction.

We use this fact to prove that for every $A \in \mathcal{A}(K_4)$ with $A \not\leq K_1$ we must have $[A, Z(K_1)] \neq 1$. Assume the contrary. Then $[< A^{G_1} >, Z(K_1)] = 1$. Since $K_4/K_1 \simeq \mathbb{Z}_4$, we get that at least $[U, Z(K_1)] = 1$ where U is the preimage of $(H_1/K_1)' \simeq L_3(7)$. Now $H_{14}/K_1 \simeq (\mathbb{Z}_4 * SL_2(7)) : \mathbb{Z}_2$ contains an automorphism α of U normalizing $Z(K_4)$. By $Z(K_4) \leq Z(K_1)$, we hold $Z(K_4) \lhd < U, \alpha >= H_1$, a contradiction. The same argument can be used to prove $[A, \Omega_1(Z(K_1))] \neq 1$ since $Z(K_4) \leq Z(K_1)$ implies $\Omega_1(Z(K_4)) \leq \Omega_1(Z(K_1))$.

Using $K_4/K_1 \simeq \mathbb{Z}_4$, we hold $\mid A : A \cap K_1 \mid= 2$ if $A \not\leq K_1$, $A \in \mathcal{A}(K_4)$. Since A is maximal, we get $\mid (A \cap K_1)\Omega_1(Z(K_1)) \mid \leq \mid A \mid$. Thus we have $\frac{|A \cap K_1||\Omega_1(Z(K_1))|}{|\Omega_1(Z(K_1)) \cap A|} \leq \mid A \mid$ which implies $\frac{|\Omega_1(Z(K_1))|}{|\Omega_1(Z(K_1)) \cap A|} \leq \frac{|A|}{|A \cap K_1|} = 2$. Using $[A, \Omega_1(Z(K_1))] \neq 1$, this leads to $\mid \Omega_1(Z(K_1)) : \Omega_1(Z(K_1)) \cap A \mid= 2$ proving that A induces transvections on $\Omega_1(Z(K_1))$. Let now $a \in A - \Omega_1(Z(K_1))$ and let $d \in G_1$ with $o(d) = p$, $2 \neq p$ a prime and $d^a = d^{-1}$. Since a is a transvection and $< a, d >$ is a dihedral group, we hold that $\mid \Omega_1(Z(K_1)) : C_{\Omega_1(Z(K_1))}(< d >) \mid= 4$ yielding $o(d) = 3$. But this contradicts the fact that, because $(H_1/K_1)' \simeq L_3(7)$, the involution a inverts also elements of order seven. Thereby we have a contradiction to $A \not\leq K_1$, so $A \leq K_1$ implying the final contradiction, namely $J(K_4) = J(K_1)$, proving the theorem. $\square$

Using the MAGMA [2] function for coset enumeration we establish the following theorem:

Theorem 2.9.2 *Let* $G :=< z, X, Y, Z, \rho, x, v_1, v_2, \nu, u, i, f_1, f_2, f_3, f_4, f_5, \tilde{a}, \rho_n, \tau, \beta, \rho_1 \mid z^4 = X^2 = Y^2 = Z^4 = \rho^3 = 1, x^2 = z^2, z^X = z^{-1}, x^X = x^{-1}, [x, z] = [z^{-1}x, Z] = 1, x^{Z^2} = x^{-1}, Z^{x^2} = Z^{-1}, (xZ)^3 = 1, (z^2)^{xZ} = z^{-1}X, x = (XY)^2, z^Y = z^{-1}, [z, \rho] = [X, \rho] = 1, \rho^x = \rho^{-1}, (ZZ^Y)^3 = 1, [\rho, xZ] = 1, (\rho z^{-1}XY)^7 = 1, [X, a] = 1, a^2 = z^{-1}(XY)^2, [a, z] = 1, a^Y = a^{-1}, [x, a] = 1, [\rho, \rho^a] = 1, [Z, a] = 1, i = z^{-1}X, \nu = ((\rho z^{-1}XY)^2)^{z^{-1}X(\rho^{-1})^Y}, u = xZX, v_2 = \nu^u, v_1 = v_2^{-1}v_2^{\nu}, \nu^{\rho} = \nu^4, [\rho, i] = 1, (\nu x)^3 = x^2, \nu^i = \nu^{-1}, [\nu, x^2] = 1, x^i = x^{-1}, v_1^u = v_1^{-1}, v_2^u = \nu, \nu^u = v_2, [\rho, u] = 1, (x^2)^u = x^2 i, (xx^u)^3 = 1, f_1 = (XY)^2, a = \tilde{a}f_2f_5, X = \beta^{f_3f_5}, Y = a\beta, \rho = (\tau f_4^{-1}z)^{X\tau}, [f_1, z] = [f_2, z] = [f_3, z] = [f_5, z] = [\tilde{a}, z] = [\rho_n, z] = [\tau, z] = 1, z^{\beta} = z^{-1}, f_1^2 = z^2, f_2^2 = z^2, f_3^2 = z^2, f_5^2 = z^2, [f_1, f_2] = z^2, [f_1, f_3] = 1, [f_1, f_5] = z^2, [f_2, f_3] = z^2, [f_2, f_5] = 1, [f_3, f_5] = 1, \tilde{a}^2 = \rho_n^3 = \tau^2 = 1, (\tilde{a}\rho_n)^3 = 1, \rho_n^{\tau} = \rho_n^{-1}, (\tau\rho_n\tilde{a})^5 = 1, f_4 = z\tilde{a}\tilde{a}^{\rho_n}, f_1^{\rho_n} = f_1f_3z^{-1}, f_2^{\rho_n} = f_5, f_3^{\rho_n} = f_1^{-1}, f_5^{\rho_n} = f_5f_2z^{-1}, f_1^{\tau} = f_2^{-1}, f_3^{\tau} = f_5, [f_1, \tilde{a}] = 1, f_3^{\tilde{a}} = f_3^{-1}, f_2^{\tilde{a}} = f_2f_3, f_5^{\tilde{a}} = f_1f_3f_5z^{-1}, \beta^2 = 1, \rho_n^{\beta} = \rho_n^{-1}, \tilde{a}^{\beta} = f_5^{-1}f_2^{-1}f_1z, f_1^{\beta} = f_1^{-1}, f_3^{\beta} = f_1f_3z^{-1}, f_2^{\beta} = f_3f_4z^{-1}, f_5^{\beta} = \tilde{a}f_1f_3f_4z^2, (\tau\tau^{\beta})^3 = 1 >$. *Then* $G \simeq O'N$.

□

As a corollary we obtain by Proposition 1.2.4:

Corollary 2.9.3 *The Buekenhout geometry for the O'Nan sporadic group is simply connected.*

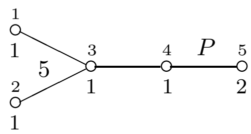
□

Chapter 3

Generators and relations for the Ivanov-Shpectorov geometry

3.1 The Ivanov-Shpectorov geometry for the O'Nan sporadic group

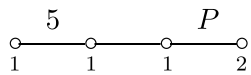
In this chapter we will give generators and relations for the universal completion of the amalgam related to the Ivanov-Shpectorov geometry for the groups $O'N$ and $3O'N$. To recall, this is a geometry of rank five with the following with the following Buekenhout diagram:



Also recall that the geometry of $3O'N$ is a triple cover of the $O'N$-geometry where $Z(3O'N)$ acts as a group of deck transformations. The maximal parabolics are $G_1 \simeq J_1$, $G_2 \simeq M_{11}$ and $G_5 \simeq (\mathbb{Z}_4 * Q_8 * Q_8) : A_5$ (G_3 and G_4 will be constructed later on). In difference to the Buekenhout geometry, we can construct a presentation for this geometry successively using simply connected geometries for $L_2(11)$, M_{11}, J_1 and $\mathbb{Z}_2^5 : A_5$.

3.2 A presentation of J_1

It is known by [15], that the group $G \simeq J_1$ acts flag-transitively on a rank four geometry Γ with the following Buekenhout diagram:



The maximal parabolic subgroups are $G_1 \simeq L_2(11)$, $G_2 \simeq \mathbb{Z}_2 \times A_5$, $G_3 \simeq S_3 \times D_{10}$ and $G_4 \simeq \mathbb{Z}_2 \times A_5$, read from the left to the right in the diagram. Furthermore we have $B = G_{1234} \simeq \mathbb{Z}_2$ and $B = Z(G_4)$ (see e.g. [12]).

It is known from [15], that Γ is 3-simply connected. We use Γ to construct a presentation for J_1. The following amalgam corresponds to Γ (see e.g. [12]): $G_{12} \simeq A_5$, $G_{13} \simeq D_{12}$, $G_{14} \simeq D_{12}$, $G_{23} \simeq D_{12}$, $G_{24} \simeq \mathbb{Z}_2^3$, $G_{34} \simeq D_{20}$, $G_{123} \simeq S_3$, $G_{124} \simeq G_{134} \simeq G_{234} \simeq \mathbb{Z}_2^2$ and $B \simeq \mathbb{Z}_2$.

We set $B =:< z >$, $G_{123} =:< z, t >$, $G_{124} =:< z, b >$, $G_{134} =:< z, a_n >$ and $G_{234} =:< z, a >$. Thereby we hold the following relations:

$$\mathcal{R}_o : z^2 = a^2 = a_n^2 = b^2 = t^3 = 1,$$

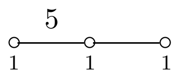
$$\mathcal{R}_m : [z, a] = [z, a_n] = [z, b] = 1, t^z = t^{-1}.$$

The diagram yields the following relations:

$$\mathcal{R}_d : [a, b] = [a, t] = [a_n, t] = (aa_n)^5 = (a_n b)^3 = (tb)^5 = 1.$$

3.2.1 Additional relations for G_4

The residue of G_4 in Γ has the Coxeter diagram:



The corresponding Coxeter group C is isomorphic to $\mathbb{Z}_2 \times A_5$ such that $Z(C)$ acts non-trivially on the geometry. Hence, using the relations obtained so far, we have $< z, a, a_n, b > \simeq \mathbb{Z}_2^2 \times A_5$.

Let $C =< a_1, a_2, a_3 \mid a_1^2 = a_2^2 = a_3^2 = (a_1 a_2)^5 = (a_2 a_3)^3 = 1 >$. Then $o(a_1 a_2 a_3) = 10$ and therefore we have $< a_1, a_2, a_3 \mid a_1^2 = a_2^2 = a_3^2 = (a_1 a_2)^5 = (a_2 a_3)^3 = (a_1 a_2 a_3)^5 = 1 > \simeq A_5$ by adding a $(2, 3, 5)$-relation to the Coxeter relations.

Since $Z(G_4) =< z >$ acts trivially on the residue of G_4, we have to distinguish two cases, namely, $a_n \in G_4'$ and $a_n \notin G_4'$. If $a_n \notin G_4'$ $(a \in G_4')$, we can assume w.l.o.g. that a and b are also not contained (are contained) in G_4'. Thus we can add either the relation

$\mathcal{R}_{(2,3,5)}.1 : (zaa_n b)^5 = 1$

or

$\mathcal{R}_{(2,3,5)}.2 : (aa_n b)^5 = 1.$

In both cases we hold $< z, a, a_n, b > \simeq \mathbb{Z}_2 \times A_5$. The correct relation will be distinguished by the amalgamation with G_3.

3.2.2 Additional relations for G_1

We have to ensure that $G_{12} =< z, b, t >\simeq A_5$. Using the relations obtained so far we hold $< z, b, t \mid z^2 = b^2 = t^3 = 1, (tb)^5 = 1, [z, b] = 1, t^z = t^{-1} >\simeq \mathbb{Z}_2 \times A_5$ where $< t, b >\simeq A_5$. We identify t with the element (123) of A_5, z with (23)(45) and b with (24)(35). Using this identification we hold $tb = (14253)$ and $[b, t] = (15423)$, thus $z = tb[b, t]^3 = bt[b, t]^2$. Therefore we have to add the relation

$\mathcal{R}_A : z = bt[b, t]^2$.

Using the relations $\mathcal{R}_d$, we find $G_{13} =< z, t, b >\simeq D_{12} \simeq< z, a_n, b >= G_{14}$. By [18], this amalgam determines the group $G_1 \simeq L_2(11)$ because it is the amalgam of a simply connected geometry.

3.2.3 Amalgamation of G_3 and G_4

Using the relations $\mathcal{R}_o$, $\mathcal{R}_m$, $\mathcal{R}_d$ and $\mathcal{R}_A$, we easily see that $a \in Z(< z, b, t, a >)$ and $< z, b, t, a >\simeq \mathbb{Z}_2 \times A_5$. Furthermore we have $G_3 =< z, t, a, a_n >\simeq S_3 \times D_{10}$ where $< z, t >$ and $< a, a_n >$ are the direct factors of G_3. It remains to analyze whether $a_n \in G_4'$ or not.

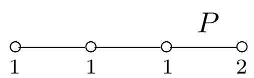
Using the subgroup lattice of J_1 (given in e.g. [12]), we see that J_1 contains a single conjugacy class of subgroups D_{20}. Given such a subgroup U of J_1, its two normal subgroups of shape D_{10} are non-conjugate in J_1. One of these two classes correspond to the direct factors of subgroups of shape $S_3 \times D_{10}$. These subgroups have no supergroup A_5 inside J_1. Thus a and a_n are not contained in G_4'. Together with the fact that Γ is simply connected [15] this yields the following lemma.

Lemma 3.2.1 *Let* $J :=< a, a_n, b, t, z \mid \mathcal{R}_o \cup \mathcal{R}_m \cup \mathcal{R}_d \cup \mathcal{R}_A \cup \mathcal{R}_{(2,3,5)}.1 >$. *Then* $J \simeq J_1$.

□

3.3 A presentation of M_{11}

By [4] it is known, that the Mathieu group M_{11} acts transitively on a geometry Γ, related to its 3-transitive action on 12 points, with the following diagram:



The corresponding maximal subgroups of $G \simeq M_{11}$ are $G_1 \simeq L_2(11)$, $G_2 \simeq S_5$, $G_3 \simeq S_3 \times S_3$ and $G_4 \simeq GL_2(3)$, read from the left to the right in the diagram (see e.g. [6]). It is shown in [18], that Γ is simply connected. Again, we use this geometry to get a presentation for G.

By the previous section, we hold $G_{12} \simeq A_5$, $G_{13} \simeq D_{12}$, $G_{14} \simeq D_{12}$, $G_{123} \simeq S_3$, $G_{124} \simeq \mathbb{Z}_2^2 \simeq G_{234}$ and $B = G_{1234} \simeq \mathbb{Z}_2$.

Since $G_4 \simeq GL_2(3)$, we find $G_{24} \simeq D_8$, $G_{34} \simeq D_{12}$, $G_{234} \simeq \mathbb{Z}_2^2$ and $B = Z(G_4)$ because we have $GL_2(3)$ acting on a geometry with the Coxeter diagram for S_4. This shows that $G_{23} \simeq D_{12}$. As in the last section we set $B =:< z >$, $G_{123} =:< z, t >$, $G_{124} =:< z, b >$ and $G_{134} =:< z, a_n >$. Moreover we set $G_{234} =:< z, v >$. Therefore we find the following relations:

$\mathcal{R}'_o : z^2 = v^2 = a_n^2 = b^2 = t^3 = 1$,

$\mathcal{R}'_m : [z, v] = [z, a_n] = [z, b] = 1, t^z = t^{-1}$.

The diagram yields the following relations:

$\mathcal{R}'_d : [v, t] = [a_n, t] = (a_n b)^3 = (v a_n)^3 = (tb)^5 = 1, (vb)^2 = z$.

Since $G_{12} =< z, t, b >\simeq A_5$, we add again the relation

$\mathcal{R}'_A : z = bt[b, t]^2$

in order to get $G_1 =< z, b, t, a_n >\simeq L_2(11)$.

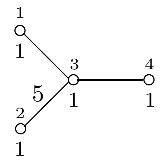
Using the relations $\mathcal{R}'_o$, $\mathcal{R}'_m$ and $\mathcal{R}'_d$, we already see that $G_4 =< z, b, a_n, v >\simeq GL_2(3)$. Furthermore it is easy to see that these relations give $G_2 =< z, t, b, v >\simeq S_5$. Obviously, we have $[t, v a_n] = 1$, which is enough to prove $G_3 =< z, t, a_n, v >\simeq S_3 \times S_3$. Then by the results of [18], we have the following lemma:

Lemma 3.3.1 *Let $M :=< z, t, b, a_n, v \mid \mathcal{R}'_o \cup \mathcal{R}'_m \cup \mathcal{R}'_d \cup \mathcal{R}'_A >$. Then $M \simeq M_{11}$.*

□

3.4 Some geometry for $\mathbb{Z}_2^5 : A_5$

It is known by [17], that a maximal parabolic subgroup P of the group $\mathbb{Z}_4 L_3(4)$ acts flag-transitively on a geometry Γ having the following Coxeter diagram:



From the diagram of the rank five geometry of Ivanov and Shpectorov [17] we draw that $P \cap M_{11} \simeq GL_2(3)$, so $\Omega_1(Z(P))$ is contained in the Borel subgroup of this geometry. Therefore we can assume that P is a maximal parabolic in $\mathbb{Z}_2 L_3(4)$. Thus P is isomorphic to $\mathbb{Z}_2^5 : A_5$ such that $Z(P) \simeq \mathbb{Z}_2$, $O_2(P)$ is an indecomposable module for A_5 where $O_2(P)/Z(P)$ is the natural $L_2(4)$-module.

Then the maximal parabolic subgroups of the pair (Γ, P) are: $G_1 \simeq A_5$, $G_2 \simeq S_4$, $G_3 \simeq \mathbb{Z}_2^3$ and $G_4 \simeq A_5$ (the numbering is denoted above the diagram nodes). Furthermore this implies $G_{12} \simeq S_3 \simeq G_{24}$, $G_{13} \simeq \mathbb{Z}_2^2 \simeq G_{23}$ and $G_{14} \simeq D_{10}$. Clearly, all minimal parabolic subgroups are isomorphic to $\mathbb{Z}_2$. We set $< b >:= G_{123}$, $< a_n >:= G_{124}$, $< a >:= G_{134}$ and $< v >:= G_{234}$ and let C be the corresponding (infinite) Coxeter group. Thus we obtain the following relations:

$\tilde{\mathcal{R}}_o : a^2 = a_n^2 = b^2 = v^2 = 1,$
$\tilde{\mathcal{R}'}_c : [a,b] = [a,v] = [b,v] = (aa_n)^5 = (a_nb)^3 = (a_nv)^3 = 1.$

In view of the aim to give generators and relations for the Ivanov-Shpectorov geometry, we denote $\tilde{\mathcal{R}'}_c$ by:

$\tilde{\mathcal{R}}_c : [a,b] = [a,v] = (aa_n)^5 = (a_nb)^3 = (va_n)^3 = 1, (vb)^2 = 1.$

Then $C =< a, a_n, b, v \mid \tilde{\mathcal{R}}_o \cap \tilde{\mathcal{R}}_c >$. Again, we have to ensure that $G_1$and G_4 are both isomorphic to A_5. Thus we have to diminish the order of the two Coxeter elements aa_nb and va_na. Therefore we have to add

$\tilde{\mathcal{R}}_{(2,3,5)} : (aa_nb)^5 = (va_na)^5 = 1.$

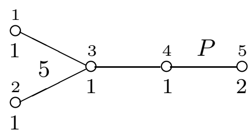
Using the Magma [2] program for coset enumeration, we get the following lemma:

Lemma 3.4.1 *Let $P :=< a, a_n, b, v \mid \tilde{\mathcal{R}}_o \cup \tilde{\mathcal{R}}_c \cup \tilde{\mathcal{R}}_{(2,3,5)} >$. Then P is isomorphic to a maximal parabolic of $\mathbb{Z}_2L_3(4)$.*

□

3.5 Generators and relations for the Ivanov-Shpectorov geometry

We recall that the geometry Γ of Ivanov and Shpectorov for the O'Nan group has the following diagram:



The maximal parabolic subgroups G_1, G_2 and G_5 of the pair (Γ, G) (where $G \in \{O'N, 3O'N\}$) are $G_1 \simeq J_1$, $G_2 \simeq M_{11}$ and G_5 is isomorphic to a maximal parabolic of $\mathbb{Z}_4L_3(4)$.

Let P_1 be a maximal parabolic in $\mathbb{Z}_4L_3(4)$. According to the last section, we get generators and relations for P acting on the geometry Γ_5 simply by adding a center which acts trivially such that $G_{15} \simeq \mathbb{Z}_2 \times A_5 \simeq G_{45}$ and $G_{25} \simeq GL_2(3)$. Thus we add a new generator z and transform the relation $(vb)^2 = 1$ in $\tilde{\mathcal{R}}_c$ in $(vb)^2 = z$. Since P_1 has to contain a subgroup $\mathbb{Z}_2 \times A_5$ of $G_1 \simeq J_1$, we change $\tilde{\mathcal{R}}_{(2,3,5)}$ to

$\bar{\mathcal{R}}_{(2,3,5)} : (zaa_nb)^5 = (va_na)^5 = 1.$

Furthermore we add

$\bar{\mathcal{R}}_z : z^2 = [a,z] = [b,z] = [a_n,z] = [v,z] = 1.$

We set $\bar{\mathcal{R}}_o := \tilde{\mathcal{R}}_o$ and

$\bar{\mathcal{R}}_c : [a,b] = [a,v] = (aa_n)^5 = (a_nb)^3 = (va_n)^3 = 1, (vb)^2 = z.$

Then we have the following:

Lemma 3.5.1 *Let $P_1 :=< z, a, a_n, b, v \mid \bar{\mathcal{R}}_o \cup \bar{\mathcal{R}}_c \cup \bar{\mathcal{R}}_{(2,3,5)} >$. Then P is isomorphic to a maximal parabolic of $\mathbb{Z}_4 L_3(4)$.*

□

We are now able to give generators and relations for the Ivanov-Shpectorov geometry. First we set $\mathcal{R}_O := \mathcal{R}_o \cup \mathcal{R}'_o \cup \bar{\mathcal{R}}_o$, thus:

$\mathcal{R}_O : z^2 = a^2 = a_n^2 = b^2 = v^2 = t^3 = 1$.

Then we put $\mathcal{R}_M := \mathcal{R}_m \cup \mathcal{R}'_m$, thus:

$\mathcal{R}_M : [z, a] = [z, a_n] = [z, b] = [z, v] = 1, t^z = t^{-1}$.

We set $\mathcal{R}_D := \mathcal{R}_d \cup \mathcal{R}'_d \cup \bar{\mathcal{R}}_c$, thus:

$\mathcal{R}_D : [a, b] = [a, t] = [a_n, t] = [v, t] = [a, v] = (aa_n)^5 = (a_n b)^3 = (va_n)^3 = (tb)^5 = 1, (vb)^2 = z$.

We recall that $\mathcal{R}_A = \mathcal{R}'_A$ and

$\mathcal{R}_A : z = bt[b, t]^2$.

Finally, we set $\mathcal{R}_{(2,3,5)} := \bar{\mathcal{R}}_{(2,3,5)}$, thus:

$\mathcal{R}_{(2,3,5)} : (zaa_n b)^5 = (va_n a)^5 = 1$.

Then we get that

$U :=< z, a, a_n, b, v, t \mid \mathcal{R}_O \cup \mathcal{R}_M \cup \mathcal{R}_D \cup \mathcal{R}_A \cup \mathcal{R}_{(2,3,5)} >$ is the universal completion of the amalgam of G_1, G_2 and G_5. Using MAGMA [2] for coset enumeration, we hold that $G_3 =< z, a, b, v, t > \simeq \mathbb{Z}_2 \times S_5$ and $G_4 =< z, a, a_n, v, t > \simeq S_3 \times A_5$. So U is the universal completion of the amalgam corresponding to the Ivanov-Shpectorov geometry.

Theorem 3.5.2 *Let Γ' be a flag-transitive geometry with the same Buekenhout diagram as Γ. Assume furthermore that for a flag-transitive automorphism group H we have $H_1/K_1 \simeq J_1$, $H_2/K_2 \simeq M_{11}$ and $H_5/K_5 \simeq \mathbb{Z}_2^5 : A_5$ (a maximal parabolic in $\mathbb{Z}_2 L_3(4)$) where K_i denotes the kernel of the action of H_i on the corresponding residue. Then $K_1 = K_2 = 1$ and $B = K_5 \simeq \mathbb{Z}_2$.*

Proof. Clearly, we have that K_i is a subgroup of the Borel group B of Γ' in H for all i. Since the Borel subgroup of the geometry for H_5/K_5 is trivial, we hold $B = K_5$. We have that $H_{12}/K_{12} \simeq L_2(11)$. We have $K_{12}/K_i \triangleleft H_{12}/K_i$ for $i = 1, 2$, thus $K_{12} = K_i = 1$ since $H_{12}/K_i \simeq L_2(11)$. This implies the assertion. □

Using the *Adaptive Coset Enumerator* ACE, version 3 [13], we establish the following theorem:

Theorem 3.5.3 *Let $G :=< a, a_n, b, v, t, z \mid a^2 = a_n^2 = b^2 = v^2 = t^3 = z^2 = 1, [z, a] = [z, a_n] = [z, b] = [z, v] = 1, t^z = t^{-1}, [a, b] = [a, t] = [a_n, t] = [v, t] = [a, v] = 1, (a_n a)^5 = (a_n b)^3 = (a_n v)^3 = (tb)^5 = 1, (vb)^2 = z, z = bt[b, t]^2, (zaa_n b)^5 = (aa_n v)^5 = 1 >$. Then $G \simeq 3O'N$.*

□

As a corollary we hold by Proposition 1.2.4:

Corollary 3.5.4 *The 3-fold cover of the Ivanov-Shpectorov geometry for the sporadic O'Nan group is universal.*

$\square$

Chapter 4

Constructing an irreducible representation for the Buekenhout geometry

4.1 Introduction

In this chapter we construct 154×154-matrices over $GF(3)$ for the generators of the amalgam related to the Buekenhout geometry given in Chapter 2. These matrices provide an irreducible representation of that amalgam. This representation for the O'Nan group has been constructed in [23] using the computer. We are going to give a construction which is largely done by hand. In particular we do not use the fact that the group $O'N$ is the universal completion of the amalgam.

We start with the representation of the group $L_3(7) : \mathbb{Z}_2$, identify the generators z, X, Y, Z and ρ, and then construct a matrix a satisfying all the required relations.

The representation for $L_3(7) : \mathbb{Z}_2$ will split for that group as the direct sum of a 1-dimensional, a 57-dimensional and a 96-dimensional module all being irreducible.

In order to construct the representation for $L_3(7) : \mathbb{Z}_2$, we use the canonical generators and relations given in Chapter 2. To recall:

Set $G_1 :=< v_1, v_2, \nu, \rho, x, u, i | v_1^7 = v_2^7 = \nu^7 = 1, \rho^3 = x^4 = i^2 = 1, [v_1, v_2] = [v1, \nu] = 1, v_2^\nu = v_2 v_1, v_1^\rho = v_1^2, v_2^\rho = v_2^4, v_1^x = v_2, v_2^x = v_1^{-1}, [v_1, i] = 1, v_2^i = v_2^{-1}, \nu^\rho = \nu^4, \rho^x = \rho^{-1}, [\rho, i] = 1, x^i = x^{-1}, (\nu x)^3 = x^2, \nu^i = \nu^{-1}, [\nu, x^2] = 1, v_1^u = v_1^{-1}, v_2^u = \nu, \nu^u = v_2, [\rho, u] = 1, (x^2)^u = x^2 i, (xx^u)^3 = 1 >$.

Then $G_1 \simeq L_3(7) : \mathbb{Z}_2$. More exactly, we have $P_1 :=< v_1, v_2, \nu, \rho, x, i > \simeq \mathbb{Z}_7^2 : SL_2(7) : \mathbb{Z}_2$, where $< v_1, v_2 >= O_7(P_1)$, $< \nu, \rho, x, i > \simeq SL_2(7) : \mathbb{Z}_2$ and $< \nu, \rho, x > \simeq SL_2(7)$. Furthermore the pair (P_1, P_1^u) consists of two incident maximal parabolic subgroups of $L_3(7)$.

4.2 Constructing matrices for $L_3(7) : \mathbb{Z}_2$

4.2.1 The irreducible 57-dimensional $GF(3)$-module

It is known that $L_3(7) : \mathbb{Z}_2$ has an irreducible 57-dimensional $GF(3)$-module V_{57} (see e.g. [22]). We construct a representation simply by calculating matrices satisfying the required relations and do not consider the construction of a specific representation. As above we set $G_1 :=< v_1, v_2, \nu, \rho, x, u, i >$, $P_1 :=< v_1, v_2, \nu, \rho, x, i >$ and $W := O_7(P_1) =< v_1, v_2 >$. Then we have $V_{57} \mid_W = C_{V_{57}}(W) \oplus [V_{57}, W]$ where $[V_{57}, W] = \bigoplus_{j=1}^{8} C_{[V_{57},W]}(H_j)$, H_j the hyperplanes in W.

Since $\frac{x^7-1}{x-1}$ is irreducible over $GF(3)$, the smallest representation of an element of order seven is six dimensional and hence we hold $dimC_{V_{57}}(W) = 9$ and $dimC_{[V_{57},W]}(H_j) = 6$.

We number the hyperplanes in W as follows: $H_1 :=< v_1 >$, $H_j :=< (j-2)v_1 + v_2 >$ for $j = 2, 3, \ldots, 8$. Moreover we set

$$J := \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -1 & -1 & -1 & -1 & -1 & -1 \end{pmatrix}.$$

This implies the following approach:

$$v_1 = \begin{pmatrix} I_9 & & & & & & & & \\ & I_6 & & & & & & 0 & \\ & & J & & & & & & \\ & & & J & & & & & \\ & & & & J & & & & \\ & & & & & J & & & \\ & 0 & & & & & J & & \\ & & & & & & & J & \\ & & & & & & & & J \end{pmatrix}, \quad v_2 = \begin{pmatrix} I_9 & & & & & & & & \\ & J & & & & & & 0 & \\ & & I_6 & & & & & & \\ & & & J^6 & & & & & \\ & & & & J^5 & & & & \\ & & & & & J^4 & & & \\ & 0 & & & & & J^3 & & \\ & & & & & & & J^2 & \\ & & & & & & & & J \end{pmatrix}.$$

We set $\mathcal{C} := C_{[V_{57},W]}(H_1)$. Since $[v_1, \nu] = 1$, ν acts on $\mathcal{C}$. Using our generators and relations, we have that $t := x^2 \in Z(< \nu, \rho, x, i >)$, thus t inverts v_1 and v_2 and centralizes ν. Therefore t acts on $\mathcal{C}$ as well as v_2 because $[v_1, v_2] = 1$. Since $< \nu, \rho, x, i >$ acts transitively on the hyperplanes of W, t inverts v_2 on $\mathcal{C}$. Moreover $v_1 = v_2^{\nu} v_2^{-1}$ imlying that $[\nu, v_2] = 1$ on $\mathcal{C}$. Since t inverts v_2 and centralizes ν on $\mathcal{C}$, ν and v_2 cannot induce the same subgroup of order seven on $\mathcal{C}$, thus we can assume that $\nu = I_6$ on $\mathcal{C}$ because $GL_6(3)$ does not contain a subgroup $\mathbb{Z}_7^2$.

We identify v_2 on $\mathcal{C}$ with the permutation (1234567) where the numbers one to six represent the canonical basis vectors and seven their negative sum. Since $v_2^{\rho} = v_2^4$ and

$v_2^t = v_2^{-1}$, we can identify t with (27)(54)(36) and ρ with (253)(674). Thus we have the following on $\mathcal{C}$:

$$t \sim C := \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & -1 & -1 & -1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}, \rho \sim B := \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & -1 & -1 & -1 & -1 & -1 \end{pmatrix}$$

If we now set $C_{[V_{57},W]}(H_k)^\nu =: C_{[V_{57},W]}(H_{k\nu})$, we hold the identification $\nu \sim$ (2345678) and therefore the following on $[V_{57}, W]$:

$$\nu = \begin{pmatrix} I_6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I_6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I_6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I_6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I_6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & I_6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & I_6 \\ 0 & I_6 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad t = x^2 = diag(C)$$

We approach i, ρ and x on $[V_{57}, W]$ by the following:

$i = (A_{ij})_{i,j=1,\ldots 8}$, $\rho := (R_{ij})_{i,j=1,\ldots 8}$ and $x := (X_{ij})_{i,j=1,\ldots 8}$. Where A_{ij}, R_{ij}, X_{ij} are elements of $GF(3)^{6\times 6}$.

Then, using the relation $v_1 i = i v_1$, we hold:

$$v_1 i = \begin{pmatrix} A_{11} & A_{12} & A_{13} & A_{14} & A_{15} & A_{16} & A_{17} & A_{18} \\ JA_{21} & JA_{22} & JA_{23} & JA_{24} & JA_{25} & JA_{26} & JA_{27} & JA_{28} \\ \vdots & & & \ddots & & & & \vdots \end{pmatrix} =$$

$$\begin{pmatrix} A_{11} & A_{12}J & A_{13}J & A_{14}J & A_{15}J & A_{16}J & A_{17}J & A_{18}J \\ A_{21} & A_{22}J & A_{23}J & A_{24}J & A_{25}J & A_{26}J & A_{27}J & A_{28}J \\ \vdots & & & \ddots & & & & \vdots \end{pmatrix} = i v_1.$$

Therefore we get $A_{12} = A_{13} = A_{14} = \ldots = A_{18} = 0$, $A_{21} = A_{31} = A_{41} = \ldots = A_{81} = 0$ and $A_{ij}J = JA_{ij}$ for all A_{ij} except A_{11}.

The relation $v_2 i = i v_2^{-1}$ leads to:

$$v_2 i = \begin{pmatrix} JA_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & A_{22} & A_{23} & A_{24} & A_{25} & A_{26} & A_{27} & A_{28} \\ 0 & J^6A_{32} & J^6A_{33} & J^6A_{34} & J^6A_{35} & J^6A_{36} & J^6A_{37} & J^6A_{38} \\ \vdots & & & \ddots & & & & \vdots \end{pmatrix} =$$

$$\begin{pmatrix} A_{11}J^6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & A_{22} & A_{23}J & A_{24}J^2 & A_{25}J^3 & A_{26}J^4 & A_{27}J^5 & A_{28}J^6 \\ 0 & A_{32} & A_{33}J & A_{34}J^2 & A_{35}J^3 & A_{36}J^4 & A_{37}J^5 & A_{38}J^6 \\ \vdots & & & \ddots & & & & \vdots \end{pmatrix} = iv_2.$$

Together with $A_{ij}J = JA_{ij}$ from above, this shows that $A_{ij} = 0$ except for A_{11}, A_{22}, A_{38}, A_{47}, A_{56}, A_{65}, A_{74} and A_{83} which are therefore elements of $GL_6(3)$. Furthermore the relation $\nu^i = \nu^{-1}$ yields $A_{22} = A_{38} = \ldots = A_{83}$. Moreover we have that $J^{A_{11}} = J^{-1}$, $[A_{22}, J] = 1$ and $A_{11}^2 = A_{22}^2 = I_6$. The relation $it = ti$ proves that $[A_{11}, C] = [A_{22}, C] = 1$. Now $\mid C_{GL_6(3)}(< J, C >) \mid = 2 \cdot 13$ and $\mid N_{GL_6(3)}(< J >) \cap C_{GL_6(3)}(C) \mid = 2^2 \cdot 3 \cdot 13$ having a normal 2-Sylow subgroup. Therefore we have $A_{11} = \pm C$ and $A_{22} = \pm I_6$ and choose $A_{11} := -C$ and $A_{22} := -I_6$. Thus we hold:

$$i = \begin{pmatrix} -C & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -I_6 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I_6 \\ 0 & 0 & 0 & 0 & 0 & 0 & -I_6 & 0 \\ 0 & 0 & 0 & 0 & 0 & -I_6 & 0 & 0 \\ 0 & 0 & 0 & 0 & -I_6 & 0 & 0 & 0 \\ 0 & 0 & 0 & -I_6 & 0 & 0 & 0 & 0 \\ 0 & 0 & -I_6 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

To compute ρ, we start with the relation $v_1\rho = \rho v_1^2$ leading to

$$v_1\rho = \begin{pmatrix} R_{11} & R_{12} & R_{13} & R_{14} & R_{15} & R_{16} & R_{17} & R_{18} \\ JR_{21} & JR_{22} & JR_{23} & JR_{24} & JR_{25} & JR_{26} & JR_{27} & JR_{28} \\ \vdots & & & \ddots & & & & \vdots \end{pmatrix} =$$

$$\begin{pmatrix} R_{11} & R_{12}J^2 & R_{13}J^2 & R_{14}J^2 & R_{15}J^2 & R_{16}J^2 & R_{17}J^2 & R_{18}J^2 \\ R_{21} & R_{22}J^2 & R_{23}J^2 & R_{24}J^2 & R_{25}J^2 & R_{26}J^2 & R_{27}J^2 & R_{28}J^2 \\ \vdots & & & \ddots & & & & \vdots \end{pmatrix} = \rho v_1^2.$$

Hence $R_{12} = R_{13} = \ldots = R_{18} = 0$ and $R_{21} = R_{31} = \ldots = R_{81} = 0$. Similarily, using $v_2\rho = \rho v_2^4$, we hold $R_{23} = R_{24} = \ldots R_{28} = 0$ and $R_{32} = R_{42} = \ldots = R_{82} = 0$. By the relation $\nu\rho = \rho\nu^4$, we deduce that $R_{ij} = 0$ except for R_{11}, R_{22} and $R_{36} = R_{43} = R_{57} = R_{64} = R_{78} = R_{85} = R_{22}$. Thus $R_{11}, R_{22} \in GL_6(3)$. Moreover $v_1\rho = \rho v_1^2$ yields $J^{R_{22}} = J^2$ and $v_2\rho = \rho v_2^4$ yields $J^{R_{11}} = J^4$. Exploiting $\rho i = i\rho$, we hold $[R_{11}, C] = 1$. Also, since ρ should be of order three, we get $o(R_{11}) = o(R_{22}) = 3$. Thus we can choose $R_{11} := B$ and $R_{22} := B^{-1}$ leading to

$$\rho = \begin{pmatrix} B & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & B^{-1} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & B^{-1} & 0 & 0 \\ 0 & 0 & B^{-1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & B^{-1} & 0 \\ 0 & 0 & 0 & B^{-1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & B^{-1} \\ 0 & 0 & 0 & 0 & B^{-1} & 0 & 0 & 0 \end{pmatrix}.$$

We compute x in a similar way. We start using the reation $v_1x = xv_2$:

$$v_1x = \begin{pmatrix} X_{11} & X_{12} & X_{13} & X_{14} & X_{15} & X_{16} & X_{17} & X_{18} \\ JX_{21} & JX_{22} & JX_{23} & JX_{24} & JX_{25} & JX_{26} & JX_{27} & JX_{28} \\ JX_{31} & JX_{32} & JX_{33} & JX_{34} & JX_{35} & JX_{36} & JX_{37} & JX_{38} \\ \vdots & & & \ddots & & & & \vdots \end{pmatrix} =$$

$$\begin{pmatrix} X_{11}J & X_{12} & X_{13}J^6 & X_{14}J^5 & X_{15}J^4 & X_{16}J^3 & X_{17}J^2 & X_{18}J \\ X_{21}J & X_{22} & X_{23}J^6 & X_{24}J^5 & X_{25}J^4 & X_{26}J^3 & X_{27}J^2 & X_{28}J \\ X_{31}J & X_{32} & X_{33}J^6 & X_{34}J^5 & X_{35}J^4 & X_{36}J^3 & X_{37}J^2 & X_{38}J \\ \vdots & & & \ddots & & & & \vdots \end{pmatrix} = xv_2.$$

This proves $X_{11} = X_{13} = \ldots = X_{18} = 0$ and $X_{22} = X_{32} = \ldots = X_{82} = 0$ and therefore $X_{12}, X_{21} \in GL_6(3)$. By $v_2x = xv_1^{-1}$ we hold:

$$v_2x = \begin{pmatrix} 0 & JX_{12} & 0 & 0 & 0 & 0 & 0 & 0 \\ X_{21} & 0 & X_{23} & X_{24} & X_{25} & X_{26} & X_{27} & X_{28} \\ J^{-1}X_{31} & 0 & J^{-1}X_{33} & J^{-1}X_{34} & J^{-1}X_{35} & J^{-1}X_{36} & J^{-1}X_{37} & J^{-1}X_{38} \\ \vdots & & & \ddots & & & & \vdots \end{pmatrix} =$$

$$\begin{pmatrix} 0 & X_{12}J^{-1} & 0 & 0 & 0 & 0 & 0 & 0 \\ X_{21} & 0 & X_{23}J^{-1} & X_{24}J^{-1} & X_{25}J^{-1} & X_{26}J^{-1} & X_{27}J^{-1} & X_{28}J^{-1} \\ X_{31} & 0 & X_{33}J^{-1} & X_{34}J^{-1} & X_{35}J^{-1} & X_{36}J^{-1} & X_{37}J^{-1} & X_{38}J-1 \\ \vdots & & & \ddots & & & & \vdots \end{pmatrix} = xv_1^{-1}.$$

Thus $X_{21} = X_{31} = \ldots = X_{81} = 0$ from the first column and $X_{23} = X_{24} = \ldots = X_{28} = 0$ from the second row. The third row shows $X_{3j}J^{-1} = J^{-1}X_{3j}$ implying $X_{3j}J = JX_{3j}$. Using this information and the third row in $v_1x = xv_2$, we get $X_{33} = X_{34} = \ldots = X_{37} = 0$ and $X_{38} \neq 0$.

As a next step we use $\rho x \rho = x$ ($\rho^x = \rho^{-1}$) and find

$$x = \begin{pmatrix} 0 & X_{12} & 0 & 0 & 0 & 0 & 0 & 0 \\ X_{21} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & X_{38} \\ 0 & 0 & 0 & 0 & B^{-1}X_{38}B^{-1} & 0 & 0 & 0 \\ 0 & 0 & X_{53} & X_{54} & X_{55} & X_{56} & X_{57} & X_{58} \\ 0 & 0 & 0 & 0 & 0 & 0 & BX_{38}B & 0 \\ 0 & 0 & BX_{56}B & BX_{53}B & BX_{57}B & BX_{54}B & BX_{58}B & BX_{55}B \\ 0 & 0 & B^{-1}X_{54}B^{-1} & B^{-1}X_{56}B^{-1} & B^{-1}X_{58}B^{-1} & B^{-1}X_{53}B^{-1} & B^{-1}X_{55}B^{-1} & B^{-1}X_{57}B^{-1} \end{pmatrix}$$

where $[X_{12}, B] = [X_{21}, B] = 1$ because $X_{12}, X_{21} \in GL_6(3)$. Furthermore since x has to be invertible, $X_{38} \in GL_6(3)$.

Since $x^2 = t = diag(C)$, we hold $X_{12}X_{21} = C = X_{21}X_{12}$, $X_{38}B^{-1}X_{54}B^{-1} = C$, $X_{38}B^{-1}X_{5j}B^{-1} = 0$ for $j \in \{3, 5, 6, 7, 8\}$. Thus $X_{5j} = 0$ except for $X_{54} = BCX_{38}^{-1}B = CBX_{38}^{-1}B$ since $[B, C] = 1$ and

$$x = \begin{pmatrix} 0 & X_{12} & 0 & 0 & 0 & 0 & 0 & 0 \\ X_{21} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & X_{38} \\ 0 & 0 & 0 & 0 & B^1X_{38}B^1 & 0 & 0 & 0 \\ 0 & 0 & 0 & CBX_{38}^{-1}B & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & BX_{38}B & 0 \\ 0 & 0 & 0 & 0 & 0 & CB^{-1}X_{38}^{-1}B^{-1} & 0 & 0 \\ 0 & 0 & CX_{38}^{-}1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

furthermore $[X_{38}, C] = 1$. The last relation has to hold since $x^2 = diag(C)$ implies $X_{54} = BCX_{38}B$ as well as $X_{54} = BX_{38}CB$. Now X_{38} also commutes with J as seen above. Thus $X_{38} \in C_{GL_6(3)}(< C, J >)$. By $(\nu x)^3 = t$, we get the relation $X_{12}X_{38}X_{21} = C$, hence $X_{38} = I_6$ since $X_{12}X_{21} = C = X_{21}X_{12}$. The relation $v_2 x = x v_1^{-1}$ shows that $J^{X_{12}} = J^{-1}$. Therefore $[X_{12}, X_{21}] = 1$ leads to $X_{21} \in C_{GL_6(3)}(< C, J, B >)$, $X_{12} \in CC_{GL_6(3)}(< C, J, B >)$. Now $C_{GL_6(3)}(< C, J, B >) = \{I_6, -I_6\}$ yields $X_{12} = \pm C$ and $X_{21} = \pm I_6$ and we choose $X_{12} = C$ and therefore $X_{21} = I_6$. Thus

$$x = \begin{pmatrix} 0 & C & 0 & 0 & 0 & 0 & 0 & 0 \\ I_6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & I_6 \\ 0 & 0 & 0 & 0 & B & 0 & 0 & 0 \\ 0 & 0 & 0 & CB^{-1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & B^{-1} & 0 \\ 0 & 0 & 0 & 0 & 0 & CB & 0 & 0 \\ 0 & 0 & C & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

The relation $xi = ix^{-1} = tix$ is also fulfilled.

Set $\tilde{C} := C_{V_{57}}(W)$ with $W =< v_1, v_2 >$ and we compute ν, i, ρ and x on $\tilde{C}$. Recall that $\mathcal{C} = C_{[V_{57},W]}(v_1)$. Since $v_1 = I_9 = v_2$ on $\tilde{C}$, we use the 15-dimensional space $U = \tilde{C} \oplus \mathcal{C}$ in order to have more information. Clearly, ν acts on $\tilde{C}$, thus we can choose a basis of U such that

$$v_2 = \begin{pmatrix} 1\,0\,0 & 0 & 0 \\ 0\,1\,0 & 0 & 0 \\ 0\,0\,1 & 0 & 0 \\ 0\,0\,0 & I_6 & 0 \\ 0\,0\,0 & 0 & J \end{pmatrix} \text{ and } \nu = \begin{pmatrix} 1\,0\,0 & 0 & 0 \\ 0\,1\,0 & 0 & 0 \\ 0\,0\,1 & 0 & 0 \\ 0\,0\,0 & J & 0 \\ 0\,0\,0 & 0 & I_6 \end{pmatrix}$$

because, by $\nu \sim_{G_1} v_1$, we have $dimC_{V_{57}}(\nu) = 15$ and $dimC_{[V_{75},W]}(\nu) = 12$ implying that $C_{V_{57}}(S)$ with $S =< v_1, v_2, \nu >$ is three dimensional. Furthermore we have $N_{L_3(7):\mathbb{Z}_2}(S) =< S, i, t, \rho, u >=: N$ acts on $C_{V_{57}}(S)$. Since i inverts both, v_2 and ν, we find that i has the following shape on U:

$$i = \begin{pmatrix} A_{11} & 0 & 0 \\ 0 & A_{21} & 0 \\ 0 & 0 & A_{33} \end{pmatrix},$$

where $A_{11} \in GL_3(3)$ and $A_{22}, A_{33} \in GL_6(3)$. Moreover we hold $A_{ii}^2 = 1$ and $J^{A_{22}} = J^{A_{33}} = J^{-1}$. Therefore we choose $A_{22} := C$ and from the previous calculations we have $A_{33} = -C$ because A_{33} determines the action of i on $\mathcal{C}$ computed above. Since $v_2^\rho = v_2^4$, $\nu^\rho = \nu^4$ and $[\rho, i] = 1$ we hold the following for ρ on U:

$$\rho = \begin{pmatrix} R_{11} & 0 & 0 \\ 0 & B & 0 \\ 0 & 0 & B \end{pmatrix},$$

where $R_{22} \in GL_6(3)$ was chosen to be B, $R_{11} \in GL_3(3)$ and $[A_{11}, R_{11}] = 1$. Also, by $[i, u] = 1$ and $\mathbb{Z}_7^{1+2} : (\mathbb{Z}_3 \times D_8) \simeq N$, we get that $< i, u, t > \simeq D_8$ and $i \in Z(< i, u, t >)$. Thereby $A_{11} \notin Z(GL_3(3))$[1] . Moreover we hold that t is of the following shape on U:

$$t = \begin{pmatrix} T_{11} & 0 & 0 \\ 0 & T_{22} & 0 \\ 0 & 0 & C \end{pmatrix},$$

where $T_{11} \in GL_3(3)$, $T_{22} \in GL_6(3)$. Since $[u, t] \neq 1$, we get that $T_{11} \notin Z(GL_3(3))$. This yields $R_{11} = I_3$ because $< A_{11}, T_{11} > \simeq \mathbb{Z}_2^2$ and the centralizer of such a group in $GL_3(3)$ is isomorphic to $\mathbb{Z}_2^3$. Since t centralizes ρ, i and ν, we therefore hold $T_{22} \in C_{GL_6(3)}(< C, B, J >) = Z(GL_6(3))$, thus $T_{22} = \pm I_6$ so we choose $T_{22} = -I_6$.

[1]Note that $\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}$ is not a square in $GL_3(3)$

Now $< A_{11}, T_{11} >$ must fix a vector of the space $GF(3)^3$. Therefore we choose

$$A_{11} = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 1 & 1 \end{pmatrix} \text{ and } T_{11} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}.$$

In order to compute x on $\tilde{C}$, we procced as follows. Let $x = (a_{ij})_{1 \leq i,j \leq 9}$. Exploiting the relations $\rho x = x\rho^{-1}$, $xi = itx$, $x^2 = t$ and $(\nu x)^3 = t$, we get that

$$x = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & \epsilon & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & \epsilon & \epsilon & 0 & \epsilon & 0 \\ 0 & 0 & 0 & 1 & 2 & 2 & 0 & 2 & 0 \\ 0 & 0 & \epsilon & 0 & 1 & 1 & 2 & 1 & 2 \\ 0 & 0 & \epsilon & 1 & 2 & 2 & 1 & 2 & 0 \\ 0 & -\epsilon & -\epsilon & 1 & 2 & 1 & 0 & 2 & 0 \\ 0 & 0 & \epsilon & 1 & 2 & 2 & 0 & 2 & 1 \\ 0 & -\epsilon & -\epsilon & 1 & 2 & 2 & 0 & 1 & 0 \end{pmatrix}, \ \epsilon \in \{1, 2\}.$$

Thus we choose

$$x = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 2 & 2 & 0 & 2 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & 2 & 1 & 2 \\ 0 & 0 & 1 & 1 & 2 & 2 & 1 & 2 & 0 \\ 0 & 2 & 2 & 1 & 2 & 1 & 0 & 2 & 0 \\ 0 & 0 & 1 & 1 & 2 & 2 & 0 & 2 & 1 \\ 0 & 2 & 2 & 1 & 2 & 2 & 0 & 1 & 0 \end{pmatrix}.$$

We start to compute u on U. Since u normalizes S, u acts on U and we approach u by

$$u = \begin{pmatrix} A & 0 & 0 \\ 0 & A_{11} & A_{12} \\ 0 & A_{21} & A_{22} \end{pmatrix},$$

where $A \in GL_3(3)$ and $A_{ij} \in GL_6(3)$. Then the relation $v_2 u = u\nu$ yields

$$\begin{pmatrix} A & 0 & 0 \\ 0 & A_{11} & A_{12} \\ 0 & JA_{21} & JA_{22} \end{pmatrix} = \begin{pmatrix} A & 0 & 0 \\ 0 & A_{11}J & A_{12} \\ 0 & A_{21}J & A_{22} \end{pmatrix},$$

hence $A_{11} = A_{22} = 0$. Exploiting $iu = ui$, we get

$$A = \begin{pmatrix} a_{11} & a_{12} & 0 \\ a_{31} & a_{32} + a_{33} & 0 \\ a_{31} & a_{32} & a_{33} \end{pmatrix},$$

$CA_{12} + A_{12}C = 0$ and $CA_{21} + A_{21}C = 0$.

Now we use the relation $tu = uti$ and get on $C_{V_{57}}(S)$:

$$\begin{pmatrix} a_{11} & a_{12} & 0 \\ -a_{31} & -a_{32} - a_{33} & 0 \\ 2a_{31} & -a_{32} & -a_{33} \end{pmatrix} = \begin{pmatrix} -a_{11} & a_{12} & 0 \\ -a_{31} & a_{32} + a_{33} & 0 \\ -a_{31} & a_{32} - a_{33} & -a_{33} \end{pmatrix},$$

thus $a_{11} = 0$ and $a_{32} + a_{33} = 0$. Hence we hold

$$A = \begin{pmatrix} 0 & a_{12} & 0 \\ a_{31} & 0 & 0 \\ a_{31} & -a_{33} & a_{33} \end{pmatrix}.$$

Also $u^2 = 1$ implies $A^2 = 1$ which gives us

$$A = \begin{pmatrix} 0 & a_{12} & 0 \\ a_{12} & 0 & 0 \\ a_{12} & -a_{33} & a_{33} \end{pmatrix}.$$

The entries of A will be determined using the Weyl relation. Using $[u, \rho] = 1$, we obtain $[B, A_{12}] = [B, A_{21}] = 1$. Also, $u^2 = 1$ implies $A_{12} = A_{21}^{-1}$. Together with $[J, A_{12}] = 1$ and $CA_{12} = -A_{12}C$ this yields

$$A_{12} = \pm \begin{pmatrix} -1 & 1 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 & 0 & 1 \\ -1 & -1 & 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & -1 & 1 & 1 \\ -1 & 0 & -1 & -1 & 1 & 0 \\ 0 & -1 & 0 & -1 & -1 & 1 \end{pmatrix}.$$

Notice that this implies $A_{21} = A_{12}^{-1} = -A_{12}$.

We approach u on $[V_{57}, v_1]$ as follows. Set $u = (a_{ij})_{1 \le i,j \le 7}$ where $a_{ij} \in GL_6(3)$. Using the relations $v_2 u = u\nu$ and $\nu u = uv_2$, we hold

$$u = \begin{pmatrix} a & a & a & a & a & a & a \\ a & Ja & J^2a & J^3a & J^4a & J^5a & J^6a \\ a & J^2a & J^4a & J^6a & Ja & J^3a & J^5a \\ a & J^3a & J^6a & J^2a & J^5a & Ja & J^4a \\ a & J^4a & Ja & J^5a & J^2a & J^6a & J^3a \\ a & J^5a & J^3a & Ja & J^6a & J^4a & J^2a \\ a & J^6a & J^5a & J^4a & J^3a & J^2a & Ja \end{pmatrix}, J^a = J^{-1}, a \in GL_6(3).$$

Thereby $u^2 = 1$ implies $a^2 = 1$. Furthermore $tu = uti$ gives $Ca = -aC$ and $\rho u = u\rho$ implies $[B, a] = 1$. Thus we get that

$$a = \pm \begin{pmatrix} 1 & -1 & -1 & 0 & -1 & 0 \\ 1 & 1 & -1 & 1 & -1 & -1 \\ 0 & 1 & 0 & 1 & 1 & -1 \\ 1 & 0 & 1 & 1 & -1 & 0 \\ -1 & 0 & 0 & 1 & -1 & -1 \\ 1 & 1 & -1 & 0 & 0 & 1 \end{pmatrix}.$$

Checking the Weyl relation $(xx^u)^3 = 1$ leads to the following possibilities (for A_{12} and a we just give the sign of the above matrices):

- $a_{12} = -1, a_{33} = -1, A_{12} : +, a : +,$
- $a_{12} = -1, a_{33} = -1, A_{12} : -, a : +,$
- $a_{12} = 1, a_{33} = -1, A_{12} : +, a : +,$
- $a_{12} = 1, a_{33} = -1, A_{12} : -, a : +.$

Of course, multiplication of u with -1 does not change the relation, thus we have eight possibilities. Therefore we choose $a_{12} = a_{33} = -1$ and

$$A_{12} = \begin{pmatrix} -1 & 1 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 & 0 & 1 \\ -1 & -1 & 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & -1 & 1 & 1 \\ -1 & 0 & -1 & -1 & 1 & 0 \\ 0 & -1 & 0 & -1 & -1 & 1 \end{pmatrix}, a = \begin{pmatrix} 1 & -1 & -1 & 0 & -1 & 0 \\ 1 & 1 & -1 & 1 & -1 & -1 \\ 0 & 1 & 0 & 1 & 1 & -1 \\ 1 & 0 & 1 & 1 & -1 & 0 \\ -1 & 0 & 0 & 1 & -1 & -1 \\ 1 & 1 & -1 & 0 & 0 & 1 \end{pmatrix}.$$

4.2.1.1 Summary

We summarize the above results. Set

$$J = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ -1 & -1 & -1 & -1 & -1 & -1 \end{pmatrix}, C = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & -1 & -1 & -1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix},$$

$$B = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ -1 & -1 & -1 & -1 & -1 & -1 \end{pmatrix},$$

as matrices in $GL_6(3)$. Furthermore we set

$$\nu_9 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & J \end{pmatrix}, \rho_9 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & B \end{pmatrix}, i_9 = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & C \end{pmatrix}$$

and

$$x_9 = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & -1 & -1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 & -1 & 1 & -1 \\ 0 & 0 & 1 & 1 & -1 & -1 & 1 & -1 & 0 \\ 0 & -1 & -1 & 1 & -1 & 1 & 0 & -1 & 0 \\ 0 & 0 & 1 & 1 & -1 & -1 & 0 & -1 & 1 \\ 0 & -1 & -1 & 1 & -1 & -1 & 0 & 1 & 0 \end{pmatrix}$$

as matrices in $GL_9(3)$. Moreover let

$$\nu_{48} = \begin{pmatrix} I_6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I_6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I_6 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I_6 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I_6 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & I_6 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & I_6 \\ 0 & I_6 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \rho_{48} = \begin{pmatrix} B & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & B^{-1} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & B^{-1} & 0 & 0 \\ 0 & 0 & B^{-1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & B^{-1} & 0 \\ 0 & 0 & 0 & B^{-1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & B^{-1} \\ 0 & 0 & 0 & 0 & B^{-1} & 0 & 0 & 0 \end{pmatrix},$$

$$i_{48} = \begin{pmatrix} -C & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -I_6 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -I_6 \\ 0 & 0 & 0 & 0 & 0 & 0 & -I_6 & 0 \\ 0 & 0 & 0 & 0 & 0 & -I_6 & 0 & 0 \\ 0 & 0 & 0 & 0 & -I_6 & 0 & 0 & 0 \\ 0 & 0 & 0 & -I_6 & 0 & 0 & 0 & 0 \\ 0 & 0 & -I_6 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

and

$$x_{48} = \begin{pmatrix} 0 & C & 0 & 0 & 0 & 0 & 0 & 0 \\ I_6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & I_6 \\ 0 & 0 & 0 & 0 & B & 0 & 0 & 0 \\ 0 & 0 & 0 & CB^{-1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & B^{-1} & 0 \\ 0 & 0 & 0 & 0 & 0 & CB & 0 & 0 \\ 0 & 0 & C & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

as matrices in $GL_{48}(3)$. We set

$$v_{1,57} = \begin{pmatrix} I_9 &&&&&&&& \\ & I_6 &&&&&&& \\ && J &&&&&& \\ &&& J &&&&& \\ &&&& J &&&& \\ &&&&& J &&& \\ &&&&&& J && \\ &&&&&&& J & \\ &&&&&&&& J \end{pmatrix}, \quad v_{2,57} = \begin{pmatrix} I_9 &&&&&&&& \\ & J &&&&&&& \\ && I_6 &&&&&& \\ &&& J^6 &&&&& \\ &&&& J^5 &&&& \\ &&&&& J^4 &&& \\ &&&&&& J^3 && \\ &&&&&&& J^2 & \\ &&&&&&&& J \end{pmatrix},$$

$$\nu_{57} = \begin{pmatrix} \nu_9 & 0 \\ 0 & \nu_{48} \end{pmatrix}, \rho_{57} = \begin{pmatrix} \rho_9 & 0 \\ 0 & \rho_{48} \end{pmatrix}, x_{57} = \begin{pmatrix} x_9 & 0 \\ 0 & x_{48} \end{pmatrix} \text{ and } i_{57} = \begin{pmatrix} i_9 & 0 \\ 0 & i_{48} \end{pmatrix}.$$

Then we have proved the following:

Lemma 4.2.1 *We have* $GL_{57}(3) \geq < v_{1,57}, v_{2,57}, \nu_{57}, \rho_{57}, x_{57}, i_{57} > \simeq \mathbb{Z}_7^2 : SL_2(7) : \mathbb{Z}_2$.

□

Now we put

$$A = \begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ -1 & 1 & -1 \end{pmatrix} \in GL_3(3), A_{12} = \begin{pmatrix} -1 & 1 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 1 & 0 & 1 \\ -1 & -1 & 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & -1 & 1 & 1 \\ -1 & 0 & -1 & -1 & 1 & 0 \\ 0 & -1 & 0 & -1 & -1 & 1 \end{pmatrix} \in GL_6(3).$$

We set

$$u_{15} = \begin{pmatrix} A & 0 & 0 \\ 0 & 0 & A_{12} \\ 0 & -A_{12} & 0 \end{pmatrix}.$$

Furthermore

$$a = \begin{pmatrix} 1 & -1 & -1 & 0 & -1 & 0 \\ 1 & 1 & -1 & 1 & -1 & -1 \\ 0 & 1 & 0 & 1 & 1 & -1 \\ 1 & 0 & 1 & 1 & -1 & 0 \\ -1 & 0 & 0 & 1 & -1 & -1 \\ 1 & 1 & -1 & 0 & 0 & 1 \end{pmatrix} \in GL_6(3)$$

and

$$u_{42} = \begin{pmatrix} a & a & a & a & a & a & a \\ a & Ja & J^2a & J^3a & J^4a & J^5a & J^6a \\ a & J^2a & J^4a & J^6a & Ja & J^3a & J^5a \\ a & J^3a & J^6a & J^2a & J^5a & Ja & J^4a \\ a & J^4a & Ja & J^5a & J^2a & J^6a & J^3a \\ a & J^5a & J^3a & Ja & J^6a & J^4a & J^2a \\ a & J^6a & J^5a & J^4a & J^3a & J^2a & Ja \end{pmatrix}.$$

If we set

$$u_{57} = \begin{pmatrix} u_{15} & 0 \\ 0 & u_{42} \end{pmatrix},$$

then the following holds:

Lemma 4.2.2 *We have* $GL_{57}(3) \geq < v_{1,57}, v_{2,57}, \nu_{57}, \rho_{57}, x_{57}, i_{57}, u_{57} > \simeq L_3(7) : \mathbb{Z}_2$. *Moreover these matrices provide an irreducible 57-dimensional representation of the group* $L_3(7) : \mathbb{Z}_2$.

□

4.2.2 The irreducible 96-dimensional $GF(3)$-module

By [22], we see that $L_3(7) : \mathbb{Z}_2$ has an irreducible 96-dimensional $GF(3)$-module V_{96}. This module splits for $W = O_7(P_1)$ as $V_{96} = C_{V_{96}}(W) \bigoplus_{j=1}^{8} C_{[V_{96},W]}(H_j)$ using the notation introduced above. Since $6 \mid dimC_{[V_{96},W]}(H_j)$, we find that $V_{96} = \bigoplus_{j=1}^{8} C_{V_{96}}(H_j)$ with $dimC_{V_{96}}(H_j) = 12$. We set

$$T := \begin{pmatrix} J^{-1} & 0 \\ 0 & J \end{pmatrix}, Y := \begin{pmatrix} 0 & I_6 \\ I_6 & 0 \end{pmatrix} \text{ and } S := \begin{pmatrix} J & 0 \\ 0 & J \end{pmatrix}.$$

We set

$$v_{1,96} := \begin{pmatrix} I_{12} & & & & & & & \\ & T & & & & & & \\ & & T & & 0 & & & \\ & & & T & & & & \\ & & & & T & & & \\ & & 0 & & & T & & \\ & & & & & & T & \\ & & & & & & & T \end{pmatrix} \text{ and } v_{2,96} := \begin{pmatrix} T & & & & & & & \\ & I_{12} & & & & & & \\ & & T^6 & & & 0 & & \\ & & & T^5 & & & & \\ & & & & T^4 & & & \\ & & 0 & & & T^3 & & \\ & & & & & & T^2 & \\ & & & & & & & T \end{pmatrix}.$$

Moreover we set $t_{96} = x_{96}^2 := diag(Y)$ and

$$\nu_{96} := \begin{pmatrix} S & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & I_{12} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I_{12} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & I_{12} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & I_{12} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & I_{12} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & I_{12} \\ 0 & I_{12} & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Furthermore we set

$$\tilde{C} := \begin{pmatrix} C & 0 \\ 0 & C \end{pmatrix}, D := \begin{pmatrix} 0 & C \\ C & 0 \end{pmatrix} \text{ and } R := \begin{pmatrix} B & 0 \\ 0 & B \end{pmatrix}.$$

Now we proceed as for the 57-dimensional module. That is we compute matrices which satisfy the relations for our canonical generators for $L_3(7) : \mathbb{Z}_2$ but we are not considering a particular representation. This means that we construct the matrices successively using all matrices obtained so far. Then we try to determine every matrix as far as possible and choose appropriate matrices when we have more than one choice.

Then similar computations as for the 57-dimensional module lead to

$$i_{96} := \begin{pmatrix} \tilde{C} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & D & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & D \\ 0 & 0 & 0 & 0 & 0 & 0 & D & 0 \\ 0 & 0 & 0 & 0 & 0 & D & 0 & 0 \\ 0 & 0 & 0 & 0 & D & 0 & 0 & 0 \\ 0 & 0 & 0 & D & 0 & 0 & 0 & 0 \\ 0 & 0 & D & 0 & 0 & 0 & 0 & 0 \end{pmatrix},$$

$$\rho_{96} := \begin{pmatrix} R & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & R^{-1} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & R^{-1} & 0 & 0 \\ 0 & 0 & R^{-1} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & R^{-1} & 0 \\ 0 & 0 & 0 & R^{-1} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & R^{-1} \\ 0 & 0 & 0 & 0 & R^{-1} & 0 & 0 & 0 \end{pmatrix} \text{ and}$$

$$x_{96} := \begin{pmatrix} 0 & I_{12} & 0 & 0 & 0 & 0 & 0 & 0 \\ Y & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & S^{-1} \\ 0 & 0 & 0 & 0 & (RSR)^{-1} & 0 & 0 & 0 \\ 0 & 0 & 0 & YRSR & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & RS^{-1}R & 0 \\ 0 & 0 & 0 & 0 & 0 & Y(RS^{-1}R)^{-1} & 0 & 0 \\ 0 & 0 & YS & 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

Thus we have established the following lemma

Lemma 4.2.3 *We have* $GL_{96}(3) \geq < v_{1,96}, v_{2,96}, \nu_{96}, \rho_{96}, x_{96}, i_{96} > \simeq \mathbb{Z}_7^2 : SL_2(7) : \mathbb{Z}_2$.

□

The matrix u_{96} is also computed similarily. Here we obtain

$$u_{96} := \begin{pmatrix} \tilde{U} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & a & a & a & a & a & a & a \\ 0 & a & Ta & T^2a & T^3a & T^4a & T^5a & T^6a \\ 0 & a & T^2a & T^4a & T^6a & Ta & T^3a & T^5a \\ 0 & a & T^3a & T^6a & T^2a & T^5a & Ta & T^4a \\ 0 & a & T^4a & Ta & T^5a & T^2a & T^6a & T^3 \\ 0 & a & T^5a & T^3a & Ta & T^6a & T^4a & T^2a \\ 0 & a & T^6a & T^5a & T^4a & T^3a & T^2a & Ta \end{pmatrix},$$

with

$$\tilde{U} := \begin{pmatrix} \pm C & 0 \\ 0 & \pm I_6 \end{pmatrix} \text{ and } a := \begin{pmatrix} bC & b^C \\ b & Cb \end{pmatrix} = \begin{pmatrix} bC & b^3 \\ b & Cb \end{pmatrix}$$

where $b \in C_{GL_6(3)}(< B, J >)$, $o(b) = 8$. Thus we find

$$b = \begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ -1 & -1 & -1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 0 & 1 & 1 \\ -1 & 0 & -1 & -1 & -1 & 0 \\ 0 & -1 & 0 & -1 & -1 & 0 \end{pmatrix}^i, i \in \{1, 3, 5, 7\}.$$

Checking the Weyl relation leads to the following:

$$\tilde{U} = \begin{pmatrix} C & 0 \\ 0 & I_6 \end{pmatrix} \text{ and}$$

$$b = \begin{pmatrix} -1 & -1 & -1 & 0 & -1 & 0 \\ 0 & -1 & -1 & -1 & 0 & -1 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & -1 & -1 & -1 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}.$$

Of course, multiplication with $-I_{96}$ does not change then relation giving us $-u_{96}$ as a second possibility. Thus we choose

$$u_{96} := \begin{pmatrix} \tilde{U} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & a & a & a & a & a & a & a \\ 0 & a & Ta & T^2a & T^3a & T^4a & T^5a & T^6a \\ 0 & a & T^2a & T^4a & T^6a & Ta & T^3a & T^5a \\ 0 & a & T^3a & T^6a & T^2a & T^5a & Ta & T^4a \\ 0 & a & T^4a & Ta & T^5a & T^2a & T^6a & T^3 \\ 0 & a & T^5a & T^3a & Ta & T^6a & T^4a & T^2a \\ 0 & a & T^6a & T^5a & T^4a & T^3a & T^2a & Ta \end{pmatrix},$$

with

$$\tilde{U} = \begin{pmatrix} C & 0 \\ 0 & I_6 \end{pmatrix} \text{ and } a = \begin{pmatrix} bC & b^3 \\ b & Cb \end{pmatrix},$$

with

$$b = \begin{pmatrix} -1 & -1 & -1 & 0 & -1 & 0 \\ 0 & -1 & -1 & -1 & 0 & -1 \\ 1 & 1 & 0 & 0 & 0 & 1 \\ -1 & 0 & 0 & -1 & -1 & -1 \\ 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \end{pmatrix}.$$

Then the following lemma holds:

Lemma 4.2.4 *We have* $GL_{96}(3) \geq < v_{1,96}, v_{2,96}, \nu_{96}, \rho_{96}, x_{96}, i_{96}, u_{96} > \simeq L_3(7) : \mathbb{Z}_2$. *Moreover these matrices provide an irreducible 96-dimensional representation of the group* $L_3(7) : \mathbb{Z}_2$.

$\square$

4.2.3 The 154-dimensional representation of $L_3(7) : \mathbb{Z}_2$

We gather the information obtained in the previous subsections and set

$$v_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & v_{1,57} & 0 \\ 0 & 0 & v_{1,96} \end{pmatrix}, v_2 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & v_{2,57} & 0 \\ 0 & 0 & v_{2,96} \end{pmatrix}, \nu = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \nu_{57} & 0 \\ 0 & 0 & \nu_{96} \end{pmatrix}, \rho = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \rho_{57} & 0 \\ 0 & 0 & \rho_{96} \end{pmatrix},$$

$$x = \begin{pmatrix} 1 & 0 & 0 \\ 0 & x_{57} & 0 \\ 0 & 0 & x_{96} \end{pmatrix}, i = \begin{pmatrix} 1 & 0 & 0 \\ 0 & i_{57} & 0 \\ 0 & 0 & i_{96} \end{pmatrix} \text{ and } u = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -u_{57} & 0 \\ 0 & 0 & -u_{96} \end{pmatrix}.$$

This form for u is chosen because it ensures that u and x^2 have the same Jordan form. Since we want to construct a representation of $O'N$, this is necessary. Then the following holds:

Lemma 4.2.5 *We have $GL_{154}(3) \geq < v_1, v_2, \nu, \rho, x, i, u > \simeq L_3(7) : \mathbb{Z}_2$. Moreover the module splits as $V_{154} = V_1 \oplus V_{57} \oplus V_{96}$ with $dim V_k = k$ and V_k is an irreducible module for $L_3(7) : \mathbb{Z}_2$ ($k = 1, 57, 96$).*

□

4.3 The construction of the generator a

In this section we construct the remaining generator a. In order to do so, we use the identification of the geometrical generators for $L_3(7) : \mathbb{Z}_2$ coming from the amalgam of the Buekenhout geometry as words in the canonical ones given in Chapter 2. Thus we keep the generator ρ and set $x = (XY)^2$. Then $z = (ut)^{xu}$ where $t = x^2$ and $X = zi$. Furthermore we have $Y = ie$ with $e = (xx^{\nu^3})^{x^\nu}$ and $Z = (x^{-1})^u$. For our further considerations, we need the following two lemmas:

Lemma 4.3.1 *Let $G_1 =< z, X, Y, Z, \rho > \simeq L_3(7) : \mathbb{Z}_2$ be as in the amalgam for the Buekenhout geometry. Then $C_{G_1'}(a) =< zX, x, Z, Z^Y > \simeq L_2(7)$.*

Proof. By construction, we have for $\mathbb{Z}_2 \times PGL_2(7) \simeq < z, X, Y, Z >=: H \leq G_1$ that $a^2 = z^{-1}x \in Z(H)$. Moreover H is a subgroup of index two in the parabolic $G_2 \simeq (\mathbb{Z}_4 \times L_2(7)) : \mathbb{Z}_2$ of the Buekenhout geometry, which itself containes a. Thereby we hold that $C_{G_1'}(a)$ is isomorphic to a subgroup of $L_2(7)$ because $Y \in G_1'$ and $a^Y = a^{-1}$. Furthermore the relations of Chapter 2 yield $x, zX, Z \in C_{G_1'}(a)$ and $< x, zX, Z > \simeq S_4$. Also, since $a^Y = a^{-1}$, $Z^Y \in C_{G_1'}(a) - < x, zX, Z >$ because Y is an automorphism of $H' \simeq L_2(7)$ and $o(ZZ^Y) = 3$ proving the assertion. □

We set $G_4 :=< z, X, Y, \rho, a > \simeq \mathbb{Z}_4 L_3(4) : \mathbb{Z}_2$. Then the 154-dimensional module splits for G_4 as $V_{154} = C_{V_{154}}(z) \oplus [V_{154}, z] = C_{V_{154}}(z) \oplus C_{[V_{154},z]}(z^2) \oplus [V_{154}, z^2]$.

Computing the dimensions of the eigenspaces of z shows that we have $dimC_{V_{154}}(z) = 30$, $dimC_{[V_{154},z]}(z^2) = 44$ and $dim[V_{154}, z^2] = 80$. We set $V_{30} := C_{V_{154}}(z)$, $V_{44} := C_{[V_{154},z]}(z^2)$ and $V_{80} := [V_{154}, z^2]$.

Lemma 4.3.2 *Let $G =< z, a, X, Y, Z, \rho >$ be a completion of the amalgam of the Buekenhout geometry. Assume that G has a 154-dimensional $GF(3)G$-module such that the matrices for z, X, Y, Z and ρ are as above. Then z and a have the same Jordan form.*

Proof. Set $G_{14} =< z, \rho, X, Y >$ Using MAGMA [2] to compute the indecomposable summands of V_{154} as a G_{14}-module, we hold that $V_{30}|_{G_{14}} = V_6 \oplus V_9 \oplus V_{15}$, $V_{44}|_{G_{14}} = V_{1,1} \oplus V_{1,2} \oplus V_{12} \oplus V_{15,1} \oplus V_{15,2}$ and $V_{80}|_{G_{14}} = V_8 \oplus V_{24,1} \oplus V_{24,2} \oplus V_{24,3}$ with $dimV_{i,\epsilon} = i$. Moreover one of the 24-dimensional submodules is irreducible. Using the 3-modular characters of $L_3(4)$ as given in [22], we hold therefore that in V_{30} two irreducible 15-dimensional $L_3(4) : \mathbb{Z}_2$-modules are involved, in V_{80} there are an irreducible 8-dimensional (4-dim. over $GF(9)$) and an irreducible 72-dimensional (36-dim. over $GF(9)$) $\mathbb{Z}_4 L_3(4) : \mathbb{Z}_2$-module involved and that V_{44} is an irreducible $\mathbb{Z}_2 L_3(4) : \mathbb{Z}_2$-module (22-dim over $GF(9)$)[2].

Using the generators and relations for G_4 as in Chapter 2, we find that a is not a square in G_4 but z^2a is. Thus the characters we find in [22] are the characters of z^2a. Furthermore the 72-dimensional module admits another involutory automorphism of $L_3(4)$, namely the field automorphism of $GF(4)$. This automorphism fuses two of the classes of elements of order four in $L_3(4)$. This implies that z^2a is an element of type $4A$ in the notation of [22]. The information gathered so far proves that $dim(V_{154}(a^2, -1) \cap V_{80}) = 40$, $dim(V_{154}(a^2, -1) \cap V_{44}) = 24$, $dim(V_{154}(a^2, -1) \cap V_{30}) = 16$, thus $dimV_{154}(a^2, -1) = 80$ by [22]. Moreover, since z^2a is a $4A$-element, we hold by [22] that $dim(V_{154}(a, 1) \cap V_{80}) = 16$, $dim(V_{154}(a, 1) \cap V_{44}) = 8$ and $dim(V_{154}(a, -1) \cap V_{80}) = 24$, $dim(V_{154}(a, -1) \cap V_{44}) = 12$. On V_{30} we have that XY is an element of type $4B$ or $4C$. We also get $tr(XY) = -1$ on V_{30}. By [22], this implies that w.l.o.g. $tr(XY) = 3$ on $V_{15,1}$ and $tr(XY) = -1$ on $V_{15,2}$. Since $z = 1$ on $V_{15,1}$, we have $tr(a) = tr(z^2a)$ on $V_{15,1}$. Therefore we get that $tr(a) = -1$ on $V_{15,1}$ using [22] because z^2a is of type $4A$. Straightforward calculations yield $dim(V_{154}(a, 1) \cap V_{30}) \in \{3, 5\}$ and $dim(V_{154}(a, -1) \cap V_{30}) \in \{4, 2\}$. Thus either we find $dimV_{154}(a, 1) = 30$ and $dimV_{154}(a, -1) = 44$ or $dimV_{154}(a, 1) = 32$ and $dimV_{154}(a, -1) = 42$. Set $L =< zX, x, Z, Z^Y >= C_{G_1'}(a)$. Then L acts on $C_{V_{154}}(a^2) = V_{154}(a^2, 1)$. We use MAGMA [2] to compute the indecomposable summands of $V_{154}(a^2, 1)|_L$ and obtain $V_{154}(a^2, 1)|_L = W_{7,1} \oplus W_{7,2} \oplus W_{15,1} \oplus W_{15,2} \oplus W_{15,3} \oplus W_{15,4}$ with $dimW_{i,\epsilon} = i$. This proves $dimV_{154}(a, 1) = 30$ and $dimV_{154}(a, -1) = 44$, hence the assertion holds. □

These two lemmas provide a basis to construct the remaining generator a. The construction of a will now consist of constructing a *suitable* a on V_{44} and extend this

[2] Recall that G_4 is the group $4_2L_3(4) : 2_1$ in notation of [8] and [22].

with elements of $C_{G_1'}(a) \simeq L_2(7)$ to the full module.

4.3.1 Computing a on V_{44}

We proceed as follows. Using MAGMA [2], we compute the Jordan form of z by typing

`J, T:=JordanForm(z);`.

The function `JordanForm()` of MAGMA returns two values, the Jordan form J and a transformation matrix T such that $J = TzT^{-1}$. We give the matrix T as MAGMA input in the appendix. Then we have

$$J = \begin{pmatrix} -I_{44} & 0 & 0 \\ 0 & I_{30} & 0 \\ 0 & 0 & J_{80} \end{pmatrix}, \text{ where } J_{80} = \begin{pmatrix} \begin{matrix} 0 & 1 \\ -1 & 0 \end{matrix} & 0 & \cdots \\ 0 & \begin{matrix} 0 & 1 \\ -1 & 0 \end{matrix} & \ddots \\ \vdots & \ddots & \ddots \end{pmatrix} \in GL_{80}(3).$$

Now, we reduce any matrix $M \in \{X, Y, \rho\}$ and take the upper left 44×44-submatrix of TMT^{-1} by typing

`M44:=Submatrix(T*M*T^-1, 1,1, 44,44);`.

We store these matrices in a sequence Q in MAGMA. Using the presentation of $\mathbb{Z}_2 L_3(4) : \mathbb{Z}_2$ of Chapter 2 (here we set $z^2 = 1$), we can now induce the 44-dimensional Module for $M :=< z, X, Y, \rho >$ to $G :=< z, X, Y, \rho, a >$ as follows:

`W:=GModule(M, Q);`[3]

`V:=Induction(W, G);`

Using the Meataxe-program as installed in MAGMA, we can reduce V and obtain a matrix $a_M \in GL_{44}(3)$ for a, such that $< -I_{44}, X_M, Y_M, \rho_M, a_M > \simeq \mathbb{Z}_2 L_3(4) : \mathbb{Z}_2$.

We conjugate these matrices in $GL_{44}(3)$ on X_{44}, Y_{44}, and ρ_{44}. Set $a_{44,0}$ to be the corresponding conjugate of a_M. We need to construct *suitable* $C_{GL_{44}(3)}(< X_{44}, Y_{44}, \rho_{44} >)$-conjugates. This is achieved as follows. Using the representation for $L_3(7) : \mathbb{Z}_2$, we see that the points (objects stabilized by a $L_3(7) : \mathbb{Z}_2$) of the Buekenhout geometry correspond to certain 1-dimensional subspaces. Now lines of the Buekenhout geometry (objects stabilized by $(\mathbb{Z}_4 \times L_2(7)) : \mathbb{Z}_2$) have exactly two points. Let $G_1 =< z, X, Y, \rho, Z >$ and p_1 the point of the geometry fixed by G_1. Since $a \in G_2 =< a, z, X, Y, Z > \simeq (\mathbb{Z}_4 \times L_2(7)) : \mathbb{Z}_2$, we have that $p_1 a$ is collinear to p_1 and a interchanges p_1 and $p_1 a$. Hence, as a matrix, a has to fuse the two 1-dimensional submodules of G_1 and G_1^a. By construction, the 1-dimensional submodule belonging to G_1 is $< b_1 >$ where b_1 is the first standard basis vector of the 154-dimensional module. We set $M_t := TMT^{-1}$ for $M \in \{X, Y, \rho, Z\}$, $J = TzT^{-1}$, $G_{1,t} :=< J, X_t, Y_t, \rho_t, Z_t >$ and $C :=< JX_t, (X_t Y_t)^2, Z_t, Z_t^{Y_t} > \simeq L_2(7)$. Since C has to centralize a and $C \leq G_{1,t}'$ we get $C \leq (G_{1,t}^{a_t})'$ and the 1-dimensional submodule for $G_{1,t}^{a_t}$ is in $C_{V_{154}}(C)$. Now $C_{V_{154}}(C)$ can easily be computed as the intersection of the eigenspaces $V_{154}(\alpha, 1)$ where

[3]The order in Q must be the same as the generators for H, i.e., z, X, Y and ρ, where the $-I_{44}$ is the corresponding matrix for z

$\alpha \in \{JX_t, (X_tY_t)^2, Z_t, Z_t^{Y_t}\}$. This intersection has dimension four and is contained in $V_{44} = V_{154}(J, -1)$. Because $[X, a] = 1$, we must have $X \in G_{1,t}^{a_t} - (G_{1,t}^{a_t})'$ and the relation $Y^a = a^2Y$ implies that $Y_t \in G'_{1,t}$ but $Y_t \in G_{1,t}^{a_t} - (G_{1,t}^{a_t})'$. Thereby we hold that the 1-dimensional submodule for $G_{1,t}^{a_t}$ is inside $V_{154}(X_t, -1) \cap V_{154}(Y_t, -1)$. Thus we compute $W := C_{V_{154}}(C) \cap V_{154}(X_t, -1) \cap V_{154}(Y_t, -1)$ and hold that $W =< b_2, b_3 >$ where $b_2 =$

$$(0, 1, 2, 2, 2, 1, 2, 2, 2, 1, 2, 0, 0, 2, 1, 0, 1, 0, 0, 2, 1, 1, 0, 1, 0, 2, 2, 0, 0, \dots)$$

and $b_3 =$

$$(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 2, 2, 2, 2, 1, 0, 1, 0, 0,\\ 1, 0, 2, 1, 0, 0, \dots).$$

We can now regard b_1, b_2 and b_3 as vectors in a 44-dimensional space and conjugate $a_{44,0}$ in $C_{GL_{44}(3)}(< X_{44}, Y_{44}, \rho_{44} >)$ such that $b_1 a_{44,0}^k \in < b_2, b_3 >$ using MAGMA. As a result we get eight *suitable* candidates $a_{44,i}$.

The matrices X_{44}, Y_{44}, ρ_{44} and $a_{44,i}$ are given explicitly in the appendix as well as the generators of $C_{GL_{44}(3)}(< X_{44}, Y_{44}, \rho_{44} >)$.

Let us return briefly to W. We find by multiplying with T^{-1} that $b_2T^{-1} =$

$$(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 2, 1, 1, 1, 0, 1, 1,\\ 1, 2, 1, 1, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 1, 2, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0,\\ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 2, 1, 2, 2,\\ 2, 2, 1, 0, 0, 0, 0, 2, 2, 0, 1, 2, 2, 1, 1, 1, 1, 0, 2, 0, 1, 2, 0, 2, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0)$$

and $b_3T^{-1} =$

$$(0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 1, 0, 0, 0, 1, 0, 0, 0, 0,\\ 2, 2, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 2, 2, 1, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0,\\ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 1,\\ 0, 0, 0, 1, 1, 1, 0, 1, 0, 0, 0, 1, 2, 0, 2, 0, 1, 2, 2, 2, 2, 0, 2, 2, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0).$$

So neither b_2 nor b_3 is contained in the 57-dimensional or 96-dimensional submodule for $G_{1,t}$. Therefore we get an irreducible module for $G =< z, X, Y, Z, \rho, a >$.

4.3.2 Extending a

We give an algorithm to extend the suitable candidates $a_{44,i}$ obtained in the previous subsection.

4.3.2.1 Description of the algorithm

The algorithm constructs a Jordan basis B_a of V_{154} for a and a matrix τ whose j-th row is the j-th vector in B_a. Then $a = T^{-1}\tau^{-1}J\tau T$, where J is the Jordan form of z and T is the transformation matrix from above, i.e., $TzT^{-1} = J$. We work with the transformed matrices $X_t := TXT^{-1}$, $Y_t := TYT^{-1}$, $\rho_t := T\rho T^{-1}$, $Z_t := TZT^{-1}$ and J. Furthermore we set $C :=< JX_t, (X_tY_t)^2, Z_t, Z_t^{Y_t} >\simeq L_2(7)$ which should centralize a by Lemma 4.3.1. Also we construct a 154×154-matrix A_t whose upper left 44×44-submatrix is one of the suitable $a_{44,i}$s obtained above and the rest is simply I_{110}. Moreover V_{44} is now identified with the subspace of V_{154} generated by the first 44 standard basis vectors.

We now compute the eigenspace $E_2 := V_{154}(A_t, -1) \cap V_{44}$ and a basis B_2 of E_2. It turns out that $dimE_2 = 12$. The vectors of B_2 are included as the first 12 vectors in a sequence $B_{a,2}$ which shall become a basis of $V_{154}(a, -1)$. We now extend this sequence in the following way. For a vector $b \in B_2$ and an element $k \in C$ we check whether $bk \in< B_{a,2} >$. If $bk \notin< B_{a,2} >$, we store bk as a new element in $B_{a,2}$. This is repeated until $dim < B_{a,2} >= 44$.

The same process has to be performed for $E_1 := V_{154}(A_t, 1) \cap V_{44}$ with basis B_1 and a sequence $B_{a,1}$. It turns out here that $\mid B_1 \mid= 8$ and we stop if $dim < B_{a,1} >= 30$.

The next step is to construct a partial basis $B_{a,3}$ corresponding to the $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$-boxes. For this we compute $E_3 := V_{154}(X_t, -1) \cap V_{154}(Y_t, 1) \cap V_{44}$ with basis B_3, and $E_4 := V_{154}(X_t, 1) \cap V_{154}(Y_t, -1) \cap V_{44}$ with basis B_4. Since $a^2 = z^{-1}(XY)^2$ has to hold on V_{154} (and holds correspondingly on V_{44}), a cannot have eigenvectors in E_3 and E_4 (since z inverts all vectors in V_{44}). Because $[a_{44,i}, X_{44}] = 1$ and $a_{44,i}^{Y_{44}} = a_{44,i}^{-1}$, we get $E_3A_t = V_{154}(X_t, -1) \cap V_{154}(Y_t, -1) \cap V_{44}$ and $E_4A_t = V_{154}(X_t, 1) \cap V_{154}(Y_t, 1) \cap V_{44}$. Furthermore as a vector space $V_{44} := E_1 \oplus E_2 \oplus E_3 \oplus E_3A_t \oplus E_4 \oplus E_4A_t$, and $dimE_3 = dimE_4 = 6$.

We store in a sequence $B_{a,3}$ firstly the vector pairs $(b, bA_t), b \in B_3$ and secondly the pairs $(b, bA_t), b \in B_4$. Then we run the same loop as above taking the first, third up to the 23-th vector in $B_{a,3}$, thus we check whether for a vector b_i of these and some $k \in C$ we have $b_ik \notin< B_{a,3} >$ and then append b_ik and b_iA_tk to $B_{a,3}$ until $\mid B_{a,3} \mid= 80$.

The last part of the algorithm constructs B_a as a sequence of vectors in V_{154} simply by appending the vectors in $B_{a,2}$, $B_{a,1}$ and $B_{a,3}$ (in this order) to B_a. Then the i-th vector in B_a is put as the i-th row of a matrix τ and we hold $a := T^{-1}\tau^{-1}J\tau T$ such that $< a, z, X, Y, \rho >\simeq \mathbb{Z}_4L_3(4) : \mathbb{Z}_2$ and $[a, Z] = 1$.

Remark. Since this algorithm does not make use of any algorithm in MAGMA more profound than matrix multiplication (one could even store the elements of C as a set of matrices), the extension process of $a_{44,i}$ can be seen as a *computer free* process in the author's opinion.

4.3.2.2 The algorithm

We give the algorithm in MAGMA statements. For $M \in \{X, Y, \rho\}$, `Mt` is the matrix M_t from above, `a44i` is one of the suitable $a_{44,i}$'s.

```
C:=sub<GL(154,3)|J*Xt,(Xt*Yt)^2,Zt, Zt^Yt>;
```

```
V:=VectorSpace(GF(3), 154);
B:=Basis(V);
B44:=[B|];
for k in [1..44] do
    Append(~B44, B[k]);
end for;
V44:=sub<V|B44>;
M154:=MatrixAlgebra(GF(3), 154);
At:=M154!1;
InsertBlock(~At, a44i, 1,1);
Ba:=[V|];
```

This part constructs C, V, the natural basis for V, V_{44}, the matrix A_t and initializes the sequence B_a which shall become the Jordan basis corresponding to a.

```
E2:=Eigenspace(At, 2) meet V44;
B2:=Basis(E2);
Ba2:=[V|];
for k in B2 do
    Append(~Ba2, k);
end for;
for j in [1..12] do
    if # Ba2 ne 44 then
        for k in C do
            if Ba2[j]*k notin sub<V|Ba2> then
                Append(~Ba2, Ba2[j]*k);
            end if;
        end for;
    else break;
    end if;
end for;
```

This part creates the basis $B_{a,2}$ of $V_{154}(a,-1)$.

```
E1:=Eigenspace(At, 1) meet V44;
B1:=Basis(E1);
Ba1:=[V|];
for k in B1 do
    Append(~Ba1, k);
end for;
for j in [1..8] do
    if # Ba1 ne 30 then
        for k in C do
            if Ba1[j]*k notin sub<V|Ba1> then
                Append(~Ba1, Ba1[j]*k);
            end if;
```

```
        end for;
      else break;
      end if;
   end for;
```

This part creates the basis $B_{a,1}$ of $V_{154}(a,1)$.

```
   E3:=Eigenspace(Xt, 2) meet Eigenspace(Yt, 1) meet V44;
   B3:=Basis(E3);
   Ba3:=[V|];
   for k in B3 do
      Append(~Ba3, k);
      Append(~Ba3, k*At);
   end for;
   E4:=Eigenspace(Xt, 1) meet Eigenspace(Yt, 2) meet V44;
   B4:=Basis(E4);
   for k in B4 do
      Append(~Ba3, k);
      Append(~Ba3, k*At);
   end for;
   for j in [0..11] do
      if # Ba3 ne 80 then
        for k in C do
          if Ba3[2*j+1]*k notin sub<V|Ba3> then
            Append(~Ba3, Ba3[2*j+1]*k);
            Append(~Ba3, Ba3[2*j+2]*k);
          end if;
        end for;
      else break;
      end if;
   end for;
```

This part creates the basis $B_{a,3}$ consisting of the vector pairs which belong to the $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$-boxes.

```
   for k in Ba2 do
      Append(~Ba, k);
   end for;
   for k in Ba1 do
      Append(~Ba, k);
   end for;
   for k in Ba3 do
      Append(~Ba, k);
   end for;
```

```
tau:=M154!0;
for k in [1..154] do
   tau[k]:=Ba[k];
end for;
a:=T^-1*tau^-1*J*tau*T;
a:=G!a;
```

This final part constructs the Jordan basis B_a, the matrix τ and the matrix a which is given as a MAGMA input in the appendix. The construction of the matrix a finishes the proof of the following theorem:

Theorem 4.3.3 *Let $\mathcal{A}$ be the amalgam of the Buekenhout geometry for $O'N$. Then every completion G of $\mathcal{A}$ has an irreducible 154-dimensional $GF(3)$-module.*

□

Chapter 5

Construction of the Ivanov-Shpectorov Geometry out of the Buekenhout Geometry

In this chapter we show that a completion of the amalgam related to the Buekenhout geometry is also a completion of the amalgam of the Ivanov-Shpectorov geometry. This is done without using the fact that $O'N$ is a completion of both amalgams.

We fix the following notation. $G :=< a, z, \rho, X, Y, Z >$ and $G_4 :=< a, z, \rho, X, Y > \simeq \mathbb{Z}_4 L_3(4) : 2_1$. Using the generators for G_4 as given in Chapter 2, we have seen that $E_1 :=< f_1, f_2, f_3, f_5, z > \simeq \mathbb{Z}_4 * Q_8 * Q_8$, also $\bar{P_1} :=< E_1, a, \rho_1, f_4 >=< E_1, \tilde{a}, \rho_n, \tau > \simeq (\mathbb{Z}_4 * Q_8 * Q_8) : A_5$ with $E_1 = O_2(\bar{P_1})$. Moreover, if $\bar{P_2} := \bar{P_1}^X$, then $< \bar{P_1}, \bar{P_2} >= G_4' \simeq \mathbb{Z}_4 L_3(4)$. Thus $O_2(\bar{P_1} \cap \bar{P_2}) \in Syl_2(G_4')$ and $S :=< O_2(\bar{P_1} \cap \bar{P_2}), X > \in Syl_2(G_4)$. Using the relations of Chapter 2, we get that $O_2(\bar{P_1} \cap \bar{P_2}) =< f_1, f_2, f_3, f_4, f_5, z, a >$ since $X = \beta^{f_3 f_5}$, and therefore $S =< f_1, f_2, f_3, f_4, f_5, z, a, X >$. Furthermore the relation $f_4 = zXYf_2f_1z^2$ implies $Y \in S$.

5.1 G has a subgroup $L \simeq \mathbb{Z}_4^3 L_3(2)$

The aim of this section is to establish the following lemma:

Lemma 5.1.1 *Let G be the completion of the amalgam $\mathcal{A}$ related to the Buekenhout geometry for $O'N$. Then G has a subgroup $L \simeq \mathbb{Z}_4^3 L_3(2)$.*

Again, note that this lemma will be proved without using the group $O'N$. We start to prove:

Lemma 5.1.2 *Set $b := f_4 f_5^{-1}$. Then $F :=< a, b, z > \simeq \mathbb{Z}_4^3$ and $F \lhd S$.*

Proof. Since $f_4, f_5 \in G_4'$, we get $[z, b] = 1$. Using the relations of Chapter 2, we obtain $[a, b] = [a, f_5] f_1 f_3 z^{-1}$. By $[f_2, f_5] = 1$, we hold $[a, f_5] = [f_2 f_5 a, f_5] = f_1 f_3 z$ thus F is abelian and $f_3 \in F$. Moreover $o(b) = 4$. The fact $f_3 \notin < z, a >$ implies $F \simeq \mathbb{Z}_4^3$ since $b^2 = f_4^{-1} f_5 f_4 f_5^{-1} = [f_4, f_5^{-1}] = [f_4, f_5] = z f_3$ ($f_5^2 = z^2$). We need to prove that F is normal in S. Clearly, we have $S =< E_1, X >$ and $f_1 \in F$ (since $a^2 = z^{-1} f_1$). Using the relations as given in Chapter 2, we see that $a^X = a^{-1}$, $a^{f_2} = a f_3$ and $a^{f_5} = a f_1 f_3 z$ since $f_3 = [a, f_2]$ and $[a, f_5] = f_1 f_3 z$. Using $[f_2, f_4] = f_1^{-1}$, $f_1^{f_2} = f_1^{f_5} = f_1^{-1}$ and $[f_2, f_5] = 1$, we compute $[b, f_2] = f_1^{-1}$, hence $b^{f_2} = b f_1^{-1}$. Now $[b, f_5] = [f_5, f_4^{-1}] = [f_4, f_5] = f_3 z$, thus $b^{f_5} = b^{-1}$. By $b = f_4 f_5^{-1}$, we hold $b^X = f_5 a f_4 f_3 f_1 z^{-1}$. Using $[f_5, a] = z f_1 f_3^{-1}$, $[f_3, f_4] = 1$ and $[f_1, f_4] = z^2$, we get $b^X = a f_5 f_4 z^2 = a b^{-1} z^2$, proving the lemma. □

Lemma 5.1.3 *Set $P_1 :=< S, \rho_n >$ and $U :=< F, f_2, f_4 >$. Then $P_1 \simeq \mathbb{Z}_4^3 S_4$, $F \lhd P_1$ and $U = O_2(P_1)$.*

Proof. Using our relations, we hold $a^{\rho_n} = a^{-1} b^{-1} z$ and $b^{\rho_n} = a z^2$. Since $[\rho_n, z] = 1$, we have $F \lhd P_1$.

We show $| U |= 2^8$ and $U \lhd P_1$. Using the relations of Chapter 2, we get $[f_2, f_4] = f_1^{-1} \in F$ and $(f_2 f_4)^2 = [f_2, f_4]$. In particular we hold $o(f_2 f_4) = 8$. Furthermore $(f_2 f_4)^{f_2} = f_4 f_2 = f_4^{-1} f_2^{-1}$, hence $< f_2, f_4 >\simeq Q_{16}$. Since F is abelian and of exponent four, $|< f_2, f_4 > \cap F |= 4$ and $| U |= 2^8$. In particular we have $F \lhd U$ and U/F is elementary abelian of order four.

We show $U \lhd P_1$. Our relations prove the following: $f_2^{\rho_n} = f_5 = b^{-1} f_4$, $f_4^{\rho_n} = b^{-1} a z f_4 f_2$. This shows that $< f_2, f_4, \rho_n > F/F \simeq A_4$. Moreover $Y \in S$ and $\rho_n \rho_n^Y = z^2 a^{-1} b$ and the lemma is proved. □

Lemma 5.1.4 *Set $g := (ZX)^2$, $x := (b^2)^{g^{-1}}$, $P_2 :=< S, x >$ and $W :=< F, X, Y >$. Then $(z^2)^x = z^2 a^2$, $(a^2 z^2)^x = z^2$, $P_2 \simeq \mathbb{Z}_4^3 S_4$, $F \lhd P_2$ and $W = O_2(P_2)$.*

Proof. The action of x on $< z^2, a^2 >$ is verified using the matrices of the previous chapter. Since $o(XY) = 8$, we have $< X, Y >\simeq D_{16}$. Moreover $f_1 = (XY)^2$ and as above we hold $< X, Y > \cap F =< f_1 >$, so $| W |= 2^8$ and W/F is again elementary abelian of order four.

Using the relations of Chapter 2 and the matrices, we get $X^x = X b^2 a^{-1}$, $Y^x = XY a^2 b^{-1}$, $z^x = z b^2 a^{-1}$, $b^x = z^2 a b$ and, by construction, $[a, x] = 1$. Thus we have $W \lhd P_2$.

We get $o(x f_2) = 12$. Set $y := (x f_2)^4 \notin S$. Then one verifies $y y^x = z^2 a^2 \in F$ ($x \notin W$), proving the lemma. □

Corollary 5.1.5 *We have $P_1 \cap P_2 = S$, $S \simeq \mathbb{Z}_4^3 D_8$. Moreover U and W are the preimages of the elementary abelian groups of order four in S/F.*

□

Lemma 5.1.6 *$L :=< P_1, P_2 >\simeq \mathbb{Z}_4^3 L_3(2)$.*

Proof. By the previous lemmas it remains to show that the Weyl relation holds in L. Clearly, $Y^{\rho_n} \notin P_2$, $x \notin P_1$ and we verify, using the matrices, that $o(x Y^{\rho_n}) = 12$, $(x Y^{\rho_n})^3 = z a^{-1} b^{-1}$. □

5.2 Construction of the maximal parabolic groups of the Ivanov-Shpectorov geometry

In this section we construct the maximal parabolic subgroups $\bar{G_1} \simeq J_1$, $\bar{G_2} \simeq M_{11}$ and $\bar{G_5} \simeq (\mathbb{Z}_4 * Q_8 * Q_8) : A_5$. The group L constructed above contains a subgroup $N \simeq \mathbb{Z}_2^3 : \mathbb{Z}_7 : \mathbb{Z}_3$. We construct a group $H \simeq \mathbb{Z}_2 \times A_5$ using $\bar{P_1}$ such that $< N, H > \simeq J_1$.

By the previous section, $\Omega_1(F) \leq O_2(\bar{P_1}) \cap O_2(\bar{P_2})$. Therefore $\Omega_1(F)$ is not contained in any subgroup of $\bar{P_i}$ of shape $\mathbb{Z}_2 \times A_5$. Set $\alpha := \rho_1^X$ then $\alpha \in \bar{P_2} - \bar{P_1}$ and $\bar{G_5} := \bar{P_1}^{\alpha}$. Then $\Omega_1(F) \cap O_2(\bar{G_5}) = < z^2 >$.

Lemma 5.2.1 *Set $i := (f_2 f_5)^X$. Moreover set $A := \tilde{a}$, $B := z^2 i^{\rho_n}$ and $A_n := z^2 i^{\rho_1}$. Then $U := < A, A_n, B > \simeq \mathbb{Z}_2 \times A_5$, $U' = < z^2 A, z^2 A_n, z^2 B >$ and $U \leq \bar{P_1}$.*

Proof. Obviously, we have $U \leq \bar{P_1}$. Since $[f_2, f_5] = 1$, we have $i^2 = 1$. Using the relations obtained in Chapter 2, we compute $i = a f_2 f_3 f_5 \notin E_1$. Furthermore $A^2 = (a f_2 f_5)^2 = z^2 f_2 f_3 f_5 f_3 f_2 f_5 = (f_2 f_5)^2 = 1$ and $A_n^2 = 1$. Using the matrices, we get $[A, B] = 1$, $o(AA_n) = 5$, $o(BA_n) = 3$ and $o(z^2 A A_n B) = 5$, hence $U \simeq \mathbb{Z}_2 \times A_5$ as in Chapter 3. $\square$

Lemma 5.2.2 $< z^2, A, B >^{\alpha} = \Omega_1(F)$.

Proof. Using the matrices, we verify the following identities: $A^{\alpha} = b^2$ and $B^{\alpha} = a^2 b^2$. Since $[z, \alpha] = 1$, the assertion follows. $\square$

We set $\bar{A} := A^{\alpha}$, $\bar{B} := B^{\alpha}$, $\bar{A_n} := A_n^{\alpha}$, $\bar{G_{15}} := U^{\alpha}$.

Lemma 5.2.3 *Let $y := (x f_2)^4$. Then $N_1 := < \Omega_1(F), \rho_n, y > \simeq \mathbb{Z}_2^3 : (\mathbb{Z}_7 : \mathbb{Z}_3)$.*

Proof. As in the proof of Lemma 5.1.4, we have $o(y) = 3$. Then the following holds: $s := \rho_n y$ is of order seven, $[\rho_n, y] = s^{-1}$ and $y \rho_n = s^2$, proving the lemma. $\square$

We identify $\bar{A}$ with (12)(34) and $\bar{B}$ with (13)(24) in A_5. Then we can identify $\bar{A_n}$ with (15)(24). Set $d := \bar{B}\bar{A_n}$, then we identify d with (135). With this identification, we have $\bar{d} := d^{(\bar{A}\bar{A_n})^{\bar{B}}} = (234)$. Therefore $K := < \bar{A}, \bar{B}, \bar{d} > \simeq \mathbb{Z}_2 \times A_4$. Since $\rho_n, \bar{d} \in P_1$, we have $\rho_n \sim_{P_1} \bar{d}$. Set $\delta := X f_2 a b^{-1}$. Then we verify $\rho_n^{\delta} = \bar{d}$, hence we set $N := N_1^{\delta}$. This implies $\bar{G_{15}} \cap N = K$. We prove the following lemma:

Lemma 5.2.4 $\bar{G_1} := < N, \bar{G_{15}} > \simeq J_1$.

Proof. We show $\bar{G_1} \simeq J_1$ using the generators and relations of the Ivanov geometry as given in Chapter 3. Set $\bar{s} := s^{\alpha}$. Then we have $(z^2)^{\bar{s}} = z^2 \bar{A}\bar{B}$, $(z^2)^{\bar{s}^{-1}} = z^2 \bar{B} \in \bar{G_{15}}'$ and $\bar{B}^{\bar{s}^{-1}} = z^2 \bar{A}$. Moreover we find $o(z^2 \bar{A}\bar{B}\bar{A_n}) = 5$. We set $\tilde{A} := z^2 \bar{A}\bar{B}$, $\tilde{B} := \bar{B}$ and $\tilde{A_n} := \bar{A_n}$. Then $\bar{G_{15}} = < \tilde{A}, \tilde{B}, \tilde{A_n} >$ and $(\tilde{A}, \tilde{B}, \tilde{A_n})$ satisfies all the required relations.

We set $\bar{t} := \tilde{B}\tilde{A_n}$. Since $(z^2)^{\bar{s}^{-1}} = z^2\tilde{B}$, z^2 inverts $\bar{t}^{\bar{s}}$ and $\bar{t}^{\tilde{A}\bar{s}}$. Using the matrices, we find $o([\tilde{A_n}, \bar{t}^{\bar{s}}]) = 5$. Thus we set $\tilde{t} := \bar{t}^{\tilde{A}\bar{s}}$ and compute $[\tilde{A_n}, \tilde{t}] = 1$. Moreover $[\tilde{A}, \tilde{t}] = 1$ by construction, $o(\tilde{B}\tilde{t}) = 5$ and $z^2 = \tilde{B}\tilde{t}[\tilde{B}, \tilde{t}]^2$ finishing the proof. □

Let P be a maximal parabolic subgroup of $L_3(4)$, $j \in P - O_2(P)$ an involution. Then $C_P(j) \simeq \mathbb{Z}_2^4$ and $C_P(j) \cap O_2(P) \simeq \mathbb{Z}_2^2$. In the extension $\mathbb{Z}_4L_3(4)$ the elements j and ij are conjugate, where i denotes the central involution. Thus the centralizer of j in $\mathbb{Z}_4P$ is of order 32. This group has the structure $\mathbb{Z}_2 \times (\mathbb{Z}_4 * D_8)$.

We compute $C := C_{\bar{G_5}}(\tilde{A})$. Clearly, $z \in C$, $\Omega_1(F) \leq C$ and $< z, \Omega_1(F) > \simeq \mathbb{Z}_4 \times \mathbb{Z}_2^2$. Furthermore we find $[\tilde{A}, f_3^\alpha] = 1$, thus $C = < z, \Omega_1(F), f_3^\alpha >$ and $D_8 \simeq < \tilde{B}, f_3^\alpha > \lhd C$.

In order to find the remaining generator $\tilde{v}$ to construct $\bar{G_2} = < \tilde{A_n}, \tilde{B}, \tilde{t}, \tilde{v} > \simeq M_{11}$ (generators as in Chapter 3), we need to compute $C_{<\tilde{B}, f_3^\alpha>}(\tilde{t})$ since the relations $[\tilde{A}, \tilde{v}] = [\tilde{t}, \tilde{v}] = 1$ have to hold. Moreover $< \tilde{B}, f_3^\alpha > = < \tilde{B}, \tilde{v} >$ must be fulfilled.

Lemma 5.2.5 *Set $\tilde{v} := \tilde{B}f_3^\alpha$. Then $\bar{G_2} := < \tilde{A_n}, \tilde{B}, \tilde{t}, \tilde{v} > \simeq M_{11}$.*

Proof. The involutions in $< \tilde{B}, f_3^\alpha >$ are the following: z^2 (the central involution), $\tilde{B}$, $\tilde{B}^{f_3^\alpha}$, $\tilde{B}f_3^\alpha$ and $\tilde{B}^{f_3^\alpha}f_3^\alpha$. Using the matrices, we get $[\tilde{t}, x] = 1$ only for $x = \tilde{B}f_3^\alpha$. Therefore $\tilde{v} \in \{\tilde{B}f_3^\alpha, \tilde{A}\tilde{B}f_3^\alpha\}$. Then we compute $o(\tilde{A_n}\tilde{A}\tilde{B}f_3^\alpha) = 6$ and $o(\tilde{A_n}\tilde{B}f_3^\alpha) = 3$. With respect to the relations as given in Chapter 3, we set $\tilde{v} := \tilde{B}f_3^\alpha$ and the lemma is proved. □

Corollary 5.2.6 *The groups $\bar{G_1}$, $\bar{G_2}$ and $\bar{G_5}$ are the end-parabolic groups of the Ivanov-Shpectorov geometry.*

□

Remark. The amalgam $(G_4, L, \bar{G_1})$ has been used by Lempken to construct $O'N$ [20].

Appendix A

The transformation matrix T

We give the matrix T which transforms the generator z in its Jordan form $J = TzT^{-1}$. Since $T \in GL_{154}(3)$, we display T only as a MAGMA input.

```
 T:=GL(154,3)![ 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 2, 1, 1, 1, 1, 1, 2, 1, 1, 0, 2, 1, 0, 0, 0, 0, 2, 1, 0, 0, 0, 0, 0, 0, 1, 2, 0, 0, 0, 0, 1, 2,
0, 2, 2, 2, 2, 2, 1, 2, 2, 2, 2, 2, 1, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 2, 0, 0, 0, 1, 0, 2, 0, 0, 1, 1, 1, 1, 2, 1,
1, 1, 1, 1, 2, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 2, 1, 1, 1, 1, 1, 2, 1,
1, 1, 0, 2, 0, 1, 0, 0, 0, 2, 0, 1, 0, 0, 2, 2, 2, 2, 1, 2, 2, 2, 2, 2, 1, 2, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 1, 2, 0, 0,
0, 0, 1, 2, 0, 0, 0, 0, 0, 0, 2, 1, 0, 0, 0, 0, 2, 1, 0, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 2, 0, 2, 0, 0, 1, 0, 0, 2, 0, 0, 1, 0, 0, 0, 0, 0,
2, 1, 0, 0, 0, 0, 2, 1, 0, 0, 1, 0, 0, 2, 0, 0, 1, 0, 0, 2, 0, 0, 2, 1, 0, 0, 0, 0, 2, 1, 0, 0, 2, 2, 2, 1, 2, 2, 2, 2, 2, 1, 2,
2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1,
1, 1, 1, 1, 1, 1, 1, 1, 1, 2, 1, 0, 0, 0, 0, 2, 1, 0, 0, 0, 0, 2, 0, 0, 0, 1, 0, 2, 0, 0, 0, 1, 0, 2, 0, 0, 0, 0, 1, 2, 0, 0, 0,
0, 1, 2, 0, 1, 0, 0, 0, 2, 0, 1, 0, 0, 0, 2, 0, 0, 1, 0, 0, 2, 0, 0, 1, 0, 0, 1, 2, 2, 2, 2, 2, 1, 2, 2, 2, 2, 2, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 2, 1, 0, 0, 0, 0, 2, 0, 0, 0, 0, 1, 0, 0, 0, 0,
0, 1, 0, 0, 1, 0, 0, 0, 2, 0, 1, 0, 0, 0, 2, 0, 0, 2, 0, 1, 0, 0, 0, 2, 0, 1, 0, 0, 0, 0, 2, 0, 1, 0, 0, 0, 2, 0, 1, 1, 1, 2, 1,
1, 1, 1, 1, 2, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 1, 2, 2, 2, 2, 2, 1, 2, 2, 2, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 1, 0, 0, 0, 2, 0, 1, 0, 0, 0, 2, 2, 2, 2, 2, 1, 2, 2, 2, 2, 2, 1, 2, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 1, 2,
0, 0, 0, 0, 1, 2, 0, 0, 0, 0, 0, 0, 2, 1, 0, 0, 0, 0, 2, 1, 0, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 2, 0, 1, 0, 0, 0, 2, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 2, 1, 0, 2,
0, 0, 0, 1, 0, 2, 0, 0, 0, 1, 0, 0, 1, 0, 2, 0, 0, 0, 1, 0, 2, 0, 0, 0, 0, 1, 0, 2, 0, 0, 0, 1, 0, 2, 2, 2, 1, 2, 2, 2, 2, 2, 1,
2, 2, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 2, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 2, 0, 1, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 1, 2, 0, 0, 0, 0, 1,
2, 0, 0, 0, 0, 0, 0, 2, 1, 0, 0, 0, 0, 2, 1, 0, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0,
1, 0, 0, 0, 2, 0, 1, 0, 0, 2, 2, 2, 2, 1, 2, 2, 2, 2, 2, 1, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 2, 0, 0, 1,
0, 0, 2, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 2, 0, 0, 0, 1, 0, 2, 0, 0, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 2, 1, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 1, 0, 0, 0, 0, 2, 1, 0, 0, 0, 0, 0, 0, 1, 2, 0, 0, 0, 0, 1, 2, 0, 2, 2, 2, 2, 2, 1, 2, 2, 2,
2, 2, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 2, 0, 0, 0, 1, 0, 2, 0, 0, 1, 0, 2, 0, 0, 0, 1, 0, 2, 0, 0,
0, 0, 1, 0, 2, 0, 0, 0, 1, 0, 2, 2, 2, 1, 2, 2, 2, 2, 2, 1, 2, 2, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 2, 1, 1, 1, 1, 1, 2,
1, 1, 1, 1, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 2, 0, 0, 2, 0, 0, 0, 1, 0, 2, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
1, 1, 1, 2, 1, 1, 1, 1, 1, 2, 1, 1, 0, 2, 1, 0, 0, 0, 0, 2, 1, 0, 0, 0, 0, 0, 0, 1, 2, 0, 0, 0, 0, 1, 2, 0, 2, 2, 2, 2, 2, 1, 2,
2, 2, 2, 2, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 2, 0, 0, 0, 1, 0, 2, 0, 0, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 2, 1, 0, 0,
0, 0, 0, 2, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 2, 1, 1, 1, 1, 1, 2, 1, 1, 1, 0, 2, 0, 1, 0, 0,
0, 2, 0, 1, 0, 0, 2, 2, 2, 2, 1, 2, 2, 2, 2, 2, 1, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 2, 0, 0, 0, 0, 1, 2, 0, 0, 0, 0,
0, 0, 2, 1, 0, 0, 0, 0, 2, 1, 0, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 2, 0, 0, 2, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 1, 0, 2, 0, 0, 0, 1, 0, 2, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 1, 2, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 1, 0, 2, 0, 0, 0, 1, 0, 0, 1, 0, 2, 0, 0, 0, 1, 0, 2, 0, 0, 0, 0, 1, 0, 2, 0,
```

```
0, 0, 1, 0, 2, 2, 2, 1, 2, 2, 2, 2, 2, 1, 2, 2, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 2, 0, 0, 1, 0, 0,
2, 0, 0, 0, 1, 2, 0, 0, 0, 0, 1, 2, 0, 2, 2, 2, 2, 2, 1, 2, 2, 2, 2, 2, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 2, 0, 0,
0, 1, 0, 2, 0, 0, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 2, 1, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 2, 0, 2, 1, 0, 0, 0, 0, 2, 1, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 2, 0, 0, 0, 1, 0, 2, 0, 0, 0, 0, 1, 0, 2, 0, 0, 0, 1, 0, 2, 2, 2, 1, 2, 2,
2, 2, 2, 1, 2, 2, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 2, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 2, 0,
0, 2, 0, 0, 0, 1, 0, 2, 0, 0, 0, 1, 0, 0, 1, 0, 2, 0, 0, 0, 1, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0,
0, 2, 0, 1, 0, 0, 0, 2, 2, 2, 2, 2, 1, 2, 2, 2, 2, 2, 1, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 2, 0, 0, 0, 0, 1, 2, 0, 0,
0, 0, 0, 0, 2, 1, 0, 0, 0, 0, 2, 1, 0, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 2, 0, 0, 2, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 2, 0, 1, 0, 0,
0, 2, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 2, 0, 0, 0, 1, 0, 2, 0, 0, 2, 2, 1, 2, 2, 2, 2, 2, 1, 2,
2, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 2, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 2, 0, 0, 2, 0, 0, 0,
1, 0, 2, 0, 0, 0, 1, 0, 0, 1, 0, 2, 0, 0, 0, 1, 0, 2, 0, 0, 0, 0, 1, 0, 2, 0, 0, 0, 1, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 1, 2, 0, 0, 0, 0, 1, 2, 0, 0, 0, 0, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0,
1, 0, 0, 2, 0, 0, 1, 0, 0, 0, 0, 0, 2, 1, 0, 0, 0, 0, 2, 1, 0, 0, 1, 0, 0, 2, 0, 0, 1, 0, 0, 2, 0, 0, 2, 1, 0, 0, 0, 0, 2, 1, 0,
0, 2, 2, 2, 1, 2, 2, 2, 2, 2, 1, 2, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 2, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 2, 1, 2,
2, 2, 2, 2, 1, 2, 2, 2, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 2, 0, 1, 0, 0, 0, 2, 0, 0, 2, 0, 1, 0, 0, 0, 2, 0,
1, 0, 0, 0, 0, 2, 0, 1, 0, 0, 0, 2, 0, 1, 1, 1, 2, 1, 1, 1, 1, 1, 2, 1, 1, 1, 0, 0, 0, 2, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 2, 0, 0, 1, 0, 0, 2, 0, 2, 2, 2, 2, 2, 1, 2, 2, 2, 2, 2, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 1, 0, 2, 0, 0, 0, 1, 0, 2, 0, 0, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 2, 1, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 2, 0, 2, 1, 0,
0, 0, 0, 2, 1, 0, 0, 0, 0, 0, 0, 1, 2, 0, 0, 0, 0, 1, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1,
0, 0, 0, 0, 2, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 2, 0, 0, 0, 0, 1, 2, 0, 0, 0, 0, 1, 0, 0,
0, 2, 0, 1, 0, 0, 0, 2, 0, 1, 0, 0, 0, 0, 2, 1, 0, 0, 0, 0, 2, 1, 0, 2, 0, 0, 0, 1, 0, 2, 0, 0, 0, 1, 0, 0, 2, 0, 0, 1, 0, 0, 2,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 2, 0, 1, 0, 0, 0, 2, 0, 1, 0,
0, 0, 0, 2, 1, 0, 0, 0, 0, 2, 1, 0, 2, 0, 0, 0, 1, 0, 2, 0, 0, 0, 1, 0, 0, 2, 0, 0, 1, 0, 0, 2, 0, 0, 2, 1, 1, 1, 1, 1, 2, 1, 1,
1, 1, 1, 2, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 1, 2, 0, 0, 0, 0, 1, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 2, 1, 0, 0, 0, 0, 2, 1, 0, 2, 0, 0, 0, 1, 0, 2, 0, 0, 0, 1, 0, 0, 2, 0, 0, 1, 0,
0, 2, 0, 0, 2, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 2, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 1, 2, 0, 0, 0, 0, 1, 2, 0, 0, 0, 0, 1, 0, 0,
0, 2, 0, 1, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 2, 2, 2, 2, 2, 1,
2, 2, 2, 2, 2, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 2, 1, 0, 0, 0, 0, 2, 1, 0, 0, 0, 0, 2, 0, 0, 0, 1, 0, 2, 0, 0, 0, 1, 0, 2, 0,
0, 0, 0, 1, 2, 0, 0, 0, 0, 1, 2, 0, 1, 0, 0, 0, 2, 0, 1, 0, 0, 0, 2, 0, 0, 1, 0, 0, 2, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 2, 0, 0, 0, 1, 0, 2, 0, 0, 0, 1, 0, 0, 2, 0, 0, 1, 0, 0, 2, 0, 0, 2,
1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 2, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 1, 2, 0, 0, 0, 0, 1, 2, 0, 0, 0, 0, 1, 0, 0, 0, 2, 0, 1, 0,
0, 0, 2, 0, 1, 0, 0, 0, 0, 2, 1, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
1, 0, 0, 2, 0, 0, 1, 0, 0, 2, 0, 0, 2, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 2, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 1, 2, 0, 0, 0, 0, 1,
2, 0, 0, 0, 0, 1, 0, 0, 0, 2, 0, 1, 0, 0, 0, 2, 0, 1, 0, 0, 0, 0, 2, 1, 0, 0, 0, 0, 2, 1, 0, 2, 0, 0, 0, 1, 0, 2, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 2, 1, 0, 0, 0, 0,
2, 1, 0, 0, 0, 0, 2, 0, 0, 0, 1, 0, 2, 0, 0, 0, 1, 0, 2, 0, 0, 0, 0, 1, 2, 0, 0, 0, 0, 1, 2, 0, 1, 0, 0, 0, 2, 0, 1, 0, 0, 0, 2,
0, 0, 1, 0, 0, 2, 0, 0, 1, 0, 0, 1, 2, 2, 2, 2, 2, 1, 2, 2, 2, 2, 2, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 1, 1, 0, 1, 0,
0, 1, 1, 0, 0, 1, 1, 0, 0, 0, 2, 2, 2, 2, 0, 0, 2, 2, 2, 2, 1, 0, 0, 1, 1, 0, 1, 0, 1, 0, 0, 1, 2, 0, 2, 0, 0, 2, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 2, 2, 2, 0, 0, 2, 2, 2, 2, 1, 0, 1, 0, 0, 1, 1, 0, 0, 1, 1, 0, 1, 0, 0, 1,
1, 0, 1, 0, 1, 0, 0, 1, 0, 0, 2, 2, 2, 2, 2, 0, 0, 2, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0,
1, 1, 0, 1, 0, 0, 1, 1, 0, 0, 0, 2, 2, 2, 2, 1, 0, 1, 0, 0, 1, 1, 0, 1, 0, 0, 1, 0, 0, 2, 2, 2, 2, 1, 0, 0, 1, 1, 0, 0, 0, 1, 1,
1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 1, 0, 0, 0, 2, 2, 2, 2, 1, 0, 1, 0, 0, 1, 1, 0, 1,
0, 0, 1, 0, 0, 2, 2, 2, 2, 1, 0, 0, 1, 1, 0, 0, 0, 1, 1, 1, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 2, 0, 0,
0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 2, 2, 2, 1, 0, 0, 1, 1, 0, 1, 0,
0, 1, 1, 0, 1, 0, 1, 0, 0, 1, 0, 0, 2, 2, 2, 2, 2, 0, 0, 2, 2, 0, 0, 0, 2, 2, 2, 2, 1, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 2, 2, 2, 1, 0, 0, 1, 1, 0, 1, 0, 1, 0, 0, 1, 0, 0, 2, 2, 2, 2, 2, 0, 0, 2, 2, 0, 0, 0,
2, 2, 2, 2, 1, 0, 1, 0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 1, 1,
0, 0, 1, 1, 0, 1, 0, 1, 0, 0, 1, 2, 0, 2, 0, 0, 2, 1, 0, 1, 0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 0, 2, 2, 2, 2, 0, 0, 2, 2, 2, 2, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 1, 0, 0, 0, 2, 2, 2, 2, 1, 0, 0, 1, 1, 0, 0, 0, 1, 1, 1, 1, 1,
```

0, 0, 1, 1, 0, 0, 0, 2, 2, 2, 2, 1, 0, 1, 0, 0, 1, 1, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0,
1, 0, 1, 0, 0, 1, 1, 0, 1, 0, 0, 1, 2, 0, 2, 0, 0, 2, 1, 0, 1, 0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 0, 2, 2, 2, 2, 0, 0, 2, 2, 2, 2, 1,
0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 1, 0, 1, 0, 0, 1, 1, 0, 0, 0, 1, 1, 1, 1,
1, 0, 0, 1, 1, 0, 0, 0, 2, 2, 2, 2, 1, 0, 1, 0, 0, 1, 1, 0, 1, 0, 0, 1, 0, 0, 2, 2, 2, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0,
1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 2, 0, 0, 2, 1, 0, 1, 0, 0,
1, 1, 0, 1, 0, 0, 1, 1, 0, 1, 0, 0, 1, 1, 0, 1, 0, 0, 1, 1, 0, 1, 0, 0, 1, 1, 0, 1, 0, 0, 1, 1, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 2, 2, 0, 0, 0, 2, 2, 2, 2, 0, 0, 2, 2, 2, 2, 0, 0, 2, 2, 2, 2, 0, 0, 2, 2, 2,
2, 0, 0, 2, 2, 2, 2, 0, 0, 2, 2, 2, 2, 0, 0, 2, 2, 2, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1,
1, 1, 1, 0, 0, 1, 1, 0, 1, 0, 0, 1, 1, 0, 1, 0, 0, 1, 1, 0, 1, 0, 0, 1, 1, 0, 1, 0, 0, 1, 1, 0, 1, 0, 0, 1, 1, 0, 1, 0, 0, 1, 1,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0,
0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 2, 1, 0, 0, 0, 0, 2, 1, 0, 2, 0, 0, 0, 1, 0, 2, 0, 0,
0, 1, 0, 0, 2, 0, 0, 1, 0, 0, 2, 0, 0, 2, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 2, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 1, 2, 0, 0, 0, 0,
1, 2, 0, 0, 0, 0, 1, 0, 0, 0, 2, 0, 1, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 2, 0, 1, 0, 0, 0,
2, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 1, 2, 0, 0, 0, 0, 1, 2, 0, 0, 0, 0, 0, 0, 2, 1, 0, 0, 0, 0, 2, 1, 0, 1, 1, 1, 1, 1,
2, 1, 1, 1, 1, 1, 2, 0, 0, 2, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 2, 0, 1, 0, 0, 0, 2, 0, 1, 0, 0, 2, 2, 2, 2, 1, 2, 2, 2, 2, 2, 1, 2,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 2, 0, 0, 1, 0, 0, 2, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 2,
0, 0, 0, 1, 0, 2, 0, 0, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 2, 1, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 2, 0, 2, 1, 0, 0, 0, 0, 2, 1, 0, 0,
0, 0, 0, 0, 1, 2, 0, 0, 0, 0, 1, 2, 0, 2, 2, 2, 2, 2, 1, 2, 2, 2, 2, 2, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
1, 0, 2, 0, 0, 0, 1, 0, 2, 0, 0, 1, 0, 2, 0, 0, 0, 1, 0, 2, 0, 0, 0, 0, 1, 0, 2, 0, 0, 0, 1, 0, 2, 2, 2, 1, 2, 2, 2, 2, 2, 1, 2,
2, 2, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 1, 2, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 2, 0, 0, 2, 0, 0, 0,
1, 0, 2, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 2, 0, 0, 0, 1, 0, 2, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0,
1, 0, 0, 1, 2, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 2, 0, 0, 2, 0, 0, 0, 1, 0, 2, 0, 0, 0, 1, 0, 0, 1, 0,
2, 0, 0, 0, 1, 0, 2, 0, 0, 0, 0, 1, 0, 2, 0, 0, 0, 1, 0, 2, 2, 2, 1, 2, 2, 2, 2, 2, 1, 2, 2, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 1, 0, 0, 2, 0, 0, 1, 0, 0, 2, 0, 0, 0, 1, 2, 0, 0, 0, 0, 1, 2, 0, 2, 2, 2, 2, 2, 1, 2, 2, 2, 2, 2, 1, 0, 0, 1,
0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 2, 0, 0, 0, 1, 0, 2, 0, 0, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 2, 1, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0,
0, 2, 0, 2, 1, 0, 0, 0, 0, 2, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 2, 0, 0, 0, 1, 0, 2, 0, 0, 0,
0, 1, 0, 2, 0, 0, 0, 1, 0, 2, 2, 2, 1, 2, 2, 2, 2, 2, 1, 2, 2, 2, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 1, 2, 1, 1, 1, 1, 1, 2, 1,
1, 1, 1, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 2, 0, 0, 2, 0, 0, 0, 1, 0, 2, 0, 0, 0, 1, 0, 0, 1, 0, 2, 0, 0, 0, 1, 0, 2, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 2, 0, 0, 0, 1, 0, 2, 0, 0, 2, 2, 1, 2, 2, 2, 2, 2, 1, 2, 2, 2, 0, 0, 0, 1, 0, 0, 0, 0,
0, 1, 0, 0, 1, 2, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 2, 0, 0, 2, 0, 0, 0, 1, 0, 2, 0, 0, 0, 1, 0, 0, 1,
0, 2, 0, 0, 0, 1, 0, 2, 0, 0, 0, 0, 1, 0, 2, 0, 0, 0, 1, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 2, 0, 0, 0, 0, 1,
2, 0, 0, 0, 0, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 0, 2, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 2, 0, 0, 1, 0, 0, 2, 0, 0, 1, 0, 0, 0,
0, 0, 2, 1, 0, 0, 0, 0, 2, 1, 0, 0, 1, 0, 0, 2, 0, 0, 1, 0, 0, 2, 0, 0, 2, 1, 0, 0, 0, 0, 2, 1, 0, 0, 2, 2, 2, 1, 2, 2, 2, 2, 2,
1, 2, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 2, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 2, 1, 2, 2, 2, 2, 2, 1, 2, 2, 2, 2, 0,
0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 0, 0, 2, 0, 1, 0, 0, 0, 2, 0, 0, 2, 0, 1, 0, 0, 0, 2, 0, 1, 0, 0, 0, 0, 2, 0, 1, 0, 0,
0, 2, 0, 1, 1, 1, 2, 1, 1, 1, 1, 1, 2, 1, 1, 1, 0, 0, 0, 2, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 1, 0, 0, 2, 0, 0, 1, 0, 0, 2, 0, 2, 2, 2, 2, 2, 1, 2, 2, 2, 2, 2, 1, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 2, 0, 0, 0,
1, 0, 2, 0, 0, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 2, 1, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 2, 0, 2, 1, 0, 0, 0, 0, 2, 1, 0, 0, 0, 0, 0,
0, 1, 2, 0, 0, 0, 0, 1, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 2, 1, 1, 1, 1, 1,
2, 1, 1, 1, 1, 1, 2, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 1, 2, 0, 0, 0, 0, 1, 2, 0, 0, 0, 0, 1, 0, 0, 0, 2, 0, 1, 0, 0, 0, 2, 0, 1,
0, 0, 0, 0, 2, 1, 0, 0, 0, 0, 2, 1, 0, 2, 0, 0, 0, 1, 0, 2, 0, 0, 0, 1, 0, 0, 2, 0, 0, 1, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 1, 0, 0, 0, 2, 0, 1, 0, 0, 0, 2, 0, 1, 0, 0, 0, 0, 2, 1, 0, 0, 0, 0, 2,
1, 0, 2, 0, 0, 0, 1, 0, 2, 0, 0, 0, 1, 0, 0, 2, 0, 0, 1, 0, 0, 2, 0, 0, 2, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 2, 0, 0, 0, 0, 0, 2,
0, 0, 0, 0, 0, 1, 2, 0, 0, 0, 0, 1, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 2,
1, 0, 2, 0, 0, 0, 1, 0, 2, 0, 0, 0, 1, 0, 0, 1, 0, 2, 0, 0, 0, 1, 0, 2, 0, 0, 0, 0, 1, 0, 2, 0, 0, 0, 1, 0, 2, 2, 2, 1, 2, 2, 2,
2, 2, 1, 2, 2, 2, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 1, 2, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 2, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 1, 0, 2, 0, 0, 0, 1, 0, 2, 0, 0, 0, 1, 0, 0, 2, 0,

0, 1, 0, 0, 2, 0, 0, 2, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 2, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 1, 2, 0, 0, 0, 0, 1, 2, 0, 0, 0, 0,
1, 0, 0, 0, 2, 0, 1, 0, 0, 0, 2, 0, 1, 0, 0, 0, 0, 2, 1, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0,
0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 0, 2, 0, 0, 1, 0, 0, 2, 0, 0, 2, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 2, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0,
0, 1, 2, 0, 0, 0, 0, 1, 2, 0, 0, 0, 0, 1, 0, 0, 0, 2, 0, 1, 0, 0, 0, 2, 0, 1, 0, 0, 0, 0, 2, 1, 0, 0, 0, 0, 2, 1, 0, 2, 0, 0, 0,
1, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 2, 0, 0, 0,
0, 0, 1, 2, 0, 0, 0, 0, 1, 2, 0, 0, 0, 0, 1, 0, 0, 0, 2, 0, 1, 0, 0, 0, 2, 0, 1, 0, 0, 0, 0, 2, 1, 0, 0, 0, 0, 2, 1, 0, 2, 0, 0,
0, 1, 0, 2, 0, 0, 0, 1, 0, 0, 2, 0, 0, 1, 0, 0, 2, 0, 0, 2, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0,
0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 2, 2, 2, 1, 0, 0, 1, 1, 0, 1, 0, 0, 1, 1, 0, 1, 0,
1, 0, 0, 1, 0, 0, 2, 2, 2, 2, 2, 0, 0, 1, 1, 0, 0, 0, 2, 2, 2, 2, 1, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 2, 2, 2, 2, 1, 0, 0, 1, 1, 0, 1, 0, 1, 0, 0, 1, 0, 0, 2, 2, 2, 2, 2, 0, 0, 1, 1, 0, 0, 0, 2, 2, 2, 2, 1, 0,
1, 0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 1, 1, 0, 0, 1, 1, 0, 1,
0, 1, 0, 0, 1, 2, 0, 1, 0, 0, 1, 1, 0, 1, 0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 0, 2, 2, 2, 2, 0, 0, 2, 2, 2, 2, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 1, 0, 0, 0, 2, 2, 2, 2, 1, 0, 0, 1, 1, 0, 1, 0, 2, 2, 2, 2, 1, 0, 0, 1, 1, 0, 0,
0, 2, 2, 2, 2, 1, 0, 1, 0, 0, 1, 1, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 1,
1, 0, 1, 0, 0, 1, 2, 0, 1, 0, 0, 1, 1, 0, 1, 0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 0, 2, 2, 2, 2, 0, 0, 2, 2, 2, 2, 1, 0, 0, 1, 1, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 1, 0, 1, 0, 0, 1, 1, 0, 1, 0, 2, 2, 2, 2, 1, 0, 0, 1, 1, 0,
0, 0, 2, 2, 2, 2, 1, 0, 1, 0, 0, 1, 1, 0, 1, 0, 0, 1, 0, 0, 2, 2, 2, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0,
0, 2, 0, 0, 1, 1, 0, 0, 0, 2, 2, 2, 2, 0, 0, 2, 2, 2, 2, 0, 0, 2, 2, 2, 2, 0, 0, 2, 2, 2, 2, 0, 0, 2, 2, 2, 2, 0, 0, 2, 2, 2, 2,
0, 0, 2, 2, 2, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 2, 2, 2, 2, 1, 0, 0, 1, 1, 0, 1, 0, 0, 1, 1,
0, 1, 0, 0, 1, 1, 0, 1, 0, 0, 1, 1, 0, 1, 0, 0, 1, 1, 0, 1, 0, 0, 1, 1, 0, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 2, 1, 1, 1, 1, 1, 2, 1, 1, 0, 2, 1, 0, 0, 0, 0, 2, 1, 0, 0, 0, 0, 0, 0, 1, 2, 0,
0, 0, 0, 1, 2, 0, 2, 2, 2, 2, 2, 1, 2, 2, 2, 2, 2, 1, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1, 0, 2, 0, 0, 0, 1, 0, 2, 0, 0, 1,
1, 1, 1, 2, 1, 1, 1, 1, 1, 2, 1, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 2, 1, 1,
1, 1, 1, 2, 1, 1, 1, 0, 2, 0, 1, 0, 0, 0, 0, 2, 0, 1, 0, 0, 2, 2, 2, 2, 1, 2, 2, 2, 2, 2, 1, 2, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 1,
0, 1, 2, 0, 0, 0, 0, 1, 2, 0, 0, 0, 0, 0, 0, 2, 1, 0, 0, 0, 0, 2, 1, 0, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 2, 0, 0, 2, 0, 0, 0, 0,
0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 2, 1, 0, 0, 0, 0, 2, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1,
0, 0, 1, 0, 0, 0, 2, 0, 1, 0, 0, 0, 2, 0, 0, 2, 0, 1, 0, 0, 0, 2, 0, 1, 0, 0, 0, 0, 2, 0, 1, 0, 0, 0, 2, 0, 1, 1, 1, 2, 1, 1, 1,
1, 1, 2, 1, 1, 1, 0, 0, 0, 2, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 2, 0, 0, 2, 1, 2, 2, 2, 2, 2, 1, 2, 2, 2, 2, 2, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 1, 0, 0, 0, 2, 0, 1, 0, 0, 0, 2, 2, 2, 2, 2, 1, 2, 2, 2, 2, 2, 1, 2, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 1, 2, 0, 0,
0, 0, 1, 2, 0, 0, 0, 0, 0, 0, 2, 1, 0, 0, 0, 0, 2, 1, 0, 1, 1, 1, 1, 1, 2, 1, 1, 1, 1, 1, 2, 0, 0, 0, 2, 0, 0, 0, 0, 0, 2, 0, 0,
0, 2, 0, 1, 0, 0, 0, 2, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 2, 0, 2, 0, 0, 0, 1, 1, 2, 2, 2, 2, 2, 2, 1, 1, 1, 1, 1, 1, 1, 2, 0, 0, 1, 2, 2, 1, 0,
0, 2, 2, 1, 2, 2, 2, 2, 1, 2, 1, 1, 1, 1, 0, 0, 1, 0, 2, 0, 0, 0, 2, 0, 1, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 1, 0, 0, 2, 1, 0,
0, 0, 0, 1, 2, 0, 0, 0, 0, 2, 1, 0, 0, 0, 0, 1, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1,
0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 1, 0, 0, 0, 1, 0, 2, 0, 0, 1, 0, 2, 0, 0, 0, 2, 0, 1, 0, 0, 0, 0, 2, 0,
1, 0, 0, 0, 1, 0, 2, 0, 0, 2, 2, 2, 1, 2, 2, 1, 1, 1, 2, 1, 1, 2, 0, 0, 1, 2, 2, 1, 0, 0, 2, 1, 1, 0, 2, 0, 1, 0, 0, 0, 1, 0, 2,
0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 2, 0, 0, 1, 0, 0, 1, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 1, 0, 0, 1, 0, 0, 2, 0, 0, 0, 0, 2, 1, 0, 0, 0, 0,
1, 2, 0, 0, 0, 0, 2, 1, 0, 0, 0, 0, 1, 2, 0, 0, 0, 1, 2, 2, 2, 2, 2, 2, 1, 1, 1, 1, 1, 1, 2, 0, 0, 1, 2, 2, 1, 0, 0, 2, 2, 1, 2,
2, 2, 2, 1, 2, 1, 1, 1, 1, 0, 0, 1, 0, 2, 0, 0, 0, 0, 2, 0, 1, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 1, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,

```
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 2, 0, 0, 2, 0, 0, 1, 0,
0, 0, 1, 0, 2, 0, 0, 0, 2, 0, 1, 2, 1, 2, 2, 2, 2, 1, 2, 1, 1, 1, 1, 0, 1, 0, 1, 1, 2, 0, 2, 0, 2, 2, 1, 1, 2, 0, 0, 0, 0, 2, 1,
0, 0, 0, 0, 0, 0, 0, 0, 2, 1, 0, 0, 0, 0, 1, 2, 0, 0, 0, 0, 2, 1, 0, 0, 0, 0, 1, 2, 0, 0, 1, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 2, 0,
0, 0, 2, 0, 1, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 1, 0, 0, 2, 2, 2, 2, 1, 2, 1, 1, 1, 1, 2, 1, 1, 0, 1, 1, 2, 0, 2, 0, 2, 2, 1, 0, 0,
2, 0, 0, 1, 0, 0, 1, 0, 0, 2, 0, 2, 2, 2, 2, 1, 2, 1, 1, 1, 1, 2, 1, 2, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 2, 0, 0, 2, 0, 0, 1, 0, 0,
1, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 1, 1, 2, 1, 1, 1, 2, 2, 1, 2, 2, 2, 0, 0, 2, 1, 0, 0, 0, 0, 1, 2, 0, 0, 1, 2, 2, 2, 2, 2, 2, 1, 1, 1, 1, 1, 1, 1, 2, 0, 0, 1,
2, 2, 1, 0, 0, 2, 2, 1, 2, 2, 2, 2, 1, 2, 1, 1, 1, 1, 0, 0, 1, 0, 2, 0, 0, 0, 2, 0, 1, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 1, 0,
0, 2, 1, 0, 0, 0, 0, 1, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 1, 0, 0, 0, 1, 0, 2, 0, 0, 0, 0, 2, 0, 1, 0, 0, 0, 1, 0, 2, 0, 0, 2, 2, 2, 1, 2, 2, 1, 1, 1, 2, 1,
1, 2, 0, 0, 1, 2, 2, 1, 0, 0, 2, 1, 1, 0, 2, 0, 1, 0, 0, 0, 1, 0, 2, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 2, 0, 0, 0, 2, 0, 0, 1,
0, 0, 1, 0, 0, 2, 1, 0, 2, 0, 0, 0, 2, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 2, 0, 0, 2, 0, 0, 1, 0, 2, 1, 2, 2, 2, 2, 1, 2, 1, 1, 1, 1, 0, 1, 0, 1,
1, 2, 0, 2, 0, 2, 2, 1, 1, 2, 0, 0, 0, 0, 2, 1, 0, 0, 0, 0, 0, 0, 0, 0, 2, 1, 0, 0, 0, 0, 1, 2, 0, 0, 0, 0, 2, 1, 0, 0, 0, 0, 1,
2, 0, 0, 1, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 1, 0, 2, 0, 0, 0, 2, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 2, 1, 1, 1, 1, 2, 1, 2, 2, 2, 2, 2, 2, 2, 1, 2, 2, 1, 1, 1,
2, 1, 1, 2, 0, 0, 1, 2, 2, 1, 0, 0, 2, 1, 1, 0, 2, 0, 1, 0, 0, 0, 1, 0, 2, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 2, 0, 0, 0, 2, 0,
0, 1, 0, 0, 1, 0, 0, 2, 1, 0, 2, 0, 0, 0, 2, 0, 1, 0, 0, 0, 0, 2, 0, 1, 0, 0, 0, 1, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 2, 0, 0, 0, 0, 2, 1, 0, 0, 0, 1, 1,
1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 0, 1, 2, 2, 0, 2, 0, 2, 1, 1, 0, 1, 0, 2, 0, 0, 0, 1, 0, 1, 0, 0, 0, 2, 0, 0, 0, 1, 2, 0, 0, 0, 0,
2, 1, 0, 0, 2, 0, 0, 0, 1, 0, 1, 0, 0, 0, 2, 0, 1, 2, 0, 0, 0, 0, 2, 1, 0, 0, 0, 2, 0, 0, 1, 0, 0, 1, 0, 0, 2, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0,
0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 2, 0, 0, 0,
2, 0, 1, 0, 0, 2, 2, 2, 2, 1, 2, 1, 1, 1, 1, 2, 1, 1, 0, 1, 1, 2, 0, 2, 0, 2, 2, 1, 0, 0, 2, 0, 0, 1, 0, 0, 1, 0, 0, 2, 0, 2, 2,
2, 2, 1, 2, 1, 1, 1, 1, 2, 1, 2, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 2, 0, 0, 2, 0, 0, 1, 0, 0, 1, 0, 0, 2, 0, 0, 0, 2, 0, 0, 0, 0, 0,
1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
1, 2, 2, 2, 2, 2, 2, 1, 1, 1, 1, 1, 0, 2, 0, 0, 0, 1, 0, 1, 0, 0, 0, 2, 0, 0, 0, 1, 2, 0, 0, 0, 0, 2, 1, 0, 0, 2, 0, 0, 0, 1, 0,
1, 0, 0, 0, 2, 0, 1, 2, 0, 0, 0, 0, 2, 1, 0, 0, 0, 2, 0, 0, 1, 0, 0, 1, 0, 0, 2, 0, 0, 1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 0, 1,
2, 2, 0, 2, 0, 2, 1, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 2, 2, 2, 2, 2, 1, 1, 1, 1, 1, 1, 2, 1, 2, 2, 0, 2, 0, 2, 1, 1, 0, 1, 0,
0, 2, 1, 0, 0, 0, 0, 1, 2, 0, 0, 0, 0, 1, 0, 0, 2, 0, 0, 2, 0, 0, 1, 0, 0, 0, 0, 2, 0, 1, 0, 0, 0, 1, 0, 2, 0, 0, 2, 0, 1, 0, 0,
0, 1, 0, 2, 0, 1, 1, 1, 2, 1, 1, 2, 2, 2, 1, 2, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0,
2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 1, 2, 0, 2, 0, 2, 2, 1, 2, 0, 0, 1, 0, 0, 1, 0, 0, 2, 0, 0, 2, 0, 1, 0, 0,
0, 1, 0, 2, 0, 0, 0, 2, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 2, 2, 0, 0, 0, 1, 0, 1, 0, 0, 0, 2, 0, 2, 1, 0, 0, 0, 0, 1, 2, 0, 0, 0, 0,
1, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 1, 2, 2, 2, 2, 2, 2, 1, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
1, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
```

0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 1, 2, 0, 2, 0, 2, 2, 1, 0, 2, 0, 1, 0, 0, 0, 1, 0, 2, 0,
0, 0, 2, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 2, 2, 0, 0, 0, 1, 0, 1, 0, 0, 0, 2, 0, 2, 1, 0, 0, 0, 0, 1, 2, 0, 0, 0, 0, 1, 0, 0, 0, 0,
0, 2, 0, 0, 0, 0, 0, 1, 2, 2, 2, 2, 2, 2, 1, 1, 1, 1, 1, 2, 0, 0, 1, 0, 0, 1, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 2, 2, 0, 2, 0, 2, 1, 1, 0, 1, 2, 1, 0,
0, 0, 0, 1, 2, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 1, 2, 2, 2, 2, 2, 2, 1, 1, 1, 1, 1, 2, 0, 0, 1, 0, 0, 1, 0, 0, 2,
0, 0, 2, 0, 1, 0, 0, 0, 1, 0, 2, 0, 0, 0, 2, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 2, 2, 0, 0, 0, 1, 0, 1, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 1, 2, 2, 1, 0,
0, 2, 1, 1, 2, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 2, 2, 0, 0, 0, 1, 0, 1, 0, 0, 0, 2, 0, 2, 1, 0, 0, 0, 0, 1, 2, 0, 0, 0, 0, 1, 0, 0,
0, 0, 0, 2, 0, 0, 0, 0, 0, 1, 2, 2, 2, 2, 2, 2, 1, 1, 1, 1, 1, 2, 0, 0, 1, 0, 0, 1, 0, 0, 2, 0, 0, 2, 0, 1, 0, 0, 0, 1, 0, 2, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1,
1, 2, 0, 0, 1, 2, 2, 1, 0, 0, 2, 2, 0, 0, 0, 1, 0, 1, 0, 0, 0, 2, 0, 2, 1, 0, 0, 0, 0, 1, 2, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 2, 0,
0, 0, 0, 0, 1, 2, 2, 2, 2, 2, 2, 1, 1, 1, 1, 1, 2, 0, 0, 1, 0, 0, 1, 0, 0, 2, 0, 0, 2, 0, 1, 0, 0, 0, 1, 0, 2, 0, 0, 0, 2, 0, 0,
0, 0, 1, 1, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 1, 2, 2, 0, 2, 0, 2, 1, 1, 0, 1, 0, 1, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 1, 2, 2, 2, 2, 2, 2, 1, 1, 1, 1, 1, 2,
0, 0, 1, 0, 0, 1, 0, 0, 2, 0, 0, 2, 0, 1, 0, 0, 0, 1, 0, 2, 0, 0, 0, 2, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 2, 2, 0, 0, 0, 1, 0, 1, 0,
0, 0, 2, 0, 2, 1, 0, 0, 0, 0, 1, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 2, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
2, 1, 2, 2, 2, 2, 2, 2, 2, 1, 2, 0, 1, 2, 1, 2, 1, 0, 2, 1, 1, 2, 2, 1, 1, 0, 0, 0, 2, 1, 2, 1, 0, 0, 1, 2, 1, 2, 2, 1, 1, 2, 2,
0, 2, 1, 2, 0, 1, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 2, 1, 0, 0, 0, 0, 0, 1, 0, 0, 2, 1, 2, 1, 1, 2, 1, 0, 2, 1, 2, 1, 1, 2, 2, 0, 1, 2, 2, 1, 1, 0, 2, 1, 2, 0,
1, 2, 0, 0, 2, 1, 2, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 2, 1, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 2, 1, 1, 1, 1, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 1, 0, 0, 0, 0, 0, 1, 2, 1, 2, 0, 1, 2, 1, 2, 2, 1, 1, 0, 0, 0, 2, 1, 2, 1, 0, 0, 0, 1,
2, 1, 2, 2, 1, 1, 2, 2, 0, 2, 1, 2, 0, 1, 2, 0, 0, 0, 0, 0, 0, 1, 2, 1, 0, 2, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 2, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 2, 0, 0, 0, 0, 2, 0, 1, 2, 2, 1, 1, 0, 1, 2, 1, 0, 2, 1, 2, 1,
2, 0, 1, 2, 0, 0, 1, 2, 1, 2, 1, 2, 2, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 2, 1, 1, 2, 2, 0, 0, 0, 2, 1, 2, 1, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 2, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 2, 0, 0, 0, 0, 2, 0, 0, 1, 2, 2, 1, 1, 0, 2,
1, 2, 0, 1, 2, 0, 0, 1, 2, 1, 2, 1, 2, 2, 1, 1, 0, 0, 0, 0, 0, 0, 0, 2, 1, 1, 2, 2, 0, 0, 0, 2, 1, 2, 1, 1, 2, 1, 0, 2, 1, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 2, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 1, 0, 0, 1, 0, 0, 0,
0, 0, 2, 1, 2, 1, 2, 1, 2, 0, 1, 2, 0, 0, 2, 1, 2, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 2, 1, 2, 1, 2, 1, 0, 2, 1, 2, 1, 1, 2, 2, 0, 1,
2, 2, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
1, 2, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2,
1, 0, 0, 1, 0, 0, 0, 2, 1, 2, 0, 1, 2, 2, 1, 1, 2, 2, 0, 2, 1, 2, 0, 1, 2, 0, 0, 0, 0, 0, 0, 1, 2, 1, 0, 2, 1, 1, 2, 2, 1, 1, 0,
0, 0, 2, 1, 2, 1, 0, 0, 1, 2, 1, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,

0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 1, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 2, 1, 0, 1, 0, 0, 0, 0, 0, 0, 2, 1, 2, 1, 0, 0, 2, 1, 2, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 2, 1, 2, 1, 2, 1, 0, 2,
1, 2, 1, 1, 2, 2, 0, 1, 2, 2, 1, 1, 0, 2, 1, 2, 0, 1, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 1, 2, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 1, 0, 1, 0, 0, 0, 0, 2, 1, 2, 0, 1, 2, 2, 1, 2, 0, 1, 2, 0, 0, 0, 0, 0, 0, 1, 2, 1, 0,
2, 1, 1, 2, 2, 1, 1, 0, 0, 0, 2, 1, 2, 1, 0, 0, 1, 2, 1, 2, 2, 1, 1, 2, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 1, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 2, 1, 2, 0, 0, 1,
2, 1, 2, 0, 0, 1, 2, 1, 2, 0, 0, 1, 2, 1, 2, 0, 0, 1, 2, 1, 2, 0, 0, 1, 2, 1, 2, 0, 0, 1, 2, 1, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 1,
1, 2, 2, 0, 2, 1, 1, 2, 2, 0, 2, 1, 1, 2, 2, 0, 2, 1, 1, 2, 2, 0, 2, 1, 1, 2, 2, 0, 2, 1, 1, 2, 2, 0, 2, 1, 1, 2, 2, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 2, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 1, 2, 1, 0, 2, 1, 1, 2, 1, 0, 2, 1, 1, 2, 1, 0, 2, 1, 1, 2, 1, 0, 2, 1, 1, 2, 1, 0, 2, 1, 1, 2, 1, 0, 2, 1, 1, 2,
1, 0, 2, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
2, 0, 0, 1, 2, 2, 0, 0, 0, 0, 0, 0, 2, 1, 1, 0, 1, 0, 2, 1, 1, 0, 1, 0, 2, 1, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 2, 1, 1, 0, 1, 0, 2,
1, 1, 0, 1, 0, 2, 1, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 1, 1, 2, 0, 0, 1, 0, 0, 0, 0, 0, 0, 2, 1, 1, 0, 1, 0, 2, 1, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 2, 1, 1, 0, 1, 0,
2, 1, 1, 0, 1, 0, 2, 1, 1, 0, 1, 0, 2, 1, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 2, 2, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 1, 1, 0, 1, 0, 2, 1, 1, 0, 1,
0, 2, 1, 1, 0, 1, 0, 2, 1, 1, 0, 1, 0, 2, 1, 1, 0, 1, 0, 2, 1, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 2, 0, 0, 0, 0, 0, 0, 2, 1, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 1, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2,
0, 1, 0, 0, 0, 1, 0, 2, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 2, 0, 0, 0, 2, 0, 0, 1, 0, 0, 1, 0, 0, 2, 1, 0, 2, 0, 0, 0, 2, 0, 1, 0, 0,
0, 0, 2, 0, 1, 0, 0, 0, 1, 0, 2, 0, 0, 2, 2, 2, 1, 2, 2, 1, 1, 1, 2, 1, 1, 2, 0, 0, 1, 2, 2, 1, 0, 0, 2, 1, 1, 0, 2, 0, 1, 0, 0,
0, 1, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 2, 0, 0, 1, 0, 0, 1, 0, 0, 2, 0, 0, 1, 0, 2, 0, 0, 0, 2, 0, 1, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 1, 0, 0, 0, 2, 1,
0, 0, 0, 0, 1, 2, 0, 0, 0, 0, 2, 1, 0, 0, 0, 0, 1, 2, 0, 0, 1, 2, 2, 2, 2, 2, 2, 1, 1, 1, 1, 1, 1, 1, 2, 0, 0, 1, 2, 2, 1, 0, 0,

```
2, 2, 1, 2, 2, 2, 2, 1, 2, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 1, 0, 0, 0, 1, 0, 2, 0, 0, 0, 2, 0, 0, 1, 0, 0, 1, 0, 0, 2, 1, 0, 2, 0, 0, 0, 2, 0, 1,
0, 0, 0, 0, 2, 0, 1, 0, 0, 0, 1, 0, 2, 0, 0, 2, 2, 2, 1, 2, 2, 1, 1, 1, 2, 1, 1, 2, 0, 0, 1, 2, 2, 1, 0, 0, 2, 1, 1, 0, 2, 0, 1,
0, 0, 0, 1, 0, 2, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 1, 0, 0, 1, 0, 0, 2, 0, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 1, 0, 0,
2, 1, 0, 0, 0, 0, 1, 2, 0, 0, 0, 0, 2, 1, 0, 0, 0, 0, 1, 2, 0, 0, 1, 2, 2, 2, 2, 2, 2, 1, 1, 1, 1, 1, 1, 1, 2, 0, 0, 1, 2, 2, 1,
0, 0, 2, 2, 1, 2, 2, 2, 2, 1, 2, 1, 1, 1, 1, 0, 0, 1, 0, 2, 0, 0, 0, 2, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 2, 1, 1, 2, 2, 2, 1, 2, 2, 0, 0, 1, 0, 0, 0, 0,
0, 2, 0, 0, 0, 0, 0, 0, 1, 0, 2, 0, 0, 0, 2, 0, 1, 2, 1, 2, 2, 2, 2, 1, 2, 1, 1, 1, 1, 0, 1, 0, 1, 1, 2, 0, 2, 0, 2, 2, 1, 1, 2,
0, 0, 0, 0, 2, 1, 0, 0, 0, 0, 0, 0, 0, 0, 2, 1, 0, 0, 0, 0, 1, 2, 0, 0, 0, 0, 2, 1, 0, 0, 0, 0, 1, 2 ];
```

Appendix B

The 44-dimensional matrices

We present the reduced matrices on $V_{154}(z^2, 1)$. These are the upper left 44×44-submatrices of TMT^{-1}, $M \in \{X, Y, \rho\}$.

$$
X_{44} = \begin{pmatrix}
2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 1 & 2 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 2 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 2 & 2 & 2 & 2 & 2 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 2 & 0 & 2 & 0 & 2 & 1 & 2 & 2 & 1 & 0 & 1 & 0 & 0 & 2 & 2 & 2 & 1 & 0 & 2 & 1 & 2 & 0 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 2 & 2 & 2 & 0 & 2 & 1 & 2 & 2 & 1 & 2 & 1 & 0 & 1 & 2 & 2 & 2 & 1 & 2 & 1 & 0 & 1 & 2 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 2 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 2 & 1 & 1 & 2 & 1 & 2 & 0 & 2 & 1 & 1 & 1 & 2 & 0 & 1 & 2 & 1 & 0 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 2 & 1 & 1 & 0 & 0 & 2 & 1 & 1 & 2 & 1 & 2 & 0 & 2 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 2 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 2 & 2 & 2 & 2 & 2 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 1 & 2 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 1 & 0 & 2 & 0 & 0 & 0 & 0 & 2 & 1 & 0 & 0 & 1 & 2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 2 & 1 & 2 & 0 & 0 & 1 & 2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}
$$

$$
Y_{44} = \left(\begin{array}{cccccccccccccccccccccccccccccccccccccccccccc}
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&2&1&2&2&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&2&2&2&0&2&1&2&2&1&2&1&0&1&2&2&2&1&2&1&0&1&2&0&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&2&0&0&0&0&0&0&0&0&0&2&0&0&0&0&2&1&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&2&0&0&2&0&0&0&0&0&0&2&0&0&0&0&0&2&0&0&0&2&2&2&2&2&2&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&1&2&2&1&2&1&1&2&1&1&2&2&1&0&0&0&2&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&0&0&1&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&1&1&2&2&1&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&1&0&1&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&1&1&2&1&2&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&2&0&2&0&0&0&0&0&0&0&2&0&0&0&1&1&2&1&1&1&2&2&2&2&2&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&1&2&0&2&0&1&0&2&2&0&2&0&1&1&2&0&2&0&1&2&0&0&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&2&0&2&2&1&2&1&1&0&0&0&1&1&0&0&2&0&1&1&0&2&1&1&0&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&1&0&2&1&2&2&2&0&2&2&2&0&0&1&0&0&2&0&0&1&1&2&1&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&2&0&1&2&1&1&1&0&1&1&1&0&0&2&0&1&1&0&2&2&2&2&2&2&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&2&0&0&0&1&1&1&0&1&1&1&0&0&0&0&1&1&0&0&2&2&1&2&2&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&2&2&2&0&1&1&2&0&1&2&0&1&0&0&1&0&2&1&1&0&1&1&2&1&1&2&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&2&0&2&0&0&1&2&2&1&2&1&0&1&2&2&2&1&0&2&1&2&0&1&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&2&2&1&2&2&0&0&1&2&2&1&2&1&0&1&2&2&2&2&0&0&2&0&1&2&0&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&0&2&2&2&2&2&2&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&1&0&1&0&0&1&2&2&1&2&2&0&1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&2&1&2&1&1&1&0&2&2&2&2&0&2&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&2&1&1&1&1&2&2&2&2&1&2&1&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&2&2&0&2&0&0&1&2&2&1&1&1&1&1&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2
\end{array}\right)
$$

$$\rho_{44} = \left(\begin{array}{*{44}{c}}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 2 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 2 & 2 & 2 & 2 & 2 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 2 & 2 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 2 & 2 & 1 & 2 & 2 & 0 & 0 & 1 & 2 & 2 & 1 & 2 & 1 & 0 & 1 & 2 & 2 & 2 & 2 & 0 & 0 & 2 & 0 & 1 & 2 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 2 & 1 & 1 & 2 & 1 & 2 & 0 & 2 & 1 & 1 & 1 & 2 & 0 & 1 & 2 & 1 & 0 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 2 & 1 & 1 & 2 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 2 & 2 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 2 & 2 & 2 & 2 & 2 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 2 & 0 & 0 & 2 & 0 & 2 & 0 & 0 & 0 & 1 & 2 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 2 & 2 & 2 & 2 & 2 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 1 & 2 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 2 & 1 & 2 & 0 & 1 & 2 & 1 & 0 & 0 & 2 & 2 & 2 & 2 & 0 & 1 & 1 & 2 & 0 & 0 & 2 & 2 & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 2 & 0 & 1 & 0 & 1 & 0 & 1 & 2 & 2 & 0 & 0 & 2 & 1 & 1 & 0 & 0 & 2 & 2 & 1 & 2 & 2 & 2 & 1 & 0 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2 & 1 & 0 & 1 & 0 & 1 & 2 & 1 & 1 & 2 & 0 & 2 & 0 & 0 & 1 & 1 & 1 & 2 & 0 & 1 & 2 & 1 & 0 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 2 & 2 & 2 & 0 & 2 & 1 & 2 & 2 & 1 & 2 & 1 & 0 & 1 & 2 & 2 & 2 & 1 & 0 & 2 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 1 & 2 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 2 & 2 & 1 & 2 & 1 & 0 & 2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 2 & 1 & 2 & 1 & 2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 2 & 2 & 1 & 2 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 0 & 2 & 0 & 0 & 2 & 1 & 1 & 2 & 1 & 1 & 0 & 2 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{array}\right)$$

Appendix C

The 44-dimensional matrices obtained by the Meataxe

We present the 44-dimensional matrices X_M, Y_M, ρ_M and a_M obtained by using the Meataxe program in Magma [2]. We give also a matrix t_{44}, obtained using the Magma function `IsIsomorphic` for modules such that $t_{44}M_M t_{44}^{-1} = M_{44}$ where $M_M \in \{X_M, Y_M, \rho_M\}$ and M_{44} is the corresponding matrix in $\{X_{44}, Y_{44}, \rho_{44}\}$.

$$X_M = \begin{pmatrix}
0 0 1 0 1 1 2 2 2 0 0 2 1 1 2 0 2 0 0 2 0 0 0 2 0 1 1 1 0 1 2 2 1 2 1 2 2 2 0 0 2 1 2 1 \\
1 2 2 1 2 0 0 0 0 1 0 0 1 1 0 0 2 0 2 1 0 1 2 1 2 2 1 0 0 0 1 0 2 0 2 2 0 0 1 1 1 1 2 2 \\
1 2 2 1 0 0 1 1 0 0 1 0 1 0 0 2 0 2 0 2 2 0 2 1 2 0 0 0 0 2 1 1 2 2 0 1 2 0 0 0 0 0 2 2 \\
0 0 1 0 1 1 0 1 1 2 2 2 2 1 1 0 1 2 1 2 0 0 1 1 2 0 0 0 0 1 2 2 0 0 0 0 0 2 0 2 1 1 0 1 \\
2 1 0 1 1 1 0 1 1 1 2 2 0 0 1 0 1 1 2 2 2 2 2 2 1 1 2 1 0 2 2 1 0 2 2 1 1 1 2 2 1 1 0 2 \\
1 1 2 0 2 1 0 2 0 1 1 2 0 2 0 2 1 1 2 2 2 1 2 1 0 2 2 1 0 2 1 1 0 0 2 0 0 1 1 2 1 1 0 0 \\
1 0 2 1 0 1 2 1 0 0 2 1 1 2 1 0 1 2 1 2 2 1 0 2 1 2 2 2 0 2 2 1 0 2 1 0 2 1 2 0 2 1 1 2 \\
0 0 0 2 2 1 2 1 1 1 0 1 1 1 1 1 1 2 0 1 2 1 1 2 0 0 0 1 0 1 0 2 1 0 1 2 0 1 1 1 0 1 0 2 \\
1 1 1 1 0 0 0 2 2 2 2 2 2 2 1 1 1 1 1 0 0 2 1 1 0 1 0 2 0 2 0 1 0 0 1 1 1 1 1 0 1 1 1 0 \\
1 2 2 0 0 1 1 0 2 2 1 1 0 2 2 2 2 2 0 0 0 1 2 0 0 1 2 1 0 1 0 2 0 2 0 2 2 1 2 0 0 0 1 1 \\
1 2 2 1 0 2 1 0 1 0 1 0 0 0 0 2 1 0 2 1 0 1 1 1 2 2 1 1 0 1 2 2 2 0 0 0 2 0 0 0 0 1 0 1 \\
0 2 1 2 1 1 1 2 1 2 0 1 2 2 0 1 0 1 2 1 2 1 1 1 2 0 0 2 0 1 0 2 2 0 2 0 0 2 2 2 2 2 2 0 \\
0 0 0 0 0 1 0 1 1 0 2 2 0 2 0 2 0 2 0 2 2 2 0 2 1 1 1 0 0 1 2 2 0 0 1 2 0 2 0 0 0 2 1 2 \\
0 0 2 0 0 1 2 1 1 1 2 0 1 1 0 2 0 1 2 1 0 0 2 2 1 0 0 2 0 2 0 1 0 0 1 2 1 1 0 1 1 2 1 0 \\
0 1 2 1 2 0 1 2 1 0 2 2 0 2 0 1 1 2 2 0 1 1 1 0 0 2 2 2 0 0 0 0 1 0 1 2 1 1 1 2 0 1 1 1 \\
0 0 0 0 1 1 2 0 0 2 1 1 0 1 2 2 2 2 2 2 0 2 2 1 2 2 0 0 0 2 2 1 2 1 0 2 1 2 1 2 0 2 0 2 \\
2 1 0 2 2 1 1 0 0 0 1 2 2 2 1 1 0 2 0 0 2 0 2 0 1 1 0 0 0 2 2 1 1 1 1 1 1 0 0 1 2 0 2 0 \\
2 2 1 0 2 0 1 2 0 1 2 1 0 0 0 1 1 0 1 0 2 1 0 0 0 1 0 2 0 0 1 0 1 0 1 0 2 1 1 1 1 0 1 2 \\
0 0 2 1 0 0 1 1 0 0 1 0 2 1 0 2 1 1 0 0 1 1 0 2 0 2 0 2 0 2 0 1 1 1 2 0 2 1 0 1 0 1 2 2 \\
1 2 1 1 2 2 2 0 0 1 2 1 2 2 1 2 2 0 1 1 2 1 0 0 2 0 2 2 0 1 2 2 1 2 0 2 1 0 2 1 0 2 2 0 \\
2 2 0 1 0 1 1 0 2 0 1 1 1 2 0 1 1 0 1 2 2 1 2 0 1 0 2 2 0 0 2 0 0 0 1 2 2 2 2 2 2 2 2 0 \\
0 2 1 0 2 0 2 2 1 0 2 1 0 2 0 0 2 2 1 0 0 0 0 1 1 2 0 2 0 0 0 0 2 0 0 2 1 0 2 0 1 1 2 0 \\
0 1 2 1 0 0 1 2 0 0 2 1 1 0 2 0 2 2 2 2 0 1 1 0 1 0 0 0 0 0 1 0 0 2 1 1 1 2 0 2 1 0 2 1 \\
2 0 1 2 0 1 0 1 2 2 2 0 2 2 0 0 0 2 0 2 2 0 2 0 0 1 2 2 0 0 1 0 0 0 2 2 1 1 2 1 1 1 0 0 \\
0 0 0 1 2 1 2 0 0 1 1 1 2 0 1 1 0 0 0 1 1 0 0 0 2 2 1 0 0 2 1 1 0 1 2 0 2 2 0 0 1 1 0 0 \\
2 2 2 0 1 1 0 2 0 1 0 2 1 2 1 2 1 0 1 2 0 1 1 1 2 2 2 1 0 2 1 1 2 1 2 2 2 0 1 2 0 1 2 1 \\
0 1 1 1 2 1 1 0 0 0 1 2 2 0 2 0 0 1 2 0 0 2 0 2 2 0 0 2 0 2 0 1 2 0 0 0 2 1 2 1 0 0 2 0 \\
1 0 2 2 2 0 2 0 2 0 0 0 0 0 0 0 2 0 0 1 0 1 1 0 2 2 1 0 0 2 2 1 2 1 2 2 1 2 2 0 2 2 2 0 \\
1 2 1 0 0 0 1 2 0 2 2 1 0 2 1 0 1 2 2 2 2 1 0 1 1 0 1 0 2 2 0 1 1 0 1 2 0 0 1 2 0 2 0 2 \\
0 2 0 0 0 1 2 2 2 0 0 2 1 0 0 0 2 0 1 2 1 2 2 1 0 1 2 1 0 0 0 2 1 0 0 1 0 1 0 0 1 2 2 2 \\
1 1 1 0 1 2 2 2 0 2 1 0 2 2 2 2 2 0 0 1 1 0 1 1 0 1 2 1 0 0 0 0 2 0 2 2 2 0 0 2 2 2 2 1 \\
2 1 2 1 2 1 0 0 0 2 1 0 1 2 0 1 2 0 2 2 0 1 2 0 0 2 0 0 0 2 0 0 1 0 1 0 2 2 0 1 0 2 2 1 \\
0 1 0 1 1 2 0 2 1 2 1 0 2 0 0 1 1 1 2 0 0 1 1 2 0 0 0 0 0 0 2 0 0 2 1 2 0 0 1 2 1 1 2 1 \\
2 2 0 2 2 2 1 0 1 2 2 0 2 0 2 0 0 0 0 1 0 1 0 2 0 2 0 1 0 2 2 1 2 0 2 0 0 0 1 0 0 1 0 0 \\
2 1 0 0 1 1 1 0 1 2 1 0 0 0 2 2 1 1 1 1 2 1 2 1 0 0 1 0 0 2 1 1 0 2 2 1 1 2 1 1 1 1 0 1 \\
1 2 0 0 1 1 2 1 0 2 2 2 2 0 1 2 0 0 2 0 1 0 1 1 1 0 0 1 0 2 0 1 0 0 1 2 0 1 2 0 1 2 0 1 \\
0 2 0 0 0 1 2 1 0 0 2 2 0 2 2 0 1 1 0 2 0 1 1 2 1 2 2 0 0 2 2 1 2 2 2 0 2 2 0 0 2 2 1 2 \\
2 1 0 0 2 2 2 1 0 0 0 2 0 2 0 2 2 1 0 2 2 1 2 1 2 1 2 1 0 0 1 0 2 0 1 0 2 1 1 1 1 0 0 1 \\
0 1 0 2 0 1 0 2 1 1 0 0 2 0 2 0 2 2 0 1 1 1 2 0 2 0 2 0 0 2 2 1 1 0 1 1 1 1 0 0 1 1 2 0 \\
1 1 2 2 1 0 1 1 1 2 1 1 2 1 2 1 1 1 0 0 1 0 2 0 0 0 1 2 0 1 2 2 1 1 1 0 0 0 0 2 1 2 1 0 \\
0 1 0 0 1 1 2 1 2 0 2 1 2 0 2 2 0 2 0 0 0 0 2 2 2 2 0 2 0 2 2 1 0 1 2 0 1 0 2 1 2 2 2 0 \\
1 0 1 2 2 2 2 2 0 1 1 0 1 1 0 0 0 1 1 1 2 1 2 0 2 2 2 1 0 2 0 1 1 1 0 1 2 1 0 0 2 1 0 2 \\
1 0 1 0 1 1 1 1 2 0 2 2 1 1 1 0 2 0 1 0 1 0 1 1 2 2 2 0 0 0 2 0 2 2 0 0 0 1 0 0 1 0 1 0 \\
1 0 0 1 1 0 1 2 0 1 2 1 1 1 0 0 0 2 1 2 2 0 1 1 1 2 0 1 0 0 1 0 1 2 1 2 1 2 1 2 1 0 0 1
\end{pmatrix}$$

$$Y_M = \begin{pmatrix}
0&2&0&1&0&2&0&1&0&1&1&1&0&0&1&1&0&1&0&2&0&0&1&1&2&0&0&2&2&2&1&1&0&2&1&1&2&0&0&1&1&0&0&2\\
1&0&2&0&2&0&1&2&0&2&1&1&0&2&0&0&0&0&2&0&1&1&0&0&2&2&1&1&1&1&2&2&1&0&2&0&0&1&1&0&2&0&0&0\\
0&0&2&0&0&2&2&1&2&0&2&0&0&2&2&0&0&1&1&2&2&0&2&0&2&1&0&1&0&2&1&2&2&2&2&1&1&2&1&2&2&1&0&0\\
2&0&0&1&2&1&2&1&0&2&2&0&1&1&1&2&0&1&2&2&0&1&2&1&1&2&0&1&1&1&0&2&1&0&0&0&1&1&1&1&0&0&0&2\\
2&2&2&1&0&2&1&2&2&1&2&2&1&0&1&0&2&0&1&0&1&1&0&2&2&0&2&0&2&1&1&1&2&1&1&2&0&2&2&0&2&2&1&0\\
2&0&1&2&2&2&0&2&1&2&1&2&0&1&2&1&1&2&0&1&0&0&2&0&0&1&0&1&1&1&0&2&2&0&0&1&2&0&2&0&2&2&1&1\\
2&0&2&2&0&0&0&2&1&0&2&0&1&2&2&2&0&0&1&1&1&2&0&0&1&2&2&2&1&1&2&0&1&0&0&0&1&1&2&1&0&0&1&0\\
1&1&1&1&1&1&2&0&0&1&2&0&0&2&0&0&2&2&2&2&1&1&1&1&0&0&2&2&0&2&0&1&0&0&2&0&1&0&0&0&0&1&2&1\\
2&2&2&1&2&0&1&0&1&0&2&2&1&2&0&2&0&2&1&2&0&0&0&1&2&2&1&2&1&2&2&0&1&0&1&2&1&0&0&1&1&2&2&2\\
0&1&2&2&2&1&1&1&0&1&2&1&2&0&0&2&0&0&1&0&2&1&0&2&1&1&2&0&2&2&1&2&0&2&2&0&2&2&1&0&2&1&2&2\\
1&0&1&2&1&2&0&0&2&0&0&1&0&2&0&1&2&2&2&0&1&2&0&2&2&1&0&2&1&2&2&2&0&0&0&0&1&1&1&2&1&0&0&0\\
0&1&0&2&1&1&0&1&0&2&1&0&2&0&1&1&2&0&1&1&2&2&0&2&2&1&2&0&1&2&1&0&0&1&2&2&1&2&1&2&1&2&2&1\\
2&0&0&1&2&2&2&2&1&1&0&0&2&2&1&0&0&0&2&2&2&2&2&0&0&0&1&2&1&2&0&0&0&0&1&0&1&1&2&1&1&1&1&2\\
2&1&2&2&1&1&1&2&2&1&1&2&0&0&1&1&2&1&0&1&1&0&0&1&0&0&2&0&2&2&0&0&1&2&1&1&2&0&1&0&2&2&2&2\\
2&0&2&1&1&2&1&0&0&1&1&1&1&2&2&0&0&2&2&2&0&1&0&1&0&1&1&1&0&2&0&2&1&2&2&1&2&2&1&2&1&2&1&1\\
2&2&2&0&1&2&2&2&0&1&0&0&2&2&1&1&2&1&2&1&0&0&0&1&2&0&0&1&2&2&0&1&0&2&1&0&0&2&0&1&0&1&1&2\\
1&1&0&0&0&2&0&2&1&1&0&1&2&1&0&0&2&2&0&0&1&1&2&2&1&2&0&2&2&0&2&1&0&0&1&1&1&1&1&0&1&2&0&0\\
1&1&1&2&0&2&1&1&2&0&2&0&0&0&2&0&2&1&0&1&2&1&1&0&1&1&1&1&0&1&0&1&2&1&2&1&0&1&1&2&2&1&0&2\\
2&1&0&2&0&2&2&1&2&2&1&1&1&2&2&1&1&1&0&1&0&1&0&0&1&0&2&1&2&1&1&2&2&1&1&1&1&1&1&1&2&2&0&1\\
0&2&0&1&0&1&0&2&1&1&0&0&1&2&1&2&1&1&0&0&1&2&0&2&1&1&1&2&0&0&2&1&1&1&2&1&0&0&2&1&2&0&0&2\\
0&0&2&2&1&1&0&1&0&2&1&2&0&0&0&1&2&2&1&1&0&0&2&0&2&1&0&0&0&2&1&2&1&0&1&2&0&0&2&0&2&0&1&1\\
0&1&2&0&1&1&2&2&2&2&2&2&0&0&0&0&0&0&1&0&2&0&0&1&0&2&1&0&0&0&2&0&0&2&0&2&0&1&1&0&1&2&0&2\\
1&2&1&2&2&1&0&0&0&2&1&2&2&0&1&2&1&2&2&1&2&0&0&2&2&0&1&0&1&0&0&2&1&0&1&2&1&2&0&1&0&0&2&1\\
1&1&2&1&0&2&2&2&2&0&2&1&1&2&0&1&2&1&2&0&0&1&2&2&0&0&2&2&0&1&2&1&1&1&0&2&0&1&0&1&2&2&2&0\\
1&0&2&1&2&1&1&1&2&1&2&2&2&2&2&1&1&2&0&0&2&2&0&0&0&0&2&2&0&0&1&2&0&1&1&1&0&0&0&0&2&1&2&1\\
2&2&1&1&2&0&2&0&2&1&0&0&1&1&2&2&0&2&2&0&1&1&2&2&2&2&2&0&1&0&1&1&0&1&0&0&1&1&1&1&2&1&1&2\\
2&0&0&1&1&1&2&1&2&1&1&2&1&0&2&0&0&2&2&0&0&2&1&0&1&1&0&0&0&0&1&2&0&1&1&1&2&2&0&1&2&2&2&0\\
1&2&1&1&2&1&2&2&0&0&1&2&2&2&2&2&2&0&1&0&2&0&1&1&0&2&2&1&2&2&0&0&2&2&2&2&2&0&1&0&0&0&1&0\\
0&2&2&0&2&2&1&2&1&2&0&2&2&2&2&0&1&2&2&2&0&2&0&2&0&1&2&0&2&1&1&0&1&1&1&0&0&1&2&0&0&2&0&1\\
0&0&1&0&2&0&1&2&1&0&1&1&1&0&0&1&0&2&2&1&0&2&2&2&2&1&0&2&1&2&0&2&1&2&0&0&0&1&0&0&0&1&1&0\\
1&0&2&2&1&2&2&2&1&0&2&1&0&1&2&0&1&1&2&0&2&0&0&2&0&2&0&2&0&0&0&2&1&1&0&0&1&0&1&1&2&1&0&0\\
2&2&1&2&1&1&2&1&1&2&0&1&0&0&0&2&2&0&0&0&2&0&2&2&1&0&1&2&2&1&2&0&1&1&2&0&2&0&2&2&1&2&2&2\\
1&1&0&0&0&0&1&2&1&0&2&1&1&1&0&2&2&2&0&0&2&1&1&1&0&2&0&2&0&2&1&0&2&2&1&2&2&1&1&2&1&0&0&1\\
1&0&0&2&0&1&1&1&0&1&1&1&1&2&2&0&1&0&2&1&1&2&0&2&2&2&0&2&0&2&1&2&1&1&1&1&1&2&1&0&1&0&2&0\\
1&1&0&1&2&1&0&2&1&1&1&2&0&2&1&2&0&0&1&0&1&0&1&0&1&0&0&0&2&1&2&0&0&2&0&1&1&2&1&0&0&2&2&2\\
0&1&0&2&0&2&1&2&1&2&1&2&1&0&2&0&1&0&0&1&1&1&0&1&2&2&0&1&0&2&2&0&2&1&0&0&0&2&2&1&2&2&2&2\\
1&2&0&1&1&0&1&2&2&2&0&1&2&2&2&0&0&1&1&0&2&1&2&2&1&0&2&0&1&2&2&2&0&0&0&1&1&0&1&2&1&1&1&0\\
0&0&1&1&2&2&2&0&0&1&1&0&2&0&1&2&1&1&1&2&0&1&2&2&0&2&1&2&2&1&0&1&0&0&0&1&0&1&2&0&1&0&1&0\\
1&2&1&0&0&2&0&2&0&0&0&0&1&2&2&2&1&0&1&0&0&1&1&2&2&1&1&1&2&1&2&0&0&1&0&1&1&2&0&2&2&2&0&0\\
2&2&0&1&1&2&2&1&1&0&1&1&2&2&1&0&0&2&1&0&1&2&1&2&0&2&0&1&2&2&0&2&2&0&1&1&0&0&2&0&0&2&2&1\\
0&2&1&0&1&1&1&2&1&0&1&2&0&2&0&1&2&1&2&1&1&2&0&1&1&2&2&1&1&0&1&0&2&0&0&1&0&0&2&1&1&1&1&1\\
1&0&0&2&2&0&0&1&2&1&0&1&1&0&2&1&2&2&1&2&1&0&1&1&1&2&2&1&0&2&1&1&2&2&0&1&0&2&0&2&1&1&1&1\\
0&1&1&2&2&1&1&0&0&2&1&2&1&0&0&0&0&1&0&1&2&2&2&2&2&0&2&1&1&1&2&2&0&1&2&1&2&2&0&0&0&0&2&2\\
0&0&1&2&1&1&2&2&1&1&0&0&0&0&1&2&0&0&0&1&0&0&0&2&2&2&1&0&1&1&1&2&2&0&0&1&1&1&0&2&2&1&1&0
\end{pmatrix}$$

$$\rho_M = \begin{pmatrix}
1&1&0&2&1&0&2&0&0&1&1&0&0&2&2&1&1&1&0&0&2&2&1&0&1&2&2&0&1&2&1&0&2&2&1&1&2&0&0&1&2&0&1&1 \\
0&0&1&0&0&1&2&2&2&0&0&2&0&1&1&2&1&0&0&1&0&2&0&2&2&1&2&0&2&1&1&1&0&1&0&2&0&0&0&0&2&1&2&1 \\
0&1&2&2&1&1&1&0&1&1&0&0&1&0&1&1&0&2&1&2&2&1&1&0&1&0&2&1&2&1&2&2&1&2&1&1&0&0&2&0&1&2&2&0 \\
0&1&1&0&1&2&0&2&2&2&1&1&1&2&0&0&0&1&0&2&1&2&1&1&2&0&1&1&2&0&2&1&0&1&2&1&0&1&1&2&2&0&0&0 \\
2&1&2&1&0&2&1&0&0&0&0&1&2&0&0&1&2&0&0&2&1&1&0&2&0&0&1&2&0&1&0&1&2&0&0&0&1&0&2&2&1&1&1&2 \\
0&1&2&0&0&0&0&1&0&1&1&2&0&1&2&0&2&0&1&1&2&1&2&2&0&0&2&1&0&1&2&1&0&2&2&1&2&2&0&1&2&2&0&0 \\
1&1&2&0&2&1&0&1&1&2&2&1&2&0&0&1&2&2&1&1&2&2&0&1&0&1&1&0&1&2&1&2&2&1&2&0&2&2&1&2&0&2&2&1 \\
2&0&1&2&0&0&2&1&2&2&0&1&2&0&1&2&1&1&0&1&1&0&1&2&1&0&2&1&2&1&2&0&0&2&0&0&1&0&2&1&1&0&0&2 \\
0&2&1&1&0&0&1&2&1&1&2&1&0&2&2&2&1&0&1&2&0&0&2&2&1&2&0&2&1&2&1&2&0&1&2&2&1&2&0&0&0&2&2&0 \\
2&2&2&2&2&1&1&0&0&2&0&2&0&0&0&0&2&2&0&1&1&1&0&0&1&0&0&2&2&2&1&0&2&2&1&1&2&2&0&0&1&1&2&2 \\
0&1&1&2&2&0&2&2&1&0&0&1&0&1&2&2&2&2&0&2&1&1&1&1&1&0&2&1&1&2&1&0&2&0&1&1&1&0&2&2&1&1&0&0 \\
2&0&2&0&1&1&2&0&1&0&2&0&0&0&0&2&1&1&0&0&0&0&1&0&1&0&0&2&1&1&0&1&1&1&2&1&1&0&2&2&0&0&0&1 \\
1&2&1&0&0&1&0&0&0&2&1&2&1&2&0&0&0&2&2&1&0&1&1&1&2&1&2&0&0&0&2&0&2&2&0&2&2&0&1&1&2&2&1&1 \\
2&0&2&2&0&2&2&1&0&0&1&1&0&2&2&0&0&2&2&0&0&1&2&0&1&1&0&2&1&1&1&2&2&1&0&2&1&2&1&0&1&0&2&0 \\
2&1&1&2&2&1&1&2&1&1&1&2&2&2&0&2&2&2&0&0&1&2&0&0&2&0&2&1&0&0&2&2&0&2&2&1&0&2&2&0&2&2&0&0 \\
1&1&1&1&2&1&2&1&0&1&1&1&0&2&1&2&2&2&0&0&2&0&0&2&0&1&0&1&0&1&1&1&0&1&1&0&1&0&1&1&0&2&1&0 \\
0&0&2&1&0&0&1&1&2&0&1&1&2&0&0&2&1&2&0&2&0&2&2&1&2&1&2&1&0&0&1&0&0&1&0&0&1&2&2&1&0&0&1&2 \\
1&0&1&0&0&0&2&2&0&0&1&0&0&0&2&1&1&1&1&1&2&2&0&1&1&0&0&1&2&2&2&1&1&2&2&2&1&0&2&1&2&2&0&2 \\
0&0&2&2&1&0&1&0&0&2&1&0&1&1&1&0&1&1&0&1&1&0&2&0&0&1&1&1&1&2&0&2&1&2&1&0&2&1&2&1&0&1&0&1 \\
0&0&0&1&1&0&2&2&2&2&2&1&2&2&0&0&2&2&2&0&1&1&1&0&2&2&0&2&2&1&0&1&1&2&0&0&1&1&2&1&2&1&1&2 \\
0&0&0&1&1&1&2&0&1&2&0&1&0&0&0&1&0&2&0&1&0&1&2&0&1&2&2&1&0&2&0&0&0&1&1&2&1&1&1&2&0&0&0&1 \\
1&2&2&1&1&1&1&2&1&1&2&0&2&0&1&0&1&1&2&2&1&0&2&1&1&0&0&2&2&0&0&1&1&2&0&1&1&0&2&0&1&1&0&2 \\
2&1&0&1&1&0&2&2&2&2&0&2&0&0&2&2&1&1&1&2&0&1&0&1&0&0&1&0&0&1&0&0&1&2&1&1&1&0&0&1&0&0&1&2 \\
2&2&0&2&2&1&1&1&0&2&0&0&2&1&0&2&1&0&0&1&1&2&0&0&0&2&2&2&0&1&2&1&2&2&0&1&0&1&2&0&1&1&1&0 \\
2&1&2&2&2&2&0&2&0&1&0&2&1&0&0&0&1&2&1&0&1&2&0&0&1&2&1&2&0&1&1&2&2&0&0&1&0&2&2&2&1&2&0&1 \\
0&0&1&2&1&2&1&1&0&2&1&0&2&1&2&1&0&2&1&2&0&1&0&1&1&2&0&2&1&0&0&1&1&0&0&2&1&0&1&1&0&0&1&0 \\
2&2&0&0&2&2&0&1&1&1&1&2&1&2&0&0&0&2&2&0&1&2&0&0&0&1&1&1&0&1&0&0&0&2&2&0&1&0&0&1&2&1&0&2 \\
0&2&1&0&1&0&2&1&0&0&2&0&1&1&2&2&1&2&1&0&1&0&2&2&2&2&2&0&2&2&1&0&0&1&2&2&1&2&2&0&0&1&0&1 \\
0&2&0&0&1&2&1&2&0&2&2&1&1&1&1&0&2&1&2&0&2&2&1&0&1&2&2&0&2&2&2&2&1&0&1&1&1&0&1&0&0&2&2&2 \\
0&1&0&2&2&1&1&0&2&2&2&1&1&0&2&1&1&1&2&1&0&2&0&1&1&0&1&0&1&2&2&0&1&1&2&2&2&1&0&1&2&1&0&1 \\
1&0&1&1&1&1&0&2&1&2&2&2&1&0&0&1&0&0&1&2&2&0&1&2&0&1&1&1&0&2&1&1&1&0&2&2&2&2&2&1&2&2&1&0 \\
1&2&1&0&2&1&1&1&1&2&1&0&0&2&1&0&1&1&1&0&1&2&2&1&1&1&0&0&0&1&0&0&0&2&0&2&1&0&2&0&2&0&0&1 \\
0&0&0&2&2&2&2&2&0&2&1&1&1&0&0&1&0&1&1&2&2&0&1&0&2&2&1&0&2&0&1&2&1&1&1&0&1&1&0&2&1&0&2&0 \\
0&2&0&1&1&0&0&0&1&0&1&0&2&2&0&1&2&2&0&0&2&1&2&1&0&1&0&0&2&1&0&0&0&2&0&1&1&1&1&1&2&1&1&1 \\
2&0&0&0&2&2&0&2&2&2&0&1&0&2&2&0&0&0&2&0&0&1&1&0&1&2&0&0&0&1&1&2&2&2&1&2&2&1&2&0&0&0&2&1 \\
0&0&2&2&0&2&1&2&2&0&0&0&1&1&1&1&2&0&0&2&2&1&0&1&0&1&1&1&2&1&2&2&0&0&0&2&1&0&0&1&2&1&0&2 \\
2&2&1&2&1&1&2&0&2&1&1&1&1&1&0&1&0&2&2&2&1&2&0&0&1&0&2&2&1&0&1&1&0&0&2&0&0&1&2&0&2&1&0&0 \\
2&1&1&0&1&1&1&1&1&1&1&0&0&2&2&1&1&1&0&0&2&1&2&1&0&2&0&2&0&1&2&0&2&1&1&0&1&0&2&2&1&0&1&0 \\
0&0&0&1&0&2&1&2&0&0&2&1&0&0&0&0&0&1&2&2&2&1&1&2&1&0&0&1&0&1&2&1&2&1&0&0&0&0&1&1&2&0&0&2 \\
0&2&1&1&0&1&1&1&2&2&2&2&1&0&0&2&1&1&1&0&0&2&0&1&2&0&2&2&1&2&0&2&0&1&0&1&1&0&1&0&0&1&2&0 \\
1&0&2&1&2&1&0&2&2&2&1&1&2&2&2&2&2&0&1&2&0&1&2&1&2&1&1&2&1&2&1&2&1&2&0&0&1&0&0&1&0&0&1&1 \\
0&2&1&1&1&0&2&1&0&2&1&0&1&1&1&0&0&0&0&0&2&1&0&0&1&2&1&1&2&2&0&0&0&1&1&2&2&0&1&1&2&2&2&1 \\
1&1&1&2&0&2&0&2&0&0&2&2&0&0&0&1&1&2&1&1&1&2&0&2&2&1&0&0&1&1&2&1&0&0&2&0&0&2&2&1&1&1&0&0 \\
0&1&1&0&0&1&1&2&0&2&2&1&2&2&1&0&2&2&2&2&2&1&1&1&0&0&1&0&2&2&2&0&2&2&2&0&2&1&1&0&1&1&0&0
\end{pmatrix}$$

$$
a_M = \begin{pmatrix}
2&0&0&2&2&1&2&0&0&0&0&0&0&1&1&0&0&0&0&1&1&0&2&0&1&2&1&2&2&0&0&0&1&0&2&2&1&0&0&1&1&0&1&0 \\
1&2&1&0&2&0&0&1&1&2&0&0&1&2&0&0&2&2&1&0&2&1&2&0&2&0&1&0&0&2&0&1&0&2&2&0&2&0&2&0&0&2&1&2 \\
0&0&2&0&0&1&0&1&0&1&0&2&1&1&2&1&1&2&0&2&0&0&2&0&0&1&1&2&2&2&2&1&1&1&1&1&0&0&0&2&1&1&1&0 \\
2&1&0&1&2&0&2&1&2&1&1&1&1&0&0&0&1&0&1&2&0&2&2&2&1&0&2&2&2&2&2&2&0&1&2&1&2&1&1&2&2&1&1&0 \\
0&2&2&2&0&0&1&0&0&0&1&2&1&1&1&0&1&0&2&1&1&0&2&2&2&1&1&0&0&1&0&0&2&0&2&0&2&0&2&2&2&2&0&1 \\
1&2&2&2&2&2&0&0&2&1&1&2&0&2&2&0&2&2&1&1&2&1&1&2&0&2&0&2&2&0&0&0&2&2&2&0&1&0&2&1&0&2&0&2 \\
1&1&2&0&1&2&0&0&1&0&1&0&2&1&1&0&1&1&1&1&1&2&2&0&2&2&0&0&0&1&2&1&1&2&1&1&2&0&1&1&2&0&2&1 \\
0&0&0&2&0&1&2&1&1&2&0&1&2&0&1&2&0&1&2&0&2&0&1&1&0&0&1&1&1&2&1&2&1&1&2&0&1&0&0&1&0&1&2&2 \\
0&0&2&0&1&1&1&2&1&0&2&0&2&0&2&0&2&1&2&2&2&1&0&0&0&2&1&1&0&1&0&0&1&1&2&1&0&1&0&0&2&2&2&1 \\
1&2&0&2&2&0&0&2&2&0&2&0&0&2&2&0&0&0&1&1&0&1&2&0&2&0&1&2&2&0&2&0&2&2&0&2&1&1&2&0&0&2&0&1 \\
2&2&0&2&1&0&0&2&0&2&1&0&1&2&1&1&0&2&1&2&0&2&0&1&2&0&0&0&1&2&1&2&1&0&0&2&2&1&1&2&1&2&0&1 \\
1&1&2&0&2&1&0&2&1&0&1&2&2&2&0&0&0&2&2&1&0&1&2&2&1&1&1&2&1&2&0&1&1&0&1&0&1&0&2&1&1&2&1&1 \\
1&1&2&0&2&0&2&2&0&1&2&1&2&1&2&1&0&0&2&1&0&1&2&0&2&2&2&2&1&2&1&2&0&1&1&0&0&0&0&2&1&2&2&0 \\
1&2&2&0&1&0&1&0&0&1&1&1&0&2&1&0&2&0&0&2&0&0&0&0&1&0&0&2&0&2&0&0&0&0&0&0&1&1&2&2&1&1&2&0 \\
2&1&1&1&0&0&0&2&0&0&1&1&1&2&1&2&0&0&1&1&0&2&2&0&0&2&0&2&1&2&0&2&0&2&0&1&0&2&2&0&1&2&1&2 \\
2&1&0&2&1&0&2&2&1&2&1&0&2&0&1&2&1&2&1&2&1&0&2&2&0&0&1&1&0&1&0&2&2&0&0&1&0&2&0&2&2&0&2&1 \\
1&0&1&2&2&2&0&2&0&2&0&2&0&0&1&2&1&0&0&0&0&2&1&2&2&0&2&2&2&2&0&2&1&0&0&2&0&0&1&2&0&2&2&1 \\
0&0&2&0&1&0&1&0&0&1&0&1&2&0&0&0&2&1&2&1&0&0&2&2&0&1&1&1&0&1&2&0&0&1&2&1&2&1&0&2&0&0&0&2 \\
2&1&0&0&0&2&2&2&0&0&0&1&2&1&0&0&0&0&0&1&2&2&0&1&1&2&0&0&2&2&0&1&0&1&2&0&2&1&1&2&2&1&0&1 \\
1&0&1&2&2&1&1&0&0&2&2&2&1&1&2&0&2&1&2&0&1&1&2&1&1&2&2&1&1&1&2&2&1&1&1&0&2&2&1&0&1&0&2&0 \\
0&1&2&2&0&1&2&0&1&2&2&0&0&0&1&0&2&1&0&1&1&2&0&1&1&2&0&0&2&1&1&0&0&2&0&0&2&2&2&1&1&0&0&1 \\
2&0&1&1&0&1&1&2&0&1&1&1&0&2&1&0&1&1&2&0&1&2&0&1&2&2&0&1&0&0&2&1&2&1&2&0&0&0&0&1&2&1&2&1 \\
1&2&2&2&2&2&0&2&2&1&1&1&2&0&1&1&2&0&1&2&1&0&2&2&1&0&1&1&2&0&0&1&0&2&1&2&1&1&0&2&2&2&2&2 \\
1&1&0&2&0&1&2&2&0&1&1&0&0&2&1&2&2&2&1&2&0&1&2&0&0&1&1&1&0&1&2&0&0&1&1&1&2&0&1&1&0&2&1&1 \\
2&2&1&1&1&2&0&1&2&0&1&2&0&1&0&0&2&0&1&1&0&1&1&0&1&1&2&0&1&0&1&0&0&2&0&0&0&0&0&1&1&2&2&0 \\
1&0&1&0&1&2&1&0&2&0&1&2&2&0&2&2&0&0&0&1&1&0&1&2&0&2&0&2&0&0&2&2&1&1&0&0&1&0&2&1&0&1&2&0 \\
0&1&2&2&1&0&0&0&0&2&1&2&1&2&0&0&1&2&2&2&1&2&1&0&1&2&0&2&1&2&0&2&2&2&1&1&0&2&1&1&0&2&1&0 \\
1&2&1&0&1&1&2&1&1&1&1&2&1&1&0&0&1&0&0&1&0&1&0&1&0&1&1&0&1&1&0&1&2&0&1&0&1&2&0&1&2&2&0&0 \\
0&2&0&0&2&1&1&2&0&2&2&0&2&1&1&0&1&1&0&0&0&2&1&2&0&1&2&1&0&2&1&1&0&0&1&0&1&0&2&1&2&1&1&0 \\
2&2&2&0&1&1&1&2&0&1&2&0&2&0&1&0&0&1&2&2&0&2&1&2&1&0&1&1&2&0&2&0&0&1&2&1&0&0&2&2&2&2&2&0 \\
0&2&1&1&0&2&1&0&2&1&2&2&1&2&1&0&2&2&1&0&0&0&0&1&1&2&2&2&0&0&2&2&2&0&0&2&2&2&1&0&1&1&0&0 \\
0&1&2&0&2&2&1&2&2&1&2&2&0&1&2&2&2&1&1&2&1&2&1&1&0&1&0&2&1&1&1&2&0&1&0&0&1&0&1&2&1&1&2&0 \\
1&0&1&2&2&0&1&2&2&0&1&1&1&2&1&0&2&0&0&0&1&1&2&0&2&0&1&0&2&1&1&1&2&0&0&0&0&2&1&2&1&0&0&0 \\
0&1&2&0&1&1&0&2&0&1&2&1&1&0&0&0&2&2&1&2&0&0&2&2&2&0&0&0&2&1&1&2&2&2&2&0&2&0&0&2&1&0&2&2 \\
0&2&1&0&2&1&0&0&1&0&1&2&0&2&0&1&1&2&0&0&2&0&2&2&2&2&2&2&1&0&1&0&1&1&0&0&1&1&0&2&0&2&1&0 \\
2&1&0&0&1&1&2&1&1&2&0&1&1&2&0&1&0&1&2&1&0&1&2&0&2&0&0&2&1&0&1&1&1&0&1&0&2&1&0&1&1&1&0&1 \\
2&2&2&2&1&0&2&0&1&0&2&2&0&0&2&2&1&1&0&2&1&0&2&2&2&2&1&1&1&1&2&1&2&2&2&2&1&0&0&0&1&1&2&2 \\
0&1&0&0&1&0&2&1&1&0&1&1&1&0&0&1&2&0&0&1&0&0&0&0&2&0&1&2&2&1&0&2&2&1&2&1&2&1&1&0&0&1&0&2 \\
0&2&1&1&2&1&0&0&1&1&2&0&0&1&1&1&1&1&1&1&0&2&2&2&1&0&2&1&0&1&1&2&2&0&1&0&1&2&0&0&2&0&2&1 \\
1&0&2&2&0&2&1&1&0&0&2&0&2&2&0&1&0&2&0&2&2&1&0&0&1&1&0&0&0&1&2&0&1&0&0&0&2&2&1&0&0&0&0&2 \\
2&1&2&0&0&1&2&2&1&1&0&0&1&1&1&1&0&0&0&1&1&2&2&0&2&0&0&1&1&0&0&1&1&2&2&1&0&1&1&2&2&2&0&0 \\
2&0&2&0&2&0&0&2&1&0&2&1&0&1&1&1&1&2&2&2&0&0&2&1&2&2&2&1&2&1&2&2&1&1&2&1&0&2&0&1&0&2&1&2 \\
1&1&1&2&0&2&1&2&0&2&2&1&0&2&2&1&1&0&0&0&2&1&1&0&2&0&0&2&2&2&0&0&2&1&2&1&0&1&2&1&1&0&2&0 \\
0&0&2&0&2&0&0&1&0&2&1&2&2&0&0&2&1&1&0&0&1&2&2&1&2&1&2&0&0&1&0&0&1&0&1&0&1&2&2&1&0&0&0&0
\end{pmatrix}
$$

$$t_{44} = \left(\begin{array}{*{44}{c}}
1&0&2&0&1&2&1&1&2&2&0&2&1&1&2&2&2&1&1&0&2&2&1&0&1&1&0&2&1&1&2&2&0&2&0&2&1&0&2&1&2&0&2&0 \\
0&0&1&2&2&2&2&0&2&1&2&1&0&1&1&0&0&1&1&1&0&1&1&2&1&0&2&2&0&2&2&1&0&2&2&0&0&2&2&1&1&2&1&1 \\
1&0&0&1&2&2&0&2&1&1&0&1&0&0&2&0&2&1&1&1&1&2&1&0&2&0&2&0&2&0&0&1&0&2&1&0&2&0&2&0&2&0&2&0 \\
1&1&0&1&0&0&0&2&2&1&1&0&2&0&1&0&1&0&0&1&0&0&1&0&1&0&0&2&1&2&2&1&1&2&1&2&1&0&2&0&2&1&0&2 \\
2&1&2&2&0&1&0&2&0&2&1&2&2&0&0&1&0&0&0&1&2&1&0&1&1&0&0&1&2&2&2&2&0&1&0&2&0&0&2&2&2&2&1&1 \\
1&1&2&0&2&2&2&0&1&1&2&1&1&1&0&1&2&0&1&0&2&1&1&0&0&1&1&2&0&2&2&0&2&1&1&0&2&2&0&1&0&0&2&1 \\
1&1&2&0&1&2&0&1&0&2&2&0&2&2&2&0&0&0&2&0&2&2&2&0&2&2&0&0&1&0&0&1&0&1&2&0&0&2&1&1&0&2&0&1 \\
1&2&0&2&1&2&2&0&1&0&2&2&2&0&2&0&0&0&0&0&2&2&0&2&2&0&1&1&2&0&0&2&2&2&2&0&1&1&1&1&2&1&0&0 \\
2&1&0&0&0&2&2&1&2&1&1&2&2&2&0&2&2&2&0&2&2&2&1&2&2&2&2&1&2&0&1&2&1&2&2&1&2&0&2&1&1&0&2&0 \\
0&0&1&1&1&0&2&2&1&2&0&2&0&1&1&2&0&1&1&1&2&2&2&1&2&1&0&0&2&0&1&2&1&1&2&0&0&2&2&0&2&0&1&0 \\
1&2&0&2&2&0&0&1&0&1&0&1&0&1&2&1&2&0&0&2&0&2&1&0&1&1&1&1&0&2&1&2&0&2&0&0&1&2&2&1&2&2&2&0 \\
0&0&0&2&1&0&0&0&0&0&0&1&1&1&2&2&2&0&0&1&2&2&2&0&1&1&1&1&2&0&1&0&1&1&2&0&2&2&0&1&0&1&2&1 \\
0&2&0&0&1&2&2&0&0&1&0&2&1&2&0&1&0&1&0&1&2&1&1&1&2&0&2&0&2&0&2&2&1&1&0&0&2&1&2&0&2&0&2&2 \\
1&2&2&1&0&2&0&2&1&0&1&2&2&2&2&2&0&1&1&2&1&2&0&1&0&2&2&0&0&0&0&2&1&0&2&0&0&2&2&1&0&0&2&0 \\
0&0&0&2&2&0&1&1&0&1&1&2&2&0&2&1&0&2&2&0&2&0&0&1&0&1&0&1&1&1&1&2&0&1&0&1&2&2&1&2&1&0&1&0 \\
0&0&2&1&2&2&0&0&1&1&1&1&2&2&0&0&2&1&0&2&0&1&0&2&2&0&0&0&2&0&0&1&0&0&1&2&1&1&2&1&1&0&0&0 \\
2&2&0&2&0&0&1&0&1&2&1&2&0&1&1&2&1&1&0&1&0&0&1&1&2&2&0&2&1&1&1&1&1&2&1&0&1&1&1&1&2&2&1&1 \\
2&1&1&0&1&1&1&1&1&0&2&2&1&0&1&0&0&1&2&0&1&1&0&2&1&0&0&2&0&2&0&0&1&0&1&1&1&0&0&1&2&0&2&1 \\
0&1&0&0&0&1&1&2&1&1&2&1&2&0&2&1&2&1&0&2&2&2&2&1&1&2&0&1&0&1&2&0&0&2&0&0&0&0&1&1&2&2&1&2 \\
1&0&1&0&0&2&1&2&1&1&0&1&2&0&2&2&2&2&2&2&1&1&0&2&0&2&2&2&1&2&1&0&0&0&1&0&0&1&1&1&1&0&1&0 \\
2&0&1&0&1&2&1&1&0&0&0&2&1&0&0&1&2&0&1&0&1&0&2&1&0&0&1&1&0&0&1&0&0&2&0&2&1&0&1&1&1&2&1&1 \\
0&1&1&1&1&1&0&2&2&1&2&1&2&2&0&1&0&2&0&2&0&2&1&0&0&1&2&1&1&0&2&2&2&0&1&0&0&2&1&0&1&0&0&0 \\
0&0&0&2&2&1&2&2&0&1&0&0&0&0&1&1&0&0&1&1&0&0&1&0&1&1&0&0&1&0&0&2&0&0&2&2&1&1&2&1&1&0&1&1 \\
0&0&0&1&1&2&1&0&2&1&1&2&1&2&0&1&2&0&2&0&1&2&1&0&0&1&1&1&1&0&1&1&1&1&0&2&2&2&0&2&2&1&0&2 \\
2&1&2&0&2&2&2&2&0&2&0&0&0&2&0&1&0&0&0&2&0&0&2&1&2&2&2&1&2&0&1&1&0&1&1&1&0&2&1&1&0&0&2&1 \\
1&2&1&1&0&1&0&1&2&1&1&1&0&2&1&1&1&1&2&0&0&2&1&0&1&1&1&2&1&0&1&1&1&0&0&1&2&2&1&2&2&0&1&2 \\
0&2&0&2&1&2&2&1&0&1&0&2&1&0&1&2&2&0&1&2&1&1&2&2&1&1&0&2&1&1&0&1&2&0&0&1&1&1&1&2&2&2&2&2 \\
0&2&2&1&0&2&1&0&0&1&2&1&0&0&0&1&1&0&0&2&0&0&2&0&1&2&0&0&0&0&2&1&1&1&0&1&2&2&2&0&1&1&0&0 \\
0&2&0&2&0&1&0&0&0&2&1&2&0&1&2&0&2&2&0&0&2&0&1&1&1&2&2&1&1&1&1&0&1&1&2&1&1&0&2&1&1&0&1&0 \\
2&1&2&1&0&0&0&2&0&2&2&2&0&2&0&1&1&2&2&0&0&0&2&1&0&2&1&2&0&2&1&1&1&2&2&2&0&2&0&2&1&1&0&0 \\
1&2&1&1&0&1&0&0&1&2&1&0&2&0&1&2&2&1&2&1&0&1&0&2&2&1&0&1&1&0&1&0&0&0&1&1&0&0&0&0&0&1&1&2 \\
1&2&0&1&1&2&0&0&1&1&2&2&2&1&2&0&0&1&2&2&1&1&1&1&1&2&2&2&0&0&0&2&1&0&2&0&1&1&2&2&0&0&0&2 \\
0&0&2&1&1&0&1&1&1&1&1&2&0&0&0&2&2&2&2&2&2&0&0&0&2&1&0&2&0&1&2&2&0&2&2&0&1&1&0&1&0&1&2&1 \\
1&2&1&2&0&1&2&0&2&1&1&1&2&0&2&1&0&0&1&2&1&1&0&0&0&1&1&1&2&0&0&1&2&2&1&2&0&1&2&0&1&1&0&0 \\
2&0&2&2&0&0&1&2&0&1&2&0&1&0&0&2&2&0&0&0&1&1&0&1&1&1&2&0&1&2&2&0&2&2&0&1&0&2&1&0&2&1&2&0 \\
1&2&0&1&1&0&0&0&2&2&0&2&0&0&2&0&0&0&0&1&2&1&2&0&0&1&2&1&0&0&2&0&0&1&1&1&2&0&0&1&0&1&0&1 \\
2&1&0&1&2&1&1&0&0&0&1&1&1&2&2&2&0&2&1&1&1&1&2&1&0&0&1&0&0&1&0&0&1&0&2&1&1&2&0&2&1&0&0&0 \\
0&1&1&2&0&2&2&2&0&2&1&1&1&2&2&1&1&0&0&2&0&2&1&1&1&1&2&2&2&1&2&1&0&0&0&0&2&1&2&0&1&2&2&2 \\
1&1&0&1&1&0&1&1&2&2&1&0&2&2&1&0&2&1&2&2&2&2&0&1&1&0&0&2&2&0&1&0&1&2&2&0&1&1&2&1&1&0&1&0 \\
0&0&2&1&0&0&2&1&1&1&1&1&0&0&2&0&0&0&0&1&1&2&0&0&0&1&1&1&2&2&1&2&2&0&1&0&2&1&1&2&0&1&0&0 \\
0&1&2&2&0&1&1&0&2&1&1&1&0&2&1&1&1&1&0&2&2&2&0&1&2&2&2&1&0&0&0&0&0&1&0&0&1&1&0&1&0&1&2&2 \\
1&1&0&1&0&2&1&1&2&2&0&2&2&2&0&2&2&2&2&0&2&1&2&2&2&0&2&2&0&0&1&0&2&2&2&0&0&1&0&0&0&1&2&0 \\
1&0&1&1&2&0&2&0&2&0&0&0&2&2&1&2&2&2&1&1&1&2&0&1&0&0&0&0&0&0&1&0&2&1&1&1&2&0&2&1&0&0&1&1 \\
1&0&1&2&2&2&1&2&0&0&1&1&0&0&2&2&0&1&1&2&1&2&1&2&2&2&2&1&0&0&0&0&1&0&0&1&2&1&1&2&0&0&2&0
\end{array}\right)$$

Appendix D

The generators of $C_{GL_{44}(3)}(< X_{44}, Y_{44}, \rho_{44} >)$

We present the generators of $C_{GL_{44}(3)}(H)$ with $H :=< X_{44}, Y_{44}, \rho_{44} >$. These matrices have been achieved as follows. We use MAGMA [2] to determine the indecomposable summands of $V_{44}|_{<X_{44},Y_{44},\rho_{44}>}$. We hold that $V_{44}|_{<X_{44},Y_{44},\rho_{44}>} = V_{1,A} \oplus V_{1,B} \oplus V_{12} \oplus V_{15,A} \oplus V_{15,B}$ where $dimV_{i,K} = i$. Moreover $V_{1,A} \not\cong V_{1,B}$ and $V_{15,A} \not\cong V_{15,B}$ and $V_{15,B}$ is a tensor product of $V_{15,A}$ with a 1-dimensional module [21]. Furthermore we have that V_{12} is irreducible but not absolutely irreducible. It is a 6-dimensional module over $GF(9)$ (see e.g. [22]). Therefore the centralizer of $H|_{V_{12}}$ has order at most eight. According to [21], we have that $| C_{GL_{15}(3)}(H|_{V_{15,A}}) |=| C_{GL_{15}(3)}(H|_{V_{15,B}}) |= 2 \cdot 3$. This proves that $| C_{GL_{44}(3)}(H) | \leq 2^7 \cdot 3^2$.

We now use the MAGMA program `IsIsomorphic` for modules to hold matrices $t_i \neq t_{44}$ $(i = 1, 2, 3, 4, 5)$ to conjugate the generators obtained by the meataxe. Then $s_i = t_i t_{44}^{-1}$ centralizes X_{44}, Y_{44} and ρ_{44}. We display these matrices as s_i on the following pages. Using MAGMA, we hold $|< s_1, s_2, s_3, s_4, s_5 >|= 2^7 \cdot 3^2$ thus $< s_1, s_2, s_3, s_4, s_5 >= C_{GL_{44}(3)}(H)$.

$$s_1 = \begin{pmatrix}
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&1&0&0&0&0&1&0&1&2&1&2&0&0&2&1&1&1&2&2&0&0&1&2&2&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&2&2&1&0&1&0&1&0&2&0&1&1&2&0&2&0&1&2&2&2&1&1&0&0&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&0&0&2&1&0&1&0&0&2&1&1&2&0&2&2&2&1&0&1&1&1&2&0&2&2&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&2&1&2&0&1&0&0&2&2&0&1&1&2&0&1&2&1&1&2&1&2&0&1&0&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&1&1&2&0&0&0&2&1&1&2&0&1&1&2&0&2&2&0&1&0&2&1&2&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&1&2&2&0&2&1&1&1&0&1&1&0&0&2&0&0&0&1&0&1&1&0&1&0&0&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&2&0&0&2&1&2&0&2&1&1&2&2&0&0&0&0&1&2&2&1&1&0&2&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&2&1&1&0&0&0&2&0&0&0&1&0&2&0&2&2&0&0&0&1&1&1&2&0&0&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&2&1&0&1&0&0&2&0&1&2&2&2&2&2&1&2&0&0&0&0&2&1&2&0&2&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&2&2&2&0&0&1&2&1&1&0&0&0&2&2&2&1&2&0&1&0&2&1&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&2&0&0&0&0&2&0&2&1&1&1&0&0&1&2&2&2&1&1&0&0&2&1&1&0&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&0&2&0&2&0&2&0&1&0&2&1&1&0&1&0&2&1&1&1&2&2&0&0&2&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&2&0&0&0&0&0&2&1&1&2&0&1&1&2&2&0&1&1&0&0&2&0&2&2&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&2&2&2&0&2&0&2&1&1&0&0&0&2&1&0&0&1&0&2&2&0&2&1&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&1&2&2&0&1&0&2&1&2&1&1&2&0&1&0&2&1&2&0&1&1&0&1&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&2&2&1&1&0&0&2&2&2&0&2&2&0&0&1&2&0&0&2&0&2&2&0&2&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&1&0&1&0&1&2&1&1&1&2&2&1&0&0&2&0&1&2&0&2&0&1&2&2&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&0&0&1&2&0&0&0&2&1&2&2&0&1&0&1&1&2&0&0&2&0&1&2&0&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&0&0&1&2&1&0&0&1&0&0&1&1&2&2&0&1&1&1&2&2&1&2&1&1&2&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&1&0&2&2&1&0&0&2&0&2&1&1&0&2&2&1&2&1&0&2&2&1&2&1&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&2&0&0&0&1&0&1&1&2&0&2&1&0&1&0&2&2&2&1&2&0&1&1&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&2&2&1&1&0&0&0&0&0&2&0&0&1&1&1&1&1&2&0&2&1&1&2&2&1&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&1&2&1&0&0&0&0&2&1&1&1&0&2&2&2&1&1&2&1&1&1&0&0&1&0&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&2&0&0&0&1&0&1&1&2&0&2&1&0&1&0&2&2&2&2&2&0&0&1&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&2&0&2&0&0&1&2&2&0&2&1&0&0&2&1&0&2&1&2&2&0&0&1&0&0&1&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&1&0&0&1&0&1&2&1&2&0&0&2&1&1&0&2&0&1&1&2&0&0&2&1&2&2&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&2&1&0&0&0&0&0&2&0&0&1&1&1&1&2&0&1&1&1&1&0&0&0&1&2&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1\\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2
\end{pmatrix}$$

$$
s_2 = \begin{pmatrix}
2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&2&2&0&2&2&0&2&2&2&2&1&1&0&2&0&2&2&1&1&2&0&0&0&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&2&2&2&0&0&2&0&0&1&2&0&2&1&0&2&2&1&2&1&1&2&1&1&1&1&2&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&2&2&1&0&2&0&0&2&2&2&0&0&1&0&1&1&0&2&1&0&1&1&1&2&0&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&1&2&2&2&0&2&2&0&0&2&1&0&1&2&0&2&1&2&1&2&0&1&2&2&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&2&0&2&2&0&0&1&1&0&1&0&2&2&2&0&2&1&2&0&2&1&0&2&1&1&2&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&1&0&0&1&0&0&2&2&2&1&2&1&1&0&0&1&1&0&0&0&1&2&2&0&1&2&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&1&0&1&0&0&2&1&1&2&0&1&0&1&0&1&2&0&1&0&1&1&0&0&0&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&1&2&0&2&0&1&0&1&0&1&1&1&2&2&2&0&1&0&1&2&1&2&0&0&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&1&2&2&0&0&2&1&1&0&1&2&1&1&1&2&0&0&1&0&1&0&1&2&2&2&1&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&2&2&1&1&0&1&2&2&0&2&1&1&1&0&0&0&2&1&0&0&2&1&0&1&1&2&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&1&0&1&1&0&1&1&0&1&1&2&1&2&2&0&1&0&1&1&2&2&1&0&0&0&2&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&1&2&1&0&0&1&0&0&2&1&0&2&2&0&1&1&2&1&2&2&1&2&2&2&2&1&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&2&2&2&0&2&1&2&2&1&0&1&2&2&0&1&1&2&0&0&1&0&1&0&0&0&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&2&2&1&1&0&1&0&0&0&0&0&1&0&2&0&1&1&2&2&2&1&2&0&0&0&2&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&1&1&1&0&0&2&2&1&1&2&0&0&1&0&0&1&2&1&0&0&1&2&2&0&0&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&2&0&0&2&0&1&1&1&1&2&1&2&2&0&0&0&2&0&0&0&2&1&1&0&2&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&1&0&1&1&1&2&0&1&2&1&1&2&1&0&1&0&1&2&0&0&1&2&2&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&1&2&2&0&2&2&1&0&2&1&0&0&1&1&0&1&2&0&1&1&2&0&1&2&0&2&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&1&2&0&2&0&0&2&0&1&0&2&0&2&1&2&2&0&1&1&1&1&0&1&1&0&1&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&1&1&0&1&0&0&2&0&1&1&0&0&2&2&0&0&0&0&2&2&1&0&0&2&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&2&2&2&0&2&2&0&0&0&0&1&0&0&1&1&2&2&2&1&1&0&2&0&1&1&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&2&0&2&0&1&2&2&1&0&1&1&1&2&0&0&2&0&0&2&2&1&2&0&2&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&1&1&1&2&2&1&2&0&0&2&1&2&0&1&0&0&0&2&1&2&2&0&2&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&2&2&2&0&2&2&0&0&0&0&1&0&0&1&1&2&2&2&1&1&2&2&0&2&1&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&1&1&2&2&1&0&0&2&1&1&1&0&1&1&1&0&1&0&0&1&0&0&2&0&2&2&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&1&2&2&0&0&1&2&1&2&0&2&1&2&1&1&1&2&1&1&1&0&2&1&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&1&2&2&0&1&2&1&0&0&1&0&2&1&1&1&1&0&1&2&1&0&2&0&1&0&2&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&2&1&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&2&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&2&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&1&0&1&1&0&1&2&2&1&2&2&0&1&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&1&0&0&0&0&1&1&2&1&2&1&2&2&2&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&2&0&1&0&0&1&0&1&2&0&0&2&1&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&2&0&1&0&0&0&1&1&2&0&0&2&1&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&2&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&2&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&1&1&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&2&1&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&2&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1
\end{pmatrix}
$$

$$s_3 = \begin{pmatrix}
1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&2&2&2&2&0&2&1&0&1&0&1&0&1&1&1&1&2&1&0&2&1&2&2&1&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&1&0&2&2&2&0&2&2&2&1&2&0&0&2&0&0&2&2&1&0&1&1&0&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&2&2&2&2&2&2&2&0&2&0&1&2&1&1&1&2&2&2&2&0&2&0&2&1&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&1&0&1&0&0&1&2&0&1&0&1&2&0&0&0&1&2&1&0&0&2&2&2&1&0&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&2&2&1&0&0&0&1&2&2&0&1&2&1&1&1&2&2&0&0&1&1&1&1&2&0&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&2&1&2&0&1&1&1&1&1&1&0&1&0&1&1&1&0&2&0&0&1&2&2&1&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&2&1&1&0&2&2&1&2&2&0&1&1&2&0&1&0&1&1&1&2&2&0&1&1&0&2&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&2&1&0&1&0&2&0&2&0&1&0&0&1&2&2&0&0&0&1&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&1&0&1&0&2&2&1&2&0&0&0&2&2&2&0&2&1&1&0&1&0&2&1&0&2&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&2&2&2&2&1&1&2&1&1&1&0&1&1&0&1&1&0&0&1&0&1&1&1&0&1&1&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&1&1&1&0&1&2&0&2&0&1&0&2&2&2&2&1&2&0&1&2&1&1&2&2&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&2&2&0&1&1&1&0&1&1&1&1&1&0&0&1&0&0&1&1&2&0&2&2&0&2&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&2&0&2&2&0&2&1&0&1&0&1&1&1&1&1&2&1&1&2&0&1&1&1&1&1&2&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&2&2&2&2&1&1&1&0&1&2&2&1&0&2&1&0&1&2&1&2&2&0&0&1&2&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&1&0&2&1&0&1&2&2&0&0&2&2&2&0&2&1&0&0&1&0&0&1&2&2&1&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&1&2&1&0&1&2&2&2&2&2&0&2&0&2&1&2&0&1&0&0&2&1&1&2&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&1&2&1&0&1&1&1&0&1&0&2&0&0&0&1&1&1&1&1&0&2&2&1&0&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&2&2&2&0&2&1&0&0&0&1&1&1&0&0&0&1&1&1&1&0&0&0&0&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&2&0&1&1&2&2&0&0&0&2&2&1&2&0&2&2&0&0&2&2&2&2&0&2&2&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&2&1&2&2&0&2&1&1&2&1&2&1&1&2&1&1&2&2&1&2&0&0&1&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&2&2&0&0&2&2&2&0&2&2&2&0&1&0&1&1&2&0&2&2&2&0&0&0&0&1&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&1&0&2&1&0&1&2&2&1&1&1&1&0&1&2&2&1&1&0&0&1&0&0&1&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&2&1&2&1&2&0&1&1&0&1&0&2&1&1&2&1&2&1&1&2&2&2&2&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&2&2&0&0&2&2&2&0&2&2&2&0&1&0&1&1&2&0&2&2&0&0&0&2&0&1&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&2&1&0&2&1&2&1&0&1&2&0&0&1&2&0&0&2&2&1&2&0&0&1&0&2&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&2&1&2&1&0&2&2&1&2&2&0&0&0&1&1&2&1&1&0&2&1&0&0&0&2&2&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&1&0&1&0&1&2&1&0&1&1&0&1&0&0&0&2&0&0&1&0&2&2&0&1&2&0&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&2&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&0&2&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&2&0&2&1&0&2&1&1&2&1&1&0&2&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&2&0&0&0&0&1&2&1&2&1&2&1&1&1&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&0&1&0&2&0&0&1&0&2&1&0&0&1&2&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&0&1&0&2&0&0&0&1&2&1&0&0&1&2&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&1&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&2&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&0&2&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&2&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1
\end{pmatrix}$$

$$s_4 = \begin{pmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 2 & 2 & 1 & 2 & 0 & 2 & 1 & 0 & 0 & 0 & 0 & 2 & 2 & 1 & 2 & 0 & 0 & 1 & 0 & 2 & 2 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 2 & 2 & 1 & 2 & 0 & 2 & 1 & 1 & 2 & 0 & 0 & 2 & 2 & 1 & 2 & 0 & 0 & 1 & 0 & 2 & 2 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 1 & 0 & 2 & 2 & 2 & 0 & 2 & 1 & 1 & 0 & 0 & 0 & 2 & 0 & 1 & 2 & 0 & 0 & 2 & 1 & 0 & 0 & 1 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 2 & 2 & 0 & 1 & 1 & 2 & 1 & 0 & 1 & 2 & 2 & 0 & 0 & 0 & 1 & 1 & 2 & 1 & 0 & 0 & 2 & 2 & 0 & 2 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 1 & 1 & 2 & 0 & 1 & 2 & 1 & 1 & 0 & 2 & 2 & 0 & 2 & 1 & 1 & 1 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 1 & 2 & 0 & 2 & 0 & 2 & 1 & 1 & 0 & 0 & 0 & 2 & 0 & 1 & 2 & 0 & 0 & 0 & 2 & 1 & 1 & 2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 2 & 2 & 1 & 1 & 2 & 2 & 0 & 2 & 2 & 0 & 1 & 1 & 2 & 0 & 0 & 2 & 0 & 2 & 2 & 0 & 0 & 2 & 2 & 1 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 0 & 2 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 1 & 2 & 1 & 2 & 2 & 1 & 2 & 2 & 2 & 0 & 2 & 0 & 1 & 1 & 2 & 0 & 2 & 2 & 1 & 2 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 2 & 1 & 0 & 0 & 1 & 2 & 0 & 0 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2 & 0 & 1 & 1 & 2 & 1 & 0 & 1 & 2 & 1 & 0 & 0 & 0 & 1 & 1 & 2 & 1 & 0 & 0 & 2 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 2 & 0 & 0 & 1 & 1 & 2 & 1 & 0 & 1 & 2 & 2 & 2 & 0 & 0 & 1 & 1 & 2 & 1 & 0 & 0 & 2 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 1 & 2 & 1 & 2 & 1 & 1 & 0 & 1 & 0 & 0 & 2 & 0 & 1 & 1 & 2 & 0 & 2 & 2 & 1 & 2 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 2 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 2 & 2 & 1 & 0 & 1 & 1 & 0 & 2 & 1 & 0 & 0 & 1 & 2 & 0 & 0 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 2 & 0 & 1 & 1 & 2 & 2 & 2 & 2 & 2 & 0 & 1 & 2 & 2 & 0 & 1 & 2 & 0 & 2 & 2 & 0 & 0 & 2 & 2 & 1 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 2 & 2 & 0 & 2 & 1 & 1 & 1 & 0 & 1 & 2 & 2 & 0 & 0 & 0 & 1 & 1 & 2 & 1 & 0 & 0 & 0 & 1 & 2 & 2 & 1 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2 & 0 & 0 & 1 & 2 & 1 & 0 & 1 & 2 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 1 & 2 & 0 & 0 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 2 & 1 & 1 & 0 & 2 & 2 & 0 & 2 & 0 & 1 & 2 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 2 & 2 & 0 & 0 & 1 & 2 & 1 & 0 & 1 & 2 & 2 & 0 & 0 & 0 & 1 & 1 & 2 & 1 & 1 & 0 & 1 & 2 & 0 & 0 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 2 & 2 & 1 & 0 & 2 & 2 & 2 & 2 & 2 & 0 & 1 & 1 & 2 & 0 & 0 & 2 & 0 & 2 & 2 & 1 & 2 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 1 & 2 & 1 & 2 & 0 & 2 & 1 & 1 & 0 & 0 & 0 & 2 & 2 & 1 & 2 & 0 & 0 & 0 & 2 & 1 & 2 & 2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 2 & 2 & 0 & 2 & 1 & 1 & 1 & 0 & 1 & 2 & 2 & 0 & 0 & 0 & 1 & 0 & 2 & 1 & 0 & 0 & 1 & 2 & 2 & 1 & 1 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 2 & 2 & 1 & 0 & 2 & 2 & 2 & 2 & 2 & 0 & 1 & 1 & 2 & 0 & 0 & 2 & 0 & 2 & 2 & 0 & 0 & 1 & 2 & 2 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 1 & 2 & 1 & 2 & 0 & 2 & 1 & 1 & 0 & 0 & 0 & 2 & 2 & 1 & 2 & 0 & 0 & 2 & 2 & 1 & 0 & 2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 2 & 0 & 0 & 1 & 2 & 1 & 0 & 1 & 2 & 0 & 0 & 0 & 0 & 1 & 1 & 2 & 1 & 0 & 0 & 2 & 2 & 0 & 2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 2 & 0 & 0 & 0 & 1 & 2 & 1 & 0 & 1 & 2 & 2 & 1 & 0 & 0 & 1 & 1 & 2 & 1 & 0 & 0 & 2 & 2 & 0 & 2 & 2 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 2 & 1 & 1 & 0 & 2 & 2 & 0 & 2 & 0 & 1 & 1 & 2 & 0 & 2 & 0 & 2 & 1 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 1 & 0 & 1 & 2 & 0 & 1 & 2 & 2 & 1 & 2 & 2 & 0 & 1 & 2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 1 & 0 & 0 & 0 & 2 & 1 & 2 & 1 & 2 & 1 & 2 & 2 & 2 & 2 & 2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 2 & 0 & 1 & 0 & 0 & 2 & 0 & 1 & 2 & 0 & 0 & 2 & 1 & 2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 2 & 0 & 1 & 0 & 0 & 0 & 2 & 1 & 2 & 0 & 0 & 2 & 1 & 2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 2 & 0 & 0 & 0 & 2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 2 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{pmatrix}$$

$$
s_5 = \begin{pmatrix}
2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&2&2&1&0&2&2&1&0&1&1&2&0&2&2&1&1&2&1&0&1&0&1&0&1&2&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&1&1&2&2&0&2&0&0&0&2&2&1&1&0&2&0&1&2&1&1&0&0&1&1&0&1&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&2&2&2&1&0&0&0&0&0&1&0&1&2&2&1&1&2&0&1&0&0&1&1&2&1&2&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&1&2&2&1&1&1&2&0&0&2&1&1&0&0&1&0&0&0&0&0&0&0&1&1&2&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&2&2&0&1&2&0&0&0&2&1&2&0&1&2&1&2&2&1&2&2&2&2&1&1&0&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&1&2&2&2&2&2&1&2&1&0&0&0&2&0&1&2&0&2&1&0&0&1&2&2&1&2&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&1&0&1&0&0&2&0&0&0&0&0&0&0&2&2&2&1&1&1&1&2&1&0&2&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&2&0&0&2&0&0&0&2&0&2&1&1&2&1&1&2&0&0&2&0&1&2&0&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&2&1&0&0&2&0&0&0&2&1&2&0&0&1&2&1&0&0&2&2&2&0&0&2&0&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&1&0&1&1&0&0&2&2&0&2&0&2&2&0&0&2&2&1&2&0&0&1&2&0&1&2&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&1&2&0&1&1&2&0&2&2&0&0&1&1&2&2&1&2&0&2&0&2&0&2&1&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&2&1&1&1&0&1&0&0&0&1&1&1&2&0&1&0&2&1&2&2&0&0&2&2&0&2&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&2&0&2&1&1&0&0&2&0&1&2&0&2&2&1&1&0&0&0&0&1&1&0&1&1&2&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&1&1&1&0&0&0&0&0&0&0&0&0&2&0&0&1&2&2&1&2&2&0&2&0&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&1&0&2&0&2&1&0&0&2&0&2&0&0&0&2&1&0&2&1&0&0&1&0&2&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&2&1&1&1&1&0&2&1&2&0&0&0&1&0&2&0&0&1&2&0&0&2&1&1&2&1&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&2&0&2&0&0&1&0&0&0&2&1&0&0&0&0&0&2&0&2&0&2&0&2&0&1&2&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&1&2&2&2&1&0&0&2&1&1&2&1&1&0&2&0&0&1&1&2&0&0&2&0&0&1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&2&0&0&2&0&0&2&0&1&2&0&1&2&1&1&2&0&1&1&1&2&0&1&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&2&2&1&1&1&0&0&2&0&2&0&2&1&1&2&1&1&1&1&2&0&0&2&0&1&0&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&2&2&0&1&0&2&0&0&0&2&2&0&2&0&1&1&2&2&1&1&1&2&1&2&2&0&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&1&2&1&1&1&0&2&1&0&1&1&2&0&2&2&1&1&0&0&0&0&2&2&0&0&1&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&2&2&2&2&0&1&0&2&1&2&1&0&2&1&2&1&1&1&0&2&2&2&0&0&2&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&2&2&0&1&0&2&0&0&0&2&2&0&2&0&1&1&2&2&1&1&2&2&1&1&2&0&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&1&1&0&1&0&0&0&0&0&0&0&0&2&1&1&1&0&2&2&1&2&2&2&2&1&2&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&2&1&2&1&0&2&2&2&0&2&1&0&2&0&2&1&0&0&1&1&2&0&2&0&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&1&0&1&0&0&0&2&0&0&0&1&2&0&2&1&1&1&1&0&0&1&2&0&1&2&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&2&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&0&2&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&2&0&0&0&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&2&0&2&1&0&2&1&1&2&1&1&0&2&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&2&0&0&0&1&2&1&2&1&2&1&1&1&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&0&1&0&2&0&0&1&0&2&1&0&0&1&2&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&0&1&0&2&0&0&0&1&2&1&0&0&1&2&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&1&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&1&0&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&2&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&0&2&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&2&2&0&0 \\
0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1
\end{pmatrix}
$$

Appendix E

The suitable candidates $a_{44,i}$

We present the suitable candidates $a_{44,i}$ obtained by conjugating the $t_{44}a_Mt_{44}^{-1}$ in $C_{GL_{44}(3)}(< X_{44}, Y_{44}, \rho_{44} >$ in the way described in Chapter 4.

$$a_{44,1} = \begin{pmatrix}
0 & 2 & 1 & 1 & 1 & 2 & 1 & 1 & 1 & 2 & 1 & 0 & 0 & 1 & 2 & 0 & 2 & 0 & 0 & 1 & 2 & 2 & 0 & 2 & 0 & 1 & 1 & 0 & 0 & 2 & 1 & 1 & 1 & 1 & 2 & 0 & 2 & 0 & 0 & 2 & 0 & 1 & 2 & 0 \\
0 & 2 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 0 & 1 & 2 & 1 & 1 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 2 & 2 & 0 & 0 & 2 & 2 & 2 & 0 & 0 & 2 & 2 & 2 & 1 & 2 & 0 & 1 & 0 & 2 & 2 & 2 & 1 & 1 & 0 \\
2 & 1 & 2 & 0 & 2 & 0 & 0 & 1 & 1 & 2 & 1 & 2 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 2 & 0 & 0 & 2 & 2 & 0 & 1 & 0 & 1 & 1 & 1 & 2 & 0 & 0 & 0 & 1 & 2 & 1 & 1 & 0 & 1 & 2 & 0 & 2 \\
0 & 2 & 2 & 2 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 2 & 2 & 2 & 0 & 2 & 1 & 0 & 2 & 2 & 0 & 0 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 1 & 2 & 2 & 0 & 2 & 0 & 0 & 2 & 0 & 0 & 2 & 1 & 1 & 1 \\
1 & 2 & 0 & 0 & 2 & 1 & 0 & 2 & 0 & 2 & 1 & 2 & 0 & 2 & 0 & 2 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 2 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 2 & 1 & 2 & 2 & 0 & 2 & 2 & 2 & 2 & 0 & 2 & 0 \\
2 & 0 & 1 & 2 & 2 & 0 & 1 & 1 & 2 & 0 & 2 & 1 & 2 & 2 & 0 & 2 & 1 & 0 & 1 & 2 & 2 & 1 & 2 & 1 & 0 & 1 & 1 & 0 & 2 & 2 & 1 & 1 & 0 & 0 & 1 & 0 & 2 & 0 & 1 & 0 & 2 & 2 & 2 & 2 \\
0 & 1 & 1 & 1 & 2 & 0 & 2 & 2 & 2 & 1 & 2 & 2 & 2 & 1 & 2 & 1 & 1 & 1 & 1 & 2 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 2 & 1 & 1 & 0 & 2 & 0 & 1 & 0 & 2 & 1 & 2 & 0 & 2 & 0 & 1 & 1 \\
0 & 0 & 2 & 1 & 1 & 0 & 2 & 2 & 1 & 0 & 0 & 2 & 0 & 1 & 1 & 0 & 2 & 0 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 2 & 2 & 1 & 2 & 0 & 1 & 0 & 2 & 0 & 1 & 2 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 2 \\
0 & 2 & 2 & 2 & 1 & 0 & 1 & 0 & 0 & 1 & 2 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 1 & 1 & 1 & 2 & 0 & 1 & 2 & 0 & 0 & 1 & 1 & 1 & 0 & 2 & 2 & 2 & 1 & 0 & 0 & 2 & 0 & 1 & 1 & 2 & 2 & 0 \\
0 & 2 & 0 & 2 & 1 & 0 & 1 & 1 & 1 & 0 & 2 & 1 & 0 & 0 & 0 & 2 & 1 & 2 & 1 & 1 & 2 & 2 & 1 & 1 & 0 & 2 & 1 & 0 & 1 & 1 & 2 & 0 & 2 & 2 & 1 & 0 & 1 & 2 & 2 & 1 & 2 & 2 & 0 & 0 \\
1 & 2 & 2 & 0 & 2 & 1 & 1 & 2 & 1 & 1 & 1 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 2 & 1 & 1 & 1 & 1 & 1 & 2 & 0 & 2 & 2 & 1 & 2 & 1 & 2 & 1 & 0 & 1 & 2 & 1 & 2 & 2 & 0 & 0 & 0 & 2 & 1 \\
2 & 0 & 2 & 1 & 2 & 0 & 2 & 0 & 2 & 2 & 0 & 0 & 2 & 0 & 1 & 0 & 2 & 2 & 0 & 2 & 2 & 0 & 1 & 2 & 2 & 1 & 1 & 0 & 0 & 2 & 2 & 2 & 2 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 2 & 0 & 0 & 0 \\
0 & 1 & 0 & 2 & 1 & 0 & 2 & 2 & 1 & 2 & 0 & 1 & 1 & 2 & 1 & 0 & 1 & 1 & 2 & 1 & 1 & 0 & 0 & 1 & 0 & 2 & 2 & 1 & 2 & 0 & 1 & 2 & 1 & 1 & 0 & 0 & 0 & 2 & 2 & 2 & 2 & 2 & 2 & 1 \\
2 & 2 & 2 & 1 & 0 & 1 & 2 & 0 & 1 & 1 & 1 & 0 & 2 & 1 & 0 & 0 & 1 & 2 & 0 & 0 & 0 & 1 & 2 & 0 & 0 & 2 & 1 & 0 & 0 & 0 & 2 & 1 & 0 & 2 & 0 & 0 & 2 & 0 & 1 & 2 & 1 & 1 & 1 & 1 \\
2 & 1 & 0 & 2 & 1 & 1 & 2 & 2 & 1 & 2 & 0 & 1 & 0 & 0 & 2 & 0 & 0 & 2 & 0 & 2 & 2 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 2 & 2 & 1 & 0 & 1 & 1 & 2 & 0 & 1 & 0 & 2 & 1 & 0 \\
1 & 0 & 2 & 0 & 1 & 0 & 0 & 0 & 1 & 2 & 1 & 2 & 0 & 2 & 0 & 0 & 2 & 0 & 0 & 2 & 2 & 1 & 1 & 0 & 1 & 2 & 0 & 2 & 2 & 2 & 1 & 1 & 0 & 0 & 1 & 0 & 2 & 1 & 1 & 2 & 1 & 1 & 2 & 2 \\
1 & 2 & 2 & 1 & 1 & 1 & 2 & 0 & 0 & 0 & 0 & 2 & 2 & 2 & 0 & 1 & 1 & 0 & 1 & 2 & 2 & 1 & 0 & 0 & 0 & 2 & 0 & 1 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 2 & 2 & 1 & 0 & 2 & 2 \\
1 & 2 & 0 & 2 & 0 & 1 & 2 & 2 & 2 & 0 & 2 & 2 & 2 & 2 & 0 & 2 & 2 & 2 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 2 & 2 & 1 & 1 & 2 & 0 & 2 & 1 & 0 & 0 & 0 & 1 & 2 & 1 & 2 & 1 & 0 & 1 \\
2 & 1 & 2 & 1 & 2 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 2 & 0 & 0 & 0 & 2 & 1 & 2 & 0 & 2 & 1 & 0 & 0 & 0 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 2 & 0 & 1 & 0 & 0 & 2 & 1 & 1 & 1 \\
2 & 2 & 1 & 2 & 2 & 1 & 1 & 1 & 0 & 2 & 0 & 0 & 2 & 0 & 0 & 2 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 2 & 2 & 1 & 2 & 1 & 0 & 2 & 1 & 1 & 1 & 1 & 2 & 0 & 2 & 0 & 0 & 2 & 0 & 1 & 2 & 0 \\
0 & 1 & 0 & 0 & 0 & 1 & 2 & 0 & 1 & 1 & 1 & 1 & 0 & 1 & 0 & 0 & 2 & 0 & 0 & 1 & 2 & 0 & 1 & 2 & 2 & 0 & 2 & 2 & 2 & 1 & 2 & 1 & 1 & 2 & 1 & 0 & 2 & 1 & 1 & 0 & 2 & 0 & 2 & 0 \\
2 & 1 & 1 & 0 & 1 & 1 & 2 & 0 & 1 & 0 & 1 & 0 & 1 & 2 & 0 & 2 & 2 & 1 & 0 & 2 & 0 & 2 & 2 & 2 & 0 & 1 & 0 & 2 & 1 & 1 & 2 & 2 & 0 & 2 & 1 & 2 & 1 & 0 & 0 & 0 & 2 & 1 & 1 & 0 \\
2 & 1 & 1 & 1 & 1 & 2 & 1 & 2 & 2 & 0 & 0 & 1 & 0 & 0 & 2 & 0 & 2 & 1 & 1 & 1 & 2 & 1 & 0 & 1 & 1 & 0 & 0 & 2 & 1 & 0 & 2 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 & 0 & 2 & 2 \\
0 & 2 & 2 & 0 & 2 & 0 & 2 & 1 & 1 & 0 & 1 & 2 & 1 & 2 & 2 & 2 & 0 & 2 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 2 & 2 & 2 & 0 & 0 & 0 & 2 & 2 & 2 & 2 & 1 & 0 & 1 & 1 & 2 & 1 & 0 & 1 \\
1 & 1 & 0 & 2 & 1 & 1 & 2 & 0 & 2 & 1 & 2 & 1 & 1 & 0 & 0 & 1 & 0 & 1 & 2 & 1 & 0 & 2 & 0 & 2 & 2 & 2 & 1 & 1 & 1 & 1 & 2 & 2 & 0 & 2 & 1 & 2 & 1 & 0 & 0 & 0 & 2 & 1 & 1 & 0 \\
0 & 0 & 2 & 2 & 2 & 2 & 1 & 0 & 0 & 1 & 2 & 2 & 1 & 0 & 2 & 1 & 0 & 1 & 0 & 2 & 0 & 2 & 0 & 2 & 1 & 1 & 1 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 2 & 1 & 0 & 0 & 1 & 1 & 1 & 2 \\
2 & 2 & 1 & 1 & 0 & 2 & 1 & 1 & 1 & 1 & 2 & 0 & 2 & 0 & 2 & 1 & 0 & 2 & 1 & 1 & 1 & 0 & 2 & 0 & 2 & 0 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 & 1 & 2 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
2 & 1 & 1 & 1 & 1 & 2 & 2 & 1 & 1 & 2 & 2 & 2 & 1 & 1 & 2 & 1 & 1 & 0 & 0 & 2 & 2 & 1 & 2 & 0 & 1 & 2 & 0 & 0 & 0 & 1 & 2 & 2 & 2 & 2 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 2 & 1 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 2 & 1 & 2 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 1 & 2 & 2 & 2 & 1 & 1 & 2 & 2 & 1 & 0 & 2 & 1 & 1 & 2 & 1 & 1 & 1 & 0 & 1 & 2 & 0 & 2 & 2 & 2 & 1 \\
0 & 0 & 0 & 1 & 1 & 2 & 0 & 2 & 2 & 1 & 2 & 0 & 2 & 2 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 1 & 1 & 2 & 2 & 1 & 2 & 1 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 2 & 1 & 0 & 1 & 0 & 0 & 0 & 2 \\
0 & 2 & 2 & 1 & 1 & 2 & 0 & 1 & 1 & 1 & 2 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 2 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 2 & 0 & 1 & 1 & 2 & 0 & 2 & 2 & 0 & 1 & 1 & 2 & 1 & 1 & 2 & 2 & 2 & 2 \\
1 & 0 & 1 & 0 & 0 & 0 & 2 & 1 & 1 & 0 & 1 & 2 & 2 & 0 & 1 & 0 & 2 & 0 & 2 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 2 & 2 & 2 & 0 & 1 & 1 & 2 & 0 & 2 & 1 & 1 & 1 & 0 & 2 & 1 & 0 & 0 & 2 \\
1 & 2 & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 1 & 2 & 1 & 2 & 2 & 1 & 0 & 2 & 0 & 0 & 2 & 2 & 1 & 2 & 2 & 0 & 1 & 2 & 0 & 2 & 2 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
1 & 1 & 0 & 1 & 1 & 2 & 0 & 1 & 1 & 1 & 1 & 0 & 2 & 0 & 1 & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 2 & 1 & 0 & 2 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 2 & 2 & 1 \\
2 & 1 & 1 & 2 & 2 & 0 & 0 & 2 & 2 & 2 & 0 & 2 & 2 & 1 & 2 & 2 & 0 & 1 & 1 & 0 & 0 & 0 & 2 & 2 & 2 & 0 & 2 & 2 & 1 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 2 & 1 & 2 & 2 & 1 & 2 & 2 & 1 \\
0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 2 & 2 & 0 & 2 & 2 & 0 & 1 & 1 & 1 & 1 & 0 & 2 & 1 & 0 & 2 & 2 & 1 & 0 & 0 & 0 & 2 & 0 & 2 & 2 & 0 & 2 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 2 & 0 & 2 \\
2 & 0 & 0 & 1 & 2 & 2 & 0 & 0 & 1 & 1 & 2 & 0 & 0 & 1 & 2 & 0 & 1 & 1 & 2 & 2 & 1 & 1 & 1 & 1 & 2 & 0 & 1 & 2 & 0 & 1 & 0 & 2 & 2 & 2 & 0 & 1 & 0 & 1 & 2 & 0 & 1 & 0 & 0 & 2 \\
0 & 0 & 0 & 1 & 2 & 0 & 1 & 1 & 1 & 2 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 2 & 1 & 1 & 0 & 1 & 0 & 2 & 0 & 1 & 1 & 2 & 1 & 1 & 1 & 0 & 0 & 0 & 1 & 2 & 2 & 2 & 1 \\
0 & 2 & 2 & 2 & 0 & 0 & 1 & 1 & 2 & 2 & 2 & 0 & 2 & 1 & 0 & 1 & 2 & 1 & 1 & 0 & 1 & 0 & 1 & 2 & 2 & 2 & 0 & 0 & 2 & 2 & 2 & 2 & 2 & 0 & 2 & 1 & 2 & 2 & 2 & 2 & 2 & 2 & 2 & 2 \\
0 & 1 & 1 & 2 & 2 & 2 & 1 & 0 & 1 & 1 & 2 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 2 & 0 & 0 & 0 & 1 & 1 & 1 & 2 & 0 & 2 & 2 & 1 & 1 & 2 & 2 & 1 & 1 & 1 & 2 & 0 & 2 & 2 & 2 & 2 & 1 \\
0 & 2 & 2 & 0 & 0 & 0 & 2 & 1 & 1 & 2 & 2 & 0 & 1 & 0 & 0 & 1 & 0 & 2 & 0 & 0 & 2 & 0 & 1 & 0 & 1 & 2 & 1 & 2 & 1 & 0 & 0 & 1 & 0 & 2 & 2 & 0 & 1 & 1 & 0 & 0 & 0 & 2 & 2 & 2 \\
0 & 1 & 0 & 2 & 2 & 2 & 0 & 0 & 2 & 1 & 2 & 1 & 2 & 1 & 0 & 1 & 2 & 1 & 2 & 1 & 1 & 1 & 0 & 2 & 2 & 2 & 1 & 1 & 0 & 1 & 2 & 2 & 1 & 2 & 2 & 2 & 0 & 0 & 0 & 2 & 1 & 2 & 0 & 2 \\
0 & 0 & 0 & 2 & 2 & 1 & 1 & 1 & 1 & 2 & 2 & 0 & 0 & 2 & 0 & 1 & 2 & 0 & 2 & 1 & 1 & 1 & 0 & 0 & 1 & 2 & 2 & 2 & 1 & 0 & 2 & 0 & 1 & 1 & 1 & 2 & 1 & 2 & 1 & 0 & 2 & 2 & 1 & 1 \\
0 & 1 & 1 & 1 & 1 & 2 & 1 & 1 & 0 & 0 & 2 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 1 & 2 & 0 & 2 & 1 & 1 & 2 & 0 & 0 & 1 & 2 & 1 & 2 & 1 & 2 & 1 & 1 & 1 & 0 & 0 & 2 & 2 & 2 & 2 & 2 & 1
\end{pmatrix}$$

$$a_{44,2} = \begin{pmatrix}
0&2&1&1&1&2&1&1&1&2&1&0&0&1&2&0&2&0&0&1&2&2&0&2&0&1&1&0&0&1&2&2&2&2&1&0&1&0&0&1&0&2&1&0 \\
0&2&0&1&1&0&0&1&1&0&1&2&1&1&0&1&1&1&1&1&0&2&2&0&0&2&2&2&0&0&1&1&1&2&1&0&2&0&1&1&1&2&2&1 \\
2&1&2&0&2&0&0&1&1&2&1&2&1&0&0&1&1&1&0&0&2&0&0&2&2&0&1&0&2&2&2&1&0&0&0&2&1&2&2&0&2&1&0&1 \\
0&2&2&2&1&1&1&0&0&1&0&2&2&2&0&2&1&0&2&2&0&0&2&2&2&2&2&2&1&1&2&1&1&0&1&0&0&1&0&0&1&2&2&1 \\
1&2&0&0&2&1&0&2&0&2&1&2&0&2&0&2&0&0&0&1&1&1&0&2&0&1&1&1&0&0&0&0&1&2&1&1&0&1&1&1&1&0&1&2 \\
2&0&1&2&2&0&1&1&2&0&2&1&2&2&0&2&1&0&1&2&2&1&2&1&0&1&1&0&1&1&2&2&0&0&2&0&1&0&2&0&1&1&1&1 \\
0&1&1&1&2&0&2&2&2&1&2&2&2&1&2&1&1&1&1&2&1&0&0&1&1&1&0&1&1&2&2&0&1&0&2&0&1&2&1&0&1&0&2&0 \\
0&0&2&1&1&0&2&2&1&0&0&2&0&1&1&0&2&0&1&1&1&1&1&0&0&2&2&1&1&0&2&0&1&0&2&1&0&2&2&0&0&0&0&0 \\
0&2&2&2&1&0&1&0&0&1&2&2&0&0&0&2&0&0&1&1&1&2&0&1&2&0&0&1&2&2&0&1&1&1&2&0&0&1&0&2&2&1&1&1 \\
0&2&0&2&1&0&1&1&1&0&2&1&0&0&0&2&1&2&1&1&2&2&1&1&0&2&1&0&2&2&1&0&1&1&2&0&2&1&1&2&1&1&0&2 \\
1&2&2&0&2&1&1&2&1&1&1&0&2&0&0&0&0&0&2&1&1&1&1&1&2&0&2&2&2&1&2&1&2&0&2&1&2&1&1&0&0&0&1&0 \\
2&0&2&1&2&0&2&0&2&2&0&0&2&0&1&0&2&2&0&2&2&0&1&2&2&1&1&0&0&1&1&1&1&0&2&2&2&0&2&0&1&0&0&0 \\
0&1&0&2&1&0&2&2&1&2&0&1&1&2&1&0&1&1&2&1&1&0&0&1&0&2&2&1&1&0&2&1&2&2&0&0&0&1&1&1&1&1&1&0 \\
2&2&2&1&0&1&2&0&1&1&1&0&2&1&0&0&1&2&0&0&0&1&2&0&0&2&1&0&0&0&1&2&0&1&0&0&1&0&2&1&2&2&2&2 \\
2&1&0&2&1&1&2&2&1&2&0&1&0&0&2&0&0&2&0&2&2&1&1&1&1&0&0&0&2&0&0&1&1&2&0&2&2&1&0&2&0&1&2&1 \\
1&0&2&0&1&0&0&0&1&2&1&2&0&2&0&0&2&0&0&2&2&1&1&0&1&2&0&2&1&1&2&2&0&0&2&0&1&2&2&1&2&2&1&2 \\
1&2&2&1&1&1&2&0&0&0&0&2&2&2&0&1&1&0&1&2&2&1&0&0&0&2&0&1&2&0&2&0&2&0&0&2&2&2&1&1&2&0&1&1 \\
1&2&0&2&0&1&2&2&2&0&2&2&2&2&0&2&2&2&0&1&1&1&1&0&1&0&2&2&2&2&1&0&1&2&0&0&0&2&1&2&1&2&0&2 \\
2&1&2&1&2&1&0&1&1&0&0&1&0&2&0&0&0&2&1&2&0&2&1&0&0&0&1&2&0&0&0&0&0&2&1&1&0&2&0&0&1&2&2&0 \\
2&2&1&2&2&1&1&1&0&2&0&0&2&0&0&2&1&0&1&0&0&0&0&2&2&1&2&1&0&1&2&2&2&2&1&0&1&0&0&1&0&2&1&0 \\
0&1&0&0&0&1&2&0&1&1&1&1&0&1&0&0&2&0&0&1&2&0&1&2&2&0&2&2&1&2&1&2&2&1&2&0&1&2&2&0&1&0&1&1 \\
2&1&1&0&1&1&2&0&1&0&1&0&1&2&0&2&2&1&0&2&0&2&2&2&0&1&0&2&2&2&1&1&0&1&2&1&2&0&0&0&1&2&2&2 \\
2&1&1&1&1&2&1&2&2&0&0&1&0&0&2&0&2&1&1&1&2&1&0&1&1&0&0&2&2&0&1&0&0&0&0&1&0&0&0&2&0&0&1&1 \\
0&2&2&0&2&0&2&1&1&0&1&2&1&2&2&2&0&2&1&1&1&0&0&0&0&1&2&2&1&0&0&0&1&1&1&1&2&0&2&2&1&2&0&0 \\
1&1&0&2&1&1&2&0&2&1&2&1&1&0&0&1&0&1&2&1&0&2&0&2&2&2&1&1&2&2&1&1&0&1&2&1&2&0&0&0&1&2&2&2 \\
0&0&2&2&2&2&1&0&0&1&2&2&1&0&2&1&0&1&0&2&0&2&0&2&1&1&1&1&0&2&2&0&0&2&2&2&1&2&0&0&2&2&2&0 \\
2&2&1&1&0&2&1&1&1&1&2&0&2&0&2&1&0&2&1&1&1&0&2&0&2&0&0&1&2&2&2&2&2&0&0&2&1&0&0&0&0&2&0&2 \\
2&1&1&1&1&2&2&1&1&2&2&2&1&1&2&1&1&0&0&2&2&1&2&0&1&2&0&0&0&2&1&1&1&1&2&0&2&0&0&2&0&1&2&0 \\
0&2&1&2&2&0&1&2&1&2&2&0&0&1&0&1&1&2&2&0&1&2&0&1&0&1&1&1&2&0&0&2&1&2&2&2&0&1&1&2&1&1&1&1 \\
0&2&2&1&1&2&2&0&1&2&2&0&1&0&0&1&2&1&1&0&0&1&1&1&2&1&2&0&0&0&2&1&2&0&1&1&2&1&2&0&2&2&2&1 \\
0&0&0&1&1&2&2&1&2&2&2&0&0&2&0&1&1&2&0&1&0&0&2&1&0&2&1&2&0&2&1&1&1&0&1&2&1&2&0&0&1&1&1&1 \\
2&2&1&2&2&1&0&1&2&0&0&1&1&2&2&2&0&2&0&1&1&2&2&2&1&0&1&0&1&1&0&2&1&1&0&2&1&1&2&1&0&2&2&0 \\
2&1&0&0&1&1&0&0&0&0&2&1&2&1&1&2&0&1&0&0&1&1&2&1&1&0&2&1&0&2&2&2&0&1&0&0&0&0&0&0&1&0&1&0 \\
2&1&2&1&1&2&2&1&2&2&0&0&1&2&2&2&2&0&0&1&0&1&1&2&0&2&2&1&1&2&2&0&2&1&2&1&1&1&0&2&0&1&1&2 \\
1&1&1&0&0&1&2&0&1&1&1&1&1&1&1&0&2&1&1&1&0&1&0&0&2&0&1&0&0&1&0&2&2&2&1&1&2&1&1&1&0&1&1&2 \\
0&2&2&1&1&0&2&1&1&1&1&1&1&2&2&1&1&1&2&2&2&1&0&0&0&0&0&2&1&1&1&0&2&0&1&1&1&0&0&2&0&1&2&0 \\
1&2&2&1&0&2&2&2&2&2&2&0&0&1&1&2&1&1&0&2&2&0&1&1&2&0&2&0&2&2&2&0&1&0&1&2&0&1&1&2&0&2&2&0 \\
0&2&2&1&0&1&1&1&2&1&1&0&2&1&0&2&2&1&1&0&0&0&0&1&0&0&2&2&1&1&0&2&1&2&2&2&0&0&2&0&1&1&1&1 \\
0&0&0&0&2&1&1&1&1&1&2&0&1&1&0&1&0&1&1&1&2&1&1&0&2&1&0&2&1&0&1&0&1&1&0&2&2&2&1&1&1&1&1&1 \\
0&2&2&1&1&1&2&0&2&2&1&2&2&0&0&2&2&2&0&1&0&0&0&2&2&2&1&0&2&2&1&1&2&2&1&1&1&2&0&2&2&2&2&0 \\
0&0&0&2&2&1&0&1&2&1&2&0&2&2&0&1&2&0&2&1&1&1&1&2&0&1&2&0&0&1&2&2&2&0&0&1&1&1&2&2&2&1&1&2 \\
0&1&2&0&0&2&2&2&1&2&2&2&1&1&0&1&0&1&0&0&2&0&2&0&2&1&2&1&2&2&1&0&0&0&0&0&0&0&2&1&0&1&2&1 \\
0&2&2&0&0&0&1&1&2&1&2&0&0&0&0&1&0&2&0&0&2&0&2&2&0&1&1&0&0&1&1&1&0&2&2&0&1&2&0&2&1&1&0&1 \\
0&1&1&1&1&2&1&1&0&0&2&0&0&1&0&1&1&1&1&2&0&2&1&1&2&0&0&1&1&2&1&2&1&2&2&2&0&0&1&1&1&1&1&2
\end{pmatrix}$$

$$a_{44,3} = \begin{pmatrix}
0&1&2&2&2&1&2&2&2&1&2&0&0&2&1&0&1&0&0&2&1&1&0&1&0&2&2&0&0&1&2&2&2&2&1&0&1&0&0&1&0&2&1&0\\
0&2&0&1&1&0&0&1&1&0&1&2&1&1&0&1&1&1&1&1&0&2&2&0&0&2&2&2&0&0&2&2&2&1&2&0&1&0&2&2&2&1&1&2\\
1&1&2&0&2&0&0&1&1&2&1&2&1&0&0&1&1&1&0&0&2&0&0&2&2&0&1&0&1&1&1&2&0&0&0&1&2&1&1&0&1&2&0&2\\
0&2&2&2&1&1&1&0&0&1&0&2&2&2&0&2&1&0&2&2&0&0&2&2&2&2&2&2&2&2&1&2&2&0&2&0&0&2&0&0&2&1&1&2\\
2&2&0&0&2&1&0&2&0&2&1&2&0&2&0&2&0&0&0&1&1&1&0&2&0&1&1&1&0&0&0&0&2&1&2&2&0&2&2&2&2&0&2&1\\
1&0&1&2&2&0&1&1&2&0&2&1&2&2&0&2&1&0&1&2&2&1&2&1&0&1&1&0&2&2&1&1&0&0&1&0&2&0&1&0&2&2&2&2\\
0&1&1&1&2&0&2&2&2&1&2&2&2&1&2&1&1&1&1&2&1&0&0&1&1&1&0&1&2&1&1&0&2&0&1&0&2&1&2&0&2&0&1&0\\
0&0&2&1&1&0&2&2&1&0&0&2&0&1&1&0&2&0&1&1&1&1&0&0&2&2&1&2&0&1&0&2&0&1&2&0&1&1&0&0&0&0&0&0\\
0&2&2&2&1&0&1&0&0&1&2&2&0&0&0&2&0&0&1&1&1&2&0&1&2&0&0&1&1&1&0&2&2&2&1&0&0&2&0&1&1&2&2&2\\
0&2&0&2&1&0&1&1&1&0&2&1&0&0&0&2&1&2&1&1&2&2&1&1&0&2&1&0&1&1&2&0&2&2&1&0&1&2&2&1&2&2&0&1\\
2&2&2&0&2&1&1&2&1&1&1&0&2&0&0&0&0&0&2&1&1&1&1&1&2&0&2&2&1&2&1&2&1&0&1&2&1&2&2&0&0&0&2&0\\
1&0&2&1&2&0&2&0&2&2&0&0&2&0&1&0&2&2&0&2&2&0&1&2&2&1&1&0&0&2&2&2&2&0&1&1&1&0&1&0&2&0&0&0\\
0&1&0&2&1&0&2&2&1&2&0&1&1&2&1&0&1&1&2&1&1&0&0&1&0&2&2&1&2&0&1&2&1&1&0&0&0&2&2&2&2&2&2&0\\
1&2&2&1&0&1&2&0&1&1&1&0&2&1&0&0&1&2&0&0&0&1&2&0&0&2&1&0&0&0&2&1&0&2&0&0&2&0&1&2&1&1&1&1\\
1&1&0&2&1&1&2&2&1&2&0&1&0&0&2&0&0&2&0&2&2&1&1&1&1&0&0&0&1&0&0&2&2&1&0&1&1&2&0&1&0&2&1&2\\
2&0&2&0&1&0&0&0&1&2&1&2&0&2&0&0&2&0&0&2&2&1&1&0&1&2&0&2&2&2&1&1&0&0&1&0&2&1&1&2&1&1&2&1\\
2&2&2&1&1&1&2&0&0&0&0&2&2&2&0&1&1&0&1&2&2&1&0&0&0&2&0&1&1&0&1&0&1&0&0&1&1&1&2&2&1&0&2&2\\
2&2&0&2&0&1&2&2&2&0&2&2&2&2&0&2&2&2&0&1&1&1&1&0&1&0&2&2&1&1&2&0&2&1&0&0&0&1&2&1&2&1&0&1\\
1&1&2&1&2&1&0&1&1&0&0&1&0&2&0&0&0&2&1&2&0&2&1&0&0&0&1&2&0&0&0&0&0&1&2&2&0&1&0&0&2&1&1&0\\
1&2&1&2&2&1&1&1&0&2&0&0&2&0&0&2&1&0&1&0&0&0&0&2&2&1&2&1&0&2&1&1&1&1&2&0&2&0&0&2&0&1&2&0\\
0&1&0&0&0&1&2&0&1&1&1&1&0&1&0&0&2&0&0&1&2&0&1&2&2&0&2&2&2&1&2&1&1&2&1&0&2&1&1&0&2&0&2&2\\
1&1&1&0&1&1&2&0&1&0&1&0&1&2&0&2&2&1&0&2&0&2&2&2&0&1&0&2&1&1&2&2&0&2&1&2&1&0&0&0&2&1&1&1\\
1&1&1&1&1&2&1&2&2&0&0&1&0&0&2&0&2&1&1&1&2&1&0&1&1&0&0&2&1&0&2&0&0&0&0&2&0&0&0&1&0&0&2&2\\
0&2&2&0&2&0&2&1&1&0&1&2&1&2&2&2&0&2&1&1&1&0&0&0&0&1&2&2&2&0&0&0&2&2&2&2&1&0&1&1&2&1&0&0\\
2&1&0&2&1&1&2&0&2&1&2&1&1&0&0&1&0&1&2&1&0&2&0&2&2&2&1&1&1&1&2&2&0&2&1&2&1&0&0&0&2&1&1&1\\
0&0&2&2&2&2&1&0&0&1&2&2&1&0&2&1&0&1&0&2&0&2&0&2&1&1&1&1&0&1&1&0&0&1&1&1&2&1&0&0&1&1&1&0\\
1&2&1&1&0&2&1&1&1&1&2&0&2&0&2&1&0&2&1&1&1&0&2&0&2&0&0&1&1&1&1&1&1&0&0&1&2&0&0&0&0&1&0&1\\
1&1&1&1&1&2&2&1&1&2&2&2&1&1&2&1&1&0&0&2&2&1&2&0&1&2&0&0&0&1&2&2&2&2&1&0&1&0&0&1&0&2&1&0\\
0&1&2&1&1&0&2&1&2&1&1&0&0&2&0&2&2&1&1&0&2&1&0&2&0&2&2&2&2&0&0&2&1&2&2&2&0&1&1&2&1&1&1&1\\
0&1&1&2&2&1&1&0&2&1&1&0&2&0&0&2&1&2&2&0&0&2&2&2&1&2&1&0&0&0&2&1&2&0&1&1&2&1&2&0&2&2&2&1\\
0&0&0&2&2&1&1&2&1&1&1&0&0&1&0&2&2&1&0&2&0&0&1&2&0&1&2&1&0&2&1&1&1&0&1&2&1&2&0&0&1&1&1&1\\
2&1&2&1&1&2&0&2&1&0&0&2&2&1&1&1&0&1&0&2&2&1&1&1&2&0&2&0&1&1&0&2&1&1&0&2&1&1&2&1&0&2&2&0\\
2&2&0&0&2&2&0&0&0&0&1&2&1&2&2&1&0&2&0&0&2&2&1&2&2&0&1&2&0&2&2&2&0&1&0&0&0&0&0&0&1&0&1&0\\
2&2&1&2&2&1&1&2&1&1&0&0&2&1&1&1&1&0&0&2&0&2&2&1&0&1&1&2&1&2&2&0&2&1&2&1&1&1&0&2&0&1&1&2\\
1&2&2&0&0&2&1&0&2&2&2&2&2&2&2&0&1&2&2&2&0&2&0&0&1&0&2&0&0&1&0&2&2&2&1&1&2&1&1&1&0&1&1&2\\
0&1&1&2&2&0&1&2&2&2&2&2&2&1&1&2&2&2&1&1&1&2&0&0&0&0&0&1&1&1&1&0&2&0&1&1&1&0&0&2&0&1&2&0\\
1&1&1&2&0&1&1&1&1&1&1&0&0&2&2&1&2&2&0&1&1&0&2&2&1&0&1&0&2&2&2&0&1&0&1&2&0&1&1&2&0&2&2&0\\
0&1&1&2&0&2&2&2&1&2&2&0&1&2&0&1&1&2&2&0&0&0&0&2&0&0&1&1&1&1&0&2&1&2&2&2&0&0&2&0&1&1&1&1\\
0&0&0&0&1&2&2&2&2&2&1&0&2&2&0&2&0&2&2&2&1&2&2&0&1&2&0&1&1&0&1&0&1&1&0&2&2&2&1&1&1&1&1&1\\
0&1&1&2&2&2&1&0&1&1&2&1&1&0&0&1&1&1&0&2&0&0&0&1&1&1&2&0&2&2&1&1&2&2&1&1&1&2&0&2&2&2&2&0\\
0&0&0&1&1&2&0&2&1&2&1&0&1&1&0&2&1&0&1&2&2&2&2&1&0&2&1&0&0&1&2&2&2&0&0&1&1&1&2&2&2&1&1&2\\
0&2&1&0&0&1&1&1&2&1&1&1&2&2&0&2&0&2&0&0&1&0&1&0&1&2&1&2&2&2&1&0&0&0&0&0&0&0&2&1&0&1&2&1\\
0&1&1&0&0&0&2&2&1&2&1&0&0&0&0&2&0&1&0&0&1&0&1&1&0&2&2&0&0&1&1&1&0&2&2&0&1&2&0&2&1&1&0&1\\
0&2&2&2&2&1&2&2&0&0&1&0&0&2&0&2&2&2&2&1&0&1&2&2&1&0&0&2&1&2&1&2&1&2&2&2&0&0&1&1&1&1&1&2
\end{pmatrix}$$

$$a_{44,4} = \begin{pmatrix}
0\,1\,2\,2\,2\,1\,2\,2\,2\,1\,2\,0\,0\,2\,1\,0\,1\,0\,0\,2\,1\,1\,0\,1\,0\,2\,2\,0\,0\,2\,1\,1\,1\,1\,2\,0\,2\,0\,0\,2\,0\,1\,2\,0 \\
0\,2\,0\,1\,1\,0\,0\,1\,1\,0\,1\,2\,1\,1\,0\,1\,1\,1\,1\,1\,0\,2\,2\,0\,0\,2\,2\,2\,0\,0\,1\,1\,1\,2\,1\,0\,2\,0\,1\,1\,1\,2\,2\,0 \\
1\,1\,2\,0\,2\,0\,0\,1\,1\,2\,1\,2\,1\,0\,0\,1\,1\,1\,0\,0\,2\,0\,0\,2\,2\,0\,1\,0\,2\,2\,2\,1\,0\,0\,0\,2\,1\,2\,2\,0\,2\,1\,0\,1 \\
0\,2\,2\,2\,1\,1\,1\,0\,0\,1\,0\,2\,2\,2\,0\,2\,1\,0\,2\,2\,0\,0\,2\,2\,2\,2\,2\,2\,1\,1\,2\,1\,1\,0\,1\,0\,0\,1\,0\,0\,1\,2\,2\,2 \\
2\,2\,0\,0\,2\,1\,0\,2\,0\,2\,1\,2\,0\,2\,0\,2\,0\,0\,0\,1\,1\,1\,0\,2\,0\,1\,1\,1\,0\,0\,0\,0\,1\,2\,1\,1\,0\,1\,1\,1\,1\,0\,1\,0 \\
1\,0\,1\,2\,2\,0\,1\,1\,2\,0\,2\,1\,2\,2\,0\,2\,1\,0\,1\,2\,2\,1\,2\,1\,0\,1\,1\,0\,1\,1\,2\,2\,0\,0\,2\,0\,1\,0\,2\,0\,1\,1\,1\,1 \\
0\,1\,1\,1\,2\,0\,2\,2\,2\,1\,2\,2\,2\,1\,2\,1\,1\,1\,1\,2\,1\,0\,0\,1\,1\,1\,0\,1\,1\,2\,2\,0\,1\,0\,2\,0\,1\,2\,1\,0\,1\,0\,2\,2 \\
0\,0\,2\,1\,1\,0\,2\,2\,1\,0\,0\,2\,0\,1\,1\,0\,2\,0\,1\,1\,1\,1\,1\,0\,0\,2\,2\,1\,1\,0\,2\,0\,1\,0\,2\,1\,0\,2\,2\,0\,0\,0\,0\,1 \\
0\,2\,2\,2\,1\,0\,1\,0\,0\,1\,2\,2\,0\,0\,0\,2\,0\,0\,1\,1\,1\,2\,0\,1\,2\,0\,0\,1\,2\,2\,0\,1\,1\,1\,2\,0\,0\,1\,0\,2\,2\,1\,1\,0 \\
0\,2\,0\,2\,1\,0\,1\,1\,1\,0\,2\,1\,0\,0\,0\,2\,1\,2\,1\,1\,2\,2\,1\,1\,0\,2\,1\,0\,2\,2\,1\,0\,1\,1\,2\,0\,2\,1\,1\,2\,1\,1\,0\,0 \\
2\,2\,2\,0\,2\,1\,1\,2\,1\,1\,1\,0\,2\,0\,0\,0\,0\,0\,2\,1\,1\,1\,1\,1\,2\,0\,2\,2\,2\,1\,2\,1\,2\,0\,2\,1\,2\,1\,1\,0\,0\,0\,1\,2 \\
1\,0\,2\,1\,2\,0\,2\,0\,2\,2\,0\,0\,2\,0\,1\,0\,2\,2\,0\,2\,2\,0\,1\,2\,2\,1\,1\,0\,0\,1\,1\,1\,1\,0\,2\,2\,2\,0\,2\,0\,1\,0\,0\,0 \\
0\,1\,0\,2\,1\,0\,2\,2\,1\,2\,0\,1\,1\,2\,1\,0\,1\,1\,2\,1\,1\,0\,0\,1\,0\,2\,2\,1\,1\,0\,2\,1\,2\,2\,0\,0\,0\,1\,1\,1\,1\,1\,1\,2 \\
1\,2\,2\,1\,0\,1\,2\,0\,1\,1\,1\,0\,2\,1\,0\,0\,1\,2\,0\,0\,0\,1\,2\,0\,0\,2\,1\,0\,0\,0\,1\,2\,0\,1\,0\,0\,1\,0\,2\,1\,2\,2\,2\,2 \\
1\,1\,0\,2\,1\,1\,2\,2\,1\,2\,0\,1\,0\,0\,2\,0\,0\,2\,0\,2\,2\,1\,1\,1\,1\,0\,0\,0\,2\,0\,0\,1\,1\,2\,0\,2\,2\,1\,0\,2\,0\,1\,2\,0 \\
2\,0\,2\,0\,1\,0\,0\,0\,1\,2\,1\,2\,0\,2\,0\,0\,2\,0\,0\,2\,2\,1\,1\,0\,1\,2\,0\,2\,1\,1\,2\,2\,0\,0\,2\,0\,1\,2\,2\,1\,2\,2\,1\,1 \\
2\,2\,2\,1\,1\,1\,2\,0\,0\,0\,0\,2\,2\,2\,0\,1\,1\,0\,1\,2\,2\,1\,0\,0\,0\,2\,0\,1\,2\,0\,2\,0\,2\,0\,0\,2\,2\,2\,1\,1\,2\,0\,1\,1 \\
2\,2\,0\,2\,0\,1\,2\,2\,2\,0\,2\,2\,2\,2\,0\,2\,2\,2\,0\,1\,1\,1\,1\,0\,1\,0\,2\,2\,2\,2\,1\,0\,1\,2\,0\,0\,0\,2\,1\,2\,1\,2\,0\,2 \\
1\,1\,2\,1\,2\,1\,0\,1\,1\,0\,0\,1\,0\,2\,0\,0\,0\,2\,1\,2\,0\,2\,1\,0\,0\,0\,1\,2\,0\,0\,0\,0\,0\,2\,1\,1\,0\,2\,0\,0\,1\,2\,2\,2 \\
1\,2\,1\,2\,2\,1\,1\,1\,0\,2\,0\,0\,2\,0\,0\,2\,1\,0\,1\,0\,0\,0\,0\,2\,2\,1\,2\,1\,0\,1\,2\,2\,2\,2\,1\,0\,1\,0\,0\,1\,0\,2\,1\,0 \\
0\,1\,0\,0\,0\,1\,2\,0\,1\,1\,1\,1\,0\,1\,0\,0\,2\,0\,0\,1\,2\,0\,1\,2\,2\,0\,2\,2\,1\,2\,1\,2\,2\,1\,2\,0\,1\,2\,2\,0\,1\,0\,1\,0 \\
1\,1\,1\,0\,1\,1\,2\,0\,1\,0\,1\,0\,1\,2\,0\,2\,2\,1\,0\,2\,0\,2\,2\,2\,0\,1\,0\,2\,2\,2\,1\,1\,0\,1\,2\,1\,2\,0\,0\,0\,1\,2\,2\,0 \\
1\,1\,1\,1\,1\,2\,1\,2\,2\,0\,0\,1\,0\,0\,2\,0\,2\,1\,1\,1\,2\,1\,0\,1\,1\,0\,0\,2\,2\,0\,1\,0\,0\,0\,0\,1\,0\,0\,0\,2\,0\,0\,1\,1 \\
0\,2\,2\,0\,2\,0\,2\,1\,1\,0\,1\,2\,1\,2\,2\,2\,0\,2\,1\,1\,1\,0\,0\,0\,0\,1\,2\,2\,1\,0\,0\,0\,1\,1\,1\,1\,2\,0\,2\,2\,1\,2\,0\,2 \\
2\,1\,0\,2\,1\,1\,2\,0\,2\,1\,2\,1\,1\,0\,0\,1\,0\,1\,2\,1\,0\,2\,0\,2\,2\,2\,1\,1\,2\,2\,1\,1\,0\,1\,2\,1\,2\,0\,0\,0\,1\,2\,2\,0 \\
0\,0\,2\,2\,2\,2\,1\,0\,0\,1\,2\,2\,1\,0\,2\,1\,0\,1\,0\,2\,0\,2\,0\,2\,1\,1\,1\,1\,0\,2\,2\,0\,0\,2\,2\,2\,1\,2\,0\,0\,2\,2\,2\,1 \\
1\,2\,1\,1\,0\,2\,1\,1\,1\,1\,2\,0\,2\,0\,2\,1\,0\,2\,1\,1\,1\,0\,2\,0\,2\,0\,0\,1\,2\,2\,2\,2\,2\,0\,0\,2\,1\,0\,0\,0\,0\,2\,0\,2 \\
1\,1\,1\,1\,1\,2\,2\,1\,1\,2\,2\,2\,1\,1\,2\,1\,1\,0\,0\,2\,2\,1\,2\,0\,1\,2\,0\,0\,0\,2\,1\,1\,1\,1\,2\,0\,2\,0\,0\,2\,0\,1\,2\,0 \\
0\,0\,2\,0\,0\,2\,2\,0\,1\,2\,1\,0\,0\,2\,0\,2\,2\,0\,0\,2\,1\,1\,1\,2\,2\,1\,1\,2\,0\,2\,1\,1\,2\,1\,1\,1\,0\,1\,2\,0\,2\,2\,2\,1 \\
0\,0\,0\,2\,2\,1\,0\,1\,1\,2\,1\,0\,1\,1\,0\,2\,0\,2\,2\,2\,0\,0\,2\,2\,1\,1\,2\,1\,1\,2\,0\,0\,0\,2\,0\,0\,2\,1\,0\,1\,0\,0\,0\,2 \\
0\,1\,1\,2\,2\,1\,0\,2\,2\,2\,1\,0\,0\,0\,0\,2\,2\,0\,1\,0\,0\,2\,0\,2\,2\,2\,1\,0\,1\,1\,2\,0\,2\,2\,0\,1\,1\,2\,1\,1\,2\,2\,2\,2 \\
1\,0\,2\,0\,0\,0\,1\,2\,2\,0\,2\,1\,1\,0\,2\,0\,1\,0\,1\,0\,1\,1\,0\,0\,0\,0\,1\,1\,2\,0\,1\,1\,2\,0\,2\,1\,1\,1\,0\,2\,1\,0\,0\,2 \\
1\,1\,0\,0\,1\,1\,0\,0\,0\,0\,2\,1\,2\,1\,1\,2\,0\,1\,0\,0\,1\,1\,2\,1\,1\,0\,2\,1\,0\,2\,2\,2\,0\,1\,0\,0\,0\,0\,0\,0\,1\,0\,1\,0 \\
1\,2\,0\,2\,2\,1\,0\,2\,2\,2\,2\,0\,1\,0\,2\,0\,0\,1\,1\,0\,0\,0\,2\,0\,2\,2\,2\,2\,2\,1\,0\,2\,0\,0\,1\,0\,1\,1\,1\,0\,1\,2\,2\,1 \\
2\,2\,2\,1\,1\,0\,0\,1\,1\,1\,0\,1\,1\,2\,1\,1\,0\,2\,2\,0\,0\,0\,1\,1\,1\,0\,1\,1\,1\,0\,1\,1\,0\,1\,0\,0\,2\,1\,2\,2\,1\,2\,2\,1 \\
0\,0\,0\,2\,2\,2\,0\,2\,1\,1\,0\,1\,1\,0\,2\,2\,2\,2\,0\,1\,2\,0\,1\,1\,2\,0\,0\,0\,2\,0\,2\,2\,0\,2\,0\,0\,1\,0\,1\,0\,1\,2\,0\,2 \\
2\,0\,0\,2\,1\,1\,0\,0\,2\,2\,1\,0\,0\,2\,1\,0\,2\,2\,1\,1\,2\,2\,2\,2\,1\,0\,2\,1\,0\,1\,0\,2\,2\,2\,0\,1\,0\,1\,2\,0\,1\,0\,0\,2 \\
0\,0\,0\,2\,1\,0\,2\,2\,2\,1\,0\,0\,2\,2\,0\,0\,0\,2\,2\,2\,0\,2\,1\,2\,2\,0\,2\,0\,2\,0\,1\,1\,2\,1\,1\,1\,0\,0\,0\,1\,2\,2\,2\,1 \\
0\,1\,1\,1\,0\,0\,2\,2\,1\,1\,1\,0\,1\,2\,0\,2\,1\,2\,2\,0\,2\,0\,2\,1\,1\,1\,0\,0\,2\,2\,2\,2\,2\,0\,2\,1\,2\,2\,2\,2\,2\,2\,2\,2 \\
0\,2\,2\,1\,1\,1\,2\,0\,2\,2\,1\,2\,2\,0\,0\,2\,2\,2\,0\,1\,0\,0\,0\,2\,2\,2\,1\,0\,2\,2\,1\,1\,2\,2\,1\,1\,1\,2\,0\,2\,2\,2\,2\,1 \\
0\,1\,1\,0\,0\,0\,1\,2\,2\,1\,1\,0\,2\,0\,0\,2\,0\,1\,0\,0\,1\,0\,2\,0\,2\,1\,2\,1\,1\,0\,0\,1\,0\,2\,2\,0\,1\,1\,0\,0\,0\,2\,2\,2 \\
0\,2\,0\,1\,1\,1\,0\,0\,1\,2\,1\,2\,1\,2\,0\,2\,1\,2\,1\,2\,2\,2\,0\,1\,1\,1\,2\,2\,0\,1\,2\,2\,1\,2\,2\,2\,0\,0\,0\,2\,1\,2\,0\,2 \\
0\,0\,0\,1\,1\,2\,2\,2\,2\,1\,1\,0\,0\,1\,0\,2\,1\,0\,1\,2\,2\,2\,0\,0\,2\,1\,1\,1\,1\,0\,2\,0\,1\,1\,1\,2\,1\,2\,1\,0\,2\,2\,1\,1 \\
0\,2\,2\,2\,2\,1\,2\,2\,0\,0\,1\,0\,0\,2\,0\,2\,2\,2\,2\,1\,0\,1\,2\,2\,1\,0\,0\,2\,2\,1\,2\,1\,2\,1\,1\,1\,0\,0\,2\,2\,2\,2\,2\,1
\end{pmatrix}$$

$$
a_{44,5} = \begin{pmatrix}
0 1 2 2 2 1 2 2 2 1 2 0 0 2 1 0 1 0 0 2 1 1 0 1 0 2 2 0 0 1 2 2 2 2 1 0 1 0 0 1 0 2 1 0 \\
0 2 0 1 1 0 0 1 1 0 1 2 1 1 0 1 1 1 1 1 0 2 2 0 0 2 2 2 0 0 2 2 2 1 2 0 1 0 2 2 2 1 1 0 \\
1 1 2 0 2 0 0 1 1 2 1 2 1 0 0 1 1 1 0 0 2 0 0 2 2 0 1 0 1 1 1 2 0 0 0 1 2 1 1 0 1 2 0 2 \\
0 2 2 2 1 1 1 0 0 1 0 2 2 2 0 2 1 0 2 2 0 0 2 2 2 2 2 2 2 2 1 2 2 0 2 0 0 2 0 0 2 1 1 1 \\
2 2 0 0 2 1 0 2 0 2 1 2 0 2 0 2 0 0 0 1 1 1 0 2 0 1 1 1 0 0 0 0 2 1 2 2 0 2 2 2 2 0 2 0 \\
1 0 1 2 2 0 1 1 2 0 2 1 2 2 0 2 1 0 1 2 2 1 2 1 0 1 1 0 2 2 1 1 0 0 1 0 2 0 1 0 2 2 2 2 \\
0 1 1 1 2 0 2 2 2 1 2 2 2 1 2 1 1 1 1 2 1 0 0 1 1 1 0 1 2 1 1 0 2 0 1 0 2 1 2 0 2 0 1 1 \\
0 0 2 1 1 0 2 2 1 0 0 2 0 1 1 0 2 0 1 1 1 1 1 0 0 2 2 1 2 0 1 0 2 0 1 2 0 1 1 0 0 0 0 2 \\
0 2 2 2 1 0 1 0 0 1 2 2 0 0 0 2 0 0 1 1 1 2 0 1 2 0 0 1 1 1 0 2 2 2 1 0 0 2 0 1 1 2 2 0 \\
0 2 0 2 1 0 1 1 1 0 2 1 0 0 0 2 1 2 1 1 2 2 1 1 0 2 1 0 1 1 2 0 2 2 1 0 1 2 2 1 2 2 0 0 \\
2 2 2 0 2 1 1 2 1 1 1 0 2 0 0 0 0 0 2 1 1 1 1 1 2 0 2 2 1 2 1 2 1 0 1 2 1 2 2 0 0 0 2 1 \\
1 0 2 1 2 0 2 0 2 2 0 0 2 0 1 0 2 2 0 2 2 0 1 2 2 1 1 0 0 2 2 2 2 0 1 1 1 0 1 0 2 0 0 0 \\
0 1 0 2 1 0 2 2 1 2 0 1 1 2 1 0 1 1 2 1 1 0 0 1 0 2 2 1 2 0 1 2 1 1 0 0 0 2 2 2 2 2 2 1 \\
1 2 2 1 0 1 2 0 1 1 1 0 2 1 0 0 1 2 0 0 0 1 2 0 0 2 1 0 0 0 2 1 0 2 0 0 2 0 1 2 1 1 1 1 \\
1 1 0 2 1 1 2 2 1 2 0 1 0 0 2 0 0 2 0 2 2 1 1 1 1 0 0 0 1 0 0 2 2 1 0 1 1 2 0 1 0 2 1 0 \\
2 0 2 0 1 0 0 0 1 2 1 2 0 2 0 0 2 0 0 2 2 1 1 0 1 2 0 2 2 2 1 1 0 0 1 0 2 1 1 2 1 1 2 2 \\
2 2 2 1 1 1 2 0 0 0 0 2 2 2 0 1 1 0 1 2 2 1 0 0 0 2 0 1 1 0 1 0 1 0 0 1 1 1 2 2 1 0 2 2 \\
2 2 0 2 0 1 2 2 2 0 2 2 2 2 0 2 2 2 0 1 1 1 1 0 1 0 2 2 1 1 2 0 2 1 0 0 0 1 2 1 2 1 0 1 \\
1 1 2 1 2 1 0 1 1 0 0 1 0 2 0 0 0 2 1 2 0 2 1 0 0 0 1 2 0 0 0 0 0 1 2 2 0 1 0 0 2 1 1 1 \\
1 2 1 2 2 1 1 1 0 2 0 0 2 0 0 2 1 0 1 0 0 0 0 2 2 1 2 1 0 2 1 1 1 1 2 0 2 0 0 2 0 1 2 0 \\
0 1 0 0 0 1 2 0 1 1 1 1 0 1 0 0 2 0 0 1 2 0 1 2 2 0 2 2 2 1 2 1 1 2 1 0 2 1 1 0 2 0 2 0 \\
1 1 1 0 1 1 2 0 1 0 1 0 1 2 0 2 2 1 0 2 0 2 2 2 0 1 0 2 1 1 2 2 0 2 1 2 1 0 0 0 2 1 1 0 \\
1 1 1 1 1 2 1 2 2 0 0 1 0 0 2 0 2 1 1 1 2 1 0 1 1 0 0 2 1 0 2 0 0 0 0 2 0 0 0 1 0 0 2 2 \\
0 2 2 0 2 0 2 1 1 0 1 2 1 2 2 2 0 2 1 1 1 0 0 0 0 1 2 2 2 0 0 0 2 2 2 2 1 0 1 1 2 1 0 1 \\
2 1 0 2 1 1 2 0 2 1 2 1 1 0 0 1 0 1 2 1 0 2 0 2 2 2 1 1 1 1 2 2 0 2 1 2 1 0 0 0 2 1 1 0 \\
0 0 2 2 2 2 1 0 0 1 2 2 1 0 2 1 0 1 0 2 0 2 0 2 1 1 1 1 0 1 1 0 0 1 1 1 2 1 0 0 1 1 1 2 \\
1 2 1 1 0 2 1 1 1 1 2 0 2 0 2 1 0 2 1 1 1 0 2 0 2 0 0 1 1 1 1 1 1 0 0 1 2 0 0 0 0 1 0 1 \\
1 1 1 1 1 2 2 1 1 2 2 2 1 1 2 1 1 0 0 2 2 1 2 0 1 2 0 0 0 1 2 2 2 2 1 0 1 0 0 1 0 2 1 0 \\
0 0 1 0 0 1 1 0 2 1 2 0 0 1 0 1 1 0 0 1 2 2 2 1 1 2 2 1 0 2 1 1 2 1 1 1 0 1 2 0 2 2 2 1 \\
0 0 0 1 1 2 0 2 2 1 2 0 2 2 0 1 0 1 1 1 0 0 1 1 2 2 1 2 1 2 0 0 0 2 0 0 2 1 0 1 0 0 0 2 \\
0 2 2 1 1 2 0 1 1 1 2 0 0 0 0 1 1 0 2 0 0 1 0 1 1 1 2 0 1 1 2 0 2 2 0 1 1 2 1 1 2 2 2 2 \\
2 0 1 0 0 0 2 1 1 0 1 2 2 0 1 0 2 0 2 0 2 2 0 0 0 0 2 2 2 0 1 1 2 0 2 1 1 1 0 2 1 0 0 2 \\
2 2 0 0 2 2 0 0 0 0 1 2 1 2 2 1 0 2 0 0 2 2 1 2 2 0 1 2 0 2 2 2 0 1 0 0 0 0 0 0 1 0 1 0 \\
2 1 0 1 1 2 0 1 1 1 1 0 2 0 1 0 0 2 2 0 0 0 1 0 1 1 1 1 2 1 0 2 0 0 1 0 1 1 1 0 1 2 2 1 \\
1 1 1 2 2 0 0 2 2 2 0 2 2 1 2 2 0 1 1 0 0 0 2 2 2 0 2 2 1 0 1 1 0 1 0 0 2 1 2 2 1 2 2 1 \\
0 0 0 1 1 1 0 1 2 2 0 2 2 0 1 1 1 1 0 2 1 0 2 2 1 0 0 0 2 0 2 2 0 2 0 0 1 0 1 0 1 2 0 2 \\
1 0 0 1 2 2 0 0 1 1 2 0 0 1 2 0 1 1 2 2 1 1 1 1 2 0 1 2 0 1 0 2 2 2 0 1 0 1 2 0 1 0 0 2 \\
0 0 0 1 2 0 1 1 1 2 0 0 1 1 0 0 0 1 1 1 0 1 2 1 1 0 1 0 2 0 1 1 2 1 1 1 0 0 0 1 2 2 2 1 \\
0 2 2 2 0 0 1 1 2 2 2 0 2 1 0 1 2 1 1 0 1 0 1 2 2 2 0 0 2 2 2 2 2 0 2 1 2 2 2 2 2 2 2 2 \\
0 1 1 2 2 2 1 0 1 1 2 1 1 0 0 1 1 1 0 2 0 0 0 1 1 1 2 0 2 2 1 1 2 2 1 1 1 2 0 2 2 2 2 1 \\
0 2 2 0 0 0 2 1 1 2 2 0 1 0 0 1 0 2 0 0 2 0 1 0 1 2 1 2 1 0 0 1 0 2 2 0 1 1 0 0 0 2 2 2 \\
0 1 0 2 2 2 0 0 2 1 2 1 2 1 0 1 2 1 2 1 1 1 0 2 2 2 1 1 0 1 2 2 1 2 2 2 0 0 0 2 1 2 0 2 \\
0 0 0 2 2 1 1 1 1 2 2 0 0 2 0 1 2 0 2 1 1 1 0 0 1 2 2 2 1 0 2 0 1 1 1 2 1 2 1 0 2 2 1 1 \\
0 1 1 1 1 2 1 1 0 0 2 0 0 1 0 1 1 1 1 2 0 2 1 1 2 0 0 1 2 1 2 1 2 1 1 1 0 0 2 2 2 2 2 1
\end{pmatrix}
$$

$$a_{44,6} = \begin{pmatrix}
0&1&2&2&2&1&2&2&2&1&2&0&0&2&1&0&1&0&0&2&1&1&0&1&0&2&2&0&0&2&1&1&1&1&2&0&2&0&0&2&0&1&2&0\\
0&2&0&1&1&0&0&1&1&0&1&2&1&1&0&1&1&1&1&1&0&2&2&0&0&2&2&2&0&0&1&1&1&2&1&0&2&0&1&1&1&2&2&1\\
1&1&2&0&2&0&0&1&1&2&1&2&1&0&0&1&1&1&0&0&2&0&0&2&2&0&1&0&2&2&2&1&0&0&0&2&1&2&2&0&2&1&0&1\\
0&2&2&2&1&1&1&0&0&1&0&2&2&2&0&2&1&0&2&2&0&0&2&2&2&2&2&2&1&1&2&1&1&0&1&0&0&1&0&0&1&2&2&1\\
2&2&0&0&2&1&0&2&0&2&1&2&0&2&0&2&0&0&0&1&1&1&0&2&0&1&1&1&0&0&0&0&1&2&1&1&0&1&1&1&1&0&1&2\\
1&0&1&2&2&0&1&1&2&0&2&1&2&2&0&2&1&0&1&2&2&1&2&1&0&1&1&0&1&1&2&2&0&0&2&0&1&0&2&0&1&1&1&1\\
0&1&1&1&2&0&2&2&2&1&2&2&2&1&2&1&1&1&1&2&1&0&0&1&1&1&0&1&1&2&2&0&1&0&2&0&1&2&1&0&1&0&2&0\\
0&0&2&1&1&0&2&2&1&0&0&2&0&1&1&0&2&0&1&1&1&1&1&0&0&2&2&1&1&0&2&0&1&0&2&1&0&2&2&0&0&0&0&0\\
0&2&2&2&1&0&1&0&0&1&2&2&0&0&0&2&0&0&1&1&1&2&0&1&2&0&0&1&2&2&0&1&1&1&2&0&0&1&0&2&2&1&1&1\\
0&2&0&2&1&0&1&1&1&0&2&1&0&0&0&2&1&2&1&1&2&2&1&1&0&2&1&0&2&2&1&0&1&1&2&0&2&1&1&2&1&1&0&2\\
2&2&2&0&2&1&1&2&1&1&1&0&2&0&0&0&0&0&2&1&1&1&1&1&2&0&2&2&2&1&2&1&2&0&2&1&2&1&1&0&0&0&1&0\\
1&0&2&1&2&0&2&0&2&2&0&0&2&0&1&0&2&2&0&2&2&0&1&2&2&1&1&0&0&1&1&1&1&0&2&2&2&0&2&0&1&0&0&0\\
0&1&0&2&1&0&2&2&1&2&0&1&1&2&1&0&1&1&2&1&1&0&0&1&0&2&2&1&1&0&2&1&2&2&0&0&0&1&1&1&1&1&1&0\\
1&2&2&1&0&1&2&0&1&1&1&0&2&1&0&0&1&2&0&0&0&1&2&0&0&2&1&0&0&0&1&2&0&1&0&0&1&0&2&1&2&2&2&2\\
1&1&0&2&1&1&2&2&1&2&0&1&0&0&2&0&0&2&0&2&2&1&1&1&1&0&0&0&2&0&0&1&1&2&0&2&2&1&0&2&0&1&2&1\\
2&0&2&0&1&0&0&0&1&2&1&2&0&2&0&0&2&0&0&2&2&1&1&0&1&2&0&2&1&1&2&2&0&0&2&0&1&2&2&1&2&2&1&2\\
2&2&2&1&1&1&2&0&0&0&0&2&2&2&0&1&1&0&1&2&2&1&0&0&0&2&0&1&2&0&2&0&2&0&0&2&2&2&1&1&2&0&1&1\\
2&2&0&2&0&1&2&2&2&0&2&2&2&2&0&2&2&2&0&1&1&1&1&0&1&0&2&2&2&2&1&0&1&2&0&0&0&2&1&2&1&2&0&2\\
1&1&2&1&2&1&0&1&1&0&0&1&0&2&0&0&0&2&1&2&0&2&1&0&0&0&1&2&0&0&0&0&0&2&1&1&0&2&0&0&1&2&2&0\\
1&2&1&2&2&1&1&1&0&2&0&0&2&0&0&2&1&0&1&0&0&0&0&2&2&1&2&1&0&1&2&2&2&2&1&0&1&0&0&1&0&2&1&0\\
0&1&0&0&0&1&2&0&1&1&1&1&0&1&0&0&2&0&0&1&2&0&1&2&2&0&2&2&1&2&1&2&2&1&2&0&1&2&2&0&1&0&1&1\\
1&1&1&0&1&1&2&0&1&0&1&0&1&2&0&2&2&1&0&2&0&2&2&2&0&1&0&2&2&2&1&1&0&1&2&1&2&0&0&0&1&2&2&2\\
1&1&1&1&1&2&1&2&2&0&0&1&0&0&2&0&2&1&1&1&2&1&0&1&1&0&0&2&2&0&1&0&0&0&0&1&0&0&0&2&0&0&1&1\\
0&2&2&0&2&0&2&1&1&0&1&2&1&2&2&2&0&2&1&1&1&0&0&0&0&1&2&2&1&0&0&0&1&1&1&1&2&0&2&2&1&2&0&0\\
2&1&0&2&1&1&2&0&2&1&2&1&1&0&0&1&0&1&2&1&0&2&0&2&2&2&1&1&2&2&1&1&0&1&2&1&2&0&0&0&1&2&2&2\\
0&0&2&2&2&2&1&0&0&1&2&2&1&0&2&1&0&1&0&2&0&2&0&2&1&1&1&1&0&2&2&0&0&2&2&2&1&2&0&0&2&2&2&0\\
1&2&1&1&0&2&1&1&1&1&2&0&2&0&2&1&0&2&1&1&1&0&2&0&2&0&0&1&2&2&2&2&2&0&0&2&1&0&0&0&0&2&0&2\\
1&1&1&1&1&2&2&1&1&2&2&2&1&1&2&1&1&0&0&2&2&1&2&0&1&2&0&0&0&2&1&1&1&1&2&0&2&0&0&2&0&1&2&0\\
0&2&1&2&2&0&1&2&1&2&2&0&0&1&0&1&1&2&2&0&1&2&0&1&0&1&1&1&2&0&0&2&1&2&2&2&0&1&1&2&1&1&1&1\\
0&2&2&1&1&2&2&0&1&2&2&0&1&0&0&1&2&1&1&0&0&1&1&1&2&1&2&0&0&0&2&1&2&0&1&1&2&1&2&0&2&2&2&1\\
0&0&0&1&1&2&2&1&2&2&2&0&0&2&0&1&1&2&0&1&0&0&2&1&0&2&1&2&0&2&1&1&1&0&1&2&1&2&0&0&1&1&1&1\\
1&2&1&2&2&1&0&1&2&0&0&1&1&2&2&2&0&2&0&1&1&2&2&2&1&0&1&0&1&1&0&2&1&1&0&2&1&1&2&1&0&2&2&0\\
1&1&0&0&1&1&0&0&0&0&2&1&2&1&1&2&0&1&0&0&1&1&2&1&1&0&2&1&0&2&2&2&0&1&0&0&0&0&0&0&1&0&1&0\\
1&1&2&1&1&2&2&1&2&2&0&0&1&2&2&2&2&0&0&1&0&1&1&2&0&2&2&1&1&2&2&0&2&1&2&1&1&1&0&2&0&1&1&2\\
2&1&1&0&0&1&2&0&1&1&1&1&1&1&1&0&2&1&1&1&0&1&0&0&2&0&1&0&0&1&0&2&2&2&1&1&2&1&1&1&0&1&1&2\\
0&2&2&1&1&0&2&1&1&1&1&1&1&2&2&1&1&1&2&2&2&1&0&0&0&0&0&2&1&1&1&0&2&0&1&1&1&0&0&2&0&1&2&0\\
2&2&2&1&0&2&2&2&2&2&2&0&0&1&1&2&1&1&0&2&2&0&1&1&2&0&2&0&2&2&2&0&1&0&1&2&0&1&1&2&0&2&2&0\\
0&2&2&1&0&1&1&1&2&1&1&0&2&1&0&2&2&1&1&0&0&0&0&1&0&0&2&2&1&1&0&2&1&2&2&2&0&0&2&0&1&1&1&1\\
0&0&0&0&2&1&1&1&1&1&2&0&1&1&0&1&0&1&1&1&2&1&1&0&2&1&0&2&1&0&1&0&1&1&0&2&2&2&1&1&1&1&1&1\\
0&2&2&1&1&1&2&0&2&2&1&2&2&0&0&2&2&2&0&1&0&0&0&2&2&2&1&0&2&2&1&1&2&2&1&1&1&2&0&2&2&2&2&0\\
0&0&0&2&2&1&0&1&2&1&2&0&2&2&0&1&2&0&2&1&1&1&1&2&0&1&2&0&0&1&2&2&2&0&0&1&1&1&2&2&2&1&1&2\\
0&1&2&0&0&2&2&2&1&2&2&2&1&1&0&1&0&1&0&0&2&0&2&0&2&1&2&1&2&2&1&0&0&0&0&0&0&0&2&1&0&1&2&1\\
0&2&2&0&0&0&1&1&2&1&2&0&0&0&0&1&0&2&0&0&2&0&2&2&0&1&1&0&0&1&1&1&0&2&2&0&1&2&0&2&1&1&0&1\\
0&1&1&1&1&2&1&1&0&0&2&0&0&1&0&1&1&1&1&2&0&2&1&1&2&0&0&1&1&2&1&2&1&2&2&2&0&0&1&1&1&1&1&2
\end{pmatrix}$$

$$a_{44,7} = \left(\begin{array}{*{44}{c}}
0&2&1&1&1&2&1&1&1&2&1&0&0&1&2&0&2&0&0&1&2&2&0&2&0&1&1&0&0&2&1&1&1&1&2&0&2&0&0&2&0&1&2&0\\
0&2&0&1&1&0&0&1&1&0&1&2&1&1&0&1&1&1&1&1&0&2&2&0&0&2&2&2&0&0&2&2&2&1&2&0&1&0&2&2&2&1&1&2\\
2&1&2&0&2&0&0&1&1&2&1&2&1&0&0&1&1&1&0&0&2&0&0&2&2&0&1&0&1&1&1&2&0&0&0&1&2&1&1&0&1&2&0&2\\
0&2&2&2&1&1&1&0&0&1&0&2&2&2&0&2&1&0&2&2&0&0&2&2&2&2&2&2&2&2&1&2&2&0&2&0&0&2&0&0&2&1&1&2\\
1&2&0&0&2&1&0&2&0&2&1&2&0&2&0&2&0&0&0&1&1&1&0&2&0&1&1&1&0&0&0&0&2&1&2&2&0&2&2&2&2&0&2&1\\
2&0&1&2&2&0&1&1&2&0&2&1&2&2&0&2&1&0&1&2&2&1&2&1&0&1&1&0&2&2&1&1&0&0&1&0&2&0&1&0&2&2&2&2\\
0&1&1&1&2&0&2&2&2&1&2&2&2&1&2&1&1&1&1&2&1&0&0&1&1&1&0&1&2&1&1&0&2&0&1&0&2&1&2&0&2&0&1&0\\
0&0&2&1&1&0&2&2&1&0&0&2&0&1&1&0&2&0&1&1&1&1&1&0&0&2&2&1&2&0&1&0&2&0&1&2&0&1&1&0&0&0&0&0\\
0&2&2&2&1&0&1&0&0&1&2&2&0&0&0&2&0&0&1&1&1&2&0&1&2&0&0&1&1&1&0&2&2&2&1&0&0&2&0&1&1&2&2&2\\
0&2&0&2&1&0&1&1&1&0&2&1&0&0&0&2&1&2&1&1&2&2&1&1&0&2&1&0&1&1&2&0&2&2&1&0&1&2&2&1&2&2&0&1\\
1&2&2&0&2&1&1&2&1&1&1&0&2&0&0&0&0&0&2&1&1&1&1&1&2&0&2&2&1&2&1&2&1&0&1&2&1&2&2&0&0&0&2&0\\
2&0&2&1&2&0&2&0&2&2&0&0&2&0&1&0&2&2&0&2&2&0&1&2&2&1&1&0&0&2&2&2&2&0&1&1&1&0&1&0&2&0&0&0\\
0&1&0&2&1&0&2&2&1&2&0&1&1&2&1&0&1&1&2&1&1&0&0&1&0&2&2&1&2&0&1&2&1&1&0&0&0&2&2&2&2&2&2&0\\
2&2&2&1&0&1&2&0&1&1&1&0&2&1&0&0&1&2&0&0&0&1&2&0&0&2&1&0&0&0&2&1&0&2&0&0&2&0&1&2&1&1&1&1\\
2&1&0&2&1&1&2&2&1&2&0&1&0&0&2&0&0&2&0&2&2&1&1&1&1&0&0&0&1&0&0&2&2&1&0&1&1&2&0&1&0&2&1&2\\
1&0&2&0&1&0&0&0&1&2&1&2&0&2&0&0&2&0&0&2&2&1&1&0&1&2&0&2&2&2&1&1&0&0&1&0&2&1&1&2&1&1&2&1\\
1&2&2&1&1&1&2&0&0&0&0&2&2&2&0&1&1&0&1&2&2&1&0&0&0&2&0&1&1&0&1&0&1&0&0&1&1&1&2&2&1&0&2&2\\
1&2&0&2&0&1&2&2&2&0&2&2&2&2&0&2&2&2&0&1&1&1&1&0&1&0&2&2&1&1&2&0&2&1&0&0&0&1&2&1&2&1&0&1\\
2&1&2&1&2&1&0&1&1&0&0&1&0&2&0&0&0&2&1&2&0&2&1&0&0&0&1&2&0&0&0&0&0&1&2&2&0&1&0&0&2&1&1&0\\
2&2&1&2&2&1&1&1&0&2&0&0&2&0&0&2&1&0&1&0&0&0&0&2&2&1&2&1&0&2&1&1&1&1&2&0&2&0&0&2&0&1&2&0\\
0&1&0&0&0&1&2&0&1&1&1&1&0&1&0&0&2&0&0&1&2&0&1&2&2&0&2&2&2&1&2&1&1&2&1&0&2&1&1&0&2&0&2&2\\
2&1&1&0&1&1&2&0&1&0&1&0&1&2&0&2&2&1&0&2&0&2&2&2&0&1&0&2&1&1&2&2&0&2&1&2&1&0&0&0&2&1&1&1\\
2&1&1&1&1&2&1&2&2&0&0&1&0&0&2&0&2&1&1&1&2&1&0&1&1&0&0&2&1&0&2&0&0&0&0&2&0&0&0&1&0&0&2&2\\
0&2&2&0&2&0&2&1&1&0&1&2&1&2&2&2&0&2&1&1&1&0&0&0&0&1&2&2&2&0&0&0&2&2&2&2&1&0&1&1&2&1&0&0\\
1&1&0&2&1&1&2&0&2&1&2&1&1&0&0&1&0&1&2&1&0&2&0&2&2&2&1&1&1&1&2&2&0&2&1&2&1&0&0&0&2&1&1&1\\
0&0&2&2&2&2&1&0&0&1&2&2&1&0&2&1&0&1&0&2&0&2&0&2&1&1&1&1&0&1&1&0&0&1&1&1&2&1&0&0&1&1&1&0\\
2&2&1&1&0&2&1&1&1&1&2&0&2&0&2&1&0&2&1&1&1&0&2&0&2&0&0&1&1&1&1&1&1&0&0&1&2&0&0&0&0&1&0&1\\
2&1&1&1&1&2&2&1&1&2&2&2&1&1&2&1&1&0&0&2&2&1&2&0&1&2&0&0&0&1&2&2&2&2&1&0&1&0&0&1&0&2&1&0\\
0&1&2&1&1&0&2&1&2&1&1&0&0&2&0&2&2&1&1&0&2&1&0&2&0&2&2&2&2&0&0&2&1&2&2&2&0&1&1&2&1&1&1&1\\
0&1&1&2&2&1&1&0&2&1&1&0&2&0&0&2&1&2&2&0&0&2&2&2&1&2&1&0&0&0&2&1&2&0&1&1&2&1&2&0&2&2&2&1\\
0&0&0&2&2&1&1&2&1&1&1&0&0&1&0&2&2&1&0&2&0&0&1&2&0&1&2&1&0&2&1&1&1&0&1&2&1&2&0&0&1&1&1&1\\
1&1&2&1&1&2&0&2&1&0&0&2&2&1&1&1&0&1&0&2&2&1&1&1&2&0&2&0&1&1&0&2&1&1&0&2&1&1&2&1&0&2&2&0\\
1&2&0&0&2&2&0&0&0&0&1&2&1&2&2&1&0&2&0&0&2&2&1&2&2&0&1&2&0&2&2&2&0&1&0&0&0&0&0&0&1&0&1&0\\
1&2&1&2&2&1&1&2&1&1&0&0&2&1&1&1&1&0&0&2&0&2&2&1&0&1&1&2&1&2&2&0&2&1&2&1&1&1&0&2&0&1&1&2\\
2&2&2&0&0&2&1&0&2&2&2&2&2&2&2&0&1&2&2&2&0&2&0&0&1&0&2&0&0&1&0&2&2&2&1&1&2&1&1&1&0&1&1&2\\
0&1&1&2&2&0&1&2&2&2&2&2&2&1&1&2&2&2&1&1&1&2&0&0&0&0&0&1&1&1&1&0&2&0&1&1&1&0&0&2&0&1&2&0\\
2&1&1&2&0&1&1&1&1&1&1&0&0&2&2&1&2&2&0&1&1&0&2&2&1&0&1&0&2&2&2&0&1&0&1&2&0&1&1&2&0&2&2&0\\
0&1&1&2&0&2&2&2&1&2&2&0&1&2&0&1&1&2&2&0&0&0&0&2&0&0&1&1&1&1&0&2&1&2&2&2&0&0&2&0&1&1&1&1\\
0&0&0&0&1&2&2&2&2&2&1&0&2&2&0&2&0&2&2&2&1&2&2&0&1&2&0&1&1&0&1&0&1&1&0&2&2&2&1&1&1&1&1&1\\
0&1&1&2&2&2&1&0&1&1&2&1&1&0&0&1&1&1&0&2&0&0&0&1&1&1&2&0&2&2&1&1&2&2&1&1&1&2&0&2&2&2&2&0\\
0&0&0&1&1&2&0&2&1&2&1&0&1&1&0&2&1&0&1&2&2&2&2&1&0&2&1&0&0&1&2&2&2&0&0&1&1&1&2&2&2&1&1&2\\
0&2&1&0&0&1&1&1&2&1&1&1&2&2&0&2&0&2&0&0&1&0&1&0&1&2&1&2&2&2&1&0&0&0&0&0&0&0&2&1&0&1&2&1\\
0&1&1&0&0&0&2&2&1&2&1&0&0&0&0&2&0&1&0&0&1&0&1&1&0&2&2&0&0&1&1&1&0&2&2&0&1&2&0&2&1&1&0&1\\
0&2&2&2&2&1&2&2&0&0&1&0&0&2&0&2&2&2&2&1&0&1&2&2&1&0&0&2&1&2&1&2&1&2&2&2&0&0&1&1&1&1&1&2
\end{array}\right)$$

$$a_{44,8} = \begin{pmatrix}
0 2 1 1 1 2 1 1 1 2 1 0 0 1 2 0 2 0 0 1 2 2 0 2 0 1 1 0 0 1 2 2 2 2 1 0 1 0 0 1 0 2 1 0 \\
0 2 0 1 1 0 0 1 1 0 1 2 1 1 0 1 1 1 1 1 0 2 2 0 0 2 2 2 0 0 1 1 1 2 1 0 2 0 1 1 1 2 2 0 \\
2 1 2 0 2 0 0 1 1 2 1 2 1 0 0 1 1 1 0 0 2 0 0 2 2 0 1 0 2 2 2 1 0 0 0 2 1 2 2 0 2 1 0 1 \\
0 2 2 2 1 1 1 0 0 1 0 2 2 2 0 2 1 0 2 2 0 0 2 2 2 2 2 2 1 1 2 1 1 0 1 0 0 1 0 0 1 2 2 2 \\
1 2 0 0 2 1 0 2 0 2 1 2 0 2 0 2 0 0 0 1 1 1 0 2 0 1 1 1 0 0 0 0 1 2 1 1 0 1 1 1 1 0 1 0 \\
2 0 1 2 2 0 1 1 2 0 2 1 2 2 0 2 1 0 1 2 2 1 2 1 0 1 1 0 1 1 2 2 0 0 2 0 1 0 2 0 1 1 1 1 \\
0 1 1 1 2 0 2 2 2 1 2 2 2 1 2 1 1 1 1 2 1 0 0 1 1 1 0 1 1 2 2 0 1 0 2 0 1 2 1 0 1 0 2 2 \\
0 0 2 1 1 0 2 2 1 0 0 2 0 1 1 0 2 0 1 1 1 1 1 0 0 2 2 1 1 0 2 0 1 0 2 1 0 2 2 0 0 0 0 1 \\
0 2 2 2 1 0 1 0 0 1 2 2 0 0 0 2 0 0 1 1 1 2 0 1 2 0 0 1 2 2 0 1 1 1 2 0 0 1 0 2 2 1 1 0 \\
0 2 0 2 1 0 1 1 1 0 2 1 0 0 0 2 1 2 1 1 2 2 1 1 0 2 1 0 2 2 1 0 1 1 2 0 2 1 1 2 1 1 0 0 \\
1 2 2 0 2 1 1 2 1 1 1 0 2 0 0 0 0 0 2 1 1 1 1 1 2 0 2 2 2 1 2 1 2 0 2 1 2 1 1 0 0 0 1 2 \\
2 0 2 1 2 0 2 0 2 2 0 0 2 0 1 0 2 2 0 2 2 0 1 2 2 1 1 0 0 1 1 1 1 0 2 2 2 0 2 0 1 0 0 0 \\
0 1 0 2 1 0 2 2 1 2 0 1 1 2 1 0 1 1 2 1 1 0 0 1 0 2 2 1 1 0 2 1 2 2 0 0 0 1 1 1 1 1 1 2 \\
2 2 2 1 0 1 2 0 1 1 1 0 2 1 0 0 1 2 0 0 0 1 2 0 0 2 1 0 0 0 1 2 0 1 0 0 1 0 2 1 2 2 2 2 \\
2 1 0 2 1 1 2 2 1 2 0 1 0 0 2 0 0 2 0 2 2 1 1 1 1 0 0 0 2 0 0 1 1 2 0 2 2 1 0 2 0 1 2 0 \\
1 0 2 0 1 0 0 0 1 2 1 2 0 2 0 0 2 0 0 2 2 1 1 0 1 2 0 2 1 1 2 2 0 0 2 0 1 2 2 1 2 2 1 1 \\
1 2 2 1 1 1 2 0 0 0 0 2 2 2 0 1 1 0 1 2 2 1 0 0 0 2 0 1 2 0 2 0 2 0 0 2 2 2 1 1 2 0 1 1 \\
1 2 0 2 0 1 2 2 2 0 2 2 2 2 0 2 2 2 0 1 1 1 1 0 1 0 2 2 2 2 1 0 1 2 0 0 0 2 1 2 1 2 0 2 \\
2 1 2 1 2 1 0 1 1 0 0 1 0 2 0 0 0 2 1 2 0 2 1 0 0 0 1 2 0 0 0 0 0 2 1 1 0 2 0 0 1 2 2 2 \\
2 2 1 2 2 1 1 1 0 2 0 0 2 0 0 2 1 0 1 0 0 0 0 2 2 1 2 1 0 1 2 2 2 2 1 0 1 0 0 1 0 2 1 0 \\
0 1 0 0 0 1 2 0 1 1 1 1 0 1 0 0 2 0 0 1 2 0 1 2 2 0 2 2 1 2 1 2 2 1 2 0 1 2 2 0 1 0 1 0 \\
2 1 1 0 1 1 2 0 1 0 1 0 1 2 0 2 2 1 0 2 0 2 2 2 0 1 0 2 2 2 1 1 0 1 2 1 2 0 0 0 1 2 2 0 \\
2 1 1 1 1 2 1 2 2 0 0 1 0 0 2 0 2 1 1 1 2 1 0 1 1 0 0 2 2 0 1 0 0 0 0 1 0 0 0 2 0 0 1 1 \\
0 2 2 0 2 0 2 1 1 0 1 2 1 2 2 2 0 2 1 1 1 0 0 0 0 1 2 2 1 0 0 0 1 1 1 1 2 0 2 2 1 2 0 2 \\
1 1 0 2 1 1 2 0 2 1 2 1 1 0 0 1 0 1 2 1 0 2 0 2 2 2 1 1 2 2 1 1 0 1 2 1 2 0 0 0 1 2 2 0 \\
0 0 2 2 2 2 1 0 0 1 2 2 1 0 2 1 0 1 0 2 0 2 0 2 1 1 1 1 0 2 2 0 0 2 2 2 1 2 0 0 2 2 2 1 \\
2 2 1 1 0 2 1 1 1 1 2 0 2 0 2 1 0 2 1 1 1 0 2 0 2 0 0 1 2 2 2 2 2 0 0 2 1 0 0 0 0 2 0 2 \\
2 1 1 1 1 2 2 1 1 2 2 2 1 1 2 1 1 0 0 2 2 1 2 0 1 2 0 0 0 2 1 1 1 1 2 0 2 0 0 2 0 1 2 0 \\
0 0 2 0 0 2 2 0 1 2 1 0 0 2 0 2 2 0 0 2 1 1 1 2 2 1 1 2 0 2 1 1 2 1 1 1 0 1 2 0 2 2 2 1 \\
0 0 0 2 2 1 0 1 1 2 1 0 1 1 0 2 0 2 2 2 0 0 2 2 1 1 2 1 1 2 0 0 0 2 0 0 2 1 0 1 0 0 0 2 \\
0 1 1 2 2 1 0 2 2 2 1 0 0 0 0 2 2 0 1 0 0 2 0 2 2 2 1 0 1 1 2 0 2 2 0 1 1 2 1 1 2 2 2 2 \\
2 0 2 0 0 0 1 2 2 0 2 1 1 0 2 0 1 0 1 0 1 1 0 0 0 0 1 1 2 0 1 1 2 0 2 1 1 1 0 2 1 0 0 2 \\
2 1 0 0 1 1 0 0 0 0 2 1 2 1 1 2 0 1 0 0 1 1 2 1 1 0 2 1 0 2 2 2 0 1 0 0 0 0 0 0 1 0 1 0 \\
2 2 0 2 2 1 0 2 2 2 2 0 1 0 2 0 0 1 1 0 0 0 2 0 2 2 2 2 2 1 0 2 0 0 1 0 1 1 1 0 1 2 2 1 \\
1 2 2 1 1 0 0 1 1 1 0 1 1 2 1 1 0 2 2 0 0 0 1 1 1 0 1 1 1 0 1 1 0 1 0 0 2 1 2 2 1 2 2 1 \\
0 0 0 2 2 2 0 2 1 1 0 1 1 0 2 2 2 2 0 1 2 0 1 1 2 0 0 0 2 0 2 2 0 2 0 0 1 0 1 0 1 2 0 2 \\
1 0 0 2 1 1 0 0 2 2 1 0 0 2 1 0 2 2 1 1 2 2 2 2 1 0 2 1 0 1 0 2 2 2 0 1 0 1 2 0 1 0 0 2 \\
0 0 0 2 1 0 2 2 2 1 0 0 2 2 0 0 0 2 2 2 0 2 1 2 2 0 2 0 2 0 1 1 2 1 1 1 0 0 0 1 2 2 2 1 \\
0 1 1 1 0 0 2 2 1 1 1 0 1 2 0 2 1 2 2 0 2 0 2 1 1 1 0 0 2 2 2 2 2 0 2 1 2 2 2 2 2 2 2 2 \\
0 2 2 1 1 1 2 0 2 2 1 2 2 0 0 2 2 2 0 1 0 0 0 2 2 2 1 0 2 2 1 1 2 2 1 1 1 2 0 2 2 2 2 1 \\
0 1 1 0 0 0 1 2 2 1 1 0 2 0 0 2 0 1 0 0 1 0 2 0 2 1 2 1 1 0 0 1 0 2 2 0 1 1 0 0 0 2 2 2 \\
0 2 0 1 1 1 0 0 1 2 1 2 1 2 0 2 1 2 1 2 2 2 0 1 1 1 2 2 0 1 2 2 1 2 2 2 0 0 0 2 1 2 0 2 \\
0 0 0 1 1 2 2 2 2 1 1 0 0 1 0 2 1 0 1 2 2 2 0 0 2 1 1 1 1 0 2 0 1 1 1 2 1 2 1 0 2 2 1 1 \\
0 2 2 2 2 1 2 2 0 0 1 0 0 2 0 2 2 2 2 1 0 1 2 2 1 0 0 2 2 1 2 1 2 1 1 1 0 0 2 2 2 2 2 1
\end{pmatrix}$$

Appendix F

The matrix a

We display the matrix a as a MAGMA input. This matrix has been obtained by applying the algorithm of Chapter 4 to the matrix $a_{44,1}$ of Appendix E.

```
 a:=G![ 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 2, 0, 0, 2, 2,
0, 1, 1, 1, 1, 1, 0, 2, 2, 2, 2, 2, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 1, 0, 0, 2, 2, 0, 1, 0, 0, 2, 2, 0, 1, 0, 0, 2, 2, 0, 1, 0,
0, 2, 2, 0, 2, 2, 0, 2, 1, 2, 2, 2, 0, 2, 1, 2, 0, 1, 1, 2, 2, 0, 0, 1, 1, 2, 2, 0, 1, 2, 0, 1, 2, 2, 1, 2, 0, 1, 2, 2, 2, 1, 0,
0, 2, 1, 2, 1, 0, 0, 2, 1, 2, 2, 2, 1, 1, 0, 2, 2, 2, 1, 1, 0, 0, 1, 0, 2, 0, 1, 0, 1, 0, 2, 0, 1, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 2, 0, 0, 1, 0, 0, 1, 2, 0, 2, 2, 2, 2, 2, 0, 2, 2, 2, 2, 0, 0, 1, 0, 0, 1, 0,
0, 2, 0, 0, 2, 1, 2, 1, 0, 2, 1, 1, 2, 1, 0, 2, 1, 1, 2, 1, 0, 2, 1, 1, 2, 1, 0, 2, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0,
1, 1, 1, 1, 1, 0, 1, 1, 1, 1, 0, 2, 2, 0, 0, 0, 0, 2, 2, 0, 0, 0, 2, 2, 2, 2, 2, 0, 2, 2, 2, 2, 2, 0, 2, 0, 2, 2, 2, 2, 2, 0, 2,
2, 2, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 2, 1, 0, 2, 1, 1, 2, 0, 0, 2, 2,
0, 0, 2, 0, 0, 1, 0, 0, 2, 0, 0, 1, 0, 0, 2, 0, 0, 1, 0, 0, 2, 0, 0, 1, 1, 2, 0, 0, 2, 2, 1, 2, 1, 0, 2, 1, 2, 1, 2, 0, 1, 2, 1,
0, 2, 1, 1, 2, 2, 0, 1, 2, 2, 1, 2, 0, 1, 0, 1, 1, 1, 0, 2, 0, 2, 2, 0, 2, 0, 0, 1, 0, 0, 1, 0, 0, 2, 0, 0, 2, 0, 1, 1, 1, 0, 1,
0, 2, 2, 2, 1, 1, 2, 2, 2, 1, 2, 2, 1, 1, 1, 2, 1, 1, 1, 2, 1, 1, 2, 2, 2, 1, 2, 2, 2, 0, 1, 1, 0, 1, 1, 0, 2, 2, 0, 2, 0, 0, 1,
0, 1, 0, 0, 2, 2, 0, 1, 2, 1, 1, 1, 1, 1, 2, 1, 0, 2, 1, 0, 0, 2, 1, 2, 1, 2, 1, 0, 2, 2, 1, 2, 1, 1, 1, 0, 0, 1, 2, 0, 2, 0, 2,
1, 2, 1, 1, 1, 1, 2, 1, 1, 2, 2, 0, 1, 2, 1, 0, 2, 1, 2, 1, 2, 0, 1, 2, 1, 0, 1, 2, 0, 0, 2, 0, 2, 1, 0, 0, 0, 1, 0, 2, 0, 1, 0,
2, 0, 1, 0, 2, 1, 2, 0, 2, 2, 1, 2, 1, 0, 1, 1, 2, 2, 1, 0, 1, 1, 0, 1, 2, 0, 2, 2, 0, 1, 0, 2, 1, 1, 0, 2, 0, 1, 2, 2, 0, 0, 1,
0, 1, 2, 1, 0, 2, 0, 2, 1, 2, 2, 1, 2, 0, 2, 0, 1, 2, 1, 0, 1, 0, 0, 0, 2, 1, 0, 0, 0, 1, 1, 0, 0, 0, 0, 2, 1, 0, 0, 0, 0, 0, 0,
0, 0, 0, 1, 2, 1, 2, 1, 2, 2, 0, 2, 0, 0, 0, 0, 1, 2, 0, 0, 0, 0, 2, 1, 0, 2, 1, 1, 0, 1, 0, 0, 0, 2, 1, 2, 1, 1, 0, 2, 1, 1, 2,
2, 0, 1, 2, 2, 1, 2, 1, 0, 2, 2, 1, 1, 2, 0, 1, 1, 2, 0, 1, 2, 1, 1, 1, 0, 2, 1, 2, 2, 2, 0, 2, 2, 2, 1, 0, 0, 1, 1, 1, 2, 0, 2,
1, 2, 0, 0, 1, 1, 2, 1, 0, 0, 2, 2, 1, 0, 1, 1, 2, 1, 2, 0, 2, 2, 1, 2, 2, 2, 1, 0, 0, 1, 1, 1, 2, 0, 0, 1, 1, 0, 0, 0, 2, 2, 2,
0, 0, 0, 1, 0, 0, 1, 2, 2, 1, 2, 2, 0, 2, 2, 1, 2, 0, 1, 2, 0, 0, 0, 1, 2, 0, 1, 2, 1, 2, 0, 1, 1, 2, 1, 0, 2, 1, 1, 2, 0, 0, 2,
2, 0, 0, 2, 2, 1, 1, 1, 2, 0, 2, 0, 2, 1, 2, 0, 1, 1, 2, 1, 1, 2, 1, 0, 1, 2, 2, 1, 2, 0, 2, 0, 2, 1, 1, 1, 0, 0, 1, 2, 2, 2, 0,
2, 2, 1, 1, 1, 0, 1, 1, 2, 2, 2, 0, 2, 1, 2, 1, 0, 2, 1, 2, 1, 2, 0, 1, 2, 0, 1, 1, 0, 1, 1, 0, 2, 2, 0, 2, 2, 2, 0, 1, 1, 0, 1,
1, 0, 2, 2, 0, 0, 1, 0, 1, 2, 2, 0, 2, 0, 2, 1, 1, 0, 0, 1, 2, 0, 2, 0, 0, 2, 1, 0, 1, 0, 0, 1, 0, 0, 0, 2, 2, 2, 1, 0, 0, 0, 1,
2, 0, 2, 1, 2, 2, 2, 2, 1, 2, 0, 0, 2, 2, 1, 2, 1, 1, 1, 1, 2, 1, 0, 1, 0, 1, 2, 1, 2, 1, 0, 2, 2, 1, 2, 0, 1, 2, 1, 2, 2, 2, 0,
0, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 2, 1, 0, 0, 2, 0, 1, 1, 0, 0, 1, 0, 2, 2, 2, 0, 0, 1, 0, 0, 1, 0, 0, 2, 0, 0, 1, 2, 1, 0, 2, 1,
2, 1, 2, 0, 1, 2, 1, 2, 0, 0, 0, 1, 2, 1, 0, 0, 0, 2, 2, 0, 2, 1, 2, 1, 1, 0, 1, 2, 1, 2, 0, 2, 2, 2, 0, 2, 0, 1, 1, 1, 0, 1, 2,
2, 1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 0, 0, 0, 2, 0, 0, 2, 0, 0, 1, 1, 2, 1, 1, 1, 1, 2, 1, 2, 1, 0, 2, 2, 1, 1, 1, 0, 0, 2, 1, 2, 0,
1, 2, 0, 0, 0, 1, 2, 0, 1, 2, 2, 0, 2, 0, 2, 1, 1, 2, 2, 0, 2, 1, 1, 2, 2, 0, 0, 0, 1, 1, 0, 2, 0, 0, 2, 2, 0, 1, 2, 2, 0, 1, 0,
0, 1, 1, 0, 2, 0, 0, 2, 0, 1, 2, 1, 0, 1, 0, 2, 1, 2, 0, 0, 1, 1, 2, 0, 0, 0, 2, 2, 1, 0, 0, 0, 0, 1, 1, 1, 2, 0, 0, 2, 2, 2, 1,
1, 1, 0, 2, 2, 0, 2, 2, 0, 1, 1, 0, 0, 0, 1, 0, 0, 2, 0, 0, 2, 0, 0, 1, 1, 1, 2, 0, 1, 1, 2, 2, 1, 0, 2, 2, 0, 0, 2, 2, 0, 0, 1,
0, 0, 2, 1, 2, 1, 1, 1, 1, 1, 2, 1, 2, 0, 1, 1, 2, 2, 1, 1, 0, 1, 2, 2, 1, 1, 0, 2, 1, 1, 0, 1, 0, 0, 0, 0, 2, 1, 0, 1, 2, 1, 0,
2, 1, 1, 2, 2, 2, 0, 0, 0, 0, 2, 0, 1, 1, 0, 0, 1, 0, 2, 2, 0, 2, 2, 2, 1, 2, 0, 1, 1, 1, 2, 1, 0, 1, 0, 0, 1, 2, 0, 2, 0, 0, 2,
1, 0, 0, 1, 0, 0, 2, 0, 0, 2, 0, 0, 1, 0, 1, 1, 2, 2, 1, 0, 2, 2, 1, 1, 2, 0, 0, 1, 2, 2, 2, 0, 0, 2, 1, 1, 1, 1, 1, 1, 1, 2, 0,
2, 2, 2, 2, 1, 0, 1, 0, 0, 2, 1, 2, 2, 0, 0, 1, 2, 1, 0, 0, 2, 0, 0, 0, 1, 2, 2, 2, 0, 0, 0, 1, 2, 0, 1, 2, 1, 1, 1, 1, 2, 1, 1,
1, 0, 0, 1, 2, 1, 0, 2, 1, 1, 2, 1, 2, 0, 1, 1, 2, 0, 2, 0, 2, 2, 1, 2, 2, 2, 2, 2, 1, 0, 0, 1, 1, 0, 2, 1, 0, 0, 0, 0, 1, 2, 0,
0, 0, 0, 0, 2, 2, 0, 1, 0, 0, 1, 1, 0, 2, 0, 2, 1, 1, 1, 1, 0, 1, 2, 2, 2, 2, 2, 2, 0, 2, 0, 0, 1, 1, 0, 1, 0, 0, 1, 0, 2, 1, 2,
1, 2, 0, 1, 2, 1, 2, 1, 2, 1, 2, 2, 2, 2, 1, 2, 1, 1, 1, 1, 2, 1, 0, 2, 1, 2, 1, 2, 0, 1, 2, 1, 0, 0, 0, 2, 0, 2, 0, 0, 0, 1, 0,
0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 2, 2, 2, 2, 0, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 2, 0, 1, 0, 0, 1, 2, 0, 2, 0, 0, 2, 2, 0, 2,
0, 0, 2, 2, 0, 1, 0, 0, 1, 0, 0, 0, 0, 0, 1, 2, 0, 1, 1, 2, 1, 2, 0, 1, 1, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 2, 1, 0,
1, 2, 1, 2, 1, 0, 1, 2, 2, 1, 0, 1, 1, 2, 2, 1, 0, 1, 1, 2, 1, 0, 2, 2, 2, 0, 1, 0, 2, 2, 2, 0, 2, 0, 0, 1, 1, 1, 2, 0, 0, 1, 1,
1, 2, 1, 2, 0, 0, 1, 2, 1, 2, 0, 0, 1, 1, 2, 0, 1, 2, 1, 1, 2, 0, 1, 2, 1, 0, 2, 0, 0, 1, 0, 1, 1, 1, 1, 0, 0, 1, 0, 1, 0, 1, 0,
2, 1, 1, 2, 0, 2, 2, 0, 0, 2, 0, 2, 2, 2, 0, 0, 0, 1, 2, 2, 2, 0, 2, 2, 2, 1, 1, 1, 1, 1, 1, 1, 0, 0, 1, 1, 0, 1, 1, 0, 2, 2, 2,
1, 1, 1, 0, 2, 1, 1, 1, 0, 1, 2, 2, 0, 2, 0, 2, 1, 1, 2, 2, 0, 2, 0, 1, 0, 0, 1, 1, 1, 2, 0, 1, 1, 1, 1, 2, 0, 0, 0, 1, 2, 0, 1,
1, 2, 1, 1, 2, 0, 0, 2, 2, 1, 1, 2, 0, 1, 0, 1, 2, 1, 1, 2, 2, 1, 0, 1, 2, 0, 0, 1, 1, 1, 1, 2, 1, 2, 0, 1, 1, 2, 1, 0, 2, 0, 0,
2, 0, 1, 0, 0, 1, 2, 0, 2, 0, 0, 1, 0, 0, 2, 2, 0, 2, 0, 0, 0, 0, 0, 2, 0, 2, 2, 2, 2, 1, 2, 0, 1, 0, 0, 0, 1, 0, 1, 2, 2, 2, 1,
1, 2, 0, 0, 1, 2, 2, 2, 1, 0, 2, 2, 1, 1, 0, 2, 2, 1, 1, 1, 2, 1, 2, 1, 2, 1, 2, 2, 2, 1, 2, 2, 0, 2, 0, 0, 0, 1, 2, 0, 1, 2, 1,
0, 2, 1, 2, 2, 1, 2, 2, 0, 1, 1, 2, 2, 2, 2, 2, 2, 2, 1, 2, 2, 1, 0, 1, 0, 1, 2, 0, 2, 0, 0, 1, 2, 0, 1, 0, 2, 1, 0, 2, 2, 0, 1,
2, 2, 1, 1, 1, 0, 2, 1, 0, 2, 0, 0, 1, 0, 2, 0, 1, 2, 0, 2, 0, 2, 2, 0, 0, 1, 0, 1, 2, 2, 2, 2, 0, 1, 2, 1, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 2, 0, 0, 1, 0, 2, 2, 0, 0, 1, 0, 1, 1, 0, 0, 0, 2, 2, 2, 2, 2, 0, 1, 1, 2, 1, 1, 0, 1, 0,
1, 0, 0, 2, 1, 2, 1, 0, 2, 1, 1, 2, 1, 0, 2, 1, 0, 0, 2, 0, 0, 1, 2, 1, 0, 0, 1, 1, 1, 0, 2, 0, 0, 1, 1, 0, 0, 2, 0, 2, 2, 1, 2,
2, 0, 0, 0, 2, 2, 1, 1, 1, 0, 0, 1, 0, 0, 2, 2, 1, 0, 0, 2, 2, 0, 0, 1, 0, 0, 2, 2, 1, 2, 2, 1, 1, 2, 1, 1, 1, 2, 2, 2, 2, 1, 1,
```

1, 0, 2, 0, 2, 0, 0, 1, 2, 0, 1, 0, 1, 0, 0, 1, 1, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 2, 1,
1, 2, 1, 1, 0, 2, 2, 2, 2, 2, 0, 0, 1, 1, 0, 0, 0, 0, 2, 2, 0, 0, 1, 1, 0, 2, 0, 0, 1, 1, 2, 1, 0, 2, 1, 1, 2, 1, 0, 2, 1, 2, 1,
0, 0, 1, 1, 0, 0, 2, 0, 0, 1, 1, 0, 0, 2, 0, 2, 1, 0, 2, 0, 0, 1, 0, 2, 2, 1, 1, 1, 2, 1, 2, 2, 0, 0, 2, 1, 0, 0, 2, 2, 0, 0, 1,
0, 0, 2, 2, 1, 2, 2, 1, 1, 0, 0, 1, 0, 0, 2, 2, 2, 1, 1, 1, 0, 2, 1, 1, 1, 2, 2, 2, 0, 1, 0, 1, 0, 2, 0, 2, 0, 0, 1, 0, 2, 0, 0,
2, 0, 0, 1, 1, 0, 2, 0, 0, 0, 0, 0, 2, 0, 2, 2, 2, 2, 2, 1, 1, 1, 2, 1, 2, 2, 1, 1, 0, 2, 1, 2, 2, 2, 1, 0, 2, 1, 1, 2, 0, 0, 0,
1, 0, 1, 2, 2, 1, 2, 0, 1, 0, 0, 2, 1, 2, 2, 0, 2, 2, 1, 2, 1, 2, 2, 1, 2, 1, 0, 2, 1, 0, 0, 0, 1, 2, 0, 1, 1, 2, 2, 2, 2, 2, 2,
1, 2, 2, 0, 1, 0, 1, 0, 1, 2, 2, 2, 2, 1, 2, 2, 0, 1, 0, 2, 1, 0, 0, 2, 0, 0, 1, 2, 1, 1, 1, 0, 2, 1, 2, 2, 0, 1, 2, 2, 2, 0, 1,
2, 0, 2, 0, 2, 0, 0, 1, 0, 0, 1, 2, 2, 2, 2, 0, 2, 2, 0, 0, 1, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 0, 1, 2, 0, 2, 2, 2, 2,
0, 0, 0, 0, 0, 0, 2, 0, 0, 2, 2, 0, 1, 0, 2, 2, 2, 2, 1, 0, 2, 2, 2, 2, 2, 0, 0, 2, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
0, 0, 0, 0, 0, 1, 2, 0, 1, 1, 2, 1, 2, 0, 1, 1, 2, 2, 0, 1, 2, 0, 1, 2, 0, 1, 2, 0, 1, 0, 2, 0, 2, 1, 2, 0, 2, 0, 2, 1, 2, 1, 0,
1, 1, 2, 2, 1, 0, 1, 1, 2, 2, 2, 0, 1, 1, 2, 2, 2, 0, 1, 1, 2, 2, 2, 2, 0, 1, 0, 2, 2, 2, 0, 1, 0, 2, 1, 0, 2, 0, 1, 2, 1, 0, 2,
0, 1, 2, 0, 2, 2, 0, 1, 2, 0, 2, 0, 2, 1, 0, 2, 1, 1, 2, 0, 0, 1, 0, 1, 0, 1, 1, 0, 1, 1, 0, 1, 0, 2, 1, 0, 1, 1, 1, 0, 0, 1, 1,
1, 1, 0, 0, 1, 1, 1, 0, 2, 1, 0, 1, 1, 1, 0, 1, 1, 0, 2, 1, 1, 2, 2, 0, 1, 2, 2, 0, 2, 0, 2, 2, 2, 1, 1, 1, 0, 2, 1, 1, 1, 0, 1,
2, 1, 0, 1, 1, 1, 1, 2, 1, 1, 2, 1, 1, 2, 2, 0, 2, 0, 0, 2, 1, 1, 2, 1, 0, 2, 1, 2, 1, 1, 1, 2, 1, 1, 2, 0, 1, 2, 0, 0, 2, 2, 0,
2, 1, 2, 1, 0, 0, 1, 2, 2, 1, 0, 1, 2, 1, 2, 2, 0, 1, 2, 0, 0, 2, 1, 2, 1, 1, 2, 1, 0, 2, 2, 0, 1, 2, 1, 0, 2, 1, 2, 0, 2, 2, 2,
2, 2, 0, 0, 0, 0, 0, 2, 1, 1, 1, 2, 1, 0, 1, 2, 1, 2, 1, 0, 2, 1, 1, 0, 2, 0, 2, 1, 1, 0, 2, 0, 1, 2, 1, 2, 1, 2, 1, 1, 1, 2, 1,
1, 2, 1, 0, 2, 1, 0, 0, 0, 1, 2, 0, 2, 1, 2, 1, 2, 2, 2, 1, 2, 2, 0, 2, 2, 2, 1, 1, 2, 2, 1, 1, 2, 0, 0, 0, 0, 1, 1, 1, 1, 2, 2,
2, 2, 1, 2, 0, 0, 2, 2, 2, 1, 0, 0, 1, 0, 2, 0, 0, 1, 1, 1, 1, 2, 1, 2, 2, 2, 1, 0, 0, 0, 2, 2, 0, 1, 0, 1, 1, 2, 0, 0, 0, 0, 1,
1, 1, 1, 2, 0, 2, 0, 0, 1, 1, 0, 1, 1, 0, 1, 2, 2, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 2, 0, 0, 1, 0, 1,
0, 1, 2, 1, 0, 1, 1, 1, 2, 1, 0, 1, 1, 0, 0, 1, 0, 1, 0, 1, 0, 1, 0, 0, 2, 2, 1, 0, 0, 1, 1, 0, 0, 2, 0, 0, 1, 1, 2, 1, 0, 2, 1,
1, 2, 1, 0, 2, 1, 2, 1, 2, 1, 2, 2, 0, 2, 0, 1, 0, 1, 2, 1, 0, 0, 0, 0, 1, 2, 2, 2, 1, 1, 2, 1, 0, 2, 2, 1, 1, 1, 2, 0, 1, 0, 0,
1, 1, 0, 1, 2, 2, 1, 0, 2, 2, 1, 1, 2, 1, 1, 0, 0, 2, 1, 1, 1, 1, 1, 2, 2, 1, 2, 1, 2, 2, 1, 2, 2, 0, 2, 0, 1, 2, 0, 2, 1, 1, 0,
1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0, 2, 0, 0, 1, 0, 0, 0, 1, 0, 1, 2, 1, 2, 2, 1, 2, 2, 1, 2, 2, 1, 2, 0, 0, 0, 1, 0,
1, 1, 0, 2, 0, 0, 1, 0, 0, 2, 0, 0, 1, 2, 1, 0, 0, 1, 1, 1, 2, 1, 0, 2, 1, 1, 2, 1, 0, 2, 1, 0, 2, 0, 1, 0, 1, 2, 1, 2, 1, 2, 2,
1, 2, 2, 2, 1, 1, 2, 1, 0, 0, 0, 0, 1, 1, 2, 0, 1, 0, 2, 1, 0, 2, 2, 1, 2, 1, 0, 2, 2, 1, 0, 1, 1, 0, 1, 2, 2, 1, 1, 1, 1, 1, 1,
2, 1, 1, 0, 0, 2, 1, 2, 2, 0, 2, 2, 2, 1, 2, 1, 2, 0, 2, 1, 0, 2, 1, 2, 0, 1, 2, 2, 0, 2, 2, 2, 2, 2, 0, 0, 0, 0, 0, 1, 2, 0, 1,
0, 0, 2, 2, 0, 1, 0, 1, 1, 1, 0, 1, 2, 2, 1, 1, 0, 1, 2, 2, 2, 2, 0, 1, 0, 1, 1, 2, 0, 1, 0, 0, 0, 0, 0, 1, 2, 0, 1, 2, 1, 0, 2,
1, 2, 1, 2, 2, 0, 2, 2, 1, 2, 1, 2, 2, 1, 1, 2, 0, 0, 0, 2, 2, 1, 1, 2, 2, 2, 2, 2, 1, 2, 0, 0, 1, 1, 1, 1, 2, 0, 1, 0, 2, 0, 0,
0, 2, 2, 2, 1, 0, 2, 2, 2, 1, 0, 0, 1, 1, 1, 1, 2, 1, 1, 1, 2, 0, 0, 0, 0, 2, 2, 0, 1, 0, 0, 2, 0, 0, 1, 1, 0, 1, 1, 1, 1, 2, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 2, 0, 0, 0, 0, 2, 2, 0, 1, 0, 0, 1, 1, 0, 1, 0, 1, 0, 0, 1, 2, 0, 2, 1, 1, 2, 0, 0, 2, 2,
2, 2, 1, 0, 2, 1, 1, 2, 0, 0, 2, 2, 2, 2, 1, 1, 0, 0, 0, 2, 1, 1, 0, 0, 0, 2, 0, 1, 2, 1, 1, 1, 0, 1, 2, 1, 1, 1, 1, 0, 1, 2, 2,
2, 1, 0, 1, 2, 2, 2, 2, 0, 1, 0, 2, 2, 2, 0, 1, 0, 2, 2, 1, 1, 1, 1, 1, 0, 1, 1, 1, 1, 1, 0, 1, 2, 1, 1, 0, 0, 1, 2, 1, 1, 0, 0,
1, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 1, 0, 2, 2, 0, 1, 1, 0, 2, 2, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 2, 0, 0, 0, 1, 1, 1,
1, 1, 0, 0, 0, 2, 0, 1, 2, 2, 0, 1, 0, 0, 0, 2, 0, 2, 2, 1, 1, 2, 1, 0, 0, 0, 0, 1, 1, 2, 1, 0, 2, 1, 0, 2, 0, 0, 2, 0, 1, 1, 1,
0, 1, 1, 1, 0, 1, 0, 0, 1, 1, 0, 1, 0, 0, 0, 1, 1, 1, 0, 1, 2, 1, 1, 0, 1, 0, 0, 2, 1, 1, 0, 1, 2, 0, 2, 0, 0, 0, 2, 2, 0, 0, 1,
2, 2, 1, 2, 1, 1, 0, 2, 1, 0, 1, 2, 0, 2, 0, 1, 1, 0, 1, 0, 1, 1, 0, 0, 0, 2, 1, 1, 0, 1, 2, 0, 2, 1, 1, 2, 0, 1, 0, 0, 2, 1, 1,
1, 0, 0, 2, 1, 1, 0, 0, 1, 1, 2, 1, 1, 0, 1, 0, 1, 2, 2, 0, 2, 2, 2, 1, 0, 0, 1, 0, 0, 0, 2, 0, 1, 2, 0, 0, 2, 2, 2, 0, 2, 2, 0,
0, 0, 1, 0, 2, 2, 0, 0, 2, 2, 0, 0, 0, 1, 2, 0, 0, 2, 2, 2, 2, 1, 1, 1, 2, 2, 2, 1, 0, 0, 1, 2, 1, 0, 1, 2, 2, 2, 0, 2, 0, 2, 1,
2, 0, 0, 0, 1, 1, 2, 0, 1, 1, 1, 1, 2, 0, 2, 2, 2, 2, 2, 0, 0, 1, 0, 1, 0, 2, 1, 1, 1, 2, 2, 1, 1, 1, 1, 2, 2, 2, 0, 0, 0, 1, 2,
2, 1, 1, 1, 0, 1, 1, 0, 0, 2, 0, 2, 1, 2, 0, 2, 0, 2, 1, 0, 0, 0, 0, 0, 0, 2, 2, 1, 1, 1, 0, 0, 0, 1, 2, 2, 2, 0, 1, 1, 1, 0, 1,
0, 0, 1, 2, 2, 2, 1, 0, 2, 2, 1, 0, 2, 0, 0, 2, 2, 0, 1, 2, 0, 1, 2, 0, 2, 0, 1, 0, 1, 0, 2, 2, 2, 1, 1, 2, 2, 1, 2, 2, 2, 0, 1,
0, 0, 1, 0, 1, 2, 1, 1, 1, 2, 1, 2, 0, 1, 2, 1, 1, 0, 0, 0, 2, 2, 1, 2, 0, 1, 2, 1, 1, 2, 2, 0, 0, 1, 1, 2, 2, 2, 0, 1, 0, 1, 2,
1, 0, 0, 1, 0, 1, 0, 0, 2, 1, 0, 2, 2, 0, 0, 0, 1, 0, 2, 0, 0, 0, 0, 0, 2, 2, 2, 2, 2, 0, 1, 0, 0, 2, 1, 1, 1, 0, 1, 1, 2, 1, 1,
1, 0, 0, 1, 0, 2, 0, 2, 2, 2, 1, 0, 0, 0, 0, 2, 1, 1, 1, 0, 2, 2, 2, 2, 2, 1, 0, 2, 2, 0, 1, 0, 0, 1, 2, 1, 0, 2, 0, 0, 2, 2, 0,
1, 0, 2, 1, 2, 0, 0, 1, 0, 2, 1, 0, 1, 2, 2, 2, 2, 1, 0, 0, 2, 0, 2, 0, 1, 0, 0, 1, 2, 1, 2, 0, 1, 2, 0, 1, 2, 1, 1, 1, 2, 1, 2,
0, 1, 2, 1, 1, 0, 0, 0, 2, 1, 1, 2, 2, 2, 0, 1, 1, 2, 2, 0, 0, 0, 1, 0, 1, 0, 0, 1, 0, 1, 2, 1, 0, 0, 0, 1, 0, 2, 0, 2, 1, 0, 2,
2, 0, 2, 2, 2, 0, 1, 0, 0, 0, 0, 0, 2, 2, 1, 1, 2, 1, 1, 1, 0, 2, 1, 1, 1, 0, 2, 2, 2, 1, 0, 0, 0, 0, 1, 0, 2, 0, 0, 0, 2, 2, 1,
2, 2, 0, 2, 0, 2, 1, 0, 0, 1, 0, 1, 2, 2, 0, 2, 2, 0, 0, 2, 1, 0, 2, 0, 0, 0, 2, 2, 2, 0, 1, 2, 1, 1, 1, 1, 1, 0, 1, 1, 0, 2, 0,
2, 1, 0, 0, 0, 0, 2, 2, 2, 2, 1, 0, 0, 1, 2, 1, 1, 1, 1, 2, 2, 2, 2, 0, 2, 1, 2, 0, 0, 1, 2, 2, 2, 0, 1, 1, 1, 1, 2, 0, 0, 0, 1,
1, 2, 0, 0, 1, 0, 1, 0, 2, 2, 2, 2, 2, 2, 0, 1, 1, 1, 2, 2, 2, 1, 1, 1, 2, 2, 1, 1, 1, 1, 0, 1, 1, 0, 0, 0, 1, 2, 2, 2, 0, 2, 0,
2, 1, 0, 0, 2, 0, 2, 1, 2, 2, 1, 1, 1, 0, 0, 0, 0, 0, 0, 2, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 1, 1, 0, 0,
0, 2, 0, 0, 2, 1, 0, 1, 2, 2, 1, 1, 0, 0, 2, 2, 0, 2, 0, 2, 2, 2, 2, 0, 0, 1, 0, 0, 1, 1, 0, 1, 0, 0, 1, 1, 2, 1, 0, 1, 0, 1, 2,
1, 0, 1, 0, 1, 0, 1, 2, 2, 2, 1, 0, 1, 2, 2, 2, 0, 0, 2, 0, 0, 2, 0, 0, 2, 0, 0, 2, 0, 2, 1, 0, 2, 0, 0, 2, 1, 0, 2, 0, 0, 0, 1,
1, 0, 0, 0, 0, 1, 1, 0, 0, 2, 0, 1, 1, 0, 0, 2, 0, 1, 1, 0, 0, 1, 2, 1, 2, 0, 0, 1, 2, 1, 2, 0, 0, 2, 2, 2, 2, 0, 1, 2, 2, 2, 2,
0, 1, 1, 1, 0, 1, 0, 0, 0, 1, 0, 0, 1, 1, 2, 0, 0, 1, 0, 0, 0, 0, 0, 0, 2, 1, 1, 1, 1, 2, 0, 2, 1, 1, 0, 2, 1, 1, 1, 2, 1, 2,
0, 2, 2, 2, 2, 0, 0, 1, 0, 0, 1, 2, 1, 1, 0, 2, 0, 1, 2, 2, 1, 2, 2, 1, 1, 1, 2, 1, 2, 0, 2, 2, 0, 1, 1, 2, 2, 0, 0, 2, 0, 2, 0,
0, 1, 1, 0, 0, 2, 2, 1, 2, 2, 2, 0, 0, 0, 0, 0, 0, 1, 2, 2, 2, 0, 0, 2, 0, 0, 1, 1, 0, 0, 1, 2, 0, 2, 2, 1, 0, 1, 2, 0, 0, 1, 1,
1, 1, 1, 0, 0, 2, 1, 1, 2, 1, 2, 2, 2, 0, 2, 2, 2, 0, 2, 1, 2, 2, 1, 2, 0, 2, 2, 0, 2, 1, 2, 0, 1, 2, 0, 2, 1, 0, 0, 0, 2, 0, 0,
0, 0, 2, 1, 2, 0, 2, 2, 1, 2, 1, 0, 0, 2, 0, 2, 2, 0, 1, 2, 2, 1, 2, 1, 2, 1, 0, 2, 2, 2, 0, 0, 2, 2, 2, 1, 1, 1, 0, 1, 0, 1, 1,
1, 2, 0, 1, 2, 1, 1, 2, 0, 1, 2, 1, 1, 1, 2, 0, 1, 1, 1, 1, 0, 0, 2, 1, 0, 2, 2, 2, 1, 0, 2, 1, 1, 2, 2, 0, 0, 1, 2, 2, 1, 2, 1,
2, 0, 2, 2, 0, 2, 1, 0, 1, 0, 0, 0, 0, 1, 2, 0, 1, 1, 0, 0, 1, 2, 2, 2, 0, 2, 2, 2, 0, 0, 1, 1, 1, 0, 1, 1, 0, 1, 1, 2, 0, 0, 2,
1, 0, 1, 1, 0, 0, 2, 2, 0, 1, 1, 2, 0, 2, 2, 0, 1, 1, 0, 0, 1, 0, 2, 1, 2, 0, 2, 0, 2, 2, 0, 2, 1, 1, 2, 1, 2, 0, 0, 1, 1, 2, 1,
2, 2, 2, 0, 0, 1, 0, 0, 0, 0, 0, 0, 1, 2, 0, 2, 0, 2, 2, 1, 0, 2, 1, 1, 1, 2, 2, 1, 2, 0, 1, 0, 1, 2, 2, 2, 2, 1, 0, 0, 1, 1, 2,
2, 1, 2, 0, 2, 1, 2, 0, 2, 1, 2, 2, 0, 2, 2, 1, 0, 2, 2, 1, 0, 2, 1, 0, 0, 2, 2, 2, 2, 2, 1, 0, 2, 1, 1, 1, 1, 0, 0, 1, 2, 2, 1,
2, 1, 1, 1, 0, 0, 2, 0, 2, 0, 1, 0, 2, 1, 2, 0, 2, 1, 1, 1, 1, 0, 0, 2, 0, 1, 0, 2, 2, 2, 0, 1, 1, 1, 1, 1, 1, 0, 1, 1, 0, 2, 2,
0, 2, 2, 1, 0, 1, 2, 1, 1, 1, 0, 0, 1, 1, 2, 1, 0, 1, 0, 0, 0, 1, 2, 1, 1, 0, 1, 2, 1, 1, 0, 0, 2, 0, 0, 0, 0, 0, 2, 1, 0, 2, 1,
1, 1, 0, 1, 2, 0, 0, 1, 0, 1, 2, 2, 1, 1, 1, 2, 1, 2, 2, 2, 1, 2, 0, 2, 2, 1, 0, 0, 1, 1, 2, 0, 2, 2, 1, 0, 1, 2, 0, 2, 1, 2, 0,
0, 2, 2, 2, 2, 2, 2, 2, 1, 0, 2, 1, 1, 1, 0, 0, 1, 2, 2, 1, 0, 2, 1, 1, 0, 0, 2, 0, 2, 0, 2, 1, 2, 1, 1, 1, 2, 1, 1, 1, 1, 1, 0, 1, 0,
2, 1, 2, 0, 2, 2, 0, 1, 1, 1, 0, 2, 0, 1, 0, 2, 0, 2, 2, 1, 0, 2, 1, 0, 1, 2, 0, 0, 0, 0, 2, 0, 0, 0, 0, 2, 1, 2, 0, 1, 1, 2, 0,
2, 1, 1, 1, 0, 0, 1, 2, 1, 2, 0, 2, 0, 1, 2, 1, 1, 1, 1, 2, 0, 0, 1, 2, 2, 2, 0, 2, 0, 2, 0, 2, 1, 1, 2, 0, 1, 2, 1, 1, 2, 0, 1, 2,
1, 1, 1, 0, 0, 2, 1, 1, 1, 2, 0, 1, 0, 2, 1, 1, 2, 2, 1, 0, 2, 2, 2, 1, 2, 1, 2, 0, 2, 2, 0, 0, 1, 2, 2, 1, 0, 0, 0, 1, 2, 0, 0,
2, 1, 0, 1, 0, 2, 2, 0, 2, 2, 2, 1, 1, 0, 0, 1, 2, 1, 1, 0, 1, 1, 2, 0, 0, 1, 1, 1, 0, 1, 0, 0, 2, 2, 0, 0, 0, 2, 1, 0, 1, 2, 1,
0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 1, 2, 0, 1, 1, 1, 1, 0, 0, 1, 2, 2, 1, 2, 0, 2, 2, 2,
2, 0, 0, 1, 0, 0, 1, 0, 0, 2, 0, 0, 2, 1, 0, 1, 2, 2, 2, 1, 0, 1, 2, 2, 2, 1, 2, 1, 0, 1, 0, 1, 2, 1, 0, 1, 0, 2, 1, 1, 1, 1, 2,
2, 1, 1, 1, 1, 2, 0, 1, 0, 1, 0, 0, 0, 1, 0, 1, 0, 0, 1, 2, 1, 2, 2, 2, 1, 2, 1, 2, 2, 2, 0, 1, 2, 2, 1, 1, 0, 1, 2, 2, 1, 1, 1,
0, 1, 2, 0, 0, 1, 0, 1, 2, 0, 0, 0, 2, 0, 0, 1, 0, 2, 0, 2, 0, 0, 2, 0, 2, 0, 1, 0, 1, 1, 1, 0, 1, 0, 0, 2, 1, 2, 0, 0, 0, 0, 0,
2, 0, 0, 2, 0, 2, 0, 2, 2, 2, 1, 0, 2, 0, 0, 0, 2, 0, 2, 2, 1, 2, 0, 1, 1, 0, 0, 0, 2, 0, 1, 1, 2, 2, 1, 2, 1, 0, 1, 1, 1, 2, 0,
1, 1, 1, 1, 0, 0, 2, 1, 1, 2, 0, 1, 2, 1, 1, 2, 0, 1, 2, 0, 1, 0, 0, 2, 2, 2, 2, 2, 1, 0, 2, 0, 0, 0, 1, 0, 2, 1, 2, 1, 1, 1, 2,
1, 2, 2, 1, 0, 2, 0, 1, 2, 1, 0, 0, 0, 1, 0, 2, 0, 2, 1, 0, 0, 1, 1, 0, 1, 1, 1, 2, 2, 2, 1, 1, 0, 0, 0, 0, 2, 1, 1, 1, 0, 2, 2,
1, 1, 2, 2, 0, 1, 1, 0, 2, 0, 2, 0, 2, 2, 1, 1, 1, 1, 0, 0, 1, 0, 1, 0, 2, 1, 2, 0, 2, 2, 1, 0, 2, 2, 0, 2, 1, 2, 0, 2, 0, 0, 2,

2, 1, 1, 2, 1, 1, 1, 0, 0, 0, 0, 0, 2, 0, 1, 2, 1, 0, 1, 0, 2, 2, 1, 2, 0, 2, 2, 1, 0, 0, 1, 1, 1, 1, 1, 2, 1, 2, 0, 1, 0, 1, 2,
2, 1, 1, 0, 1, 2, 1, 2, 1, 1, 2, 0, 0, 0, 2, 2, 1, 1, 0, 1, 2, 2, 2, 0, 0, 0, 1, 1, 0, 0, 1, 1, 2, 1, 2, 0, 1, 1, 0, 1, 2, 2, 0,
0, 2, 1, 0, 0, 2, 2, 0, 0, 0, 2, 0, 1, 2, 0, 1, 0, 0, 1, 0, 0, 2, 1, 1, 1, 2, 1, 0, 1, 2, 1, 1, 0, 0, 2, 0, 1, 0, 0, 0, 0, 0, 0,
0, 0, 0, 1, 1, 2, 0, 0, 1, 2, 1, 1, 0, 2, 0, 1, 1, 0, 0, 0, 2, 0, 1, 2, 1, 1, 2, 2, 0, 2, 2, 2, 2, 0, 0, 1, 0, 0, 1, 1, 1, 2, 2,
2, 2, 2, 2, 0, 0, 2, 0, 0, 2, 2, 0, 1, 1, 1, 2, 2, 1, 2, 2, 1, 1, 1, 2, 1, 2, 1, 0, 0, 1, 0, 2, 1, 0, 2, 0, 1, 0, 2, 1, 1, 1, 1,
2, 2, 1, 0, 2, 0, 0, 0, 1, 2, 0, 1, 1, 0, 0, 0, 0, 0, 2, 1, 2, 0, 2, 1, 0, 0, 0, 2, 0, 1, 1, 0, 2, 0, 2, 0, 0, 0, 0, 1, 2, 2, 1,
2, 2, 1, 2, 2, 0, 1, 1, 2, 2, 1, 1, 1, 1, 0, 0, 1, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 2, 2, 1, 1, 2, 2, 1, 2, 0, 0, 2, 1,
2, 2, 2, 2, 2, 1, 0, 0, 1, 2, 0, 2, 2, 2, 2, 0, 0, 1, 0, 0, 1, 0, 2, 1, 1, 1, 1, 0, 2, 2, 0, 1, 1, 2, 2, 0, 0, 2, 0, 1, 1, 1, 2,
1, 2, 1, 2, 2, 1, 2, 2, 1, 0, 2, 0, 1, 0, 1, 0, 0, 1, 0, 2, 2, 1, 0, 2, 0, 0, 2, 1, 1, 1, 1, 2, 0, 0, 0, 0, 0, 2, 0, 1, 2, 0, 1,
1, 0, 0, 2, 0, 1, 1, 1, 2, 0, 2, 1, 0, 0, 0, 1, 2, 2, 1, 0, 2, 0, 2, 0, 0, 1, 1, 2, 2, 1, 1, 2, 2, 1, 2, 2, 0, 1, 1, 0, 1, 0, 1,
0, 1, 1, 2, 0, 2, 0, 2, 2, 0, 2, 2, 1, 0, 1, 2, 1, 1, 0, 1, 2, 0, 2, 0, 0, 2, 1, 2, 2, 0, 1, 2, 2, 0, 2, 1, 1, 2, 2, 2, 0, 0, 0,
2, 0, 0, 2, 1, 1, 2, 1, 2, 2, 1, 0, 0, 1, 1, 2, 2, 1, 2, 0, 2, 0, 1, 0, 1, 2, 2, 1, 1, 1, 2, 1, 2, 2, 1, 1, 2, 0, 0, 1, 1, 0, 1,
2, 1, 1, 2, 2, 2, 0, 0, 0, 2, 2, 1, 1, 0, 1, 2, 1, 2, 0, 1, 0, 1, 1, 0, 0, 1, 0, 2, 1, 0, 0, 2, 1, 0, 1, 2, 2, 0, 1, 2, 0, 1, 0,
0, 2, 0, 0, 0, 2, 0, 1, 2, 1, 0, 1, 2, 1, 0, 0, 2, 1, 1, 1, 1, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0,
2, 0, 0, 2, 0, 0, 1, 0, 0, 1, 2, 0, 2, 2, 2, 2, 0, 0, 1, 2, 2, 1, 2, 0, 1, 1, 1, 1, 1, 0, 1, 0, 0, 1, 2, 0, 1, 1, 1, 2, 2, 0, 1,
1, 1, 2, 1, 2, 2, 1, 2, 1, 1, 2, 2, 1, 2, 1, 2, 2, 2, 0, 2, 2, 2, 2, 2, 0, 2, 2, 2, 0, 0, 0, 1, 2, 2, 0, 0, 0, 1, 2, 1, 1, 0, 0,
2, 2, 1, 1, 0, 0, 2, 2, 1, 2, 0, 0, 0, 1, 1, 2, 0, 0, 0, 1, 1, 1, 1, 1, 0, 1, 1, 1, 1, 1, 0, 1, 2, 1, 2, 0, 0, 0, 2, 1, 2, 0, 0,
0, 0, 2, 2, 2, 1, 1, 0, 1, 0, 0, 0, 0, 0, 2, 1, 2, 0, 0, 0, 0, 2, 0, 2, 2, 1, 2, 1, 0, 0, 0, 2, 0, 1, 1, 1, 2, 0, 1, 1, 0, 0, 0,
2, 0, 2, 2, 2, 2, 1, 0, 2, 0, 0, 2, 0, 2, 0, 2, 0, 1, 0, 1, 2, 0, 0, 1, 2, 1, 1, 0, 0, 1, 2, 0, 1, 2, 0, 1, 2, 0, 1, 2, 2, 1, 1,
2, 2, 0, 2, 1, 2, 1, 0, 0, 0, 1, 1, 1, 1, 1, 0, 1, 2, 2, 2, 0, 2, 0, 0, 2, 2, 0, 1, 1, 2, 1, 2, 1, 1, 1, 1, 1, 0, 2, 1, 2, 0, 2,
1, 0, 1, 2, 0, 1, 1, 1, 0, 0, 1, 0, 2, 0, 0, 2, 0, 2, 1, 0, 1, 1, 2, 2, 1, 1, 2, 1, 1, 1, 2, 2, 1, 0, 0, 2, 0, 2, 0, 2, 2, 0, 0,
1, 0, 2, 1, 2, 1, 2, 1, 0, 1, 0, 0, 0, 0, 0, 2, 0, 1, 2, 1, 1, 1, 0, 2, 0, 0, 2, 2, 1, 2, 0, 2, 1, 2, 0, 0, 2, 2, 1, 0, 2, 0, 2,
0, 2, 0, 1, 2, 1, 0, 0, 1, 1, 1, 1, 2, 2, 0, 1, 0, 1, 2, 0, 2, 0, 1, 2, 0, 1, 2, 1, 2, 0, 2, 2, 1, 0, 1, 2, 2, 2, 1, 2, 1, 0, 0,
1, 0, 0, 2, 2, 0, 2, 2, 1, 2, 0, 0, 1, 1, 2, 1, 2, 1, 2, 0, 1, 1, 1, 0, 1, 1, 0, 1, 2, 2, 2, 0, 0, 2, 2, 2, 1, 1, 0, 2, 1, 1, 1,
2, 0, 0, 1, 0, 2, 0, 1, 2, 1, 0, 0, 1, 0, 0, 0, 0, 2, 0, 0, 0, 0, 0, 0, 1, 1, 2, 0, 0, 1, 1, 1, 2, 2, 2, 2, 0, 0, 1, 0, 0, 1, 2,
0, 2, 2, 2, 2, 0, 1, 2, 1, 1, 2, 1, 1, 0, 0, 0, 2, 2, 1, 1, 0, 2, 0, 0, 2, 2, 0, 2, 2, 2, 2, 1, 1, 2, 0, 0, 1, 2, 0, 2, 0, 1, 2,
0, 0, 1, 0, 0, 1, 0, 0, 2, 2, 0, 2, 2, 0, 0, 1, 0, 0, 2, 1, 1, 2, 2, 2, 2, 2, 0, 2, 0, 0, 2, 2, 0, 1, 1, 2, 2, 1, 0, 2, 0, 0, 1,
0, 0, 0, 1, 1, 0, 0, 1, 2, 2, 1, 1, 1, 2, 1, 2, 1, 2, 0, 0, 0, 2, 0, 0, 2, 0, 1, 2, 0, 1, 0, 2, 0, 2, 1, 0, 0, 2, 0, 0, 0, 0, 1,
0, 0, 0, 0, 0, 0, 0, 2, 0, 2, 2, 1, 0, 2, 1, 1, 1, 1, 0, 0, 1, 0, 0, 1, 2, 0, 2, 2, 2, 2, 2, 2, 1, 0, 0, 1, 0, 2, 1, 2, 2, 2, 1,
2, 2, 1, 2, 0, 2, 2, 1, 1, 2, 0, 0, 2, 2, 0, 2, 2, 1, 2, 0, 0, 1, 0, 0, 1, 2, 0, 2, 0, 0, 2, 2, 0, 0, 1, 0, 1, 0, 0, 2, 2, 2, 2,
2, 2, 0, 2, 0, 0, 2, 1, 1, 2, 1, 2, 2, 1, 0, 2, 0, 0, 2, 2, 0, 1, 1, 1, 0, 0, 1, 2, 0, 0, 1, 0, 0, 0, 2, 1, 2, 0, 0, 0, 2, 1, 1,
1, 2, 1, 2, 0, 1, 0, 2, 0, 2, 0, 0, 2, 0, 1, 2, 1, 2, 2, 1, 1, 2, 0, 0, 1, 1, 1, 1, 0, 0, 1, 2, 2, 1, 0, 1, 2, 2, 1, 1, 2, 1, 2,
0, 0, 0, 2, 0, 0, 2, 1, 1, 2, 2, 2, 2, 0, 1, 2, 2, 0, 2, 0, 0, 2, 1, 2, 1, 1, 0, 1, 2, 0, 2, 1, 0, 0, 1, 1, 0, 2, 0, 2, 0, 1, 0,
1, 2, 0, 2, 0, 1, 1, 2, 2, 0, 1, 2, 0, 2, 2, 1, 0, 1, 2, 0, 1, 2, 1, 1, 0, 0, 1, 0, 0, 1, 2, 2, 2, 1, 2, 2, 0, 0, 1, 1, 2, 2, 2,
0, 2, 2, 1, 1, 1, 0, 1, 1, 0, 1, 2, 1, 2, 0, 1, 2, 2, 2, 1, 1, 0, 1, 2, 2, 2, 0, 0, 0, 1, 0, 2, 0, 1, 2, 1, 1, 1, 2, 0, 1, 1, 0,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 0, 0, 1, 1, 0, 1, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 2, 0, 2, 2, 2, 2, 1, 0, 0, 2, 2, 0,
1, 0, 1, 2, 2, 1, 0, 0, 2, 0, 0, 2, 1, 2, 2, 1, 2, 1, 1, 2, 2, 1, 2, 1, 2, 0, 1, 1, 1, 2, 2, 0, 1, 1, 1, 2, 0, 2, 1, 2, 0, 0, 0,
2, 1, 2, 0, 0, 1, 2, 2, 2, 0, 1, 1, 2, 2, 2, 0, 1, 1, 0, 0, 0, 2, 2, 1, 0, 0, 0, 2, 2, 0, 0, 0, 0, 2, 2, 0, 0, 0, 0, 2, 2, 2, 2,
2, 0, 2, 1, 2, 2, 2, 0, 2, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 0, 0, 1, 2, 1, 2, 2, 0, 0, 0, 0, 2, 0, 0, 2, 0, 2, 2, 1, 0, 0, 0, 0, 0,
0, 2, 1, 1, 0, 2, 0, 0, 0, 1, 0, 0, 1, 2, 0, 2, 2, 2, 2, 2, 1, 1, 1, 2, 1, 2, 0, 2, 1, 1, 0, 0, 2, 1, 1, 1, 1, 1, 2, 0, 0, 1, 0,
0, 1, 2, 0, 2, 0, 0, 2, 2, 0, 2, 2, 2, 2, 1, 1, 2, 0, 2, 1, 2, 2, 1, 2, 0, 2, 0, 1, 0, 2, 1, 2, 0, 2, 0, 0, 0, 0, 1, 2, 2, 1, 1,
1, 0, 0, 0, 0, 2, 1, 1, 2, 0, 1, 0, 0, 1, 0, 1, 2, 2, 2, 1, 1, 2, 2, 1, 1, 1, 2, 2, 2, 1, 0, 0, 0, 0, 0, 2, 2, 0, 0, 1, 0, 1, 0,
0, 2, 2, 0, 0, 2, 2, 1, 1, 1, 0, 2, 1, 2, 0, 0, 0, 0, 2, 0, 0, 0, 2, 1, 2, 0, 1, 1, 0, 1, 0, 1, 0, 0, 2, 2, 2, 1, 2, 1, 0, 2, 2,
2, 1, 2, 2, 1, 2, 1, 0, 2, 0, 2, 2, 0, 2, 2, 1, 2, 1, 0, 0, 1, 2, 0, 1, 2, 0, 1, 2, 0, 1, 2, 0, 1, 2, 1, 1, 0, 0, 1, 0, 1, 2, 0,
2, 0, 0, 1, 1, 0, 0, 0, 2, 0, 2, 1, 0, 1, 2, 0, 0, 1, 0, 0, 0, 2, 2, 2, 0, 2, 0, 0, 0, 2, 0, 1, 2, 0, 1, 1, 0, 0, 0, 1, 2, 0, 2,
2, 2, 1, 2, 1, 2, 1, 2, 2, 1, 2, 0, 0, 2, 1, 1, 2, 2, 2, 0, 0, 1, 1, 2, 0, 2, 1, 1, 1, 2, 1, 2, 0, 0, 2, 2, 1, 1, 2, 2, 2, 1, 0,
1, 2, 0, 2, 0, 2, 2, 0, 0, 2, 2, 1, 0, 2, 0, 0, 1, 2, 0, 2, 0, 0, 0, 0, 0, 2, 2, 2, 2, 0, 0, 1, 0, 0, 1, 1, 2, 1, 2, 1, 1, 2, 1,
2, 0, 1, 2, 0, 2, 0, 1, 1, 2, 2, 0, 1, 0, 2, 0, 2, 0, 1, 2, 1, 0, 0, 1, 1, 0, 2, 1, 1, 1, 2, 2, 2, 0, 2, 1, 2, 2, 1, 1, 0, 0, 0,
2, 0, 0, 1, 2, 1, 0, 0, 0, 1, 0, 1, 2, 1, 0, 0, 0, 2, 0, 1, 2, 0, 1, 1, 2, 1, 0, 0, 2, 1, 0, 0, 1, 1, 1, 1, 0, 2, 1, 0, 2, 1, 1,
0, 0, 2, 1, 1, 1, 2, 0, 1, 0, 2, 2, 1, 1, 0, 0, 1, 1, 2, 2, 1, 0, 1, 0, 2, 1, 2, 1, 1, 1, 0, 0, 1, 1, 1, 0, 1, 2, 0, 0, 0, 2, 0,
2, 1, 0, 0, 2, 0, 0, 0, 0, 1, 2, 1, 1, 0, 0, 0, 1, 2, 1, 1, 1, 2, 1, 0, 1, 0, 1, 1, 2, 2, 0, 1, 0, 1, 2, 0, 2, 0, 2, 1, 0, 0, 1,
1, 0, 2, 0, 2, 0, 1, 2, 2, 0, 2, 1, 2, 0, 2, 1, 1, 1, 2, 2, 0, 0, 1, 2, 1, 2, 1, 1, 0, 0, 0, 2, 1, 0, 0, 0, 2, 0, 0, 0, 1, 0, 1,
2, 1, 0, 0, 2, 1, 0, 1, 2, 0, 1, 1, 0, 2, 1, 0, 2, 1, 0, 0, 1, 1, 1, 1, 1, 2, 0, 1, 0, 2, 1, 0, 0, 2, 1, 1, 0, 2, 1, 2, 2, 2, 0,
1, 2, 1, 0, 0, 0, 2, 0, 0, 0, 0, 0, 2, 1, 2, 0, 2, 0, 2, 0, 2, 0, 0, 1, 2, 2, 2, 1, 2, 1, 1, 1, 2, 2, 1, 2, 0, 2, 0, 1, 1, 1, 0,
0, 1, 0, 1, 1, 2, 0, 2, 0, 1, 2, 0, 1, 2, 0, 1, 2, 0, 1, 2, 0, 1, 0, 1, 2, 0, 0, 1, 2, 1, 1, 0, 0, 0, 2, 0, 2, 1, 2, 0, 0, 1, 1,
0, 0, 0, 0, 2, 2, 2, 0, 1, 2, 0, 0, 1, 0, 1, 2, 0, 1, 1, 0, 2, 0, 0, 0, 2, 2, 2, 2, 1, 2, 1, 0, 0, 0, 1, 2, 0, 0, 0, 2, 1, 1, 2,
2, 1, 2, 2, 1, 2, 2, 0, 2, 1, 1, 1, 2, 2, 0, 0, 1, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 2, 2, 0, 0, 0, 0, 2, 2, 0, 0, 0, 2,
2, 2, 2, 1, 0, 2, 1, 1, 2, 0, 0, 2, 2, 2, 2, 2, 0, 2, 1, 1, 2, 1, 0, 1, 0, 0, 1, 1, 0, 0, 1, 1, 0, 0, 1, 2, 1, 1, 1, 0, 1, 2, 1,
1, 1, 1, 1, 0, 0, 0, 2, 1, 1, 0, 0, 0, 2, 2, 2, 1, 1, 2, 0, 2, 2, 1, 1, 2, 0, 2, 0, 2, 0, 0, 0, 2, 0, 2, 0, 0, 0, 1, 2, 2, 2, 1,
1, 1, 2, 2, 2, 1, 1, 0, 1, 1, 0, 0, 0, 0, 1, 1, 0, 0, 0, 1, 0, 1, 1, 0, 2, 1, 0, 1, 1, 0, 2, 2, 0, 1, 1, 1, 2, 2, 0, 1, 1, 1, 2,
0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 1, 1, 1, 0, 1, 1, 1, 1, 1, 0, 1, 0, 2, 0, 0, 2, 1, 1, 2, 1, 0, 2, 2, 1, 0, 0, 0, 0, 2, 0, 2,
2, 1, 1, 2, 0, 1, 0, 0, 0, 0, 2, 0, 1, 2, 1, 1, 0, 1, 0, 0, 0, 1, 1, 1, 0, 1, 1, 1, 0, 1, 0, 0, 0, 1, 1, 1, 0, 1, 2, 0, 2, 2,
1, 0, 2, 0, 2, 2, 1, 0, 2, 2, 2, 0, 1, 1, 2, 1, 2, 0, 1, 0, 1, 1, 1, 1, 1, 0, 1, 0, 2, 0, 2, 2, 2, 1, 1, 2, 0, 1, 2, 1, 1, 0, 0,
2, 0, 2, 0, 1, 2, 2, 1, 0, 0, 0, 0, 1, 2, 2, 1, 2, 1, 1, 2, 1, 1, 0, 1, 0, 0, 0, 0, 2, 2, 1, 1, 0, 1, 0, 2, 1, 0, 0, 1, 0, 2, 1,
0, 0, 1, 0, 0, 0, 2, 2, 2, 2, 2, 0, 0, 0, 1, 2, 0, 2, 2, 0, 0, 2, 2, 2, 0, 0, 0, 1, 0, 0, 2, 2, 2, 0, 0, 0, 2, 0, 1, 2, 0, 1, 2,
2, 2, 0, 2, 0, 2, 1, 2, 0, 1, 0, 0, 1, 2, 1, 1, 1, 1, 2, 2, 2, 0, 2, 2, 1, 1, 1, 0, 0, 0, 0, 0, 1, 0, 2, 1, 0, 1, 0, 0, 2, 1,
0, 1, 0, 1, 0, 0, 1, 0, 0, 0, 1, 1, 2, 0, 0, 1, 2, 2, 2, 0, 0, 1, 0, 2, 2, 0, 1, 1, 2, 1, 0, 1, 0, 2, 2, 0, 0, 0, 0, 1, 1, 2, 0,
0, 0, 0, 0, 1, 2, 2, 0, 0, 2, 1, 0, 2, 1, 1, 1, 2, 2, 2, 1, 0, 2, 2, 2, 2, 1, 0, 2, 2, 2, 0, 1, 0, 0, 1, 1, 2, 2, 1, 2, 2, 1, 0,
2, 2, 2, 1, 2, 0, 2, 0, 1, 0, 2, 0, 1, 2, 0, 1, 1, 0, 2, 0, 0, 2, 1, 1, 0, 0, 0, 2, 2, 1, 2, 0, 1, 2, 2, 1, 2, 0, 1, 2, 0, 1, 2,
1, 1, 1, 0, 2, 2, 2, 1, 0, 0, 0, 0, 1, 0, 2, 0, 1, 0, 0, 0, 2, 2, 2, 2, 1, 1, 1, 0, 2, 2, 1, 2, 0, 0, 0, 1, 1, 0, 0, 0, 0, 0, 1,
0, 2, 2, 1, 1, 2, 2, 1, 1, 1, 1, 1, 0, 1, 2, 0, 0, 2, 1, 2, 0, 1, 1, 2, 2, 2, 0, 1, 1, 1, 2, 2, 0, 0, 1, 2, 0, 1, 2, 2, 2, 1, 0,
1, 0, 0, 1, 2, 0, 1, 0, 0, 1, 2, 2, 0, 1, 0, 0, 1, 2, 1, 0, 0, 2, 0, 1, 0, 1, 2, 2, 2, 2, 0, 0, 1, 0, 2, 2, 0, 1, 0, 2, 1, 1, 0,
2, 0, 0, 2, 2, 1, 2, 0, 1, 2, 1, 1, 0, 0, 0, 2, 0, 1, 2, 1, 1, 1, 2, 1, 2, 0, 1, 2, 0, 0, 0, 1, 0, 2, 0, 2, 2, 2, 1, 0, 2, 2, 2,
1, 1, 1, 0, 1, 0, 0, 0, 2, 0, 0, 1, 1, 0, 0, 0, 2, 2, 1, 2, 0, 2, 1, 1, 2, 2, 1, 0, 0, 0, 1, 0, 2, 2, 0, 0, 2, 1, 2, 1, 1, 1, 1,
0, 1, 0, 1, 1, 1, 2, 2, 0, 1, 1, 2, 2, 2, 0, 0, 0, 1, 1, 2, 2, 0, 2, 0, 1, 2, 2, 0, 2, 2, 1, 2, 2, 0, 2, 2, 0, 0, 2, 2, 2, 2, 2,
0, 2, 1, 0, 0, 1, 1, 0, 1, 1, 0, 0, 1, 2, 1, 1, 1, 0, 0, 0, 2, 2, 2, 0, 0, 2, 1, 0, 2, 2, 0, 2, 1, 2, 0, 0, 1, 2, 2, 2, 0, 1, 1,
1, 2, 2, 2, 1, 0, 0, 1, 2, 1, 0, 0, 0, 0, 0, 0, 0, 2, 2, 1, 1, 1, 0, 0, 2, 1, 0, 1, 1, 0, 2, 1, 0, 1, 0, 0, 1, 1, 2, 0, 0, 1, 0,
0, 1, 0, 0, 1, 0, 2, 2, 0, 0, 1, 2, 2, 2, 0, 0, 2, 2, 0, 0, 0, 1, 1, 2, 1, 0, 1, 0, 0, 0, 1, 2, 2, 0, 1, 1, 2, 0, 0, 1, 0, 0, 0,
2, 0, 1, 2, 2, 1, 0, 0, 1, 1, 2, 2, 0, 0, 0, 0, 0, 0, 1, 2, 0, 0, 2, 2, 0, 1, 1, 1, 0, 1, 2, 0, 0, 2, 0, 2, 1, 0, 0, 1, 0, 1, 0,
2, 2, 2, 0, 2, 2, 1, 0, 0, 1, 1, 1, 0, 2, 2, 2, 2, 0, 0, 0, 2, 2, 0, 0, 1, 2, 0, 0, 0, 2, 2, 2, 2, 2, 1, 2, 1, 2, 0, 0, 0, 2, 2,

1, 2, 0, 0, 0, 1, 1, 1, 0, 1, 0, 1, 0, 2, 2, 1, 0, 0, 1, 2, 2, 2, 2, 2, 0, 0, 1, 2, 0, 1, 0, 2, 1, 1, 0, 0, 2, 2, 2, 1, 2, 2, 0,
1, 1, 2, 2, 2, 0, 2, 0, 0, 0, 1, 1, 1, 0, 2, 1, 2, 2, 2, 2, 1, 2, 0, 0, 0, 2, 1, 0, 0, 2, 2, 0, 2, 2, 0, 2, 1, 0, 2, 1, 2, 0, 1,
1, 1, 1, 2, 2, 0, 0, 0, 1, 2, 1, 2, 1, 1, 0, 2, 0, 1, 1, 2, 1, 1, 1, 0, 0, 0, 0, 1, 1, 0, 1, 2, 0, 0, 1, 1, 1, 0, 1, 2, 2, 0, 0,
0, 0, 1, 1, 0, 0, 2, 1, 1, 2, 1, 2, 0, 1, 0, 1, 2, 2, 1, 0, 1, 1, 2, 1, 0, 2, 2, 0, 0, 2, 1, 2, 2, 0, 2, 1, 1, 0, 1, 1, 1, 1, 2,
2, 2, 1, 1, 1, 2, 2, 1, 2, 2, 2, 2, 2, 2, 2, 2, 1, 2, 2, 1, 2, 1, 2, 1, 2, 0, 0, 2, 1, 2, 1, 1, 0, 2, 1, 1, 0, 1, 0, 0, 0, 1, 1,
2, 2, 0, 2, 0, 0, 0, 1, 1, 0, 1, 1, 1, 0, 2, 2, 0, 1, 1, 1, 0, 0, 2, 2, 0, 0, 2, 0, 0, 1, 1, 2, 2, 1, 2, 1, 1, 0, 1, 2, 2, 1, 1,
2, 1, 1, 1, 1, 0, 1, 2, 1, 1, 0, 2, 2, 2, 0, 0, 2, 0, 2, 2, 0, 1, 2, 0, 2, 0, 1, 2, 2, 1, 1, 2, 1, 1, 2, 1, 1, 1, 0, 0, 2, 0, 0,
2, 2, 2, 1, 2, 2, 2, 1, 2, 0, 1, 0, 2, 1, 1, 1, 1, 0, 0, 0, 1, 1, 0, 1, 2, 1, 0, 0, 1, 0, 1, 1, 0, 0, 0, 2, 1, 1, 1, 1, 2, 2, 0,
1, 1, 2, 2, 1, 2, 2, 1, 1, 0, 2, 0, 0, 0, 2, 2, 1, 0, 0, 2, 2, 1, 0, 0, 0, 0, 1, 1, 0, 0, 0, 0, 2, 1, 1, 1, 2, 1, 0, 2, 0, 1, 2,
0, 1, 2, 2, 2, 0, 2, 2, 2, 2, 2, 0, 1, 2, 0, 2, 1, 1, 1, 1, 0, 1, 2, 1, 0, 1, 2, 1, 0, 0, 1, 0, 1, 1, 1, 2, 0, 1, 0, 0, 1, 2, 0,
2, 0, 2, 1, 1, 0, 1, 0, 2, 2, 2, 2, 2, 2, 2, 2, 1, 1, 1, 0, 0, 2, 0, 2, 2, 1, 1, 0, 1, 1, 1, 1, 1, 1, 2, 0, 0, 0, 2, 2, 1, 2, 0,
1, 0, 0, 0, 1, 0, 2, 0, 1, 1, 0, 0, 1, 1, 1, 2, 1, 2, 0, 1, 1, 1, 1, 1, 1, 2, 0, 0, 2, 2, 0, 2, 1, 1, 1, 2, 2, 0, 1, 2, 0, 0, 1,
1, 0, 2, 2, 2, 1, 2, 0, 2, 2, 0, 0, 0, 0, 1, 2, 0, 1, 0, 0, 2, 2, 2, 0, 1, 2, 2, 1, 1, 1, 0, 1, 2, 0, 0, 2, 1, 0, 0, 2, 2, 2, 1,
0, 2, 1, 2, 0, 0, 2, 2, 0, 2, 1, 2, 0, 1, 0, 1, 2, 2, 2, 1, 1, 1, 1, 0, 2, 2, 0, 0, 1, 1, 2, 1, 0, 2, 2, 2, 2, 1, 0, 2, 0, 2, 0,
0, 1, 2, 2, 0, 0, 1, 2, 1, 1, 2, 1, 2, 2, 1, 1, 2, 0, 0, 1, 1, 0, 2, 1, 2, 0, 1, 1, 1, 2, 0, 2, 1, 2, 2, 1, 2, 0, 1, 1, 1, 1, 2,
2, 0, 1, 0, 0, 1, 1, 1, 2, 2, 1, 2, 2, 0, 2, 2, 2, 0, 0, 2, 1, 2, 2, 0, 2, 2, 1, 0, 0, 1, 2, 0, 0, 2, 0, 2, 0, 0, 0, 0, 2, 0, 0,
0, 2, 2, 2, 1, 1, 2, 0, 2, 0, 0, 1, 1, 0, 1, 1, 0, 0, 1, 0, 2, 1, 2, 2, 0, 1, 1, 2, 2, 2, 2, 0, 0, 2, 1, 1, 2, 2, 1, 1, 2, 1, 2,
2, 0, 1, 0, 0, 2, 2, 0, 1, 1, 0, 2, 1, 2, 0, 1, 2, 2, 1, 0, 2, 2, 1, 2, 2, 2, 1, 2, 0, 2, 2, 0, 0, 1, 1, 0, 0, 1, 0, 1, 2, 2, 0,
2, 0, 2, 2, 0, 0, 2, 1, 1, 2, 2, 0, 2, 0, 2, 2, 1, 1, 1, 2, 1, 0, 0, 0, 2, 2, 2, 2, 2, 0, 2, 1, 2, 1, 1, 2, 0, 0, 0, 1, 1, 0, 1,
1, 2, 2, 2, 2, 0, 1, 2, 1, 0, 2, 1, 1, 0, 2, 0, 0, 1, 0, 0, 0, 1, 0, 2, 1, 1, 2, 0, 0, 2, 2, 1, 1, 0, 0, 0, 0, 0, 0, 2, 1, 0, 0,
1, 1, 2, 2, 0, 2, 0, 0, 2, 0, 2, 0, 2, 0, 1, 0, 1, 0, 1, 0, 1, 1, 0, 1, 0, 0, 1, 2, 0, 0, 2, 2, 0, 0, 0, 2, 2, 0, 1, 0, 2, 2, 2,
2, 2, 2, 2, 2, 2, 1, 0, 1, 2, 0, 0, 0, 2, 2, 1, 2, 0, 0, 2, 1, 2, 0, 0, 0, 0, 1, 0, 2, 2, 1, 0, 1, 1, 1, 0, 1, 2, 2, 0, 0, 1, 2,
0, 0, 1, 2, 2, 2, 0, 0, 2, 2, 2, 1, 0, 1, 0, 2, 1, 1, 2, 2, 0, 2, 0, 0, 2, 2, 0, 1, 1, 2, 1, 2, 2, 2, 2, 1, 0, 1, 1, 1, 0, 2, 2,
0, 0, 0, 1, 2, 0, 0, 1, 1, 1, 1, 2, 0, 1, 0, 1, 1, 2, 1, 0, 2, 0, 2, 2, 1, 1, 0, 0, 0, 2, 1, 2, 1, 0, 2, 0, 2, 1, 2, 2, 1, 1, 2,
2, 2, 0, 0, 0, 1, 1, 0, 2, 1, 2, 2, 1, 1, 0, 0, 0, 1, 1, 0, 1, 0, 1, 2, 2, 0, 2, 0, 1, 0, 1, 2, 0, 2, 1, 1, 2, 1, 1, 0, 2, 2, 0,
0, 2, 1, 0, 1, 1, 2, 1, 1, 0, 1, 1, 1, 2, 1, 2, 2, 0, 2, 1, 2, 2, 1, 2, 2, 1, 2, 2, 2, 1, 1, 1, 2, 2, 1, 2, 1, 2, 2, 2, 2, 2, 2,
1, 2, 1, 1, 0, 2, 2, 1, 2, 0, 0, 2, 0, 1, 1, 2, 2, 0, 1, 1, 0, 1, 0, 0, 2, 0, 0, 0, 2, 2, 0, 2, 2, 2, 1, 1, 2, 2, 1, 0, 1, 0, 0,
2, 2, 0, 1, 1, 2, 2, 1, 1, 0, 1, 2, 2, 0, 2, 2, 2, 0, 0, 1, 1, 1, 2, 2, 0, 0, 0, 2, 1, 0, 1, 2, 2, 0, 1, 1, 0, 1, 1, 2, 0, 1, 2,
2, 1, 2, 2, 0, 1, 2, 0, 1, 1, 0, 0, 2, 0, 1, 2, 1, 1, 2, 1, 2, 2, 1, 2, 0, 1, 0, 2, 2, 2, 1, 2, 0, 0, 0, 1, 1, 0, 0, 2, 1, 1, 1,
1, 0, 1, 1, 0, 0, 0, 1, 2, 1, 0, 0, 1, 2, 0, 1, 1, 2, 2, 2, 1, 1, 1, 1, 2, 2, 0, 0, 0, 2, 2, 1, 2, 2, 1, 1, 0, 0, 0, 0, 0, 1, 1,
1, 0, 0, 2, 2, 1, 0, 0, 0, 0, 1, 2, 2, 2, 1, 2, 1, 1, 1, 0, 2, 1, 2, 1, 0, 1, 0, 0, 0, 1, 2, 1, 0, 0, 2, 0, 1, 1, 1, 2, 1, 0, 0,
1, 2, 1, 2, 1, 2, 1, 1, 2, 2, 2, 2, 1, 0, 0, 1, 0, 0, 2, 1, 0, 1, 0, 2, 2, 2, 2, 2, 0, 2, 1, 1, 0, 1, 0, 0, 2, 0, 2, 2, 2, 2, 2,
1, 1, 1, 1, 1, 1, 2, 0, 2, 1, 1, 0, 1, 1, 0, 1, 0, 0, 0, 1, 0, 0, 2, 2, 1, 2, 0, 1, 1, 1, 2, 1, 0, 2, 0, 1, 1, 0, 1, 1, 2, 0, 0,
2, 2, 0, 1, 1, 1, 1, 2, 2, 0, 1, 2, 0, 2, 0, 2, 1, 1, 1, 2, 1, 2, 0, 2, 2, 0, 1, 1, 0, 2, 2, 0, 0, 0, 0, 2, 1, 0, 2, 0, 0, 0, 1,
0, 2, 1, 0, 1, 2, 0, 2, 0, 0, 2, 0, 0, 1, 2, 0, 1, 1, 1, 2, 0, 0, 1, 2, 1, 2, 2, 1, 2, 0, 0, 2, 1, 2, 1, 0, 2, 2, 2, 1, 0, 2, 1,
2, 0, 0, 1, 0, 2, 2, 2, 2, 2, 0, 0, 1, 1, 2, 0, 1, 2, 2, 0, 0, 1, 0, 2, 0, 2, 0, 2, 2, 1, 1, 2, 0, 1, 2, 1, 1, 2, 1, 2, 0, 1, 1,
1, 2, 0, 1, 1, 0, 2, 1, 2, 0, 1, 1, 1, 1, 0, 2, 1, 2, 2, 1, 1, 1, 1, 2, 2, 1, 2, 2, 0, 1, 0, 0, 0, 0, 2, 1, 2, 2, 2, 2, 0, 2, 2,
2, 1, 2, 0, 0, 2, 0, 0, 2, 2, 1, 0, 0, 2, 0, 0, 0, 0, 1, 0, 0, 0, 1, 0, 1, 1, 0, 2, 2, 2, 0, 0, 1, 1, 0, 0, 2, 2, 0, 2, 2, 1, 2,
0, 2, 1, 1, 2, 1, 1, 0, 0, 0, 1, 1, 0, 0, 2, 2, 0, 2, 1, 1, 0, 1, 2, 2, 1, 1, 2, 2, 2, 0, 1, 2, 2, 1, 0, 1, 1, 0, 2, 1, 2, 1, 2,
0, 2, 2, 0, 2, 2, 1, 2, 2, 1, 0, 1, 2, 2, 0, 0, 0, 1, 1, 0, 0, 2, 1, 1, 2, 2, 0, 2, 0, 2, 2, 0, 0, 1, 2, 1, 0, 0, 0, 2, 0, 2, 2,
1, 1, 2, 1, 2, 1, 1, 2, 2, 2, 2, 2, 2, 0, 1, 1, 2, 2, 2, 2, 0, 0, 0, 1, 1, 0, 1, 1, 0, 2, 0, 0, 0, 1, 2, 1, 0, 2, 1, 0, 0, 0, 1,
2, 1, 2, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 1, 1, 2, 2, 2, 1, 0, 0, 1, 1, 0, 1, 1, 0, 1, 1, 2, 0, 0, 0, 2, 2, 1, 0, 1, 1, 0, 0, 1, 1,
0, 0, 1, 0, 2, 1, 0, 0, 1, 1, 2, 2, 2, 2, 2, 1, 0, 1, 2, 0, 0, 0, 1, 0, 2, 2, 2, 2, 0, 0, 0, 2, 2, 0, 1, 1, 2, 2, 2, 1, 2, 0, 1,
1, 1, 1, 2, 1, 0, 2, 0, 2, 0, 1, 1, 1, 0, 2, 2, 0, 1, 2, 1, 0, 2, 2, 0, 0, 0, 2, 2, 0, 0, 1, 2, 1, 0, 1, 0, 1, 1, 1, 1, 2, 1, 2,
1, 2, 2, 2, 1, 2, 0, 0, 0, 2, 0, 1, 1, 1, 2, 0, 1, 1, 1, 1, 2, 0, 2, 2, 0, 2, 2, 1, 1, 2, 1, 1, 2, 1, 0, 2, 0, 2, 2, 0, 2, 1, 1,
2, 0, 0, 1, 1, 2, 1, 2, 2, 1, 1, 2, 1, 1, 2, 2, 0, 0, 0, 1, 0, 0, 1, 0, 0, 2, 0, 2, 1, 0, 2, 2, 1, 1, 0, 2, 0, 1, 0, 1, 2, 0, 2,
1, 1, 2, 1, 1, 0, 1, 2, 2, 0, 0, 0, 0, 1, 1, 0, 0, 1, 1, 0, 1, 0, 0, 0, 1, 1, 2, 2, 2, 0, 1, 0, 2, 2, 0, 0, 1, 2, 2, 0, 1, 0, 1,
1, 1, 1, 1, 1, 2, 0, 0, 0, 2, 1, 1, 1, 2, 2, 2, 0, 2, 2, 1, 1, 1, 2, 0, 0, 1, 1, 0, 1, 0, 0, 0, 0, 2, 1, 1, 2, 2, 2, 2, 1, 2, 2,
0, 1, 2, 0, 2, 0, 2, 0, 1, 1, 1, 0, 1, 0, 0, 2, 2, 0, 0, 2, 2, 0, 1, 1, 2, 2, 0, 0, 2, 0, 1, 2, 2, 2, 0, 1, 1, 0, 1, 1, 2, 0, 2,
2, 1, 1, 0, 2, 1, 0, 2, 0, 2, 0, 1, 1, 2, 2, 1, 1, 1, 1, 0, 0, 2, 0, 1, 2, 1, 1, 2, 1, 2, 2, 0, 1, 2, 0, 2, 0, 1, 2, 2, 1, 2, 0,
2, 2, 1, 1, 2, 2, 2, 2, 0, 2, 2, 1, 1, 2, 1, 2, 0, 2, 2, 2, 1, 0, 0, 1, 0, 0, 0, 2, 1, 0, 0, 2, 2, 1, 2, 0, 2, 1, 0, 0, 2, 2,
0, 2, 0, 2, 1, 2, 0, 0, 1, 2, 1, 2, 0, 2, 2, 0, 2, 0, 1, 2, 1, 2, 2, 2, 1, 2, 0, 0, 0, 2, 1, 2, 0, 1, 0, 1, 2, 2, 1, 0, 1, 0, 0,
0, 2, 0, 1, 2, 0, 1, 0, 0, 1, 2, 0, 2, 2, 1, 1, 2, 1, 0, 2, 2, 1, 1, 2, 0, 1, 2, 0, 1, 1, 1, 0, 0, 1, 1, 2, 0, 1, 2, 1, 0, 0, 1,
0, 0, 2, 0, 2, 2, 2, 2, 1, 1, 2, 0, 2, 1, 1, 0, 1, 0, 2, 2, 2, 2, 1, 1, 2, 2, 1, 0, 1, 0, 2, 0, 1, 0, 1, 1, 1, 1, 2, 1, 2, 0, 0,
2, 0, 2, 1, 0, 0, 1, 0, 1, 0, 2, 1, 1, 0, 0, 1, 1, 1, 2, 2, 1, 0, 2, 1, 0, 1, 0, 1, 0, 1, 1, 2, 1, 2, 0, 2, 0, 1, 0, 2, 0, 0, 0,
1, 2, 2, 2, 2, 1, 2, 2, 1, 0, 0, 1, 2, 1, 0, 1, 2, 0, 2, 1, 2, 0, 2, 0, 0, 2, 2, 2, 0, 1, 2, 1, 1, 1, 1, 0, 2, 2, 0, 1, 2, 2, 0,
1, 1, 0, 0, 1, 2, 2, 2, 0, 1, 1, 0, 0, 2, 1, 0, 1, 1, 2, 0, 0, 1, 2, 0, 0, 1, 2, 2, 0, 0, 1, 0, 2, 0, 2, 0, 2, 0, 0, 1, 1, 2, 1,
0, 2, 2, 2, 2, 0, 0, 2, 2, 1, 0, 0, 1, 2, 0, 0, 2, 2, 2, 1, 0, 1, 0, 1, 1, 0, 1, 2, 1, 0, 2, 2, 0, 1, 1, 1, 2, 0, 2, 0, 2, 0, 0,
0, 1, 0, 1, 1, 2, 0, 2, 2, 0, 0, 2, 1, 0, 1, 0, 0, 0, 1, 2, 0, 2, 0, 2, 2, 0, 1, 1, 1, 0, 0, 1, 0, 2, 2, 0, 2, 0, 1, 0, 0, 2, 2,
2, 2, 0, 0, 1, 1, 0, 0, 2, 2, 1, 1, 2, 0, 0, 1, 0, 0, 2, 2, 2, 0, 0, 1, 0, 1, 0, 2, 0, 2, 1, 1, 2, 0, 0, 1, 2, 2, 1, 1, 2, 0, 0,
0, 2, 2, 0, 2, 2, 2, 1, 2, 0, 2, 2, 0, 2, 2, 1, 2, 2, 0, 1, 1, 0, 2, 1, 2, 0, 1, 2, 2, 1, 1, 1, 2, 0, 2, 1, 0, 1, 1, 0, 2, 0, 1,
2, 0, 2, 2, 1, 1, 2, 1, 1, 2, 1, 1, 2, 0, 1, 2, 1, 1, 1, 1, 0, 1, 0, 1, 0, 0, 0, 1, 0, 1, 1, 0, 1, 1, 2, 0, 1, 2, 0, 2, 2, 2, 0,
2, 2, 2, 2, 2, 0, 1, 1, 2, 2, 0, 1, 1, 0, 0, 0, 1, 0, 0, 0, 2, 1, 2, 1, 0, 2, 0, 0, 0, 0, 0, 0, 0, 0, 2, 2, 1, 1, 1, 2, 0, 0, 2,
2, 2, 2, 0, 0, 2, 0, 2, 0, 2, 2, 0, 0, 1, 0, 0, 0, 1, 1, 0, 2, 2, 0, 2, 2, 1, 2, 0, 0, 2, 2, 0, 1, 2, 0, 0, 0, 2, 2, 2, 2, 2, 1,
0, 0, 0, 2, 2, 0, 1, 0, 2, 2, 2, 2, 2, 0, 1, 1, 1, 1, 1, 1, 2, 2, 2, 1, 0, 1, 1, 1, 0, 2, 2, 1, 0, 2, 0, 2, 2, 2, 0, 0, 0, 2, 2,
0, 1, 2, 1, 0, 0, 1, 0, 1, 1, 1, 2, 0, 0, 1, 2, 1, 2, 2, 1, 2, 0, 0, 1, 2, 1, 2, 1, 2, 2, 0, 1, 1, 1, 1, 0, 2, 0, 1, 1, 1, 2, 0,
1, 1, 0, 1, 1, 2, 2, 1, 0, 2, 1, 2, 0, 1, 1, 1, 2, 0, 1, 0, 2, 1, 2, 2, 1, 1, 1, 2, 0, 0, 1, 1, 1, 2, 2, 1, 1, 0, 0, 0, 1, 0, 0,
1, 0, 0, 1, 2, 0, 2, 1, 1, 1, 2, 2, 0, 0, 2, 1, 1, 2, 1, 2, 0, 1, 0, 1, 2, 0, 0, 0, 1, 1, 0, 1, 0, 1, 2, 2, 0, 0, 0, 1, 1, 2, 2,
0, 1, 1, 0, 1, 0, 0, 0, 1, 2, 2, 0, 2, 0, 1, 0, 2, 2, 1, 1, 2, 0, 0, 0, 1, 0, 1, 1, 1, 1, 2, 0, 2, 2, 1, 1, 2, 1, 1, 1, 2, 2, 0,
1, 0, 0, 0, 0, 1, 2, 0, 0, 1, 1, 2, 1, 2, 2, 0, 1, 2, 1, 1, 2, 2, 2, 2, 0, 1, 0, 1, 0, 2, 2, 2, 0, 1, 0, 0, 2, 2, 0, 1, 1, 2, 2,
1, 0, 0, 1, 1, 0, 1, 1, 2, 1, 2, 1, 0, 0, 1, 0, 0, 2, 1, 1, 0, 1, 0, 1, 2, 2, 1, 0, 0, 2, 0, 2, 0, 2, 0, 0, 1, 1, 1, 2, 1, 1, 2,
1, 1, 1, 0, 0, 2, 0, 2, 0, 1, 2, 2, 1, 2, 2, 0, 1, 2, 0, 2, 2, 2, 2, 2, 0, 2, 0, 2, 2, 1, 1, 2, 0, 2, 2, 2, 2, 2, 2, 1, 1, 2, 1,
0, 2, 1, 0, 0, 2, 1, 0, 0, 1, 0, 0, 0, 2, 2, 0, 2, 0, 2, 1, 2, 0, 2, 1, 2, 1, 2, 0, 2, 2, 2, 1, 2, 0, 0, 1, 2, 2, 2, 1, 2, 0, 0,
2, 0, 1, 2, 1, 0, 0, 1, 2, 1, 0, 2, 0, 2, 1, 1, 2, 2, 2, 0, 2, 1, 1, 1, 0, 2, 1, 1, 0, 0, 2, 1, 0, 0, 1, 2, 0, 2, 2, 1, 1, 0, 2,
2, 0, 2, 2, 0, 0, 2, 1, 1, 0, 2, 1, 1, 0, 2, 2, 2, 2, 0, 1, 2, 2, 2, 2, 1, 1, 1, 0, 0, 2, 0, 2, 1, 0, 2, 2, 2, 2, 2, 0, 2, 1, 1,
0, 1, 0, 2, 0, 1, 0, 1, 1, 2, 2, 1, 0, 0, 0, 2, 0, 2, 1, 1, 1, 1, 2, 1, 2, 2, 1, 1, 0, 0, 1, 0, 0, 1, 0, 1, 0, 2, 1, 0, 1, 0, 1,
1, 1, 2, 2, 1, 0, 0, 2, 0, 1, 0, 2, 0, 1, 1, 2, 1, 2, 2, 2, 1, 2, 2, 1, 0, 0, 0, 1, 2, 2, 0, 0, 2, 1, 2, 0, 2, 1, 0, 1, 2, 1, 1,
1, 0, 1, 0, 1, 0, 2, 1, 0, 0, 2, 1, 2, 0, 0, 2, 0, 0, 2, 2, 1, 0, 2, 1, 0, 2, 2, 0, 1, 1, 2, 2, 1, 1, 1, 2, 2, 1, 2, 2, 0, 0, 2,
1, 0, 1, 0, 2, 0, 2, 0, 0, 1, 2, 2, 0, 0, 1, 0, 2, 2, 2, 2, 2, 0, 0, 1, 1, 2, 0, 1, 2, 0, 0, 2, 0, 0, 2, 2, 1, 0, 1, 1, 0, 1, 2,
1, 2, 2, 1, 0, 1, 0, 1, 2, 0, 2, 0, 2, 0, 2, 2, 0, 1, 1, 1, 2, 0, 2, 2, 0, 0, 0, 0, 1, 0, 1, 0, 0, 1, 2, 0, 2, 0, 2, 1, 0, 1, 0,

1, 0, 0, 1, 0, 2, 0, 2, 2, 0, 1, 1, 2, 0, 1, 0, 2, 0, 0, 1, 1, 1, 2, 0, 0, 1, 1, 0, 0, 1, 1, 0, 2, 2, 2, 2, 1, 1, 2, 2, 2, 0, 1,
0, 1, 0, 1, 1, 2, 2, 0, 1, 2, 0, 2, 2, 1, 0, 2, 1, 1, 1, 0, 2, 1, 1, 0, 0, 1, 0, 0, 2, 2, 1, 2, 2, 2, 1, 2, 0, 2, 2, 2, 0, 1, 2,
2, 1, 0, 1, 1, 0, 2, 1, 0, 1, 1, 0, 2, 0, 1, 1, 2, 0, 2, 1, 1, 2, 1, 1, 2, 1, 1, 2, 0, 2, 2, 1, 1, 1, 1, 0, 1, 0, 1, 2, 0, 1, 2,
1, 1, 1, 0, 1, 1, 2, 1, 0, 0, 0, 1, 0, 2, 0, 2, 2, 2, 2, 0, 1, 2, 0, 2, 2, 0, 1, 1, 0, 0, 0, 2, 0, 1, 1, 2, 2, 1, 1, 0, 2, 1, 0,
2, 1, 0, 2, 1, 2, 2, 2, 2, 1, 2, 1, 1, 1, 0, 2, 1, 1, 1, 2, 0, 1, 2, 0, 2, 0, 0, 0, 0, 0, 1, 2, 1, 2, 2, 2, 2, 0, 0, 0, 0, 2, 2,
0, 1, 2, 2, 2, 1, 1, 0, 2, 0, 0, 2, 2, 2, 2, 1, 2, 2, 2, 2, 2, 0, 2, 0, 2, 1, 0, 0, 0, 1, 2, 2, 0, 0, 0, 0, 1, 0, 2, 1, 1, 0, 1,
2, 1, 0, 2, 2, 0, 1, 0, 2, 0, 0, 2, 0, 1, 2, 0, 2, 0, 1, 0, 1, 0, 0, 2, 0, 1, 2, 1, 1, 1, 1, 0, 2, 0, 1, 1, 0, 0, 1, 0, 1, 2, 2,
0, 1, 1, 0, 1, 0, 1, 2, 2, 0, 0, 1, 1, 2, 0, 2, 0, 1, 0, 1, 2, 1, 2, 2, 2, 2, 2, 0, 1, 1, 2, 2, 2, 0, 1, 2, 1, 1, 1, 1, 2, 0, 1,
0, 1, 2, 0, 1, 0, 0, 1, 0, 1, 2, 0, 1, 2, 0, 2, 0, 2, 2, 2, 2, 2, 2, 1, 2, 0, 1, 2, 1, 1, 2, 2, 0, 1, 0, 0, 0, 2, 2, 1, 1, 1, 1,
2, 0, 1, 2, 1, 0, 0, 0, 0, 0, 0, 2, 2, 2, 2, 0, 0, 0, 2, 0, 1, 1, 1, 0, 1, 2, 1, 1, 0, 1, 1, 2, 1, 1, 0, 0, 0, 2, 1, 0, 0, 2, 2,
2, 0, 2, 0, 0, 2, 1, 1, 1, 2, 0, 0, 0, 2, 0, 0, 2, 1, 2, 0, 1, 0, 0, 1, 2, 0, 1, 1, 2, 0, 1, 1, 1, 1, 2, 1, 0, 1, 1, 2, 2, 1, 1,
0, 0, 1, 2, 1, 1, 2, 1, 0, 0, 0, 0, 2, 1, 0, 1, 0, 2, 0, 0, 2, 2, 1, 2, 2, 0, 0, 1, 0, 1, 2, 1, 0, 1, 2, 2, 0, 2, 2, 1, 2, 2, 1,
1, 2, 2, 0, 2, 0, 1, 0, 2, 1, 2, 1, 1, 2, 0, 2, 0, 0, 0, 0, 1, 2, 1, 0, 0, 1, 2, 2, 2, 2, 0, 1, 1, 0, 0, 0, 2, 0, 1, 0, 1, 1, 0,
0, 1, 2, 0, 0, 0, 1, 2, 1, 1, 1, 1, 0, 2, 0, 0, 0, 0, 1, 2, 0, 1, 0, 0, 0, 0, 0, 1, 1, 1, 2, 1, 1, 0, 0, 0, 0, 2, 0, 2, 2, 0, 1,
1, 1, 2, 2, 0, 1, 2, 2, 0, 2, 1, 2, 1, 0, 2, 0, 0, 2, 1, 0, 2, 2, 2, 2, 1, 2, 1, 0, 2, 2, 0, 2, 2, 2, 0, 1, 2, 2, 0, 1, 1, 0, 2,
2, 0, 1, 1, 0, 1, 1, 2, 1, 1, 2, 2, 2, 1, 1, 1, 2, 1, 0, 2, 1, 0, 1, 1, 1, 2, 0, 1, 2, 2, 2, 2, 0, 1, 2, 0, 0, 1, 2, 1, 2, 2, 2,
0, 2, 2, 0, 0, 2, 0, 2, 2, 2, 2, 1, 1, 2, 0, 0, 1, 2, 1, 2, 2, 1, 2, 1, 1, 0, 0, 0, 2, 2, 0, 0, 0, 0, 1, 2, 0, 1, 1, 2, 2, 1, 1,
1, 2, 0, 2, 0, 0, 2, 2, 2, 1, 0, 2, 0, 0, 2, 1, 2, 1, 0, 2, 0, 1, 0, 1, 1, 0, 2, 1, 1, 1, 0, 0, 2, 0, 2, 0, 1, 1, 1, 2, 2, 1, 2,
1, 0, 2, 0, 2, 2, 2, 1, 2, 0, 1, 1, 1, 0, 1, 0, 2, 0, 0, 0, 1, 1, 2, 2, 0, 0, 2, 2, 1, 1, 2, 1, 2, 1, 0, 2, 0, 1, 2, 1, 0, 0, 2,
0, 0, 2, 0, 2, 2, 2, 1, 1, 2, 2, 0, 2, 2, 0, 2, 2, 0, 0, 0, 0, 0, 0, 2, 0, 2, 2, 2, 0, 2, 1, 0, 1, 0, 1, 1, 2, 0, 2, 0, 2, 2, 0,
0, 2, 2, 2, 2, 2, 0, 2, 2, 2, 0, 0, 0, 2, 1, 2, 2, 1, 0, 0, 2, 0, 0, 2, 0, 1, 0, 2, 2, 0, 0, 1, 1, 2, 1, 0, 1, 2, 1, 2, 2, 0, 2,
0, 0, 1, 0, 0, 0, 2, 1, 2, 1, 2, 0, 0, 1, 2, 2, 0, 1, 2, 1, 0, 0, 2, 1, 1, 0, 0, 0, 0, 0, 1, 1, 2, 1, 0, 1, 1, 1, 2, 1, 1, 2, 2,
1, 1, 0, 2, 2, 0, 1, 1, 0, 2, 2, 0, 0, 1, 2, 0, 0, 2, 0, 1, 1, 2, 2, 0, 0, 0, 1, 2, 0, 2, 2, 2, 0, 2, 1, 0, 0, 1, 1, 0, 2, 1, 0,
0, 2, 1, 2, 1, 2, 1, 2, 0, 0, 2, 1, 2, 2, 2, 1, 0, 1, 1, 0, 0, 0, 1, 0, 1, 1, 0, 0, 1, 0, 1, 2, 2, 2, 2, 2, 2, 0, 0, 0, 0, 1, 0,
2, 2, 1, 0, 1, 2, 0, 0, 1, 1, 0, 1, 2, 0, 2, 2, 1, 2, 1, 1, 2, 2, 1, 0, 1, 2, 1, 2, 2, 1, 2, 0, 1, 1, 0, 1, 1, 2, 1, 1, 2, 0, 2,
2, 2, 0, 1, 2, 0, 0, 0, 1, 1, 1, 2, 0, 2, 0, 1, 1, 0, 1, 2, 0, 1, 2, 0, 1, 2, 1, 2, 0, 0, 0, 1, 2, 1, 2, 0, 0, 0, 2, 0, 2, 1, 0,
2, 0, 0, 0, 0, 2, 0, 0, 2, 1, 2, 1, 0, 1, 1, 1, 1, 0, 1, 1, 0, 2, 1, 0, 0, 1, 0, 1, 2, 2, 1, 2, 2, 2, 2, 2, 0, 0, 2, 2, 2, 2, 0,
0, 1, 2, 2, 0, 0, 2, 0, 2, 1, 0, 1, 1, 0, 1, 2, 1, 0, 0, 0, 1, 0, 2, 2, 0, 0, 2, 0, 1, 0, 2, 2, 0, 1, 0, 1, 0, 0, 2, 0, 1, 2, 0,
2, 0, 1, 0, 2, 0, 1, 1, 0, 0, 2, 1, 1, 1, 1, 0, 1, 1, 0, 1, 0, 1, 1, 0, 1, 2, 2, 0, 2, 0, 2, 0, 1, 0, 2, 2, 0, 0, 1, 1, 1, 2, 2,
1, 1, 1, 1, 1, 0, 2, 2, 1, 2, 1, 0, 0, 1, 2, 1, 0, 0, 0, 2, 2, 2, 1, 0, 2, 1, 0, 1, 0, 1, 0, 2, 0, 2, 0, 2, 1, 1, 1, 0, 0, 0, 0,
1, 2, 1, 0, 2, 1, 0, 2, 0, 2, 1, 1, 1, 1, 1, 2, 0, 1, 0, 0, 0, 2, 2, 1, 0, 0, 2, 2, 2, 2, 2, 1, 0, 0, 0, 0, 1, 1, 0, 1, 2, 1, 0,
0, 0, 2, 0, 1, 1, 0, 0, 0, 2, 1, 1, 0, 1, 1, 2, 1, 2, 0, 0, 2, 1, 1, 0, 0, 2, 2, 2, 0, 0, 0, 2, 1, 2, 0, 1, 2, 0, 0, 0, 2, 1, 1,
2, 0, 1, 1, 1, 0, 0, 1, 2, 0, 1, 2, 2, 1, 1, 0, 1, 1, 2, 1, 0, 1, 0, 1, 1, 2, 2, 1, 2, 0, 0, 0, 1, 1, 1, 2, 1, 2, 2, 0, 1, 2, 2,
0, 0, 1, 1, 2, 1, 1, 2, 1, 0, 2, 1, 2, 2, 0, 2, 1, 2, 2, 1, 2, 2, 1, 1, 0, 2, 0, 1, 2, 0, 1, 1, 2, 1, 2, 1, 0, 1, 0, 0, 1, 2, 2,
0, 0, 0, 0, 1, 2, 0, 0, 2, 0, 1, 0, 2, 2, 0, 1, 1, 0, 0, 0, 0, 1, 2, 1, 1, 1, 0, 0, 1, 2, 0, 0, 0, 1, 2, 0, 1, 1, 1, 0, 2, 0, 1,
1, 1, 2, 1, 1, 1, 0, 0, 0, 0, 0, 2, 2, 0, 1, 1, 1, 0, 0, 0, 0, 2, 0, 0, 2, 1, 2, 1, 0, 2, 2, 0, 1, 2, 2, 2, 2, 2, 2, 1, 2, 2, 0,
0, 2, 1, 0, 1, 0, 1, 1, 0, 1, 1, 1, 0, 2, 0, 1, 1, 2, 2, 1, 0, 1, 1, 2, 2, 1, 0, 2, 1, 0, 0, 1, 0, 1, 0, 2, 2, 2, 1, 0, 1, 0, 1,
2, 0, 2, 1, 0, 2, 1, 0, 1, 0, 2, 1, 0, 0, 0, 2, 1, 2, 1, 0, 2, 0, 2, 2, 2, 2, 2, 0, 2, 2, 0, 1, 2, 1, 2, 2, 1, 2, 1, 1, 2, 0, 0,
2, 2, 0, 0, 0, 0, 2, 1, 1, 0, 0, 0, 2, 1, 1, 1, 2, 0, 1, 2, 0, 1, 1, 2, 1, 0, 2, 0, 0, 2, 2, 0, 0, 2, 2, 2, 1, 0, 1, 1, 0, 2, 1,
2, 1, 0, 2, 0, 0, 2, 0, 1, 1, 1, 1, 1, 1, 0, 0, 2, 2, 0, 2, 2, 2, 1, 2, 2, 1, 2, 1, 0, 2, 0, 2, 2, 2, 0, 2, 0, 1, 0, 0, 0, 2, 2,
1, 1, 0, 0, 1, 1, 2, 2, 1, 2, 1, 2, 0, 1, 2, 2, 2, 2, 0, 1, 2, 0, 2, 0, 2, 0, 0, 1, 0, 0, 2, 2, 1, 1, 0, 0, 1, 0, 0, 0, 0, 0, 0,
0, 2, 1, 0, 1, 0, 1, 2, 0, 2, 2, 2, 0, 2, 0, 0, 2, 2, 2, 1, 2, 0, 2, 0, 2, 0, 0, 0, 2, 1, 2, 2, 2, 0, 2, 2, 2, 0, 2, 0, 1, 0, 2,
2, 1, 0, 0, 2, 0, 1, 0, 1, 2, 1, 2, 2, 0, 0, 1, 1, 2, 0, 0, 0, 2, 1, 2, 2, 0, 2, 0, 0, 1, 2, 0, 1, 2, 1, 0, 1, 2, 0, 0, 1, 2, 0,
0, 0, 1, 1, 2, 0, 2, 1, 1, 0, 0, 1, 0, 2, 2, 2, 1, 2, 2, 1, 1, 0, 2, 2, 1, 1, 2, 0, 2, 2, 1, 1, 2, 2, 2, 1, 2, 2, 1, 2, 2, 2, 1,
1, 0, 0, 0, 2, 0, 2, 1, 0, 1, 1, 2, 0, 1, 2, 2, 0, 1, 2, 0, 0, 0, 1, 2, 1, 2, 1, 2, 2, 2, 1, 0, 2, 1, 2, 0, 0, 2, 0, 1, 1, 0, 0,
1, 1, 1, 0, 0, 0, 1, 2, 2, 0, 0, 0, 0, 0, 1, 2, 2, 2, 2, 1, 2, 0, 0, 1, 1, 1, 0, 2, 2, 1, 0, 1, 2, 1, 1, 2, 2, 0, 1, 2, 0, 2, 2,
2, 1, 2, 0, 1, 1, 1, 0, 1, 2, 1, 2, 2, 0, 2, 2, 2, 0, 0, 1, 1, 2, 1, 1, 1, 1, 2, 0, 2, 0, 1, 2, 0, 0, 0, 1, 0, 2, 0, 1, 1, 2, 2,
2, 2, 1, 1, 1, 2, 1, 2, 1, 0, 0, 1, 0, 1, 0, 1, 2, 1, 2, 0, 0, 0, 1, 1, 2, 1, 0, 1, 2, 0, 0, 1, 2, 2, 2, 2, 1, 1, 1, 2, 1, 1, 0,
0, 1, 1, 0, 2, 2, 1, 0, 2, 1, 0, 2, 2, 1, 2, 0, 1, 2, 0, 2, 0, 2, 2, 0, 0, 2, 0, 0, 0, 0, 0, 2, 2, 0, 2, 2, 1, 0, 1, 1, 0, 1, 1,
2, 1, 1, 0, 2, 1, 2, 2, 2, 0, 2, 2, 1, 2, 1, 2, 1, 0, 1, 1, 1, 2, 0, 2, 1, 2, 1, 0, 2, 0, 0, 0, 1, 0, 1, 2, 2, 0, 2, 0, 2, 2, 0,
0, 1, 0, 0, 1, 0, 1, 0, 2, 1, 1, 2, 2, 2, 2, 2, 2, 0, 2, 2, 2, 0, 1, 2, 1, 0, 0, 0, 1, 2, 0, 0, 1, 1, 2, 2, 0, 0, 2, 1, 1, 2, 2,
2, 2, 1, 2, 0, 1, 2, 2, 1, 2, 1, 2, 0, 2, 0, 2, 0, 0, 2, 0, 0, 0, 2, 2, 1, 2, 0, 0, 2, 2, 2, 0, 0, 1, 1, 0, 2, 1, 2, 2, 0, 0, 2,
0, 2, 0, 0, 0, 0, 1, 1, 1, 0, 1, 2, 1, 2, 0, 2, 2, 1, 1, 0, 2, 2, 2, 0, 0, 1, 0, 1, 0, 1, 1, 2, 2, 1, 2, 1, 1, 1, 2, 2, 1, 1, 0,
2, 2, 2, 0, 0, 2, 2, 0, 1, 0, 1, 1, 2, 1, 2, 2, 1, 1, 2, 0, 1, 1, 2, 1, 2, 1, 0, 0, 1, 2, 2, 0, 0, 1, 2, 0, 0, 0, 1, 0, 0, 1, 2,
2, 2, 2, 0, 1, 2, 2, 2, 2, 0, 1, 2, 2, 1, 2, 0, 2, 2, 2, 0, 1, 2, 0, 0, 1, 0, 2, 2, 0, 0, 2, 2, 0, 1, 0, 2, 2, 0, 1, 2, 1, 0, 0,
1, 0, 2, 0, 0, 0, 0, 1, 1, 1, 2, 2, 0, 1, 1, 2, 2, 2, 0, 2, 2, 2, 2, 1, 2, 0, 0, 0, 1, 1, 1, 2, 2, 0, 0, 2, 2, 0, 1, 2, 1, 0, 0,
1, 1, 2, 2, 0, 1, 1, 0, 1, 2, 2, 1, 2, 1, 1, 2, 2, 0, 2, 2, 0, 1, 2, 0, 1, 0, 0, 1, 2, 2, 1, 2, 2, 2, 0, 1, 0, 0, 1, 2, 2, 1, 0,
2, 2, 0, 0, 1, 0, 2, 2, 1, 1, 0, 0, 1, 0, 2, 2, 1, 0, 2, 0, 1, 0, 0, 1, 2, 1, 2, 1, 0, 2, 1, 1, 2, 1, 2, 2, 1, 0, 1, 2, 1, 1, 2,
2, 0, 2, 0, 0, 2, 0, 0, 1, 1, 1, 0, 1, 2, 1, 0, 2, 1, 0, 0, 2, 2, 2, 2, 2, 0, 0, 2, 0, 1, 0, 0, 0, 1, 2, 2, 1, 1, 2, 1, 2, 2, 1,
2, 2, 0, 0, 1, 1, 0, 2, 2, 2, 2, 1, 2, 1, 2, 0, 2, 1, 0, 1, 1, 2, 2, 0, 1, 2, 2, 1, 0, 2, 2, 0, 2, 0, 0, 0, 1, 0, 1, 1, 2, 0, 1,
2, 2, 1, 2, 2, 2, 2, 0, 1, 0, 1, 1, 2, 1, 0, 1, 0, 2, 2, 0, 0, 2, 0, 2, 0, 0, 0, 0, 1, 1, 2, 1, 0, 2, 0, 1, 2, 0, 2, 0, 2, 1, 0,
2, 0, 2, 0, 2, 1, 0, 1, 1, 1, 1, 2, 0, 1, 1, 2, 0, 0, 2, 2, 2, 0, 1, 1, 1, 1, 2, 0, 0, 2, 2, 0, 1, 0, 2, 2, 2, 0, 0, 2, 2, 2, 0,
2, 2, 2, 1, 0, 0, 0, 0, 0, 0, 0, 2, 2, 0, 0, 1, 0, 1, 0, 0, 1, 1, 0, 2, 0, 0, 0, 1, 1, 1, 2, 1, 0, 2, 1, 2, 1, 1, 1, 1, 2, 0, 1,
1, 2, 0, 0, 1, 0, 1, 1, 2, 0, 2, 0, 0, 1, 1, 1, 0, 1, 2, 2, 0, 0, 0, 0, 2, 2, 2, 0, 0, 1, 2, 2, 2, 0, 1, 1, 2, 0, 1, 0, 2, 1, 1,
2, 1, 1, 2, 1, 2, 2, 2, 2, 1, 1, 0, 1, 1, 1, 2, 1, 1, 2, 1, 1, 1, 1, 2, 2, 1, 0, 0, 2, 2, 0, 0, 0, 2, 0, 1, 1, 2, 2, 2, 1, 2, 2,
1, 0, 0, 0, 1, 2, 2, 2, 0, 2, 2, 0, 1, 2, 1, 0, 0, 1, 2, 2, 1, 0, 0, 2, 1, 1, 1, 1, 1, 0, 1, 2, 0, 1, 0, 2, 2, 1, 2, 1, 0, 1, 2,
1, 0, 0, 2, 0, 2, 2, 1, 2, 1, 2, 1, 0, 0, 2, 1, 1, 0, 1, 2, 0, 2, 2, 0, 2, 0, 1, 0, 0, 1, 0, 0, 0, 1, 2, 0, 0, 0, 2, 0, 1, 2, 2,
2, 2, 1, 1, 0, 2, 1, 0, 2, 0, 2, 1, 2, 0, 1, 2, 2, 1, 2, 2, 2, 2, 1, 1, 0, 0, 2, 0, 0, 2, 0, 2, 1, 1, 1, 1, 1, 1, 1, 0, 2, 0, 2,
0, 2, 1, 2, 1, 0, 0, 0, 2, 2, 2, 1, 1, 2, 0, 2, 0, 2, 2, 1, 1, 1, 1, 2, 0, 2, 0, 1, 0, 1, 0, 0, 0, 1, 0, 1, 2, 1, 1, 1, 0, 2, 2,
2, 2, 0, 1, 0, 2, 1, 0, 2, 1, 1, 0, 1, 2, 2, 2, 0, 1, 2, 0, 2, 2, 0, 1, 0, 0, 1, 2, 2, 2, 0, 1, 2, 0, 2, 2, 1, 0, 2, 2, 1, 0, 0,
0, 0, 0, 2, 0, 2, 2, 0, 0, 2, 1, 1, 0, 1, 1, 2, 2, 0, 2, 2, 1, 0, 2, 2, 0, 2, 2, 1, 1, 1, 0, 2, 1, 2, 1, 1, 2, 0, 2, 1, 2, 1, 2,
1, 0, 1, 0, 1, 0, 1, 2, 2, 2, 1, 0, 2, 0, 0, 0, 1, 0, 0, 1, 0, 0, 2, 0, 2, 2, 0, 2, 2, 2, 2, 2, 0, 1, 0, 2, 1, 1, 2, 2, 2, 1, 0,
2, 1, 2, 0, 0, 0, 2, 1, 2, 2, 1, 1, 0, 1, 2, 0, 1, 1, 2, 1, 2, 0, 0, 0, 2, 0, 1, 2, 2, 1, 1, 2, 2, 1, 2, 1, 2, 0, 0, 0, 2, 0, 1,
1, 1, 0, 2, 0, 1, 1, 1, 1, 0, 0, 2, 0, 0, 2, 0, 2, 1, 1, 0, 2, 1, 2, 1, 0, 1, 2, 1, 2, 0, 0, 0, 0, 1, 1, 2, 2, 2, 0, 0, 1, 0, 2,
2, 1, 1, 0, 2, 1, 2, 1, 1, 1, 0, 1, 0, 1, 1, 2, 2, 2, 0, 0, 2, 2, 2, 2, 1, 1, 0, 2, 1, 2, 2, 1, 1, 2, 0, 1, 0, 1, 1, 2, 1, 0, 0,
1, 2, 2, 0, 1, 1, 2, 1, 2, 0, 1, 0, 0, 1, 2, 0, 0, 1, 2, 0, 0, 2, 2, 1, 0, 2, 1, 1, 1, 1, 0, 2, 1, 1, 0, 2, 0, 0, 1, 0, 2, 1, 0,
0, 0, 2, 2, 0, 1, 0, 0, 1, 0, 2, 2, 1, 1, 0, 2, 1, 0, 1, 0, 2, 0, 1, 0, 0, 0, 1, 1, 0, 0, 2, 1, 2, 2, 1, 0, 2, 2, 1, 2, 0, 0, 2,
2, 2, 0, 2, 2, 0, 0, 2, 2, 0, 1, 0, 1, 1, 1, 2, 2, 2, 2, 0, 1, 1, 0, 2, 1, 0, 0, 1, 1, 1, 2, 2, 0, 2, 2, 1, 2, 2, 1, 2, 1, 0, 1,
2, 2, 1, 2, 0, 1, 2, 0, 1, 0, 1, 2, 2, 1, 0, 2, 2, 2, 0, 1, 0, 0, 2, 1, 1, 0, 0, 1, 2, 0, 0, 1, 0, 2, 0, 1, 0, 0, 1, 2, 0, 2, 2,
1, 0, 2, 1, 2, 2, 0, 1, 2, 2, 1, 2, 1, 1, 2, 0, 1, 0, 2, 2, 1, 1, 0, 1, 0, 1, 1, 2, 2, 1, 1, 1, 0, 0, 1, 2, 1, 1, 2, 1, 1, 2, 2,

0, 1, 0, 1, 1, 0, 2, 2, 0, 2, 2, 2, 0, 1, 2, 0, 2, 2, 0, 1, 1, 0, 2, 2, 2, 2, 1, 2, 2, 0, 0, 2, 1, 0, 1, 1, 2, 2, 1, 2, 1, 2, 1,
0, 2, 2, 0, 2, 2, 2, 0, 1, 2, 2, 1, 2, 0, 1, 2, 2, 0, 0, 0, 1, 0, 1, 1, 0, 1, 1, 2, 1, 1, 2, 2, 2, 2, 0, 0, 2, 0, 2, 0, 0, 0, 1,
0, 2, 2, 0, 0, 2, 0, 1, 2, 0, 0, 0, 1, 1, 2, 1, 0, 2, 0, 2, 1, 0, 2, 0, 2, 1, 0, 2, 1, 1, 2, 2, 1, 0, 2, 2, 1, 0, 1, 1, 2, 1, 0,
0, 0, 2, 2, 2, 1, 0, 0, 1, 0, 1, 2, 1, 0, 1, 0, 0, 1, 1, 2, 0, 1, 2, 2, 2, 0, 0, 0, 0, 0, 0, 1, 1, 2, 1, 1, 0, 2, 2, 0, 0, 2, 2,
1, 1, 2, 1, 0, 2, 0, 2, 0, 0, 0, 1, 2, 0, 1, 1, 2, 0, 1, 2, 1, 1, 1, 1, 0, 2, 0, 0, 1, 1, 0, 1, 0, 1, 1, 2, 0, 0, 0, 2, 2, 2, 1,
0, 1, 2, 2, 0, 0, 1, 1, 2, 0, 1, 0, 0, 1, 2, 2, 2, 1, 2, 1, 2, 2, 2, 0, 2, 1, 1, 2, 1, 1, 2, 1, 1, 2, 1, 2, 1, 1, 0, 1, 1, 0, 0,
2, 2, 0, 0, 1, 1, 1, 2, 2, 1, 0, 2, 0, 2, 2, 1, 1, 1, 2, 1, 1, 2, 0, 2, 2, 2, 0, 1, 0, 0, 1, 0, 2, 2, 2, 2, 0, 1, 2, 1, 1, 1, 2,
0, 1, 2, 0, 0, 0, 0, 2, 2, 2, 0, 2, 1, 0, 1, 1, 2, 0, 2, 2, 1, 0, 1, 1, 2, 2, 1, 0, 0, 2, 1, 0, 2, 2, 1, 2, 1, 2, 0, 2, 0, 1, 0,
1, 0, 1, 2, 0, 2, 2, 0, 0, 0, 2, 0, 0, 1, 0, 0, 0, 1, 1, 0, 2, 1, 0, 2, 1, 2, 2, 2, 2, 1, 2, 2, 1, 2, 2, 2, 0, 2, 1, 2, 0, 1, 0,
0, 2, 0, 2, 1, 2, 1, 1, 0, 0, 2, 0, 2, 0, 2, 0, 2, 1, 1, 1, 1, 1, 2, 2, 2, 1, 1, 2, 1, 2, 1, 0, 0, 0, 1, 1, 2, 1, 1, 1, 0, 1,
1, 2, 1, 1, 1, 2, 0, 0, 1, 1, 2, 1, 0, 0, 2, 2, 0, 0, 2, 2, 0, 0, 1, 0, 2, 1, 0, 1, 0, 1, 0, 1, 1, 2, 1, 0, 1, 1, 0, 0, 0, 0, 0,
0, 2, 2, 0, 1, 0, 2, 1, 2, 1, 1, 1, 1, 2, 0, 1, 1, 2, 0, 1, 1, 2, 1, 0, 2, 0, 2, 0, 0, 0, 1, 0, 0, 0, 2, 2, 0, 2, 0, 0, 0, 1, 1,
0, 0, 2, 1, 2, 2, 2, 1, 0, 2, 0, 2, 1, 2, 2, 1, 2, 0, 2, 0, 0, 2, 2, 0, 2, 2, 2, 2, 0, 1, 2, 0, 1, 0, 0, 2, 0, 2, 0, 2, 1, 1, 2,
1, 0, 2, 0, 0, 1, 0, 1, 1, 2, 0, 2, 0, 0, 1, 1, 0, 0, 1, 2, 0, 0, 0, 0, 0, 2, 1, 1, 0, 1, 1, 1, 1, 2, 0, 1, 2, 1, 0, 1, 0, 1, 1,
1, 1, 0, 0, 0, 2, 1, 2, 1, 0, 1, 1, 2, 1, 2, 2, 0, 1, 0, 2, 0, 2, 2, 2, 1, 1, 2, 0, 0, 0, 1, 0, 2, 2, 0, 0, 1, 2, 1, 1, 2, 1, 1,
0, 0, 1, 1, 2, 2, 0, 2, 2, 1, 2, 0, 2, 0, 2, 1, 0, 2, 0, 1, 2, 2, 1, 0, 1, 1, 0, 1, 0, 1, 1, 2, 2, 2, 0, 0, 0, 0, 2, 2, 2, 1, 1,
0, 0, 2, 2, 2, 2, 0, 2, 1, 0, 1, 2, 1, 1, 0, 1, 1, 2, 0, 2, 1, 2, 0, 0, 2, 0, 2, 0, 2, 0, 2, 2, 1, 0, 0, 0, 1, 1, 1, 0, 2, 2, 2,
2, 1, 2, 1, 0, 0, 2, 0, 1, 2, 2, 0, 0, 2, 1, 2, 1, 2, 2, 1, 1, 1, 1, 1, 1, 0, 0, 0, 1, 1, 0, 1, 2, 1, 0, 1, 2, 2, 1, 1, 1, 2, 0,
1, 0, 0, 1, 1, 1, 1, 1, 1, 2, 2, 2, 2, 1, 0, 0, 0, 0, 1, 1, 1, 0, 1, 2, 1, 2, 2, 0, 0, 2, 0, 2, 1, 1, 0, 2, 1, 2, 1, 1, 2, 1, 1,
2, 0, 0, 0, 1, 2, 0, 0, 1, 1, 1, 0, 0, 0, 2, 1, 2, 0, 1, 1, 0, 1, 1, 2, 0, 2, 1, 2, 2, 2, 0, 1, 2, 1, 0, 2, 0, 1, 1, 2, 0, 0, 2,
1, 1, 2, 1, 2, 2, 0, 1, 2, 2, 2, 0, 2, 2, 1, 1, 0, 0, 0, 0, 1, 1, 2, 1, 1, 2, 1, 0, 0, 2, 0, 0, 0, 0, 1, 2, 2, 0, 2, 0, 1, 2, 0,
1, 0, 2, 0, 1, 2, 0, 2, 2, 0, 1, 1, 1, 0, 2, 1, 1, 1, 0, 0, 2, 0, 1, 0, 1, 0, 0, 2, 0, 2, 2, 2, 0, 0, 2, 0, 0, 1, 1, 2, 2, 1, 2,
1, 2, 0, 2, 1, 0, 1, 1, 0, 1, 1, 0, 2, 2, 2, 2, 1, 2, 2, 0, 0, 0, 2, 0, 2, 1, 1, 0, 1, 0, 2, 1, 1, 1, 0, 1, 1, 2, 1, 0, 1, 0, 2,
0, 0, 0, 0, 1, 1, 0, 2, 0, 2, 1, 2, 1, 2, 1, 1, 1, 0, 1, 1, 1, 0, 1, 2, 0, 0, 0, 2, 0, 2, 1, 0, 0, 2, 0, 0, 2, 0, 1, 2, 0, 0, 1,
2, 1, 1, 2, 1, 2, 0, 2, 0, 2, 2, 1, 2, 0, 2, 2, 2, 2, 2, 0, 2, 2, 1, 2, 0, 2, 2, 2, 2, 1, 0, 1, 1, 2, 1, 0, 2, 0, 0, 0, 1, 2, 0,
0, 2, 1, 1, 2, 1, 1, 2, 0, 0, 1, 1, 0, 2, 0, 0, 1, 2, 1, 0, 0, 1, 0, 1, 2, 0, 2, 2, 0, 2, 0, 1, 0, 2, 1, 0, 0, 2, 1, 0, 2, 2, 1,
2, 1, 1, 0, 0, 0, 2, 2, 0, 1, 2, 1, 0, 0, 0, 2, 1, 0, 0, 0, 2, 2, 1, 0, 1, 1, 0, 1, 0, 0, 2, 2, 1, 1, 1, 2, 0, 2, 0, 0, 1, 1, 1,
1, 0, 0, 0, 0, 0, 2, 2, 1, 1, 0, 1, 2, 1, 2, 2, 2, 1, 2, 1, 0, 0, 0, 1, 0, 1, 1, 1, 0, 1, 1, 0, 2, 1, 1, 2, 1, 2, 0, 0, 0, 0, 0,
1, 0, 1, 0, 0, 2, 0, 1, 0, 1, 1, 0, 0, 1, 2, 2, 0, 2, 2, 2, 1, 1, 0, 1, 1, 0, 1, 2, 1, 0, 2, 0, 2, 2, 1, 0, 1, 0, 1, 2, 1, 2, 1,
2, 0, 0, 0, 2, 2, 0, 0, 2, 0, 0, 0, 0, 0, 2, 2, 0, 1, 2, 0, 2, 2, 1, 0, 2, 2, 1, 0, 2, 2, 2, 2, 2, 1, 2, 1, 0, 2, 2, 0, 1, 2, 2,
0, 1, 0, 1, 2, 0, 1, 2, 1, 0, 1, 2, 2, 1, 2, 0, 2, 2, 1, 2, 2, 2, 2, 2, 0, 2, 0, 1, 1, 1, 0, 1, 0, 0, 2, 2, 0, 1, 1, 1, 2, 1, 0,
2, 1, 0, 0, 2, 1, 1, 0, 2, 0, 0, 1, 2, 1, 1, 1, 2, 2, 2, 0, 2, 2, 1, 0, 2, 2, 2, 1, 0, 0, 2, 2, 2, 0, 1, 0, 1, 0, 0, 1, 1, 0, 0,
1, 2, 0, 1, 2, 2, 0, 2, 0, 2, 2, 1, 2, 2, 1, 2, 0, 0, 0, 0, 0, 0, 1, 0, 1, 2, 1, 2, 0, 1, 1, 2, 0, 1, 2, 1, 1, 1, 1, 0, 2, 0,
0, 0, 1, 1, 1, 2, 1, 0, 2, 2, 0, 0, 0, 1, 1, 0, 0, 0, 2, 2, 0, 2, 1, 0, 2, 0, 2, 0, 0, 2, 1, 2, 2, 2, 0, 0, 2, 2, 0, 1, 2, 2, 1,
2, 0, 2, 0, 1, 0, 0, 2, 2, 2, 2, 2, 0, 1, 2, 1, 0, 2, 0, 0, 0, 2, 0, 2, 1, 1, 2, 0, 0, 1, 1, 0, 1, 0, 1, 1, 2, 0, 0, 1, 1, 0, 0,
0, 0, 0, 1, 2, 1, 0, 1, 1, 1, 1, 2, 0, 0, 1, 2, 1, 1, 0, 1, 1, 1, 1, 0, 0, 2, 1, 2, 0, 1, 0, 1, 2, 1, 1, 0, 1, 0, 1, 2, 1, 1, 1,
2, 2, 0, 0, 2, 0, 0, 1, 0, 0, 1, 1, 2, 1, 1, 0, 2, 2, 0, 0, 1, 2, 2, 2, 1, 2, 0, 2, 0, 1, 1, 2, 2, 0, 1, 2, 2, 1, 0, 1, 0, 2, 1,
0, 2, 0, 2, 2, 2, 0, 0, 0, 1, 0, 1, 0, 1, 1, 0, 0, 2, 2, 2, 2, 0, 2, 2, 2, 1, 1, 1, 1, 0, 1, 1, 2, 0, 2, 1, 0, 1, 2, 2, 0, 2, 0,
2, 0, 0, 2, 1, 2, 0, 0, 1, 1, 1, 0, 2, 2, 2, 2, 1, 0, 0, 0, 2, 2, 2, 1, 2, 0, 0, 1, 0, 2, 0, 1, 2, 0, 1, 1, 1, 2, 0, 0, 1, 1, 0,
2, 2, 2, 0, 0, 2, 2, 1, 0, 2, 0, 1, 0, 1, 2, 2, 1, 0, 2, 2, 0, 2, 1, 2, 2, 0, 0, 0, 0, 2, 1, 2, 0, 0, 0, 1, 0, 1, 2, 1, 2, 0, 0,
0, 0, 1, 1, 1, 1, 0, 2, 1, 2, 2, 0, 0, 2, 0, 2, 0, 0, 0, 1, 2, 0, 1, 1, 2, 1, 1, 2, 0, 2, 1, 2, 0, 1, 0, 1, 1, 1, 0, 0, 2, 1, 2,
2, 2, 0, 1, 0, 1, 1, 2, 0, 1, 1, 2, 0, 0, 2, 1, 2, 1, 0, 2, 0, 0, 1, 2, 2, 2, 0, 1, 1, 2, 1, 2, 2, 0, 0, 1, 1, 2, 1, 2, 2, 1, 1,
0, 0, 1, 2, 2, 0, 0, 1, 0, 0, 0, 0, 2, 1, 1, 0, 1, 0, 2, 1, 0, 1, 2, 1, 2, 2, 1, 1, 2, 1, 2, 2, 1, 2, 0, 0, 0, 2, 1, 2, 2, 0, 2,
2, 2, 2, 0, 2, 2, 0, 0, 2, 2, 2, 1, 1, 2, 2, 1, 1, 0, 2, 1, 0, 1, 1, 2, 2, 1, 2, 1, 2, 2, 2, 1, 2, 2, 0, 0, 1, 1, 0, 2, 2, 1, 0,
1, 0, 2, 1, 0, 0, 2, 0, 2, 1, 1, 0, 1, 0, 2, 0, 1, 1, 0, 1, 1, 2, 2, 0, 2, 1, 2, 1, 0, 0, 0, 1, 1, 0, 1, 1, 0, 1, 2, 0, 2, 1, 1,
1, 0, 1, 0, 0, 2, 0, 0, 2, 0, 0, 2, 0, 2, 1, 2, 1, 1, 2, 1, 2, 0, 1, 2, 0, 0, 1, 0, 2, 0, 1, 1, 2, 1, 0, 1, 1, 0, 1, 0, 0, 1, 0,
2, 1, 0, 0, 0, 1, 1, 2, 1, 0, 0, 2, 1, 0, 0, 0, 0, 2, 2, 1, 2, 0, 1, 1, 2, 1, 1, 0, 2, 2, 2, 2, 2, 1, 2, 0, 2, 2, 2, 2, 0, 1, 2,
0, 2, 0, 1, 0, 1, 0, 1, 2, 0, 2, 0, 2, 2, 1, 2, 1, 2, 1, 0, 0, 2, 1, 0, 1, 2, 1, 0, 0, 1, 0, 0, 0, 2, 2, 2, 2, 1, 0, 1, 1, 0, 2,
1, 0, 0, 0, 1, 1, 1, 2, 0, 2, 0, 1, 0, 0, 2, 2, 0, 0, 0, 0, 0, 2, 0, 0, 1, 1, 1, 1, 1, 2, 2, 2, 1, 2, 2, 1, 1, 0, 1, 2, 1, 1, 1,
0, 1, 1, 1, 0, 0, 0, 1, 0, 0, 2, 2, 2, 1, 2, 1, 0, 0, 0, 0, 0, 0, 1, 0, 1, 1, 1, 2, 1, 2, 1, 1, 0, 2, 2, 1, 0, 1, 1, 2, 2, 0, 0,
2, 2, 1, 0, 2, 0, 1, 0, 2, 2, 0, 2, 1, 0, 2, 1, 0, 1, 2, 2, 1, 1, 1, 0, 2, 0, 0, 0, 0, 0, 2, 0, 0, 2, 2, 1, 0, 2, 2, 1, 2,
0, 1, 2, 0, 2, 1, 2, 1, 0, 2, 2, 0, 2, 2, 2, 2, 2, 2, 0, 1, 2, 0, 1, 2, 0, 1, 2, 2, 0, 1, 2, 0, 2, 2, 1, 2, 1, 0, 1, 2, 2, 1, 0, 1,
1, 1, 0, 1, 2, 2, 2, 2, 0, 2, 1, 1, 2, 1, 0, 2, 0, 0, 2, 2, 0, 1, 0, 2, 0, 0, 1, 2, 1, 0, 0, 2, 1, 1, 1, 2, 1, 2, 0, 1, 2, 0, 0,
1, 2, 2, 2, 1, 0, 0, 0, 1, 0, 2, 2, 0, 2, 1, 2, 0, 0, 0, 0, 0, 1, 2, 1, 1, 2, 1, 1, 1, 0, 1, 0, 1, 0, 0, 2, 1, 0, 2,
0, 2, 0, 0, 2, 2, 2, 0, 0, 2, 1, 1, 2, 1, 2, 2, 2, 2, 0, 2, 1, 2, 0, 1, 1, 1, 2, 1, 1, 1, 1, 0, 0, 2, 1, 1, 2, 0, 0, 1, 1, 0, 1,
1, 2, 2, 0, 1, 1, 2, 2, 2, 1, 2, 0, 1, 2, 1, 0, 0, 2, 0, 2, 0, 0, 1, 2, 1, 1, 0, 2, 0, 2, 0, 0, 2, 2, 0, 0, 0, 2, 2, 1, 2, 1, 2,
1, 2, 1, 2, 0, 0, 0, 2, 2, 0, 2, 0, 1, 1, 0, 0, 2, 1, 2, 0, 1, 1, 1, 0, 1, 2, 1, 1, 1, 0, 2, 1, 2, 1, 1, 2, 1, 0, 0, 0, 1, 1, 0,
2, 2, 2, 0, 0, 1, 0, 1, 1, 0, 1, 1, 1, 2, 1, 0, 2, 2, 2, 0, 2, 1, 1, 1, 0, 2, 1, 1, 0, 2, 2, 0, 0, 1, 0, 2, 2, 2, 2, 1, 1, 0, 0,
0, 1, 1, 1, 0, 2, 0, 1, 0, 0, 1, 0, 1, 2, 0, 1, 1, 0, 2, 2, 0, 0, 1, 1, 0, 0, 2, 0, 2, 1, 2, 2, 2, 0, 1, 1, 2, 0, 2, 2, 0, 1, 2,
1, 1, 1, 1, 0, 2, 2, 1, 1, 1, 2, 2, 1, 2, 0, 2, 0, 2, 2, 1, 0, 1, 1, 2, 2, 1, 1, 1, 0, 2, 1, 0, 0, 1, 0, 2, 0, 1, 2, 2, 0, 1,
0, 2, 2, 1, 1, 1, 1, 2, 0, 0, 2, 0, 2, 0, 2, 2, 1, 2, 1, 2, 1, 2, 2, 2, 0, 1, 2, 0, 1, 2, 0, 2, 2, 0, 1, 1, 0, 1, 1, 1, 2, 0, 1,
0, 2, 2, 1, 1, 0, 0, 2, 0, 1, 1, 2, 0, 0, 0, 2, 0, 0, 0, 1, 1, 0, 1, 1, 0, 2, 0, 2, 0, 2, 1, 1, 2, 0, 1, 1, 2, 1, 0, 1, 1, 2, 0,
1, 1, 0, 2, 1, 1, 0, 0, 1, 0, 2, 2, 0, 2, 1, 2, 0, 1, 0, 1, 0, 1, 0, 0, 2, 2, 0, 2, 0, 0, 1, 0, 1, 2, 1, 1, 2, 0, 2, 1, 2, 2, 2,
2, 0, 2, 1, 1, 2, 2, 1, 0, 2, 1, 1, 1, 0, 0, 0, 0, 2, 1, 0, 0, 1, 2, 0, 1, 1, 1, 2, 1, 2, 2, 1, 2, 1, 1, 1, 0, 1, 1, 1, 2, 0, 0,
0, 2, 1, 1, 1, 1, 0, 2, 1, 1, 1, 2, 1, 0, 1, 2, 2, 0, 1, 1, 1, 2, 1, 0, 2, 2, 2, 0, 0, 0, 0, 1, 2, 1, 2, 0, 0, 1, 2, 0, 0, 1, 0,
2, 1, 0, 2, 0, 2, 1, 1, 2, 0, 1, 2, 2, 1, 1, 2, 0, 2, 0, 1, 0, 0, 1, 0, 0, 0, 2, 0, 0, 2, 0, 1, 2, 1, 0, 2, 0, 2, 1, 0, 2, 0, 0,
0, 0, 1, 1, 1, 1, 0, 0, 2, 0, 0, 0, 2, 2, 1, 1, 2, 2, 0, 2, 1, 2, 1, 0, 1, 1, 1, 1, 1, 2, 0, 1, 2, 2, 1, 2, 1, 0, 2, 2, 0, 0, 2,
2, 2, 2, 0, 1, 1, 1, 1, 0, 2, 2, 2, 2, 2, 2, 2, 1, 1, 0, 1, 0, 1, 2, 0, 2, 1, 2, 2, 2, 1, 1, 2, 2, 1, 0, 0, 1, 1, 1, 1, 1, 1, 1,
0, 1, 0, 1, 2, 1, 1, 0, 1, 1, 2, 1, 0, 0, 1, 1, 2, 2, 1, 2, 1, 1, 2, 2, 1, 2, 0, 1, 2, 2, 2, 2, 1, 1, 2, 0, 0, 1, 1, 0, 0,
2, 2, 2, 0, 2, 2, 1, 1, 2, 1, 0, 1, 0, 0, 0, 1, 2, 0, 2, 0, 0, 2, 0, 0, 2, 1, 2, 0, 2, 0, 0, 0, 2, 2, 1, 0, 2, 0, 0, 0, 1, 1, 0,
2, 0, 1, 2, 1, 2, 0, 2, 1, 0, 0, 2, 1, 0, 0, 2, 0, 1, 0, 0, 1, 2, 0, 2, 1, 0, 1, 2, 0, 1, 2, 1, 2, 1, 2, 0, 2, 0, 1, 0, 1, 1, 0,
0, 1, 2, 2, 1, 0, 0, 2, 2, 1, 2, 1, 0, 0, 1, 1, 0, 0, 0, 0, 0, 1, 1, 0, 1, 2, 2, 0, 2, 1, 1, 2, 1, 1, 2, 0, 1, 0, 0, 1, 1, 1,
0, 0, 1, 0, 2, 1, 2, 1, 0, 0, 1, 1, 0, 0, 2, 0, 2, 0, 2, 0, 0, 2, 1, 0, 1, 2, 0, 1, 1, 2, 2, 0, 0, 1, 2, 1, 1, 0, 2, 1, 1, 0,
1, 2, 2, 2, 1, 1, 1, 2, 1, 1, 0, 1, 2, 0, 2, 1, 2, 2, 1, 1, 0, 0, 2, 1, 1, 1, 2, 2, 1, 1, 1, 2, 2, 2, 2, 0, 0, 1, 1, 2, 1, 0, 2,
0, 2, 1, 0, 0, 1, 0, 0, 0, 2, 1, 0, 1, 2, 2, 1, 0, 2, 1, 0, 0, 2, 0, 1, 1, 2, 0, 2, 2, 1, 2, 0, 0, 1, 1, 0, 2, 0, 1, 0, 0, 0,
0, 0, 0, 2, 1, 2, 0, 2, 2, 0, 2, 0, 2, 0, 2, 0, 0, 2, 0, 1, 2, 1, 1, 0, 0, 1, 1, 2, 2, 1, 2, 2, 2, 0, 0, 2, 1, 1, 1, 1, 2, 1,
1, 1, 0, 2, 1, 2, 0, 1, 2, 0, 0, 1, 1, 0, 1, 0, 0, 2, 1, 1, 1, 2, 2, 2, 1, 2, 1, 1, 2, 2, 0, 1, 2, 0, 2, 0, 0, 1, 0, 1, 2, 1, 0,
0, 2, 0, 0, 2, 2, 0, 2, 1, 1, 0, 2, 0, 1, 2, 1, 2, 1, 2, 0, 0, 2, 2, 1, 2, 2, 0, 1, 1, 0, 0, 0, 0, 0, 0, 2, 2, 0, 2, 1, 1, 0, 2, 2,
2, 0, 2, 1, 0, 2, 0, 1, 2, 0, 1, 2, 2, 0, 1, 0, 0, 0, 2, 0, 1, 1, 0, 1, 1, 2, 1, 1, 0, 2, 0, 0, 0, 2, 1, 2, 1, 1, 1, 2, 1, 1, 1,
2, 2, 2, 1, 2, 1, 1, 2, 0, 2, 2, 1, 1, 0, 0, 0, 0, 1, 0, 2, 2, 0, 1, 0, 0, 1, 0, 0, 1, 1, 1, 0, 2, 2, 2, 0, 0, 1, 1, 1, 2, 0, 1,

1, 0, 2, 2, 2, 0, 1, 1, 0, 0, 2, 0, 2, 1, 2, 1, 1, 1, 1, 0, 2, 0, 2, 2, 0, 1, 2, 2, 1, 2, 0, 2, 2, 2, 1, 1, 1, 2, 1, 2, 2, 1, 1,
1, 0, 2, 2, 1, 0, 1, 0, 2, 0, 1, 2, 2, 0, 2, 1, 0, 0, 1, 0, 1, 0, 1, 1, 2, 2, 2, 2, 1, 0, 0, 0, 2, 0, 2, 0, 1, 0, 2, 0, 2, 2, 1,
2, 1, 0, 0, 2, 0, 2, 0, 2, 1, 2, 0, 1, 0, 1, 1, 0, 2, 2, 0, 2, 1, 0, 1, 1, 1, 2, 2, 2, 0, 0, 2, 1, 1, 1, 1, 0, 1, 1, 0, 0, 0, 2,
0, 0, 0, 1, 2, 0, 1, 1, 2, 2, 0, 2, 0, 2, 1, 1, 1, 0, 2, 1, 1, 1, 0, 1, 1, 2, 0, 0, 2, 1, 2, 0, 1, 0, 0, 1, 0, 2, 2, 2, 2, 0, 2,
0, 0, 0, 1, 0, 1, 0, 0, 2, 0, 2, 1, 2, 2, 1, 0, 1, 2, 2, 1, 2, 2, 1, 0, 2, 1, 2, 2, 0, 2, 1, 1, 2, 1, 0, 0, 1, 2, 1, 1, 0, 0, 0,
0, 0, 1, 2, 2, 1, 2, 1, 1, 2, 1, 0, 2, 0, 0, 2, 0, 2, 1, 0, 1, 1, 1, 0, 2, 1, 2, 0, 0, 0, 2, 2, 0, 1, 0, 2, 1, 2, 2, 1, 0, 2, 1,
0, 0, 1, 2, 2, 0, 1, 0, 0, 0, 1, 2, 0, 2, 2, 1, 1, 0, 2, 0, 2, 1, 2, 0, 0, 1, 0, 2, 1, 1, 2, 0, 2, 0, 1, 2, 0, 1, 2, 2, 0, 2, 0,
0, 2, 0, 1, 0, 0, 1, 0, 0, 2, 1, 0, 2, 0, 0, 1, 2, 1, 0, 2, 0, 0, 0, 2, 0, 0, 0, 0, 0, 1, 1, 1, 1, 0, 2, 1, 2, 1, 0, 2, 2, 1, 1,
2, 2, 0, 1, 2, 2, 1, 2, 1, 1, 1, 1, 1, 2, 2, 2, 2, 2, 0, 1, 1, 0, 2, 2, 0, 0, 1, 1, 2, 0, 1, 1, 1, 1, 1, 1, 1, 2, 2, 0, 2, 0, 2,
1, 0, 2, 0, 1, 0, 1, 0, 0, 2, 2, 1, 0, 1, 1, 1, 0, 0, 2, 0, 2, 0, 0, 2, 2, 2, 0, 0, 1, 0, 2, 2, 2, 0, 1, 0, 2, 1, 0, 1, 1, 0, 1,
0, 2, 0, 2, 1, 1, 2, 2, 1, 2, 0, 1, 1, 0, 0, 2, 2, 2, 1, 1, 2, 0, 1, 2, 1, 0, 1, 0, 2, 2, 0, 2, 2, 1, 0, 0, 2, 0, 0, 2, 0, 0, 1,
2, 0, 2, 0, 2, 2, 1, 0, 2, 1, 2, 0, 2, 0, 0, 2, 0, 1, 2, 1, 2, 0, 0, 0, 1, 1, 0, 1, 0, 0, 2, 0, 1, 0, 2, 1, 0, 0, 2, 1, 0, 1, 2,
0, 1, 0, 0, 1, 2, 0, 2, 2, 1, 1, 2, 1, 0, 1, 0, 2, 0, 0, 2, 1, 0, 2, 2, 1, 2, 1, 1, 2, 2, 2, 1, 1, 1, 1, 2, 0, 2, 2, 2, 2, 2, 2,
2, 1, 0, 2, 0, 2, 0, 2, 1, 1, 2, 0, 2, 2, 0, 2, 1, 2, 2, 2, 2, 0, 0, 0, 0, 1, 1, 0, 0, 1, 0, 2, 1, 2, 1, 0, 2, 1, 0, 1, 2, 2, 0,
2, 0, 2, 0, 0, 0, 1, 2, 1, 1, 0, 1, 1, 1, 2, 2, 2, 2, 2, 1, 1, 1, 0, 2, 1, 1, 0, 1, 0, 2, 1, 2, 2, 1, 2, 1, 1, 0, 1, 2, 1, 2, 2,
1, 1, 1, 1, 0, 0, 2, 1, 1, 1, 1, 2, 1, 0, 2, 2, 2, 2, 2, 0, 0, 0, 0, 0, 2, 1, 0, 0, 2, 1, 0, 0, 1, 0, 1, 2, 2, 1, 2, 1, 0, 0, 0,
2, 2, 1, 2, 1, 2, 0, 0, 0, 2, 0, 2, 2, 0, 0, 2, 1, 1, 2, 1, 0, 1, 1, 0, 0, 2, 1, 1, 1, 0, 1, 2, 0, 1, 2, 2, 1, 1, 1, 0, 2, 0, 1,
2, 2, 2, 0, 1, 2, 0, 2, 0, 2, 1, 1, 2, 0, 1, 1, 2, 1, 1, 0, 1, 1, 0, 0, 0, 2, 0, 0, 0, 1, 0, 2, 1, 1, 2, 0, 1, 1, 0, 0, 0, 1, 1,
1, 0, 0, 1, 2, 2, 2, 0, 2, 2, 1, 2, 2, 2, 1, 2, 2, 2, 1, 1, 0, 2, 1, 2, 0, 2, 0, 1, 0, 2, 0, 1, 2, 1, 0, 2, 1, 0, 1, 2, 0, 1, 2,
0, 0, 1, 2, 0, 0, 1, 2, 2, 0, 2, 1, 0, 0, 2, 0, 2, 0, 0, 2, 0, 2, 2, 0, 0, 1, 0, 0, 2, 1, 2, 2, 0, 2, 2, 2, 2, 2, 0, 2, 2, 2, 2,
2, 1, 2, 2, 1, 1, 2, 1, 0, 0, 2, 1, 2, 2, 1, 2, 2, 0, 1, 1, 2, 1, 1, 2, 1, 0, 0, 2, 0, 0, 2, 2, 2, 1, 1, 0, 0, 1, 0, 1, 0, 2, 0,
1, 0, 2, 1, 1, 0, 2, 2, 0, 0, 1, 0, 2, 1, 2, 1, 2, 1, 2, 2, 2, 1, 2, 0, 2, 1, 1, 1, 1, 1, 2, 2, 1, 0, 0, 2, 2, 0, 0, 1, 0, 0, 0,
0, 2, 2, 2, 0, 2, 2, 0, 0, 2, 1, 1, 1, 1, 0, 0, 2, 0, 2, 1, 0, 1, 0, 1, 0, 1, 0, 1, 1, 1, 1, 1, 2, 0, 2, 1, 1, 2, 0, 0, 0, 1, 2,
2, 1, 2, 0, 1, 0, 0, 1, 2, 0, 0, 0, 2, 0, 2, 0, 0, 2, 0, 1, 2, 0, 2, 2, 1, 0, 2, 2, 1, 0, 1, 1, 1, 1, 0, 1, 2, 2, 1, 0, 0, 0, 0,
1, 2, 2, 2, 1, 0, 1, 1, 1, 1, 0, 2, 1, 2, 0, 1, 2, 2, 1, 1, 2, 0, 2, 0, 1, 0, 2, 0, 2, 1, 2, 0, 0, 1, 0, 2, 2, 1, 1, 1, 1, 2, 0,
1, 0, 2, 2, 0, 0, 2, 0, 0, 0, 0, 1, 1, 0, 1, 0, 1, 1, 2, 1, 2, 2, 0, 0, 0, 1, 2, 0, 2, 1, 1, 2, 1, 0, 2, 0, 1, 0, 0, 0, 2, 2, 1,
2, 0, 0, 0, 1, 0, 2, 0, 1, 2, 2, 2, 0, 2, 2, 2, 2, 0, 0, 2, 0, 0, 1, 2, 0, 2, 1, 0, 0, 1, 2, 2, 0, 1, 1, 1, 0, 2, 0, 1, 1, 1, 2,
0, 1, 1, 0, 0, 1, 1, 2, 2, 1, 2, 2, 0, 0, 0, 0, 0, 0, 1, 0, 2, 1, 1, 1, 0, 2, 0, 0, 2, 2, 0, 2, 0, 2, 1, 2, 0, 2, 1, 2, 0, 1,
1, 1, 2, 1, 1, 1, 2, 1, 2, 2, 2, 2, 2, 0, 0, 2, 1, 1, 0, 2, 0, 1, 1, 0, 0, 0, 0, 0, 2, 2, 1, 2, 0, 0, 2, 0, 2, 1, 2, 0, 2, 2, 1,
1, 2, 0, 2, 2, 2, 1, 0, 0, 0, 2, 0, 0, 2, 2, 2, 1, 2, 2, 0, 2, 1, 0, 1, 0, 0, 2, 2, 1, 0, 0, 2, 2, 2, 0, 2, 2, 1, 0, 0, 2, 1, 1,
2, 2, 1, 1, 1, 1, 1, 0, 2, 1, 2, 0, 2, 0, 1, 2, 0, 2, 1, 2, 1, 2, 2, 0, 2, 0, 0, 2, 1, 0, 1, 1, 1, 1, 0, 1, 1, 1, 0, 2, 0, 2, 2,
0, 2, 1, 2, 2, 1, 1, 2, 0, 0, 2, 1, 2, 2, 2, 1, 1, 2, 2, 2, 2, 1, 1, 2, 0, 0, 1, 1, 0, 0, 2, 0, 1, 0, 2, 0, 2, 1, 1, 2, 2, 1, 2,
2, 0, 2, 0, 1, 2, 1, 1, 1, 2, 0, 1, 0, 0, 1, 2, 1, 1, 1, 1, 1, 0, 2, 1, 2, 2, 2, 1, 1, 2, 1, 1, 1, 1, 0, 1, 2, 1, 1, 0, 0, 0, 1,
0, 1, 2, 1, 1, 0, 0, 1, 0, 1, 0, 2, 1, 0, 0, 2, 1, 0, 1, 0, 0, 2, 1, 1, 0, 2, 0, 2, 2, 2, 1, 2, 0, 1, 2, 2, 0, 0, 0, 2, 1, 1, 2,
1, 0, 0, 2, 0, 1, 2, 0, 1, 2, 1, 1, 2, 0, 0, 0, 2, 2, 0, 0, 2, 0, 1, 0, 2, 0, 2, 2, 0, 1, 2, 0, 0, 0, 2, 0, 1, 1, 0, 0, 1, 1, 0,
2, 0, 1, 1, 1, 0, 2, 0, 1, 0, 0, 1, 0, 2, 2, 1, 1, 0, 0, 0, 0, 1, 0, 2, 2, 1, 1, 0, 1, 2, 0, 2, 2, 1, 0, 2, 2, 2, 0, 2, 0, 1, 0,
1, 2, 0, 2, 2, 0, 0, 1, 1, 1, 2, 1, 2, 1, 0, 1, 2, 0, 2, 2, 2, 0, 2, 1, 2, 2, 1, 0, 1, 1, 2, 1, 2, 0, 2, 0, 2, 1, 0, 0, 1, 0, 1,
2, 0, 0, 2, 2, 0, 1, 1, 2, 2, 1, 0, 1, 1, 1, 2, 1, 2, 0, 0, 0, 0, 1, 0, 2, 0, 2, 0, 0, 2, 0, 2, 0, 2, 0, 1, 1, 2, 0, 1, 2, 2, 0,
0, 2, 1, 1, 1, 2, 2, 2, 2, 1, 0, 0, 2, 1, 0, 2, 2, 1, 2, 0, 2, 1, 2, 1, 0, 0, 1, 2, 0, 1, 1, 2, 2, 0, 2, 0, 2, 1, 0, 0, 2, 0, 0,
0, 1, 1, 0, 1, 1, 0, 0, 1, 1, 0, 0, 0, 1, 0, 2, 1, 1, 2, 2, 2, 2, 0, 2, 2, 1, 1, 1, 0, 0, 1, 2, 2, 1, 1, 0, 2, 1, 2, 2, 2, 1, 2,
0, 2, 0, 1, 2, 1, 1, 2, 0, 2, 0, 1, 0, 1, 2, 0, 0, 1, 0, 2, 1, 0, 1, 2, 0, 2, 1, 0, 0, 2, 2, 0, 0, 1, 2, 2, 0, 2, 0, 0, 1, 0, 1,
1, 0, 0, 1, 0, 2, 1, 2, 0, 2, 0, 2, 2, 2, 2, 2, 0, 2, 2, 2, 2, 1, 2, 0, 0, 1, 1, 1, 2, 0, 1, 2, 2, 0, 1, 1, 0, 0, 2, 0, 2, 1, 0,
0, 1, 1, 0, 0, 0, 2, 0, 0, 1, 0, 1, 0, 2, 2, 2, 2, 1, 1, 0, 0, 2, 2, 0, 0, 1, 0, 1, 0, 2, 1, 1, 1, 2, 2, 2, 1, 2, 0, 2, 1, 2, 1,
2, 1, 2, 2, 1, 0, 0, 0, 2, 1, 1, 1, 1, 0, 0, 0, 2, 2, 2, 2, 2, 0, 0, 1, 0, 1, 1, 1, 1, 0, 0, 0, 2, 2, 0, 0, 2, 0, 1, 0, 1, 0, 1,
2, 0, 2, 1, 0, 1, 2, 1, 1, 2, 0, 0, 1, 1, 1, 1, 2, 0, 0, 1, 1, 1, 2, 1, 0, 2, 0, 0, 2, 1, 0, 1, 1, 1, 1, 1, 1, 0, 1, 1, 2, 1, 2,
0, 0, 1, 1, 1, 2, 1, 2, 0, 0, 2, 1, 0, 2, 0, 1, 2, 2, 2, 2, 2, 2, 2, 1, 1, 2, 0, 0, 0, 2, 1, 2, 0, 0, 1, 1, 2, 0, 2, 0, 1, 2, 0, 1,
2, 2, 2, 0, 0, 1, 0, 2, 1, 0, 2, 0, 2, 1, 0, 1, 0, 2, 2, 0, 2, 1, 1, 1, 1, 2, 1, 1, 0, 1, 0, 1, 0, 2, 0, 0, 0, 0, 0, 0, 1, 2, 0,
2, 1, 2, 1, 2, 2, 0, 0, 1, 0, 0, 0, 2, 1, 1, 2, 1, 0, 2, 1, 0, 2, 0, 1, 2, 2, 1, 2, 0, 0, 0, 2, 0, 0, 2, 0, 0, 2, 2, 0, 2, 2, 2,
1, 2, 0, 1, 2, 0, 0, 2, 1, 1, 2, 2, 1, 2, 0, 0, 2, 2, 2, 1, 0, 0, 0, 2, 0, 0, 2, 2, 0, 1, 1, 0, 0, 2, 0, 0, 0, 0, 0, 0, 1, 0, 1,
1, 1, 2, 1, 1, 0, 0, 1, 1, 1, 1, 0, 2, 0, 1, 1, 1, 2, 1, 1, 1, 0, 2, 1, 2, 0, 1, 2, 0, 0, 2, 1, 1, 2, 1, 2, 2, 2, 2, 0, 0, 0, 0,
2, 2, 0, 2, 0, 1, 1, 0, 2, 1, 2, 0, 2, 2, 1, 2, 0, 0, 2, 0, 2, 1, 0, 0, 0, 2, 1, 1, 2, 0, 2, 2, 2, 2, 0, 2, 1, 0, 0, 0, 2, 2, 2,
1, 0, 0, 2, 2, 2, 0, 1, 0, 0, 2, 2, 1, 1, 1, 2, 2, 1, 1, 2, 2, 1, 0, 0, 2, 1, 1, 2, 0, 1, 2, 1, 0, 1, 0, 2, 1, 0, 2, 0, 1, 2, 1,
1, 0, 1, 0, 1, 1, 2, 2, 1, 2, 0, 2, 0, 0, 0, 2, 1, 1, 1, 0, 0, 1, 1, 2, 0, 0, 1, 1, 2, 0, 2, 1, 2, 0, 0, 1, 0, 0, 2, 2, 0, 1, 1,
0, 0, 2, 2, 2, 1, 1, 2, 0, 1, 1, 2, 2, 1, 2, 0, 1, 0, 2, 0, 2, 1, 1, 1, 2, 0, 1, 2, 0, 2, 0, 1, 2, 1, 1, 1, 0, 2, 1, 0, 0, 1, 2,
1, 1, 1, 1, 1, 1, 0, 1, 2, 2, 2, 1, 1, 2, 1, 0, 1, 2, 1, 1, 2, 1, 1, 0, 0, 0, 2, 1, 0, 0, 2, 1, 0, 0, 1, 0, 1, 0, 1, 0, 2, 0, 2,
2, 0, 1, 0, 0, 2, 1, 2, 1, 1, 0, 2, 1, 1, 0, 0, 0, 1, 2, 2, 0, 1, 0, 1, 1, 1, 0, 2, 1, 2, 1, 1, 0, 2, 2, 0, 0, 0, 0, 2, 2, 2, 0,
0, 2, 0, 1, 0, 1, 1, 0, 2, 1, 0, 0, 1, 1, 0, 2, 0, 0, 2, 0, 1, 1, 0, 1, 0, 0, 1, 0, 0, 1, 1, 1, 0, 2, 0, 0, 1, 0, 2, 2, 2, 2, 1,
1, 0, 0, 2, 2, 1, 0, 2, 2, 1, 1, 0, 1, 2, 0, 1, 2, 0, 2, 2, 0, 2, 0, 2, 0, 1, 0, 2, 1, 0, 1, 2, 0, 0, 1, 1, 1, 2, 1, 2, 2, 1, 0,
1, 1, 2, 2, 2, 0, 2, 1, 2, 1, 0, 0, 1, 0, 2, 1, 2, 0, 2, 0, 0, 1, 1, 2, 2, 1, 1, 2, 0, 0, 2, 2, 1, 2, 1, 1, 2, 0, 1, 0, 2, 1, 2,
1, 1, 1, 1, 2, 2, 1, 2, 1, 0, 0, 0, 2, 0, 2, 1, 1, 2, 2, 0, 1, 2, 0, 0, 2, 2, 2, 2, 0, 0, 0, 1, 2, 1, 2, 1, 0, 0, 0, 0, 1, 0, 1,
0, 1, 2, 0, 2, 1, 0, 0, 0, 0, 0, 0, 2, 2, 2, 2, 1, 1, 1, 2, 0, 1, 0, 0, 0, 2, 2, 1, 0, 1, 2, 2, 1, 1, 2, 0, 0, 1, 0, 2, 1, 0, 0,
2, 2, 1, 0, 2, 0, 2, 0, 2, 1, 2, 2, 0, 0, 0, 2, 0, 0, 0, 1, 1, 2, 0, 0, 1, 0, 2, 1, 0, 1, 0, 2, 0, 1, 1, 1, 2, 1, 0, 2, 0, 1, 0,
2, 0, 0, 0, 2, 0, 1, 0, 0, 2, 0, 1, 0, 2, 0, 1, 2, 0, 1, 1, 1, 2, 1, 1, 2, 0, 0, 2, 2, 1, 1, 1, 1, 2, 1, 2, 1, 2, 1, 2, 1, 0, 1,
1, 2, 1, 1, 2, 2, 2, 2, 1, 0, 1, 1, 0, 2, 2, 2, 1, 0, 0, 2, 1, 2, 1, 2, 2, 0, 0, 1, 0, 2, 2, 0, 1, 1, 0, 0, 0, 2, 0, 1, 0, 1, 0,
0, 1, 2, 2, 0, 0, 0, 0, 1, 2, 1, 0, 0, 0, 0, 2, 0, 2, 0, 0, 2, 1, 0, 0, 2, 1, 0, 0, 0, 0, 0, 0, 1, 0, 1, 2, 0, 2, 2, 0, 2, 0, 0,
1, 0, 0, 0, 1, 1, 2, 2, 1, 2, 0, 0, 1, 0, 1, 0, 0, 0, 0, 2, 0, 2, 2, 0, 1, 0, 1, 0, 2, 2, 0, 1, 2, 0, 0, 1, 2, 1, 0, 1, 0, 0, 2,
1, 2, 0, 0, 1, 1, 2, 2, 0, 1, 1, 0, 0, 1, 1, 2, 2, 1, 0, 0, 2, 1, 2, 1, 1, 1, 0, 2, 1, 0, 0, 2, 2, 1, 0, 2, 0, 0, 1, 0, 1, 2, 1,
1, 1, 2, 2, 0, 1, 1, 2, 1, 1, 2, 2, 1, 1, 2, 0, 0, 1, 2, 1, 2, 2, 1, 0, 2, 0, 2, 2, 2, 2, 2, 0, 2, 2, 0, 2, 1, 2, 1, 1, 1, 0, 2,
2, 1, 2, 1, 2, 2, 0, 0, 1, 0, 2, 0, 0, 2, 2, 2, 1, 0, 2, 1, 2, 1, 1, 2, 0, 0, 2, 1, 2, 2, 1, 1, 2, 1, 2, 1, 0, 0, 2, 2, 1, 0, 0,
1, 2, 2, 2, 1, 2, 2, 1, 1, 2, 2, 1, 1, 1, 1, 0, 1, 1, 1, 1, 1, 2, 0, 0, 2, 2, 0, 0, 1, 0, 2, 0, 0, 2, 2, 1, 1, 0, 0, 2, 2, 1, 1,
0, 0, 0, 0, 0, 0, 2, 2, 0, 0, 2, 0, 0, 2, 0, 0, 1, 1, 1, 0, 1, 0, 1, 0, 1, 1, 1, 1, 0, 2, 2, 1, 2, 1, 0, 2, 1, 2, 0, 2, 0, 2, 2,
0, 0, 2, 2, 2, 2, 1, 0, 1, 0, 1, 2, 0, 2, 2, 2, 0, 1, 1, 1, 1, 2, 2, 0, 0, 2, 1, 1, 0, 2, 1, 0, 2, 1, 1, 2, 0, 0, 1, 2, 1, 2, 0,
1, 0, 0, 2, 0, 0, 2, 1, 2, 0, 2, 0, 2, 1, 1, 0, 1, 0, 2, 1, 2, 1, 1, 2, 2, 1, 2, 2, 1, 1, 0, 1, 2, 1, 0, 1, 0, 0, 0, 1, 0, 0, 2,
1, 2, 0, 1, 0, 0, 1, 2, 2, 1, 2, 0, 0, 1, 1, 0, 2, 2, 0, 0, 2, 2, 1, 1, 2, 0, 0, 1, 2, 1, 2, 0, 2, 0, 1, 2, 0, 0, 1, 2, 0, 1, 2,
2, 0, 2, 0, 2, 1, 0, 2, 0, 1, 1, 1, 0, 1, 2, 0, 0, 2, 1, 1, 0, 0, 0, 1, 0, 1, 1, 0, 0, 1, 1, 2, 2, 2, 1, 0, 2, 1, 2, 0, 0, 2, 0,
1, 1, 2, 0, 2, 2, 0, 1, 2, 2, 2, 0, 2, 1, 2, 2, 0, 1, 1, 2, 2, 2, 1, 2, 1, 1, 0, 1, 2, 1, 0, 2, 0, 1, 1, 2, 0, 2, 2, 2, 0, 1, 0,
2, 0, 0, 2, 2, 1, 0, 2, 2, 1, 1, 2, 2, 1, 0, 0, 1, 2, 2, 2, 2, 0, 0, 1, 1, 1, 1, 1, 0, 2, 1, 2, 1, 2, 2, 2, 2, 1, 0, 1, 2, 0, 2,
0, 1, 0, 2, 1, 1, 0, 2, 1, 2, 1, 2, 0, 1, 0, 2, 1, 0, 2, 2, 1, 1, 2, 2, 0, 1, 0, 1, 2, 1, 1, 1, 1, 2, 0, 1, 2, 1, 0, 2, 2, 1, 2, 2,
1, 0, 2, 2, 1, 0, 0, 0, 2, 1, 0, 0, 0, 1, 1, 2, 2, 1, 0, 0, 0, 1, 0, 2, 2, 0, 0, 1, 2, 0, 0, 1, 0, 2, 0, 0, 0, 1, 0, 0, 2, 1, 1,
1, 1, 2, 0, 2, 0, 0, 0, 2, 0, 0, 0, 1, 2, 2, 2, 1, 1, 0, 0, 0, 0, 0, 2, 0, 0, 1, 0, 0, 2, 2, 1, 2, 0, 0, 0, 0, 0, 2, 1, 0, 0, 2,

```
1, 0, 2, 2, 0, 0, 1, 2, 1, 0, 1, 2, 2, 2, 1, 0, 2, 0, 1, 2, 1, 2, 1, 0, 0, 0, 1, 2, 2, 2, 0, 1, 1, 1, 2, 2, 0, 1, 0, 1, 1, 0, 2,
1, 1, 1, 1, 0, 0, 0, 0, 0, 2, 1, 2, 1, 1, 0, 1, 0, 0, 0, 2, 2, 2, 1, 0, 2, 0, 0, 2, 2, 2, 2, 2, 1, 0, 0, 0, 0, 0, 0, 0, 2, 2, 1,
1, 1, 1, 2, 0, 1, 2, 0, 0, 1, 0, 2, 0, 1, 2, 2, 1, 1, 0, 2, 0, 2, 0, 2, 1, 0, 0, 2, 2, 1, 2, 0, 0, 0, 1, 1, 1, 2, 2, 0, 0, 0, 1,
0, 1, 0, 2, 0, 2, 0, 0, 1, 0, 2, 2, 0, 1, 0, 2, 0, 1, 1, 1, 2, 1, 0, 0, 2, 0, 1, 0, 2, 0, 0, 2, 0, 1, 0, 0, 1, 1, 0, 2, 2, 2, 1,
2, 2, 2, 0, 1, 2, 2, 0, 0, 2, 1, 0, 1, 0, 2, 1, 1, 2, 0, 2, 0, 2, 1, 0, 0, 1, 0, 1, 2, 1, 2, 0, 1, 0, 1, 2, 2, 2, 0, 0, 1, 2, 1,
2, 0, 1, 1, 2, 1, 1, 0, 0, 2, 0, 1, 0, 2, 2, 0, 1, 1, 0, 0, 0, 0, 0, 1, 2, 1, 0, 0, 1, 2, 2, 0, 2, 0, 0, 2, 1, 1, 0, 0, 0, 0, 2,
0, 0, 0, 0, 1, 0, 0, 0, 2, 1, 0, 0, 2, 0, 0, 1, 0, 0, 1, 2, 0, 2, 2, 0, 2, 0, 0, 1, 0, 1, 0, 1, 1, 2, 2, 1, 2, 2, 0, 1, 0, 1, 0,
0, 0, 0, 2, 0, 0, 0, 1, 2, 1, 0, 0, 2, 2, 0, 1, 2, 1, 0, 0, 1, 2, 1, 0, 0, 2, 2, 0, 1, 1, 2, 2, 1, 2, 2, 0, 1, 1, 0, 0, 0, 1, 2,
1, 2, 0, 2, 2, 0, 1, 2, 1, 1, 0, 1, 2, 0, 0, 0, 2, 1, 0, 1, 0, 2, 2, 1, 1, 0, 0, 2, 1, 0, 0, 1, 1, 2, 1, 2, 2, 1, 2, 1, 1, 2, 0,
0, 2, 2, 0, 2, 2, 0, 0, 2, 0, 2, 2, 2, 0, 2, 2, 1, 2, 1, 2, 1, 2, 1, 1, 1, 2, 0, 0, 2, 2, 2, 2, 2, 0, 0, 1, 0, 1, 2, 0, 0, 2, 1,
1, 0, 2, 1, 2, 1, 2, 1, 0, 0, 2, 2, 2, 2, 1, 1, 2, 1, 2, 1, 2, 2, 1, 1, 1, 0, 0, 1, 2, 2, 0, 1, 1, 1, 1, 1, 2, 2, 1, 1, 1, 1, 2,
0, 0, 1, 1, 0, 0, 2, 0, 1, 0, 0, 1, 1, 2, 2, 0, 0, 1, 1, 2, 2, 0, 0, 0, 0, 0, 0, 1, 1, 0, 1, 1, 1, 1, 2, 2, 1, 1, 1, 0, 0, 1,
0, 1, 0, 2, 1, 2, 0, 0, 1, 2, 1, 2, 0, 1, 2, 0, 0, 2, 2, 2, 1, 2, 0, 2, 0, 2, 2, 0, 2, 2, 2, 0, 2, 1, 0, 1, 0, 1, 0, 0, 2, 1, 1,
0, 1, 1, 1, 1, 2, 2, 2, 0, 0, 1, 2, 1, 2, 1, 0, 2, 1, 1, 0, 0, 2, 1, 2, 0, 2, 0, 1, 0, 0, 2, 1, 0, 2, 1, 2, 1, 2, 0, 2, 1, 1, 0,
1, 1, 0, 1, 2, 1, 1, 2, 2, 1, 2, 2, 0, 0, 2, 1, 2, 0, 0, 1, 0, 0, 0, 1, 1, 0, 0, 2, 1, 1, 2, 1, 0, 0, 0, 2, 2, 1, 1, 2, 1, 1, 0,
2, 2, 0, 0, 0, 2, 1, 2, 1, 1, 1, 1, 0, 2, 1, 2, 2, 1, 0, 2, 1, 2, 0, 1, 2, 0, 2, 1, 1, 2, 2, 2, 1, 2, 2, 1, 2, 2, 1, 0, 1, 1, 0,
0, 1, 1, 1, 0, 0, 0, 1, 2, 1, 2, 0, 0, 2, 1, 2, 2, 2, 1, 0, 2, 0, 1, 2, 2, 2, 0, 1, 1, 2, 0, 2, 1, 1, 2, 2, 2, 1, 0, 2, 1, 2, 2,
0, 1, 0, 2, 0, 1, 1, 2, 1, 1, 0, 1, 2, 1, 0, 2, 0, 0, 2, 2, 0, 2, 2, 2, 0, 1, 2, 2, 1, 0, 0, 2, 1, 0, 2, 2, 1, 0, 1, 1, 1, 1, 1,
1, 2, 2, 2, 2, 0, 0, 2, 2, 1, 2, 1, 1, 1, 1, 2, 2, 2, 2, 1, 2, 1, 1, 0, 0, 1, 1, 2, 1, 2, 2, 1, 0, 1, 1, 0, 2, 0, 1, 2, 1, 2, 2,
1, 1, 0, 1, 0, 1, 1, 2, 1, 2, 1, 2, 0, 2, 1, 1, 1, 0, 0, 2, 0, 0, 0, 2, 1, 0, 0, 1, 0, 2, 2, 1, 0, 0, 0, 0, 1, 0, 2, 0, 1, 1, 2,
2, 1, 0, 1, 0, 2, 0, 0, 2, 0, 0, 1, 2, 0, 1, 1, 1, 2, 0, 2, 0, 1, 0, 0, 2, 1, 0, 1, 2, 2, 2, 1, 0, 0, 0, 2, 0, 0, 0, 2, 0, 0, 1,
0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 2, 1, 0, 0, 2, 2, 1, 2, 0, 0, 0, 1, 2, 1, 0, 0, 2, 1, 0, 2, 2 ];
```

Bibliography

[1] Baumeister, B. *A computer-free construction of the third group of Janko.* J. Algebra **192** (1997), 780-809.

[2] Bosma, W.,Canon, J., Playoust, C. *The Magma Algebra System I: the user language.* J. Sybolic Comput. **3/4** (1997), 235-265.

[3] Buekenhout, F. (Editor) *Handbook of Incidence Geometry - Buildings and Foundations.* Elsevier, Amsterdam (1995).

[4] Buekenhout, F. *Diagram geometries for sporadic groups*; in *Finite groups coming of age.* AMS Series Contemp. Math. **45** (1985), 1-32.

[5] Buekenhout F. *Foundations of incidence geometry*; in *Handbook of Incidence Geometry* (ed. F Buekenhout). Elsevier, Amsterdam (1995), 63-106.

[6] Buekenhout, F., Leemans, D. *On a Geometry of Ivanov and Shpectorov for the O'Nan Sporadic Group.* J. Combin. Theory A **85** (1999), 148-164.

[7] Buekenhout, F. Dehon, M., Leemans, D. *On flag-transitive incidence geometries of rank 6 for the Mathieu group M_{12}*; in *Groups and Geometries* (eds. L. di Martino et al), Birkhäuser Verlag, Basel (1998), 39-54.

[8] Conway, J., et al. *An Atlas of Finite Groups.* Clarendon Press, Oxford (1985).

[9] Coxeter H.S.M., Moser, W.O.J. *Generators and Relations dor Discrete Groups.* Springer-Verlag, Berlin, Heidelberg, New York (1957).

[10] Gorenstein, D., Lyons, R., Solomon, R. *The Classification of the Finite Simple Groups, Number 2.* Math. surveys and monographs, AMS, Vol. 40 No.2, Providence, Rhode Island (1996).

[11] Gottschalk, H. *On the geometry of Baer subplanes in $PG(2,4)$.* Europ. J. Combinatorics **21** (2000), 899-912.

[12] Gottschalk, H., Leemans, D. *The Residually Weakly Primitive Geomtries of the Janko Group J_1*; in *Groups and Geometries* (eds. L. di Martino et al), Birkhäuser Verlag, Basel (1998), 65-79.

[13] Havas, G., Ramsay, C. *Coset Enumeration : ACE version 3* (1999). http://csee.uq.edu.au/~havas/ace3.tar.gz

[14] Ivanov, A.A. *Geometry of Sporadic Groups I: Petersen and Tilde Geometries.* Encyclopedia of Mathematics and its Applications **76**, Cambridge University Press, Cambridge (1999).

[15] Ivanov, A.A. *On 2-Transitive Graphs of Girth 5.* Europ. J. Combinatorics **8** (1987), 393-420.

[16] Ivanov, A.A., Meierfrankenfeld, U. *A computer-free construction of* J_4. J. Algebra **219** (1999), 113-172.

[17] Ivanov, A.A., Shpectorov, S.V. *Geometry for the O'Nan-Sims group, related to the Petersen graph* (in Russian). Usp. Mat. Nauk **41** (1986), 183-184.

[18] Ivanov, A.A., Shpectorov S.V. *Geometries for sporadic groups related to the Petersen graph. I.* Comm. Algebra **16** (1988), 925-954.

[19] Lempken, W. *On the existence and uniqueness of the sporadic simple groups* J_2 *and* J_3 *of Z. Janko.* J. Group Theory **4** (2001), 223-232.

[20] Lempken, W. *Constructing* $\hat{3}O'N$ *by means of involution centralizers.* In preparation.

[21] Lempken W. *Personal Communication.*

[22] Jansen, C., et al. *An Atlas of Brauer Characters.* Oxford Sience Publications, Oxford (1995).

[23] Jansen, C., Wilson R.A. *Two new constructions of the O'Nan group.* J. London Math. Soc. (2) **56** (1997), 579-583.

[24] O'Nan, M.E. *Some evidence for the existence of a new finite simple group.* Proc. London Math. Soc. **32** (1976), 421-479.

[25] Pasini, A. *Diagram Geometries.* Oxford University Press, Oxford (1994).

[26] Pasini, A. *Some remarks on covers and apartments*; in *Finite Geometries* (eds. C.A. Baker, L.M. Batten), Marcel Dekker, New York (1985), 233-250.

[27] Ronan, M. *On the second homotopy group of certain simplicial complexes and some combinatorial applications.* Quart. J. Math. Oxford **32** (1981), 225-233.

[28] Stroth, G. *A construction of the Held simple group.* J. Algebra **234** (2000), 707-742.

[29] Stroth, G. *Construction of sporadic groups.* In preparation.

[30] Tits, J. *Buildings of Spherical Type and Finite BN-Pairs*. Lect. Notes in Math. **386**, Springer-Verlag, Berlin-Heidelberg-New York (1974).

[31] Tits, J. *Ensembles ordonnés, immeubles et sommes amalgamées*. Bull. Soc. Math. Belg. **A38** (1986), 367-387.

Lebenslauf

Geburtstag und -ort:	13.Mai 1968 in Bad Arolsen
Schulbildung:	Grundschule Helsen von 1974 bis 1978, Christian-Rauch-Schule Arolsen von 1978 bis 1987, Abitur am 21.05.1987 mit der Durchschnittsnote 1,3 bestanden.
Zivildienst:	01.10.1987 bis 31.05.1989
Studium:	Grundstudium Mathematik (Diplom) an der Justus-Liebig-Universität Gießen vom WS 1989 bis SS 1991, Vordiplom am 05.02.1992 mit der Durchschnittsnote 1,0 bestanden.
	Hauptstudium Mathematik (Diplom) an der Justus-Liebig-Universität Gießen vom SS 1991 bis SS 1994, Mitglied im Fachbereichsrat und verschiedenen Gremien des Fachbereichs.
	Studium an der Université Libre de Bruxelles vom WS 1994 bis SS 1995 unterstützt durch ein Stipendium der Communauté française de Belgique.
	Diplom an der Justus-Liebig-Universität Gießen am 26.03.1996 mit *sehr gut* bestanden.
	Seit dem WS 1996 Promotionsstudium an der Martin-Luther-Universität Halle-Wittenberg unterstützt durch ein Graduiertenstipendium des Landes Sachsen-Anhalt. Seit Februar 1999 als wissenschaftlicher Mitarbeiter am Fachbereich Mathematik und Informatik der Martin-Luther-Universität Halle-Wittenberg im Institut für Algebra und Geometrie beschäftigt.

Erklärung

Hiermit erkläre ich an Eides statt, daß ich die vorliegende Arbeit selbständing, ohne unzulässige Hilfe Dritter und ohne Benutzung anderer als der angegebenen Quellen und Hilfsmittel angefertigt habe. Die aus anderen Werken wörtlich oder inhaltlich entnommenen Daten, Fakten und Konzepte sind unter Angabe der entprechenden Quelle als solche gekennzeichnet.

Diese Arbeit wurde bisher weder im In- noch im Ausland in gleicher oder ähnlicher Form in einem anderen Prüfungsverfahren vorgelegt.

Halle, im November 2001.

Harald Gottschalk